## **CERTIFICATION OF APPROVAL**

# Dynamic Analysis of semi-Submersible in Time Domain

Ву

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#### CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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#### ABSTRACT

Offshore platforms are categorized according to the water depth in which installed. Regarding to rising demand of deep water drilling and exploration, floating production system (FPS) is one of the best alternative due to ease of installation and transportation. The main concern for FPS is stability in wave motions. This report consists of dynamic analysis of semi-submersible both in frequency domain and time domain. Due to complexity of calculations, this analysis is accomplished by computer programming. This report mainly focused on analysis of semi-submersible as a whole structure. In literature review the types of platform are briefly described including their similarities and differences. Our focus in this project is mainly draws to three critical motions of platform, which are surge, heave and pitch. These displacements are studied under random waves. For random wave generation, P-M is chosen. The forces on body are calculated by Froude-Krylov theory. In this report the effect of water depth is investigated. The frequency domain analysis is done for two drafts and the results are compared together. For time domain analysis one case is accomplished using Newmark's Beta method and the result is compared with frequency domain method.

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# CHAPTER 1 INTRODUCTION

As the demand for oil and gas is rising nowadays, the petroleum industry inquires continues development thus deep water oil exploration plays an important role in this industry. For exploration and production of oil and gas in deep water it is inevitable to install floating platform since using fixed structure is not feasible.

The concern about semi submersible platform is stability against wave motion. As it is obvious operating a proper drilling in deep water needs structural stability of platform.

## 1.1 Background of Study

This report will concentrate on dynamic analysis of semi submersible platforms. Offshore platforms are exposed not only to the extreme conditions of the environment such as wave slam, ice impact, and fatigue, but also to accidental events such as boat impact and objects dropped off the platform. The list of such accidental events that have occurred over the past several decades is myriad. It includes ramming by a supply boat that went full ahead rather than full astern, impact from the reinforced corner of a cargo barge, and impact by a derrick barge whose mooring lines had parted. The dropped-objects category includes a number of pedestal cranes pulled off their supports when they attempted to follow the movements of a supply boat and thus exceeded their allowable radii, also drill collars, casing, a mud pump, and pile hammer.

To design of floating offshore platform there are variety of aspects should be under consideration. One of the most critical aspects is environmental issues. Some of these challenges are listed as below:

Weight control and stability became key design drivers

- Dynamic responses govern the loads on mooring and equipment
- Fatigue is an important consideration
- In some areas, the new environmental make design difficult, e.g.
  - o Large currents in deepwater of Gulf of Mexico
  - High seas and strong currents in North Atlantic
  - o Long period swells in West Africa

Installation of the platforms, mooring and decks in deepwater present new challenge

#### 1.2 Problem Statement

Design of semi submersible structures is a crucial issue since it should appreciate different aspects. In the design of semi-submersible, and its configuration in particular, a clear idea of the functions it must perform should be in hand. These will strongly influence configurational choices. Besides drilling, these functions include production, heavy lift, accommodations, operational support (surface, subsea), and even space launch.

Apart from the mission and support functions, stated simply, there are two essential functions it must of semi-submersible:

- To stably support a payload above the highest waves
- To minimally respond to waves.

To achieve these requirements we need to have accurate analysis based on structural dynamic principles.

#### 1.3 Objectives

1. To prepare a detailed literature survey about the semi-submersible technology for offshore platform.

- 2. To analyze a floating production system(FPS) dynamically in frequency domain and time domain
- 3. To compare different types of semi-submersible platforms based on dynamic response to oceanic random waves.
- 4. To determine the effect of different parameter like water depth, wave length on the FPS response.

#### 1.4 Scope of study/work

- This study consists of some researches on different types of platforms due to their characteristics similarities and differences.
- To study different types of semi-submersibles and their generations.
- To study the dynamic motions of a semi-submersible in three degree of surge, heave and pitch in frequency and time domain.

# CHAPTER 2 LITERATURE REVIEW

#### 2.1 General

An oil platform or oil rig is a large structure that is used to house workers and machinery needed to drill and/or produce oil and natural gas through wells in the ocean bed. Relying on the circumstances, the platform may be fixed to the ocean floor, consist of an artificial island, or be floating.

Mainly, oil platforms are placed on the continental shelf, though as technology improves, drilling and production operation in deeper waters becomes both feasible and profitable. A typical platform may have around thirty wellheads located on the platform and directional drilling allows reservoirs to be accessed at both different depths and at remote positions up to 5 miles (8 kilometres) from the platform.

We have different types of platform which are categorized by their functionality in depth of water.

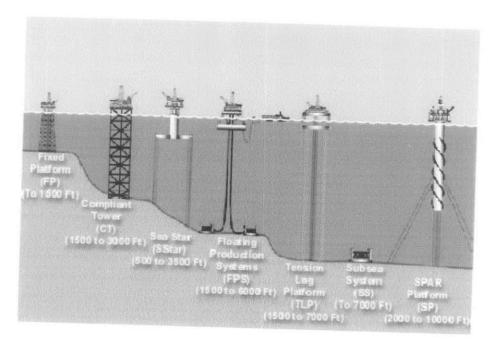


Figure 2-1: different types of platform with respect to water depth

## 2.1.1 Fixed Platforms

This kind of platform is built on concrete and/or steel legs anchored directly onto the seabed, supporting a deck with space for drilling rigs, production facilities and crew quarters. Such platforms are, by virtue of their immobility, several types of structure are used, steel jacket, concrete caisson, floating steel and even floating concrete. Steel jackets are vertical sections made of tubular steel members, and are usually piled into the seabed. Concrete caisson structures. Often they have in-built oil storage in tanks below the sea surface and these tanks were often give flotation capability, allowing them to be built close to shore. Fixed platforms are economically feasible for installation in water depths up to about 1,700 feet (520 m).



Figure 2-2: Fixed Platform

# 2.1.2 Compliant Towers

They consist of narrow towers and a piled foundation that support a conventional deck for drilling and production operations. The purpose of their design is to sustain significant lateral deflections and forces. Mostly they are used in water depths ranging from 1,500 and 3,000 feet (450 and 900 m).



Figure 2-3: Compliant Tower

#### 2.1.3 Semi-submersible Platforms

Semi submersible is having legs of sufficient buoyancy to cause the structure to float, but of weight sufficient to keep the structure upright. Semi-submersible rigs can be moved from place to place; and can be ballasted up or down by altering the amount of flooding in buoyancy tanks; they are generally anchored by cable anchors during drilling operations, though they can also be kept in place by the use of dynamic positioning. Semi-submersible can be used in depths from 600 to 6,000 feet (180 to 1,800 m).



Figure 2-4: Semi-Submersible

## 2.1.4 Jack-up Platforms

as the name suggests, are platforms that can be jacked up above the sea using legs which can be lowered like jacks. These platforms, used in relatively low depths, are designed to move from place to place, and then anchor themselves by deploying the jack-like legs.

#### 2.1.5 Drill ships

Drill ship is a maritime vessel that has been fitted with drilling apparatus. It is most often used for exploratory drilling of new oil or gas wells in deep water but can also be used for scientific drilling. It is often built on a modified tanker hull and outfitted with a dynamic positioning system to maintain its position over the well Floating are large ships equipped with processing facilities and moored to a location for a long period. The main types of floating production systems are FPSO (floating production, storage, and offloading system), FSO (floating storage and offloading system), and FSU (floating storage unit). These ships do not actually drill for oil or gas.



Figure 2-5: Drill Ship

#### 2.1.6 Tension-leg platforms

TLP consists of floating rigs tethered to the seabed in a manner that eliminates most vertical movement of the structure. TLPS are used in water depths up to about 6,000 feet (2,000 m). The "conventional" TLP is a 4-column design which looks similar to a semisubmersible, they are relatively low cost, used in water depths between 600 and 3,500 feet (200 and 1,100 m). Mini TLPs can also be used as utility, satellite or early production platforms for larger deepwater discoveries.

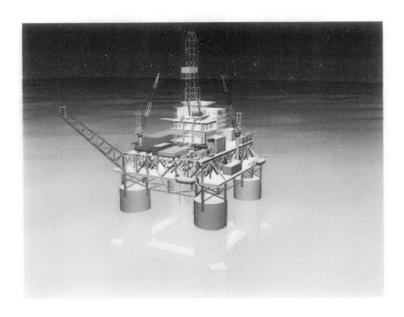


Figure 2-6: TLP

#### 2.1.7 SPAR Platforms

Spar platform moored to the seabed like the TLP, but whereas the TLP has vertical tension tethers the Spar has more conventional mooring lines. Spars have been designed in three configurations: the "conventional" one-piece cylindrical hull, the "truss spar" where the midsection is composed of truss elements connecting the upper buoyant hull (called a hard tank) with the bottom soft tank containing permanent ballast, and the "cell spar" which is built from multiple vertical cylinders. The Spar may be more economical to build for small and medium sized rigs than the TLP, and has more inherent stability than a TLP since it has a large counterweight at the bottom and does not depend on the mooring to hold it upright. It also has the ability, by use of chain-jacks attached to the mooring lines, to move horizontally over the oil field. The first production spar was Kerr-McGee's Neptune, which is a floating production facility anchored in 1,930 feet (588 m) in the Gulf of Mexico.



Figure 2-7: Spar Platform

# 2.2 History

emi-submersibles evolved from a drilling vessel type called a "submersible," which operated tting on bottom in fairly shallow water and provided a working deck well above the highest spected waves. These units transited afloat on pontoons and required "stability columns" to fely submerge to a bottom founded mode of operation. To operate in deeper water, the marine ser was developed and spread moorings were perfected allowing drilling afloat. This first plication was with barges drilling function. This was the "Bluewater." This was a Shell Oil onsored development with Bruce Collip as the inventor of record. Drilling semis may be rided into four generations:

1st Generation: Before 1971

• 2nd Generation: 1971-1980

• 3rd Generation: 1981-1984

• 4th Generation: 1984-1998

Those most recently built, those since 1998, might be called "5th Generation," but a clear distinction has not emerged. The first generation consists of a broad variety of configurations developed throughout the 1960s, beginning with the "Bluewater I" and including the notable SEDCO 135 designs and the variously configured ODECO designs. With the exception of the SEDCO 135, these designs all featured an array of multiple pontoons. Besides large diameter stability columns at the extremities, they also had many slender interior columns to support the working deck - but no diagonal trussing. The main element of global strength was the pontoon. Conversely, the SEDCO 135, a 3-column design, evolved from a Transworld submersible design and employed independent columns and footing tied together with a trussed space frame. The Forex Neptune "Pentagone 81," a 5-column, independent footing, trussed space frame design, is the culmination of this period. It is a true space frame, without trusses in a bent and also is an early user of hull-type superstructure.

The 2nd generation produced the majority of the units built. In addition to better technology exchange, this generation was stimulated by competition from drillships The structural system was somewhat unique and maintained almost entirely by a hull-type superstructure, integrally built into the tops of the stability columns. It did have a shallow truss system with diagonal bracing, but did not have the low, horizontal transverse typical of later designs. The "ZephyrlMarge Class," drilling units of the early 1970s, emulated the Mohole structural system but, upon entering service in the North Sea, the "Margie" suffered failures in the shallow truss. This event proved the necessity of the horizontal bracing and also the viability of the hull-type superstructure, without which, the "Margie" would have broken up. Another was the Offshore Company's "Chris Chenery." Without the horizontal transverses or the shallow truss, it had a hull-type superstructure, albeit heavily reinforced, it too demonstrated the strength of the hull-type super structure. The "Chris Chenery" and its single sister both have since been fitted with horizontal transverses to extend the fatigue life.

While no single design represented a complete understanding of the design principals, most 2nd generation semi-submersibles were relatively well designed from a performance point of view. An important aspect of this period was a higher level of dissemination of the design and performance knowledge. It is not a coincidence that the first USCG Regulations and the ABS Classing Rules were published in the late 1960s and that the OTC began in 1969. Most notable of this period, if only by its numbers, is the Aker H-3.0 design. Other notables include the Pacesetter and the SEDCO 700 classes.

The demarcation between the 2nd and the 3rd generation is rather sharp. In 1979, only two units were delivered and in 1980 there were none.

With respect to the increased size, payload, and higher standards of redundancy, the main features of the third generation are the presence of the twin pontoon pattern, the usage of hull-type superstructure, the well designed brace connections, and a generally thorough understanding of the design principles of semi-submersibles. Notable 3rd generation designs are the "Bingo," "Ocean Odyssey", "Scarabeo <u>5"</u> and the "Zapata Arctic". The numbers of enhanced 2nd generation designs were built during this period, particularly the Sedco 700 and Pacesetter Classes and even a few 1st generation designs. The 4th generation is rather difficult to define and is small.

the 4th generation semi-submersibles are large, suitable for harsh environment operation, and deep water capable. From a structural point of view, a marker of the 4th generation semi-submersible designs is that they rely fully on a hull-type superstructure with no bracing other than horizontals between the columns. One of the favorable aspects of this configuration is that, by the elimination of bracing, many inspection problems and the fatigue potential they represent are eliminated. The first use of this structural configuration was in the five Penrod semi-submersibles by Reineke in the early 1970s.

#### 2.3 Semi-submersible Design

Semi-submersibles consist of a deck, multiple columns and pontoons. They are "column Stabilized", meaning that the centre of gravity is above the centre of buoyancy, and the Stability is determined by the restoring moment of the columns. This contrasts with the spar platform,

which achieves stability by placing the centre of gravity below the centre of buoyancy, and the TLP, whose stability is derived from the tendons. The design of semi-submersibles depends on these principle considerations.

- Weights and CG's (cycle of steadily improving estimates)
- Hydrostatics; tank capacities
- Intact and Damaged Stability
- Current forces (mooring loads)
- Ballast System Performance
- Global Strength
- Fatigue

There may be different constraints for various load cases: operations, transit, survival, and installation. These should be identified in order to be able to check the configurations for each case. Weight estimates need to be made of all permanent payload and variable loads, including equipment and systems outfit for the functions (drilling, processing, utilities, quarters, flare, etc.). The equipment weights are to be supplanted by vendor equipment as it becomes available. The perform arrangements and calculations should be developed to support the outfit estimates (piping, access, corrosion protection, etc.). In addition, variable load requirements in amount, distribution, and with respect to the operating state should be firmly established. And, if it matters, the installation weight-states for permanently sited platforms should be determined. Before initiating the design, there should be a definitive Functions List (e.g. production, drilling, quarters), a Systems Summary, and an Equipment List (mission and support). Trial equipment and systems layouts should be made and coordinated with any constraints needed in the initial design. The constraints might include, for example:

Wind Forces (stability and mooring loads)

- Motions (sea keeping, drift and low frequency mooring loads)
- Maximum lightship draft for quayside outfitting,
- Maximum beam for canal transit or dry transportation,
- Maximum lightship weight and VCG envelope for dry transport,
- Environmental criteria for operations, transit and survival,
- Maximum lateral eccentricity of the deck load which needs to be trimmed,
- Maximum allowable motions (angles, accelerations) for each given environmental and load condition.

#### 2.4 Functions and configurations of semi-submersible

In the design of a semi-submersible, and its configuration in particular, a clear idea of the functions it must perform should be in hand. These will strongly influence configurational choices. Besides drilling, these functions include production, heavy lift, accommodations, operational support (surface, subsea), and even space launch. Apart from the mission and support functions, stated simply, there are two essential functions of a semi-submersible: To stably support a payload above the highest waves. To minimally respond to waves. These are the principal factors that establish size. It is, however, the mission functions and associated support functions that most significantly contribute to configuration.

The three main configurational components are:

- Pontoons
- Stability columns
- Deck

Figure 2.7 shows sectional views of four semi-submersible arrangements, identifying the above four components. Waterlines are shown at their typical operating state, "semi-submerged". While each has the noted components, each is distinctive. Case A is typical of 3rd generation semi-

submersibles, whereas Case B is quite typical of the 2nd generation. Similarly Cases C and D are typical of the 3rd and 4th generations respectively.

Virtually, all semi-submersibles have at least two floatation states: semi submerged (afloat on the columns) and afloat on the pontoons. The pontoons are the sole source of floatation of the semi when not semi-submerged. The stability columns are the principal elements of floatation and floatation stability while semi-submerged. Although they may function

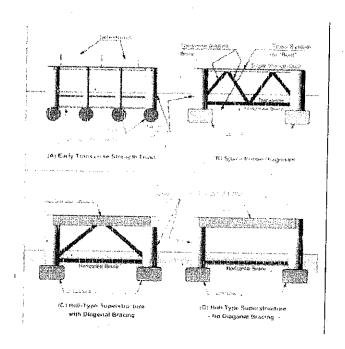


Figure 2-8:

structurally, structural strength is not the main function of the columns. It is notable that the pontoons are primarily filled with ballast when semi submerged. Beyond this, the size, submergence, proportion and spacing of the columns and pontoons are major factors in the hydrodynamic performance of semi-submersibles. Ostensibly, the deck provides the working surface for most of the semi-submersible's functions. It has the structural function to transfer the weight of the deck and its loading to the columns (and bracing). However, the deck is also a part of the overall global strength system, providing a structural connection between all of the columns.

The pontoons and columns are usually arranged and connected in a way that can provide considerable global strength. Generally the deck is likewise arranged and connected. Where this arrangement does not provide sufficient global strength, a space frame bracing system is employed (see fig. 2.9B and C). This has been very much the case in the earlier designs. However, bracing systems are problematic in that they are expensive to build.

#### 2.4.1 **Decks**

The decks of the early semi designs were a single level structures with individual deckhouses arranged with no coherent interrelated structural function. This arrangement was often referred to as a "piece of toast with lumps of butter". Support of a single deck requires a space frame bracing system and, or close column spacing. Single decks were favored in earlier semi-design because of the then limited erection resources. What has since evolved is the hull-type superstructure with integral connection to the column tops. Such a configuration can eliminate most, if not all space frame bracing. Among the advantages of the hull-type, integrally connected deck is superior strength, considerable usable interior space, and valuable floatation in damaged stability. If built with the rest of the hull in a modern shipyard. A hull-type deck is lighter, less costly, and of superior strength than other alternatives. A disadvantage, in some cases, is a necessity for mechanical ventilation and to fully outfit by a single builder. A "cousin" to the hulltype deck is the "truss-deck." It is preferred in some production applications that favour open, natural ventilation as well as historical design and fabrication practices. Particularly where there is separate fabrication, outfitting. And joining of the deck ("split construction"), the truss-type decks can be preferred because most fabricators of production decks are not equipped to build plated-structures. Similarly, design organizations that specialize in "topsides" are not experienced in working with hull-type structures. The choice of the deck type is therefore of considerable importance in configuring a semi insofar as it determines whether a split or an integrated construction will be preferred.

For clarity of terminology, the pontoon, columns, and bracing (usually) are referred to as the "hull", the "deck" being distinguished separately. With hull-type decks, there is not such a distinction.

#### 2.4.2 Colimnsl Pontoons

The number and arrangements of pontoons and columns distinguish many configurationally variants employed in the evolution of the semi. This has included as few as three to as many as a dozen or more columns. It has likewise included a simple two parallel pontoon arrangement, up to six, and even a grillage of orthogonally intersecting pontoons. As noted in the historical discussion, a few major designs featured independent footing pontoons, one for each stability column. The SEDCO 135 design, for example, had three independent pontoons; the Pentagone design had five. Figures 2.8 and 2.9 show a number of typical column and pontoon arrangements.

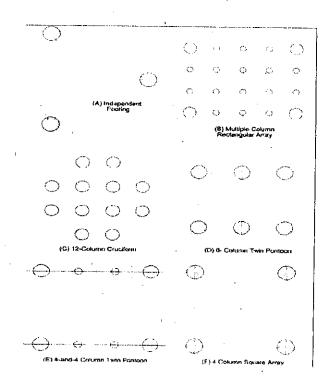


Figure 2-9: Semi-submersible column arrangements

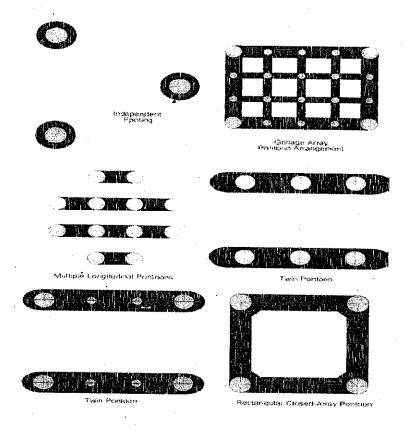


Figure 2-10: Semi-submersible pontoon arrangements

Only the 4-, 6-, and 8-column configurations continue in preference. Similarly only the twin pontoon and the closed array pontoon arrangements are currently used. A 3-column closed array pontoon (triangular) arrangement has been proposed for both FPS semi submersible and TLP applications, and offers a steel reduction opportunity. But these designs have not been successful, perhaps because of the more complex deck arrangements. The twin pontoon preference is principally because of its mobility. A preference for the 6- and 8-columns relates primarily to the twin pontoon option, and is influenced by the use of bracing systems.

A closed array, or "ring", pontoon arrangement is not very good for towing mobility, but is often preferred for a permanently sited system because it offers superior strength and an excellent potential for a braceless system. Transverse braces are not required and, with well designed column to pontoon connections, as well as special connection at the deck. The system can handle the racking loads. This is the basis of most TLP global strength systems. Fully developed hull-

type deck to the column connections offer an even greater strength potential, and allows wider column spacing.

As noted earlier, the function of the columns is to provide stability. A critical point of stability is when a semi is submerging, and when the flotation undergoes transitions from being afloat on the pontoons to being afloat on the columns. This operation is restricted to mild conditions and requires only that there be "positive GM". It limits the deck loading and otherwise discourages the particularly tall semis. For this reason, it is common to flare the columns at the pontoons to enhance stability through the critical range of drafts, Deck area is sometimes considered a sizing factor. Usually, the spacing of the columns for stability provides adequate interior space, particularly if there are two decks. Moderate deck extensions outside the column are a practical option. Sometimes, the overall width is a limiting factor. A limited maximum width has had a role in selecting the 6- and 8-column arrangement.

## 2.4.3 Bracing

Bracing configurations vary considerably. These principally include a transverse bracing, low on the columns, to resist squeeze/pry forces and, with these, a transverse diagonal bracing. The diagonal bracing is both to support the deck weight and, together with the horizontal transverse, provide the lateral racking strength. Often, a system of the horizontal diagonals is used to provide racking strength against quartering seas. A bracing system commonly found on many of the 3rd generation drilling platform. As a structural system, the strength of the space frame truss system is typically developed in parallel series of planes between columns, following civil engineering practice, called "bents." Each bent is a full truss, including the deck as a top chord and the horizontal, transverse brace as the bottom chord, all spanning between a pair of stability columns. Some use an "inverted-V" form of diagonals and some use an "inverted-W." Except

for a deck girder, the members consist of large diameter, thin walled cylinders. A well designed and well connected deck structure can eliminate the need for most bracing.

Similarly a closed array pontoon can also eliminate the need for bracing similar to a 4-column TLP. However, a twin pontoon structure will require horizontal transverses.

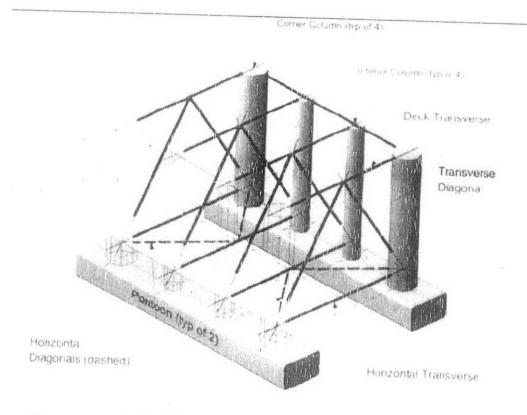


Figure 2-11: Typical 3rd generation semi-submersible bracing system

# 2.5 Theory

In this following part some theories relevant to our topic is discussed briefly. As we should have some knowledge about wave theory and oceanic forces wave theory. In this section a basic review of linear wave theory and types of hydrodynamic force theory are discussed.

# 2.5.1 Wave Theory

Ocean waves are, generally, random in nature. However, larger waves in a random wave series may be given the form of a regular wave that may be described by a deterministic theory. Even though these wave theories are idealistic, they are very useful in the design of an offshore

structures and its structural members. The wave theories that are normally applied to offshore structures are described in this section. There are several wave theories that are useful in the design of offshore structures. These theories, by necessity, are regular. Regular waves have the characteristics of having a period such that each cycle has exactly the same form. Thus the theory describes the properties of one cycle of the regular waves and these properties are invariant from cycle to cycle. There are three parameters that are needed in describing any wave theory. They are:

- > period (T) which is the time taken for two successive crests to pass a stationary point,
- ➤ Height (H) which is the vertical distance between the crest and the following trough. For a linear wave, the crest amplitude is equal to the trough amplitude, while they are unequal for a non-linear wave
- Water depth (d) represents the vertical distance from the mean water level to the mean ocean floor. For wave theories, the floor is assumed horizontal and flat. Several other quantities that are important in the water wave theory may be computed from these parameters. These critical parameters are:
- ❖ Wavelength (L) which is the horizontal distance between successive crests.
- Frequency (f) which is the reciprocal of the period.
- $\star$  Horizontal water particle velocity (u) which is the instantaneous velocity along x of a water particle.
- \* Vertical water particle velocity (v) which is the instantaneous velocity along y of a water particle.
- \* Horizontal water particle acceleration (G) which is the instantaneous acceleration along x of a water particle.
- \* Vertical water particle acceleration (G) which is the instantaneous acceleration along y of a water particle.

A wave creates a free surface motion at the mean water level acted upon by gravity. The elevation of the free surface varies with space x and time t. The simplest and most applied wave theory is the linear wave theory. For the linear wave theory, the wave has the form of a sine curve and the free surface profile is written in the following simple form:

$$\eta = aSin(kx - \omega t).$$

#### 2-1: Wave Formula

In which the quantities a,  $\omega$  and k are constants. The coefficient "a" is amplitude of the wave. The quantity  $\omega$  is the frequency of oscillation of the wave and k is called the wave number. A two-dimensional coordinate system x, y is chosen to describe the wave propagation with x in the direction of wave and y vertical. The point where the value of the profile is +a is called a crest while the point with the value -a is called the trough.

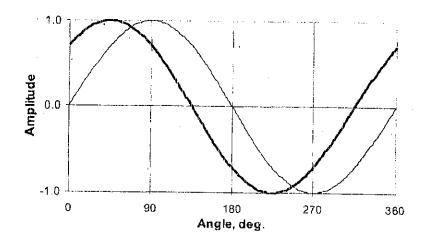


Figure 2-12: Wave Profile

#### 2.5.2 Morison equation

The basic equation for the stability of submerged objects is the Morison Equation The total horizontal wave induced force on a submerged object can be broken up into two basic parts. The Morison equation contains two forces as mentioned above. The overall equation can be shown as:

$$f = f_{inertia} + f_{drag}$$

Equation 2-2: Morison Equation

#### 2.5.2.1 Drag Force

The drag force is the predominate wave induced force on a submerged object in shallow water (Dean). The drag force is calculated using the classic drag equation (Roberson) as seen as below:

$$f_{drag} = \frac{1}{2} C_d \rho D |u| u ds$$

# **Equation 2-3: Drag Force**

Where  $C_D$  is the coefficient of drag,  $\rho$  is the density of sea water, and ds is the projected cross sectional area element as seen from the direction of flow, and u is the horizontal water particle velocity. The only unknown in the equation is the coefficient of drag. Since the coefficient of drag is dependent on the shape and surface roughness of the object, it must be determined experimentally. Results of the scale model tests are used to determine the drag coefficient.

#### 2.5.2.2 Inertia Force

The inertia force is the force imparted on the submerged object by the acceleration of the fluid past the object. The inertia force is defined by:

$$df_i = C_m \rho \frac{\pi}{4} D^2 \frac{\partial u}{\partial t} ds$$

Equation 2-4: Inertia Force

Where  $C_M$  is the coefficient of inertia,  $\rho$  is the density of sea water, and  $\frac{\partial u}{\partial t}$  is the water particle acceleration. The coefficient of inertia is based in the size and shape of the object.  $C_M$  is always greater than or equal to one.

## 2.5.3 Dynamic Analysis

A dynamic analysis is normally mandatory for every offshore structure, but can be Restricted to the main modes in the case of stiff structures.

## 2.5.3.1 Equations of Motion

The governing dynamic equations of multi-degrees-of-freedom systems can be expressed in the matrix form:

$$MX'' + CX' + KX = \vec{P}(t)$$

# **Equation 2-5: Equation of Motion**

Where:

M is the mass matrix

C is the damping matrix

K is the stiffness matrix

X, X', X'' are the displacement, velocity and acceleration vectors (function of time). P(t) is the time dependent force vector; in the most general case it may depend on the displacements of the structure also (i.e. relative motion of the structure with respect to the wave velocity in Morison equation).

#### 2.5.3.2 Mass

The mass matrix represents the distribution of masses over the structure. The added mass of water (mass of water displaced by the member and determined from potential flow theory) and the mass of marine growth. Masses are generally lumped at discrete points of the model. The mass matrix consequently becomes diagonal but local modes of vibration of single members are ignored (these modes may be important for certain members subjected to an earthquake). The selection of lumping points may significantly affect the ensuing Solution.

As a further simplification to larger models involving considerable degrees-of freedom, the system can be condensed to a few freedoms while still retaining its basic Energy distribution.

## 2.5.3.3 Damping

Damping is the most difficult to estimate among all parameters governing the dynamic response of a structure. It may consist of structural and hydrodynamic damping. Structural Damping: Structural damping is associated with the loss of energy by internal friction in the material. It increases with the order of the mode, being roughly proportional to the strain energy involved in each. Hydrodynamic Damping: Damping provided by the water surrounding the structure is commonly added to the former, but may alternatively be accounted as part of the forcing function when vibrations are close to resonance.

Viscous damping represents the most common and simple form of damping. It may have one of the following representations modal damping: a specific damping ratio  $\zeta$  expressing the percentage to critical associated with each mode (typically  $\zeta=0.5\%$  structural;  $\zeta=1.5\%$  hydrodynamic).

• proportional damping: defined as a linear combination of stiffness and mass matrices. All other types of non-viscous damping should preferably be expressed as an equivalent viscous damping matrix.

#### 2.5.3.4 Stiffness

The stiffness matrix is in all aspects similar to the one used in static analyses. Free Vibration Mode Shapes and Frequencies The first step in a dynamic analysis consists of determining the principal natural vibration mode shapes and frequencies of the undamped, multi-degree-of-freedom structure up to a given order (30<sup>th</sup> to 50<sup>th</sup>). This consists of solving the eigen-value problem:

$$KX = \lambda MX$$

## **Equation 2-6: Eigen value Equation**

For rigid structures having a fundamental vibration period well below the range of wave periods (typically less than 3 s), the dynamic behavior is simply accounted for by multiplying the time-dependent loads by a dynamic amplification

$$DAF = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\beta\zeta)^2}}$$

#### Equation 2-7: DAF Formula

Where  $\beta = \frac{T_n}{T}$  is the ratio of the period of the structure to the wave period.

# 2.5.3.5 Frequency Domain Analysis

Such analysis is most appropriate for evaluating the steady-state response of a system subjected to cyclic loadings, as the transient part of the response vanishes rapidly under the effect of damping. The loading function is developed in Fourier series up to an order  $\eta$ :

$$P(t) = \sum_{i=1}^{n} p_{i} e^{i(\omega_{1}t + \phi_{1})}$$

# Equation 2-8: Fourier series for loading Function

The plot of the amplitudes pj versus the circular frequencies  $\omega_j$  is called the amplitude power spectra of the loading. Usually, significant values of pj only occur within a narrow range of frequencies and the analysis can be restricted to it. The relationship between response and force vectors is expressed by the transfer matrix H, such as:

$$H = -M\omega^2 + IC\omega + K$$

## **Equation 2-9: Transfer Matrix**

The elements of which represent:

$$H_{j,k} = \frac{X_j}{P_k} = \frac{deflection\_in\_freedom\_j}{force\_in\_freedom\_k}$$

# **Equation 2-10: Transfer Matrix Elements**

The spectral density of response in freedom j versus force is then The fast Fourier transform (FFT) is the most efficient algorithm associated with this kind of analysis.

# 2.5.3.6 Time Domain Analysis

The response of the i-th mode may alternatively be determined by resorting to Duhamel's integral:

$$X_{j}(t) = \int_{0}^{t} P_{j}(\tau)h(t-\tau)d\tau$$

# Equation 2-11: Duhamel's integral

The overall response is then obtained by summing at each time step the individual responses over all significant modes.

#### 2.5.4 Basics of static stability

In stability investigation, the stability of structure will be observed without any hydrodynamic force. As it is mentioned before this stability is prerequisite for dynamic stability. The basic idea of checking stability is to consider righting moment which is the consequence of buoyancy and the wind heeling arm that caused by wind.

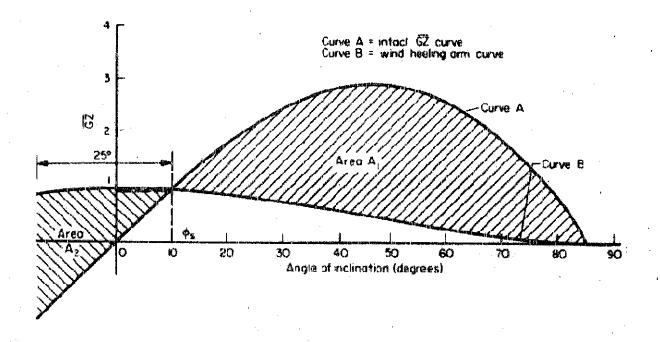


Figure 2-13: Stability criteria for beam wind and rolling

To derive wind moment there are lots equations based on type of structure and speed of wind. For righting arm curve we need to calculate several parameters like center of gravity, center of buoyancy and metacenter.

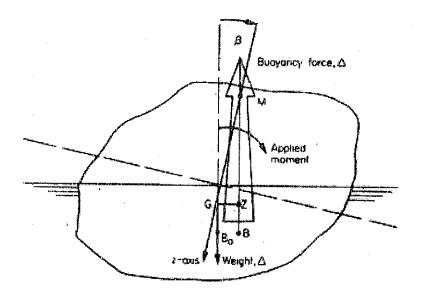


Figure 2-14: Stability of Structure

These two equations are the basic idea for calculation the parameters, In spread sheet these parameters will be calculated based on heeling angle

Equation 2-12: Basic relationship between CG,CB, MC

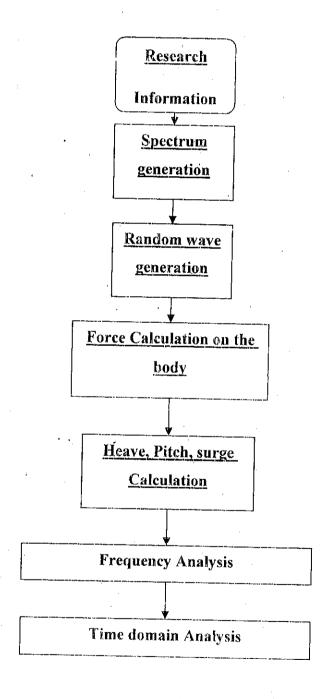
$$\overline{GM} = \overline{KB} + \overline{BM} - \overline{KG}$$

Equation 2-13: Relationship between CB, MC

$$\frac{\overline{I}}{\nabla} = \overline{BM}$$

# CHAPTER 3 METHODOLOGY

#### 3.1 Procedures



#### 3.2 General

The basic idea for this project is to prepare a simple model of semi submersible motion. According to project topic we should have analysis in both frequency and domain. As it mention before for sake of simplicity the Morison equation will be used in this project. Morison equation is the easiest and the most straight forward equation for finding the force on the structure. In first step analysis will be carried out for regular wave and in next stage will be expanded to random waves.

The semi submersible structure will be break in to different segments. However the shape of elements is not regular and contains unconformity, they will be considered symmetrical and harmonic. Submerged part of platform mainly contains two main elements which are pontoons and columns. As the pontoons have mostly rectangular cross-section and the Morison equation is applicable for cylindrical segments, a cylindrical pontoon which has same cross-section area with actual pontoon will be utilized for satisfactory of conditions.

#### 3.3 Semi-Submersible Properties

The selected semi-submersible is taken from conceptual design report. This particular semi-submersible is designed for Malaysia. The purpose of this selection is availability of all required information like structural dimension, Weight of platform, pitch and heave natural period. The cross section of this platform is rectangular. The column cross section is also rectangular. Since the cross section is symmetrical. Calculation and analysis is done for one direction. The dimension of semi-submersible is as below.

Pontoon		Column	
Length:	82.292(m)	Length	15.24(m)
Width:	82.292(m)	Width	15.24(m)
Height:	16.674(m)	Height	24.384(m)

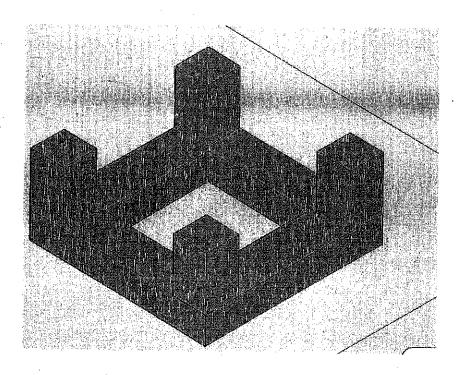


Figure 3-1: Submerged portion of Semi-Submersible

The platform has different weight depends on its draft. For this analysis operational mode is chosen due to sensitivity of operation to motions. The operational draft for this platform is 25.908(m) and the weight of structure in operational mode is 194,859(KN).

#### 3.4 Spectrum generation

The regular wave theories are applicable in a design where a single wave method is employed. This is often a common method in the design of an offshore structure. In this case an extreme wave is represented by a regular wave of the appropriate height and period. This method provides a simple analysis in determining the extreme response of an offshore structure. The random ocean wave, on the other hand, is described by an energy density spectrum. The wave energy spectrum describes the energy content of an ocean wave and its distribution over a frequency range of the random wave. Therefore, the random wave method of design may be important especially in the design of floating structures. The random wave is generally described by its statistical parameters. For this analysis P-M spectrum is chosen. The random wave generation is the following step.

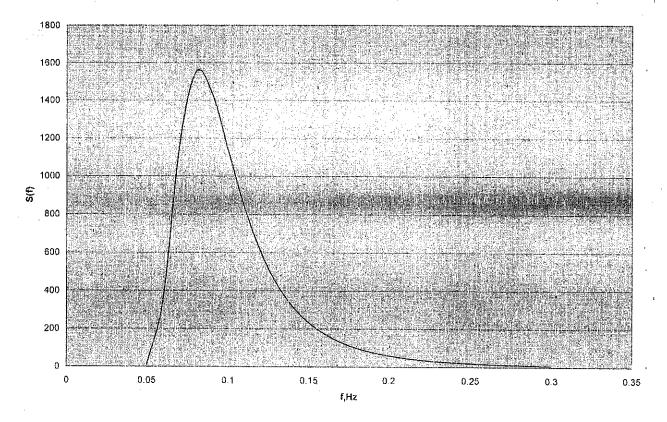


Figure 3-2: P-M Spectrum

### 3.5 Random Wave generation

The next step in hydrodynamic force calculation is random wave generation. For force calculation horizontal and vertical velocity and acceleration is needed. From energy density spectrum, wave height is calculated.

$$H(f_i) = 2\sqrt{2S(f_i)\Delta f}$$

### Equation 3-1: Wave Height Formula

The horizontal and vertical velocity and acceleration can be calculated from the below formulas.

$$u = \frac{gHk}{2\omega} \frac{\cosh(ks)}{\cos(kd)} \cos(\Theta)$$

## Equation 3-2: Horizontal velocity

$$v = \frac{gHk}{2\omega} \frac{\cosh(ks)}{\sinh(kd)} \sin(\Theta)$$

## **Equation 3-3: Vertical Velocity**

$$\frac{\partial u}{\partial t} = \frac{gHk}{2} \frac{\cosh(ks)}{\cos(kd)} \sin(\Theta)$$

## **Equation 3-4: Horizontal Acceleration**

$$\frac{\partial v}{\partial t} = -\frac{gHk}{2} \frac{\sinh(ks)}{\cos(ka')} \cos(\Theta)$$

# **Equation 3-5: Vertical Acceleration**

## 3.6 Force Calculation on the Body

In this step the horizontal, vertical velocity and moment about center of gravity are calculated. According the geometry of semi-submersible, Froude-Krylov method is the best alternative. The reason for this selection is availability of specific formula for rectangular cross section.

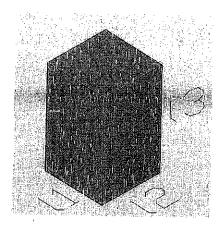


Figure 3-3: Cubic Element

The formulas for vertical and horizontal force are as below.

$$F_x = C_h \rho V \frac{\sinh(kl_3/2)}{kl_3/2} \frac{\sinh(kl_1/2)}{kl_1/2} \dot{u}_0$$

### Figure 3-4: Horizontal Force

$$F_{y} = C_{y} \rho V \frac{\sinh(kl_{3}/2)}{kl_{3}/2} \frac{\sinh(kl_{1}/2)}{kl_{1}/2} \dot{v}_{0}$$

#### Figure 3-5: Vertical Force

In these formulas " $\dot{u}_0$ ", " $\dot{v}_0$ " are horizontal and vertical acceleration of CG of the respective element.  $l_1$ ,  $l_2$ ,  $l_3$  are width, height and length of element as it shown in figure 3-3. V is the volume of the element that can be calculated simply. Since all the forces on each element can be calculated individually, the total force on the platform is obtained by adding the forces together.

## 3.7 Frequency Domain Analysis

The motion response of structure can be calculated by two methods. The first method is frequency domain analysis. This section is a brief description of this method step by step. The first step is obtaining RAO for three motions. The next step is motion spectrum for each motion which is straight forward and the last stage is calculation of motion by the time.

#### 3.7.1 RAO Calculation

RAO is calculated for each frequency. RAO is dependent on few factors. The formulas for RAO are shown as below for three motions separately.

$$RAO_{surge} = \frac{F_x/(H/2)}{[(K_{11} - m_{11}\omega^2)^2 + (C_{11}\omega^2)^2]^{\frac{1}{2}}}$$

Figure 3-6: Surge RAO

$$RAO_{heave} = \frac{F_v / (H/2)}{\left[ (K_{22} - m_{22}\omega^2)^2 + (C_{22}\omega^2)^2 \right]^{\frac{1}{2}}}$$

### Figure 3-7: Heave RAO

$$RAO_{puch} = \frac{M_{z}/(H/2)}{[(K_{33} - I_{33}\omega^{2})^{2} + (C_{33}\omega^{2})^{2}]^{\frac{1}{2}}}$$

#### Figure 3-8: Pitch RAO

In these formula  $m_{11}$ ,  $m_{22}$ ,  $I_{33}$  are added mass for surge, heave and pitch respectively. Added mass is the mass of water that is displaced with respective motion. The value for added mass can be calculated by below formulas. It should be noted that I is mass moment of inertia of structure about axis of rotation.

$$m_{11} = \frac{\pi}{4}lD^2\rho + M$$

# Figure 3-9: Surge Added Mass

$$m_{22} = \frac{\pi D^3}{12} \rho + M$$

## Figure 3-10: Heave Added Mass

$$I_{33} = m_{22} \times r^2 + I$$

# Figure 3-11: Pitch Added Mass

The value of K is stiffness for each motion, for surge this value is depends on mooring line system. But for pitch and heave the below formulas can be used.

$$k_{22} = \sqrt{\frac{\rho g A_{wp}}{2m}}$$

# Figure 3-12: Stiffness for Heave

$$k_{33} = \sqrt{\frac{gGMA_{wp}}{I}}$$

# Figure 3-13: Stiffness for Pitch

Where:

m≔ mass

GM= distance from center of gravity to metacenter

I= Second moment of inertia with respect to water plane

Awp= Water plane area

C<sub>11</sub>, C<sub>22</sub>, C<sub>33</sub> values depends on damping ratio of system. These value are vary by frequency but since the calculation of damping ratio is complicated, a constant value is assumed for every frequency.

# 3.7.2 Response Spectrum Calculation

Calculation of response spectrum is straight forward. Using the RAO values for every frequency and the energy density spectrum, the response spectrum is generated.

$$S_x(f) = S(f) \times RAO^2$$

# **Equation 3-6: Response Spectrum**

# 3.7.3 Response to Random Wave in Time

The steps for this stage are the same as random wave generation. Firstly the response amplitude is obtained from response spectrum with respect to below formula.

$$H_s(f_i) = 2\sqrt{2S_s(f_i)\Delta f}$$

Equation 3-7: Response Amplitude to Wave

Then response is achieved by using this formula.  $\phi(i)$  is random function that generates random number between 0 and  $2\pi$ .

$$\eta(t) = \sum_{i=1}^{n} \frac{H_{s,i}(f_i)}{2} Cos(kx - \omega t + \phi(i))$$

#### **Equation 3-8: Motion Equation**

#### 3.8 Newmark's Beta Method

This is one of the integration methods that is used for dynamic analysis. In this section this method is described step by step.

**Step** The stiffness matrix [K], the damping matrix [C], the mass matrix [M], the initial displacement vector  $\{X_0\}$ , the initial velocity  $\{X_0\}$  are given as the known input data.

Step2 The force  $\{F(t)\}$  is calculated for t=0

Step3 The initial acceleration vector is then calculated as below:

$$\{\ddot{X}_{0}\} = \frac{1}{M} (F_{0} - C\dot{X}_{0} - K\dot{X}_{0})$$

**Step4**  $\delta = 0.5, \alpha = 0.25(0.5 + \delta)^2$  and with integration constants as:

$$a_0 = \frac{1}{\alpha \Delta t^2}, a_1 = \frac{1}{\alpha \Delta t}, a_2 = \frac{1}{\alpha \Delta t}, a_3 = \frac{1}{2\alpha} - 1, a_4 = \frac{\delta}{\alpha} - 1, a_5 = \frac{\Delta t}{2} \left[ \frac{\delta}{\alpha} - 2 \right]$$

$$a_6 = \Delta t (1 - \delta)$$
,  $a_7 = \delta \Delta t$  and  $\Delta t$  is time step

**Step5** 
$$K^{\wedge} = K + a_0 M + a_1 C$$

Step6 For each time step, the following are calculated:

$$F_{i+\Delta t}^* = F_{i+\Delta t} + M(a_0 X_i + a_2 \dot{X}_i + a_3 \ddot{X}_i) + C(a_1 X_i + a_4 \dot{X}_i + a_5 \ddot{X}_i)$$

$$X_{i+\Delta t} = K^{-1} F_{i+\Delta t}$$

$$\ddot{X}_{\iota+\Delta\iota} = a_0 \big( X_{\iota+\Delta\iota} - X_\iota \big) - a_2 \dot{X}_\iota - a_3 \ddot{X}_\iota$$

These values are calculated at each time step and then value of evaluate until the convergence is achieved at each step.

# CHAPTER 4 RESULT AND DISCUSSION

In this chapter the results for three responses are illustrated by aid of graph and tables. This chapter consists of two sections, in first section the result for frequency analysis is shown and in next section time domain analysis is reviewed. In each section the calculation is done for different case study which is explained in the following chapter.

#### 4.1 Frequency Domain Analysis

In this section, the result for frequency domain is shown. The first case is response of semisubmersible to different significant wave heights and depth.

#### 4.1.1 Water Depth Study

In this section the motion Analysis is done for four different water depths.

Table 4-1: water Depth Case Study

Case	Water	$H_{\rm s}$
	Depth(m)	
Case A	600	6.3
Case B	900	6.3
Case C	1200	6.3
Case D	1500	6.3

# Case A:

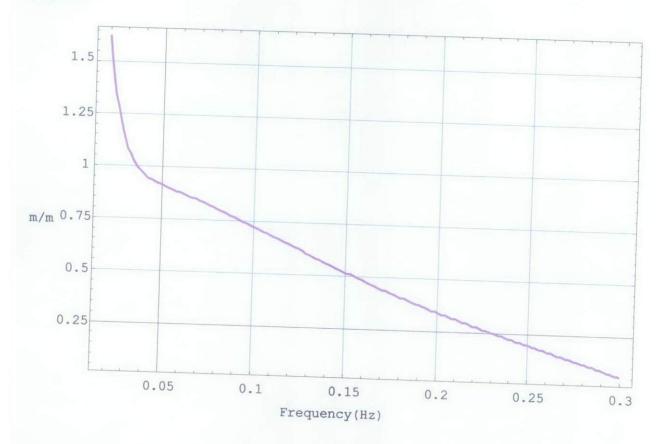


Figure 4-1: Surge RAO

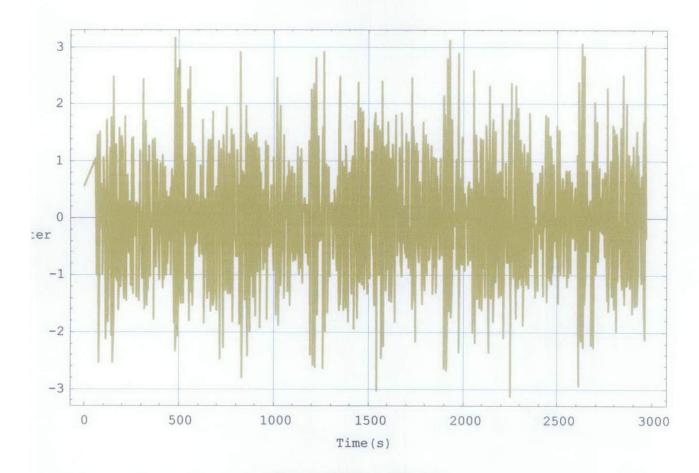


Figure 4-2: Surge Motion

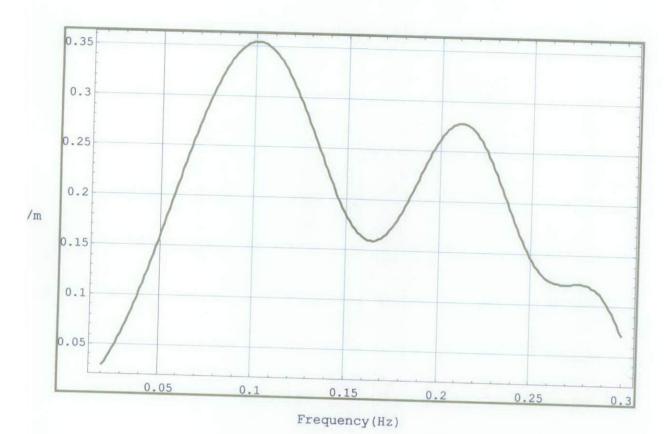


Figure 4-3: Heave RAO

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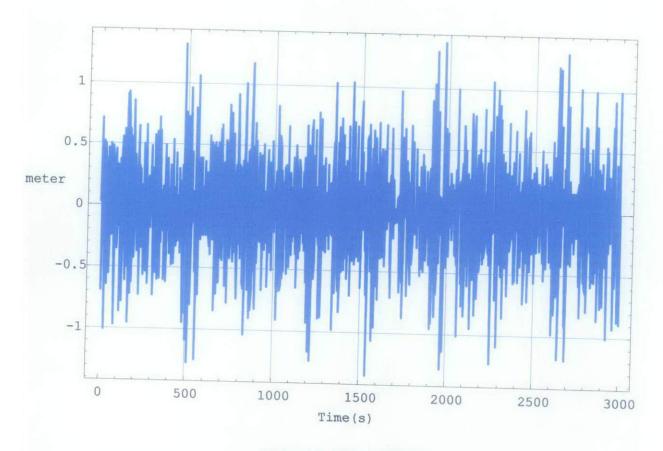


Figure 4-4: Heave RAO



Figure 4-5: Pitch RAO

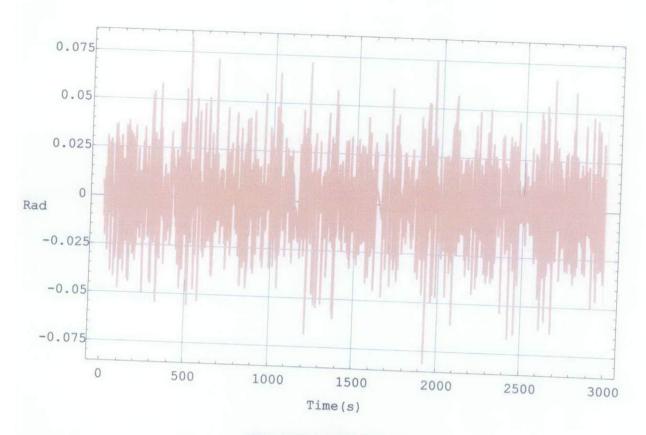


Figure 4-6: Pitch Motion

For the other cases the motion of response is illustrated only. Since the water depth does not have very significant on RAO.

#### Case B:

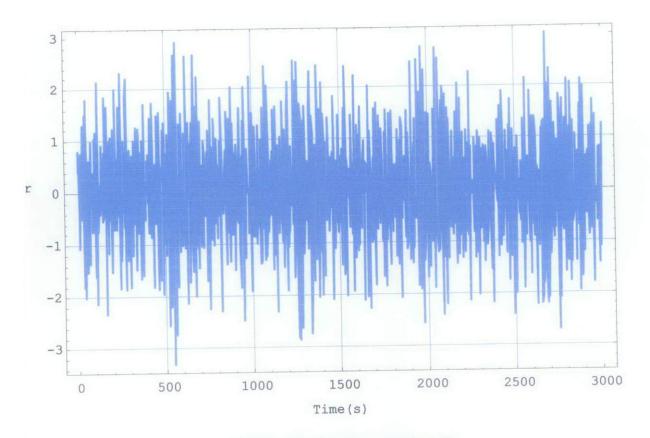


Figure 4-7: Surge Motion Case B

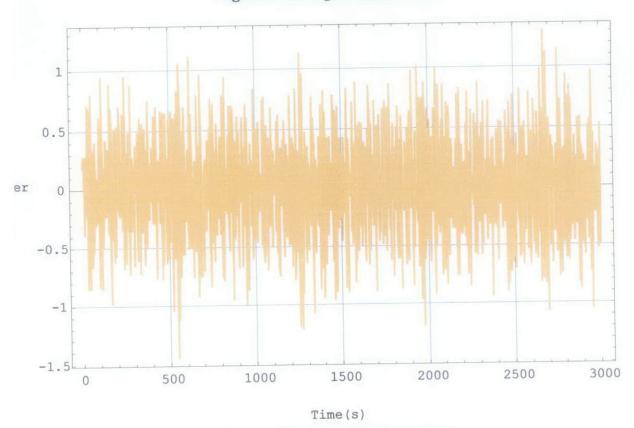


Figure 4-8: Heave Motion Case B

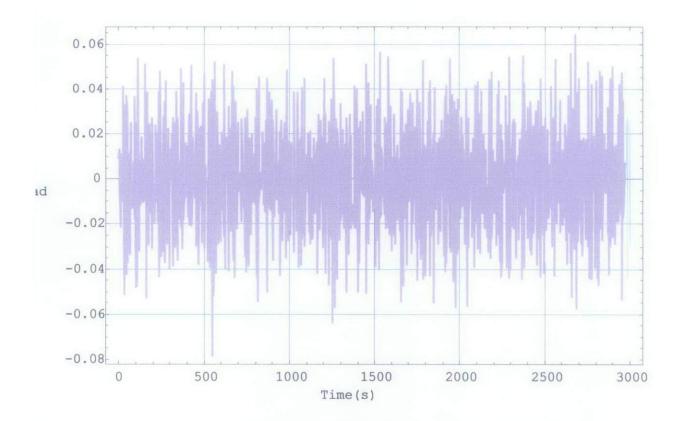


Figure 4-9: Pitch Motion Case B

Case C:

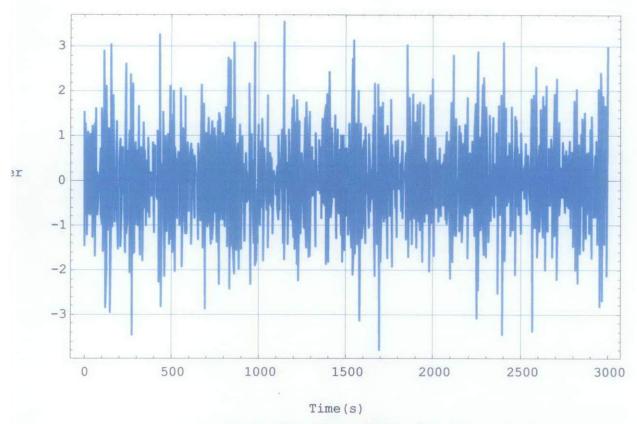


Figure 4-10: Surge Motion Case C

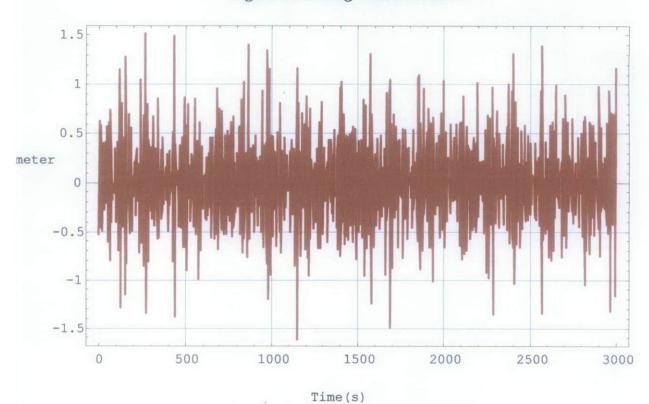


Figure 4-11: Heave Motion Case C

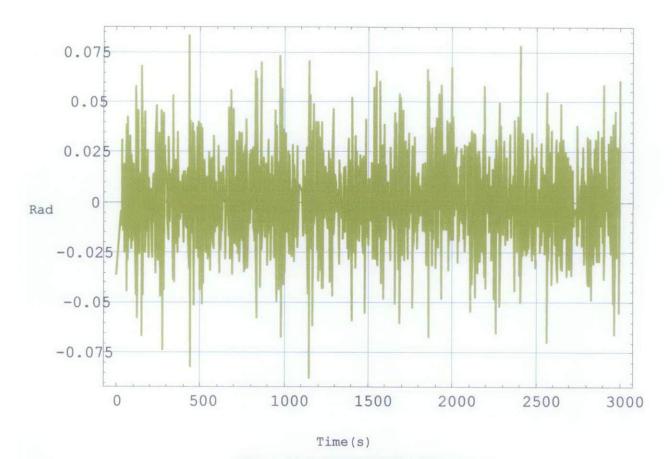


Figure 4-12: Pitch Motion Case C

Case D:

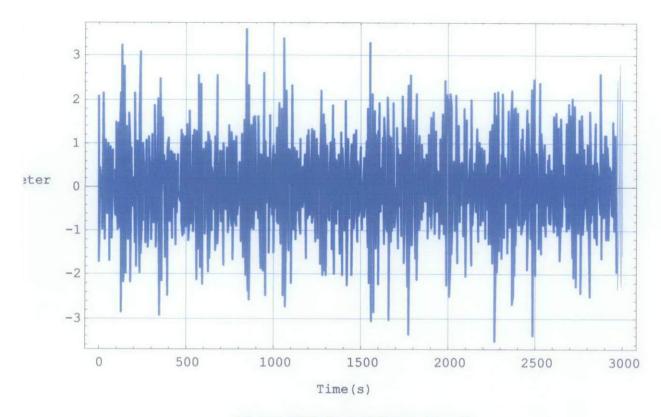


Figure 4-13: Surge Motion Case D

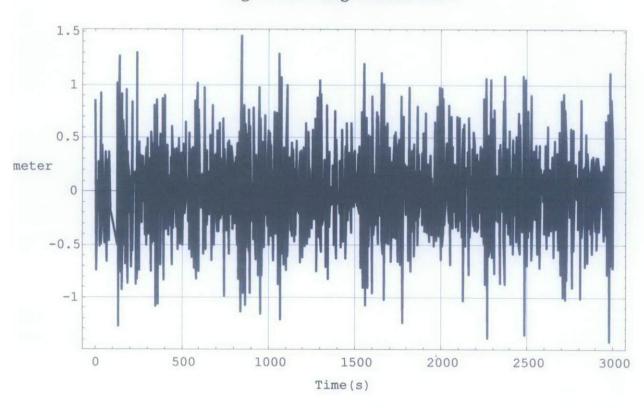


Figure 4-14: Heave Motion Case D

#### 4.1.2 Effect of Draft

In this section effect of draft on each motion is studied. For each draft result is shown as below.

Besides operational draft, survival draft is important. Survival draft for this platform is 20.36(m).

By changing draft, all the properties of system will be changed. For this system the properties for survival draft are as below.

Draft 20.36(m)

Weight 179,069(KN)

T<sub>n</sub> for Heave 18.57(s)

T<sub>n</sub> for Pitch 29.15(s)

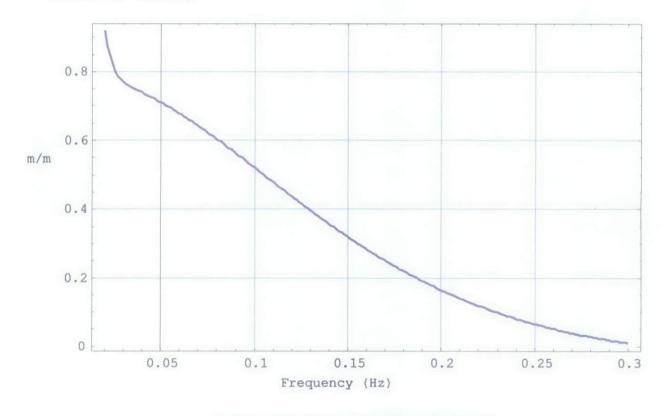


Figure 4-15: Surge RAO Survival Draft

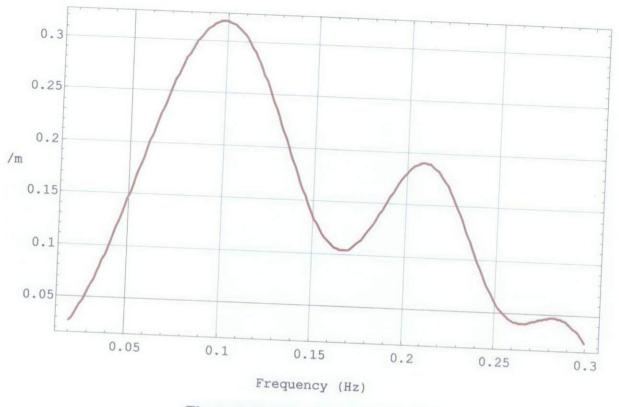


Figure 4-16: Heave RAO Survival Draft

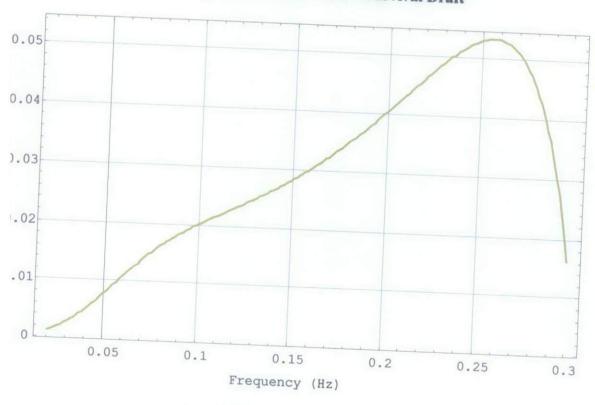


Figure 4-17: Pitch RAO Survival Draft

#### 4.2 Time Domain Analysis

In this section the calculation for motion response is revised again but with time domain approach. For this approach these three motions (surge, heave and pitch) are coupled. The steps for this method are described in methodology chapter. For same cases, the program is done frequency and time domain analysis. The comparison between two cases is conducted in discussion section.

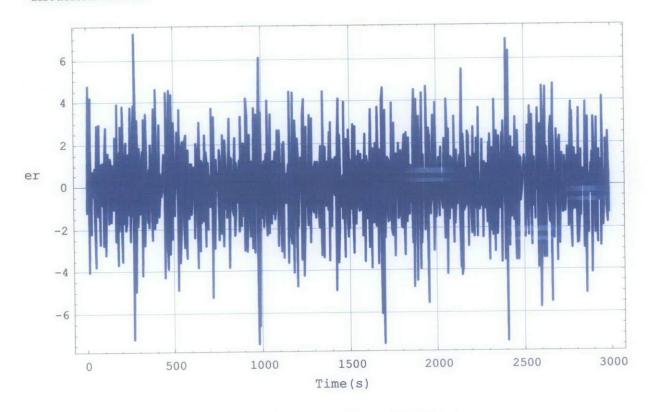


Figure 4-18: Surge Frequency Response

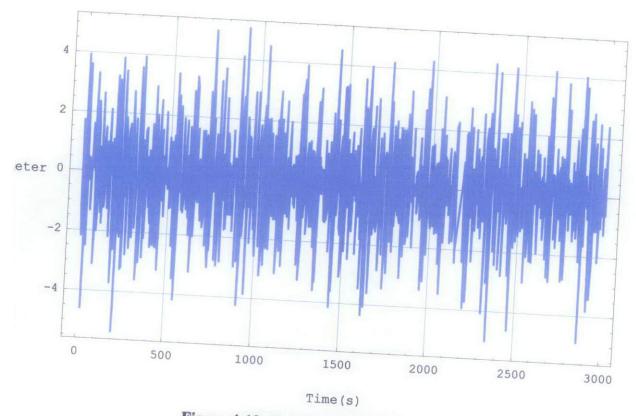


Figure 4-19: Surge Response Time Domain

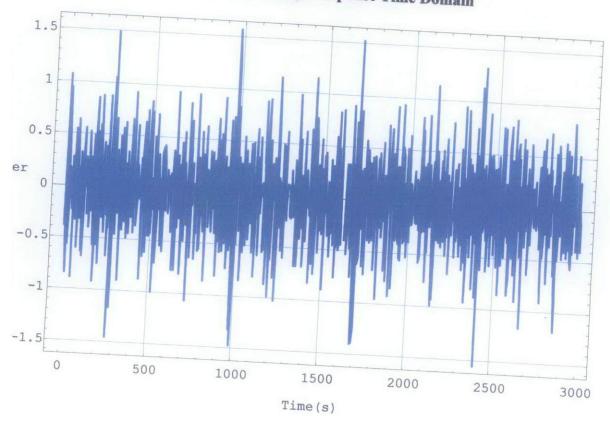


Figure 4-20: Heave Response Frequency Domain

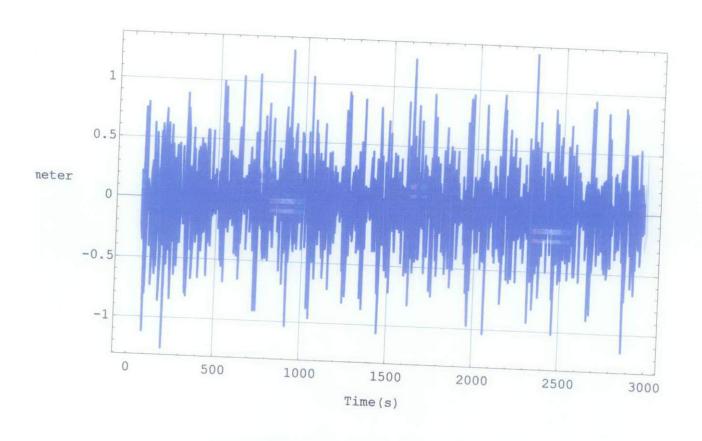


Figure 4-21: Heave Response Time Domain

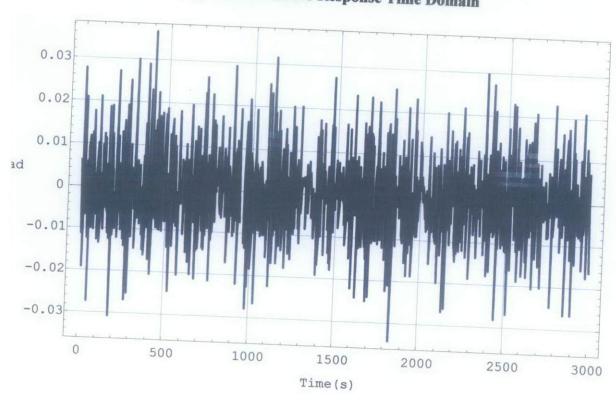


Figure 4-22: Pitch Response Frequency Domain

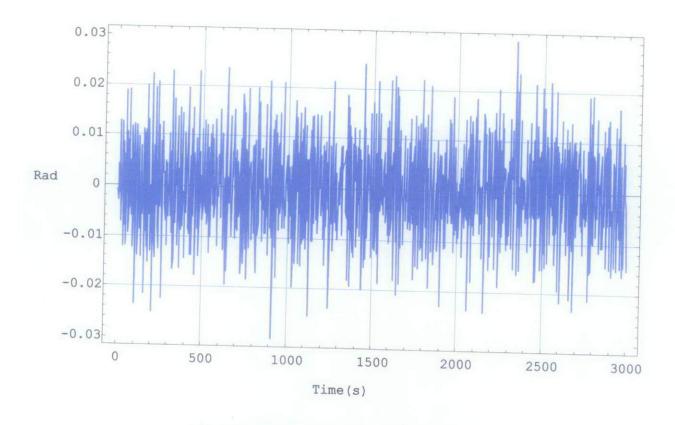


Figure 4-23: Pitch Response Time Domain

## 4.3 Discussion

In this section, the final results are compared together. Firstly frequency domain results for different water depth are discussed as below.

# 4.3.1 Water Depth Effect

Table 4-2: Water Depth effect in Frequency Domain Statistic Data

Case	Surge Mean (m)	Heave Mean (m)	Pitch Mean (Rad)	Surge Standard Deviation	Heave Standard Deviation	Pitch Standard Deviation
A	1.75764	0.362373	0.00855671	1.32062	0.275728	0.00654517
В	1.68687	0.352692	0.00865142	1.40231	0.285183	0.00643339

C	1.77429	0.367869	0.00857291	1.30701	0.26788	0.00646501
D	1.73379	0.349773	0.00853337	1.31571	0.282249	0.00642306

Table 4-3: Water Depth Effect in Frequency Domain Amplitudes

Case	Surge Maximum (m)	Heave Maximum (m)	Pitch Maximum (Rad)	Surge Minimum (m)	Heave Minimum (m)	Pitch Minimum (Rad)
<u>A</u>	6.68532	1.50891	0.035875	-6.67256	-1.50745	-0.0356382
В	6.84667	1.50348	0.0363305	-6.88711	-1.47156	-0.0370619
<u>C</u>	6.93605	1.33524	0.03503	-6.30975	-1.32733	-0.035535
D	7.16988	1.3876	0.0356523	-7.06365	-1.38549	-0.035015

From these two tables we can realize that the water depth does not have significant effect on the forces. The results shows that by increasing the maximum surge value will increase slightly but it does not have major effect on system. But the major concern in deeper is the mooring line which anchored the platform to the sea bed. The role of oceanic current should be considered in this case which is not in the scope of this project. The other issue of deeper water is the weight of mooring lines and their cost which is also not included in this report.

### 4.3.2 Effect of Draft

As it discussed in previous section, the structure draft has effect on all the properties of motion system. Since the weight of structure is in direct relationship with draft and the stiffness of pitch and heave is also affected by changing the draft, this section is allocated to this topic.

Table 4-4: Draft Effect Statistic Data

		Heave	Pitch	Surge	Heave	Pitch
Draft	Surge		Mean	S.D	S.D	S.D
(m)	Mean	Mean			(m)	(Rad)
	(m)	(m)	(Rad)	(m)	(111)	(1144)
			0.00040061	1.35823	0.279908	0.00677981
25.09	1.722.25	0.357521	0.00848861	1.33623	0.277700	
		.0.227007	0.0266375	0.524939	0.255866	0.0197674
20.36	0.665236	0.337807	0.0260373	0.524757		
			<u> </u>	L		

Table 4-5: Draft Effect Amplitudes

			Ditale	Surge	Heave	Pitch
Draft	Surge	Heave	Pitch			) ( · · · · · · · · · · · · · · · · · ·
(173)	Maximum	Maximum	Maximum	Minimum	Minimum	Minimum
(m)	(m)	(m)	(Rad)	(m)	(m)	(Rad)
		The second secon		0.15341	-1.54302	-0.0358646
25.09	8.24313	1.5849	0.0365401	-8.15241	-1,54502	0.03300.10
		11.24010	0.109219	-2.62422	-1.2379	-0.118065
20.36	2.62075	1.24018	0.109217	2.02 13.2	,	,
		<u> </u>				<b></b> -

As it is shown in tables, surge decreases significantly. In survival draft the columns submerged part is less, thus the horizontal force decreases and as the effect surge value reduces. Heave motion is not affected very much. Since the pontoon cross section does not change by changing the draft, the total vertical force is not affected.

The other critical value is pitch value. Even though horizontal force decreases by decrement in draft, but the position of center of gravity will shift upward, thus the pitch will increase.

# 4.3.3 Frequency Domain and Time domain Assessment

The purpose of this section is to compare the results from frequency domain and time domain. As it mentioned before time domain analysis is more precise method because it considers the changes in draft and stiffness. The other advantage of time domain is accomplishment of coupled motion analysis. Time domain puts the effect of different motion on each other under consideration. The statistic data is shown in below tables.

Table 4-6: Frequency and Time Domain Statistic Data

Case	Surge	Heave	Pitch	Surge	Heave	Pitch
	Mean	Mean	Mean	S.D	S.D	S.D
	(m)	(m)	(Rad)	(m)	(m)	(Rad)
Frequency	1.76826	0.368201	0.00843305	1.31726	0.27189	0.00665826
Domain						,
Time Domain	1.38721	0.319509	0.00771196	1.0748	0.251844	0.00577836
			a managana i seperanta a salama atau a managana inda managan inda managan inda managan da panama panaha a manag			

Table 4-7: Frequency and Time Domain Amplitudes

Draft	Surge	Heave	Pitch	Surge	Heave	Pitch
(m)	Maximum	Maximum	Maximum	Minimum	Minimum	Minimum
	(m)	(m)	(Rad)	(m)	(m)	(Rad)
Frequency	6.06044	1.30828	0.0388737	-6.04249	-1.33253	-0.0393902
Domain						
Time	5.9157	1.36948	0.0369864	-6.1084	-1.40459	-0.0370906
Domain						
		1				

As it obvious from these results, there is acceptable coherency between two methods. Time domain amplitudes are slightly less than frequency domain. This matter may have two sources. Firstly in time domain draft is changing by time, since the horizontal forces decreases by decrement in draft, the coupled motion is slightly less than dependent one.

The other matter that is considered in time is alternation of pitch stiffness by heave motion. This phenomenon can be studied in further studied. For surge motion the amplitude does not change by distance from wave origin, but when it is coupled with heave the effect is considerable.

# CHAPTER 5

#### CONCLUSION

This research is commenced by studying the offshore platforms. A brief literature is prepared consists if their types and their various functionality based on water depth and environmental condition. After this phase semi-submersible is taken under research in order to have better overall view of its characteristic.

Firstly, the history of semi-submersible is studied as a part of project. The sequential alternation of this platform is studied in introduction chapter. The next stage of research was a brief investigation about the effects of different factors on design and configuration of semi-submersible. This study can be found in literature review.

The general purpose of this project was dynamic analysis of semi-submersible in frequency and time domain. For dynamic analysis, surge, heave and pitch are done in this report. The other three motions are excluded from this report due to similarity. In this report, oceanic wave is the only source of hydrodynamic loading, current and wind load are excluded in this research.

The steps for dynamic analysis are described in methodology chapter by detail. Frequency domain is done at the beginning followed by time domain. Time domain analysis gives more precise and accurate results but it is more complicated. On the other hand frequency domain is straight forward and simple.

- In results and discussion chapter, frequency domain analysis is done for different water depth. As the results show, water depth does not have very significant effect on motion amplitudes.
- The other important factor is draft. In this report the motion analysis is done for two different draft, operational and survival. The results show that surge motion is more critical in operational draft due to higher load on the columns. In contrast with surge

heave is not affected significantly and it is relatively constant in two conditions. The pitch motion is more critical in survival condition. The results are available in results and discussion chapter.

> From this research we can conclude time domain analysis is necessary for floating structures like semi submersibles due to high level of uncertainty. This could be a better and more conservative analysis beside frequency domain.

# CHAPTER 6 RECOMMENDATIONS

Time domain dynamic analysis needs vast range of research. For further study, these topics are recommended.

- 1. Dynamic Analysis using diffraction theory for semi-submersible
- 2. Six degree of freedom motion analysis in random sea
- 3. The effect of damping ratio on motions in semi-submersible
- 4. Frequency domain analysis using different types of spectrum
- 5. Comparison between RAOs for different kind of platforms
- 6. Optimization in design of semi-submersible
- 7. Stiffness calculation for six degree of freedom motions
- 8. Effect of mooring lines on heave and pitch stiffness

# CHAPTER 7 APPENDIX

Code of this program is written in visual basic compatible to Excel.

## 7.1 Data Collection

Function volume(pts8, pts4, org, rpt)

Dim res1(3), res2(3), xAngle(3), yAngle(3), pts12(2, 12, 3), res(3), vect0(3), vect1(3), vect2(3) As Double

xAngle(0) = 1

xAngle(1) = 0

xAngle(2) = 0

yAngle(0) = 0

yAngle(1) = 1

yAng!e(2) = 0

res1(1) = pts8(1, 1) - pts8(0, 1)

res1(2) = pts8(1, 2) - pts8(0, 2)

res2(0) = pts8(3, 0) - pts8(0, 0)

res2(1) = pts8(3, 1) - pts8(0, 1)

res2(2) = pts8(3, 2) - pts8(0, 2)

i = vecCross(res2, yAngle, res)

If (res(0) = 0) Then

res4 = 0

Else

i = res(0) / Abs(res(0))

res4 = vecAngle(res2, yAngle) \* i

End II

i = vecCross(res1, xAngle, res)

If (res(1) = 0) Then

res3.=0

Else

i = res(1) / Abs(res(1))

res3 = vecAngle(res1, xAngle) \* i

End If

For i = 0 To 7

j = rotatem(res3, pts8(i, 0), pts8(i, 1), pts8(i, 2), rpt(0), rpt(1), rpt(2), 2, res)

j = rotatem(res3, pts8(i, 0), pts8(i, 1), pts8(i, 2), pts8(0, 0), pts8(0, 1), pts8(0, 2), 2, res)

pts12(0, i, 0) = res(0)

pts12(0, i, i) = res(1)

pts12(0, i, 2) = res(2)

j = rotatem((res4), pts12(0, i, 0), pts12(0, i, 1), pts12(0, i, 2), rpt(0), rpt(1), rpt(2), 1, res)

pts12(1, i, 0) = Round(res(0), 10)

pts12(1, i, 1) = Round(res(1), 10)

pts12(1, i. 2) = Round(res(2), 10)

Next i

For i = 0 To 3

j = rotatem(res3, pts4(i, 0), pts4(i, 1), pts4(i, 2), 0, 0, 0, 2, res)

pts12(0, i + 8, 0) = res(0)

pts12(0, i + 8, 1) = res(1)

pts12(0, i + 8, 2) = res(2)

j = rotatem(res4. pts12(0, i + 8, 0), pts12(0, i + 8, 1), pts12(0, i + 8, 2), 0, 0, 0, 1, res)

pts12(1, i + 8, 0) = Round(res(0), 10)

pts12(1, i + 8, 1) = Round(res(1), 10)

```
pts12(1, i + 8, 2) = Round(res(2), 10)
```

Next i

l = 3

For i = 0 To 11

For j = 0 To 2

$$pts12(1, i, j) = pts12(1, i, j) - org(j)$$

$$Cells(15+i, 1+j) = pts12(0, i, j)$$

Cells
$$(15 + i, 1 + j + 5) = pts12(1, i, j)$$

'MsgBox ("X res" & i & ", " & j & "is = " & pts12(0, i, j) & vbCrLf & \_

- " And " & vbCrLf & " Y res" & i & ", " & j & "is = " & pts12(0, i, j))

Nextj

Next i

$$vect0(0) = pts12(1, 9, 0) - pts12(1, 8, 0)$$

$$vect0(1) = pts12(1, 9, 1) - pts12(1, 8, 1)$$

$$vect0(2) = pts12(1, 9, 2) - pts12(1, 8, 2)$$

vect1(0) = pts12(1, 11, 0) - pts12(1, 8, 0)

$$vect1(1) = pts12(1, 11, 1) - pts12(1, 8, 1)$$

veet1(2) = pts12(1, 11, 2) - pts12(1, 8, 2)

i = vecCross(vect0, vect1, vect2)

```
d = (\text{vect2}(0) * \text{pts12}(1, 8, 0)) + (\text{vect2}(1) * \text{pts12}(1, 8, 1)) + (\text{vect2}(2) * \text{pts12}(1, 8, 2))
a = \text{vecLength(res1)}
b = \text{vecLength(res2)}
\text{volume} = ((d * b * a) / \text{vect2}(2)) - ((\text{vect2}(0) * b * a ^ 2) / (2 * \text{vect2}(2))) - ((\text{vect2}(1) * b ^ 2 * a) / 2 * \text{vect2}(2))
End Function
```

## 7.2 Mathematical functions

Function volume(pts8, pts4, org, rpt)

Dim res1(3), res2(3), xAngle(3), yAngle(3), pts12(2, 12, 3), res(3), vect0(3), vect1(3), vect2(3) As Double

xAngle(0) = 1

xAngle(1) = 0

xAngle(2) = 0

yAngle(0) = 0

yAngle(1) = 1

yAngle(2) = 0

res1(0) = pts8(1, 0) - pts8(0, 0)

res1(1) = pts8(1, 1) - pts8(0, 1)

res1(2) = pts8(1, 2) - pts8(0, 2)

res2(0) = pts8(3, 0) - pts8(0, 0)

res2(1) = pts8(3, 1) - pts8(0, 1)

res2(2) = pts8(3, 2) - pts8(0, 2)

i = vecCross(res2. yAngle, res)

If (res(0) = 0) Then

res4 = 0

Else

i = res(0) / Abs(res(0))

res4 = vecAngle(res2, yAngle) \* i

End If

i = vecCross(res), xAngle, res)

If (res(1) = 0) Then

res3 = 0

Else

i = res(1) / Abs(res(1))

res3 = vecAngle(res1, xAngle) \* i

End If

For i = 0 To 7

j = rotatem(res3, pts8(i, 0), pts8(i, 1), pts8(i, 2), rpt(0), rpt(1), rpt(2), 2, res)

```
'j = rotatem(res3, pts8(i, 0), pts8(i, 1), pts8(i, 2), pts8(0, 0), pts8(0, 1), pts8(0, 2), 2, res)

pts 12(0, i, 0) = res(0)

pts12(0, i, 1) = res(1)
```

$$\begin{split} j &= \text{rotatem}((\text{res4}), \, \text{pts}12(0, \, i, \, 0), \, \text{pts}12(0, \, i, \, 1), \, \text{pts}12(0, \, i, \, 2), \, \text{rpt}(0), \, \text{rpt}(1), \, \text{rpt}(2), \, 1, \, \text{res}) \\ \text{pts}12(1, \, i, \, 0) &= \text{Round}(\text{res}(0), \, 10) \\ \text{pts}12(1, \, i, \, 1) &= \text{Round}(\text{res}(1), \, 10) \\ \text{pts}12(1, \, i, \, 2) &= \text{Round}(\text{res}(2), \, 10) \end{split}$$

Next i

pts12(0, i, 2) = res(2)

For 
$$i = 0$$
 To 3  
 $j = \text{rotatem(res3, pts4(i, 0), pts4(i, 1), pts4(i, 2), 0, 0, 0, 2, res)}$   
 $\text{pts12}(0, i + 8, 0) = \text{res}(0)$   
 $\text{pts12}(0, i + 8, 1) = \text{res}(1)$   
 $\text{pts12}(0, i + 8, 2) = \text{res}(2)$ 

j = rotatem(res4, pts12(0, i + 8, 0), pts12(0, i + 8, 1), pts12(0, i + 8, 2), 0, 0, 0, 1, res) pts12(1, i + 8, 0) = Round(res(0), 10) pts12(1, i + 8, 1) = Round(res(1), 10) pts12(1, i + 8, 2) = Round(res(2), 10)

```
Next i
```

1 = 3

```
For i = 0 To 11

For j = 0 To 2

pts12(1, i, j) = pts12(1, i, j) - org(j)

'Cells(15 + i, 1 + j) = pts12(0, i, j)

Cells(15 + i, 1 + j + 5) = pts12(1, i, j)

'MsgBox ("X res" & i & ", " & j & "is = " & pts12(0, i, j) & vbCrLf & _

" And " & vbCrLf & " Y res" & i & ", " & j & "is = " & pts12(0, i, j))

Next j

Next i

vect0(0) = pts12(1, 9, 0) - pts12(1, 8, 0)

vect0(1) = pts12(1, 9, 1) - pts12(1, 8, 1)

vect0(2) = pts12(1, 9, 2) - pts12(1, 8, 2)
```

i = vecCross(vect0, vect1, vect2)

vect1(0) = pts12(1, 11, 0) - pts12(1, 8, 0)

vect1(1) = pts12(1, 11, 1) - pts12(1, 8, 1)

vect1(2) = pts12(1, 11, 2) - pts12(1, 8, 2)

```
d = (\text{vect2}(0) * \text{pts12}(1, 8, 0)) + (\text{vect2}(1) * \text{pts12}(1, 8, 1)) + (\text{vect2}(2) * \text{pts12}(1, 8, 2))
a = \text{vecLength}(\text{res1})
b = \text{vecLength}(\text{res2})
\text{volume} = ((d * b * a) / \text{vect2}(2)) - ((\text{vect2}(0) * b * a ^ 2) / (2 * \text{vect2}(2))) - ((\text{vect2}(1) * b ^ 2 * a) / 2 * \text{vect2}(2))
End Function
```

### 7.3 Time Domain Programming

```
\label{eq:linear_hamiltonian} $$ \prod_{K\in \mathbb{N}}[IndentingNewLine]
 (n = 200; ) [IndentingNewLine]
 \(dep = 1500;\)\[IndentingNewLine]
 \(density = 1035\\)\[IndentingNewLine]
 \comega0 = ((0.161*9.807/Hs))^.5:() [IndentingNewLine]
 \(f0 = omega0/\((2*Pi)\);\)\{IndentingNewLine}
 \adjust{1} (4lpha = 5*((Hs/4)))^2*omega()^4/9.807^2;\)[IndentingNewLine]
  \(Array[omegas. n]:\)\[IndentingNewLine]
  \comegas = Table[2*Pi*\(( .02 + \((i - 1)\))*
   step)\), {i, n}]:\)\[IndentingNewLine]
  \sqrt{\text{freq}} = \text{Table[\(( .02 + \((i - 1)\))*step)\)}, {i, n}] \(\frac{1}{1}\)\[\]{\IndentingNewLine}}
  \(Array[s, n]:\)\([IndentingNewLine]
  \(Array[k, n]:\)\[IndentingNewLine}
   \(Array[waveheight, n]:\)\[IndentingNewLine]
   \(i = 1:\)\\\IndentingNewLine\
   \slash(s = Table[alpha*9.807^2\slash((2*)])
    Pi)\)^4*freq[\([i]\)]^\\(-5\)*
```

```
\[IndentingNewLine]
 (b = Table[FindRoot[omegas]([i]))^2 =
         9.807*x*Tanh[x*dep], \{x, .5\}], \{i, n\}];\\[IndentingNewLine]
 (k = Table[b[([i, 1, 2])], (i, n)];))[IndentingNewLine]
 delta = step\[IndentingNewLine]
 \(waveheight =
  Table[2*\((2*s[\([i]\))]*delta)\)^.5,\ \{i,\,n\}];\)\[lndentingNewLine]
 waveheight[\([1]\)] = 2*\((2*s[\([i]\)]*.02)\)^.5\[IndentingNewLine]
 \(randomn = Table | Random | Real. \{0, 2*Pi\} \, \{i, n\} \\\ Indenting New Line \]
 \(coef =
  Table[\((1\V4\\((BesselJ[0, k[i]*r] - BesselJ[2, k[i]*
  r)\\\^2 + 1\\\((BesselY[0, k[i]*r] - BesselY[2, k[
     i]*r])\)^2)\)^(((-.5\)))*2/((Pi*k[
  i]*r*BesselJ[1, k[i]*r])\), {i, n}];\)\[IndentingNewLine]
 \(numbercelements = 4;\)\[IndentingNewLine] \.
 \(numberpelements = 4:\)\[IndentingNewLine]
 \(draft = 25.09:\)\[IndentingNewLine]
 \(\array\elementprop.\numbercelements + numberpelements\\)\\\
\[IndentingNewLine]
 (elementprop[1] = \{15.24, 15.24,
          8.5, 0. \((draft - 16.674)\)/2 + dep -
      draft + 16.674}:\)\[IndentingNewLine]
  \forall elementprop[2] = \{
       15.24, 15.24, 8.5.
```

0, ((draft - 16.674))/2 + dep - draft + 16.674);)

```
[IndentingNewLine]
   \(elementprop[
           31 = \{15.24, 15.24, 8.5, 82.29, ((draft - 16.674))/2 + dep - draft + (draft - 16.674) \}
16.674;:\)\[IndentingNewLine]
   \ensuremath{\mbox{(elementprop[4] = \{15.24, 15.24, 8.5, 82.29, \ensuremath{\mbox{((draft - 16.674)\)/2 + dep - 16.674)}}\ensuremath{\mbox{(elementprop[4] = \{15.24, 15.24, 8.5, 82.29, \ensuremath{\mbox{(draft - 16.674)\)/2 + dep - 16.674}\ensuremath{\mbox{(draft 
      draft + 16.674;:\)\[IndentingNewLine]
    (elementprop[5] = \{16.674, 82.292, ...
                 16.674. 0, draft + 16.674/2}:\)\[IndentingNewLine]
     \(elementprop[6] = \(\{16.674, \text{82.292}, \text{16.674}, \text{82.292}\), \dep-
        draft + 16.674/2}:\)\[IndentingNewLine]
     \langle \text{elementprop}[7] = \{82.292, 16.674, 16.674, 0, \text{dep - draft} + 16.674/2\}; \rangle 
 \[IndentingNewLine]
     \(elementprop[
                     8] = {82.292, 16.674, 16.674, 0, dep - draft + 16.674/2};\)\
  \[IndentingNewLine]
     \(hacceleration = Table[{
             9.807*k[\([i]\)]*waveheight[\([i]\)]*Cosh[k[\([i]\)]*\(elementprop[1]\)[\]
  ([5])]/((2*Cosh[k[\([i]\)]*dep])\). 9.807*k[\([
                     i]\)j*waveheight[\([i]\)]*Cosh[k[\([i]\)]*\(elementprop[
                             2]\)[\([5]\)]]/\((
                         2*Cosh[k[\([i]\)]*dep])\). 9.807*
                                            k[\([i]\)]*waveheight[\([i]\)]*Cosh[
                                         k[\langle [i] \rangle] * \langle elementprop[
                                  3]\[([5]\])\]/((2*Cosh[k[\([i]\])]*dep])), 9.807*
```

 $k[\langle [i] \rangle]$ \*waveheight[\([

```
i] \rangle ] * Cosh[k[ \langle [i] \rangle ] * \langle (elementprop[4] \rangle [ \langle [5] \rangle ] ] \wedge ((2*Cosh[k[ \langle [i] \rangle ] * (2*Cosh[k[ \langle [i] \rangle ] * (2*Cosh[k[ \langle [i] \rangle ) ] * (2*Cosh
                                                                                 dep])\). 9.807*k[\([i]\)]*waveheight[\([i]\)]*Cosh[
                                               k[\([i]\)]*\(elementprop[5]\)[\([5]\)]]/\((2*Cosh[k[\([i]\)])))
                                                      i]\)]*dep])\),
                                                         9.807*k[\([i]\)]*
                                                                            wave height [\([i]\)] * Cosh[k[\([i]\)] * \(element prop[6]\)[\([i]\)] * (element prop[6]\)[\(
([5])]/((2*Cosh[k[([i])]*
                                                                                   dep]) \), 9.807*k[\([i]\)]*waveheight[\([i]\)]*Cosh[
                                                 k[\backslash([i]\backslash)]*\backslash(elementprop[7]\backslash)[\backslash([5]\backslash)]]/\backslash((2*Cosh[k]\backslash([-1])))
                                                         i]\1]*dep])\).
                                                             9.8()7*k[\([i]\)]*
                                                                                waveheight[\([i]\)]*Cosh[k[\([i]\)]*\(elementprop[8]\)[:
  \label{eq:condition} $$ \operatorname{Table}_{\(-9.807\)*k[\([i]\)]*waveheight[\([i]\)]*} $$
                                               Sinh[k[\backslash([i]\backslash)]*\backslash(elementprop[1]\backslash)[\backslash([5]\backslash)]]/\backslash((2*Cosh[k[\backslash([5]\backslash)])))
                                                                                               i]\)]*dep])\). \(-9.807\)*
                                                      \label{lem:lemma:lemma:k[([i]\)]*Sinh[k]([i]\)]*(elementprop[
                                                     2]\)[\([5]\)]]\((
                                                 2*Cosh[k]\([i]\)]*
                                                                                   dep])\). \(-9.807\)*k[\([i]\)]*waveheight[\([i]\)]*Sinh[
                                                                                   k[\([i]\)]*\(elementprop[3]\)[\([5]\)]]/\((2*Cosh[k[\([-1])])))))
                                                                                                  i]\)]*dep])\). \(-9.807\)*k[\([
                                           i]\)]*waveheight[\([i]\)]*Sinh[k[\([i]\)]*\(elementprop[4]\)[\([5]\).
         ]] \land ((2*Cosh[k[\([i]\)]*dep])\). \(-9.807\)*k[\([i]\)]*dep]))
                i] \rangle)] * waveheight[ \langle [i] \rangle ] * Sinh[k[ \langle [i] \rangle ] * \langle elementprop[
```

5]\\[\([5]\\)]]\\((2\*Cosh[k]\\[i]\\)]\*dep])\). \(-9.807\)\*k[\([i]\)]\*waveheight[\([i]\)]\*\(elementprop[6]\)[\([5]\)]]\\((2\*Cosh[k[\([i]\)]\*\)]\*waveheight[\([i]\)]\*\\(elementprop[7]\\)[\([5]\)]]\\((2\*Cosh[k[\([i]\)]\*\\(elementprop[7]\)[\([5]\)]]\\((2\*Cosh[k[\([i]\)]\*\\(elementprop[7]\)]\\([5]\)]]\\((2\*Cosh[k[\([i]\)]\*\\(elementprop[8]\)]\\([5]\)]]\\((2\*Cosh[k[\([i]\)]\*\\(elementprop[8]\)]\\([5]\)]]\\((2\*Cosh[k[\([i]\)]\*\\(elementprop[8]\)]\\([5]\)]]\\((2\*Cosh[k[\([i]\)]]\*\\(elementprop[8]\)]\\([5]\)]]\\((2\*Cosh[k[\([i]\)]])\)

 $k[\langle [i] \rangle] * dep] \rangle$  { i, n } ]; \\[ [Indenting New Line]

\(Array[fhorizontal, 8]:\)\[IndentingNewLine]

\(Array[fvertical, 8];\)\[IndentingNewLine]

 $(ch = 2:\)[IndentingNewLine]$ 

 $(cv = 6:)\[IndentingNewLine]$ 

 $\sp = dep - draft + 21.36:\)\[IndentingNewLine]\]$ 

\(fhorizontal[1] =

 $Table[ch*density*\\(elementprop[1])]\\([1]))]*\\(elementprop[1])]\\(($ 

[2]\)]\*\(elementprop[1]\)[\([

 $\label{eq:continuity} 3]\)]*Sinh[\(elementprop[1]\)[\([3]\)]*k[\$ 

 $i] \rangle ]/2] * Sin[ \langle (elementprop[1] \rangle ) [ \langle ([1] \rangle )] * k[ \langle ([i] \rangle )]/2] \wedge$ 

 $\label{eq:continuous_loss} $$ (((\elementprop[1]))[([1]))*k[([1])) $$$ 

i]\)]/2\*\(elementprop[1]\)[\([3]\)]\*k[\([i]\)]/

2)\)\*hacceleration[\([i, 1]\)],  $\{i, n\}$ ]:\)\[IndentingNewLine]

\(fhorizontal[2] = \

 $Table[ch*density*\\(elementprop[2]\\)]\\([1]\\)]*\\(elementprop[2]\\)]\\([1]\\)[*]$ 

 $2]\rangle)]*\langle elementprop[2]\rangle)]\langle [3]\rangle)]*Sinh[\langle elementprop[2]\rangle][\langle [3]\rangle)]*k[\langle [3]\rangle)]$ 

```
i] \rangle ]/2] * Sin[ \langle (elementprop[2] \rangle ] \langle ([1] \rangle )] * k[ \langle ([i] \rangle )]/2] \wedge (( \langle ([1] \rangle )) | ([1] \rangle )) | ([1] \rangle ) | ([1] \rangle
elementprop[2] \\ \\ [\\([1]\\)]*k[\\([i]\\)]/2*\\ \\ (elementprop[2]\\)[\\([3]\\)]*
                                                                                                                                              k[\langle([i]\rangle)]/2)\rangle*
                                                                                                                          hacceleration [\([i,2]\)], \{i,n\}]; \) \([Indenting Nev.Line])
                             \label{eq:change_problem} $$ \footnote{Morizontal} = Table[ch*density*\\elementprop[3]\\)[\cline{Morizontal}] $$
                                                                                                                                       3] \rangle ]* Sinh[\elementprop[3] \rangle [\elementprop[3] \rangle ]* k[\elementprop[3] \rangle ]
                                                                                                                                                                                                                        2]*Sin[\end{center} [3]\)[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]
                                                                                                                                                                                 i] \rangle) /2] / \langle ( \langle (elementprop[3] \rangle) [ \langle [1] \rangle) ] * k | \hat{\langle} \langle [i] \rangle) ] / 
                                                                                                                                                                                                                                  2*\(elementprop[3]\)[\([3]\)]*k[\([
                                                                                                                                                                        i]\)]/2)\)*hacceleration[\([i.
                                                                                                                                                           3]\)]. {i, n}]:\)\[IndentingNewLine]
                                                             \(fhorizontal[4] = \
                                                Table [ch*density*\\(element prop [4]\\)[\\([1]\\)]*\\(element prop [4]\\)[\\([1]\\)]
                                                                                                       2] \rangle)] * \\ (elementprop[4] \rangle)[ \\ ([3] \rangle)] * Sinh[ \\ (elementprop[4] \rangle)[ \\ ([3] \rangle)] * k[ \\ ([3] \rangle)] * ([3] \rangle)[ \\ ([3] \rangle)
                                                                                                                                                                                     i] \rangle) /2] * Sin[ \langle elementprop[4] \rangle [ \langle [1] \rangle ] * k[ \langle [i] \rangle ] /2] \wedge (( \langle (1) \rangle ) ) | ( | (1) \rangle ) 
                                                   elementprop[4]\)[\([
                                                                                                                                                                                                  \label{eq:linear_prop} $$1]^*k[\([i]\)]/2*(elementprop[4]\)[\([3]\)]*k[\([i]\)]/2))^*$
                                                                                                                                                                                                                                     hacceleration[ \c ([i,4]\)], \{i,n\}]; \c ([Indenting New Line]) \c ([i,4]\) \c ([i,n]) 
                                                                                \(fhorizontal[5] =
```

 $\label{thm:contal_5} Table[ch*density*\\(elementprop_5]\\)[\\([1]\\)]^*\\(elementprop_5]\\)[\\([1]\\)]$ 

 $\label{eq:continuous} \ensuremath{\text{[2]\)]}*\(elementprop[5]\)[\([\]$ 

3])]\*Sinh[\(elementprop[

 $5] \rangle [ \langle [3] \rangle ] * k [ \langle [i] \rangle ] / 2] * Sin [ \langle [elementprop[5] \rangle ] [ \langle [1] \rangle ] * k [ \langle [1]$ 

 $[i] \rangle ]/2] \wedge ((\langle elementprop[5] \rangle) [\langle [$ 

```
1] \rangle | *k[ \langle [i] \rangle ] / 2* \langle [elementprop[5] \rangle | \langle [3] \rangle ] *k[ \langle [i] \rangle ] / 2* \langle [elementprop[5] \rangle | \langle [3] \rangle | ([i] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([3] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle | ([4] \rangle ) | / 2* \langle [elementprop[5] \rangle
                                             2)\)*hacceleration[\([i, 5]\)]. \{i, n\}]:\)\[IndentingNewLine]
\(fhorizontal[6] = Table[ch*density*\(elementprop[
                                                              6]\)[\([1]\)]*\(e\text{lementprop}[
                                                        6]\)[\([2]\)]*\(elementprop[6]\)[\([3]\)]*
                                                                        Sinh[\ensuremath{\mbox{\sc Sinh}[\ensuremath{\mbox{\sc S
                                                          3]\)]*k[\([i]\)]/2]*Sin[\(elementprop[
                                                                           6]\)[\([1]\)]*k[([i]\)]/2]/(((elementprop[6]\)[\([
                                                             1]\)]*k[\([i]\)]/
                                                                                    2*\(elementprop[6]\)[\([3]\)]*k[\([i]\)]/2)\)*
                                                                     hacceleration[\([i, 6]\\)], {i, n}];\\\[IndentingNewLine]
       \(fhorizontal[7] =
                                                Table[ch*density*\\(elementprop[2]\\)[\\([1]\\)]*\\(elementprop[2]\\)[\\((
 [2] \rangle)] * \\ (element prop [2] \rangle) [ \\ \langle [
                  3]\)]*Sinh[\(elementprop[
                                   7]\)[\([3]\)]*k[\([i]\)]/2]*
                                            Sin[\(elementprop[
                                                             7]\)[\([1]\)]*k[\([i]\)]/2]/\((\(elementprop[7]\)[\([
                                                              \label{eq:localization} $$[([i])]^2*(elementprop[7])[([3])]^*k[([i])]/$
                                                                     2)\)*hacceleration[\([i, 7]\)], \{i, n\}]:\)\[IndentingNewLine]
               \(fhorizontal[8] = Table[ch*density*\(e!ementprop[
                                              2]\)[\([1]\)]*\(elementprop[
                                                      2])[\[([2]\])]^{([2]\])}]^{[([3]\])]}^{Sinh[\(elementprop[8]\])[\]}
```

 $\label{eq:continuous} $$ ([3])^*k[\langle [i]\rangle)/2]*Sin[\langle clementprop[8]\rangle] ([1]\rangle)^*k[\langle [1]\rangle). $$$ 

 $i] \rangle) ]/2] / \langle (\langle (elementprop[8] \cdot) [ \langle ([1] \rangle) ]^*$ 

```
k[\[i]\]/2*\[elementprop[8]\]([
                                               3]\) *k[\([i]\)]/2)\) *hacceleration[\([i, 8]\)]. {i.}
                                                      n]]:\)\[IndentingNewLine]
     \(fvertical[\)] = Table|ev*density*\(elementprop[
                                         \label{eq:local_local_local_local} $$1_{\[\[1]\)}^{([1]\)}^{([2]\)}^{([2]\)}^{([3]\)}$
]*Sinh[\(elementprop[1]\)[\([3]\)]*
                                                          k[\([i]\)]/2]*Sin[\([elementprop[1]\)[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1]\)]*k[\([1
                                                                                   i]\)]/2]\((\elementprop[1]\)[\([
                                                           \label{eq:linear_linear_linear_linear} $$1]\)]*k[\([i]\)]/2*\(elementprop[1]\)[\([3]\)]*
                                                                                   k[\langle [i] \rangle]/2 \rangle *vacceleration[\([i.
                                                                              1]\)], {i. n}]:\)\[IndentingNewLine]
         \(fvertica![2] = Table[cv*density*\(elementprop[
                                                              2]\)[\([1]\)]*\(elementprop[2]\)[\([
                                                              2] \rangle ] * \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[2] \rangle [ \langle elementp
                                                                              3]\)]*k[\([i]\)]/
                                                                              2]*Sin[\ensuremath{\color=0}]\ensuremath{\color=0}]\ensuremath{\color=0}]\ensuremath{\color=0}]\ensuremath{\color=0}]\ensuremath{\color=0}]
                                                                                       2] \land ((\element prop[2]\)[\([1]\)]*
                                                                                k[\([i]\)]/2*\(elementprop[2]\)[\([3]\)]*k[\([3]\)]
                                                        i]\)]/2)\)*vacceleration[\([i, 2]\)]. {i.
                                                                                        n}].\)\[IndentingNewLine]
                \(fvertical[3] =
                                                                            Table[ev*density*\setminus(elementprop[3]\setminus)]^*([1]\setminus)]^*(elementprop[3]\setminus)]^*([1]\setminus)]^*
                                                                                            3]\)[\([2]\)]*\(elementprop[
                                                   3]\[([3]\)]*Sinh[\elementprop[3]\)[\[[3]\)]*
```

 $k[\([i]\)]/2]*Sin[\(elementprop[3]\)[\([1]\)]*k[\([i]\)]/2] \land ((\([i]\))]/2] \land ((\([i]\)))/2] \land ((\([i]\)$ 

```
\(elementprop[
        3 \le [([1])]*k[([i])]/2*(elementprop[3])[([
                     3] \) * k[\([i]\)]/2) \) * vacceleration[\([i, 3]\)], \{i.
                         n;];\)\[IndentingNewLine]
    \(fvertical[4] = Table[cv*density*\(elementprop[
                              4]\\)[\\([1]\\)]*\\(elementprop[4]\\)[\\([2]\\)]*\\(elementprop[4]\\)[\\([2]\\)]*
                          3]\)]*Sinh{\(elementprop[4]\)[\([3]\)]*k[\([i]\)]/
                                       2]*Sin[\elementprop[4]\)[\([1]\)]*k[\([i]\)]/
                                           2] \land (((elementprop[4]))[([1]))]*
                                        k[\([i]\)]/2*\(elementprop[4]\)[\([3]\)]*k[\([3]\)]
                          i]\)]/2)\)*vacceleration[\([i, 4]\)], {i,
                                           n}]:\)\[IndentingNewLine]
      \label{eq:constraint} $$ \operatorname{Table}(cv*density*(elementprop[5]))[([1]))*(elementprop[5])([1])) $$
                                            5]\)[\([
                                2] \rangle ] * \langle elementprop[5] \rangle [ \langle [3] \rangle ] * Sinh[ \langle elementprop[5] \rangle ] 
                             5]\)[\([3]\)]*k[\([i]\)]/2]*
                             Sin[\ensuremath{\langle elementprop[5] \rangle][\ensuremath{\langle [1] \rangle]}*k[\ensuremath{\langle [1] \rangle]}
                                     i] \rangle)]/2] \rangle \langle (\langle (elementprop[5] \rangle)[ \langle [1] \rangle)] *
                                     k[([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])]/2*([i])/2*([i])]/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([i])/2*([
                                         2)\)*vacceleration[\([.
                                             i, 5]\)], {i, n}};\)\[IndentingNewLine]
        \(fvertical[6] = Table[cv*]
                                           density*\(elementprop[
                                              6] \\ [\[([1]\])]^* \\ (elementprop[6]\] \\ [\[2]\])]^* \\ (elementprop[
```

6]\)[\([3]\)]\*Sinh[\(elementprop[6]\)[\([3]\)]\*k[\([

i]\)]/2]\*Sin[\(elementprop[6]\)[\([1]\)]\*

 $\label{eq:kernel} $$ k[\[i]\]/2]/((\elementprop[6]\)[\[1]\]*k[\[i]\]/2*(\elementprop[6]\)[\[i]\]/2*(\elementprop[6]\])$ 

n } ]:\)\[IndentingNewLine]

 $\label{eq:continuous} $$ \ | Table[cv*density*\\(elementprop[7])[\([1]\)]*\\(elementprop[7]\)[\([1]\)])[\([1]\)] $$$ 

 $\label{eq:continuous} \raise $$ 7]\[([2]\])^*\[([3]\])^*Sinh[\(elementprop[3]\])^* $$$ 

7]\)[\([3]\)]\*k[\([i]\)]/2]\*

 $Sin[\end{cases} $ Sin[\end{cases} $ Sin[\end{cases} $ ([1]\end{cases} ) $ $ in[\end{cases} $ ([1]\end{cases} ) $ in[\end{cases} ] $ in[\end{cases} ] $ in[\end{cases} ] $ in[\end{cases} ] $ in[\en$ 

 $\label{eq:kappa} $$ k[\[i]\]/2*\(elementprop[7]\)[\[([3]\)]*k[\[i]\]/$ 

2)\)\*vacceleration[\([

i, 7]\)], {i, n}];\)\[IndentingNewLine]

 $\label{eq:continuous} $$ \left( \operatorname{fivertical}[8] = \operatorname{Table}[\operatorname{cv*density*}(\operatorname{elementprop}[8]))[\left([1]\right)]^* \right) $$$ 

 $8] \\ [\[([2]\])] *\\ (elementprop[8]\] \\ [\[([3]\])] *Siah[\(elementprop[8]\])] \\$ 

8]\)[\([3]\)]\*

 $k[\backslash([i]\backslash)]/2]*Sin[\backslash(elementprop[8]\backslash)[\backslash([1]\backslash)]*k[\backslash([i]\backslash)]/$ 

 $2] \land ((\elementprop[8])) [\elementprop[1] \end{center} \begin{center} \begin{center} (2) \end{center} \begin{center} \begin{$ 

8]\)[\([

3]\)]\* $k[\langle [i] \rangle \rangle$ \*vacceleration[\([i, 8]\)], {i, n}]:\)\

\[IndentingNewLine]

\(Array[fmoment. 8]:\)\[IndentingNewLine]

\(\(fmoment[1]\)\(=\)\)\)

#### **CHAPTER 8**

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