FINITE ELEMENT ANALYSIS OF CONCRETE BEAM WITH POLYVINYL ALCOHOL FIBER

By

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Dissertation submitted in partial fulfillment of the requirements for the Bachelor of Engineering (Hons) (Civil Engineering)

DECEMBER 2007

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the Civil Engineering Programme Universiti Teknologi PETRONAS in partial fulfilment of the requirement for the BACHELOR OF ENGINEERING (Hons) (CIVIL ENGINEERING)

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

(MOHD FARID MOHD SAIDI)

ABSTRACT

This study presents the results of the computational modeling and analysis for evaluating the effect of Polyvinyl Alcohol Fiber (PVA) as an internal strengthening material to concrete beam. A concentrated load was applied to several two pin simply supported beam model. A finite element method was used. The results show that the internal PVA strengthening was able to increase the structural strength capacity of the beam. From the stress contours, the failure mechanism observed show that the strengthened concrete beam fail in the tension zone. The possible failure of the strengthened concrete maybe due to crushing of concrete at the bottom of beam. This type of failure generally, occurs when the stress exceed the tensile strength of cement mortar. Therefore, structural strength of this concrete is highly depending of the PVA.

ACKNOWLEDGEMENTS

I wish to thank the university, University Technology of PETRONAS, the final year project coordinator and all the people who have facilitated men in the making of this project successful. In particular, I wish to thank my supervisor, Dr. Victor R. Macam, who has always supported me from the beginning to the end of the project by guidance and attention in helping me in completing the project. His understanding and quality-based policy urged me to give the best to achieve the goals of the project.

The compliment should also go to all lectures in Civil Engineering Department for bundles of information and assistance in completing this project.

I would also like to give special thanks to my parents, Mr. Mohd Saidi Othman and Hamidah Riffin for their moral supports and others who has supported me in any ways at all, directly or indirectly during this final year project period. I am lucky to be acquainted with them.

Finally, million thanks to PETRONAS for giving me the opportunity to study in this prestige university.

It has been splendid semester, an eye-opening experience comparable to none.

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CHAPTER I

INTRODUCTION

1.1 Concrete – The Material

The word 'concrete' comes from the Latin 'concretus" meaning compounded. Today concrete is understood to consist of a graded range of stone aggregate particles bound together by a hardened cement paste. Concrete required being strong, free from excessive volume change and resistant to penetration by water. It may also need to resist chemical attack or posses a low conductivity. Concrete's strength is derived from the hydration of the cement by water. The cement constituents progressively crystallized to from a gel or paste which surrounds the aggregate particles and it binds together to a produce a conglomerate.

Concrete is a material which, although relatively strong in compression, is weak in tension and for structural member's subjects to tensile stress may reinforce by steel bars or fibers. The effectiveness of reinforced concrete and fibers as a structural material depends on the following:

- The interfacial bonding between steel or fibers and concrete which allows it to act as a composite material;
- The passivity effect of the concrete environment to inhibit steel corrosion
- The similar coefficients of thermal movement of concrete and steel

Alternatively, the concrete may be pre-compressed by applying load through tendons of high tensile steel anchored to the concrete. Under load the effect is to unload the compression and avoid significant tensile stresses.

The use of polymer materials overcomes these problems and provides equally satisfactory solutions. Polyvinyl Alcohol Fiber (PVA) are considered as one of the most suitable polymeric fibers to be used as the reinforcement instead of using steel bar and post-tensioning method. The PVA added to the Ordinary Portland Cement (OPC) is capable to enhancing the bond properties between the cement and the fibers. It believed that in the certain amount of PVA the bond strength and frictional resistance were approximately doubled for steel fibers embedded in the cement. Though not as effective as the plain steel fibers, PVA also enhanced the pull-out characteristics of brass or brass-coated steel fibers. The properties of fiber composite are sensitive to the interfacial structure between the reinforcement and the matrix.

This project leads to the question of how had the interfacial microstructure been modified by PVA. The interfacial zones adjacent to the fiber in OPC matrices have been studied for various fibers that have included steel, glass, and polypropylene. The interfacial structures were found to be substantially different from those in the bulk paste.

1.2 Problem Statement

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Numerous papers dealing with structural strengthening application of PVA have been published recently. However, most of the studies on PVA in concrete

composition are purely experimental. Therefore, they do not lend themselves to a general design procedure for at least two reasons. Firstly, these studies often fail to cover all influencing design parameters. Secondly, the design methods proposed is quite often resort to calibration factors that are only valid for those particular studies. Therefore, they are not design oriented.

So, the role of computational modeling and analysis become particularly important. Computational modeling and analysis can be utilized in most of the cases as a method to analysis and predict the behavior of structure.

1.3 Objectives of the Study

The main objectives of this study are as follows:

- To carry out a finite element analysis (FEA) of the proposed beam using STAAD-Pro version 2004
- To obtain the effect of size and place of the aggregate to the strength of beam
- To predict the ultimate load and failure stresses and hence deduce the behavior of the beam before and after failure.
- To compare the crack analysis results of size and place of the aggregate as well as PVA added

1.4 Scope of Study

It is impossible to study every parameter that will influence the behavior of the beam in linear finite element analysis. Therefore, the study will focus on the improvement and refinement of the high strength concrete beam. The overall dimension of the beam model under study is 150 mm x 640 mm. the overall thickness of the beam in this study is 100 mm. A simply supported restraint of pin is applied of two edges bottom of the beam.

There are a total three FE models that need to be modeled and analyzed in this study. The first model is purely concrete beam without PVA. This is used as a control specimen to compare the results with other model. The second and third model is concrete beam with PVA, which are the percentage of the PVA one percent and two percent were added to the cement paste.

1.5 Importance of Study

This project focuses on analysis of the application finite element method in use of plate element. By doing this project, it could help to prepare a program to check the strength of the concrete with the appropriate proportion of mixing concrete. It believed would help to ease and reduce the time to prepare the high strength concrete

CHAPTER II

LITERATURE REVIEW

2.1 Finite Element Analysis (FEA)

Finite element analysis (FEA) is a computerized method for predicting how a component/assembly will react to environmental factors such as forces, heat, and vibration. Though it is called "analysis," in the product design cycle, it is used as a virtual prototyping tool to predict what is going to happen when the product is used. Finite element analysis, as related to the mechanics of solid, is the solution of set of algebraic matrix equations that approximately the relationships between loads and deflection in static analysis and velocity, acceleration, and time in dynamic analysis.

Finite element techniques have been used successfully to model many types of elements in a concrete structure. Generally, finite element analysis will only applicable to two conditions as stated below:

- 1. Any material in the state of static
- 2. Any material which comply to the Hooke's Law

More recently, FEA application to concrete structures have improved remarkably due to research and advances in computer technology. FEA is ready to become a sufficiently practical tool for researching, designing, maintaining, and upgrading common constructed facilities. The application of FEA is in great demand as the analysis method involves visualization of the user and the result is much easier to interpret and understand. The FEA using software available in market should be tested and verified thoroughly against experimental data before full confidence can put on the reliability of the software.

One of the most important aspects of finite element modeling is the mesh design. Strain gradients across first order element are linear, which means if the mesh is too coarse then complex areas of the structures are nit modeled accurately. If the mesh size is too small, however, the number of constraints within the model will increase, this reduces deformations and increase computational costs and time. To achieve a successful model it is essential to vary the mesh in certain areas, this mesh refinement should take place in regions such as compression zones and others areas of complex behavior.

2.2 Finite Element Analysis by Other Researchers

A number of finite element models to study the behavior of concrete have been proposed by various researchers. Hamil, Baglin and Scott (1999) of University of Durham, U.K., had successfully modeled sixteen specimens of reinforced concrete beam-column connection using nonlinear finite element techniques.

Dr. Paul S. Baglin is a Senior Research Assistant at the University of Dundee had past experiences using finite element method from his doctorate, which is involved the modeling of plate reinforced concrete beams.

The paper titled "Advanced Finite Element Analysis in Structural Design" by D Iosif (2001) discussed the use of numerical methods (Finite Element Modelling) in the civil and industrial design. The author used finite element modeling to specify the behavior of finite element using stress-resultant. He also had mentioned in the paper that three representative studies are presented to illustrate the versatility of Finite Element Analysis (FEA) in solving problems associated with faults in structural design, design optimization and standard code compliance.

Shanmugam, Kumar, and Thevendran (2002) had been doing researched on double skin composite slabs (DSC). The paper deals with finite element modeling of the ultimate load behavior of DSC slabs. The study is based on the finite element analysis using ABAQUS. Twelve simply supported slabs were tested to failure under a concentrated load applied at the center. The results have shown that these slabs displayed a high degree of flexural characteristic, ultimate strength, and ductility. Close agreement has been observed between the finite element and experimental results for the ultimate loads and load-deflection responses. The finite element model was thus found to be capable of predicting the behavior of DSC slabs accurately. There are still plenty of ongoing researches on concrete structures using finite element analysis by researches all over the world.

2.3 Material Properties

A clear understanding of the way in which the component aggregate and mortar react to the applied load is an essential preliminary to full analysis of an element. One of the important properties is the stress-strain relationship, which must of necessity be the product of carefully controlled experiment. These experiment results are not generally suited to direct application and so simplification and idealization are adopted in practice. The use of computer has made it possible to reduce the amount of idealization required so that a complex analysis can be performed using material stress-strain relationship.

2.4 Finite Element Analysis of Cracking Behavior

A brief review of previous studies on the application of the finite element method to the analysis of reinforced concrete structures is presented is this section. A more detailed description of the underlying theory and the application of the finite element method to the analysis of linear and nonlinear reinforced concrete structures is presented in excellent state of-the-art reports by the American Society of Civil Engineers in 1982 (ASCE 1982) and 1985 (Meyer and Okamura, eds. 1985).

The earliest publication on the application of the finite element method to the analysis of RC structures was presented by Ngo and Scordelis (1967). In their study, simple beams were analyzed with a model in which concrete and reinforcing steel were represented by constant strain triangular elements, and a special bond link element was used to connect the steel to the concrete and describe the bond-slip effect. A linear elastic analysis was performed on beams with predefined crack patterns to determine principal stresses in concrete, stresses in steel reinforcement and bond stresses. Since the publication of this pioneering work, the analysis of reinforced concrete structures has enjoyed a growing interest and many publications have appeared. Scordelis et al. (1974) used the same approach to study the effect of shear in beams with diagonal tension cracks and accounted for the effect of stirrups, dowel shear, aggregate interlock and horizontal splitting along the reinforcing bars near the support.

Nilson (1972) introduced nonlinear material properties for concrete and steel and a nonlinear bond-slip relationship into the analysis and used an incremental load method of nonlinear analysis. Four constant strain triangular elements were combined to form a quadrilateral element by condensing out the central node. Cracking was accounted for by stopping the solution when an element reached the tensile strength, and reloading incrementally after redefining a new cracked structure. The method was applied to concentric and eccentric reinforced concrete tensile members who were subjected to loads applied at the end of the reinforcing bars and the results were compared with experimental data.

Franklin (1970) advanced the capabilities of the analytical method by developing a nonlinear analysis which automatically accounted for cracking within finite elements and the redistribution of stresses in the structure. This made it possible to trace the response of two dimensional systems from initial loading to failure in one continuous analysis. Incremental loading with iterations within each increment was used to account for cracking in the finite elements and for the nonlinear material behavior. Franklin used special frame-type elements, quadrilateral plane stress elements, axial bar members, two-dimensional bond links and tie links to study reinforced concrete frames and RC frames coupled with shear walls.

Plane stress elements were used by numerous investigators to study the behavior of reinforced concrete frame and wall systems. Nayak and Zienkiewicz (1972) conducted two dimensional stress studies which include the tensile cracking and the elasto-plastic behavior of concrete in compression using an initial stress approach. Cervenka (1970) analyzed shear walls and spandrel beams using an initial stress approach in which the elastic stiffness matrix at the beginning of the entire analysis is used in all iterations. Cervenka proposed a constitutive relationship for the composite concrete-steel material through the uncracked, cracked and plastic stages of behavior.

For the analysis of RC beams with material and geometric nonlinearities Rajagopal (1976) developed a layered rectangular plate element with axial and bending stiffness in which concrete was treated as an orthotropic material. RC beam and slab problems have also been treated by many other investigators (Lin and Scordelis 1975; Bashur and Darwin 1978; Rots et al. 1985; Barzegar and Schnobrich 1986; Adeghe and Collins 1986; Bergmann and Pantazopoulou 1988; Cervenka et al. 1990; Kwak 1990) using similar methods.

Selna (1969) analyzed beams and frames made up of one-dimensional elements with layered cross sections which accounted for progressive cracking and changing material properties through the depth of the cross section as a function of load and time. Significant advances and extensions of the finite element analysis of reinforced concrete beams and frames to include the effects of heat transfer due to fire, as well as the time-dependent effects of creep and shrinkage, were made by Becker and Bresler (1974).

Two basically different approaches have been used so far for the analysis of RC slabs by the finite element method: the modified stiffness approach and the layer approach. The former is based on an average moment-curvature relationship which reflects the various stages of material behavior, while the latter subdivides the finite element into imaginary concrete and steel layers with idealized stress-strain relations for concrete and reinforcing steel.

Experimental and analytical studies of RC slabs were conducted by Joffriet and McNeice (1971). The analyses were based on a bilinear moment-curvature relation who was derived from an empirically determined effective moment of inertia of the cracked slab section including the effect of tension stiffening. The change in bending stiffness of the elements due to cracking normal to the principal moment direction is accounted for by reducing the flexural stiffness of the corresponding element.

Dotroppe et al. (1973) used a layered finite element procedure in which slab elements were divided into layers to account for the progressive cracking through the slab thickness. Scanlon and Murray (1974) have developed a method of incorporating both cracking and time-dependent effects of creep and shrinkage in slabs. They used layered rectangular slab elements which could be cracked progressively layer by layer, and assumed that cracks propagate only parallel and perpendicular to orthogonal reinforcement. Lin and Scordelis (1975) utilized layered triangular finite elements in RC shell analysis and included the coupling between membrane and bending effects, as well as the tension stiffening effect of concrete between cracks in the model. The finite element analysis of an axis-symmetric solid under axis-symmetric loading can be readily reduced to a two-dimensional analysis. Bresler and Bertero (1968) used an axis-symmetric model to study the stress distribution in a cylindrical concrete specimen reinforced with a single plain reinforcing bar. The specimen was loaded by applying tensile loads at the ends of the bar.

In one of the pioneering early studies Rashid (1968) introduced the concept of a "smeared" crack in the study of the axis-symmetric response of pre-stressed concrete reactor structures. Rashid took into account cracking and the effects of temperature creep and load history in his analyses. Today the smeared crack approach of modeling the cracking behavior of concrete is almost exclusively used by investigators in the nonlinear analysis of RC structures, since its implementation in a finite element analysis program is more straightforward than that of the discrete crack model. Computer time considerations also favor the smeared crack model in analyses which are concerned with the global response of structures. At the same time the concerted effort of many investigators in the last 20 years has removed many of the limitations of the smeared crack model (ASCE 1982; Meyer and Okamura, eds. 1985).

Gilbert and Warner (1978) used the smeared crack model and investigated the effect of the slope of the descending branch of the concrete stress-strain relation on the behavior of RC slabs. They were among the first to point out that analytical results of the response of reinforced concrete structures are greatly influenced by the size of the finite element mesh and by the amount of tension stiffening of concrete. Several studies followed which corroborated these findings and showed the effect of mesh size (Bazant and Cedolin 1980; Bazant and Oh 1983; Kwak 1990) and tension stiffening (Barzegar and Schnobrich 1986; Leibengood et al. 1986) on the accuracy of finite element analyses of RC structures with the smeared crack model. In order to better

account for the tension stiffening effect of concrete between cracks some investigators have artificially increased the stiffness of reinforcing steel by modifying its stress-strain relationship (Gilbert and Warner 1977). Others have chosen to modify the tensile stress-strain curve of concrete by including a descending post-peak branch (Lin and Scordelis 1975; Vebo and Ghali 1977; Barzegar and Schnobrich 1986; Abdel Rahman and Hinton 1986).

In the context of the smeared crack model two different representations have emerged: the fixed crack and the rotating crack model. In the fixed crack model a crack forms perpendicular to the principal tensile stress direction when the principal stress exceeds the concrete tensile strength and the crack orientation does not change during subsequent loading. The ease of formulating and implementing this model has led to its wide-spread used in early studies (Hand et al. 1973; Lin and Scordelis 1975). Subsequent studies, however, showed that the model is associated with numerical problems caused by the singularity of the material stiffness matrix. Moreover, the crack pattern predicted by the finite element analysis often shows considerable deviations from that observed in experiments (Jain and Kennedy 1974).

The problems of the fixed crack model can be overcome by introducing a cracked shear modulus, which eliminates most numerical difficulties of the model and considerably improves the accuracy of the crack pattern predictions. The results do not seem to be very sensitive to the value of the cracked shear modulus (Vebo and Ghali 1977; Barzegar and Schnobrich 1986), as long as a value which is greater than zero is used, so as to eliminate the singularity of the material stiffness matrix and the associated numerical instability. Some recent models use a variable cracked shear modulus to represent the change in shear stiffness, as the principal stresses in the

concrete vary from tension to compression (Balakrishnan and Murray 1988; Cervenka et al. 1990).

De Borst and Nauta (1985) have proposed a model in which the total strain rate is additively decomposed into a concrete strain rate and a crack strain rate. The latter is, in turn, made up of several crack strain components. After formulating the twodimensional concrete stress-strain relation and transforming from the crack direction to the global coordinate system of the structure, a material matrix with no coupling between normal and shear stress is constructed. In spite of its relative simplicity and ease of application, this approach still requires the selection of a cracked shear modulus of concrete.

In the rotating crack model proposed by Cope et al. (1980) the crack direction is not fixed during the subsequent load history. Several tests by Vecchio and Collins (1982) have shown that the crack orientation changes with loading history and that the response of the specimen depends on the current rather than the original crack direction. In the rotating crack model the crack direction is kept perpendicular to the direction of principal tensile strain and, consequently, no shear strain occurs in the crack plane. This eliminates the need for a cracked shear modulus. A disadvantage of this approach is the difficulty of correlating the analytical results with experimental fracture mechanics research, which is at odds with the rotating crack concept. This model has, nonetheless, been successfully used in analytical studies of RC structures whose purpose is to study the global structural behavior, rather than the local effects in the vicinity of a crack (Gupta and Akbar 1983; Adeghe and Collins 1986). While the response of lightly reinforced beams in bending is very sensitive to the effect of tension stiffening of concrete, the response of RC structures in which shear plays an important role, such as over-reinforced beams and shear walls, is much more affected by the bond-slip of reinforcing steel than the tension stiffening of concrete. To account for the bond slip of reinforcing steel two different approaches are common in the finite element analysis of RC structures. The first approach makes use of the bond link element proposed by Ngo and Scordelis (1967). This element connects a node of a concrete finite element with a node of an adjacent steel element. The link element has no physical dimensions, i.e. the two connected nodes have the same coordinates.

The second approach makes use of the bond-zone element developed by de Groot et al. (1981). In this element the behavior of the contact surface between steel and concrete and of the concrete in the immediate vicinity of the reinforcing bar is described by a material law which considers the special properties of the bond zone. The contact element provides a continuous connection between reinforcing steel and concrete, if a linear or higher order displacement field is used in the discretization scheme. A simpler but similar element was proposed by Keuser and Mehlhorn (1987), who showed that the bond link element cannot represent adequately the stiffness of the steel-concrete interface.

Even though many studies of the bond stress-slip relationship between reinforcing steel and concrete have been conducted, considerable uncertainty about this complex phenomenon still exists, because of the many parameters which are involved. As a result, most finite element studies of RC structures do not account for bond-slip of reinforcing steel and many researchers express the opinion that this effect is included in the tension-stiffening model. Very little work has been done, so far, on the three-dimensional behavior of reinforced concrete systems using solid finite elements, because of the computational effort involved and the lack of knowledge of the material behavior of concrete under three dimensional stress states. Suidan and Schnobrich (1973) were the first to study the behavior of beams with 20-node three-dimensional isoparametric finite elements. The behavior of concrete in compression was assumed elasto-plastic based on the von-Mises yield criterion. A coarse finite element mesh was used in these analyses for cost reasons.

In spite of the large number of previous studies on the nonlinear finite element analysis of reinforced concrete structures, only few conclusions of general applicability have been arrived at. The inclusion of the effects of tension stiffening and bond-slip is a case in point. Since few rational models of this difficult problem have been proposed so far, it is rather impossible to assess exactly what aspects of the behavior are included in each study and what the relative contribution of each is. Similar conclusions can be reached with regard to other aspects of the finite element analysis. Even though the varying level of sophistication of proposed models is often motivated by computational cost considerations, the multitude of proposed approaches can lead to the conclusion that the skill and experience of the analyst is the most important aspect of the study and that the selection of the appropriate model depends on the problem to be solved.

Recognizing that many of the previously proposed models and methods have not been fully verified so far, it is the intent of this study to address some of the model selection issues, in particular, with regard to the effects of tension-stiffening and bondslip.

2.5.1 Behavior of Concrete

Concrete exhibits a large number of micro cracks, especially at the interfaces between coarse aggregate and mortar even before subjected to any load. The presence of these micro cracks has a great effect on the mechanical behavior of concrete, since their propagation during loading contributes to the nonlinear behavior at low stress levels and causes volumes expansion near failure. Many of these micro cracks are caused by segregation, shrinkage or thermal expansion of the mortar. Some micro cracks may develop during loading because of the difference stiffness between aggregate and mortar.

Since the aggregate-mortar interfaces has a significantly lower tensile strength than mortar. It constitutes the weakest link in the composite system. This is the primary reasons for the low tensile strength of concrete.

The response of the structure under load depends to a large extent on the stressstrain relation of the constituent materials and the magnitude of stress. Since the concrete is used mostly in compression, the stress-strain relation in compression is of primary interest. Such a relation can be obtained from cylinder test with a height to diameter ratio of two or from strain measurement in beams.

The concrete stress-strain relation exhibits nearly linear elastic response up to about 30% of the compressive strength. This is followed by gradual softening up to the concrete compressive strength when the material stiffness drops to zero. Beyond the compressive strength the concrete stress-strain relation exhibits strain softening until failure takes place by crushing.

2.6 Concrete Properties

All concrete elements except those in the top layer were modeled with the brittle cracking function. This, model considered the concrete behavior to be predominantly governed by tensile cracking. As mention earlier, the model also assumed that the compressive behavior was always linear elastic. This assumption was consistent with the observation of slab behavior during the test of where the concrete did not fail by crushing but rather separated by excessive tensile cracking due to slip.

The same observation was reported by Luttrell (1987). The concrete element in the top layer was assigned a linear elastic property where no cracking was allowed. This was done to avoid convergence problems where the model could become numerically unstable before reaching ultimate load if the crack was allowed all the way up to the top most layers.

The basic concrete properties and brittle cracking parameters were assumed based on various literature and the final values were fixed after an admissible result was obtained.

Concrete Properties	Values
Density	2400 kg/m ³
Elastic Modulus	24.8 GPa
Poisson Ratio	0.2
Cracking Failure Test	2.07 MPa
Mode I Fracture Energy	73.56 N/m
Direct Cracking Failure Displacement	1.27x10 ⁻⁵ m

Table 1: Concrete Properties

2.7 Behavior of Cracked Concrete

2.7.1 Description of a Cracked Section

The nonlinear response of concrete is often dominated by progressive cracking which results in localized cracked. Figure 2.1 depicts parts of reinforced concrete member in flexure. The member has cracked at discrete location where the concrete tensile strength was exceeded.

At the cracked section all tension is carried by the steel reinforcement. Tensile stresses are however present in the concrete between the cracks, since some tension is transferred from steel to concrete through bond. The magnitude and distribution of bond stresses between the cracks determines and distribution of tensile stresses in the concrete and reinforcing steel between the cracks.

Additional cracks can form between the initial cracks, if the tensile stress exceeds the concrete tensile strength between previously formed cracks. The final cracking state is reached when a tensile force of sufficient magnitude to form an additional crack between two existing cracks can be longer be transferred by bond from steel of concrete show the idealized distribution between cracks of bond stress, concrete tensile stress and steel stress respectively. Because of concrete is carrying some tension between the cracks the flexural rigidity is clearly greater between the cracks than at the cracks as shown Figure 2.1.



Figure 1: Distribution of Cracking

- (A). Portion of Beam
- (B). Bending Moment Distribution
- (C). Bond Stress Distribution
- (D). Concrete Tensile Stress Distribution
- (E). Steel Tensile Stress
- (F). Flexural Stiffness Distribution in Elastic Range

CHAPTER III

METHODOLOGY

3.1 Introduction

In most finite element software package like ABAQUS, ADINA, COSMOS/M, DIANA, LS-DYNA, LUCAS, STAAD-PRO, and NASTRAN provide different types of element for one-, two- or three-dimensional problems such as plane stress, plane strain, three-dimensional solid elements, straight and curved beams, and shell element. These programs are easily available in the market.

STAAD-PRO (Version 2004) has been chosen for the purpose of analyzing the concrete beam with polyvinyl alcohol fiber in this project due to its flexibility in geometry and material modeling. The chapter describes step by steps of the modeling procedures, includes all the parameters in the analysis, from the geometry modeling until the determination of the element failure. There are totally three FE models that need to be modeled and analyzed in this study. The first model is purely concrete beam without PVA. This model is used as a control specimen to compare the result with other model. The second and third model is concrete beam with PVA, which are strengthening with one percent and two percent of PVA.

3.2 General Description of STAAD-PRO

STAAD-PRO (Version 2004) is a complete, modular, self-contained finite element system. The system is capable of solving linear, non-linear, static and dynamic problems, including fields of heat transfer, fluid mechanics and electromagnetic problems.

3.3 General Description of Beam Model



Figure 2: General Model of Beam

The general view of the proposed model of the beam is as illustrated in Figure 2. All the three beams were modeled with length 640 mm and depth 150 mm. the overall thickness of the beams was 100 mm. There are an amounts of PVA should be placed in the beam. The first model without the PVA, the second one would be 1% and the third model consisted of 2% of PVA.

The beam is assumed to be simply supported at two edges. Point load was use to act on the beam until the failure criteria as noted in Table 1.

3.4 Generation of Random Aggregate Models

Random Aggregate Models require the generation of an aggregate-mortar is a realistic way. Corresponding generation algorithms should, therefore, fulfill the following requirements:

- a) The location of the aggregates particles should be random within given limits, possibly also shape and size of the particles.
- b) The spatial aggregate distribution should be uniform
- c) A given distribution of the aggregates should be exactly matched
- d) A given content should be exactly matched
- e) The achievable maximum aggregates content should be as high as in the real concrete

The first requirement implies the usage of random generator. For fulfilling the requirements c) and d) it is recommendable to generate all the particles first and then, in a second step, place them is the test volume. In this way, the designated aggregate content as well as the size distribution can be exactly matched. Furthermore, this approach is conforming to the making real concrete. The most problematic requirement is the last one.

A trivial way of placing aggregate particles in the test volume is to obtain their locations purely randomly. When a particles is placed partially outside of the test volume or when it is overlapping previously allocated particles the trial location is dismissed and a new one is obtained. Although the simplicity of such a scheme is fairly attractive, the efficiency is very poor. Such mechanisms are only capable to achieve comparably low aggregate contents, and the approach is extremely redundant and timeconsuming. The resulting random aggregate model structure is very coarse idealization which in general does not resemble real concrete.

Vervuurt used an improved mechanism which simulates a process in which circular particles are dropped into a mould. If a particle touch at least one of the earlier dropped particles or if a minimum distance to them is reached its minimum distances to them is reached its position is fixed. The reachable aggregate content can be increased by repeating the "falling" process several times and taking the deepest position found. A disadvantage of this effective algorithm is that the aggregate content cannot be exactly predetermined. Another possibility of generating an aggregate-mortar structure is to discretise the test volume or the test area similar to a finite element meshing. Then, the obtained volume or area sections are shrunk forming the aggregate particles.

obtained random aggregate structure is quite different from those in real concrete samples.

A new approach for generating a realistic aggregate-mortar structure is proposed. Instead of using the algorithm to generate the aggregate, here, in this project the manual scheme for allocating the aggregate, which has been embodied in the spreadsheet, allows to generate aggregate contents comparable to the ones in real concrete and equivalent spatial distribution. In addition, the generation mechanism successfully meets all the other requirements outlined above.



Figure 3: Random Aggregate (Model I)

Figure 3 shows 2D generation of random aggregate model created by using proposed manual generation. For a minimum particle size of 10mm the maximum aggregate volume contents which could be reached were 30% for a maximum aggregate size of 40mm, 10% and 20mm for 10%. By using a personal computer the model generation can be accomplished within a few minutes.



Figure 4: Random Aggregate (Model II)

Figure 4 shows 2D generation of random aggregate model created by using manual generation. This picture is different from the previous picture based on the place of the aggregate but the volume of the aggregate remains the same.



Figure 5: Random Aggregate Model (Model III)

Figure 5 shows 2D generation of random aggregate model created by using manual generation.

3.5 Plate Element Modeling

3.5.1 Geometry Modeling

The concrete beam is modeled as a linear two-dimensional (2-D) model. The aggregate and the PVA are also modeled in two-dimensional (2-D) as shown in Figure 6. Each node has three translational degrees of freedom whereby no rotations are considered. Element type SOLID is normally used in the analysis of structural, thermal and fluid models. The three rotational degrees of freedom are constrained at each node.



Figure 6: Finite Element Analysis Model

The major difference between the capacity of 3-D solid element model and that a 2-D shell or plate element model lies on the stress states of the material under consideration. Unlike the 3-D stress states in a solid element, the normal stress along the thickness direction in a shell element is basically neglected. As a result, the shell elements are not capable of accounting for the stress wave propagation in the target thickness direction. The solid elements have to be employed especially when the influence of normal stresses on the target failure cannot be ignored. The same type of element (i.e. SOLID) is also applied to the concrete element. The concrete is modeled using a layer of eight-node solid element with isoparametric hybrid element. Figure 6 shows the concrete element. The steel reinforcement was not fully modeled in this study. The effect of steel reinforcement was considered by converting its equivalent stiffness into concrete beam stiffness. This equivalent stiffness was added into the concrete material properties to represent the cross sectional area of reinforcement and add stiffness and strength to the concrete finite element (Mosallam, 2003).



Figure 7: Finite Element Analysis Model (3-D Model)

3.6 Material Properties

Generally, there are only two materials used for this study; concrete which is consists of aggregate and cement mortar and PVA. Therefore two element groups are created in the analysis. Linear elastic is assumed for all materials. The material properties input for both group s as follows:

Element Group 1: Aggregate (Limestone)

Characteristic strength	:	93 N/mm ²
Tensile strength	:	6 N/mm ²
Modulus of elasticity	:	39.1 x 10 ⁴ N/mm ²
Poisson's ratio	:	0.2
Strain	:	0.2

Element Group 2: Aggregate (Quartzite)

Characteristic strength	:	193 N/mm ²
Tensile strength	:	9 N/mm ²
Modulus elasticity	:	56.5 x 10 ⁴ N/mm ²
Poisson's ratio	:	0.2
Strain	:	0.22

Element Group 3: Cement Mortar

Characteristic strength	:	124.4 N/mm ²
Tensile strength	:	3.1 N/mm ²
Modulus of elasticity	:	28.4 x 10 ⁴ N/mm ²
Poisson's ratio		0.2
Strain	:	1.06
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Element Group 4: Polyvinyl Alcohol Fibers

Tensile strength	:	83 N/mm ²
Tensile strength at break	:	83 N/mm ²
Modulus of elasticity	:	$15 \text{ x } 10^4 \text{ N/mm}^2$
Poisson's ratio	:	0.048

3.7 Material Curve for Concrete

Concrete can be modeled using an isotropic material model. Concrete is generally treated as an isotropic material. Isotropic model is generally used for materials that exhibit same yield and/or creep behavior in all directions. According to Shanmugam, Kumar, and Thevendran (2002), as the analysis progressed, cracking of concrete in tensile regions introduced instability in the numerical computations, which forced analysis to stop prematurely. They had adopted stress-strain behavior of concrete in compression as shown in Figure 8.



Figure 8: Stress-strain adopted by Shanmugam, Kumar, and Thevendran (2002)

The strain at which the maximum compressive stress occurs is taken as 0.002 while the strain, at which the concrete crushes, ε_u , is taken as 0.0035. Concrete is assumed to have a maximum stress equal to $0.67f_{cu}$, where f_{cu} is the cube strength of the concrete. It is approximately equal to $0.85f_c$ ', where f_c ' is the cylinder strength of concrete. The factor 0.67 accounts for co-relation of cube strength and strength of concrete in bending as in BS 8110. The model assumes that this value is reached in an elastic fashion and stress inside concrete is directly proportional to strain at the point in consideration.

3.8 Boundary Condition and Loading

After defining the material properties for aggregate (Limestone), aggregate (Quartzite), cement paste and PVA the next step modeling is defining the boundary

conditions and loads. STAAD-Pro allows all input of constraints or loads at individual nodes and element to be done directly to the selected entities. The directions of restraints and loading are interpreted with respect to the active coordinate system. Any coordinate system can be used to specify these boundary conditions. Since the beam is assumed to be simply supported along all two edges, all the nodes along the edge of the beam is fixed translational in Z direction as shown in Figure 9. All direction of the support is set as free.



Figure 9: Simply Supported Beam

Loading conditions can be applied include point forces, distributed loads and thermal loading. For this study, point load was applied over the top surface of the concrete beam as illustrated in Figure 9.

Linear static analysis was conducted on the entire three models. The intensity of point load was increased until the result of the analysis reaches the failure criteria as shown in Table 2. The analyses were terminated and stress, strain and maximum displacement of the beam was recorded.

3.9 Termination Criteria

A successful analysis must include the termination criteria or schemes. As the load increase during the analysis process, each result of computation is checked whether failure criteria are achieved. The analysis will be terminated if the failure occurs at the model. Table 2 shows the termination criteria for the analysis in this study.

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Parameter	Strengthened Beam	Unstrengthened Beam
Maximum principal strain of concrete	0.002	0.002
Maximum principal stress of PVA	83 MPa	
Maximum principal stress of concrete	-	4.00 MPa

Analysis will be terminated if any of the output result exceeded during solution procedures. The actual failure of the beam is depending on the strain and stress occurs.

3.10 Results and Its Interpretation

All available results from the linear analysis computed will be notified to the user when command "Report Wizard" is selected. Available results will be displayed in the results window. Model strains and stresses may be listed or displayed using the commands provided in the "Results Wizard" submenu. Graphical plot results may be performed to examine deformations, displacements, stresses and mode shape of the beam model. Animation function of deformation is also available. For this study, results such as maximum and minimum principal strain and stress for the concrete and PVA are needed to determine the concrete failure in cracking and PVA, respectively.

CHAPTER IV

RESULTS AND DISCUSSION

4.1 Concrete Beam (without Polyvinyl Alcohol Fiber)

Two types of failure were observed in this case. The beam either failed by tensile splitting (cracking) when the tensile strength of $0.1f_{cu}$ is reached, or by compression crushing of the concrete when the ultimate compressive strength is exceeded of $0.8f_{cu}$. Maximum tensile stress and maximum compressive stress are recorded at failure point.

Cracking is assumed to occurred is concrete elements when the maximum principal stress, P_1 , of concrete exceeded $0.1f_{cu}$ i.e. 4.0 N/mm². $0.1f_{cu}$ is the limit of concrete tensile splitting or in other name concrete cracking. Whereas if the minimum principal, P_3 , of concrete is greater than $0.8f_{cu}$ i.e. 32 N/mm², the concrete element is assumed to have failed in crushing in the compression state.



Figure 10: Maximum Tensile Strength of Concrete Beam

At the same stage, Figure 10 shows the maximum principal stress of beam at the point load has been place and also at the both support is 11.5 N/mm² Figure 11, which is lower than the ultimate compressive strength of the concrete. Therefore, concrete crushing is likely will not occur. On the other hand, the maximum principal stress was observed at the bottom of the beam which is 4.43 N/mm², which is a bit higher than the ultimate tensile strength of the concrete. Therefore, concrete failure likely will occur.



Figure 11: Maximum Compressive Strength of Concrete Beam



Figure 12: Dimension of Beam Model

Calculation

Stresses;

 $\sigma = M/Z$ Z = I/y

First load case = 20 kN

For simply supported;

	Consider beam thickness 150mm	
$M = wL^2/8$	$I = bd^3/12$ for rectangular geometry	
= 20 kN x 0.64 m	$I = 0.64 \text{ m x } 0.15^3$	
8	12	
= 1.6 kNm	$= 0.00018 \text{ m}^4$	

$Z_b = I/y$	$\sigma_b = M/Z$	
= 0.00018 m ⁴	= <u>1.6 kNm</u>	
0.14m	0.000128 m ³	
$= 0.000128 \text{ m}^3$	= 1.244 MPa	

Based on the calculation, the stress occur at the bottom of the beam is 1.244 Mpa.

The maximum ultimate moment resistant of the beam is 1.6 kNm with the same intensity of point load. The analysis shows that the stress at the beam was 1.244 MPa which was 28% less than the value from Finite Element Analysis results. The summary of the comparison is as shown in Table 3

Table 3: Result comparison of FEA and Hand Calculation Method

Analysis Method	Result	
Finite Element Analysis (FEA)	4.43 N/mm ²	
Hand Calculation Method	1.244 N/mm ²	
Different	72%	

4.2 Concrete Beam Strengthen with PVA

Cracks occurring at the interfaces between the cement paste and aggregate due to their differences in elastic modulus, thermal coefficient and response to change in moisture content when the concrete is hardened could be the source of the local material imperfection.

In pure hardened cement paste and in fine mortar a crack can develop along a plane. The small and strong particles impose minor deviations only from an ideal fracture surface. Fracture surface of normal concrete with a maximum aggregate size. As most aggregate are stronger than the cement based matrix a crack is forced to run around the inclusions. That crack, once it meets an aggregate either has to run out of the plane and leave a blank aggregate surface behind or run in the opposite direction. In the latter case the aggregate is torn out of the matrix.

Obviously, the necessary fracture energy increase with the maximum aggregate size if the distribution remains similar. In contrast to the energy consuming crack formation in normal concrete crack in high strength concrete runs through the inclusion and forms approximately a plane as observed on fine mortar and pure hardened cement paste.



Figure 13: Maximum Principal Tension Stress of Concrete Beam with PVA

Figure 13 shows that the principal stress was observed at the bottom of the beam which is 4.32 N/mm^2 , which is reduced when PVA was putted at the critical stress at the bottom of the beam.

4.3 Summary of Result

Table 4 shown the summary of result from this computational analysis:

Parameter	Without PVA	With PVA
Load (Point Load)	20 kN	20 kN
Tensile Stress	4.43 N/mm ²	4.32 N/mm ²

Table 4: Summary of Result from Finite Element Analysis

From the FEA result, the stress for beam without PVA is 4.43 N/mm^2 . For the beam with PVA the stress was reduced to 4.32 N/mm^2 .

CHAPTER V

CONCLUSION AND RECEMMENDATION FOR FUTURE WORK

The proposed beam models have been successfully modeled using finite element approach using STAAD-Pro. For the concrete beam without internally strengthened with PVA, the failure criteria is based on concrete stresses. Concrete cracking will occur if the maximum principal stress exceeds 10% of the tensile strength of concrete whereas concrete crushing will only occur if the minimum principal stress exceeds 80% of the compressive strength of concrete.

For the concrete beam internally strengthened with PVA, the failure criteria are based on aggregate, cement mortar and PVA. Concrete crushing will occur if the minimum principal strain exceeds 0.002 whereas separation between PVA and aggregates and cement mortar will only occur if the medium principal strain exceeds 0.0045.

The conclusions that can be drawn from this study are as follows:

- The strength of the strengthened beam was increased about 8 times compared to control beam. Under such loading conditions, the strengthening was successful in increasing the structural strength of the beam and also increasing the beam thickness.
- 2. The PVA strengthening enhanced the development of more evenly distributed crack pattern.
- 3. The possible failure of the strengthened beam was due to concrete crushing. The concrete crushing occurs at the bottom of the beam (maximum tensile stress) before the separation failure occurs.
- 4. The failure will not occur at the PVA due to the small tensile stress. Therefore, the strength increase of the beam is highly depending on the strength of the concrete. By improving the concrete tensile strength, the PVA capacity can be optimized and thus increase the overall efficiency of the strengthening system.

Several suggestions are proposed for future studies of this new type of the strengthening concrete slab. The suggestions are as follows:

- Study may be conducted on other common dimension of the beam. Basically, it may be done by repeating the same procedures.
- Conduct the finite element analysis with full 3-D model including the aggregate.
- Re-analysis the beam with Non-linear Finite Element Analysis to obtain the nonlinearity behavior of the strengthening system

- Re-analysis the beam by using smaller element meshing so that the higher percentage of accuracy of stresses of the FEA results can be achieved
- Small-scale models can be done in the laboratory to validate the finite element analysis. The FEA analysis can be validated with the results from the laboratory works to prove that modeling and input-output data are correct.

The use of PVA strengthening system is well-established technique in the construction industry. Therefore, efforts to develop a better save and effective way of using this strengthening system will be beneficial to the next researches.

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