Responses of Square Tension Leg Platform Subjected To Regular Wave

by

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Dissertation submitted in partial fulfilment of
the requirements for the
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CERTIFICATION OF APPROVAL

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Civil Engineering Programme
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Approved by,

(Assoc.Prof. Dr Kurian V.John)

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TRONOH, PERAK

January 2008

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

HANNIS SALWANI BT MOHD SAZALI

ABSTRACT

This report provides the analysis of square tension leg platform (TLP) subjected to regular wave. Recent depletion of near shore oil resources is quite a big issue and therefore, this project studies on other alternatives to extract oil in deep water for water depth greater than 300m. The assumption made for this project is that the tension leg platform is subjected to a regular wave. Dynamic analysis conducted in the 'Frequency Domain Analysis'. The project objectives are to determine the forces reacted on square tension leg platform, to see the responses of square tension leg platform in the direction of surge, heave and pitch, to calculate the tension forces produced on each tether and to prove that tension leg platform is worth in deepwater exploration. Wave kinematics value is found by using Airy Wave theory while the acted forces by using Morrison equation. Overall, the responses of the TLP have been found within allowable limits, thereby confirming the suitability of TLP for deepwater application.

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My final year project would not have been a success if not for the contributions of the many people I worked with throughout the past 2 semesters. Therefore, I would like to take this opportunity to acknowledge and recognize the contributions of these individuals.

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CHAPTER 1

INTRODUCTION

1.1: Background of Study

Interest in square Tension Leg Platform (TLP)s dates back to 1960, and many studies have examined the applicability of this concept for deep water developments. Among others, Fluor Corporation, Deep Oil Technology, Aker, Mitsubishi Heavy Industries, ESSO, BP, Conoco, Saga, Amoco, IFP and Chevron are only a few companies cited here in as the pursuant of this concept with notable and publicized studies. During the seventies and eighties, especially since the Hutton TLP installation in 1984, the concept of a TLP began attracting more attention from the offshore industry as an appropriate structure for deepwater applications.

In the last two (2) decades, the economics of offshore petroleum production have changed. Reserves in deepwater began to offer significant financial incentives to justify their development. The TLP is one of the viable engineering solutions for meeting this demand. Perhaps the primary consideration in selecting this concept for the deepwater application is relative insensitivity of the TLP cost to increase in water depth. Its hull, which extends only to a limited depth, consumes the largest amount of steel. However, this is yet a fraction of the steel needed for a fixed jacket structure at the same location in great water depths. The saving in the weight of the steel combined with its excellent station-keeping characteristics make the TLP concept one of the most cost effective and practical production systems for deep water developments.

The earliest published work on TLP performance and features is by Pauling and Horton (1970). A (1/3)rd scale version of a TLP was first designed, installed and tested in sea through a joint industry project by Deep Oil Technology (DOT) in 1970s. Mercier (1982) also gives an account of some of the notable work on designs and investigations of TLP by various oil companies, drilling contractors, constructors and consulting firms. However, only in 1984 the first working TLP was successfully deployed by Conoco at the Hutton field in North Sea, United Kingdom.

Tension leg platform (TLP) is a multicolumn structure moored to the seabed by vertical tethers. It is restrained from moving vertically and rigid risers may be used. TLP is very weight sensitive. TLP is vertically moored floating structure for offshore production of oil and gas. With respect to horizontal degree of freedom (DOF), it is compliant, behaves like to a floating structure. With respect to vertical (DOF), it is stiff and resembles a fixed structure and it is not allowed to float freely.

The square tension leg platform (TLP) is a type of structural system for exploitation of oil and gas fields below sea floor and must be designed to avoid fatigue damage due to cyclic action of sea waves. The general design approach for the square TLP is not particularly different from any other compliant offshore structures. The analytical technique usually depends on the particular platform configuration.

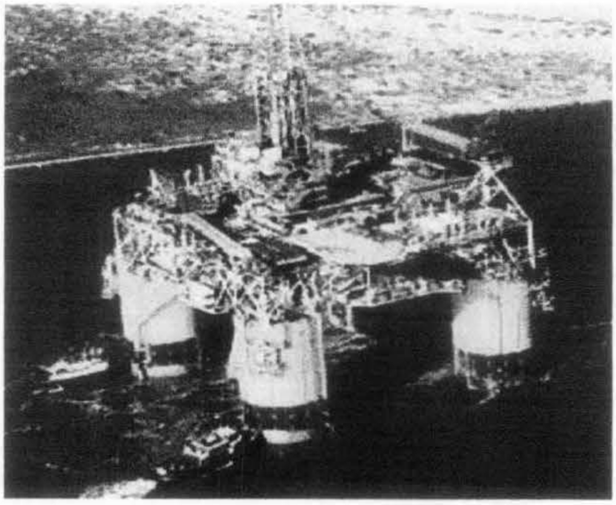

Figure 1.1: Picture of MARS Square Tension Leg Platform (sources: www.nd.edu)

What makes the dynamics of TLP unique from other floating structures is its response to the wave exciting forces. Besides the responses at the wave frequency, the platform is subjected to a high frequency tension oscillation of vertical tethers (often called springing) and a low frequency drift oscillation in surge. The overall damping of the system (including mechanical and hydrodynamic) is extremely small for both the springing and drift oscillation so that they produce significant load in tendons and significant motion in surge, respectively.

Numerous analysis and model tests have been performed on TLP which considered different aspects of platform motion and tether dynamics. It is assumed that all applied forces act at the joints of the structure. The forces acting on a member at some point other than an end must be replaced by a statically equivalent set of end forces. The forces to be used are negative to the reactions at the beam and that the applied force would cause if the beam were fixed at both ends. Each joint force is a function of time, and each has six components, including moments as components of the generalized force.

The calculation of wave forces and the determination of fluid-structure interaction may be handled either by the deterministic or the stochastic approach. The deterministic approach uses regular waves; the stochastic approach uses the effects of random waves. The Stokes wave theory is usually used to describe waves in deepwater, although it does not always provide the best fit to experimental wave data. It is used because the waves propagate without shape deformation and are periodic in space and in time. In the Stokes theory the wave amplitude of each term in the wave profile expression is not linearly related to the wave height as it true of the simpler one-term Airy theory. For this reason, the Airy theory is also widely used for deepwater wave calculations.

The wave spectrum, also called the wave spectral density function or the wave energy spectrum is used in stochastic analysis to compute the structural response. The wave spectrum most often used is the Pierson-Moskowitz spectrum.

TLP is designed to serve a number of offshore functions associated with the oil and gas production. It is considered particularly suitable for deep water applications where fixed platform costs become excessive. The displacement of the hull and the axial stiffness of the vertical tendons are chosen such that the vertical and angular natural periods are short (well below the wave periods). Some of the main advantages include minimum heave motion which consequently reduces the complexity of the well system.

Wave forces on offshore structures are calculated in three different ways:

- Morison equation
- Froude-Krylov theory
- Diffraction theory

The Morison equation assumes the force to be composed of inertia and drag forces linearly added together. The components involve an inertia (or mass) coefficient and a drag coefficient which must be determined experimentally. The Morison equation is applicable when the drag force is significant. This is usually the case when a structure is small compared to the water wave length. Structure is small when the diameter is small compared to wave length (ratio of structure diameter over wave length < 0.2).

When the drag force is small and inertia force predominates, but the structure is still relatively small the Froude – Krylov theory can be applied. It utilizes the incident wave pressure and the pressure-area method on the surface of the structure to compute the force. The advantage of this method is that for certain symmetric objects the force may be obtained in a closed form and the force coefficients are, generally, easy to determine.

When the size of the structure is comparable to the wave length, the presence of the structure is expected to alter the wave field in the vicinity of the structure. In this case the diffraction of the waves from the surface of the structure should be taken into account in the evaluation of the wave forces. It is generally known as diffraction theory.

A challenge for TLP is to keep the natural periods in heave and pitch below the range of significant wave energy. Heave period may be controlled by increasing the area of tethers to increase stiffness. Pitch period may be reduced by placing the tendons on a wide spacing to increase stiffness. However, it makes the support of the deck with large spans expensive.

TLP technology preserves many of the operational advantages of a fixed platform while reducing the cost of production in water depths up to about 1500m. Its production and maintenance operations are similar to those of fixed platforms. However, TLP are weight sensitive and may have limitations on accommodating heavy payloads.

Overall, there are many similarities between a TLP and a semisubmersible except that the mooring and the foundation systems for a TLP are unique to this concept. The structure is compliant with tendons present at each corner connecting the hull and the foundation. These tendons allow the platform to move in a horizontal plane (surge, sway and yaw) but restrict its motion in a vertical plane (heave, pitch, and roll). Buoyancy for the structure is provided by the vertical columns and horizontal pontoons making up its hull. The excess buoyancy over the platform weight ensures that the tendons are always kept in tension for all weather and loading conditions. Adequate air gap is maintained between the mean water line and the deck for all tide, wave, and motion conditions.

The deck of a TLP supports the functional requirements. It provides space for accommodation, working area, processing equipment, derrick, cranes, pumps, helideck and control room. Although the deck itself is similar to that of any conventional platform, its layout and hook-ups are quite different. It should be noted that the TLP, like a semisubmersible, is sensitive to payload increases, directly influence or be influenced by the displacement and leg spacing of TLPs are the platform response characteristics, towout stability, and barge size carrying deck for mating.

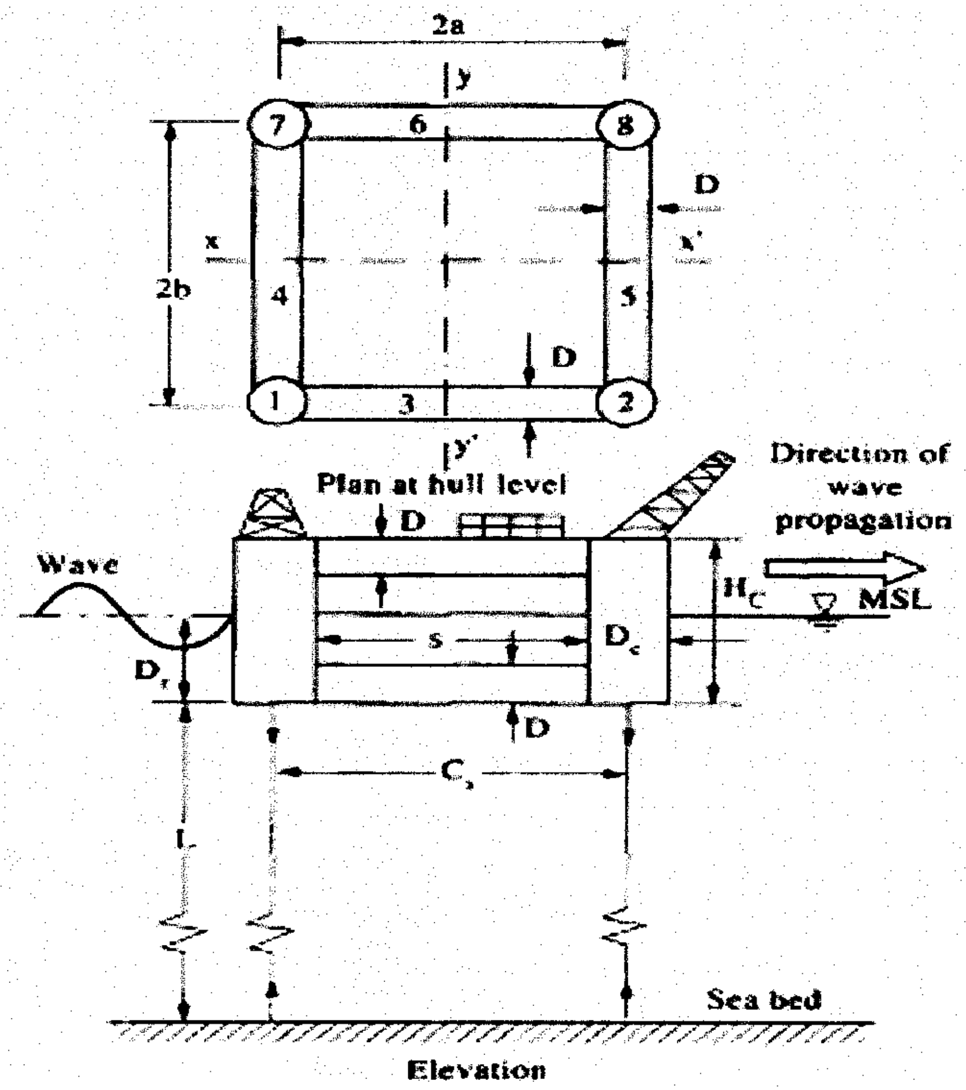

Figure 1.2: Plan and elevation of the proposed TLP model.[*Jain, 1995*]

The hull consists of the vertical columns, horizontal pontoons, and the bracings all of which can be circular, rectangular, or square in cross section. Recent and improved designs consist of larger diameter cylindrical cylinder shells for the columns and pontoons which have stiffener rings circumferentially and longitudinal stringers for a better control of the structural stability and damage resistance. Bilge and ballast systems are fitted into the space within the hull in addition to the drilling and potable water, diesel fuel, miscellaneous gear, pumps, machinery, fittings and equipment for storing, installing and monitoring the tendons.

The basic mooring system for a TLP includes tendons and connectors. Besides using solid or hollow pipes or wire ropes, one could also use high-strength materials made of Kevlar, cables, composites for tendons. Special attention must be paid to the collapse and buckling problems associated with thin-walled tubular tendons, especially in deep waters. Risers and their relevant structural components as vertical tension member can contribute to the station-keeping capability of the mooring system. These very long flexible members are complex structural entities themselves. Both tendon and riser analyses/designs make the proper design of the platform more difficult.

Construction of the platform onshore, the mooring system and the platform installation are key to the flexibility of a TLP. When the depth and location are changed, the designers need to alter these drastically. Consequently, with no great conceptual difficulty one can extend a given TLP design for an intermediate water depth to deeper water depths. The designer can achieve this objective by optimizing the weight and volume of the platform. It is important that every ton of weight saving on the topside of a TLP yields substantial reductions:

- In the fabrication costs of the hull steel
- In the required tendon pretension
- In the cost of the mooring and the foundation systems.

Excess weight and volume have the greatest impact on the cost of TLPs. As the TLPs move into the deeper waters, these two factors can impose restrictions on the mooring and foundation design.

A common feature linking all TLP designs at different water depths is the mooring system. However, the mooring system for TLPs differs significantly from most marine anchoring systems used for other floating vessels. It is permanent in the sense that it will hold the platform on station. But, the vertically oriented taut mooring system is quite rigid axially while being relatively flexible transversely. This in turn, provides a compliancy in the lateral movements (surge, sway, and yaw) while considerably suppressing the vertical motions (heave, pitch and roll) of the structure.

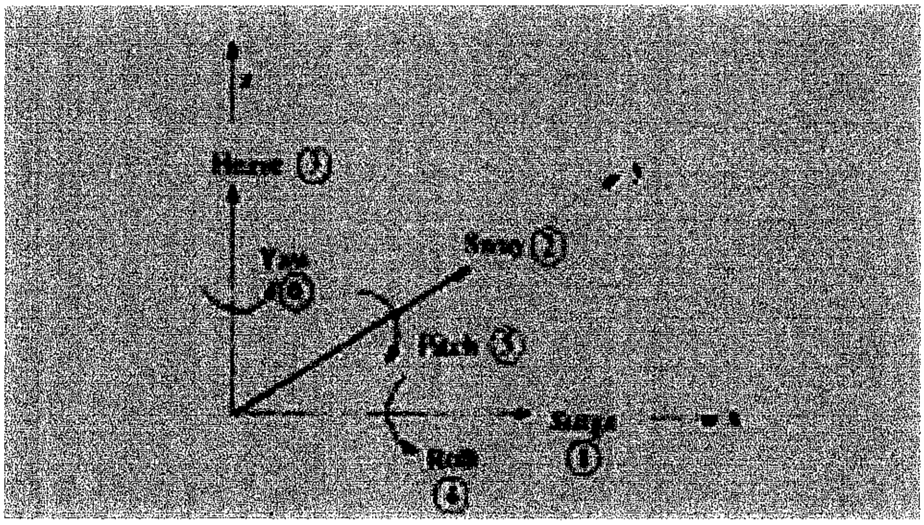

Figure 1.3: Coordination system and degree of freedom. [*Jain,1995*].

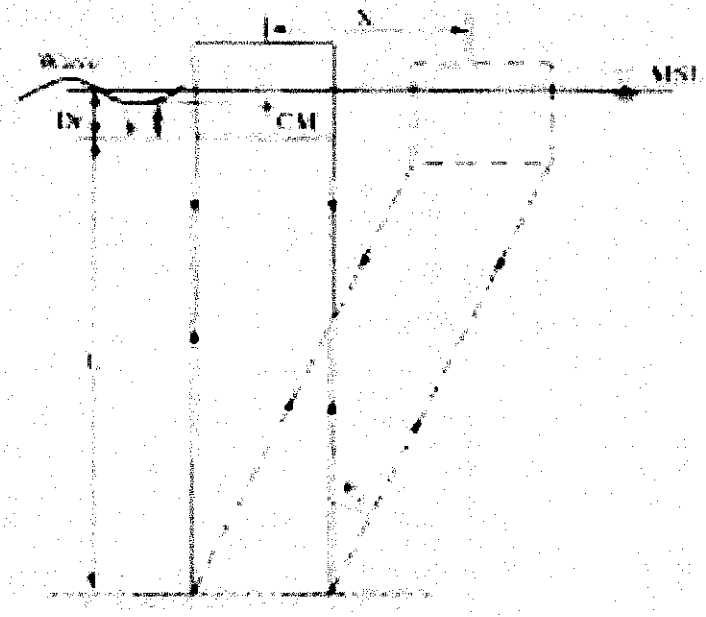

Figure 1.4: TLP with surge displacement [*Jain,1995*].

The natural periods in the horizontal modes of motion are controlled by the pretension in the mooring system and the water depth. Consequently, the designer must select the pretension and the stiffness of the mooring mechanism such that the natural periods for the lateral motions are far beyond the dominant wave excitation frequencies, whereas the periods for the motions in the vertical plane remain below the fatigue causing periods.

This fine tuning is intended to move the natural periods of the unit away from the energy intensive wave spectra to avoid amplification of the motions.

As water depth increases, the longitudinal stiffness demands may no longer be realized since the frequencies of the vertical motions begin to enter the wave frequency range. Other mooring systems such as stiffened steel tubular or composite materials can be used as an alternative solution. However, the latter brings with it a number of complexities such as the buckling, collapse, and the fatigue problems. These and other issues concerning the installation of TLP in great depths make the mooring system as one of the most problematic components of the TLP design in deepwater.

The foundation system serves as the anchorage for the tendons and therefore, keeps the platform in place. The foundation fixtures are secured to the sea bed by either tension piles or gravity base structures. Installation of the TLP foundations is an especially challenging operation for deepwater development.

The placement of a TLP in remote and hostile waters is further complicated by the fact that the storage of the reserves recovered may become an important issue. Temporary storage and tanker export facilities must be provided at times. These considerations are expected to introduce some variations in the existing TLP design concept in the near future.

1.2: Problem Statement

Recently, the depletion of near shore oil resources is quite a big issue and therefore, this project studies other technology to extract oil in deep water for water depth greater than 300m. Since fixed steel platform is expensive, this project aims to prove that the use of tension leg platform is reliable in deepwater and of low cost compared to normal fixed platform.

1.3: Objectives and Scope of Study

- To determine the wave forces acting on a square tension leg platform.
- To study the responses of square tension leg platform in the direction of surge, heave and pitch.
- To determine the variation of tether tensions in the tension leg platform (TLP).

CHAPTER 2

LITERATURE REVIEW

Tabeshpour (2006) reported the suitability of tension leg platform in deepwater exploration. Nonlinear dynamic analysis is done to determine the maximum deformations and stress of the TLP. For optimum design and control of the structure, the accurate and reliable response is needed. The analysis is done both in time and frequency domain. Pierson-Moskowitz spectrum is used based on generated random waves acted in the arbitrary direction on the structure. The hydrodynamic force is calculated using Morrison equation while the wave kinematics is calculated using Airy wave theory. In fact, Tabeshpour has calculated 'power spectral densities' (PSD), velocities and acceleration from nonlinear responses.

In contrast, Paulling and Horton (1970) used linear hydrodynamic synthesis technique to predict the platform motions and tether forces due to regular waves. Each TLP member is assumed to be cylindrical in shape with cross-sectional dimensions small in comparison to both length of the cylinder and the wavelength. Both hydrodynamic interactions between adjacent or intersecting members and free surface effect are neglected. The drag term was linearized. Indeed, the synthesis technique agreed well with experimental model results. The motions and tensions due to regular waves were shown to vary in a linear fashion with wave amplitude.

Angelides (1982) however considered the influence of hull geometry, water depth, force coefficients, pre-tension and tether stiffness on the dynamic responses of the TLP. The floating part of the TLP was modeled with six degrees of freedom as a rigid body while the tethers were signified by linear axial springs. Wave forces were evaluated using modified Morrison equation.

Faltinsen (1982) developed theoretical model for the behavior of TLP using model test programmed. The model outlines are: (i) the velocity potential solution for first- and second-order hydrodynamics, except for the slender members which were modeled with Morison's equation; (ii) Morison's theory and Newman's approximation to calculate drift forces' (iii) the large deflection three-dimensional finite element theory with forces from Morison's equation which was used for the tethers, (iv) the short-crestedness of waves, and (v) the wind and current.

Lyons (1983) compared the results of hydrodynamic analyses between two sets of large-scale model test results for wave-induced motion responses of TLPs. The results of analyses and tests showed good agreement for surge motions although discrepancies were observed for the tether tension responses at certain wave frequencies. Linear wave theory was used and hydrodynamic interference between members was neglected. The nonlinear damping was linearized by assuming an effective linear damping, which would dissipate the same amount of energy at resonance as the nonlinear damping.

Teigen (1983) presented the response of a TLP in both long-crested and short-crested waves through model tests. It was concluded that the low-frequency part of the horizontal response looked enlarged in tests carried out in long-crested seas, compared to tests carried out in short-crested seas, irrespective of the actual shape of the directional distribution.

Morgan and Malaeb (1983) investigated the dynamic response of TLPs using a deterministic analysis. The analysis was based on coupled nonlinear stiffness coefficients and closed-form inertia and drag-forcing functions using the Morison equation. The time histories of motions were presented for regular wave excitations. The nonlinear effects considered in the analysis were stiffness nonlinearity arising from coupling of various degrees of freedom, large structural displacements and hydrodynamic drag force nonlinearity arising from the square of the velocity terms. It was reported that stiffness coupling could significantly affect the behaviour of the structure and the strongest coupling found to exist between heave and surge or sway.

Spanos and Agarwal (1984) used a single degree-of-freedom model of a TLP and calculated wave forces at the structure's displaced position using the Morison equation. It was shown that by numerically integrating the equation of motion, the calculation of wave forces, on the displaced position of the structure, introduces a steady offset component in the structural response for either deterministically or stochastically described wave fields. The formulation did not involve any velocity-squared type of terms, and yet an offset component was found to be present.

Mekha et al. (1994) studied the nonlinear effect of evaluating the wave forces on a TLP up to the wave-free surface. Several approximate methods were evaluated for regular and irregular wave forces, with and without current, and compared to Stokes' second-order wave theory. The tethers were treated as massless springs providing axial and lateral stiffness at their connection with the hull. The following approximate methods were used to evaluate the wave kinematics from the mean water level to the wave free surface; hyperbolic extrapolation, linear extrapolation, stretching methods and uniform extrapolation. For a TLP subject to regular waves, the surge amplitude turns out not to be affected by the method chosen. However, the surge mean drift was very sensitive to the method used. Heave amplitude and mean offset were both affected by the method selected but were not significantly different from calculating the response to the mean water level only. The pitch response at its natural frequency was amplified at the free water surface, particularly for irregular waves, and was affected by the method selected.

Lee (1994) presented the analytical solution of the coupling problem of a 2D tension leg structure interacting with a monochromatic linear wave train. Fluid-induced drags, including form drag and inertia drag, on linearly elastic tension legs had been considered in the study. The nonlinear form drag was then replaced by a linear drag according to Lorentz's hypothesis of equivalent work. Analytical solutions showed that the inertia drag on tension legs was negligible compared to that due to the evanescent waves caused by the wave-structure interaction. However, the form drag on the legs altered the structural motion and, consequently, the wave field, especially when wave periods were close to the structure's resonant frequency.

Hahn (1994) reported the effects of wave stretching on realistic representations of the wave forces that act on offshore structures. The structures considered were modelled as linear, cantilever, stick-like systems. The lateral responses of such systems to wave forces, computed from water particle kinematics calculated by using the standard and stretching approaches, were examined. The results showed that the effects of stretching on the governing wave forces and the resulting structural responses were small, indicating that they could be ignored in design practice. It was also shown that the action of stretching could not materially influence the governing excitation and the corresponding structural response.

Duggal and Niedzwecki (1995) presented results from a large-scale experimental study of the interaction of regular and random waves with a long, flexible cylinder, exhibiting the dynamic characteristics of a TLP riser or tether in approximately 1000 m of water depth. Regular wave conditions were chosen to provide a large range of Keulegan-Carpenter numbers. Classification of the transverse response in regular waves showed similarities with results obtained by previous investigators with oscillating flow on rigid cylinders. For high Keulegan-Carpenter numbers, the response became more irregular, with response at harmonics of the incident wave frequency and at several natural frequencies of the cylinder. The greatest potential for reducing costs of a TLP in the short term is to go thoroughly through previously applied design approaches, to simplify the design and reduce the conservatism that so far have been incorporated in the TLP design to accommodate for the unproven nature of this type of platform.

According to Natvig and Vogel (1995), focus on design of future TLPs should be on the aspects of the platform geometry that affects tether loading and on the tether system itself. Their experience with a four-legged TLP has shown that the indeterminate tether system implies some very heavy cost items. The new concept of a three-legged TLP, which will be statically determinate, will not require complicated devices and the foundations can be placed with larger tolerances without affecting tether behaviour. The main aspect of three-legged TLP is that all tethers share approximately the same loads despite weather directions. With the near-equal load sharing of the three-legged TLP, the maximum load level in one group is less, thus requiring less tether cross section material than that of a four-legged TLP. Studies indicate that 12 tethers are feasible for a three-legged TLP whilst 16 would be required for a four-legged equivalent TLP. This is thus an important area for savings since tethers are important cost items.

Munkejord (1996) presented a conceptual analysis of the triangular TLP behaviour and then compared the results with data from model tests. The objective was to verify maximum tether tension, maximum platform offset, minimum air gap and tether fatigue. Aker and Saga Petroleum developed the concept of a triangular TLP, which has enabled significant savings in main steel for both hull and deck due to fewer main element intersections and effective force distributions. Munkejord (1996) summarized the design features for the triangular TLP of Aker as a statically determinate system with effective distribution of dynamic loads and fixed-length tethers. No design cases where TLP sustained a maximum storm with one tether missing were reported. No tether tension measurements required day-to-day operation and increased tolerances for the position of the foundation and increased draught and heel tolerances. No numerical study was reported on the triangular TLP. In view of the non-availability of any numerical study on the response behaviour of the triangular TLP, the present study deals with the investigation of the dynamic response of offshore TLPs under regular sea waves in the presence of current. Diffraction effects and second-order wave forces have been neglected and the evaluation of hydrodynamic forces is carried out using the modified Morison's equation with water particle kinematics using Airy's linear wave theory. The scope of the work is set to compare the structural response of a triangular-shaped TLP under regular waves in various structural degrees of freedom with that of a four-legged TLP to evaluate the viability of the former.

CHAPTER 3

METHODOLOGY

This chapter describes the methodology being done throughout two semesters. The project started with research of tension leg platform and proceeded with the calculation of frequencies domain analysis. Below are the details about the methodology carried out for the responses of square tension leg platform subjected to regular wave.

3.1 Research of TLP

All information gathered from offshore books, internet and also journals. Deep research is done to know the latest technology of TLP and the responses of TLP subjected to regular waves.

3.2 Simple dynamic rigid body analysis in frequency domain

Frequency domain analysis is performed to simplify the calculation. The simplest and most useful of all wave theories is the small amplitude wave theory. This wave theory is also known as Airy theory or sinusoidal wave theory. It is based on the assumption that the wave height is small compared to the wave length or water depth. This assumption allows the free surface boundary conditions to be linearized by dropping wave height terms which are beyond the first order. This assumption also allows the free surface conditions to be satisfied at the mean water level, rather than at the oscillating free surface.

When linearizing the drag force term in Morison's equation, the equations of motion in matrix form can be expressed as

$$\mathbf{M_v \check{\ddot{u}} + C\mathring{u} + Ku = P(v, \ddot{v})}$$

Where;

M_v = diagonal matrix of virtual mass

C = matrix for structural and viscous damping

K = square linear structural stiffness matrix

P (v, $\dot{\upsilon}$) = the load vector where (v) and ($\dot{\upsilon}$) are the water velocity and water acceleration.

$\check{\ddot{u}}$ = structural acceleration

$\mathring{u}$ = velocity

u= displacement

Pierson-Moskowitz spectrum is a formula for an energy spectrum distribution of a wind generated sea state and it is accurate recorded data. This spectrum commonly known as P-M model has since been extensively used by ocean engineers as one of the most representative for waters all over the world.

The P-M spectral model describes a fully-developed sea determined by one parameter, namely, the wind speed. The fetch and duration are considered infinite. For the applicability of such model, the wind has to blow over a large area at a nearly constant speed for many hours prior to the time when the wave record is obtained and the wind should not change its direction more than a certain specified small amount.

Response-Amplitude Operator (RAO), so called because it allows the transfer of the exciting waves into the responses of the structure. Because of the invariance of the normalized response for a linear system, the RAO is unique.

It is often found in practice that an RAO is defined as response amplitude per unit height. However, for reasons that will become clear subsequently, it is more convenient to define the RAO as the amplitude of response per unit wave amplitude. In the computation of an RAO, the waves are considered regular and a sufficient number of frequencies are chosen to cover the entire range of frequencies covered by the wave spectrum.

The RAO could be theoretical or measured. The theoretical RAO's are obtained with the help of simplified mathematical formulas. When the problem is complicated to solve analytically or when the mathematical assumptions need verification, tests are performed on a model of the prototype structure with regular waves in the controlled environment of the laboratory. The test results on model RAO's can then be scaled-up to obtain prototype RAO's.

Generally, inertial systems are linear and drag systems are nonlinear. Thus, inertia forces are linear with the wave amplitude. For a linear system then, the response function at a wave frequency can be written as;

$$\mathbf{Response\ (t) = (RAO)\ \eta(t)}$$

Where $\eta(t)$ is the wave profile as a function of time, t.

The response spectrum is defined as the response energy density of a structure due to the input wave-energy density spectrum. For a linear system, the function RAO is the squared and at a given frequency the square of the RAO is multiplied by the wave spectrum to evaluate the response spectrum value at that frequency.

$$\mathbf{S_x(f) = [RAO(w)]^2\, S(f)}$$

Where;

$S_x(f)$ = Surge response spectrum

S = the wave spectrum

f = wave frequency

If a structure is free to move in waves its motion may be critical near the resonance of the structure. Therefore, it is important to study the overall response of the structure due to a design-wave spectrum. The RAO are written relating the dynamic motion of the structure to the wave-forcing function on the structure. Then the dynamic-motion spectrum is obtained from the force spectrum, or equivalently, from the wave spectrum. If the relationship between the motion and force is linear, the conversion is relatively straightforward.

Consider that the motion of the structure in a particular direction, x, is uncoupled and be modeled by a simple linearly damped spring-mass system. If m is the total mass of the system, K is its spring constant and C is the damping coefficient, then the equation of motion is;

$$\mathbf{M_{\ddot{\upsilon}} + C_v + K_x = F_I \cos \omega t}$$

Where;

F_I = inertia force amplitude which is linear with wave height.

C_v = linear damping

x = the displacement in the motion of surge, sway, and heave.

v & $\ddot{\upsilon}$ = velocity & acceleration

$$\mathbf{x = X \cos (\omega t + \beta)}$$

where $\omega = \omega_d = (1- \xi^2)^{\wedge(1/2)}\ \omega_n$

The displacement function can be written as;

$$\mathbf{x(t) = [(F_I/(H/2)/((K-mw^2))^2 + (Cw)^2]^{1/2}]\ \eta_\beta (t)}$$

where β is the phase difference between x(t) and η (t).

This relationship can be transformed to obtain the motion spectrum in terms of the wave spectrum and RAO.

$$\mathbf{S_x(f) = [(F_I/(H/2)/((K-mw^2))^2 + (Cw)^2]^{1/2}]^2\ S(f)}$$

3.2.1 Steps in finding wave forces [Morrison equation].

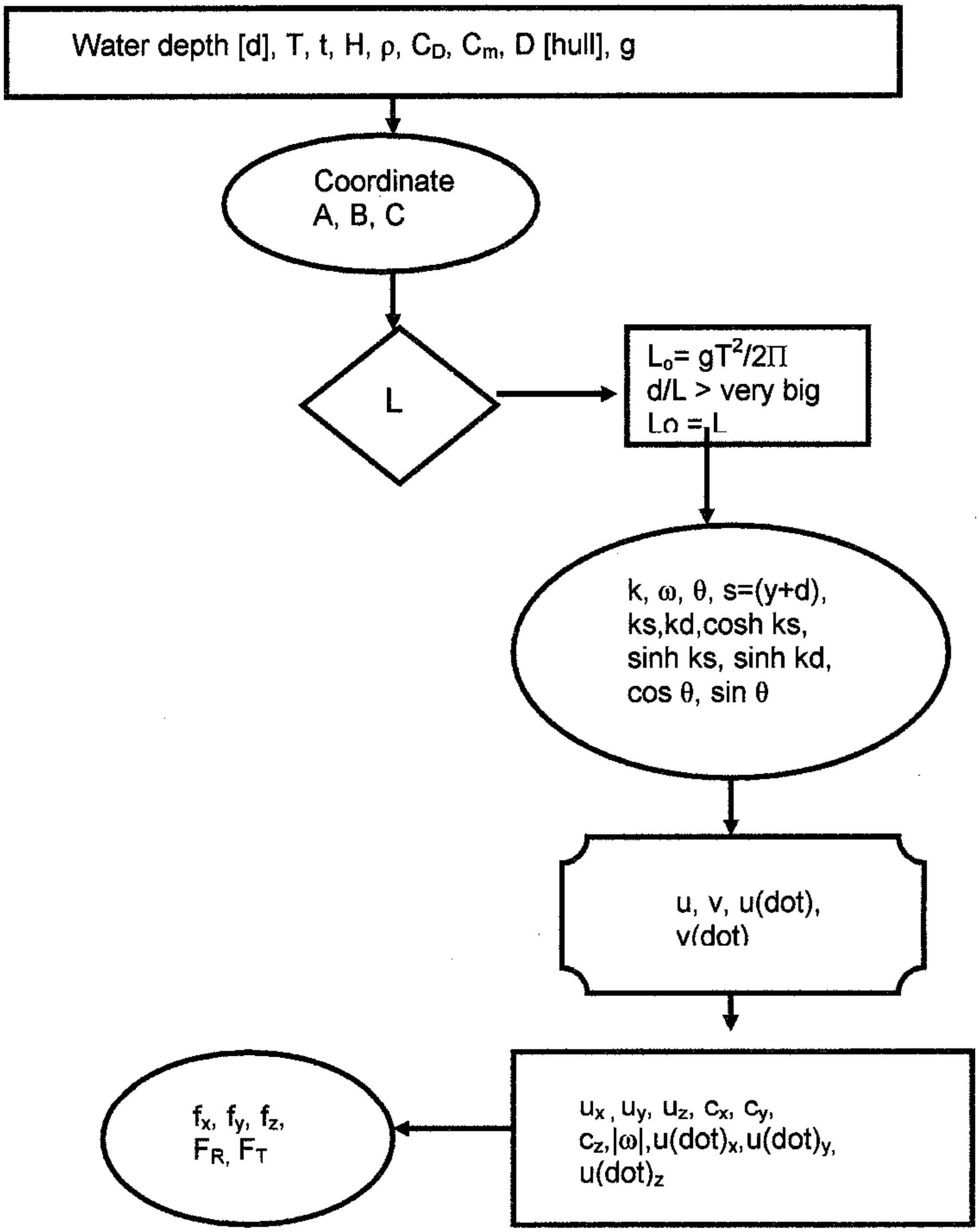

Figure 3.1: Steps in finding wave forces

3.2.2 Steps in finding P-M Spectrum

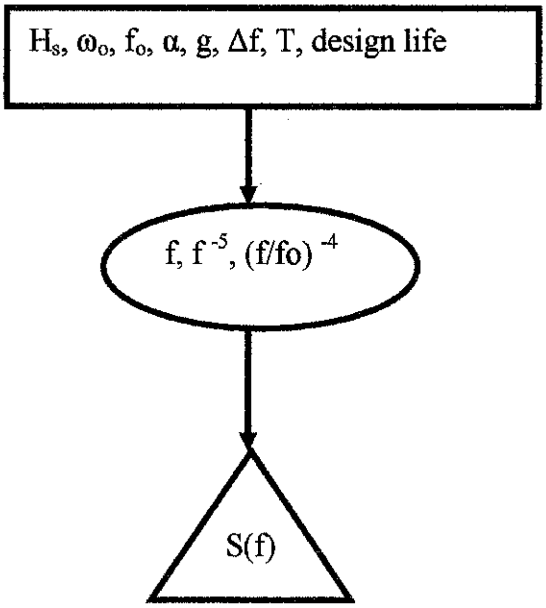

Figure 3.2: Steps in finding P-M Spectrum

3.2.3 Steps in finding wave profile

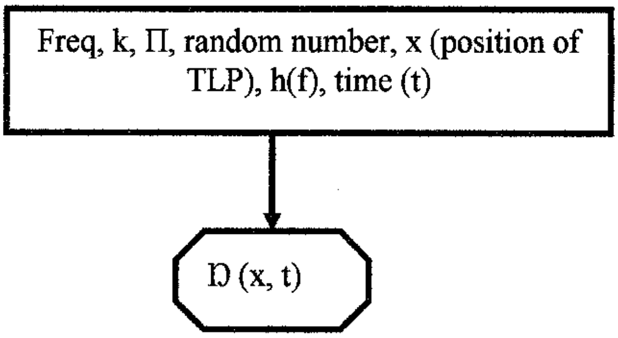

Figure 3.3: Steps in finding wave profile

3.2.4 Steps in finding surge, heave & pitch response

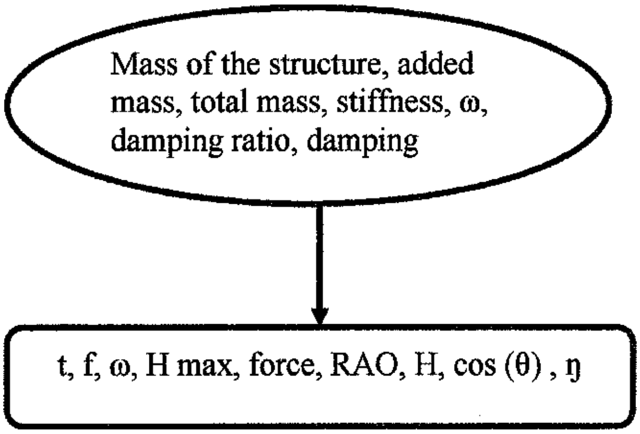

Figure 3.4: Steps in finding surge, heave and pitch response

3.2.5 Steps in finding tension in each tether

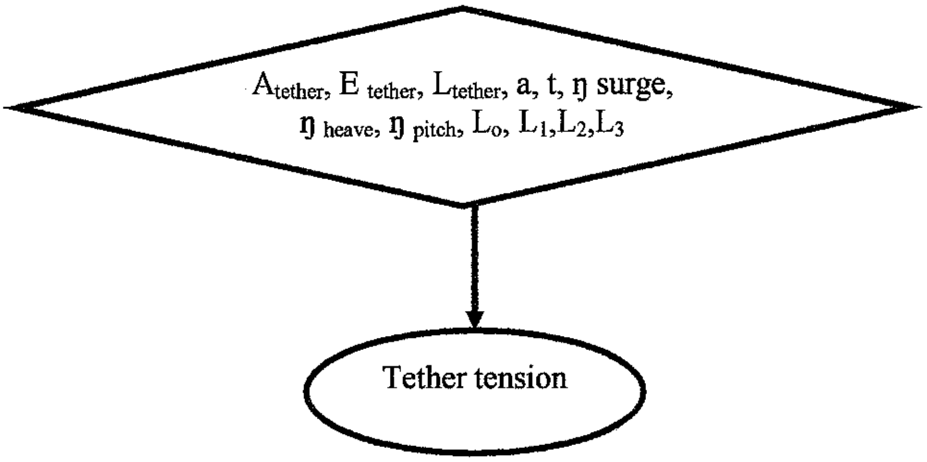

Figure 3.5: Steps in finding tether tension

CHAPTER 4:

RESULT & DISCUSSION:

This chapter summarizes the results obtained from the calculation of frequency domain analysis. The details of the calculation are attached in the appendices and the results are shown in term of graph.

Figure 4.1 shows the position of column and pontoon of the square tension leg platform (TLP). The column is in the vertical direction while the pontoon is the horizontal direction. The wave acting on the square TLP is assumed to be in zero angles. Therefore, the square TLP experienced motions in 3 degrees of freedom (surge, heave & pitch). All the columns and two (2) pontoons facing the x direction of wave experienced the wave force from the x-direction. Wave force is calculated using Morrison equation. Before completed the Morrison equation, wave kinematics is calculated such as wave velocity and wave acceleration. The result shows that wave forces induced in horizontal and vertical direction and hence stimulate the occurrence of moments towards center of gravity of square tension leg platform.

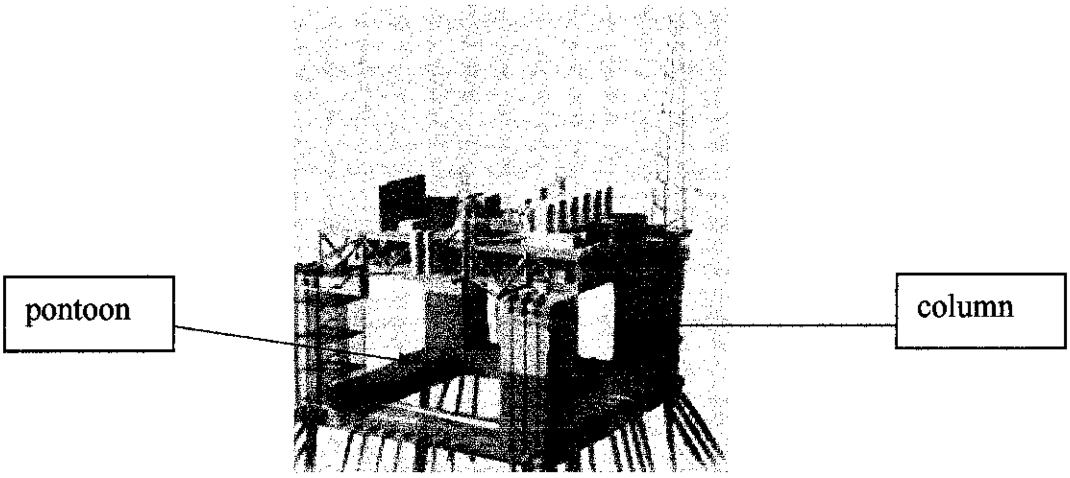

Figure 4.1: Square Tension Leg Platform (TLP)

Figure 4.2 shows that the pattern of forces as it goes deeper into the water. Force of waves develops over the distance the wind has been able to build them up. The wave force decrease from the top of the hull to the bottom of the hull. This is due to the decrease in the value of gravitational force with depth.

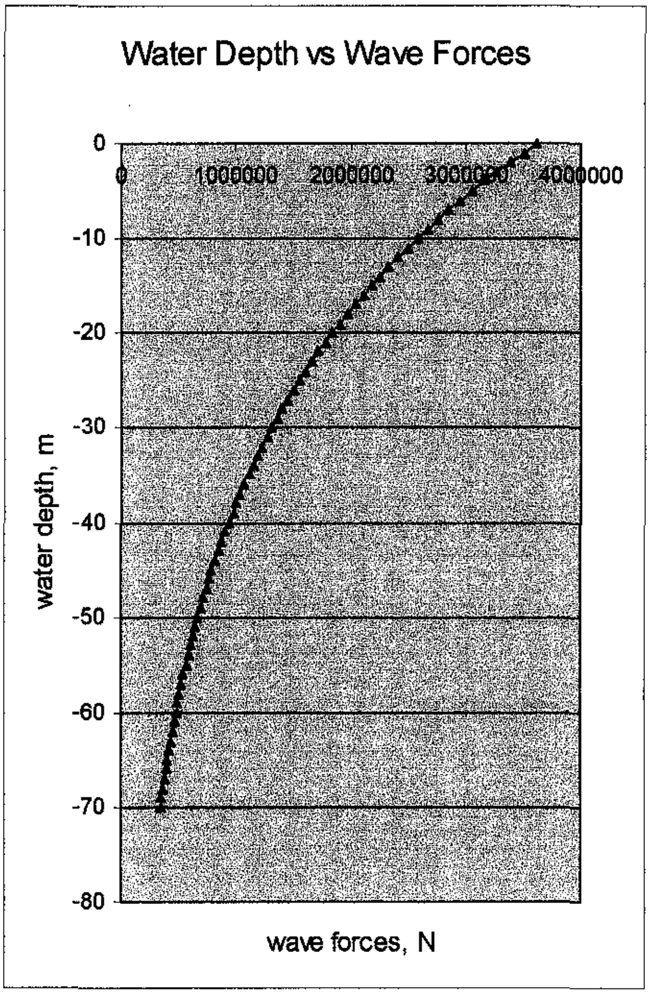

Figure 4.2: Graph of Water Depth vs Wave Forces

The graph in figure 4.3 represents the total energy content in the wave at a particular frequency. The energy density is obtained by dividing the energy ordinate at each point by the frequency increment, Δf. The spectrum generally rises sharply at low frequency end to a maximum value and then decrease rather slowly with the increase in the frequency, f. The advantage of this kind of representation is that the area under the curve gives the total energy of the wave system.

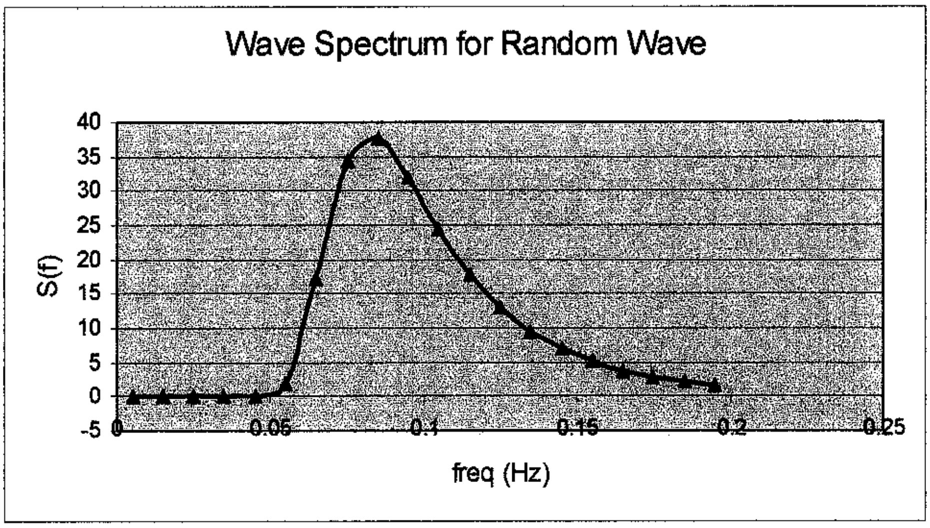

Figure 4.3: P-M Spectrum.

Figure 4.4 shows the simulation of wave in time range from 0-50 seconds. The graph represents the wave height at different frequency from P-M spectrum.

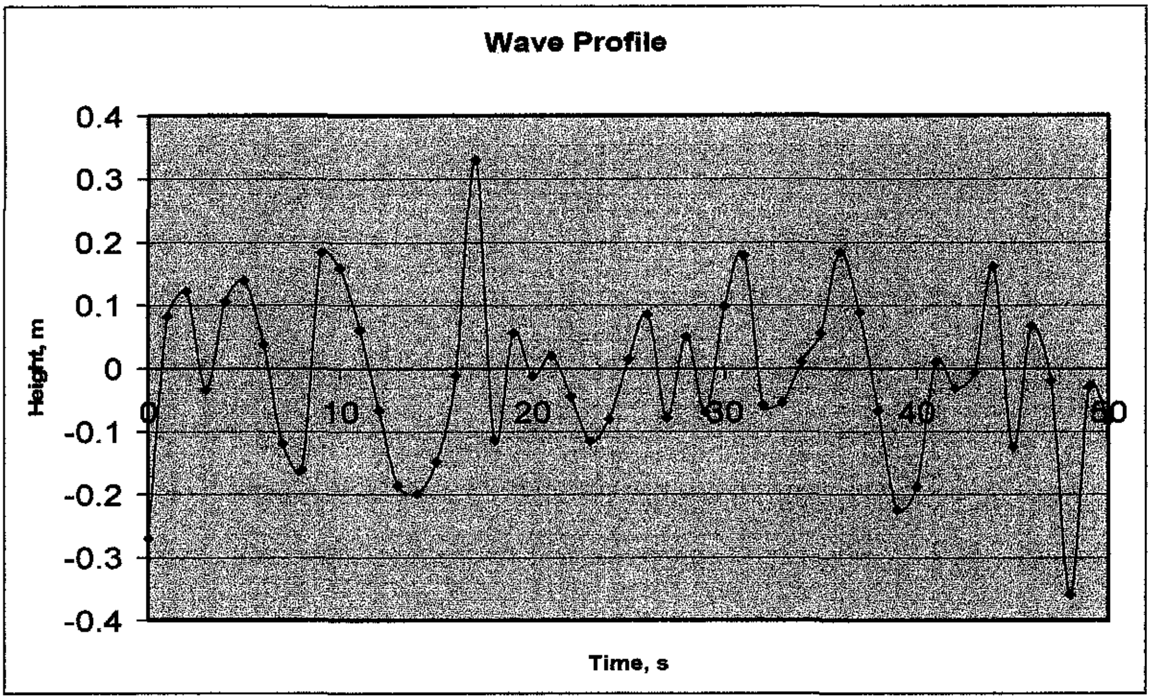

Figure 4.4: Wave Profile

Surge response shows the magnitude of motion of tension leg platform in the direction of surge with respect to time. The graph in figure 4.5 represents how much the tension leg platform will move from its original position when the wave in the direction of surge.

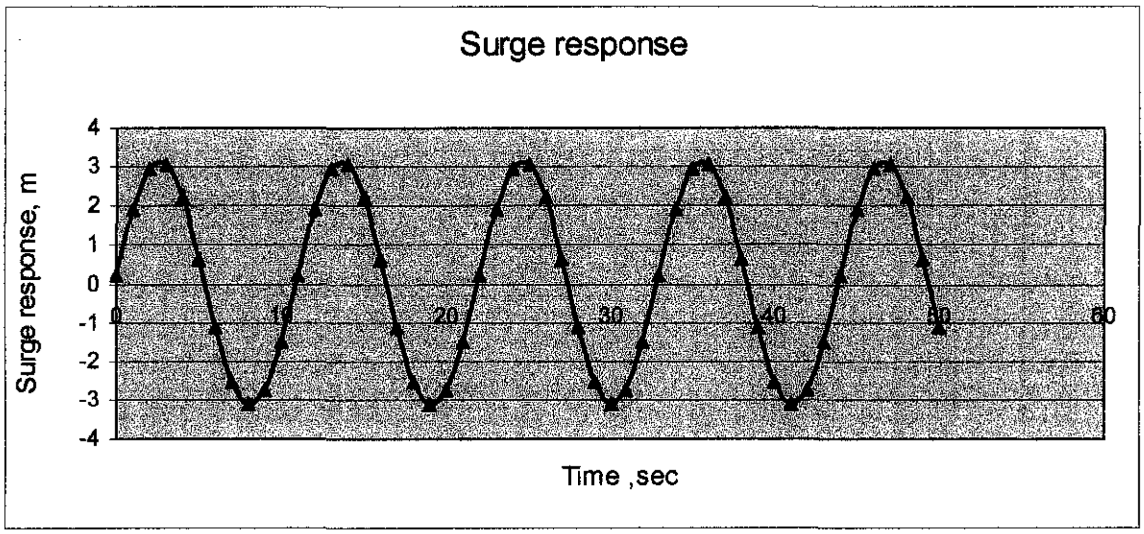

Figure 4.5: Surge Response

Figure 4.6 shows the heave response of square TLP. The heave response referred to the vertical motion of square TLP with regards to the motion of wave. The heave motion is less compared to the surge motion because square TLP is moored by tether at the seabed. Therefore, the vertical motion of square TLP is restricted by the tether.

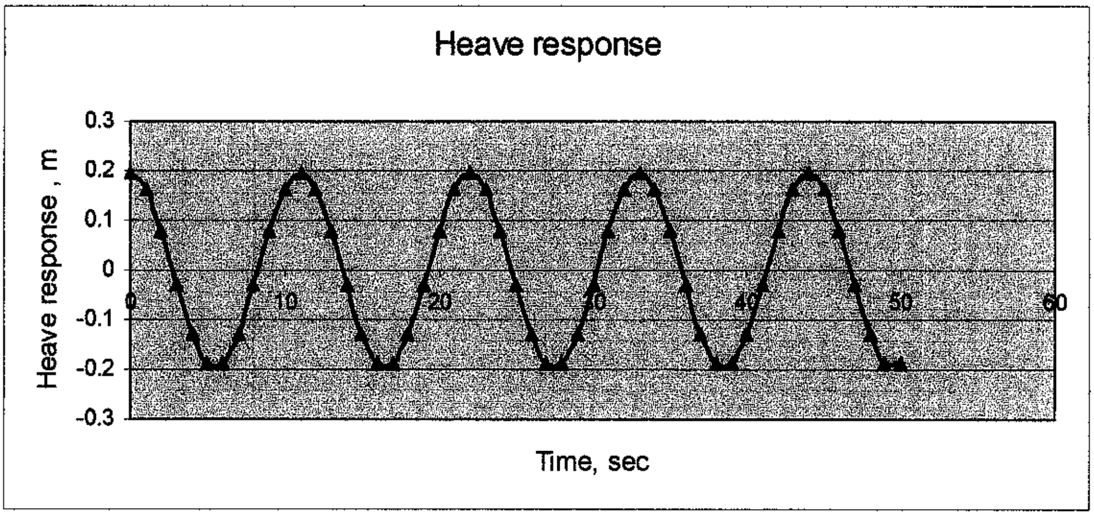

Figure 4.6: Heave Response

Pitch response referred to the rotation of square TLP. Graph in figure 4.7 shows that the maximum rotation of square TLP is around 0.08 degrees. Therefore, it is proved that the rotation only small and square TLP is safe in deep water exploration. The square TLP will not collapse when the wave attacks it.

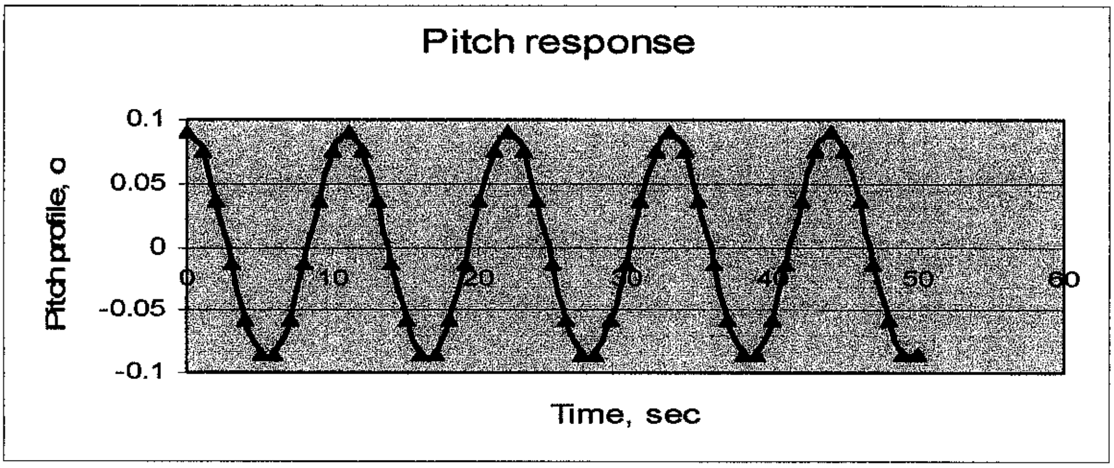

Figure 4.7: Pitch Response

Graph in figure 4.8 shows the difference tension in each tether of square TLP with respect to time. The total pretension of 16 tethers is around 55,000 KN. The vertical wave forces, reacted by the tethers, cause the tension to change with time. The tension force is kept under all condition by the excess buoyancy over weight of the platform.

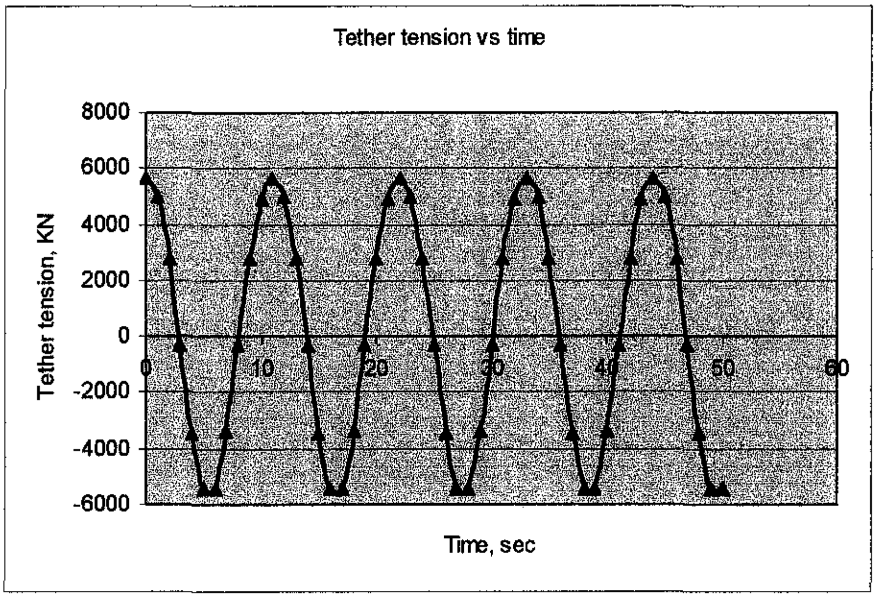

Figure 4.8: Tether Tension vs Time

Figure 4.9 shows the cost of different type of platforms with respect to water depth. It is seen that the cost of rigid platform increase tremendously with water depth. Somehow, it is different with tension leg platform (TLP). Even though the cost is increase, the value is not so significant compared to rigid platform. This is because, the part that consume most of the steel in tension leg platform only its hull. Therefore, the portion of steel needed in constructing TLP is less compared to rigid platform. Hence, it is proved that TLP is more economical and suitable for deepwater exploration.

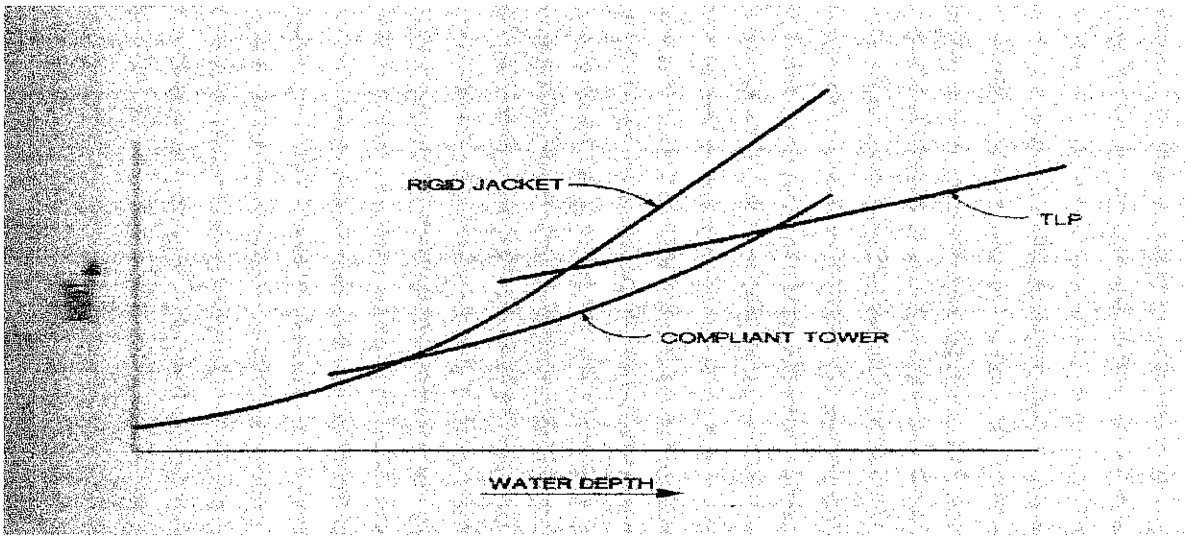

Figure 4.9: Cost Vs Water Depth

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

Conclusions:

I. Dynamic analysis of tension leg platform in frequency domain has been successfully carried out.

II. The analysis gave the responses of tension leg platform in surge, heave, and pitch as well as tether tension.

III. The maximum responses in surge (3m), heave (0.2m) and pitch (0.08°) as well as tether tension (5800 KN) have been found to be within allowable limits, thereby confirming the suitability of tension leg platform for deepwater application.

IV. Tension leg platform is economical and suitable in deepwater exploration.

Recommendations for further study:

I. Model testing on TLPs shall be conducted in our offshore laboratory so that the theoretical results can be compared with experimental results.

II. Further dynamic analysis in time domain may be conducted to be compared with the results of frequency domain analysis. Also, SACS software may be used to do analysis on TLPs.

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APPENDICES

P-M Spectrum

H_s	5.9	m
ω_o	0.5173	rad/sec
f_o	0.0823	Hz
α	0.0081	
g	9.806	m/s^2

Δf	0.01	
$(\alpha g^2/(2\Pi^4))$	0.0005	
T	11	s
$H_s = 4(m_o)^{0.5}$		
$H_{rms} = 2(2x_{mo})^{0.5}$		

f (Hz)	f^{-5}	$(f/f_o)^{-4}$	**S(f)**	**Area of S(f)**
0.005	3.2E+11	73403.932	0	0
0.015	1316872428	906.22138	0	0
0.025	102400000	117.44629	8.9418E-60	8.9418E-62
0.035	19039685.85	30.572233	2.40957E-13	2.40957E-15
0.045	5419228.099	11.187918	0.002287407	2.28741E-05
0.055	1986948.234	5.0135873	1.885558127	0.018855581
0.065	861853.0379	2.5700757	17.34567107	0.173456711
0.075	421399.177	1.4499542	34.39773108	0.343977311
0.085	225374.8089	0.878868	37.56349451	0.375634945
0.095	129235.5435	0.5632548	31.95794909	0.319579491
0.105	78352.61665	0.377435	24.44140114	0.244414011
0.115	49717.67353	0.2623059	17.90949428	0.179094943
0.125	32768	0.1879141	12.95410423	0.129541042
0.135	22301.3502	0.1381224	9.38250686	0.093825069
0.145	15601.27129	0.1037833	6.851558477	0.068515585
0.155	11177.41843	0.0794827	5.060144247	0.050601442
0.165	8176.741703	0.0618961	3.783980096	0.037839801
0.175	6092.69947	0.0489156	2.865662606	0.028656626
0.185	4614.677527	0.0391663	2.197097011	0.02197097
0.195	3546.720321	0.0317293	1.70440228	0.017044023
			[redacted]	[redacted]

Design life	25	years
Number of waves, N	(25 x 365 x 24 x 3600)/11	
	71672727.27	waves
H_s	5.800731575	m
H_{rms}	4.101736632	m
H_{max}	$[\sqrt{(\ln N)} + (0.2886/\sqrt{(\ln N)})] \times H_{rms}$	
	[redacted]	m

Surge response

mass of structure	50000000	kg
added mass,m_{11}	48780377	kg
total mass m_{11} [	98780377	kg
stiffness, k_{11}	650350	N/m
ω	0.5712	rad/sec
damping ratio,ξ	0.02	
damping,c [2mξ	2256934	N-sec/m

$(k-m\omega^2)^2$ =	9.97E+14	N^2/m^2
$(c\omega)^2$ =	1.66E+12	N^2/m^2
$((k-mw^2)^2+(cw)^2)^{0.5}$ =	31604972	N/m
$H_{max}/2$	8.86	m

t	f	ω	H_{max}	F_R (N)	RAO $_{surge}$	H $_{surge}$	k	x	cos (kx-ωt	η $_{surge}$
0	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.073471	0.230486
1	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.600988	1.885364
2	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.937695	2.941651
3	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.976689	3.063979
4	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.70559	2.213512
5	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.21047	0.660267
6	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.35147	-1.10261
7	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.80183	-2.51541
8	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.9976	-3.12959
9	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.87665	-2.75014
10	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.47736	-1.49753
11	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.073485	0.230532
12	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.601	1.885401
13	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.9377	2.941667
14	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.976686	3.063969
15	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.70558	2.213479
16	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.210456	0.660222
17	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.35149	-1.10265
18	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.80183	-2.51544
19	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.9976	-3.12959
20	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.87664	-2.75011
21	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.47735	-1.49749
22	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.0735	0.230578
23	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.601012	1.885438
24	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.937706	2.941683
25	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.976683	3.06396
26	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.705569	2.213446
27	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.210441	0.660177
28	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.3515	-1.1027
29	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.80184	-2.51547
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32	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.47733	-1.49745
33	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.073515	0.230624
34	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.601023	1.885475
35	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.937711	2.941699
36	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.97668	3.06395
37	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.705559	2.213414
38	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.210427	0.660132
39	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.35151	-1.10274
40	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.80185	-2.5155
41	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.99761	-3.1296
42	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.87663	-2.75007
43	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.47732	-1.49741
44	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.073529	0.230669
45	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.601035	1.885512
46	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.937716	2.941715
47	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.976677	3.06394
48	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.705549	2.213381
49	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	0.210412	0.660086
50	0.090909	0.5712	17.72	99148176.37	0.354075	6.27421	0.03327	45	-0.35153	-1.10278

heave response

mass of stru	50000000	kg
added mass,	40898474	kg
total mass m	90898474	kg
stiffness, k_{22}	4673320731	N/m
ω	0.5712	rad/sec
damping ratio	0.02	
damping,c [2	2076848.33	N-sec/m

$(k-m\omega^2)^2$ =	2.16E+19	N^2/m^2
$(c\omega)^2$ =	1.41E+12	N^2/m^2
$((k-mw^2)^2+(cw)^2)^{0.5}$ =	4.64E+09	N/m
$H_{max}/2$	8.86	m

t	f	ω	H_{max}	F_R (N)	RAO $_{heave}$	H $_{heave}$	k	x	cos (kx-ωt)	η $_{heave}$
0	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	1	0.194749
1	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.841253	0.163833
2	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.415413	0.080901
3	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.142319	-0.027716
4	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.654865	-0.127534
5	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.959495	-0.186861
6	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.959491	-0.18686
7	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.654854	-0.127532
8	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.142304	-0.027714
9	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.415426	0.080904
10	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.841261	0.163835
11	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	1	0.194749
12	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.841245	0.163831
13	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.415399	0.080899
14	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.142333	-0.027719
15	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.654876	-0.127536
16	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.959499	-0.186861
17	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.959487	-0.186859
18	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.654843	-0.12753
19	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.14229	-0.027711
20	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.415439	0.080906
21	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.841269	0.163836
22	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	1	0.194749
23	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.841237	0.16383
24	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.415386	0.080896
25	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.142348	-0.027722
26	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.654887	-0.127538
27	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.959503	-0.186862
28	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.959482	-0.186858
29	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.654831	-0.127528
30	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.142275	-0.027708
31	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.415453	0.080909
32	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.841277	0.163838
33	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	1	0.194749
34	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.841229	0.163828
35	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.415372	0.080893
36	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.142362	-0.027725
37	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.654898	-0.127541
38	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.959507	-0.186863
39	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.959478	-0.186857
40	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.65482	-0.127526
41	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.142261	-0.027705
42	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.415466	0.080912
43	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.841285	0.163839
44	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	1	0.194749
45	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.841221	0.163827
46	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	0.415359	0.080891
47	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.142377	-0.027728
48	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.654909	-0.127543
49	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.959511	-0.186864
50	0.09090909	0.5712	17.72	904348162.3	0.021981	0.389498	0.033272	0	-0.959474	-0.186856

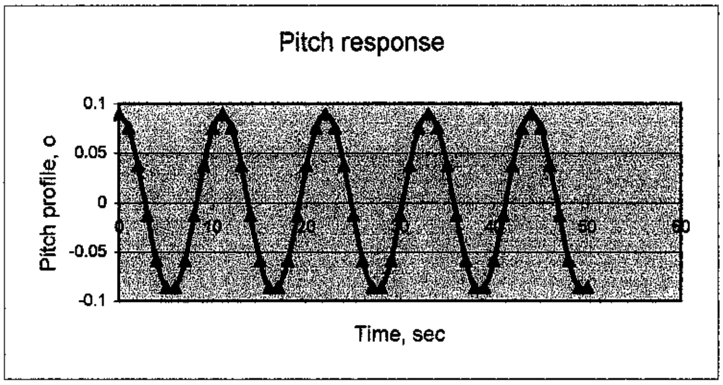
pitch response

nass of st	50000000	kg
added mas	1.54E+14	kg
otal mass	1.541E+14	kg
stiffness, k	1.15E+15	N/m
ω	0.5712	rad/sec
damping ra	0.02	
damping,c	3.52088E+12	N-sec/m

$(k-m\omega^2)^2$ =	1.20785E+30	N^2/m^2
$(c\omega)^2$ =	4.04463E+24	N^2/m^2
$((k-mw^2)^2+$	1.09902E+15	N/m
$H_{max}/2$	8.86	m

	f	ω	H_{max}	M_I (N.m)	RAO_{pitch}	H_{pitch}	k	x	cos (kx-ωt)	η_{pitch}
0	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	1	0.089747641
1	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.8412528	0.075500455
2	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.4154126	0.037282299
3	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.142319	-0.012772777
4	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.654865	-0.058772568
5	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.959495	-0.0861124
6	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.959491	-0.086112028
7	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.654854	-0.058771572
8	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.142304	-0.012771472
9	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.4154259	0.037283499
10	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.8412608	0.075501168
11	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	1	0.089747641
12	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.8412449	0.075499742
13	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.4153992	0.0372811
14	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.142333	-0.012774082
15	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.654876	-0.058773565
16	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.959499	-0.086112771
17	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.959487	-0.086111657
18	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.654843	-0.058770575
19	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.14229	-0.012770167
20	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.4154393	0.037284698
21	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.8412687	0.075501881
22	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	1	0.089747641
23	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.8412369	0.075499029
24	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.4153859	0.0372799
25	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.142348	-0.012775387
26	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.654887	-0.058774561
27	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.959503	-0.086113143
28	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.959482	-0.086111285
29	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.654831	-0.058769579
30	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.142275	-0.012768861
31	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.4154527	0.037285898
32	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.8412766	0.075502594
33	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	1	0.089747641
34	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.841229	0.075498316
35	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.4153725	0.037278701
36	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.142362	-0.012776693
37	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.654898	-0.058775558
38	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.959507	-0.086113514
39	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.959478	-0.086110914
40	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.65482	-0.058768582
41	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.142261	-0.012767556
42	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.415466	0.037287097
43	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.8412846	0.075503307
44	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	1	0.089747641
45	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.841221	0.075497603
46	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	0.4153591	0.037277501

47	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.142377	-0.012777998
48	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.654909	-0.058776555
49	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.959511	-0.086113886
50	0.090909091	0.5712	17.72	9.86E+13	0.010129531	0.17949528	0.0332724	0	-0.959474	-0.086110542



Tether tension

area of tether	0.4	m^2
E tether	2.00E+07	kN/m^2
L tether	274.56	m

AE/L	29143.7063
a	45m

												B	
t	η_{surge}	L_0	L_1	ΔL_{surge}	AE/L	η_{heave}	L_2	ΔL_{heave}	η_{pitch}	L_3	ΔL_{pitch}	$\sqrt{(l_o)^2 + (\eta_{surge}^2)}$	$\sqrt{(l_o)^2 + (\eta_{surge}^2)} - l_o$	$AE/L^*((B)+\eta_{heave})$
0	0.2304856	274.56	274.5601	9.67E-05	29143.7063	0.194748858	274.560069	6.90689E-05	0.089748	274.5600147	1.46683E-05	274.5600967	9.67432E-05	5678.522981
1	1.8853645	274.56	274.5665	0.006473	29143.7063	0.163833024	274.560049	4.88805E-05	0.0755	274.5600104	1.03808E-05	274.5664732	0.006473189	4963.354251
2	2.9416507	274.56	274.5758	0.015758	29143.7063	0.080901126	274.560012	1.19191E-05	0.037282	274.5600025	2.53127E-06	274.5757581	0.01575805	2817.006653
3	3.0639794	274.56	274.5771	0.017096	29143.7063	-0.027716425	274.560001	1.39897E-06	-0.012773	274.5600003	2.971E-07	274.5770959	0.017095858	-309.5226851
4	2.2135118	274.56	274.5689	0.008923	29143.7063	-0.127534167	274.56003	2.96201E-05	-0.058773	274.5600063	6.29046E-06	274.5689226	0.008922558	-3456.781889
5	0.6602667	274.56	274.5608	0.000794	29143.7063	-0.186860528	274.560064	6.35869E-05	-0.086112	274.5600135	1.35041E-05	274.5607939	0.000793909	-5422.670873
6	-1.102609	274.56	274.5622	0.002214	29143.7063	-0.186859721	274.560064	6.35864E-05	-0.086112	274.5600135	1.35039E-05	274.562214	0.002213983	-5381.261157
7	-2.515413	274.56	274.5715	0.011522	29143.7063	-0.127532004	274.56003	2.9619E-05	-0.058772	274.5600063	6.29024E-06	274.5715224	0.011522383	-3380.950335
8	-3.129587	274.56	274.5778	0.017836	29143.7063	-0.027713592	274.560001	1.39868E-06	-0.012771	274.5600003	2.9704E-07	274.5778358	0.017835809	-287.8752165
9	-2.750135	274.56	274.5738	0.013773	29143.7063	0.080903729	274.560012	1.19198E-05	0.037283	274.5600025	2.53143E-06	274.573773	0.013773045	2759.232091
10	-1.497531	274.56	274.5641	0.004084	29143.7063	0.163834571	274.560049	4.88814E-05	0.075501	274.5600104	1.0381E-05	274.564084	0.004083955	4893.768209
11	0.2305316	274.56	274.5601	9.68E-05	29143.7063	0.194748858	274.560069	6.90689E-05	0.089748	274.5600147	1.46683E-05	274.5600968	9.67818E-05	5678.524105
12	1.8854013	274.56	274.5665	0.006473	29143.7063	0.163831477	274.560049	4.88796E-05	0.0755	274.5600104	1.03806E-05	274.5664734	0.006473442	4963.316538
13	2.9416667	274.56	274.5758	0.015758	29143.7063	0.080898523	274.560012	1.19183E-05	0.037281	274.5600025	2.53111E-06	274.5757582	0.015758222	2816.935797
14	3.0639695	274.56	274.5771	0.017096	29143.7063	-0.027719257	274.560001	1.39925E-06	-0.012774	274.5600003	2.97161E-07	274.5770957	0.017095747	-309.6084461
15	2.2134792	274.56	274.5689	0.008922	29143.7063	-0.127536329	274.56003	2.96211E-05	-0.058774	274.5600063	6.29067E-06	274.5689223	0.008922295	-3456.852586
16	0.6602216	274.56	274.5608	0.000794	29143.7063	-0.186861334	274.560064	6.35875E-05	-0.086113	274.5600135	1.35042E-05	274.5607938	0.000793801	-5422.697524
17	-1.102653	274.56	274.5622	0.002214	29143.7063	-0.186858915	274.560064	6.35858E-05	-0.086112	274.5600135	1.35038E-05	274.5622142	0.002214157	-5381.232611
18	-2.515441	274.56	274.5715	0.011523	29143.7063	-0.127529842	274.56003	2.9618E-05	-0.058771	274.5600063	6.29003E-06	274.5715226	0.011522635	-3380.879956
19	-3.129591	274.56	274.5778	0.017836	29143.7063	-0.02771076	274.560001	1.39839E-06	-0.01277	274.5600003	2.96979E-07	274.5778358	0.017835845	-287.7916136
20	-2.750113	274.56	274.5738	0.013773	29143.7063	0.080906332	274.560012	1.19206E-05	0.037285	274.5600025	2.53159E-06	274.5737728	0.013772823	2759.301473
21	-1.49749	274.56	274.5641	0.004084	29143.7063	0.163836118	274.560049	4.88823E-05	0.075502	274.5600104	1.03812E-05	274.5640837	0.004083734	4893.806855
22	0.2305776	274.56	274.5601	9.68E-05	29143.7063	0.194748858	274.560069	6.90689E-05	0.089748	274.5600147	1.46683E-05	274.5600968	9.68204E-05	5678.525228
23	1.8854382	274.56	274.5665	0.006474	29143.7063	0.16382993	274.560049	4.88786E-05	0.075499	274.5600104	1.03804E-05	274.5664737	0.006473695	4963.278824
24	2.9416827	274.56	274.5758	0.015758	29143.7063	0.080895921	274.560012	1.19175E-05	0.03728	274.5600025	2.53094E-06	274.5757584	0.015758393	2816.86494
25	3.0639596	274.56	274.5771	0.017096	29143.7063	-0.027722089	274.560001	1.39954E-06	-0.012775	274.5600003	2.97222E-07	274.5770956	0.017095637	-309.6942071
26	2.2134465	274.56	274.5689	0.008922	29143.7063	-0.127538492	274.56003	2.96221E-05	-0.058775	274.5600063	6.29088E-06	274.568922	0.008922031	-3456.923282
27	0.6601766	274.56	274.5608	0.000794	29143.7063	-0.18686214	274.560064	6.3588E-05	-0.086113	274.5600135	1.35043E-05	274.5607937	0.000793693	-5422.724174
28	-1.102696	274.56	274.5622	0.002214	29143.7063	-0.186858109	274.560064	6.35853E-05	-0.086111	274.5600135	1.35037E-05	274.5622143	0.00221433	-5381.204063
29	-2.515468	274.56	274.5715	0.011523	29143.7063	-0.127527679	274.56003	2.9617E-05	-0.05877	274.5600063	6.28982E-06	274.5715229	0.011522887	-3380.809577
30	-3.129594	274.56	274.5778	0.017836	29143.7063	-0.027707928	274.560001	1.39811E-06	-0.012769	274.5600003	2.96918E-07	274.5778359	0.017835882	-287.7080108
31	-2.750091	274.56	274.5738	0.013773	29143.7063	0.080908935	274.560012	1.19214E-05	0.037286	274.5600025	2.53176E-06	274.5737726	0.013772601	2759.370854
32	-1.49745	274.56	274.5641	0.004084	29143.7063	0.163837665	274.560049	4.88833E-05	0.075503	274.5600104	1.03814E-05	274.5640835	0.004083513	4893.8455
33	0.2306235	274.56	274.5601	9.69E-05	29143.7063	0.194748858	274.560069	6.90689E-05	0.089748	274.5600147	1.46683E-05	274.5600969	9.6859E-05	5678.52635
34	1.885475	274.56	274.5665	0.006474	29143.7063	0.163828383	274.560049	4.88777E-05	0.075498	274.5600104	1.03802E-05	274.5664739	0.006473948	4963.241108
35	2.9416988	274.56	274.5758	0.015759	29143.7063	0.080893318	274.560012	1.19168E-05	0.037279	274.5600025	2.53078E-06	274.5757586	0.015758565	2816.794083
36	3.0639497	274.56	274.5771	0.017096	29143.7063	-0.027724922	274.560001	1.39982E-06	-0.012777	274.5600003	2.97283E-07	274.5770955	0.017095526	-309.7799681
37	2.2134138	274.56	274.5689	0.008922	29143.7063	-0.127540654	274.56003	2.96231E-05	-0.058776	274.5600063	6.2911E-06	274.5689218	0.008921768	-3456.993977
38	0.6601315	274.56	274.5608	0.000794	29143.7063	-0.186862946	274.560064	6.35886E-05	-0.086114	274.5600135	1.35044E-05	274.5607936	0.000793584	-5422.750822
39	-1.102739	274.56	274.5622	0.002215	29143.7063	-0.186857303	274.560064	6.35847E-05	-0.086111	274.5600135	1.35036E-05	274.5622145	0.002214503	-5381.175514
40	-2.515496	274.56	274.5715	0.011523	29143.7063	-0.127525516	274.56003	2.9616E-05	-0.058769	274.5600063	6.2896E-06	274.5715231	0.01152314	-3380.739198
41	-3.129597	274.56	274.5778	0.017836	29143.7063	-0.027705096	274.560001	1.39782E-06	-0.012768	274.5600003	2.96858E-07	274.5778359	0.017835918	-287.6244081
42	-2.750069	274.56	274.5738	0.013772	29143.7063	0.080911537	274.560012	1.19221E-05	0.037287	274.5600025	2.53192E-06	274.5737724	0.013772378	2759.440235
43	-1.497409	274.56	274.5641	0.004083	29143.7063	0.163839212	274.560049	4.88842E-05	0.075503	274.5600104	1.03816E-05	274.5640833	0.004083292	4893.884144
44	0.2306695	274.56	274.5601	9.69E-05	29143.7063	0.194748858	274.560069	6.90689E-05	0.089748	274.5600147	1.46683E-05	274.5600969	9.68976E-05	5678.527471
45	1.8855118	274.56	274.5665	0.006474	29143.7063	0.163826836	274.560049	4.88768E-05	0.075498	274.5600104	1.038E-05	274.5664742	0.0064742	4963.203392
46	2.9417148	274.56	274.5758	0.015759	29143.7063	0.080890715	274.560012	1.1916E-05	0.037278	274.5600025	2.53062E-06	274.5757587	0.015758737	2816.723225
47	3.0639398	274.56	274.5771	0.017095	29143.7063	-0.027727754	274.560001	1.40011E-06	-0.012778	274.5600003	2.97343E-07	274.5770954	0.017095416	-309.8657291
48	2.2133812	274.56	274.5689	0.008922	29143.7063	-0.127542816	274.56003	2.96241E-05	-0.058777	274.5600063	6.29131E-06	274.5689215	0.008921505	-3457.064671
49	0.6600865	274.56	274.5608	0.000793	29143.7063	-0.186863752	274.560064	6.35891E-05	-0.086114	274.5600135	1.35045E-05	274.5607935	0.000793476	-5422.777469
50	-1.102782	274.56	274.5622	0.002215	29143.7063	-0.186856496	274.560064	6.35842E-05	-0.086111	274.5600135	1.35035E-05	274.5622147	0.002214677	-5381.146964