Design of Active Suspension using Robust PID Controller

by

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
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Approved by,

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UNIVERSITI TEKNOLOGI PETRONAS
TRONOH, PERAK
JUNE 2010

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

Mohammad Farhan Bin Mohd Nordin

ABSTRACT

This report discusses the final analysis and basic understanding of the chosen topic, which is **Design of Active Suspension Using Robust PID Controller**. The objective of this research is to propose another method in designing an active suspension by using robust PID controller and to analyze the effectiveness of this method compared to other.

This report is divided into a few main sections for clarity and easier reference. There are **Introduction, Literature Review, Methodology and Result and Discussion.**

The Introduction part will helps the reader to have a general view on what is Robust PID Controller and how to implement it into an active suspension, in this part; it is also include the background study, problem statement, objective of study and also scope of study that being done by the writer.

The body of this report will further explain all the literature review and research that being done before regarding the same field of interest, this will include the research done by the writer on the vibration control of vehicle suspension system that have the same interest with the topic discussed in this report. The methodology part will explain on how the sequence will take place in order to complete this project within one year duration. Result and discussion shall discuss on the mathematical modeling based on two type of vehicle model that being proposed for this project. It is also included in this part the programming and simulation done in the MATLAB on how the system goes in passive suspension and also the active suspension based on two aspects which are suspension travel vs time and body mass displacement vs time. Then the validation is done by comparing the result gain from the analysis with the literature.

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LIST OF ABBREVIATIONS

<u>Notation</u>

M_1 = Quarter car body mass

M_2 = Sprung mass

K_1 = Spring Stiffness

K_2 = Tire Stiffness

C_1 = Body mass damping coefficient

C_2 = Sprung mass damping coefficient

X_1 = Body mass vertical displacement

X_2 = Sprung mass vertical displacement

W = Road profile

s = Laplace Operator

Kp = Proportional gain

Kd = Derivative gain

Ki = Integral gain

U = Actuator Force

1. INTRODUCTION

1.1 Background of Study

Active suspension system is dynamically respond to changes in the road profile because of their ability to supply energy that can be used to produce relative motion between the body and wheel. Typically active suspension systems include sensors to measure suspension variation such as body velocity, suspension displacement wheel velocity and wheel or body acceleration. Active suspension is one in which the passive components are augmented by actuators that supply additional forces that being determined by a feed back control law. There are various control strategies such as optimal state feedback, backsteeping methods, fuzzy control and more. In this paper, the research is focusing on the robust PID controller as one alternative in designing a reliable active suspension.

Several performance characteristics have to be considered in order to achieve a good suspension system. These characteristics deal with regulation of body movement, regulation of suspension movement and force distribution. Ideally the suspension should isolate the body from road disturbance and inertial disturbances associated with cornering and braking or acceleration. Furthermore the suspension should be able to minimize the vertical force transmitted to the passengers for passengers comfort. As we aware that the main problem of vehicle vibration comes from road roughness, for that reason it is necessary to find an alternative ways in controlling the vibration of the vehicle's suspension by using Robust PID controller.

Robust PID controller on the other hand is one method to obtain the feedback of certain system. The PID controller is probably the most-used feedback control design. "PID" means Proportional-Integral-Derivative, referring to the three terms operating on the error signal to produce a control signal. The desired closed loop dynamics is obtained by adjusting the three parameters K_P, K_I and K_D, often iteratively by "tuning" and without specific knowledge of a plant model. Stability can often be ensured using only the proportional term. The integral term permits the rejection of a step disturbance (often a striking specification in process control). The derivative term is used to provide damping or shaping of the response. PID controllers are the most well established class of control systems

1.2 Problem Statement

Research at the beginning had proposed a passive suspension which certainly a lot more simplified compared to the active suspension. The early design of suspension were focusing on unconstrained optimization for passive suspension which indicate the desirability of low suspension stiffness, reduced unsprung mass, and an optimum damping ratio for the best controllability. Although this optimal characteristic that being offered is an attractive choice and had been widely used, however the limitation occurred when the

1. **Suspension spring and damper do not provide energy to the suspension system, provide discomfort to the users.**

2. **Control only the motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer.**

In order to overcome this problem, active suspension had been invented.
It is also being recorded that most of the past active suspension design were developed based on the quarter model which actually suffered from a mismatched condition as the disturbance input is not in phase with the actuator input. Thus the proposed controller must be robust enough to overcome the mismatched condition so that the disturbance would not have significant effect on the performance of the system.

1.3 Objectives of Study

The main objectives of this study is to analyze and designing an active suspension by using robust PID Controller. In order to ensure the design will be able to solve the problem, the objectives are

1. **To design an active suspension which can tolerate with the road disturbance in order to give comfort to the users.**

2. **To use robust PID controller to control the suspension velocity according to the road disturbance or condition.**

A mathematical modeling will be developed based on the real active suspension by considering all the parameters inside a vehicle model. By using the mathematical modeling generated, it can be implement on the robust PID controller in order to see the result of that suspension modeling. The deliverables of this project includes a simulative study that requires the understanding of process control. This project may require the knowledge in both MATLAB and S-function. As for the initial stage, the mathematical expression that is involved in the project duration has been identified and verified.

1.4 Scope of Study

The project will cover on the mathematical modeling of vehicle suspension where the writer will analyze the modeling based on the real vehicle suspension, the study is based on the selected vehicle whereby the writer will be using the constant from this vehicle to be used as reference in the mathematical modeling. Based on that, the writer will developed the simulation based on initial constant without the implementation of the robust PID controller as the reference to compare with the design with robust PID controller. In short, it can be said that this project involves more modeling the overall system in MATLAB based on the given parameters.

2. LITERATURE REVIEW

2.1 Suspension System

The suspension system can be categorized under 3 types of suspension which are passive, semi-active and also active suspension, This is based on the external power input to the system or control bandwith (Appleyard and Wellstead, 1995)[9].A passive suspension system is a conventional system consist of a non-controlled spring and shock absorbing damper. The semi-active suspension has the same elements but the damper on the other hand has two or more selectable damping rate. An active suspension is the most advanced whereby the passive components are augmented by actuators that supply additional force. Besides these three types of suspensions system, a skyhook damper suspension system has been considered in the early design of the active suspension meanwhile inside the skyhook damper, an imaginary damper is placed between the sprung mass and the sky. The imaginary damper provides a force on the vehicle body proportional to the sprung mass absolute velocity. As a result, the sprung mass movements can be reduced without improving the tire deflections. However, this design concept is not feasible to be realized (Hrovat, 1988). Therefore, the actuator has to be placed between the sprung mass and the unsprung mass instead of the sky

2.2 Passive Suspension System

The commercial vehicles today are using passive system to control the dynamics of a vehicle's vertical motion as well as pitch and roll. Passive indicates that the suspension elements cannot supply energy to the suspension system. The passive suspension system controls the motion of the body and the wheel by limiting their relative velocities to a rate that gives the desired ride characteristics. This is achieved by using some type of damping element placed between the body and the wheels of the vehicle, such as hydraulic shock absorber. Properties of the conventional shock absorber establish the tradeoff between minimizing the body vertical acceleration and maintaining good tire-road contact force. These parameters are coupled. That is, for a comfortable ride, it is desirable to limit the body acceleration by using a soft absorber, but this allows more variation in the tire-road contact force that in turn reduces the handling performance. Also, the suspension travel, commonly called the suspension displacement, limits allowable deflection, which in turn limits the amount of relative velocity of the absorber that can be permitted. By comparison, it is desirable to reduce the relative velocity to improve handling by designing a stiffer or higher rate shock absorber. This stiffness decreases the ride quality performance at the same time increases the body acceleration, detract what is considered being good ride characteristics.

An early design for automobile suspension systems focused on unconstrained optimizations for passive suspension system which indicate the desirability of low suspension stiffness, reduced unsprung mass, and optimum damping ratio for the best controllability (Thompson, 1971). Thus the passive suspension systems, which approach optimal characteristics, had offered an attractive choice for vehicle suspension systems and had been widely used for car. However the suspension spring and damper do not provide energy to the suspension system and control only motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer. Hence, the performance of a passive suspension system is variable subject to road profiles.

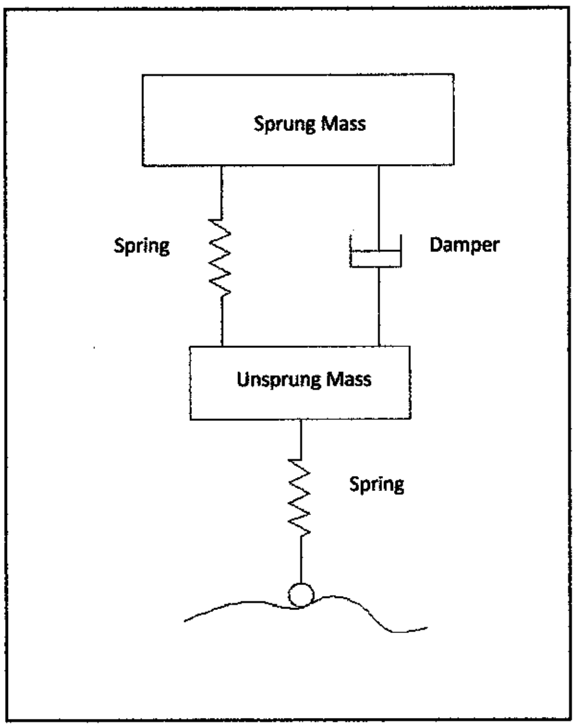


Figure 1 : Passive Suspension Sytem

2.3 Active Suspension System

Active suspension differ from the conventional passive suspensions in their ability to inject energy into the system, as well as store and dissipate it, Crolla(1988) has divided the active suspensions into two categories; the low-bandwith or soft active suspensions are characterized by an actuator that is in series with a damper and the spring. Wheel hop motion is controlled passively by the damper, so that the active function of the suspension can be restricted to body motion. Therefore, such type of suspension can only improve the ride comfort. A high-bandwith or stiff active suspension is characterized by an actuator placed in parallel with the damper and the spring. Since the actuator connects the unsprung mass to the body, it can control both the wheep hop motion as well as the body motion. The high-bandwith active suspension now can improve both the ride comfort and ride handling simultaneously. Therefore, almost all studies on the active suspension system utilized the high bandwith type.

Various types of active suspension model are reported in the literature either modeled linearly (used most) or non-linear; example are Macpherson strut suspension system (Al-Holou et al.,1999, Hong et al., 2002)[10].

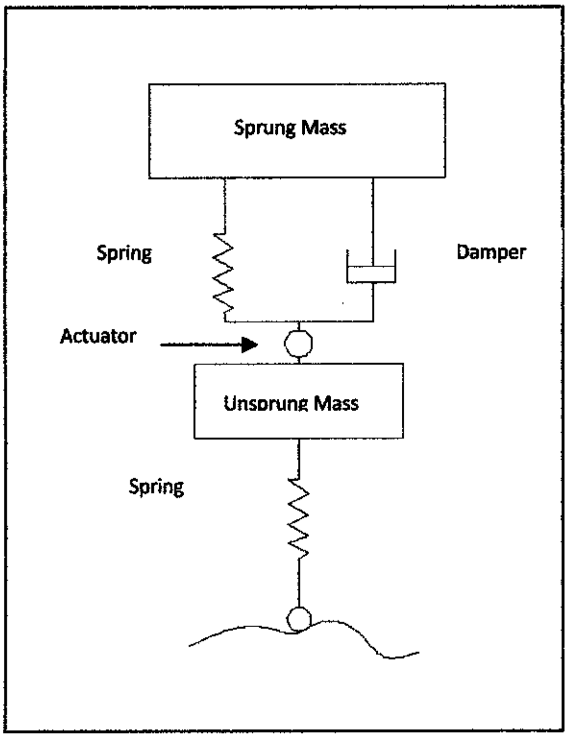


Figure 2 : A low bandwith or soft active suspension

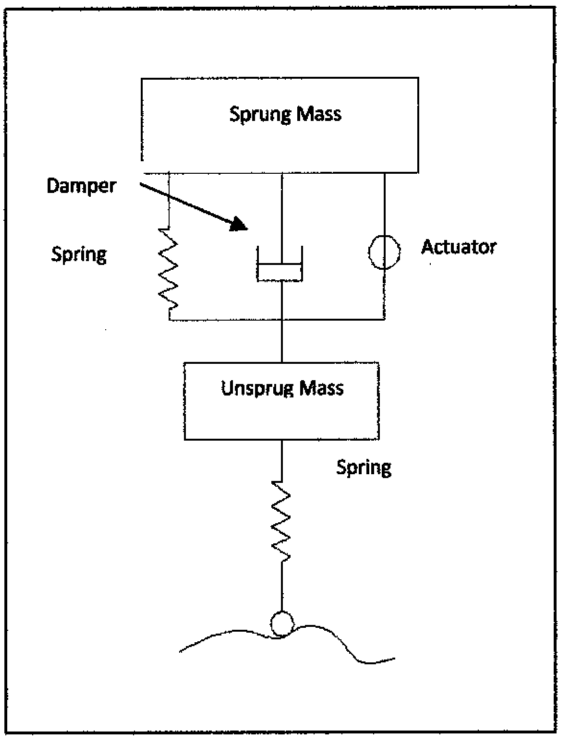


Figure 3 : A High bandwith or stiff active suspension

2.4 Vehicle Suspension System Control Strategies

In the past years, various control strategies have been proposed by numerous researchers to improve the trade-off between ride comfort and road handling. These control strategies may be grouped into techniques based on linear, nonlinear and intelligent control approaches. In the following, some of these control approaches that have been reported in the literature will be briefly presented

The most popular linear control strategy that has been used by researchers in the design of the active suspension system is based on the optimal control concept (Hrovat, 1997). Amongst the optimal concepts used are the Linear Quadratic Regulator (LQR) approach, the Linear Quadratic Gaussian (LCG) approach and the Loop Transfer Recovery (LTR) approach. These methods are based on the minimization of a linear quadratic cost function where the performance measure is a function of the states and inputs to the system.

Application of the LQR method to the active suspension system has been proposed by Hrovat(1988), Tseng and Hrovat (1990) and Esmailzadeh and Taghirad (1996)[12]. Hrovat (1988) has studied the effects of the unsprung mass on the active suspension system. The carpet plots were introduced to give a clear global view of the effect of various parameters on the systems performances, The carpet plots are the plots of the root mean square (r.m.s) values of the sprung mass acceleration and unsprung mass acceleration versus the suspension travel. The r.m.s values of all parameters are obtained from a series of simulations on different weights of the performance index. Esmailzadeh and Taghirad (1996) included the passengers's dynamics in the suspension system and the input to the system is considered as a linear force. The study utilized two approaches in selecting the performances index.

In the active suspension systems, the SMC technique was first utilized by Alleyne et al. (1993). In his work, the SMC strategy is used to control the electro-hydraulic actuator in the active suspension system. The performance of the SMC is compared to the proportional integral derivative (PID) control. The objective of the control strategy is to improve the ride quality of the vehicle using the quarter car suspension model. The ride quality is determined by observing the car body acceleration. The results showed that the proposed sliding mode controller has performed better than the PID controller in improving the ride quality but not the trade-off between ride quality and road handling.

In conclusion, there were numerous studies had been conducted regarding this system. This study involves more into deriving the mathematical expression that involves in determining the gain parameters in tuning the active suspension controller. Here are the example of studies that had been made:

" A Robust Single Input Adaptive Sliding Mode Fuzzy Logic Controller for Automative Active Suspension System"

This paper had been prepared by Ibrahim B. Kucukdemiral, Saref N. Engin, Vasfi E. Omurlu and Galip Cansever from Yildriz Technical University, Turkey, in this paper, it combines the capability of fuzzy logic with the robustness of sliding mode controller, presents prevailing results with its adaptive architecture and proves to overcome the global stability problem of the control of nonlinear systems. Effectiveness of the controller and the performance comparison are demonstrated with chosen control technique including PID and PD type self-tuning fuzzy controller on a quarter car model which consist of component-wise nonlinearities. Two major objectives that had been proposed in this paper are improve ride comfort by reducing vertical acceleration of the sprung mass and to increase holding ability of the vehicle by providing adequate suspension deflections. In this paper, it had discussing on how to model the active suspension based on the quarter car model and also the how to design a robust single-input adaptive fuzzy sliding mode controller and also the simulation result.

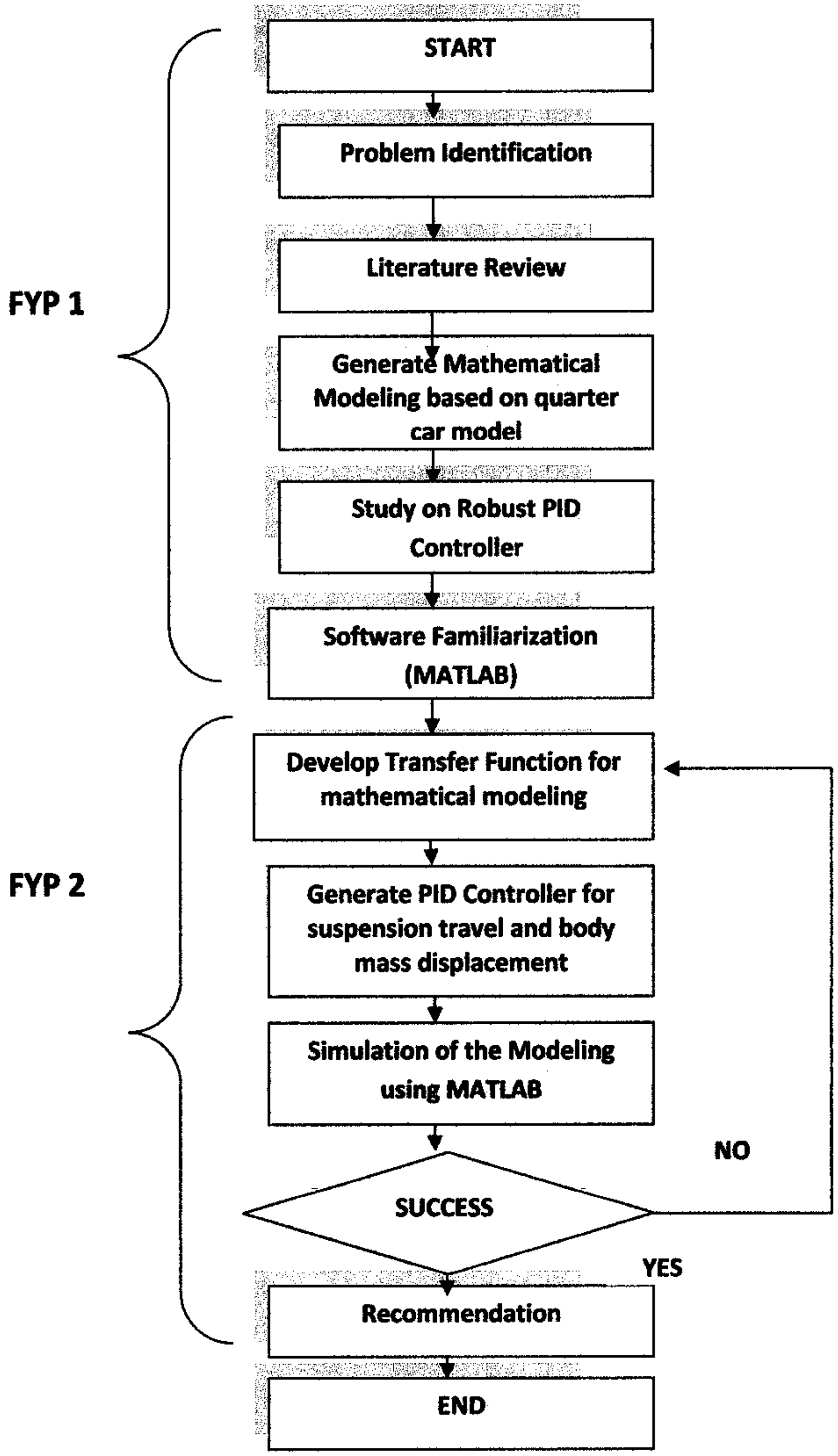
A novel single-input adaptive fuzzy sliding mode controller is proposed and can be employed to control active suspension. The strategy is robust and industry applicable since it has a single input FLC as a main controller. In order to demonstrate the effectiveness of proposed method, the controller is applied to the suspension system.

"Vibration Control of Vehicle Active Suspension System Using a New Robust Neural Network Control System"

This journal had been prepared by Ikbal Eski and Shain Yildrim from Erciyes University, Turkey, in this journal, it is stated that the main problem of vehicle vibration is comes from road roughness, thus necessary to control vibration of vehicle's suspension by using a robust artificial neural network control system scheme. Neural network based robust control system is designed to control vibration of vehicle's suspension for full suspension system. Moreover, the full vehicle system has seven degrees of freedom on the vertical direction of vehicle's chassis, on the angular variation around X-axis and on the angular variation around Y-axis.

The proposed controlled system is consisted of a robust controller, a neural controller, a model neural network of vehicle's active suspension, a standard PID controller is being used as for comparison. The author had showed how to obtain the equation for full vehicle model based on dynamic analysis.

3. METHODOLOGY



3.1 Methodology

For this project, the work has been allocated accordingly to suit both the semesters; FYP 1 and FYP 2. For FYP I, it was more emphasize on the initial works which covers the literature review, mathematical analysis and MATLAB familiarization. For the completion of project, the writer now are focusing on the FYP II progress whereby it is concentrated on developing the PID Controller on the model that being conformed to use, the writer also is now focusing on the simulation using MATLAB programming. In general, tasks that had been allocated for both FYP I and FYP II includes:

FYP I

- **Research work**
- **Literature Review**
- **Preliminary study**
- **Conceptual study**
- **Mathematical Derivation**
- **Software Familiarization**

FYP II

- **Developing Laplace equation and Transfer Function for the mathematical modeling**
- **Generate the PID controller**
- **Simulation in MATLAB**
- **PID Tuning**
- **Comparison on result between passive suspension and active suspension**
- **Validation**
- **Recommendation**

The MATLAB will be used for the simulation works and will be carried out once all the initial progress is completed.

"MATLAB is an acronym that stands for MATrix LABoratory. It is an inactive environment for performing technical computations and is considered as a standard engineering tool for most universities and industries, particularly."

Meanwhile, " Simulink features a graphical interface to numerical integration routines, where a lot of standard operations both on discrete and continuous time signals are implemented as blocks."

In addition to that, Simulink has also gained a particular impact in industry for simulating dynamical systems and designing control system.

3.2 Modeling

Based on numerous studies that had been done in completion of this project, it can be concluded that there are few models that can be used in order to simulate an active suspension design. Each of the design or model shall give us different set of variables that need to be analyze in order to get the mathematical modeling that will be use throughout the project, In short it can be said that the complex the model are, the precise the analysis will be as it is using more variables that need to be taken into account. The models that can be used are

- Full Car Model
- Half Car Model
- Quarter Car Model

In full car model analysis,in order to describe the vertical dynamics of a road vehicle which runs at a constant speed along an uneven road, 7 degrees of freedom (DOF) mathematical vehicle model is used. [I.Eski, S.Yildrim(2009)].

The analysis done were considering on the relative displacement of unsprung masses and the front left suspension mas, the rear left suspension mass, the rear right suspension mass, the front right suspension mass, the displacement of vertical motion of the vehicle, angular displacement of roll and pitch motion of the body.

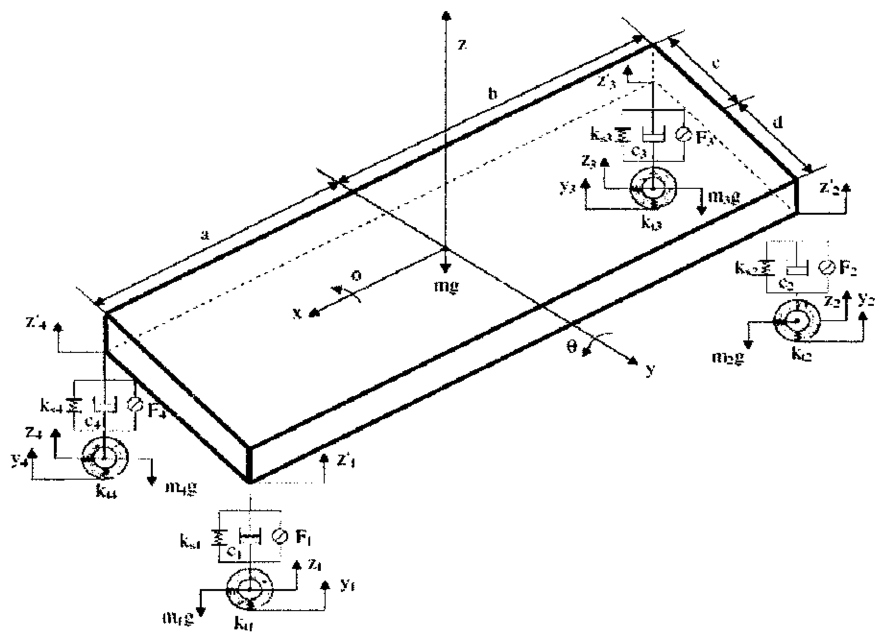


Figure 4: Full Vehicle Car Body

In half car model analysis, the controllers have been placed between sprung and unsprung masses in parallel. The vehicle model has four degrees of freedom (DOF) which are body bounce, body pitch, front wheel hop and rear wheel hop. [Yagiz and Yuksek(2001)].

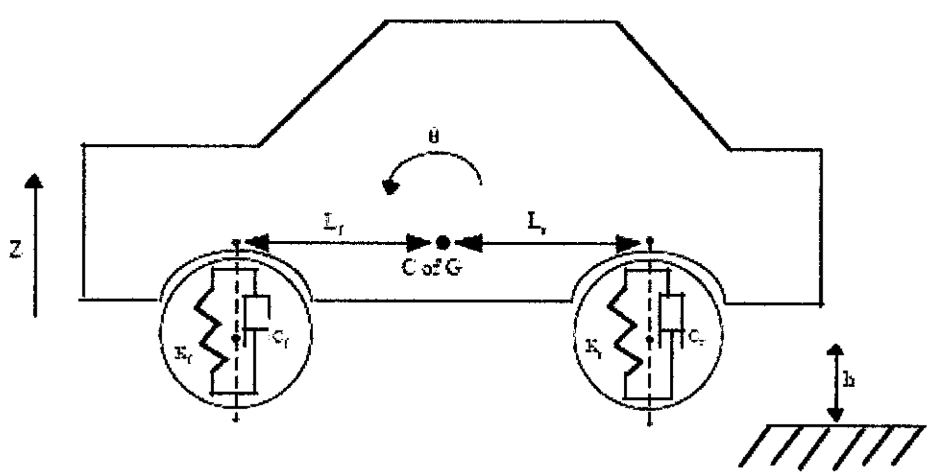


Figure 5: Half Car Model

Quarter car model can be said the simplest model among all the model that had been presented. Most of quarter car model is analyzing on two degrees of freedom (DOF) which are the vertical displacement of the sprung mass and unsprung mass. Although it have the less variables thus make it less complicated to be analyze, it is enough to develop the active suspension based on this model as it will still shows the differences between passive and active suspension, in fact, most of the past active suspension designs were develop based on this model.[Y.Md. Sam, 2004]

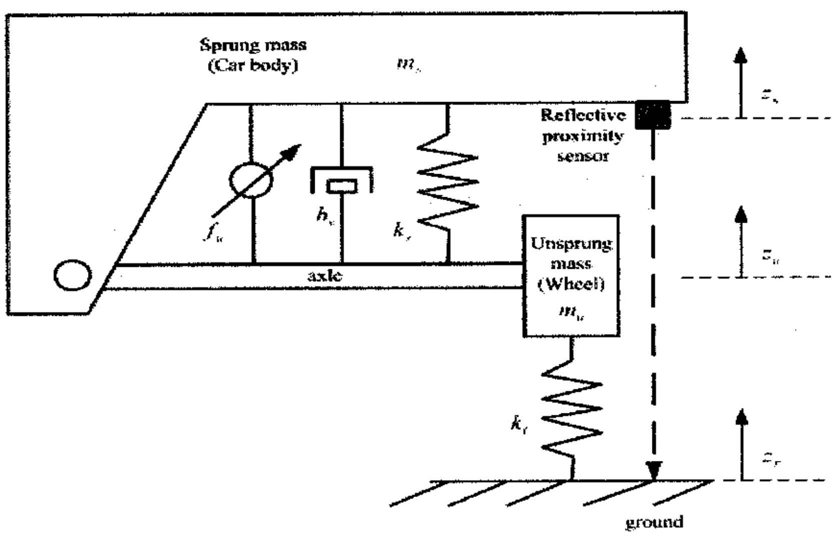


Figure 6: Quarter Car Model

3.3 Models Option

After a few discussions with FYP supervisor, the writer had come out with two most suitable designs to be use as the model for this project. The models are half car model and also quarter car model. For this first option, the writer had done the analysis for the half car model meanwhile for this option, the writer had to do the analysis on both front and rear tires, and the analysis also was based on the rotation and the translational movement of both tires after being subjected with certain road disturbance.

Option 1

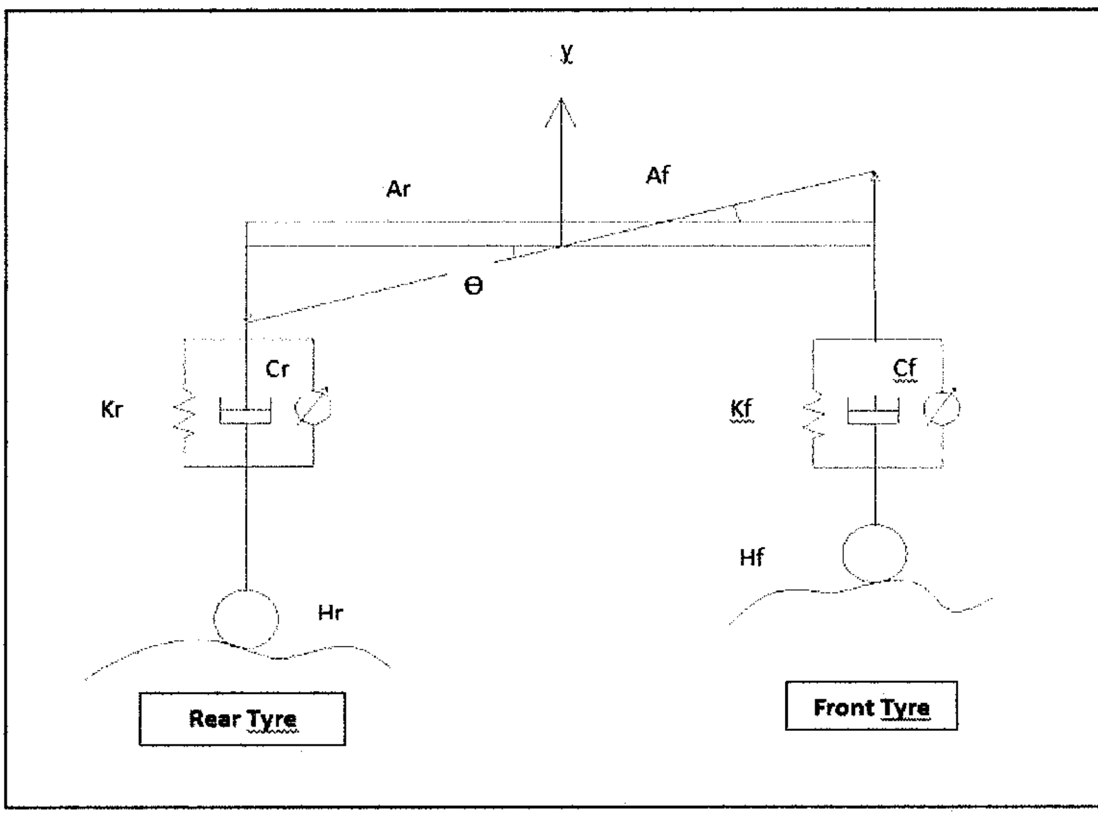


Figure 7: [Option 1]Half Car Model, 2 Degrees of Freedom

From this model, the constant to analyze such as:

Af= Length from Front Tire to Center

Ar= Length from Rear Tire to Center

Kf= Front Spring Constant

Kr= Rear Spring Constant

Hf= Front Tire Vertical Displacement

Hr= Rear Tire Vertical Displacement

Cf= Front Damper Coefficient

Cr= Rear Damper Coefficient

Option 2

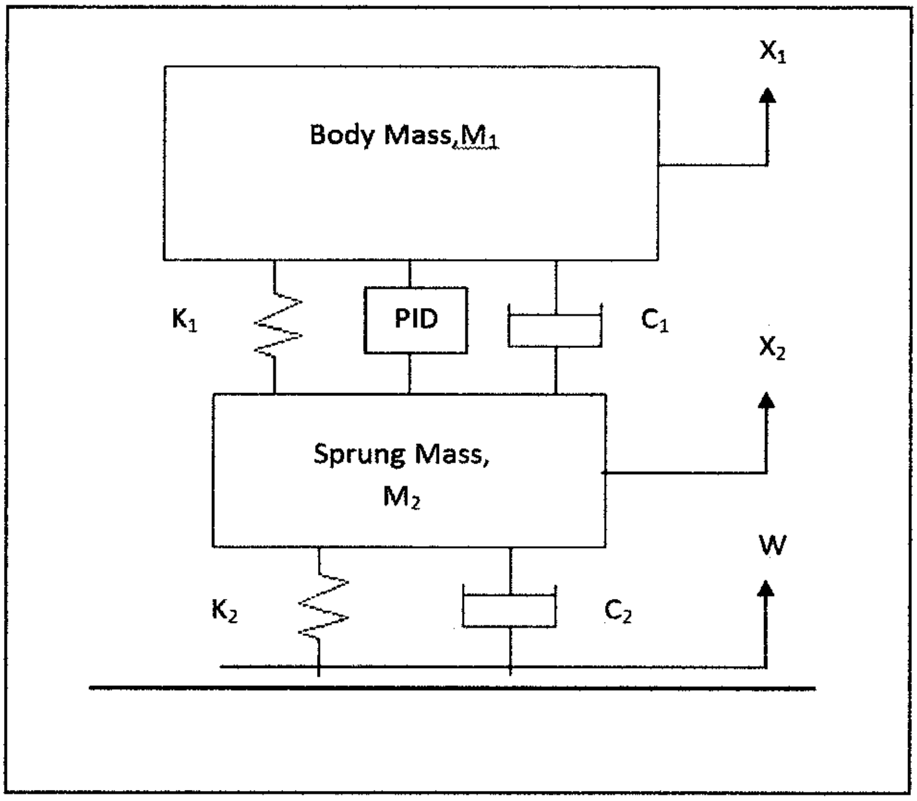


Figure 8: [Option 2] Quarter Car Model, 2 Degrees of Freedom

Based on the analysis done on both options, it is clear that for option 1, much variables need to be taken into account, thus make slightly more complicated compared to option 2, but in the other hand it provides more information thus make it more accurate and precise. Although Option 1 is able to give more accurate data, it is in the objective of this study just to analyze on how PID controller could be use to eliminate and decreasing the road disturbance, thus, it is enough to use less complicated model to analyze the response faced by the model before and after using the PID controller as the feedback controller. Thus, the writer had choose Option 2 as the model for this project

3.4 Types of Suspensions

3.4.1 Types of front suspension

3.4.1.1 Solid Beam axle

The first mass produced front suspension design was the **solid beam axle**. Just as it sounds, in the beam axle setup both of the front wheels are connected to each other by a solid axle. This style was carried over to the first automobiles from the horse drawn carriages of the past and worked well enough so that initially no other suspension even needed to be considered. In fact the beam axle can still be found today. New developments in springs, roll bars, and shocks have kept the solid axle practical for some applications[8].

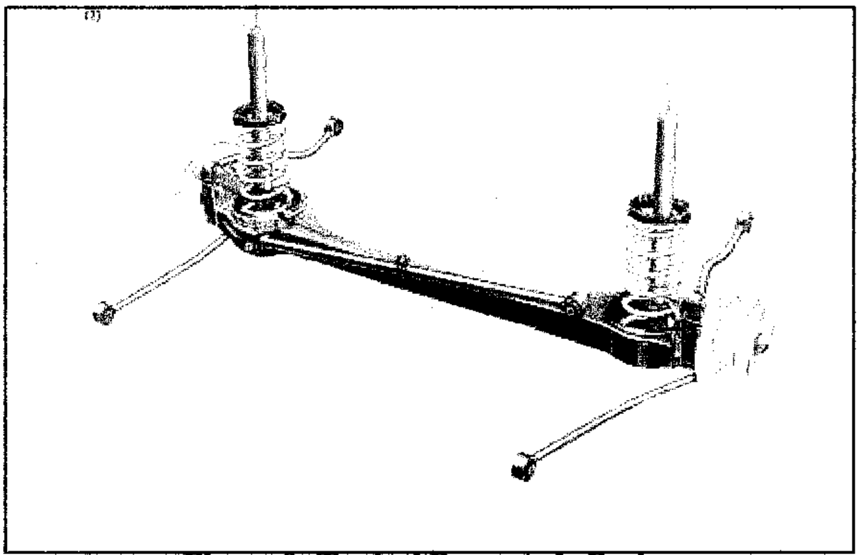


Figure 9: Typical Beam Axle Design

3.4.1.2 Swing Axle Suspensions

When the designer moved on to early attempts of designing an independent style of front suspension, one of these attempts came to be known as a **swing axle** suspension. It is, as the name suggests, set up so that the axles pivot about a location somewhere near the center of the car and allow the wheels to travel up and down through their respective arcs. This system was eventually adapted for rear suspensions as can be found on the old beetles. The disadvantage of this design is the handling problems from jacking, while it is satisfactory for some applications that can create larger cornering forces, a swing exhibits a phenomenon called jacking.Where forces act through the wheel and axle to raise the car, when this happens, the car lifts, the axle drops and there is severe loss in negative chamber, This will causes a loss in cornering power right when it is needed most[8].

3.4.1.3 Trailing Link Suspension

Another early form of front independent suspension is called the trailing link suspension. This suspension design uses a set of arms located ahead of the wheels to support the unsprung mass. In essence the wheel "trails" the suspension links. Hence the name. Since independent front suspensions were pioneered in production cars to improve the ride characteristics of vehicles as well as minimize the space needed for the suspension itself, early designs like the trailing link suspension attempted to excel in those areas of improvement. Trailing link systems like the one in the front of the old beetle were a success from the manufacturer standpoint as they did improve ride and reduce the packaging size of the suspension. However, there were some considerable drawbacks to the trailing link system when applied to vehicles that generate high cornering loads.[8]

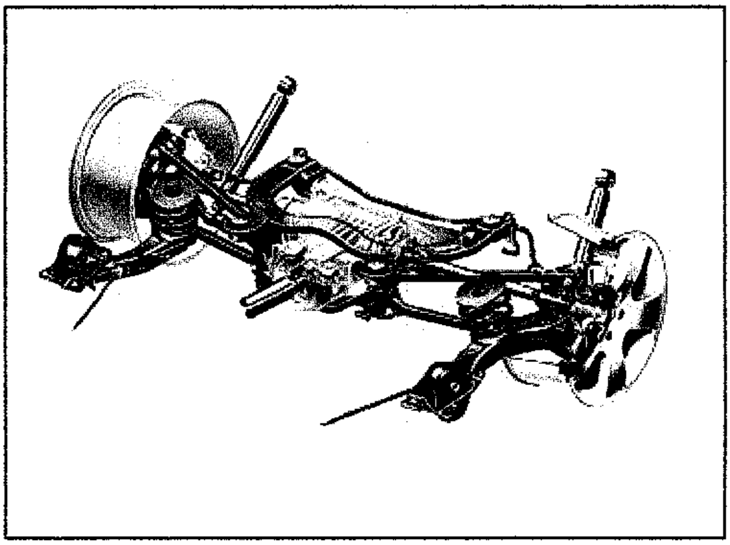


Figure 10: Trailing Link Suspension

3.4.1.4 MacPherson Front Suspension Assembly

In the 70's the MacPherson front suspension assembly became a very popular design on front wheel drive cars. This strut based system, where the spring/shock directly connects the steering knuckle to the chassis and acts as a link in the suspension, offers a simple and compact suspension package. This is perfect for small front wheel drive cars where space is tight and even allows room for the drive shaft to pass through the knuckle. Today most small cars will use this type of suspension because it is cheap, has good ride qualities, and has the compact dimensions necessary for front wheel drive cars. As with the trailing link style independent suspension, while the MacPherson assembly works very well for production road going cars, on performance cars it is less than ideal.[8]

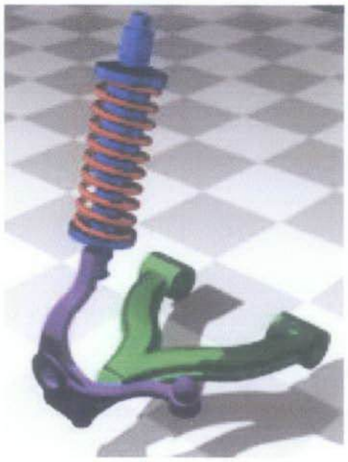


Figure 11: MacPherson Front Suspension

3.3.1.5 Double Wishbone Suspension/ Equal Length A-arm Setup

The next evolution in suspension design was to move towards the **equal length A-arm** setup. This is commonly referred to as a "double wishbone" suspension as the A shaped control arms resemble a wishbone. In this design the suspension is supported by a triangulated A-arm at the top and bottom of the knuckle. The earliest designs of the A-arm suspension included equal length upper and lower arms mounted parallel to the ground. This design has many advantages over any of the previous independent front suspension.[8]

3.4.2 Types of Active Suspension

3.4.2.1 Hydraulic actuated

Hydraulically actuated suspensions are controlled with the use of hydraulic servomechanisms. The hydraulic pressure to the servos is supplied by a high pressure radial piston hydraulic pump. Sensors continually monitor body movement and vehicle ride level, constantly supplying the computer with new data.

As the computer receives and processes data, it operates the hydraulic servos, mounted beside each wheel. Almost instantly, the servo regulated suspension generates counter forces to body lean, dive, and squat during various driving maneuvers.

In practice, the system has always incorporated the desirable self-levelling suspension and height adjustable suspension features, with the latter now tied to vehicle speed for improved aerodynamic performance, as the vehicle lowers itself at high speed.

Colin Chapman - the inventor and automotive engineer who founded Lotus Cars and the Lotus Formula One racing team - developed the original concept of computer management of hydraulic suspension in the 1980s, as a means to improve cornering in racing cars. Lotus developed a version of its 1985 Excel model with electro-hydraulic active suspension, but this was never offered to the public.

Computer Active Technology Suspension (CATS) co-ordinates the best possible balance between ride and handling by analysing road conditions and making up to 3,000 adjustments every second to the suspension settings via electronically controlled dampers.

3.4.2.2 Electromagnetic recuperative

This type of active suspension uses linear electromagnetic motors attached to each wheel independently allowing for extremely fast response and allowing for regeneration of power used through utilizing the motors as generators. This comes close to surmounting the issues with hydraulic systems with their slow response times and high power consumption. It has only recently come to light as a proof of concept model from the Bose company, the founder of which has been working on exotic suspensions for many years while he worked as an MIT professor.

Electronically controlled active suspension system (ECASS) technology was patented by the University of Texas Center for Electromechanics in the 1990s and has been developed by L-3 Electronic Systems for use on military vehicles. The ECASS-equipped HMMWV exceeded the performance specifications for all performance evaluations in terms of absorbed power to the vehicle operator, stability and handling.

4. RESULTS AND DISCUSSION

4.1 Design Parameters

In this study, the analysis is based on a model whereby the specifications needed for the constant are taken from this model.

CHASIS

SUSPENSION SYSTEM	BODY	SUSPENSION
Spring Constant, K	K_1=70 000N/m	K_2 = 450 000N/m
Damper Coefficient, C	C_1 = 400 Ns/m	C_2 = 17500Ns/m

Table 1: Suspension parameters

DIMENSIONS AND WEIGHT

Overall Length,L	3200 mm
Overall width, W	1725mm
Length from Front Tire to Center Mass, Af	1700mm
Length from Rear Tire to Center Mass, Ar	1400mm
Sprung Mass/Body Mass, M_1	3000Kg
Unsprung Mass/Suspension Mass, M_2	400Kg

Table 2: Vehicle parameters

Sources : http://www.pantaibharu.com/cars/local/car_lcl_proton_main.html

4.2 Design Requirements

For this case, the objective is to design a suspension which has below requirement:

- Overshoot = <5%
- Settling Time = <5 seconds

After being subjected with 0.08m step disturbance which means that the model will oscillate within the range +/-4mm and return to smooth ride within 5 seconds.

The specification is taken from Yokogawa SLPC-181 and 281 whereby it stated few criteria and its features.

Type	Features	Criteria
1	No overshoot	No overshoot
2	5% overshoot	ITAE minimum
3	10% overshoot	IAE minimum
4	15% overshoot	ISE minimum

Table 3: Setpoint response specification used in Yokogawa SLPC-181 and 281.

For this paper, the writer had proposed a situation when a car had been subjected with a road disturbance such as a road bump at 0.08m height. In this case the car will be replaced by the quarter car model and the step height shall be used to simulate the road disturbance.

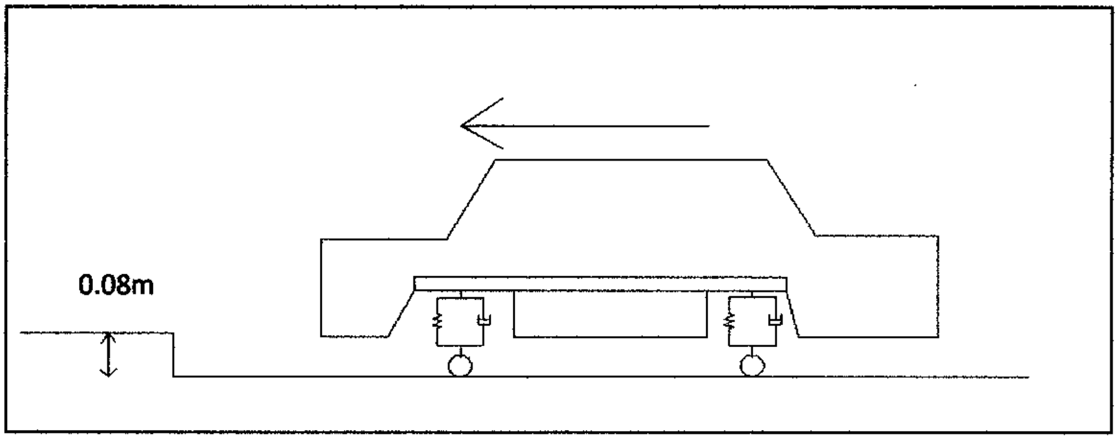


Figure 12:A car model being subjected with 0.08m step height

The writer shall be analyzing the difference between passive suspension and active suspension and also to show how PID controller could reduce the disturbance occurred. The criteria that would be taken into account are:

- **Analysis on suspension travel over time**
- **Analysis on body mass displacement over time**

4.3 Mathematical Analysis and Simulation

After having a discussion with FYP Supervisor, the writer had switches the model from using the half car model to quarter car model. This is because both of them are generating the same result by using 2 degree of freedom analysis. Since the objective of this study is to develop the PID controller in order to reduce the disturbance on the car suspension, thus it is enough to use the quarter car model as reference.

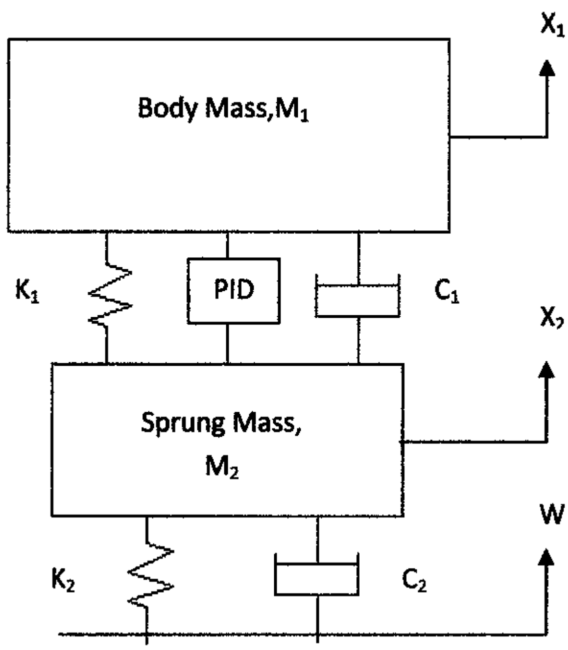
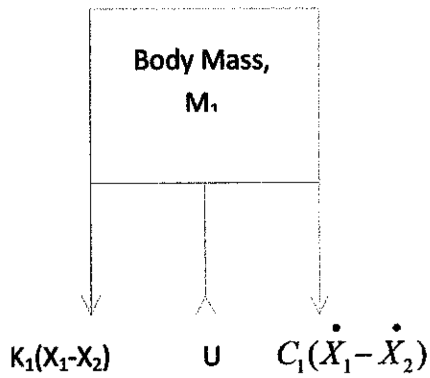


Figure 13: Quarter Car Model, 2 Degrees of Freedom

In this model, the writer are using quarter car model analysis for 2 degrees of freedom, The PID controller which acts as a feedback is located in between body mass, M_1 and Sprung Mass, M_2, this indicates that the active suspension is based on high bandwith or stiff active suspension. The type of model allows analysis in term of vertical displacement of body mass, M_1 and sprung mass, M_2.

based on above model, the mathematical analysis is as follows:



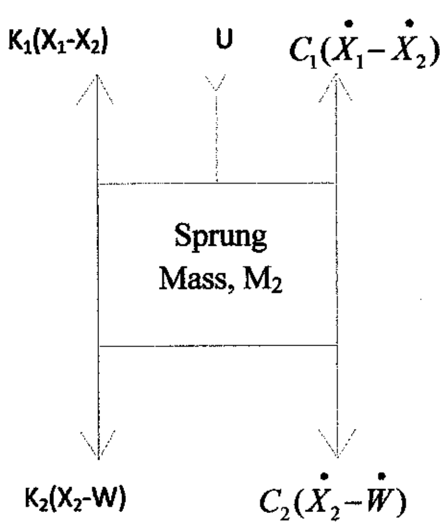
<u>Analysis on Body Mass, M_1</u>

$$+\uparrow\sum F = M_1\ddot{X}_1$$

$$-K_1(X_1-X_2)-C_1(\dot{X}_1-\dot{X}_2)+U=M_1\ddot{X}_1$$

$$M_1\ddot{X}_1+K_1(X_1-X_2)+C_1(\dot{X}_1-\dot{X}_2)-U=0$$

<u>Analysis on Suspension Mass, M_2</u>



In order to model an open loop response, the writer used state space approach

$$+\uparrow \sum F = M_2 \ddot{X}_2$$
$$K_1(X_1 - X_2) + C_1(\dot{X}_1 - \dot{X}_2) - K_2(X_2 - W) - C_2(\dot{X}_2 - \dot{W}) - U = M_2 \ddot{X}_2$$
$$M_2 \ddot{X}_2 - K_1(X_1 - X_2) - C_1(\dot{X}_1 - \dot{X}_2) + K_2(X_2 - W) + C_2(\dot{X}_2 - \dot{W}) + U = 0$$

From the above equation that we obtain, divide them with both M1 and M2 respectively.

For first equation, we obtain:

$$\ddot{X}_1 = \frac{-C_1}{M_1}(\dot{X}_1 - \dot{X}_2) - \frac{K_1}{M_1}(X_1 - X_2) + \frac{U}{M_1}$$

For second equation we obtain:

$$\ddot{X}_2 = \frac{C_1}{M_2}(\dot{X}_1 - \dot{X}_2) + \frac{K_1}{M_2}(X_1 - X_2) + \frac{C_2}{M_2}(\dot{W} - \dot{X}_2) + \frac{K_2}{M_2}(W - X_2) - \frac{U}{M_2}$$

For the third state, choose the difference between X1and X2, thus lead to **Y1=X1-X2**

$$\ddot{X}_1 = \frac{-C_1}{M_1}(\dot{Y1}) - \frac{K_1}{M_1}(Y1) + \frac{U}{M_1}$$
$$\ddot{X}_2 = \frac{C_1}{M_2}(\dot{Y1}) + \frac{C_2}{M_2}(\dot{W} - \dot{X}_2) + \frac{K_1}{M_2}(Y1) + \frac{K_2}{M_2}(W - X_2) - \frac{U}{M_2}$$

In order to obtain the expression for Y1, subtract second equation from first equation

$$\ddot{Y1} = \ddot{X1} - \ddot{X2}$$

$$\ddot{Y1} = -(\frac{C_1}{M_1} + \frac{C_1}{M_2})\dot{Y1} - (\frac{K_1}{M_1} + \frac{K_1}{M_2})Y1 - \frac{C_2}{M_2}(\dot{W} - \dot{X}_2) - \frac{K_2}{M_2}(W - X_2) + (\frac{1}{M_1} + \frac{1}{M_2})U$$

Since second derivatives cannot be used in state-space representation, integrate to get $\dot{Y1}$

$$\dot{Y1} = -(\frac{C_1}{M_1} + \frac{C_1}{M_2})Y1 - \frac{C_2}{M_2}(W - X_2) + \int(-(\frac{K_1}{M_1} + \frac{K_1}{M_2})Y1 - \frac{K_2}{M_2}(W - X_2) + (\frac{1}{M_1} + \frac{1}{M_2})U)dy$$

$\dot{Y1}$ is expressed in terms of states and input only, noted the integral as **Y2**

Thus, **Y2** is

$$\dot{Y2} = -(\frac{K1}{M1}+\frac{K1}{M2})Y1 - \frac{K2}{M2}(W - X2) + (\frac{1}{M1}+\frac{1}{M2})U$$

Since Y1=X1-X2, substitute X2 with X1-Y1

$$\dot{Y2} = -(\frac{K_1}{M_1}+\frac{K_1}{M_2})Y1 - \frac{K_2}{M_2}(W - X_1 + Y_1) + (\frac{1}{M_1}+\frac{1}{M_2})U$$

To get the state equation for **Y1**, substitute X2=X1-Y1 in $\dot{Y}1$, thus we obtain,

$$\dot{Y1} = -(\frac{C_1}{M_1}+\frac{C_1}{M_2})Y1 - \frac{C_2}{M_2}(W - X_1 + Y_1) + Y2$$

Substitute the derivative Y1 into equation of derivative X1, we obtain

$$\ddot{X1} = (\frac{-C_1C_2}{M_1M_2})X1 + (\frac{C_1}{M_1}(\frac{C_1}{M_1}+\frac{C_1}{M_2}+\frac{C_2}{M_2}) - \frac{K_1}{M_1})Y1 - (\frac{C_1}{M_1})Y2 + \frac{U}{M_1} + (\frac{C_1C_2}{M_1M_2})W$$

The states variables are X1, $\dot{X}1$, Y1 and Y2, the matrix form are:

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

$$\begin{bmatrix} \dot{X1} \\ \ddot{X1} \\ \dot{Y1} \\ \dot{Y2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-C_1C_2}{M_1M_2} & 0 & (\frac{C_1}{M_1}(\frac{C_1}{M_1}+\frac{C_1}{M_2}+\frac{C_2}{M_2}) - (\frac{K_1}{M_1})) & \frac{-C_1}{M_1} \\ \frac{C_2}{M_2} & 0 & -(\frac{C_1}{M_1}+\frac{C_1}{M_2}+\frac{C_2}{M_2}) & 1 \\ \frac{K_2}{M_2} & 0 & -(\frac{K_1}{M_1}+\frac{K_1}{M_2}+\frac{K_2}{M_2}) & 0 \end{bmatrix} \begin{bmatrix} X1 \\ \dot{X1} \\ Y1 \\ Y2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M_1} & \frac{C_1C_2}{M_1M_2} \\ 0 & \frac{-C_2}{M_2} \\ (\frac{1}{M_1}+\frac{1}{M_2}) & \frac{-K_2}{M_2} \end{bmatrix} \begin{bmatrix} U \\ W \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X1 \\ \dot{X1} \\ Y1 \\ Y2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} U \\ W \end{bmatrix}$$

4.4 Analysis on the Open Loop and Closed Loop Response for Suspension Travel, X_1-X_2

From the state space above, we can represent the open loop response when the model was being subjected with step input or without the step input. There is still no feedback controller being added up in the system, thus it is considered as open loop response. In this case step input will represent the road disturbance subjected to the quarter car model. In this case, the model is being subjected with 0.2m high step disturbance input.

In order to simulate the open loop response, the writer had programmed it using MATLAB,

```
%% Analysis of passive suspension using MATLAB

M1 = 3000; %% Body Mass
M2 = 400;%% Suspension Mass
K1= 70000;%% Body spring constant
K2= 450000;%% suspension spring constant
C1= 400;%% Body damping constant
C2= 17500;%% suspension damping constant

%% State space equation
%% Coefficient of A
A = [0,1,0,0;-(C1*C2)/(M1*M2),0,((C1/M1)*((C1/M1)+(C1/M2)+(C2/M2)))-
(K1/M1),-(C1/M1);(C2/M2),0,-((C1/M1)+(C1/M2)+(C2/M2)),1;(K2/M2),0,-
((K1/M1)+(K1/M2)+(K2/M2)),0];
%% Coefficient of B
B = [0,0;(1/M1),(C1*C2)/(M1*M2);0,-(C2/M2);(1/M1)+(1/M2),-(K2/M2)];
C = [0,0,1,0];
D = [0,0];

%% Case 1
%% Quarter Car Model without 0.08m step input
step(A,B,C,D,1)
legend('Amplitude','Time(seconds)');
title('Open Loop response to Unit step Actuated Force')

%%Case 2
%% Quarter Car Model subjected to 0.1m step input
step(A, 0.08*B,C,D,2)
legend('Amplitude','Time(seconds)');
title('Open Loop response to 0.08m step disturbance')
```

Graph 1

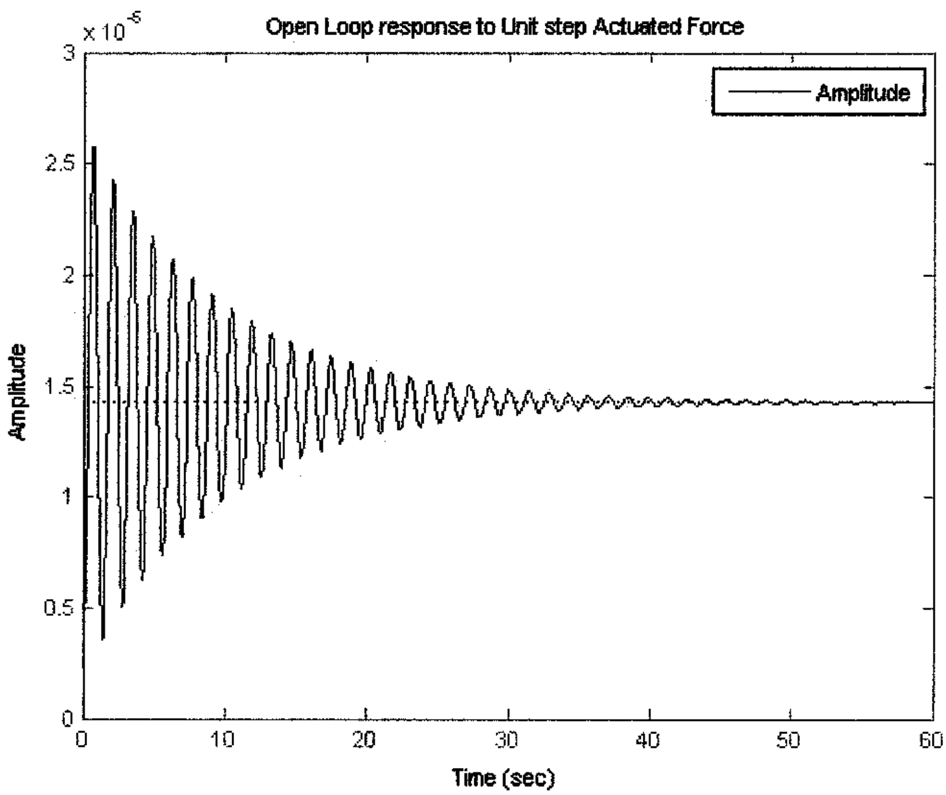
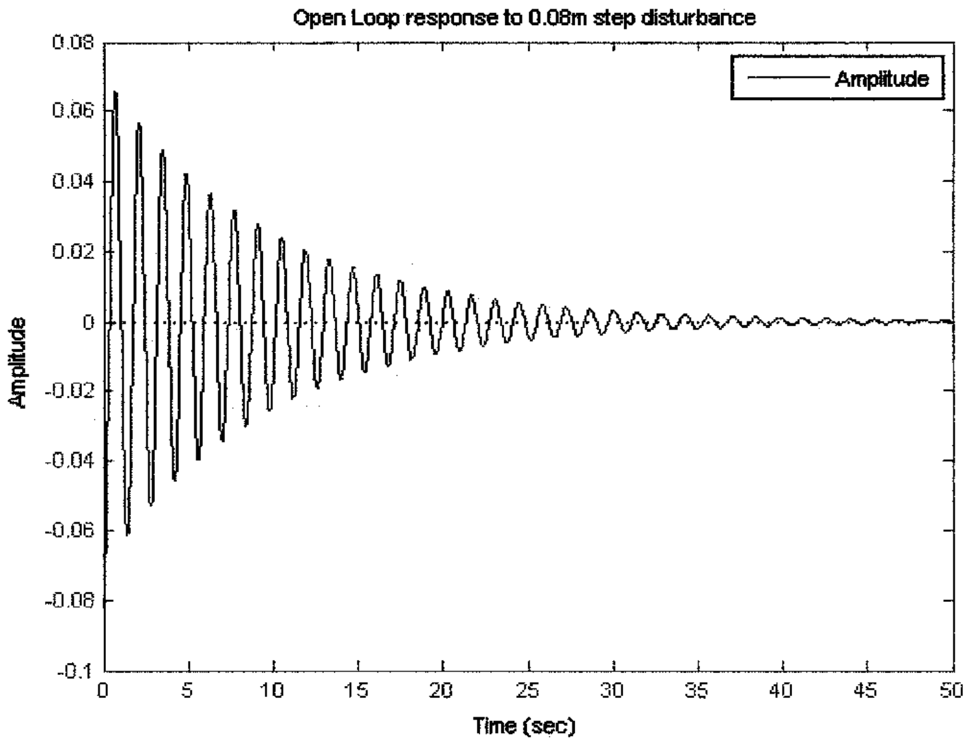


Figure 14: Open loop response to unit step actuated force

This graph show the result of open loop response without being subjected to any step disturbance, from the graph, it is observe that the system is under-damped when the unit step actuated forced is applied, we can also observe that it took a long time to reach a steady state.



Change the axis for better result:

```
axis([0 20 -0.1 0.1])
```

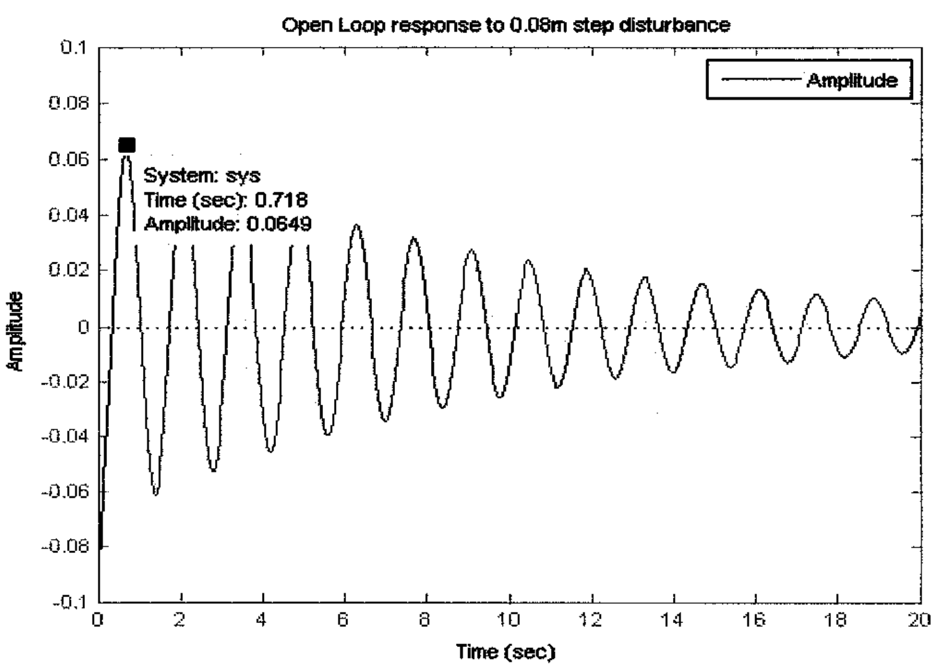


Figure 15: Open loop response with 0.08m step disturbance

From above graph, we can see that when the quarter car model is being subjected with 0.08m or 8cm bump on the road, the body of the model will be oscillated for a long time, this will caused disturbance and not a comfortable situation for the passenger inside the car. From the graph also we can observe a big overshoot and slow settling time, In order to solve this problem a feedback controller must be added up to get a small oscillation and also the oscillation can dissipated quickly.

In this work, it is focusing on using PID controller as the feedback controller. PID controller is one of the most commonly used feedback controller but yet is so reliable in giving the desired output. In order to design the PID controller for this model, first the writer has to come out with the transfer function for the model.

Transfer function can be done by doing the Laplace for the mathematical equation that we achieved before.

From the mathematical analysis, we knew that

$$M_1 \ddot{X}_1 + K_1(X_1 - X_2) + C_1(\dot{X}_1 - \dot{X}_2) - U = 0$$

$$M_2 \ddot{X}_2 - K_1(X_1 - X_2) - C_1(\dot{X}_1 - \dot{X}_2) + K_2(X_2 - W) + C_2(\dot{X}_2 - \dot{W}) + U = 0$$

By Laplace transform,

$$M_1 \ddot{X}_1 + C_1(\dot{X}_1 - \dot{X}_2) + K_1(X_1 - X_2) = U$$
$$M_1 \ddot{X}_1 + C_1 \dot{X}_1 - C_1 \dot{X}_2 + K_1 X_1 - K_1 X_2 = U$$
$$M_1 \ddot{X}_1 + C_1 \dot{X}_1 + K_1 X_1 - C_1 \dot{X}_2 - K_1 X_2 = U$$
$$(M_1 s^2 + C_1 s + K_1) X_1(s) - (C_1 s + K_1) X_2(s) = U(s)$$

$$M_2 \ddot{X}_2 - K_1(X_1 - X_2) - C_1(\dot{X}_1 - \dot{X}_2) + K_2(X_2 - W) + C_2(\dot{X}_2 - \dot{W}) + U =$$
$$M_2 \ddot{X}_2 - K_1 X_1 + K_1 X_2 - C_1 \dot{X}_1 + C_1 \dot{X}_2 + K_2 X_2 - K_2 W + C_2 \dot{X}_2 - C_2 \dot{W} + U = 0$$
$$-K_1 X_1 - C_1 \dot{X}_1 + M_2 \ddot{X}_2 + K_1 X_2 + C_1 \dot{X}_2 + K_2 X_2 + C_2 \dot{X}_2 = K_2 W + C_2 \dot{W} - U$$
$$-(C_1 s + K_1) X_1(s) + (M_2 s^2 + (C_1 + C_2)s + (K_1 + K_2)) X_2(s) = (C_2 s + K_2) W(s) - U(s)$$

The Laplace obtains from both analysis are:

$$(M_1s^2+C_1s+K_1)X_1(s)-(C_1s+K_1)X_2(s)=U(s)\text{...}(1)$$

$$-(C_1s+K_1)X_1(s)+(M_2s^2+(C_1+C_2)s+(K_1+K_2))X_2(s)=(C_2s+K_2)W(s)-U(s)\text{......}(2)$$

Arrange them in matrix form:

$$\begin{bmatrix} M_1s^2+C_1s+K_1 & -(C_1s+K_1) \\ -(C_1s+K_1) & (M_2s^2+(C_1+C_2)s+(K_1+K_2)) \end{bmatrix}\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}=\begin{bmatrix} U(s) \\ (C_2s+K_2)W(s)-U(s) \end{bmatrix}$$

$$A=\begin{bmatrix} M_1s^2+C_1s+K_1 & -(C_1s+K_1) \\ -(C_1s+K_1) & (M_2s^2+(C_1+C_2)s+(K_1+K_2)) \end{bmatrix}$$

$$\Delta = \det\begin{bmatrix} M_1s^2+C_1s+K_1 & -(C_1s+K_1) \\ -(C_1s+K_1) & M_2s_2+(C_1+C_2)s+(K_1+K_2) \end{bmatrix}$$

$$\Delta = (M_1s^2+C_1s+K_1)\bullet(M_2s^2+(C_1+C_2)s+(K_1+K_2))-(C_1s+K_1)\bullet(C_1s+K_1)$$

In order to find the transfer functions, find the inverse matrix for A and then multiple with both inputs U(s) and W(s) on the right hand side

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}=\frac{1}{\Delta}\begin{bmatrix} M_2s^2+(C_1+C_2)s+(K_1+K_2) & (C_1s+K_1) \\ (C_1s+K_1) & M_1s^2+C_1s+K_1 \end{bmatrix}\begin{bmatrix} U(s) \\ (C_2s+K_2)W(s)-U(s) \end{bmatrix}$$

Multiple with both sides

$$(M_2s^2+(C_1+C_2)s+(K_1+K_2))U(s)+(C_1s+K_1)(C_2s+K_2)W(s)-(C_1s+K_1)U(s)$$
$$\left(M_2s^2+C_1s+C_2s+K_1+K_2\right)U(s)-(C_1s+K_1)U(s)+\left(C_1C_2s^2+C_1K_2s+C_2K_1s+K_1K_2\right)W(s)$$
$$\left(M_2s^2+C_2s+K_2\right)U(s)+\left(C_1C_2s^2+C_1K_2s+C_2K_1s+K_1K_2\right)W(s)$$

$$(C_1s+K_1)U(s)+(M_1s^2+C_1s+K_1)(C_2s+K_2)W(s)-(M_1s^2+C_1s+K_1)U(s)$$
$$(C_1s+K_1)U(s)-(M_1s^2+C_1s+K_1)U(s)+\left(M_1C_2s^3+M_1K_2s^2+C_1C_2s^2+C_1K_2s+C_2K_1s+K_1K_2\right)W(s)$$
$$(-M_1s^2)U(s)+\left(M_1C_2s^3+(M_1K_2+C_1C_2)s^2+(C_1K_2+C_2K_1)s+K_1K_2\right)W(s)$$

Rearrange into matrix form,

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}=\frac{1}{\Delta}\begin{bmatrix} M_2s^2+C_2s+K_2 & \left(C_1C_2s^2+C_1K_2s+C_2K_1s+K_1K_2\right) \\ -M_1s^2 & \left(M_1C_2s^3+(M_1K_2+C_1C_2)s^2+(C_1K_2+C_2K_1)s+K_1K_2\right) \end{bmatrix}\begin{bmatrix} U(s) \\ W(s) \end{bmatrix}$$

In order to find the transfer function, zero initial condition must be assumed, thus, when we want to consider input U(s) only, set W(s) to zero, Then we get transfer function G1(s)

$$G_1(s) = \frac{X1(s) - X2(s)}{U(s)}$$
$$= \frac{(M_2 s^2 + C_2 s + K_2 - (-M_1 s^2)}{\Delta}$$
$$= \frac{M_2 s^2 + C_2 s + K_2 + M_1 s^2}{\Delta}$$
$$= \frac{(M_1 + M_2)s^2 + C_2 s + K_2}{\Delta}$$

When we want to consider input W(s) only, set U(s)=0, Thus, we get transfer function G2(s)

$$G_2(s) = \frac{X_1(s) - X_2(s)}{W(s)}$$
$$= \frac{\left(C_1C_2s^2 + C_1K_2s + C_2K_1s + K_1K_2\right) - \left(M_1C_2s^3 + (M_1K_2 + C_1C_2)s^2 + (C_1K_2 + C_2K_1)s + K_1K_2\right)}{\Delta}$$
$$= \frac{-M_1C_2s^3 - M_1K_2s^2}{\Delta}$$

$$\Delta = (M_1s^2 + C_1s + K_1)\bullet(M_2s^2 + (C_1 + C_2)s + (K_1 + K_2)) - (C_1s + K_1)\bullet(C_1s + K_1)$$
$$\Delta = (M_1M_2)s^4 + ((M_1(C_1+C_2)) + (M_2C_1))s^3 + ((M_1(K_1+K_2)) + (M_2K_1) + (C_1C_2))s^2 + ((C_1K_2) + (C_2K_1))s + K_1K_2$$

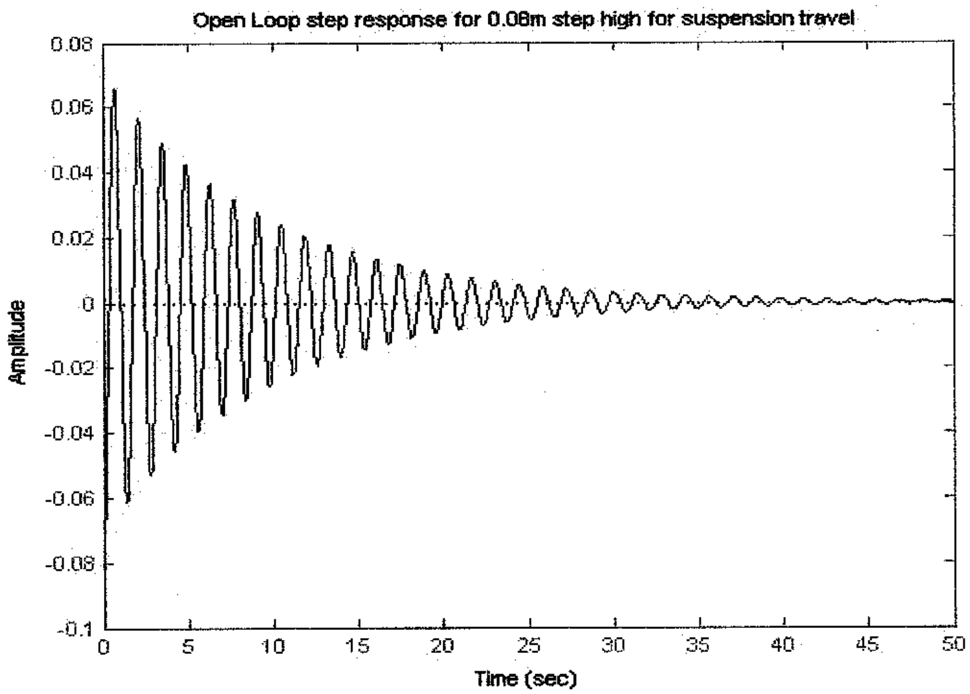
Besides using state space approach to develop the open loop response, we can also used the transfer function approach to get the same result. By Transfer Function, the MATLAB programming shall be:

```
%% define the value for each constant based on design parameters
m1=3000;%% Sprung Mass/Body Mass
m2=400;%%unsprung mass/suspension mass
k1=70000;%%spring constant for body
k2=450000;%%spring constant fo suspension
c1=400;%%Damping constant for body
c2=17500;%%Damping constant for suspension

%%define equation for each transfer function
nump=[(m1+m2) c2 k2]
denp=[(m1*m2) (m1*(c1+c2))+(m2*c1) (m1*(k1+k2))+(m2*k1)+(c1*c2)
(c1*k2)+(c2*k1) k1*k2]

num1=[-(m1*c2) -(m1*k2) 0 0]
den1=[(m1*m2) (m1*(c1+c2))+(c1*m2) (m1*(k1+k2))+(m2*k1)+(c1*c2)
(c1*k2)+(c2*k1) k1*k2]

step(0.08*num1,den1)
title('Open Loop step response for 0.08m step high for suspension
travel')
```




```
axis([0 20 -0.1 0.1])
```

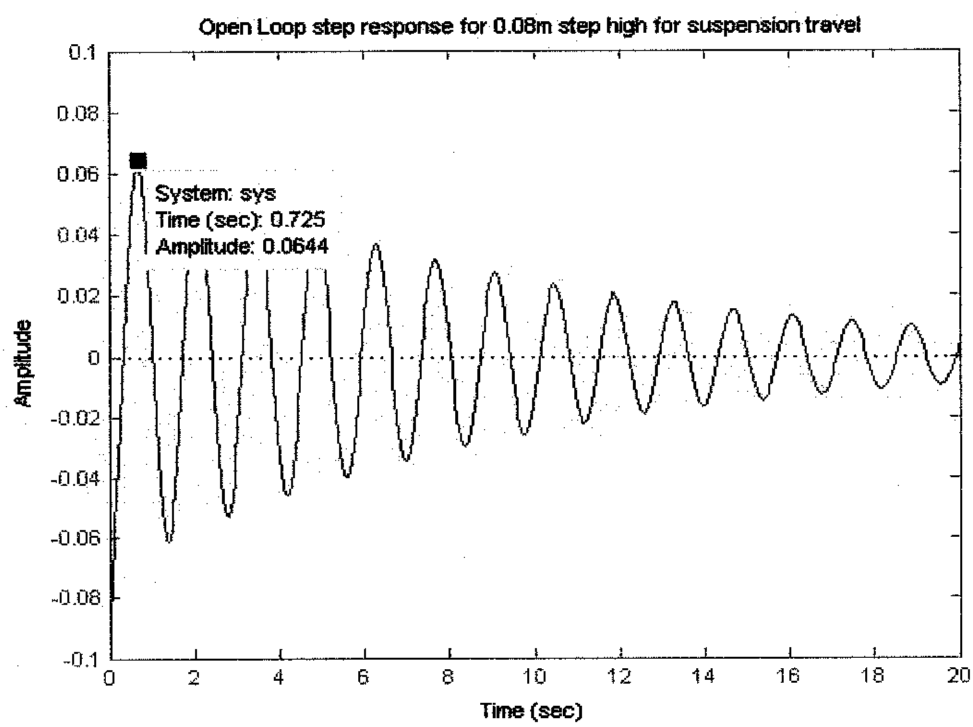


Figure 16: Open Loop Response for Suspension Travel by Transfer Function Method

From the graph above, we can see that the result gain from state space approach and transfer function approach is the same, the peak overshoot for the open loop response after being subjected with 0.08 m road disturbance is 0.0644m. Same with the result before, large oscillation and slow settling time will result in discomfort to the passenger and may cause damage to the suspension system. Thus, feedback controller must be added up to the system.

Based on above transfer functions, G1(s) and G2(s), the writer will develop the closed loop response by using Proportional, Integral and Derivative (PID) as the controller. Each element inside the PID controller will play significant role to the tuning process, in short the roles of each gain are described below [7]. From the Transfer Function, we denotes $G_1(s)$ as the actuated force input meanwhile $G_2(s)$ shall be the disturbance input

$$G_1(s)=nump/denp$$

$$G_2(s)=num1/den1$$

Proportional Control, Kp is a pure gain adjustment acting on the error signal to provide the driving input to the process. The P term in the PID – controller is used to adjust the *speed* of the system. **Integral Control, Ti,** is implemented through the introduction of an integrator. Integral control is used to provide the required accuracy for the control system. **Derivative Control, Td,** is normally introduced to increase the *damping* in the system. The derivative term also amplifies the existing noise which can cause problems including instability.

In short, controller tuning is a compromise between the requirements for fast control and the need of stable control. The stability and speed change when the PID parameter change, the changes can be summarized as follows[7]:

	Speed	**Stability**
Kp increases	increases	reduces
Ti increases	reduces	increases
Td increases	increases	increases

Table 4: Rules of thumb for the effects of the controller parameters on speed and stability in the control loop

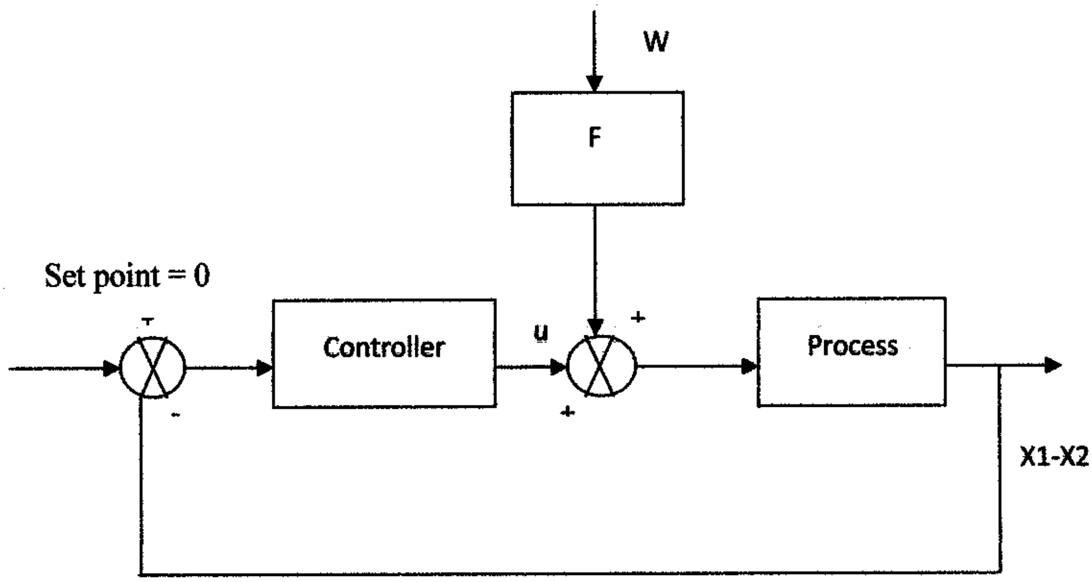


Figure 17: Schematic diagram of Closed Loop Response I

Based on developed Transfer Function G1(s) and G2(s), the schematic diagram generated is as above. F is the transfer function of road disturbance faced by the model, from schematic above

$$Process = \frac{nump}{denp}$$

$$F * Process = \frac{num1}{den1}$$

$$F * (\frac{nump}{denp}) = \frac{num1}{den1}$$

$$F = \frac{num1}{den1} \bullet \frac{denp}{nump}$$

$$den1 = denp$$

$$F = \frac{num1}{nump} = \frac{numd}{dend}$$

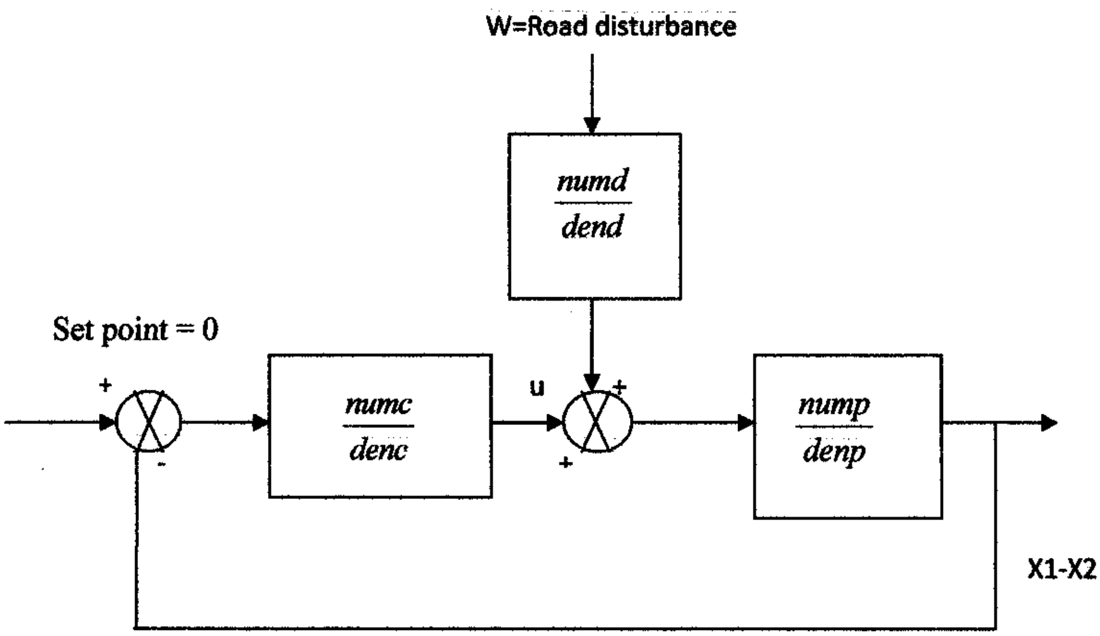


Figure 18: Schematic diagram of Closed Loop Response II

In general, transfer function for PID controller is

$$controller = \frac{numc}{denc}$$

$$Kp + \frac{Ki}{s} + Kds$$

$$\frac{Kp(s)}{s} + \frac{Ki}{s} + \frac{Kds(s)}{s}, thus$$

$$\frac{numc}{denc} = \frac{Kds^2 + Kps + Ki}{s}$$

For the first case, we are going to analyze the suspension travel of the closed loop response, In analyzing suspension travel, output that need to be use is the distance X_1-X_2. From the schematic diagram, we develop the transfer function for the suspension travel of the model, we knew that:

$$\frac{numd}{dend} = \frac{num1}{nump} = \frac{-M_1C_2s^3 - M_1K_2s^2}{(M_1+M_2)s^2 + C_2s + K_2}$$

$$\frac{numc}{denc} = \frac{Kds^2 + Kps + Ki}{s}$$

$$\frac{nump}{denp} = G1(s) = \frac{(M_1+M_2)s^2 + C_2s + K_2}{\Delta}$$

Since the output for suspension travel is the distance **X1-X2**, the transfer function is

$$\left[\frac{numd}{dend}W - \frac{numc}{denc}(X_1 - X_2)\right]\frac{nump}{denp} = X_1 - X_2$$

$$\left[\frac{numd}{dend}W - \frac{numc}{denc}(X_1 - X_2)\right] = X_1 - X_2\left[\frac{denp}{nump}\right]$$

$$\left(\frac{denp}{nump} + \frac{numc}{denc}\right)(X_1 - X_2) = \frac{numd}{dend}W$$

$$\frac{X_1 - X_2}{W} = \frac{\dfrac{numd}{dend}}{\dfrac{denp}{nump} + \dfrac{numc}{denc}}$$

$$\frac{X_1 - X_2}{W} = \frac{\dfrac{numd}{dend}}{\dfrac{denp * denc + numc * nump}{nump * denc}}$$

$$\frac{X_1 - X_2}{W} = \frac{nump * numd * denc}{dend(denp * denc + numc * nump)}$$

Since nump=dend, we can simplify above transfer function to

$$\frac{X_1 - X_2}{W} = \frac{numd * denc}{(denp * denc + numc * nump)}$$

For the beginning, assume the value for Kp,Kd and Ki as

Kd=200000

Kp=800000

Ki=500000

To simulate the closed loop response for suspension travel, programme it using MATLAB

```
%% define the value for each constant based on design parameters
m1=3000;%% Sprung Mass/Body Mass
m2=400;%%unsprung mass/suspension mass
k1=70000;%%spring constant for body
k2=450000;%%spring constant fo suspension
c1=400;%%Damping constant for body
c2=17500;%%Damping constant for suspension

%%define equation for each transfer function
nump=[(m1+m2) c2 k2]
denp=[(m1*m2) (m1*(c1+c2))+(m2*c1) (m1*(k1+k2))+(m2*k1)+(c1*c2)
(c1*k2)+(c2*k1) k1*k2]

num1=[-(m1*c2) -(m1*k2) 0 0 ]
den1=[(m1*m2) (m1*(c1+c2))+(c1*m2) (m1*(k1+k2))+(m2*k1)+(c1*c2)
(c1*k2)+(c2*k1) k1*k2]

numd=num1;
dend=nump;

%%For the beginning, we assume the value for Proportional, integral
and
%%derivative gain as

Kd=200000;
Kp=800000;
Ki=500000;

numc=[Kd,Kp,Ki];
denc=[1 0]

%%insert the transfer function for suspension travel, X1-X2
%% denotes the transfer function as numst/denst forsuspension travel,
X1-X2

numst=conv(numd,denc);
denst=polyadd(conv(denp,denc),conv(nump,numc));

%%to simulate 0.08m high step as disturbance, we need to multiply
numst with 0.08

t=0:0.05:5;
step(0.08*numst,denst,t)
axis([0 5 -0.01 0.01])
title('closed loop response with PID controller for suspension
travel')
```

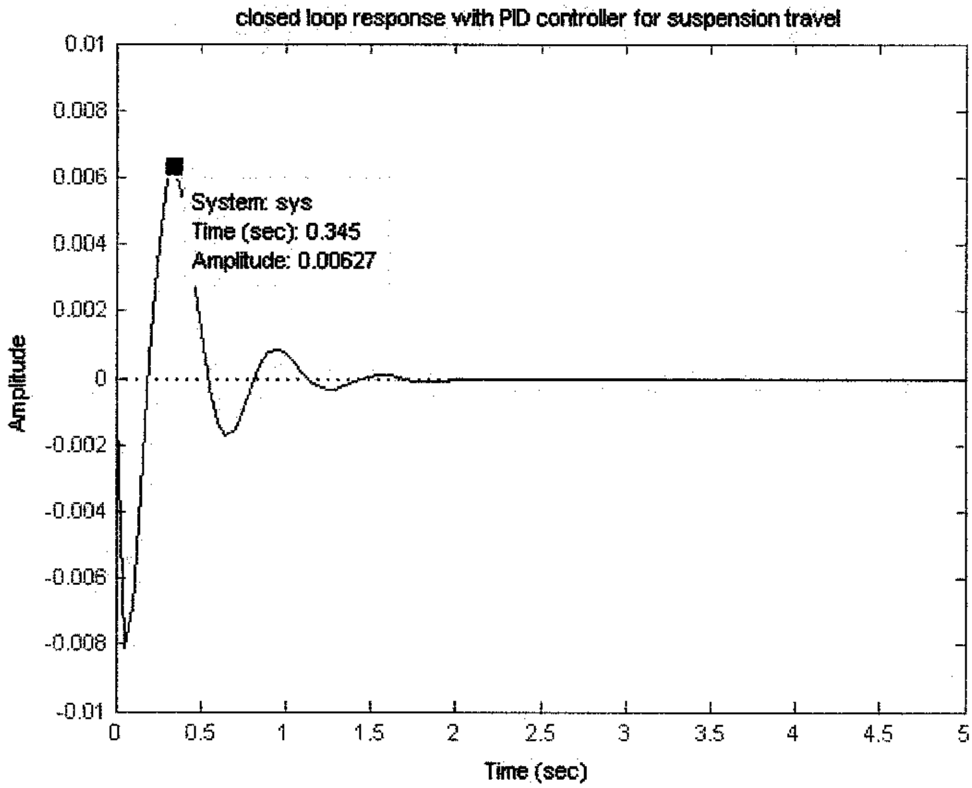


Figure 19: Closed Loop Response for Suspension Travel

From graph above, we can see that the overshoot is still exceeding the requirement which is below 5%, the percent overshoot that the writer gain is 7.5% , higher than design requirement for this PID design. From the graph, we can see that the peak overshoot is 6mm, however this design satisfy the settling time requirement which is less than 5 second whereby time taken for the suspension to dissipate the energy is around 2 seconds. In order to find the perfect PID design to satisfy all design requirements, it is needed to do PID tuning.

4.5 Analysis on the Open Loop and Closed Loop Response for Body Mass Displacement, X_1

Besides considering on the result of the suspension travel alone, the writer also would like to investigate the disturbance that occurred to the body mass displacement, X_1. In order to do the analysis on the body mass displacement, the output of X1 must be use to simulate the body mass displacement. In this case the output is change to X_1 only since we want to know the result of body mass (M1) displacement. The transfer function will be changed to

$$\left[\frac{numd}{dend}W - \frac{numc}{denc}(X_1)\right]\frac{nump}{denp} = X_1$$

In this case, the actuated force input, G1(s) or nump/denp also will be changed to

$$\frac{nump}{denp} = \frac{X_1(s)}{U(s)}$$

By determining the transfer function, we could analyze the difference between passive suspension and active suspension from body mass displacement aspect.`

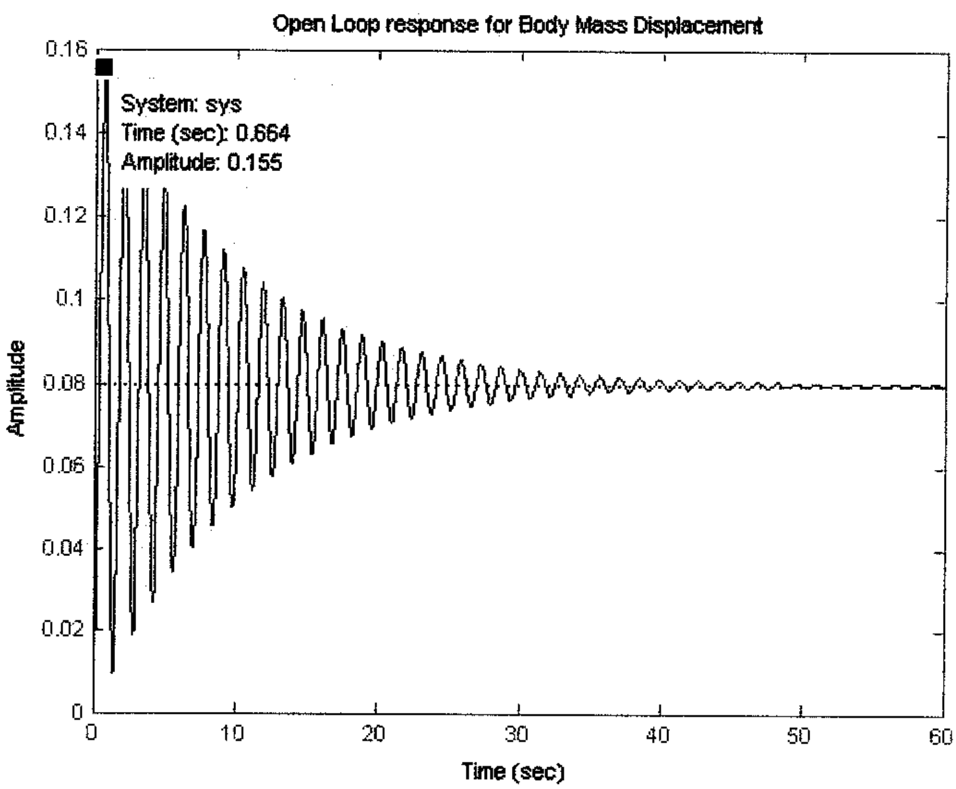
Open loop for body mass displacement, X_1

```
%% define the value for each constant based on design parameters
m1=3000;%% Sprung Mass/Body Mass
m2=400;%%unsprung mass/suspension mass
k1=70000;%%spring constant for body
k2=450000;%%spring constant fo suspension
c1=400;%%Damping constant for body
c2=17500;%%Damping constant for suspension

%%define equation for each transfer function
nump=[(m1+m2) c2 k2]
denp=[(m1*m2) (m1*(c1+c2))+(m2*c1) (m1*(k1+k2))+(m2*k1)+(c1*c2)
(c1*k2)+(c2*k1) k1*k2]

num1=[(c1*c2) ((c1*k2)+(c2*k1)) (k1*k2) ]
den1=[(m1*m2) (m1*(c1+c2))+(c1*m2) (m1*(k1+k2))+(m2*k1)+(c1*c2)
(c1*k2)+(c2*k1) k1*k2]

numd=num1;
dend=nump;
step(0.08*num1,den1)
```



Change the axis

```
step(0.08*num1,den1)
axis([0 10 -0.1 0.3])
Title('Open Loop response for Body Mass Displacement')
```

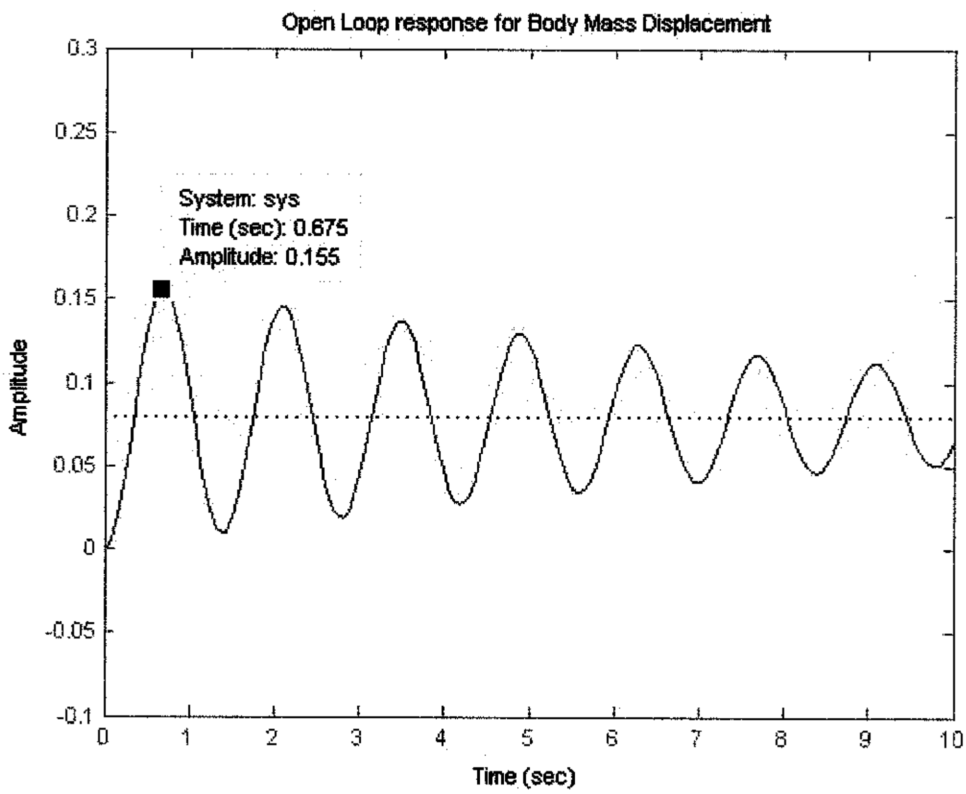


Figure 20: Open loop response for body mass displacement after being subjected with 0.08m step disturbance

From this graph, we can see that after being subjected with 0.08m step disturbance, the body mass displacement oscillates with peak overshoot of 0.155m or 155mm. This denotes that body mass displacement experience a large oscillation after such disturbance. Time taken for the body mass to return back to smooth ride also is very long.

In order to overcome the problem above, the feedback controller had been added up to the system, in order to simulate the closed loop response of the body mass displacement, X_1, we must first find the transfer function for body mass displacement, noted the output for this response is X_1.

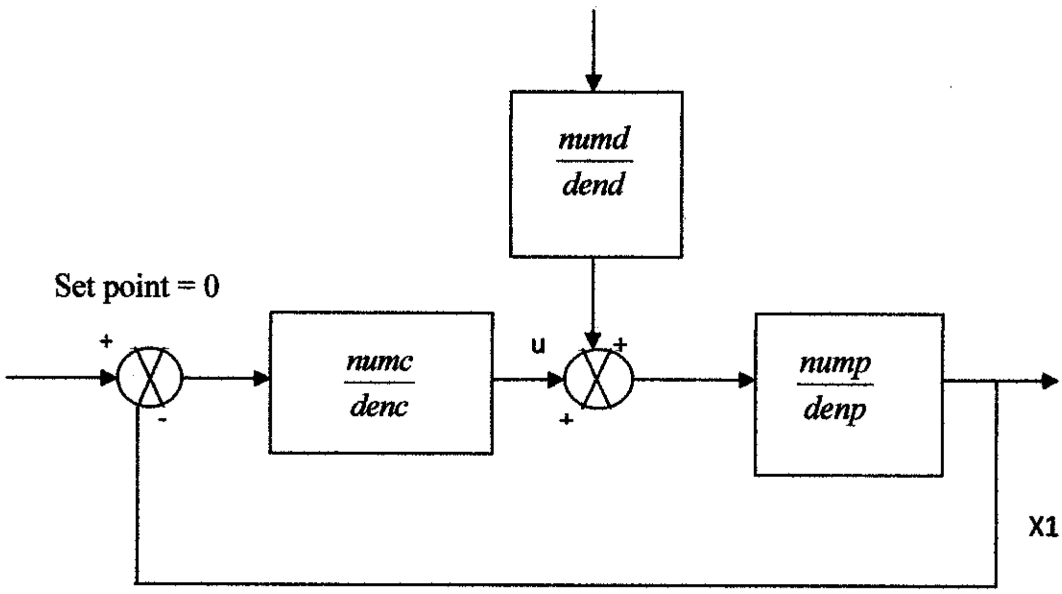


Figure 21: Schematic diagram of Closed Loop Response for Body Mass Displacement

$$\frac{numd}{dend} = \frac{num1}{nump} = \frac{C_1C_2s^2 + C_1K_2s + C_2K_1s + K_1K_2}{M_2s^2 + C_2s + K_2}$$

$$\frac{numc}{denc} = \frac{Kds^2 + Kps + Ki}{s}$$

$$\frac{nump}{denp} = G1(s) = \frac{M_2s^2 + C_2s + K_2}{\Delta}$$

$$\left[\frac{numd}{dend}W - \frac{numc}{denc}(X_1)\right]\frac{nump}{denp} = X_1$$

$$\frac{X_1}{W} = \frac{numd * denc}{(denp * denc + numc * nump)}$$

Closed loop for body mass displacement, X_1 after being subjected with road disturbance 0.08m.

```
%% define the value for each constant based on design parameters
m1=3000;%% Sprung Mass/Body Mass
m2=400;%%unsprung mass/suspension mass
k1=70000;%%spring constant for body
k2=450000;%%spring constant fo suspension
c1=400;%%Damping constant for body
c2=17500;%%Damping constant for suspension

%%define equation for each transfer function
nump=[ m2 c2 k2]
denp=[(m1*m2) (m1*(c1+c2))+(m2*c1) (m1*(k1+k2))+(m2*k1)+(c1*c2)
(c1*k2)+(c2*k1) k1*k2]

num1=[(c1*c2) ((c1*k2)+(c2*k1)) (k1*k2) ]
den1=[(m1*m2) (m1*(c1+c2))+(c1*m2) (m1*(k1+k2))+(m2*k1)+(c1*c2)
(c1*k2)+(c2*k1) k1*k2]

numd=num1;
dend=nump;

%%For the beginning, we assume the value for Proportional, integral
and
%%derivative gain as

Kd=200000;
Kp=800000;
Ki=500000;

numc=[Kd,Kp,Ki];
denc=[1 0]

%%insert the transfer function for body mass displacemnet,X1
%% denotes the transfer function as numbd/denbd for body mass
displacement

numbd=conv(numd,denc);
denbd=polyadd(conv(denp,denc),conv(nump,numc));

%%to simulate 0.08m high step as disturbance, we need to multiply
numbd with
%%0.08

t=0:0.05:5;
step(0.08*numbd,denbd,t)
axis([0 5 -0.01 0.01])
title('closed loop response with PID controller for body mass
displacement')
```

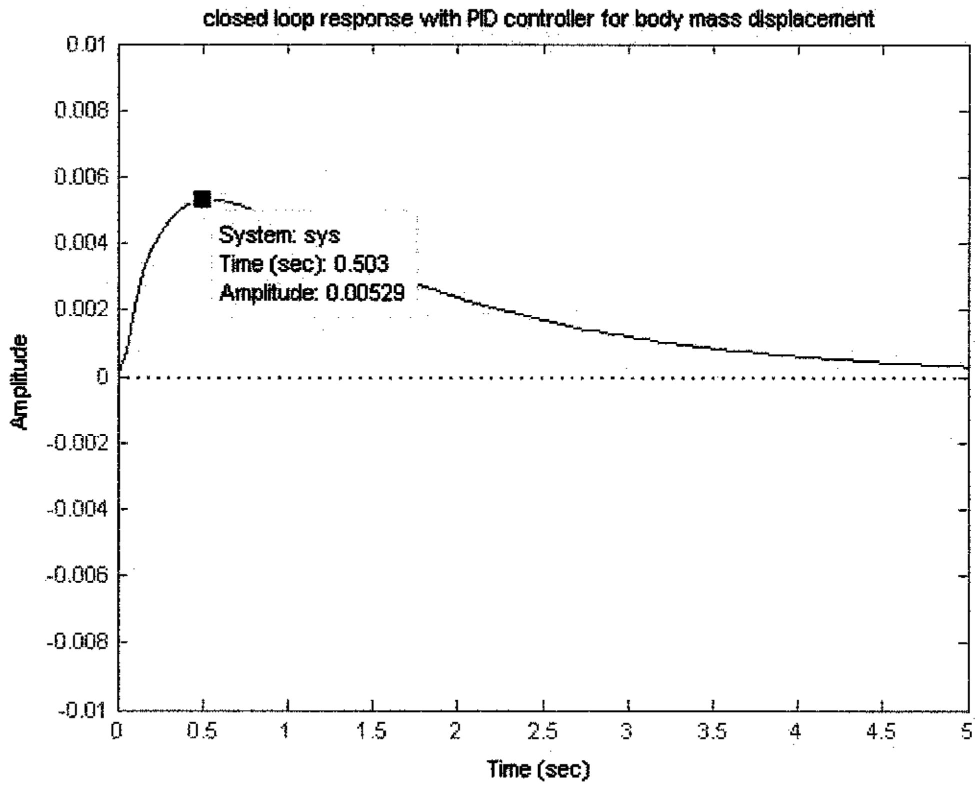


Figure 22: Closed Loop response for Body mass displacement, X_1 after being subjected with 0.08m road disturbance

From this graph, we can see that PID controller is able to reduce the oscillation occurred to the body mass displacement and return to smooth ride within the time given, but still the peak overshoot is 0.00529m or 5.29mm. This result is still exceeding the design requirement which is the oscillation must be within 4mm, thus PID tuning is needed to get the best result that comply to the design requirements.

4.6 Tuning the PID

4.6.1 Ziegler Nichols Tuning Method

From the simulation above, the writer had already proposed the closed loop response for suspension travel but the result still does not meet the design requirement, thus PID tuning need to execute in order to satisfy the design requirements and give a better result. PID tuning basically can be done by using a few method, one of the commonly used is Ziegler-Nichols method. It can be used in either original form or in modification. The methods are based on determination of some features of process dynamics. The controller parameters are then expressed in terms of the features by simple formulas. In Ziegler Nichols Tuning Method, the procedures are:

1. Select proportional control alone
2. Increase the value of the proportional gain until the point of instability is reached (sustained oscillations), the critical value of gain, $\mathbf{K_c}$, is reached.
3. Measure the period of oscillation to obtain the critical time constant, $\mathbf{T_c}$.

Control	Kp	Ti	Td
P only	0.5Kc		
PI	0.45Kc	0.833Tc	
PID tight control	0.6Kc	0.5Tc	0.125Tc
PID some overshoot	0.33Kc	0.5Tc	0.33Tc
PID no overshoot	0.2Kc	0.3Kc	0.5Tc

Table 5: Ziegler Nichols Tuning Method

This tuning method will be used to identify the best parameter for each gain in PID controller so that it could satisfy the design requirement.

4.6.2 Trial and Error Method

Based on the result acquired for both suspension travel and body mass displacement, it can be seen that it does not meeting the design requirement for the percent overshoot meanwhile for the settling time it was already satisfied which is below 5 seconds. Thus, Trial and Error method could be used to tune the PID gain in order to get the best result which satisfy both design requirements. The table below will acts as a guideline in tuning the value for proportional, integral and derivatives gains.

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

Table 6 : Relationship of Kp, Ki and Kd

The trial and error method will be done on suspension travel vs time to find the best value of Kd, Kp and Ki. Several values will be taken into account based on the table above. The values which satisfy both design requirements will be used as the value for Kp, Ki and Kd.

Ist Trial

Kd = 100000

Kp = 400000

Ki = 250000

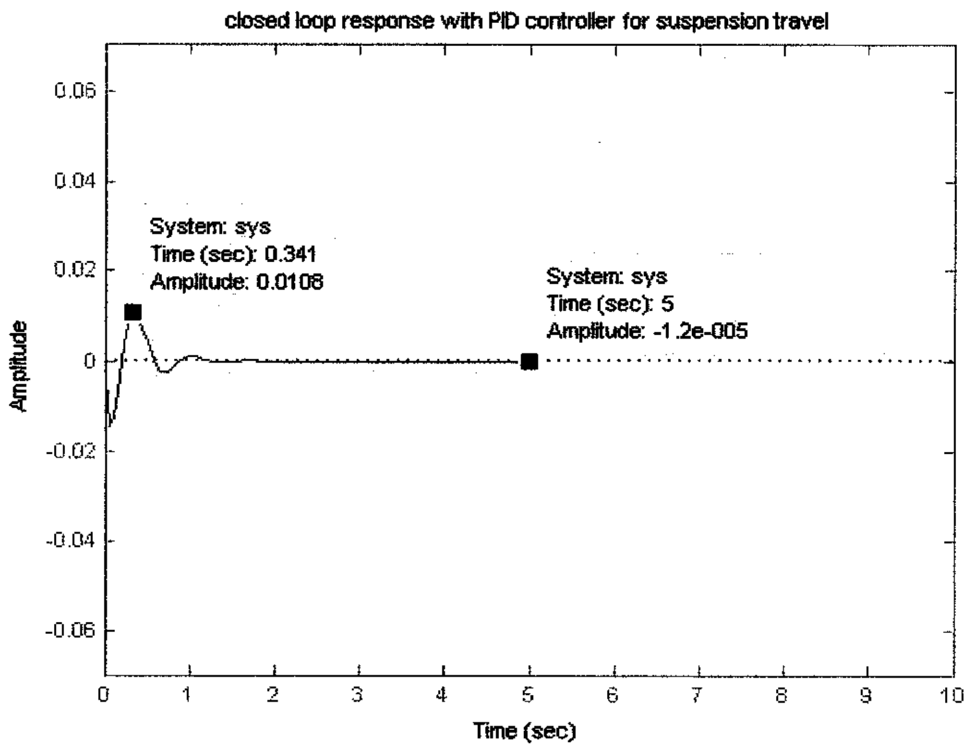


Figure 23: Closed Loop Response for Suspension Travel For Kd=100000,Kp=400000, Ki=250000.

From this value, The peak overshoot gain is 0.0108m or 10.8mm, we can see that it does not able to satisfy the design for percent overshoot which is below 5% or 4mm but it is able to satisfy the design for settling time whereby it get around 2 second to return to smooth ride. Thus, this value of Kp, Ki and Kd will be rejected

2nd Trial

Kd = 300000

Kp=1200000

Ki=800000

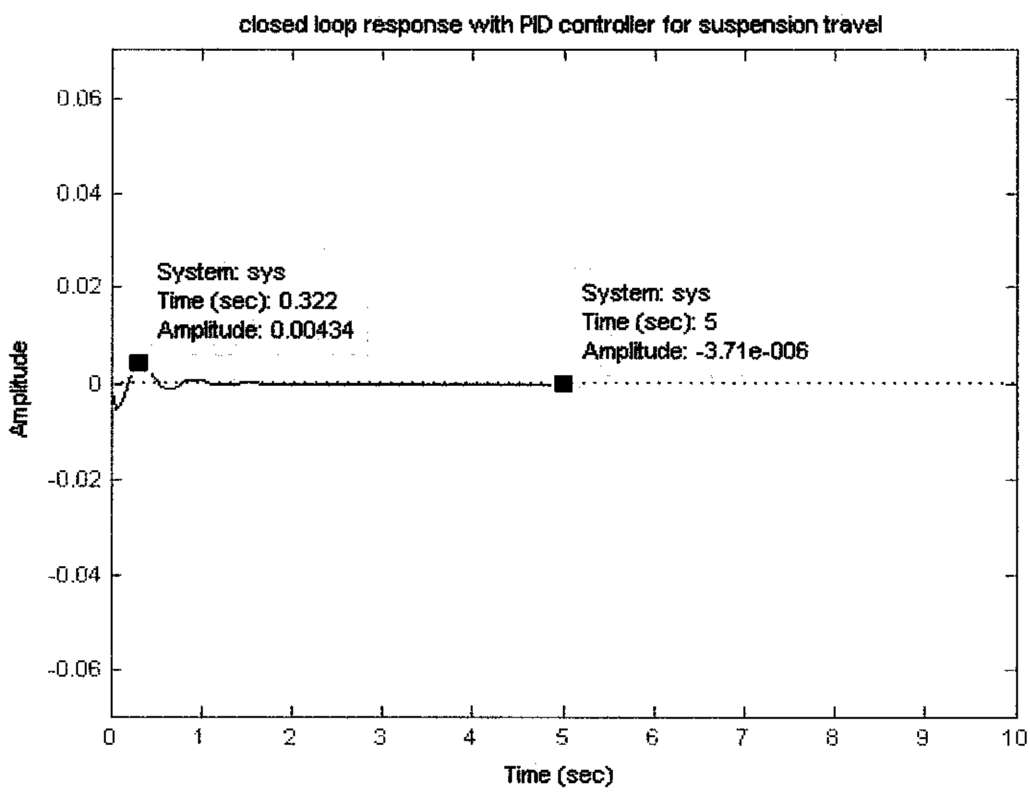


Figure 24: Closed Loop Response for Suspension Travel For Kd=300000,Kp=1200000, Ki=800000.

For this value of Kd, Kp and Ki, we can see that by increasing the value we could decrease the percent overshoot, For this design, the amplitude gain is 0.00434m or 4.34mm, although the writer was able to decrease the peak overshoot but it does not meet the requirement by 0.34mm, but the settling time is also satisfied, Thus the writer will try to increase the value of Kd, Kp and Ki to see the result.

3rd Trial

Kd = 400000

Kp = 1600000

Ki = 1000000

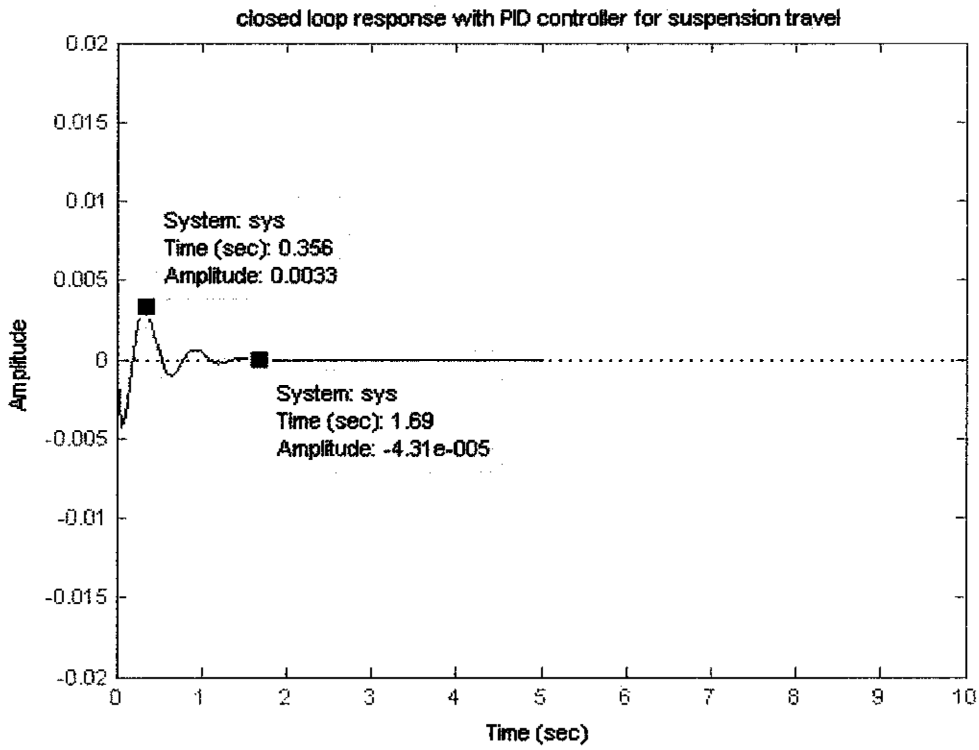


Figure 25: Closed Loop Response for Suspension Travel for Kd=400000, Kp=1600000 and Ki=1000000

Based on this value, we were able to get the desired result which satisfy both design requirement, for this gains, the peak overshoot shows the result of 0.0033m or 3.3mm, it also satisfy the settling time whereby it took less than 5 seconds to return to smooth ride. Therefore this value of Kd, Kp and Ki can be used for this design.

4.7 Final Results

By using the new value of Kd, Kp and Ki, the writer will compare the result between passive suspension and active suspension for both suspension travel vs time and body mass displacement vs time.

4.7.1 Suspension travel vs time

Passive Suspension

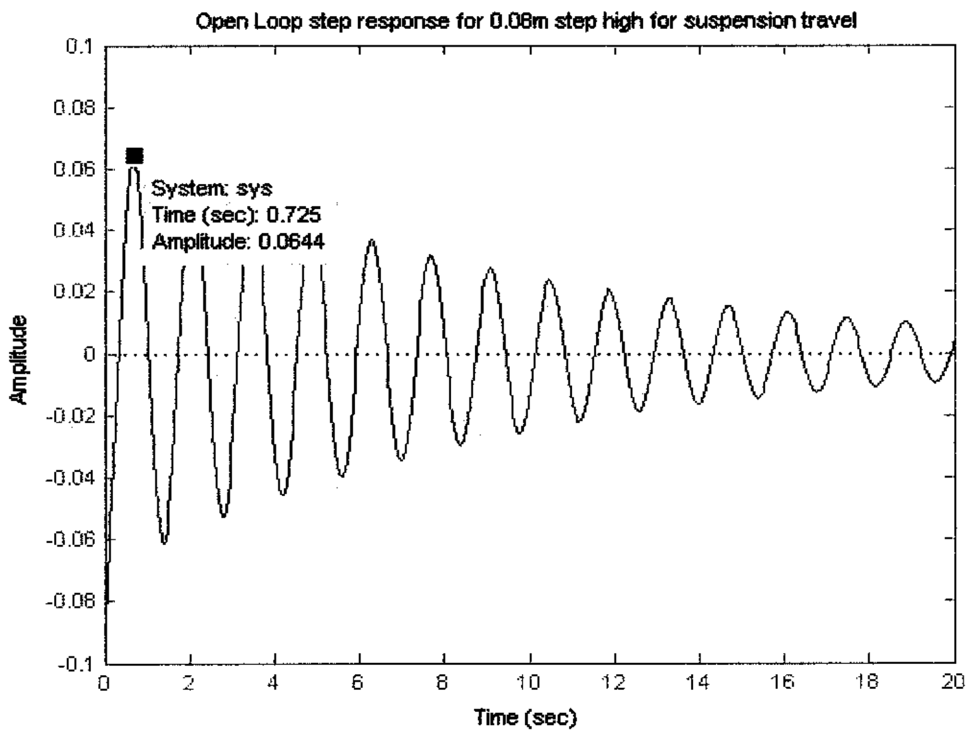


Figure 26: Passive suspension for suspension travel vs time after being subjected with 0.08m step height.

The peak overshoot is 0.0644m or 64.4mm meanwhile the settling time is very slow

Active Suspension

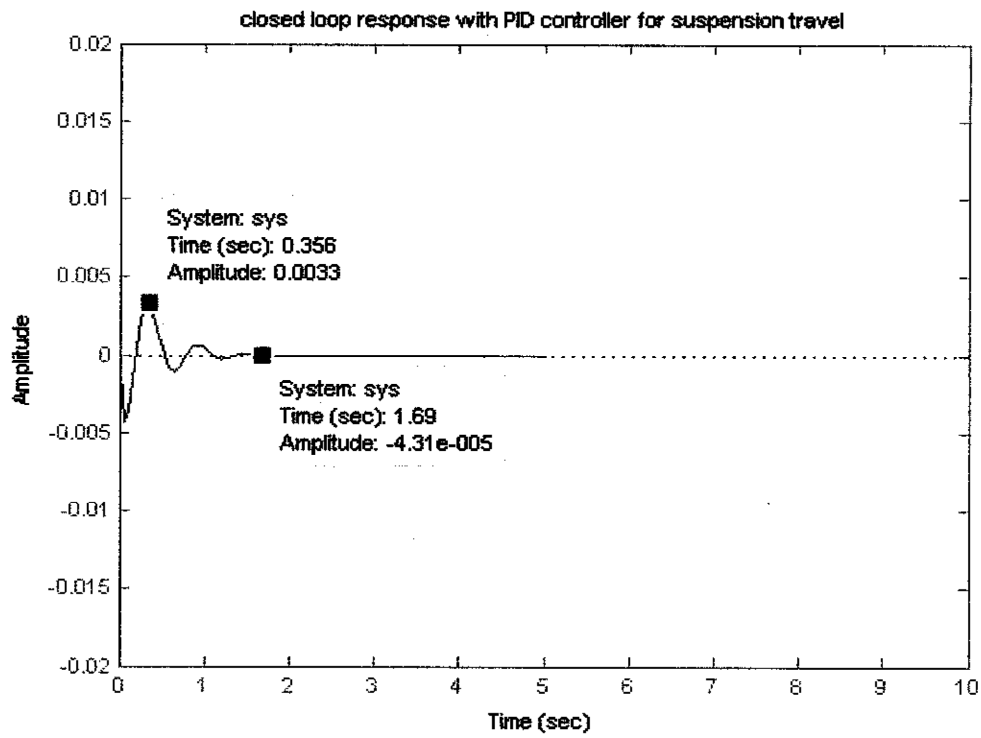


Figure 27: Active suspension for suspension travel vs time after being subjected with 0.08m step height

Implementation of PID Controller had reduced the peak overshoot to 0.0033m or 3.3mm meanwhile the settling time is below 5 second.

4.7.2 Body Mass Displacement vs time

Passive Suspension

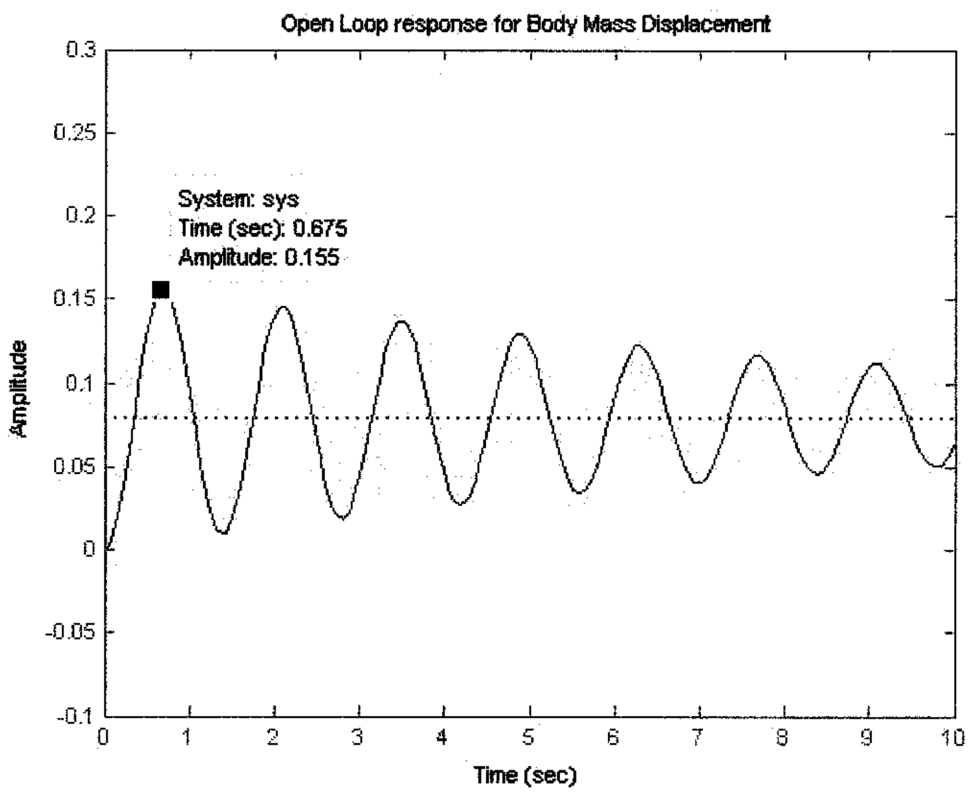


Figure 28: Passive suspension for body mass displacement vs time after being subjected with 0.08m step height.

The peak overshoot for body mass displacement vs time before the implementation of feedback controller is 0.155m or 155mm, the settling time also is very slow

Active Suspension

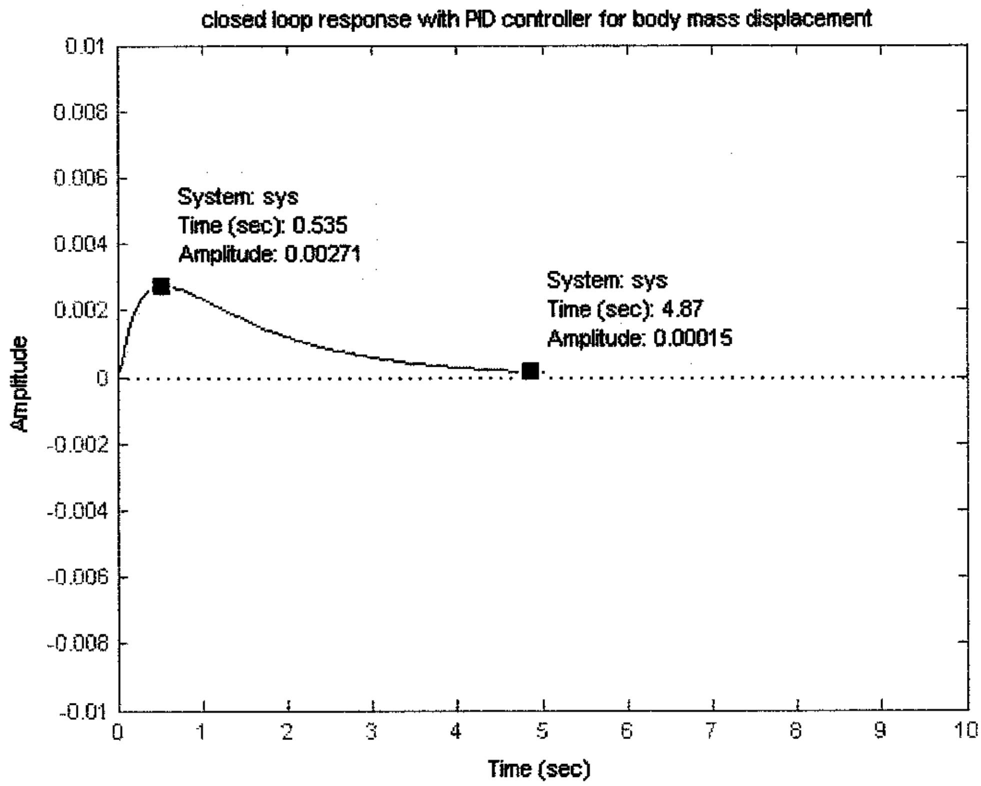


Figure 29: Active suspension for body mass displacement vs time after being subjected with 0.08m step height.

Based on both results from passive suspension and active suspension, we can see that by adding up PID controller as feedback controller we were able to decrease the peak overshoot and also the settling time for the model. This is important as to provide more comfort to the passengers and also to prevent the suspension from damage due to excessive vibration and oscillation.

Parameter	Passive Suspension	Active suspension
Suspension Travel	0.0644m	0.0033m
Body Mass Displacement	0.155m	0.00271m

Table 7: Peak Overshoot value for step height 0.08m

4.8 Validation

The purpose of the validation is to show that the result gain from this study is beneficial and can be used for further analysis on the same topic. Since the writer did not use an experimental based analysis, the validation is to compare the result gain from this study with the literature with the same topic.

The writer had chosen to compare the result based on the literature '**Development of Active Suspension System for Automobiles by using PID Controller**' by Mouleeswaran Senthil Kumar, *member, IAENG*. Based on this literature, the author is also using a quarter car model to do the analysis. However the author did not include the damping constant of wheel and tire (C_2). For this study the parameters that the author used are as follows;

M_1	Body Mass	250kg
M_2	Suspension Mass	50kg
K_1	Spring constant of suspension system	18600N/m
C_1	Damping Constant of suspension system	1000Ns/m
K_2	Spring constant of wheel and tire	196000N/m

Table 8: Parameters used in 'Development of Active Suspension System for Automobiles by using PID Controller' by Mouleeswaran Senthil Kumar[6]

The author had performed the analysis on step input and also random road input, for the step input the step height that the author used is 0.08m.

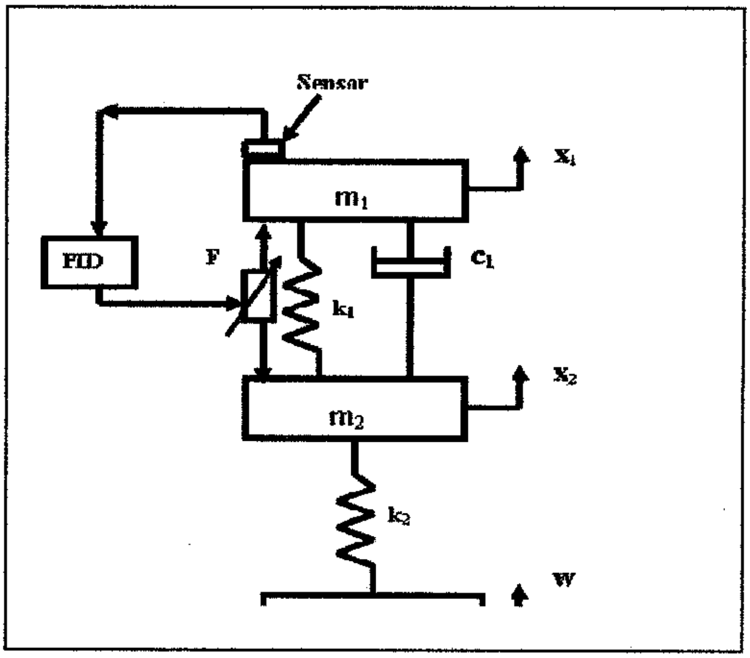


Figure 30: Model used in **"Development of Active Suspension System for Automobiles by using PID Controller' by Mouleeswaran Senthil Kumar[6]**

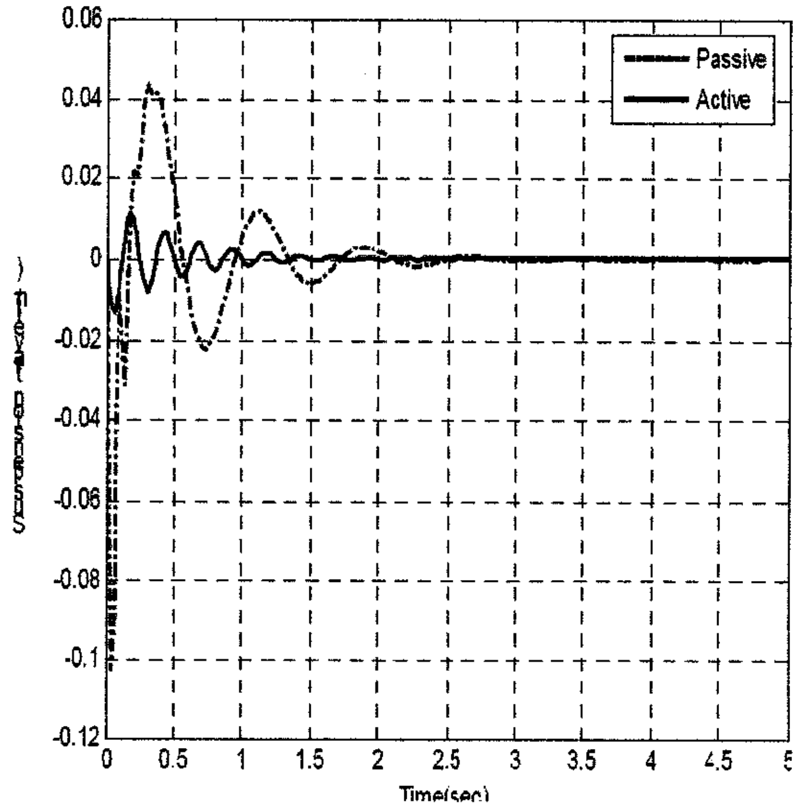


Figure 31: Graph of passive suspension and active suspension after being subjected with 0.08m step height.[6]

In order to proceed with validation, the writer had used the parameters from the journal below to be used in the MATLAB programming. The purpose is to analyze the accuracy of the programming to give the same result with the journal.

```
%% define the value for each constant based on design parameters
m1=250;%% Sprung Mass/Body Mass
m2=50;%%unsprung mass/suspension mass
k1=18600;%%spring constant for body
k2=196000;%%spring constant fo suspension
c1=1000;%%Damping constant for body
c2=0;%%Damping constant for suspension

%%define equation for each transfer function
nump=[(m1+m2) c2 k2]
denp=[(m1*m2) (m1*(c1+c2))+(m2*c1) (m1*(k1+k2))+(m2*k1)+(c1*c2)
(c1*k2)+(c2*k1) k1*k2]

num1=[-(m1*c2) -(m1*k2) 0 0 ]
den1=[(m1*m2) (m1*(c1+c2))+(c1*m2) (m1*(k1+k2))+(m2*k1)+(c1*c2)
(c1*k2)+(c2*k1) k1*k2]

numd=num1;
dend=nump;

%%For the beginning, we assume the value for Proportional, integral
and
%%derivative gain as

Kd=32060;
Kp=3055;
Ki=0.7;

numc=[Kd,Kp,Ki];
denc=[1 0]

%%insert the transfer function for suspension travel, X1-X2
%% denotes the transfer function as numst/denst for suspension travel,
X1-X2

numst=conv(numd,denc);
denst=polyadd(conv(denp,denc),conv(nump,numc));

%%to simulate 0.08m high step as disturbance, we need to multiply
numst with %%0.08

t=0:0.05:5;
step(0.08*numst,denst,t)
axis([0 5 -0.02 0.02])
title('closed loop response with PID controller for suspension
travel')
```

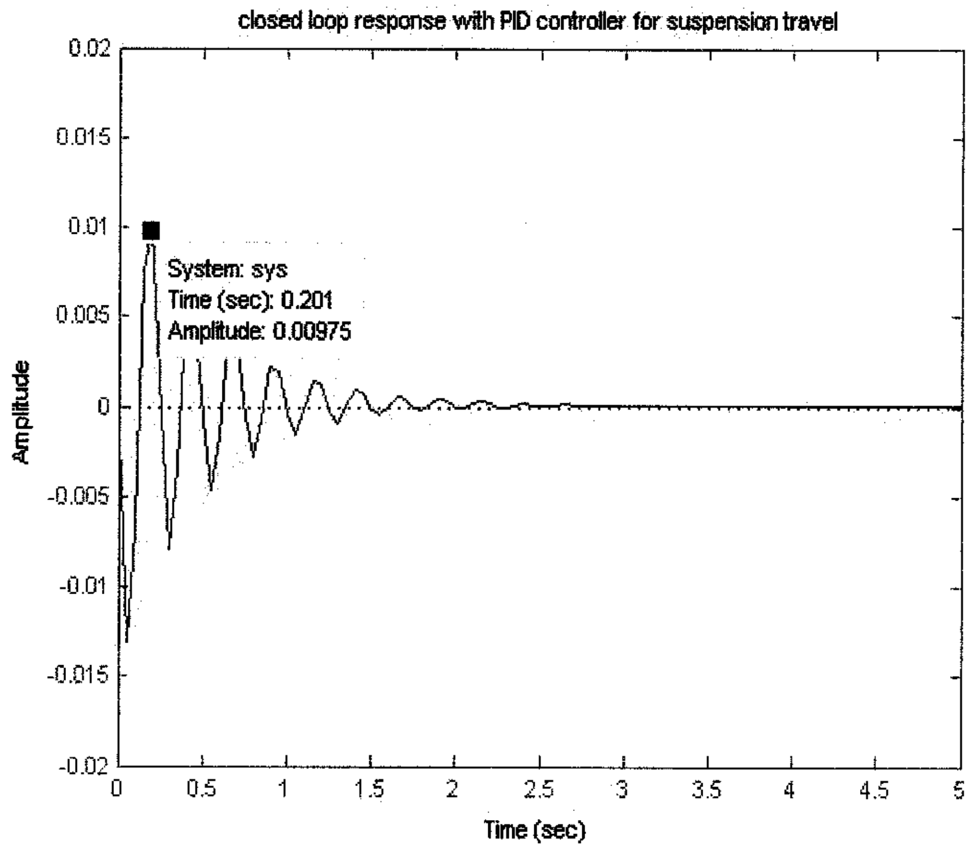


Figure 32: Closed loop response for suspension travel using the parameters from in 'Development of Active Suspension System for Automobiles by using PID Controller' by Mouleeswaran Senthil Kumar[6]

Based on the result gain from the literature and the analysis, it can be said it the result does shows a same pattern, from the literature, the author gain the value of 0.01106m or 11.06mm for the active suspension meanwhile from the analysis the writer gain the value of 0.00975m or 9.75mm for maximum elongation.

5. CONCLUSION

In conclusion, the project was able to be completed within the given time frame to achieve the required objectives. With proper planning and guidance from the Supervisor and cooperation and hard work from the student, the planned milestone and successful completion of this project had been achieved. The writer had changed the model from using the half car model to quarter car model meanwhile both of them are using 2 degrees of freedom analysis. In this dissertation draft, the writer had managed to show how the open loop system looks like after being subjected to certain road disturbance, this will acts as a reference when designing the closed loop system for the same model. The writer had come out with the design of Closed Loop response for the suspension travel, however the result of this design shows that the parameter for each gain does not comply to the design requirement, thus, the writer need to do the PID tuning in order to get the best result that satisfy all design requirements. The writer had also developed another transfer function for body mass displacement, by using the same method, the writer had analyze the difference between passive suspension and active suspension from another aspect. After doing the PID Tuning, the writer was able to find the value of Kd, Kp and Ki which satisfied all design requirements and to complete the project, the validation had been done to show that this project will be beneficial and can be used as further reference.

6. REFERENCES

[1] Donald Bastow, Geoffrey Howard, 2004, *Car Suspension and Handling,* Professional Engineering Publishing, Fourth Edition

[2] Ikbal Eski, Sahin Yildrim, 2008, *Vibration Control of vehicle active suspension system using a new robust neural network control system.* Simulation, Modelling Practice and Theory 17 (2009) 778-793, Erciyes University, Faculty of Engineering, Mechanical Engineering Department

[3] Yahaya Md Sam, Johari H.S Osman, M. Ruddin A. Ghani, 2002, *A Class of proportional-integral sliding mode control with application to active suspension system,* System and Control Letters 51 (2004) 217-223,Faculty of Electrical Engineering, University Technology of Malaysia.

[4] Maziah Mohammad, Musa Mailah, Abdul Halim Muhaimin, *Vibration Control of Mechanical Suspension System Using Active Force Control,* Department of Applied Mechanics, Faculty of Mechanical Engineering, UTM,Skudai, Johor

[5] Ibrahim B Kucukdemiral, seref N. Engin, Vasif E. Omurlu and Galip Cansever, *A Robust Single Input Adaptive Sliding Mode Fuzzy Logic Controller for Automotive Active Suspension System,* L. Wang and Y.Jin (Eds): FSKD 2005, LNAI 3613, pp.981-986,2005

[6] Mouleeswaran Senthil Kumar,2008, *Development of Active Suspension System for Automobiles using PID Controller,*WCE 2008,July 2-4,2008, London,UK. ISBN:978-988-17012-3-7.

[7] K.Astrom and T.Hagglund,1934, PID Controllers: *Theory,Design and Tuning,* Instrument Society of America(ISA),2nd Edition.

[8] Suspension Design: Types of Suspensions By Jacob Isaac-Lowry http://www.automotivearticles.com/Suspension_Design_Types_of_Suspensions.shtml

[9] Appleyard M. and Wellstead P.E (1995). Active Suspension:some background. IEEE Proc. Control Theory Application. 142(2): 123-128

[10] Al-Holou et al., 2002. *Sliding mode neural network inference fuzzy logic control for active suspension systems*. IEEE Transactions on Fuzzy Systems. v10 i2. 234-246.

[11] Mudi and Pal, 1999. *A robust self-tuning scheme for PI- and PD-type fuzzy controllers*. IEEE Transactions on Fuzzy Systems. v7. 2-16.

[12] Esmailzadeh, E. and Taghirad, H. D. (1996). *Active Vehicle Suspensions with Optimal State-Feedback Control*. Journal of Mechanical Science. pp. 1-18.