ANALYSIS OF LINEAR CONTROL

FOR NONLINEAR SYSTEM:

CONTINUOUS STIRRED TANK REACTOR (CSTR)

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CHEMICAL ENGINEERING

UNIVERSITI TEKNOLOGI PETRONAS

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by

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Dissertation submitted in partial fulfillment of

the requirements for the

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CERTIFICATION OF APPROVAL

Analysis of Linear Control for Nonlinear System:

Continuous Stirred Tank Reactor (CSTR)

by

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Approved by,

____

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TRONOH, PERAK

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

MOHD FADZLIN BIN MOHD SAHAILIN

ABSTRACT

Linear control may be favorable over nonlinear control because linear design techniques greatly facilitate the controller design process and because linear controllers impose lower requirements on the implementation and operation as compared to nonlinear controllers. It is therefore a tempting idea to use linear models and linear controller design methods also for nonlinear systems. It is for instance common practice in control engineering to use models obtained from linearization instead of complete nonlinear models. However, in order to guarantee the suitability of a linear model or the proper functioning of a linear controller in presence of the model due to linearization, a rigorous justification is required. This dissertation presents a general framework to design linear controller for nonlinear system based on linear model that guarantees stability for the nonlinear closed loop. Prior to controller design, a nominal linear model has to be derived. While the linearization is a common choice as a linear model for a nonlinear system, it does not need to be the best choice for a given region of operation.

This dissertation has two main areas of contribution. The first area is the derivation and assessment of linear model for nonlinear system and the second area is the utilization of this information for controller design. The main contribution of the first part of this dissertation is to identify a novel unifying framework for nonlinearity assessment. In the second part of this dissertation, stability conditions and controller design procedures for linear control of nonlinear systems are presented.

The results of this dissertation build a bridge between nonlinearity assessment and control theory. The key feature of the proposed methods is thereby to bring together nonlinearity measures, the development and assessment of linear models for nonlinear systems and the design of linear controllers for nonlinear systems under a unifying framework.

ACKNOWLEDGEMENT

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Above all, I would like to thank my beloved family and friends for constantly giving support, advices, and guidance from the start of my final year project until the end. They were the one who always there beside me and help me out in every way they could in the times of trouble.

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CHAPTER ONE

INTRODUCTION

It is well recognized that one of the characteristics of chemical processes that presents a challenging control problem is the inherent nonlinearity of the process. In spite of this knowledge, chemical processes have been traditionally controlled by using linear systems analysis and design tools. A major reason that the use of linear systems theory has been so pervasive is that there is an analytical solution, hence there are generally more rigorous stability and performance proofs. Also, the computational demands for linear system simulation and implementation are usually quite small when compared to a nonlinear simulation. Obviously, the use of linear system technique is quite limiting if the chemical process is highly nonlinear. Progress in nonlinear control theory, combined with computer hardware advances, now allowed advanced, nonlinear control strategies to be successfully implemented on chemical processes.

1.0 BACKGROUND OF STUDY

As stated earlier, chemical manufacturing processes present many challenging control problems, including nonlinear dynamic behavior. While there may be an extensive understanding of the behavior of nonlinear processes, satisfactory methods for their control are still evolving. The prevalent approach to date has been to use a modal of the process linearized about a steady state operating point to design a linear controller such as the classical PID algorithm. In some situations, this may be inadequate for the control of highly nonlinear processes, so the development of nonlinear controllers has featured prominently in process control in the last decade. This study is intended to give an overview on the performance of linear controller in order to control nonlinear system. Continuous stirred tank reactor (CSTR) will be used as the nonlinear model for this study simply because CSTR is one of the central components of many plants in the chemical industry and exhibit highly nonlinear dynamics, especially when consecutive and side reactions are present. If the result shows that linear controller is not suitable to be applied on nonlinear systems, nonlinear controller will be used as an alternative to the conventional controller.

1.1 PROBLEM STATEMENT

1.1.1 Problem Identification

From the system theory point of view, CSTRs belong to a class of nonlinear systems. Their mathematical models are described by sets of nonlinear differential equations. It is well known that the control of chemical reactors usually CSTRs often represent very complex problem. The control problems are due to the process nonlinearity and high sensitivity of the state and output variables to input changes. In addition, the dynamic characteristics may display a varying sign of the gain in various operating points. Evidently, the process with such properties is hardly controllable by conventional control methods, and its effective control requires application some of advanced methods.

1.1.2 Significant of Project

Through this project, general properties of nonlinear systems can be studied and level of nonlinearity for chemical reactors especially CSTRs can be identified either they are highly nonlinear, mildly nonlinear or slightly nonlinear. Furthermore, the relevancy and suitability of using linear controller to control nonlinear systems will be observed because in practice most reactor control is done with conventional linear and less frequently nonlinear designs. Therefore this project is very useful to address the problems faced by many chemical industries when come to controlling the process units.

1.2 OBJECTIVES

The main objectives of this study are:

i. To find the best method for nonlinearity assessment or measurement especially on chemical reactors.
ii. To investigate the relevancy of using typical methods, linear controller to control nonlinear models, CSTR.

1.3 SCOPE OF STUDY

The scope of study based on objectives can be simplified as below:

i. Provide methods to determine level of nonlinearity of chemical reactors.
ii. Observe the performance of linear controller using different control variables to ensure their workability on controlling nonlinear model.

1.4 RELEVANCY OF THE PROJECT

Performance of linear controller has to be illustrated to observed how it response towards changes in input using different control variables in nonlinear systems so that the limitations of linear controller can be determined as well as justify the reason why the conventional controller is not the best choice in order to control nonlinear systems especially CSTR.

1.5 FEASIBILITY OF THE PROJECT

The scope of this project is to understand the concept of nonlinearity, identify the level of nonlinearity of CSTR and determine the limitations of linear controller for CSTR. The time frame given is approximately about two semesters to complete the project. The author believed that the project will be completed in the given time frame. The tools needed to conduct the simulation are all available and provided, thus there will not be much issues to be completed the project if the author follow the dateline in the Gantt chart accordingly.

CHAPTER TWO

LITERATURE REVIEW

2.0 CRITICAL ANALYSIS OF LITERATURE

2.0.1 Nonlinearity Measure for Chemical Processes Using Gap Metric Method on Continuous Stirred Tank Reactor (CSTR)

Almost all chemical processes are inherently nonlinear in nature. Nevertheless, owing to process operation close to a steady state, most of them are treated using linear analysis and design techniques with linearity assumption in order to simplify the development, implementation, and operation of control strategies. However, there are important instances for which the linearity assumption may be violated, linear controllers are inadequate and nonlinear controllers are necessary. Therefore, methods are needed to assess the nonlinear extent of a process to decide whether a process is sufficiently nonlinear to justify a nonlinear controller or just mildly nonlinear for which a linear controller is adequate. This section explains a nonlinearity measure based on gap metric to quantify the nonlinearity degree of chemical processes, aiming to answer such question. CSTR is presented to illustrate the effectiveness to the proposed nonlinearity measure [2].

In [3], gap metric was generalized to measure the distance between two nonlinear systems, which were referred to as differential gap. Given two nonlinear system, NL1 and NL2, the differential gap was defined as [3]:

$$\delta_{\mathrm{d}}(\mathrm{NL}_1, \mathrm{NL}_2) = \max\left\{\overrightarrow{\delta_{\mathrm{d}}}(\mathrm{NL}_1, \mathrm{NL}_2), \overrightarrow{\delta_{\mathrm{d}}}(\mathrm{NL}_2, \mathrm{NL}_1)\right\} \qquad (1)$$

where

$$\overrightarrow{\delta_d}(NL_1, NL_2) = \sup_{r_1} \inf_{r_2} \delta(L_{r_1}NL_1, L_{r_2}, NL_2) \tag{2}$$

where

$L_{r_i}NL_i$ $(i = 1{,}2)$ denotes the linearization of NL_i along trajectory r_i

Based on this differential gap, a nonlinearity measure was proposed in [4], which measures the gap between a nonlinear system NL and a linear system L:

$$v_d = \inf_{L \in \Lambda} \delta_d(NL, L) = \inf_{L \in \Lambda} \sup_{r} \delta(L_r NL, L) \tag{3}$$

where L_rNL is the linearization of N along trajectory r, and Λ is a proper linear set. And further, in [4] another nonlinearity measure is defined as:

$$v_g = \sup_{p_0} \delta(L_{p0}NL, L) \tag{4}$$

where $L_{p0}NL$ is the linearization of NL at the operating point p_0. v_g is derived from v_d and is only reflects the nonlinear dynamics near an operating point while v_d is more appropriate for quantifying the nonlinearity of a system. Though v_d was expected to measure the nonlinearity of a nonlinear system theoretically, the linear system set Λ is not easy to choose. v_g will be rewrite to make it as a proper measure of nonlinearity for general nonlinear systems.

Definition 1:

$$v = \sup_{p_i p_j \epsilon \Omega} \delta(L_i NL, L_j NL) \tag{5}$$

where L_iNL, L_jNL are linearization systems of NL at two operating points p_i and p_j in the operating space Ω of nonlinear system NL. However, the calculation of v according to definition 1 requires the solution of an infinite dimensional max problem which is infeasible. Grid the entire operating space by N operating points, and the nonlinearity measure is redefined as:

Definition 2:

$$v = \max_{i,j = 1,..,N} \{\delta(L_i NL, L_j NL)\} \tag{6}$$

where $L_i NL, L_j NL$ are linearization systems of NL at the i-th operating point and j-th operating point in the operating space Ω of nonlinear system NL.

Note that the proposed definition is very sensitive to the operating space of the considered nonlinear process. The properties of v are:

i. The measure is bounded between 0 and 1.
ii. If v is close to zero, it indicates that the linearization systems in the expected operating space have similar dynamics of nonlinear system in its operating space. This implies that the nonlinear system in this operating range can be approximated by one linear system, and there exist at least one linear controller that stabilizes the nonlinear system.
iii. If v is close to 1, the linearization systems of the nonlinear system behave quite differently. This implies the dynamics of nonlinear system in the operating space are rather inconsistent. One linear controller is not able to stabilize the nonlinear system over the entire operating range and a nonlinear controller is necessary.

Consider a benchmark continuous stirred tank reactor (CSTR) process with an irreversible, first-order reaction. The dynamics of the system is described by the following nonlinear differential equations:

$$\dot{x}_1 = -x_1 + D_a.(1 - \dot{x}_1).\exp\left({x_2}/{1 + x_2/\gamma}\right) \quad (7)$$

$$\dot{x}_2 = -x_2 + B.D_a.(1 - \dot{x}_1).\exp\left({x_2}/{1 + x_2/\gamma}\right) + \beta.(u - x_2) \quad (8)$$

$$y \doteq x_2 \quad (9)$$

where x1 is the reagent conversion, x2 is the reactor temperature (output) and u is the coolant temperature (input). All variables are dimensionless. The nominal values for the constants are Da = 0.072, γ = 20, B = 8, and β = 0.3 respectively. The ranges of the variables are x1 ∈ [0,1] , x2 ∈ [0, 6] , u ∈ [−2, 2] , and y ∈ [0, 6] . The proposed nonlinearity measure is applied to this CSTR system to assess its nonlinearity degree within its operating space. First distribute N = 100 operating points in the entire operating space. Then linearize the nonlinear system around the 100 points. And 100 linear systems are formulated. Compute the gap metric values between the 100 linear systems. Finally the nonlinearity measure of CSTR in its entire operating space is calculated: v = 1. This result indicates that the dynamics of this CSTR system is quite different at different operating points within its operating space. The CSTR exhibits strong nonlinearity in its operating space. A single linear controller is not able to stabilize the nonlinear system over the entire operating range, and a nonlinear controller is necessary. In fact, the CSTR system exhibits output multiplicity, which explains the reason why the CSTR system has strong nonlinearity and confirms the nonlinearity measure v = 1.

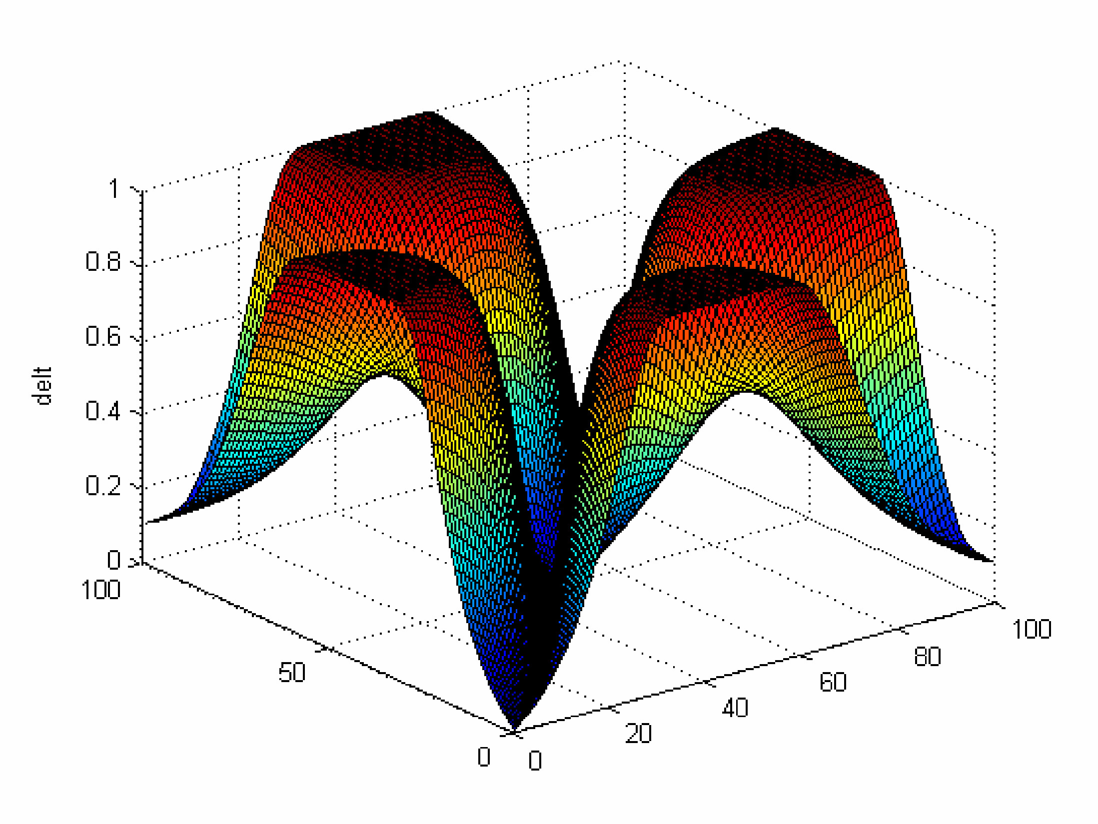


Figure 1: Gap Matrix of CSTR

2.0.2 Limitation of Linear Controller on CSTR

Linear controller such as feedback control of chemical processes that are assumed to behave linearly has a long history of research and successful industrial applications. From single-input-single-output proportional-integral-derivative (SISO PID) to multiple-input-multiple-output (MIMO PID) and even more advance model predictive control (MPC), they rely on the principle of linear process behavior. Underlying this principle are two fundamental assumptions:

i. Process dynamics are inherently linear.
ii. The controlled process will be operating closely enough to a steady state for its dynamic behavior to be considered approximately linear.

However, there are important cases for which it may be violated, such as

i. Regulator control problems where the process is highly nonlinear and frequently troubled far from its steady state by large disturbances.
ii. Servo control problems where the operating points change frequently and span a sufficiently wide range of nonlinear process dynamics.

CSTR process is expected to be characterized by highly nonlinear system. A single linear controller is unable to control CSTR system. If only one linear model is used to design a single linear controller, the closed loop system is unstable, and the output oscillates fiercely [5].

2.0.3 Linear Controller against Nonlinear Controller on CSTR

In this section, MIMO linear model predictive controller (LMPC) based on state space model and nonlinear model predictive controller based on neural network (NNMPC) are applied on CSTR. The idea is to have a good control system that will be able to give optimal performance, reject high load disturbances, and track set point changes. In order to study the performance of the two model predictive controllers, PID strategy is used as benchmark. The LMPC, NNMPC and PID strategies are used for controlling residual concentration (C_A) and reactor temperature (T) [7].

Currently, PID algorithm is the most common control algorithm used in industry. In PID control, process variable and set point must be specific. The PID controller compares the controlled variable value with the set point value to compute the error.

$$\mathrm{Error\ Value\ (E)} = \mathrm{Set\ Point\ Value} - \mathrm{Controlled\ Variable\ Measuring\ Value}$$

Depending on the error value, PID controller determines controller output value which in turn drives the process variable value towards set point. The PID controller action can be expressed as

$$U(t) = K_c \left[E(t) + \frac{1}{\tau_i} \int_0^t E(t) dt + \tau_D \frac{dE(t)}{dt} \right] \qquad (10)$$

where Kc = proportional constant, τ_i = integral time constant, τ_D = derivative time constant, E(t)= tracking error, and U(t) = controller action that will pass to the plant to adjust appropriate manipulated variable.

MPC is an important advanced control technique which can be used for difficult multivariable control problems [8],[9]. The term MPC describes a class of computer control algorithms that control the future behavior of plant through the use of explicit process model. MPC is suitable for almost any kind of problem where it displays its main strength when applied to problem with

i. Large number of manipulated and controlled variables.
ii. Changing control objectives and equipment failure.
iii. Time delays.

Recently, MPC is actually synonym to Linear Model Predictive Control (LMPC). LMPC algorithms employ linear or linearized models to obtain the predictive response of controlled process. In this work, LMPC based on state space model is used. Although LMPC is acceptable in more industrial process, but it still undesirable when the process nonlinearities are strong, operates at multi set points, and use for large disturbances rejection. Therefore nonlinear model predictive controller is more applicable and desirable to the areas of these conditions.

Nonlinear model predictive control refers to the MPC algorithm that employs more accurate nonlinear model in doing prediction and optimization. There are many different nonlinear models such as Volterra models, Polynomial Autoregressive moving average models, Hammerstein and Wiener type models, artificial neural network, and others. Neural network based model predictive controller (NNMPC) is one of the best types of nonlinear model predictive control. Neural network model of nonlinear plant is used to predict future plant performance and optimization algorithm is used to select the control input that optimizes future performance.

In order to check the ability of the controller to reject load disturbances, 10% step change in feed is applied. The close loop response of component residual concentration and reactor temperature are shown in figures (2, 3) respectively.

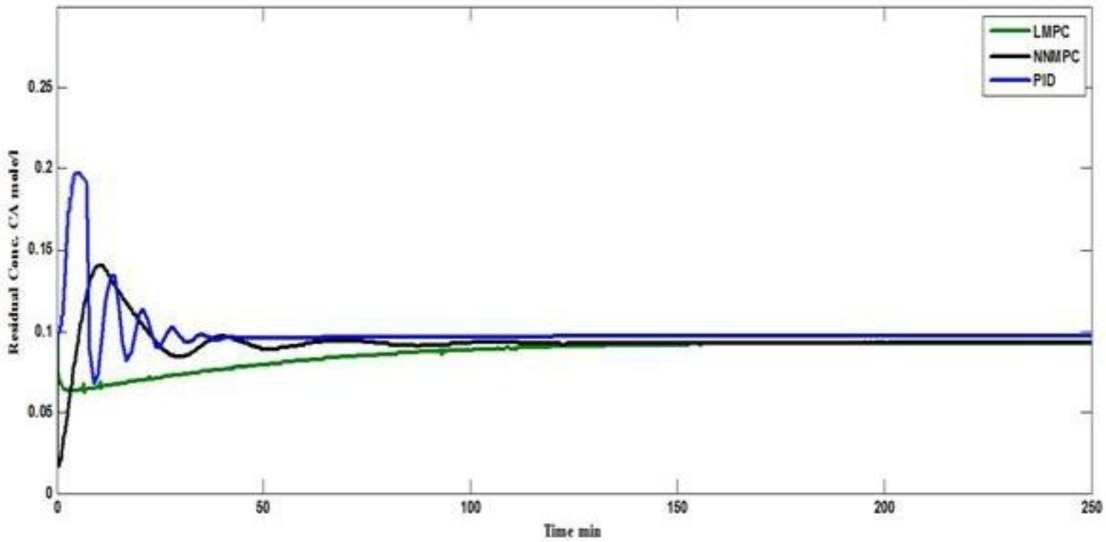


Figure 2: Close loop concentration C_A response for 10% step change in C_{A0}

Type of Controller	Response
PID	Response has overshooting with oscillation and unable to reject disturbance and return to its starting value.
LMPC	Response is slow and settled through the simulation with long time but not return to its starting value.
NNMPC	Response has overshooting and long settled time but return to its starting value.

Table 1: Close loop concentration C_A response for 10% step change in C_{A0}

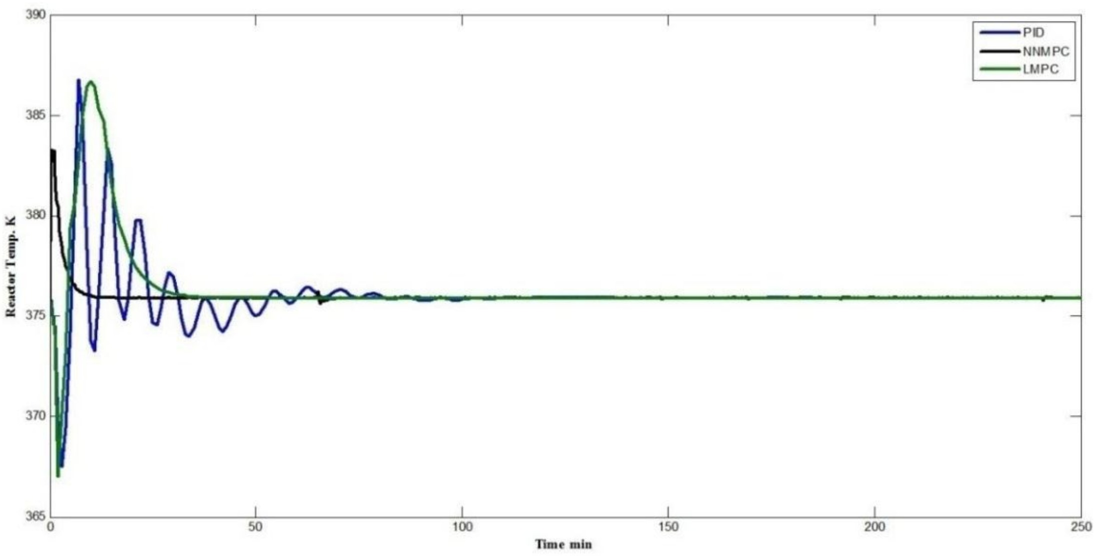


Figure 3: Close loop reactor temperature T response for 10% step change in C_{A0}

Type of Controller	Response
PID	Response has overshooting with large oscillation and has long settled time as well as able to return to starting value.
LMPC	Response has overshooting and has long settled time and return to its starting value.
NNMPC	Response has overshooting but it is settled through small time and return to the starting value.

Table 2: Close loop reactor temperature T response for 10% step change in C_{A0}

The next test is to study the ability of the controllers to track set point change.

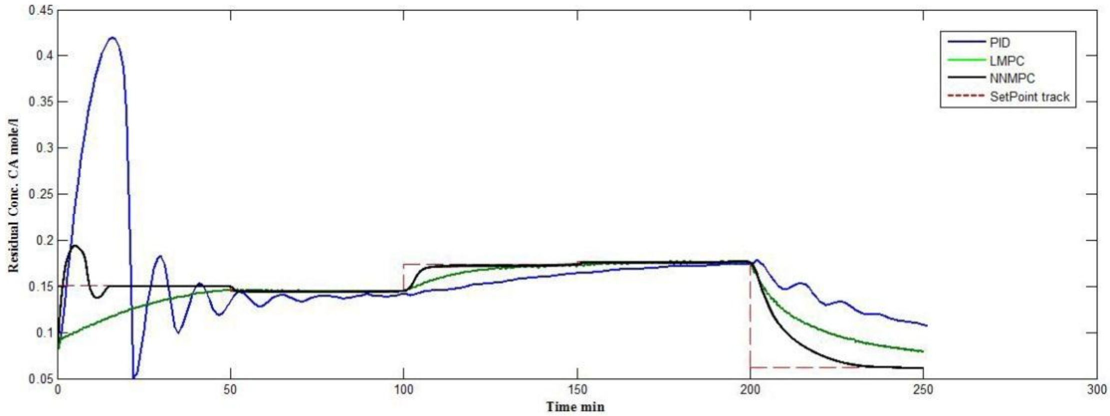


Figure 4: Close loop concentration CA response for set point tracking

Type of Controller	Response
PID	Response has overshooting in first set point, its slow response with oscillation and didn’t settled through simulation time in all set points.
LMPC	Response is slow and settled in second, third and fourth set points only.
NNMPC	Response has overshooting in first set point only, its show perfect set point tracking.

Table 3: Close loop concentration CA response for set point tracking

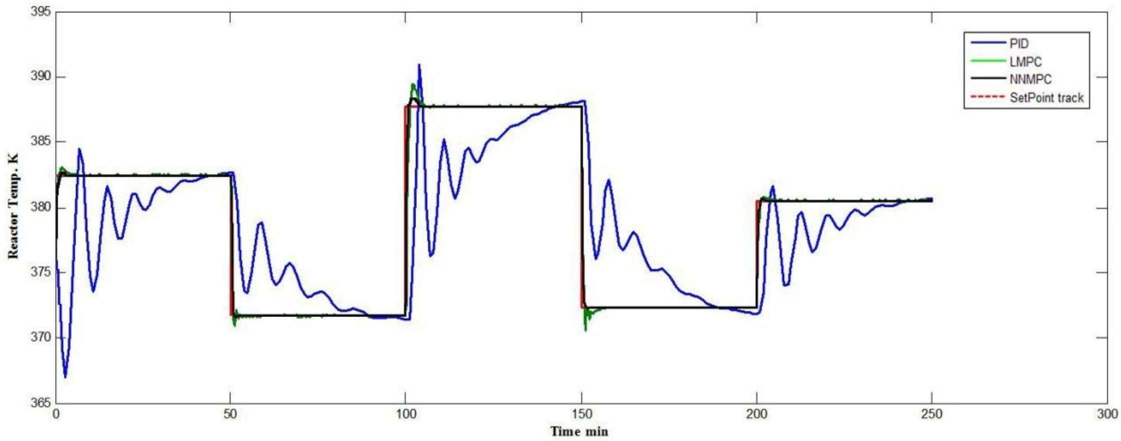


Figure 5: Close loop reactor temperature T response for set point tracking

Type of Controller	Response
PID	Response is slow and has overshooting with oscillation in all set points.
LMPC	The response is settled in in all set points with very small overshooting and show good set point tracking.
NNMPC	Response shows perfect set point tracking.

Table 4: Close loop reactor temperature T response for set point tracking

In another literature, a global Mixed Logical Dynamic (MLD) model is formulated based on three linear local models to approximate the CSTR system [2].

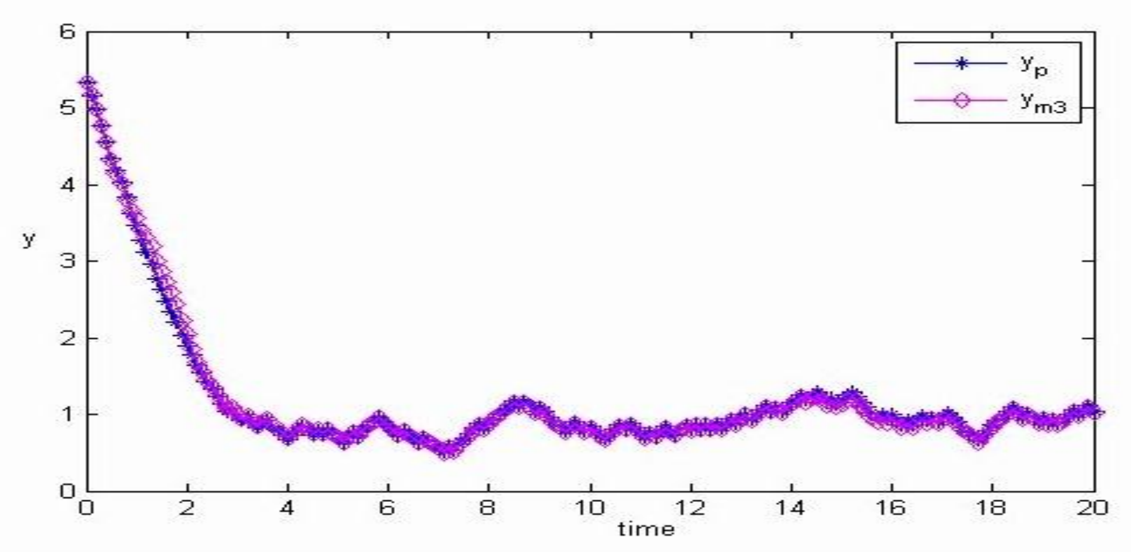


Figure 6: Open-loop Model Validation of CSTR System

Figure 6 depicts the output of the nonlinear system y_p and the output of MLD model y_{m3} under the same inputs. It is clearly seen that y_{m3} is almost coincident with y_p. So the global MLD model is good approximation to the nonlinear system in the entire operating space. MLD-MPC (Model Predictive Control) technique based on multi-linear models is employed to control this system.

CHAPTER THREE

METHODOLOGY

3.0 RESEARCH METHODOLOGY

Project Title Selection: Selection of the most appropriate title for final year project (FYP).

Research on Project: Understanding fundamental theories and concepts, performing literature review, and tool identification.

Derive Differential Equations: Compute total mass balance, component balance and energy balance that represents nonlinear CSTR.

Perform Laplace Transform and Transfer Functions: Linearized the nonlinear differential equations to develop linear CSTR.

Process Simulation: Develop CSTR model and carry out step changes to observe the response of CSTR either it shows nonlinearity characteristics or not. Develop controllers and compare performance of linear and multi-linear

Analysis of Results: Analyze the results from the process simulation software (MATLAB SIMULINK) and conduct result evaluation.

Discussion of Analysis: Discuss the findings from the results obtained and make a conclusion out of the study, determine if the objective has been achieved.

Report Writing: Compilation of all research findings, literature reviews, simulation works, and outcomes into a final report.

3.1 PROJECT ACTIVITIES

3.1.1 Determine the Model Development

For this project, the model chosen will be CSTR with cooling jacket. The reaction takes place in the CSTR is first order, exothermic and irreversible reaction.

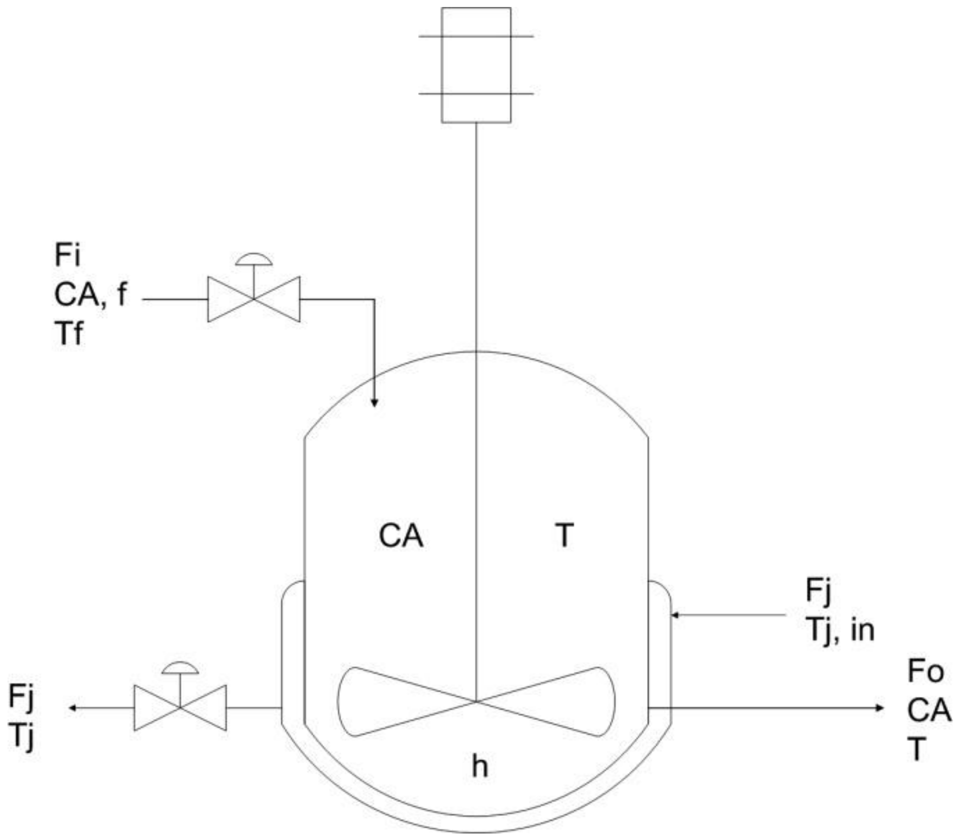


Figure 7: CSTR model

3.1.2 Derive the Equations for Nonlinear Dynamic Behavior of CSTR

The system studied is CSTR with jacket cooling in which a first-order irreversible reaction takes place:

$$A \rightarrow B$$

The reaction rate is

$$-r_A = kC_A = a \exp\left(\frac{-E}{RT_R}\right) C_A \tag{11}$$

where $-rA$ = rate of consumption of reactant A

k = specific reaction rate

C_A = concentration of reactant A in reactor

a = pre-exponential factor

E = activation energy

R = gas constant

T_R = reactor temperature

Total continuity equation:

i. Mass flow rate into reactor = $F_i\rho$
ii. Mass flow rate out of reactor = $F_o\rho$
iii. Rate of accumulation of mass within reactor = $d(\rho V)/dt$

$$d(\rho V)/dt = F_i\rho - F_o\rho$$

$$\rho\, d(V)/dt = F_i\rho - F_o\rho$$

$$dV/dt = F_i - F_o \quad , \quad F_o = \sqrt{10A_ch}$$

$$d(A_ch)/dt = F_i - \sqrt{10A_ch}$$

$$A_c\, dh/dt = F_i - \sqrt{10A_ch}$$

$$\boxed{dh/dt = F_i/A_c - \sqrt{10h/A_c}} \tag{12}$$

Component continuity equation:

i. Flow rate of component A into reactor = $F_i C_{A,f}$

ii. Flow rate of component A out of reactor = $F_o C_A$

iii. Rate of generation of component A by chemical reaction = $-\,(-r_A)V$

iv. Rate of accumulation of component A within reactor = $d(VC_A)/dt$

$$d(VC_A)/dt = F_i C_{A,f} - F_o C_A - (-r_A)V$$

$$C_A\, dV/dt + V\, dC_A/dt = F_i C_{A,f} - F_o C_A - (-r_A)V$$

$$V\, dC_A/dt = F_i C_{A,f} - F_o C_A - (-r_A)V - C_A(F_i - F_o)$$

$$V\, dC_A/dt = F_i\left(C_{A,f} - C_A\right) - (-r_A)V$$

$$dC_A/dt = F_i(C_{A,f} - C_A)/V - (-r_A)$$

$$dC_A/dt = F_i(C_{A,f} - C_A)/A_c h - (-r_A)$$

$$\boxed{dC_A/dt = F_i(C_{A,f} - C_A)/A_c h - \alpha \exp\left(-E/RT\right)C_A} \tag{13}$$

Energy balance equation:

i. Rate of energy input into reactor = $F_i \rho C_p T_f$

ii. Rate of energy out of reactor = $F_o \rho C_p T + U_i A_h (T - T_f)$

iii. Rate of energy added by exothermic reaction = $(-\Delta H) V k C_A$

iv. Rate of accumulation of energy = $d(V\rho C_p T)/dt$

v. Heat transfer area = $A_h = A_c + \pi d h$

$$d(V\rho C_p T)/dt = \rho C_p \, d(VT)/dt = \rho C_p V \, dT/dt + \rho C_p T \, dV/dt$$

$$\rho C_p \, d(VT)/dt = \rho C_p V \, dT/dt + \rho C_p T \, (F_i - F_o)$$

$$d(V\rho C_p T)/dt = F_i \rho C_p T_f - F_o \rho C_p T - U_i A_h (T - T_j) + (-\Delta H) V \alpha \exp(-E/RT) \, C_A$$

$$\rho C_p V \, dT/dt = F_i \rho C_p T_f - F_o \rho C_p T - U_i A_h (T - T_j) + (-\Delta H) V \alpha \exp(-E/RT) \, C_A - \rho C_p T \, (F_i - F_0)$$

$$\rho C_p V \, dT/dt = F_i \rho C_p (T_f - T) - U_i A_h (T - T_j) + (-\Delta H) V \alpha \exp(-E/RT) \, C_A$$

$$dT/dt = F_i/V \, (T_f - T) - U_i A_h/\rho C_p V \, (T - T_j) + (-\Delta H)/\rho C_p \, \alpha \exp(-E/RT) C_A$$

$$\boxed{dT/dt = F_i/A_c h \, (T_f - T) - U_i A_h/\rho C_p A_c h \, (T - T_j) + (-\Delta H)/\rho C_p \, \alpha \exp(-E/RT) C_A} \qquad (14)$$

3.1.3 Find the Operating Condition for CSTR

All operating conditions that will be used in this project is taken from Chemical Process Modeling and Computer Simulation written by Amiya K. Jana.

Ac = cross sectional area of reactor = 4.2822 m2

CA = concentration of reactant A in the exit stream = 8.56303 kmol/m3

CAf = concentration of reactant A in the feed stream = 10 kmol/m3

d = diameter of cylindrical reactor = 2.335 m

E = activation energy = 11 843 kcal/kmol

Fi = volumetric feed flow rate = 10 m3/h

h = height of liquid = 2.335201 m

-ΔH = heat of reaction = 5960 kcal/kmol

R = universal gas constant = 1.987 kcal/kmol. K

α = frequency factor = 34 930 800 h-1

ρ Cp = mixture density x heat capacity = 500 kcal/m3. oC

T = reactor temperature = 38.17771 oC

Tf = feed temperature = 25 oC

Tj = jacket temperature = 25 oC

Ui = overall heat transfer coefficient = 70 kcal/m2. oC. h

Integration time interval = 0.005 h

3.1.4 Derive the Equations for Linear Dynamic Behavior of CSTR

Equations for linear dynamic behavior are derived from the linearization of equation for nonlinear dynamic behavior.

Total continuity equation:

$$dh/dt = F_i/A_c - \sqrt{10h/A_c}$$

$$d(h - \bar{h})/dt = 1/A_c\,(F_i - \bar{F}_i) - \sqrt{10}/2\sqrt{A_c\bar{h}}\,(h - \bar{h})$$

$$dh'/dt = 1/A_c\,F_i' - \sqrt{10}/2\sqrt{A_c\bar{h}}\,h'$$

$$sH'(s) - h(0) = 1/A_c\,F_i'(s) - \sqrt{10}/2\sqrt{A_c\bar{h}}\,H'(s)$$

$$H'(s)\left(s + \sqrt{10}/2\sqrt{A_c\bar{h}}\right) = 1/A_c\,F_i'(s)$$

$$H'(s) = \left\{\left(2\sqrt{\bar{h}}/\sqrt{10A_c}\right)\Big/\left[\left(2\sqrt{\bar{h}A_c}/\sqrt{10}\right)s + 1\right]\right\}F_i'(s)$$

$$\boxed{H'(s) = \left[0.467045/(1.999980\,s + 1)\right]F_i'(s)} \qquad (15)$$

Component continuity equation:

$$dC_A/dt = F_iC_{Af}/A_ch - F_iC_A/A_ch - \alpha\exp(-E/RT)C_A$$

$$dC_A'/_{dt} = \left[(\overline{C_{Af}} - \overline{C_A})/_{A_c\bar{h}}\right]F_i' + \left[\overline{F_\imath}/_{A_c\bar{h}}\right]C_{Af}' - \left[\overline{F_\imath}(\overline{C_{Af}} - \overline{C_A})/_{A_c\overline{h^2}}\right]h'$$

$$- \left[\overline{F_\imath}/_{A_c\bar{h}} + \alpha \exp\left(-E/_{R\overline{T}}\right)\right]C_A'$$

$$- \left[\left(\alpha\overline{C_A}E/_{R\overline{T^2}}\right)\exp\left(-E/_{R\overline{T}}\right)\right]T'$$

$$sC_A'(s) = \left[(\overline{C_{Af}} - \overline{C_A})/_{A_c\bar{h}}\right]F_i'(s) + \left[\overline{F_\imath}/_{A_c\bar{h}}\right]C_{Af}'(s)$$

$$- \left[\overline{F_\imath}(\overline{C_{Af}} - \overline{C_A})/_{A_c\overline{h^2}}\right]H'(s) - \left[\overline{F_\imath}/_{A_c\bar{h}} + \alpha \exp\left(-E/_{R\overline{T}}\right)\right]C_A'(s)$$

$$- \left[\left(\alpha\overline{C_A}E/_{R\overline{T^2}}\right)\exp\left(-E/_{R\overline{T}}\right)\right]T'(s)$$

$$sC_A'(s) = [0.143700]F_i'(s) + [1.000020]C_{Af}'(s) - [0.615364]H'(s)$$
$$- [1.169383]C_A'(s) - [0.089181]T'(s)$$

$$C_A'(s) = \left[0.143700/_{(s + 1.169383)}\right]F_i'(s) + \left[1.000020/_{(s + 1.169383)}\right]C_{Af}'(s)$$
$$- \left[0.615364/_{(s + 1.169383)}\right]H'(s)$$
$$- \left[0.089181/_{(s + 1.169383)}\right]T'(s)$$

$$\boxed{\begin{gathered}C_A'(s) = \\ \left[0.122885/_{(0.855152\,s + 1)}\right]F_i'(s) + \left[0.855169/_{(0.855152\,s + 1)}\right]C_{Af}'(s) - \\ \left[0.526230/_{(0.855152\,s + 1)}\right]H'(s) - \left[0.076263/_{(0.855152\,s + 1)}\right]T'(s)\end{gathered}} \quad (16)$$

Energy balance equation:

$$\frac{dT}{dt} = \frac{F_i(T_f - T)}{A_c h} - \frac{U_i T}{\rho C_p h} + \frac{U_i T_j}{\rho C_p h} - \frac{U_i \pi d T}{\rho C_p A_c} + \frac{U_i \pi d T_j}{\rho C_p A_c} + \left(\frac{-\Delta H}{\rho C_p}\right) \alpha \exp\left(\frac{-E}{RT}\right) C_A$$

$$\begin{aligned}\frac{dT'}{dt} = &\left[\frac{(\bar{T}_f - \bar{T})}{A_c \bar{h}}\right] F_i' + \left[\frac{\bar{F}_i}{A_c \bar{h}}\right] T_f' \\ &+ \left[\frac{\bar{F}_i \bar{T}}{A_c \bar{h}^2} - \frac{\bar{F}_i \bar{T}_f}{A_c \bar{h}^2} + \frac{U_i \bar{T}}{\rho C_p \bar{h}^2} - \frac{U_i \bar{T}_j}{\rho C_p \bar{h}^2}\right] h' \\ &- \left[\frac{\bar{F}_i}{A_c \bar{h}} + \frac{U_i}{\rho C_p \bar{h}} + \frac{U_i \pi d}{\rho C_p A_c} \right. \\ &\left. - \left(\frac{-\Delta H}{\rho C_p}\right) \left(\frac{\alpha \overline{C_A} E}{R \bar{T}^2}\right) \exp\left(\frac{-E}{RT}\right)\right] T' \\ &+ \left[\frac{U_i}{\rho C_p \bar{h}} + \frac{U_i \pi d}{\rho C_p A_c}\right] T_j' \\ &+ \left[\left(\frac{-\Delta H}{\rho C_p}\right) \alpha \exp\left(\frac{-E}{R\bar{T}}\right)\right] C_A'\end{aligned}$$

$$\begin{aligned}sT'(s) = &-[1.317798]F_i'(s) + [1.000020]T_f'(s) + [5.981501]H'(s) \\ &- [0.236757]T'(s) + [0.299779]T_j'(s) + [2.018807]C_A'(s)\end{aligned}$$

$$\boxed{\begin{aligned}&T'(s) = \\ &-\left[\frac{5.566036}{(4.223740\, s + 1)}\right] F_i'(s) + \left[\frac{4.223824}{(4.223740\, s + 1)}\right] T_f'(s) + \\ &\left[\frac{25.264305}{(4.223740\, s + 1)}\right] H'(s) + \left[\frac{1.266189}{(4.223740\, s + 1)}\right] T_j'(s) + \\ &\left[\frac{8.526916}{(4.223740\, s + 1)}\right] C_A'(s)\end{aligned}} \quad (17)$$

3.1.5 Develop Linear and Nonlinear CSTR Model using SIMULINK

Linear CSTR model was developed using transfer function while nonlinear model constructed using integration. Due to differences in numerical and computational method between linear and nonlinear model, some adjustment need to be done so that the value of input for both CSTR models are similar and eventually the output also will converge to almost similar value. In this case, linear model need to be adjusted by subtracting the final values of input parameters with steady state values because it only consider the deviation variable of final and steady state value. Below is the CSTR model for both linear and nonlinear system where the output parameters are combined together for dynamic behavior comparison purpose. Based on the dynamic behavior study, level of nonlinearity can be estimated by looking at the plotted graph when step tests were carried out for both types of model.

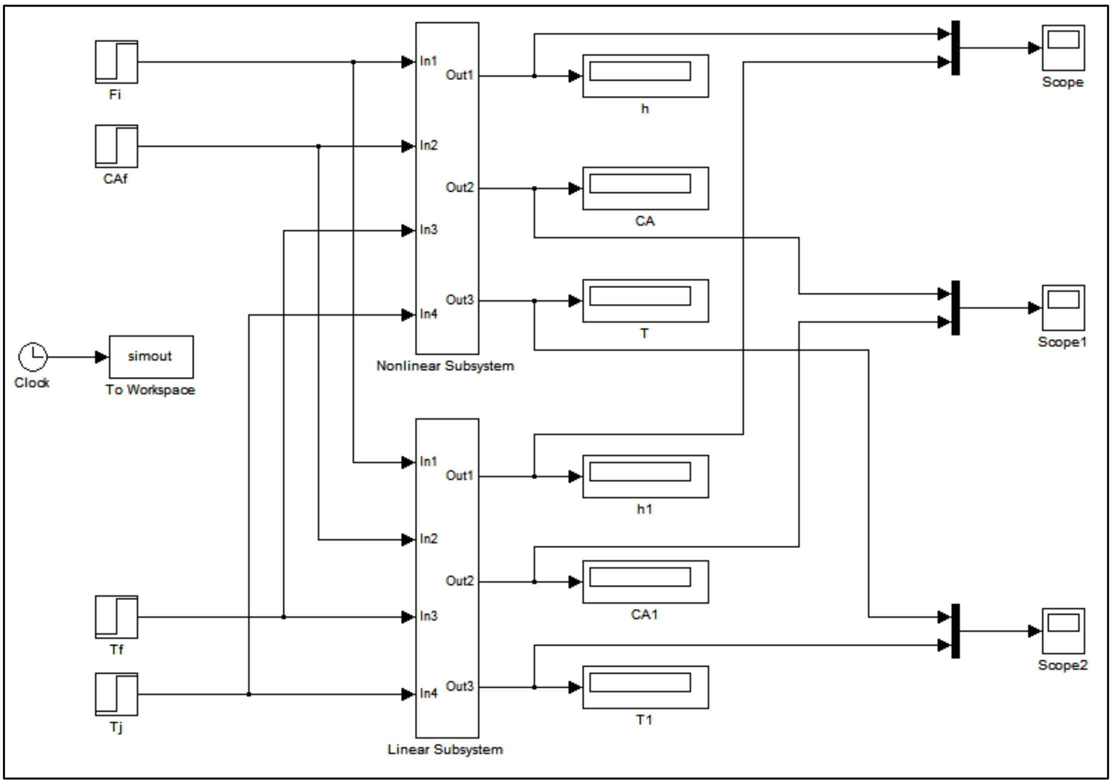


Figure 8: Nonlinear and Linearized CSTR Model

3.2 KEY MILESTONE

3.2.1 Key Milestone FYP I

No	Action Item	Remarks
1	Regular meeting with supervisor to discuss the project and prepare project proposal.	Ongoing
2	FYP Briefing	Week 2
3	Literature Search and Lab Facilities & Services Unit Briefing	Week 5
4	Submission of Extended Proposal	Week 6
5	Mid Semester Break	Week 7
6	Proposal Defense (Oral Presentation)	Week 9
7	Submission of Interim Draft Report	Week 13
8	Submission of Interim Report	Week 14

Table 5: Key milestone FYP I

3.2.2 Key Milestone FYP II

No	Action Item	Remarks
1	Regular meeting with supervisor to discuss the project	Ongoing
2	Mid Semester Break	Week 7
3	Submission of Progress Report	Week 8
4	Pre-SEDEX	Week 10
5	Submission of Technical Paper	Week 12
6	Oral Presentation	Week 13
7	Submission of Project Dissertation	Week 14

Table 6: Key milestone FYP II

3.3 GANTT CHART

3.3.1 Study Plan FYP I

No	Activities	Week													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Selection of Project Topics														
2	Preliminary Research Work														
3	Literature Review and Methodology														
4	Draft Extended Proposal														
5	Submission of Extended Proposal														
6	Proposal Defense														
7	Familiarization to MATLAB														
8	Preliminary Simulation Work														
9	Draft Interim Report														
10	Submission of Interim Report														

Work Progress

Key Milestone

3.3.2 Study Plan FYP II

No	Activities	Week													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Develop Nonlinear CSTR														
2	Develop Linear CSTR														
3	Perform Step Test for Both CSTR														
4	Submission of Progress Report														
5	Computation of Gap Metric Method														
6	Develop PID and ML-MPC Controllers														
7	Pre-SEDEX														
8	Submission of Draft Report														
9	Submission of Technical Paper														
10	Oral Presentation														
11	Submission of Project Dissertation														

Work Progress

Key Milestone

CHAPTER FOUR

RESULTS AND DISCUSSION

An important property of mathematical models of dynamic systems is the linearity property. To cope with nonlinear analysis and control problems, there are two alternative approaches. For highly nonlinear systems, special methods have to be developed that possibly rely upon certain physical properties of the application or upon mathematical properties of a certain system class. For mildly nonlinear problems, one can attempt to use linear models and linear controller design methods. However, this last approach requires a rigorous justification in order to guarantee the accuracy of a linear model or the proper function of a linear controller in presence of the nonlinear system behavior. In view of the preceding discussion, the following questions arise:

i. Given a model of a dynamic system. Is the system linear? If not, is it far from linear or close to linear?
ii. Given a control problem. Is a linear controller adequate or is a nonlinear control algorithm needed?

The two questions above can be associated with the research areas of nonlinearity assessment, linear modeling for nonlinear systems, and linear control for nonlinear systems.

4.0 NONLINEARITY ASSESSMENT FOR CSTR

Linearity is a definite property that is characterized by the superposition and homogeneity principles. If these principles are satisfied by the input-output behavior of a dynamic system, or more precisely of a model of a dynamic system, that system is called linear. Otherwise it is called nonlinear. If a mathematical model of a dynamic system is given, linearity can be checked with the model equations.

Therefore, based on the derived differential equations, it shows that CSTR is a nonlinear system.

Although the strict mathematical definition of linearity is a definite true/false property, it is sometimes interesting to ask whether a system is close to linear or far from linear. For that purpose, Desoer and Wang introduced a method which quantifies the deviation of the input-output behavior of a system from linearity as the nonlinearity measurement. Ogunnaike et al. proposed a nonlinearity measurement by comparing the local linear models corresponding to different points of the operating range. A geometric viewpoint is taken where the curvature of the steady state map is introduced as a measure of nonlinearity. The curvature measure can be extended to dynamic systems using Frechet derivatives of operators. Nikolaou and co-worker introduce an inner product for operators in order to quantify the nonlinearity of a dynamic system. The measure can be efficiently computed by Monte-Carlo-Simulations. A different approach is presented by Hahn et al., who introduce empirical controllability and observability Gramians in order to quantify the degrees of input-tostate and state-to-output nonlinearity respectively.

Nonlinear Assessment Method	Description
Desoer & Wang	Quantify the deviation of the input-output behavior of a system from linearity.
Ogunnaike et al.	Compare the local linear models corresponding to different points of the operating range.
Frechet	Introduce the curvature of the steady state map.
Nikolaou and co-worker	Introduce an inner product for operators using Monte-Carlo-Simulations.
Hahn et al.	Introduce empirical controllability and observability Gramians in order to quantify the degrees of input-tostate and state-to-output nonlinearity respectively.

Table 7: Different nonlinear assessment methods

According to those method, all of them claim that CSTR is a system that inherent high nonlinearity which agree with the gap metric method that mention in the literature review. Therefore, linear controller supposedly cannot stabilize the system. In order to prove this claim, open loop test will be carried out to observe the nonlinearity behavior as well as set points tracking to comprehend the performance of linear controller.

4.1 MODELLING FOR CSTR

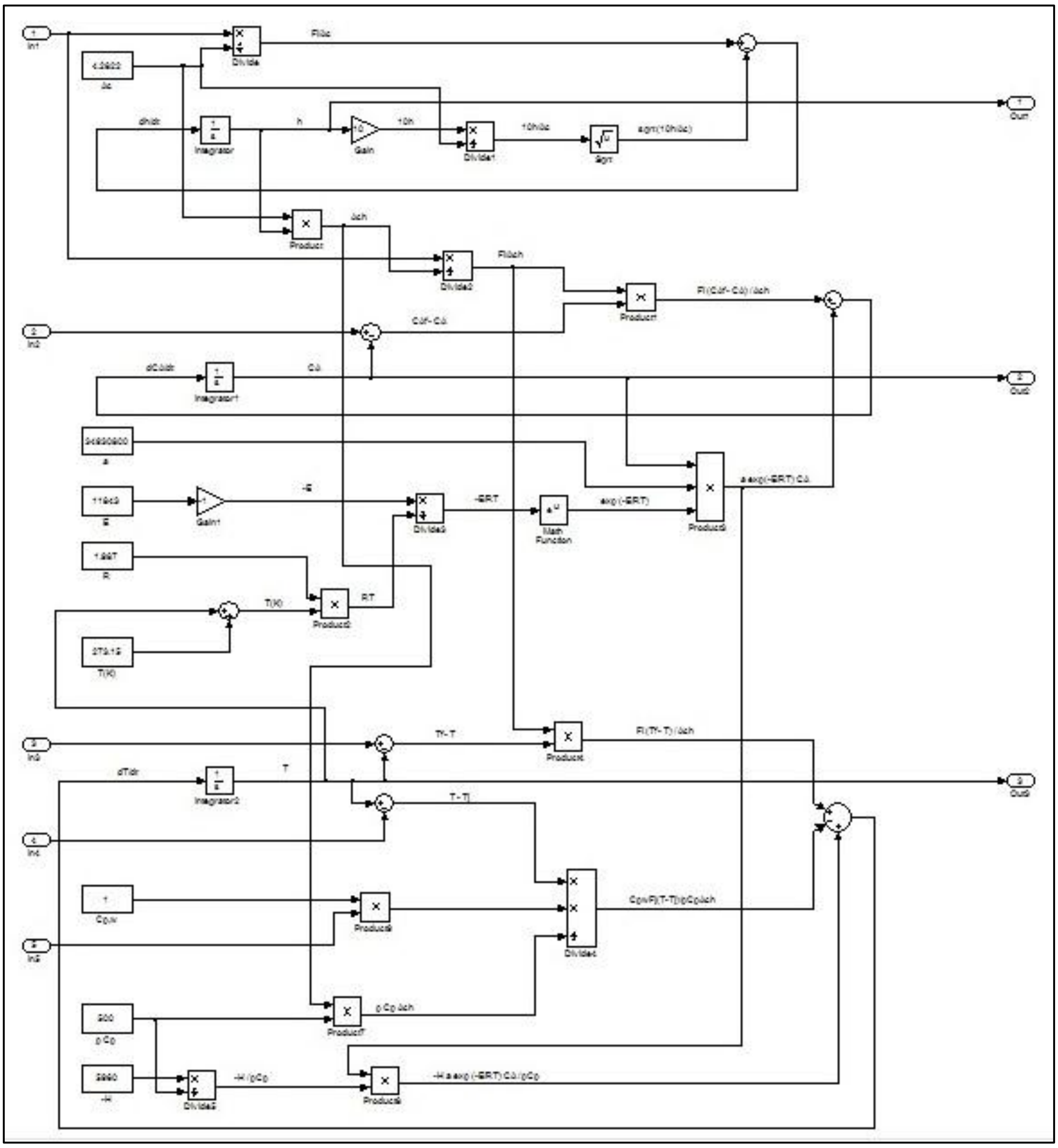


Figure 9: Nonlinear CSTR Subsystems

The most common question related to linear modeling for nonlinear system are how can a good linear model be obtained and is a dynamic system far from linear or close to linear. Usually people do not merely ask to assess the degree of nonlinearity, but ask for a good or the best possible linear model for a nonlinear system. Of course there is a strong link between nonlinearity assessment and linear modeling for nonlinear systems. All thee nonlinearity measures discussed above can be used to develop best linear model for nonlinear system. Due to the complexity of those methods, linear model for CSTR for this project is attained from linearization of nonlinear differential equations using Laplace transform.

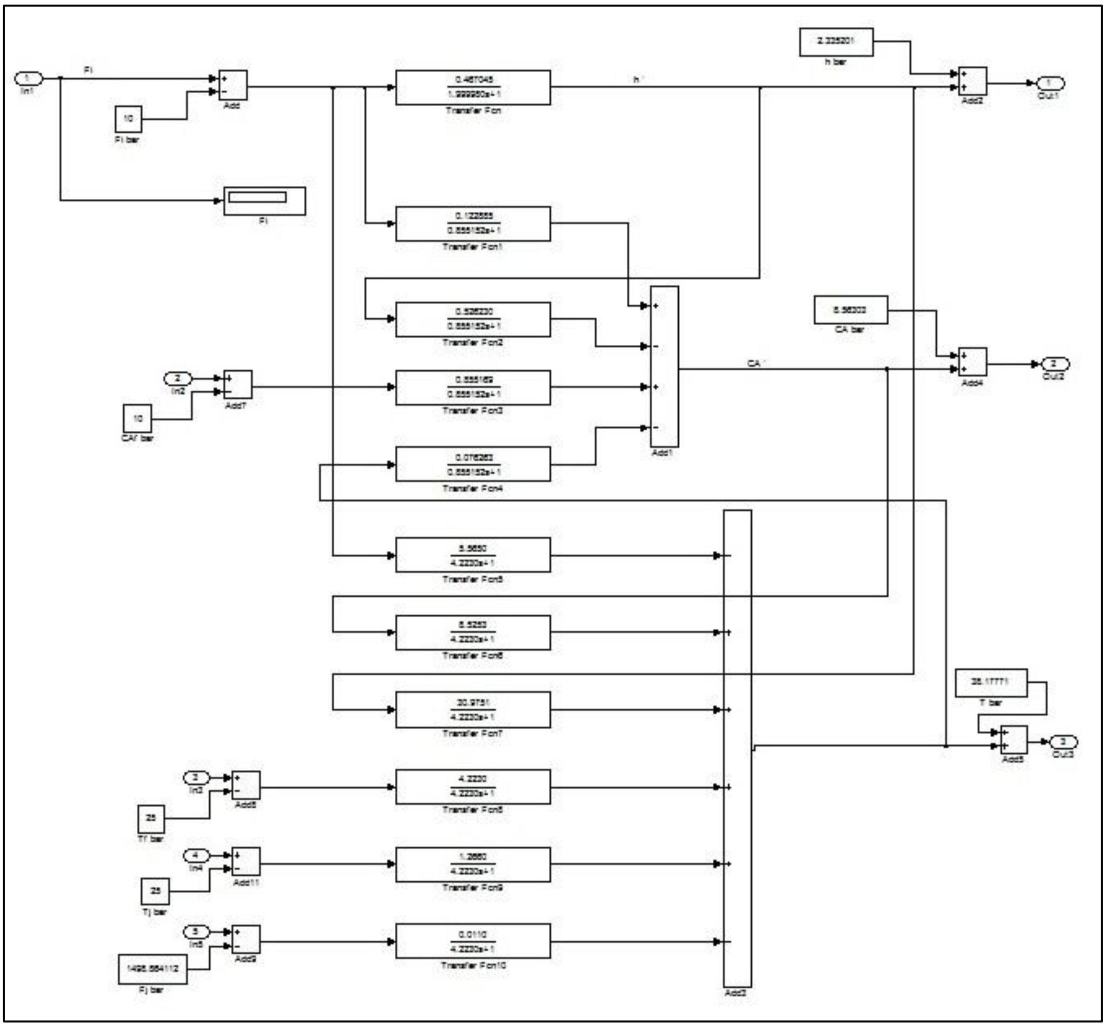


Figure 10: Linearized CSTR Subsystems

4.2 OPEN LOOP TEST ON CSTR MODEL

Step test was done in order to observe how much the difference in terms of the behavior of output when there is a change in the input between linear and nonlinear CSTR model. If the behavior and output value have not much differences, it can be deduced that linear model can be used as the alternative or representative for nonlinear CSTR. However, if the differences were too high, that indicates that CSTR is highly nonlinear whereby the linear model cannot be used to predict the trend or behavior of the nonlinear model.

4.2.1 Open Loop Test for Feed Flow Rate

Firstly, step change of input for volumetric feed flow rate was done close to the steady state value. The reason why input of volumetric feed flow rate was chosen to for step test is because it affects all the outputs of the model which are the liquid level in the reactor, concentration of reactant A in exit stream as well as reactor temperature. Therefore, volumetric feed flow rate will be used as the manipulated variable later on to control the controlled variable which is height of liquid inside the reactor.

i. Increment of 3% from initial value of feed flow rate

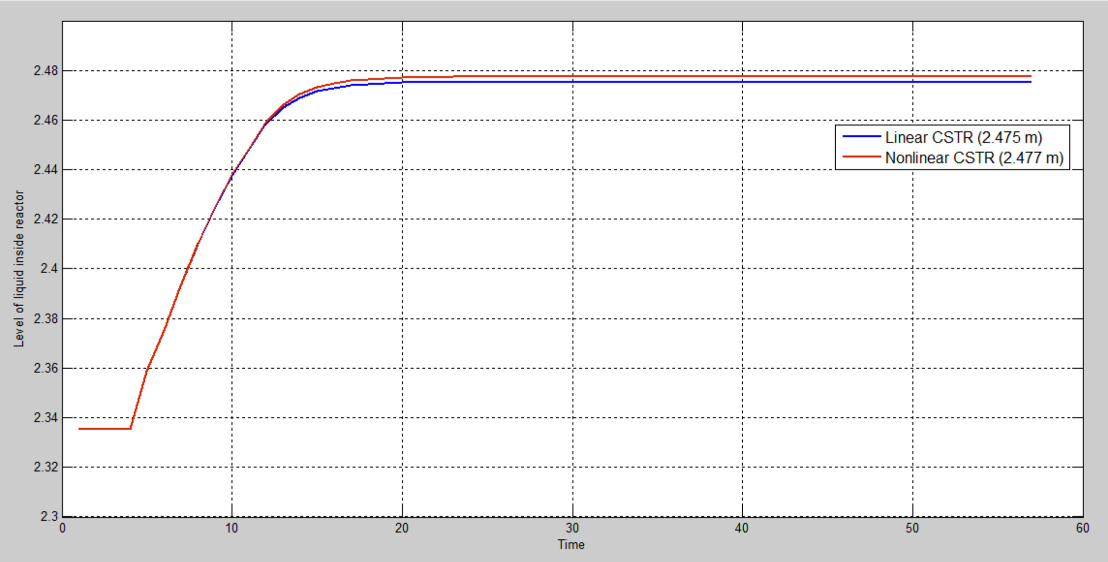


Figure 11: Changes on liquid level when step change of volumetric feed flow rate was done close to the steady state value

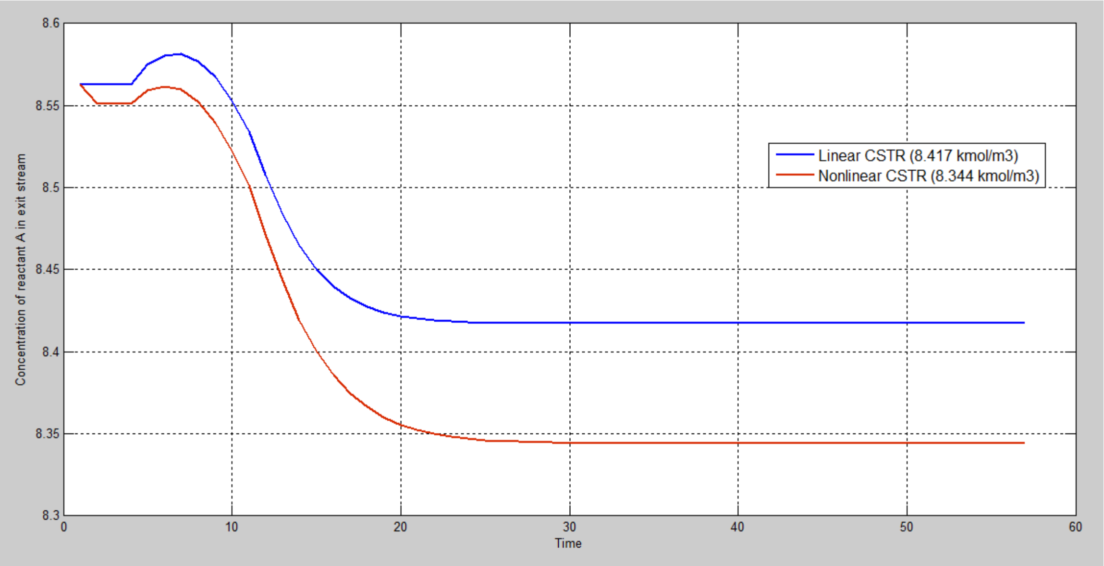


Figure 12: Changes on concentration of reactant A in exit stream when step change of volumetric feed flow rate was done close to the steady state value

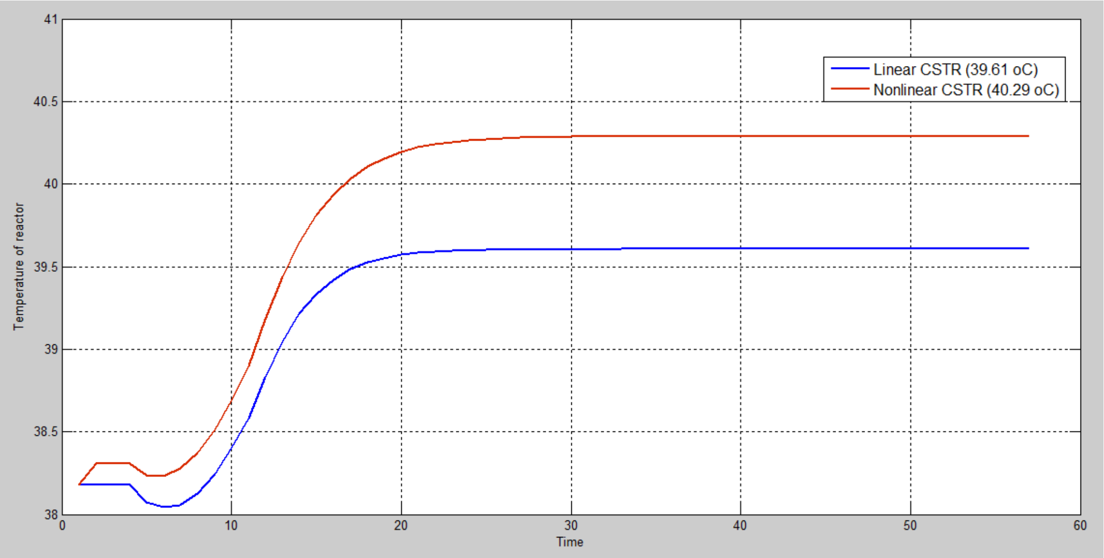


Figure 13: Changes on temperature of reactor when step change of volumetric feed flow rate was done close to the steady state value

Lastly, step change of input for volumetric feed flow rate was carried out far from the steady state value.

ii. Increment of 10% far from initial value of feed flow rate

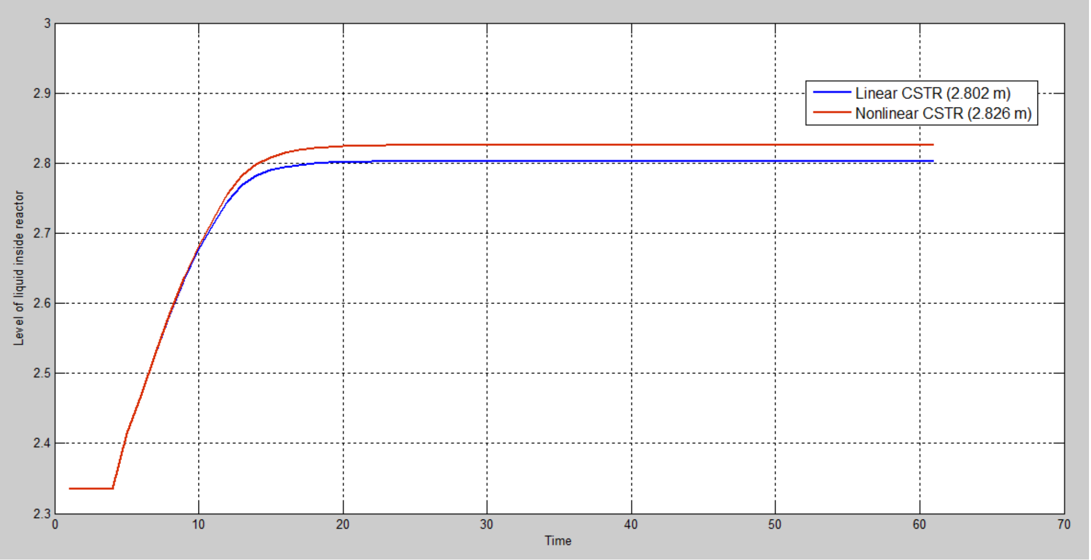


Figure 14: Changes on liquid level when step change of volumetric feed flow rate was done far from the steady state value

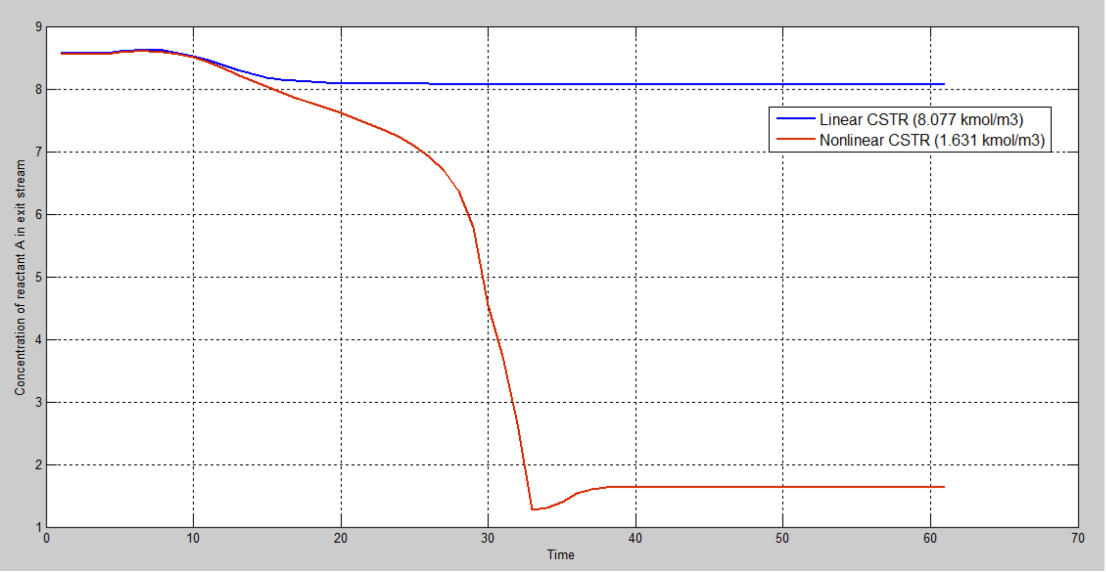


Figure 15: Changes on concentration of reactant A in exit stream when step change of volumetric feed flow rate was done far from the steady state value

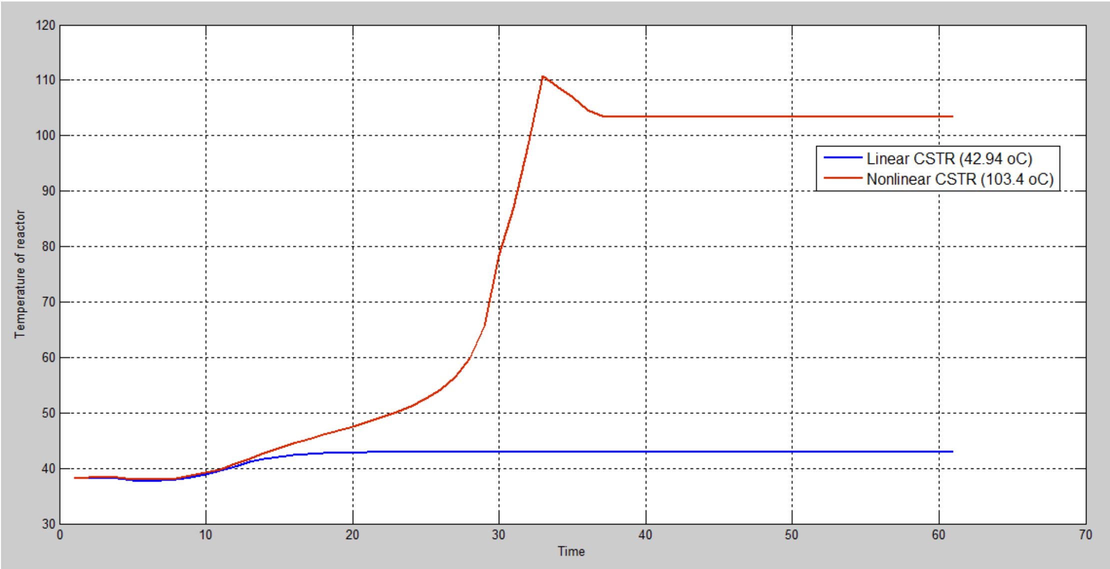


Figure 16: Changes on temperature of reactor when step change of volumetric feed flow rate was done far from the steady state value

4.2.1 Open Loop Test for Cooling Water Flow Rate

Firstly, step change of input for cooling water flow rate was done close to the steady state value. The reason why input of cooling water flow rate was chosen to for step test is because it is one of the manipulated variables to control the temperature of reactor which will directly affects the concentration of reactant A in exit stream as well as reactor temperature. Therefore, cooling water flow rate will be used as the manipulated variable later on to control the controlled variable which is temperature of reactor.

i. Decrement of 3% from initial value of cooling water flow rate

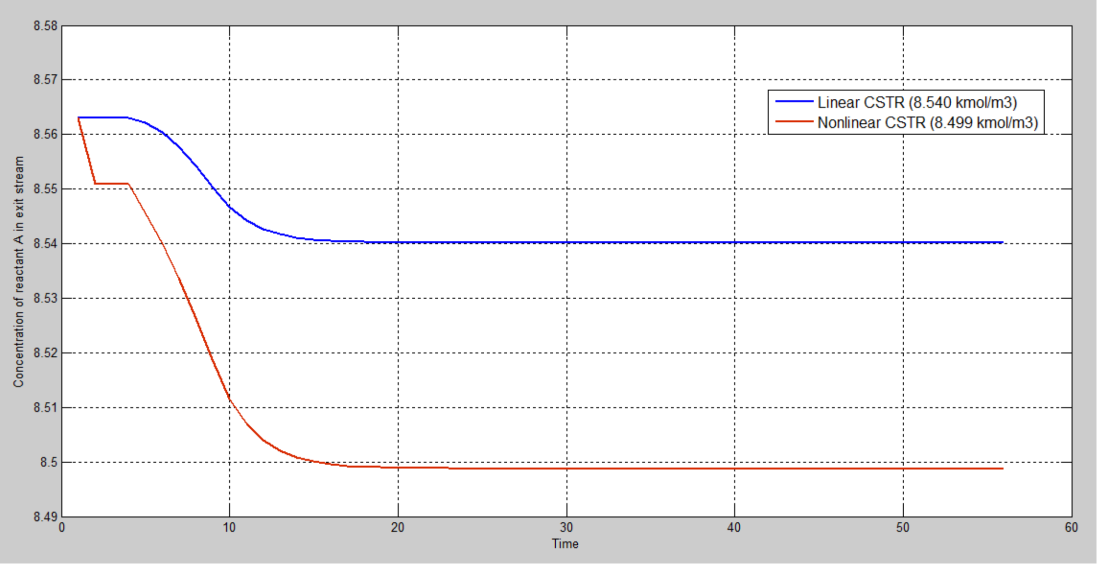


Figure 17: Changes on concentration of reactant A in exit stream when step change of cooling water flow rate was done close to the steady state value

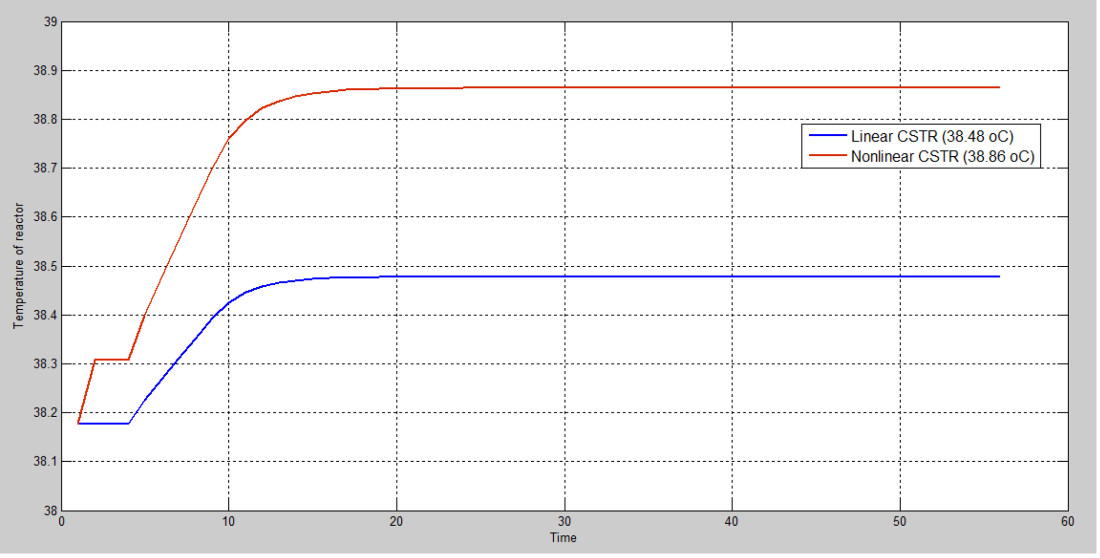


Figure 18: Changes on temperature of reactor when step change of cooling water flow rate was done close to the steady state value

ii. Decrement of 10% from initial value of cooling water flow rate

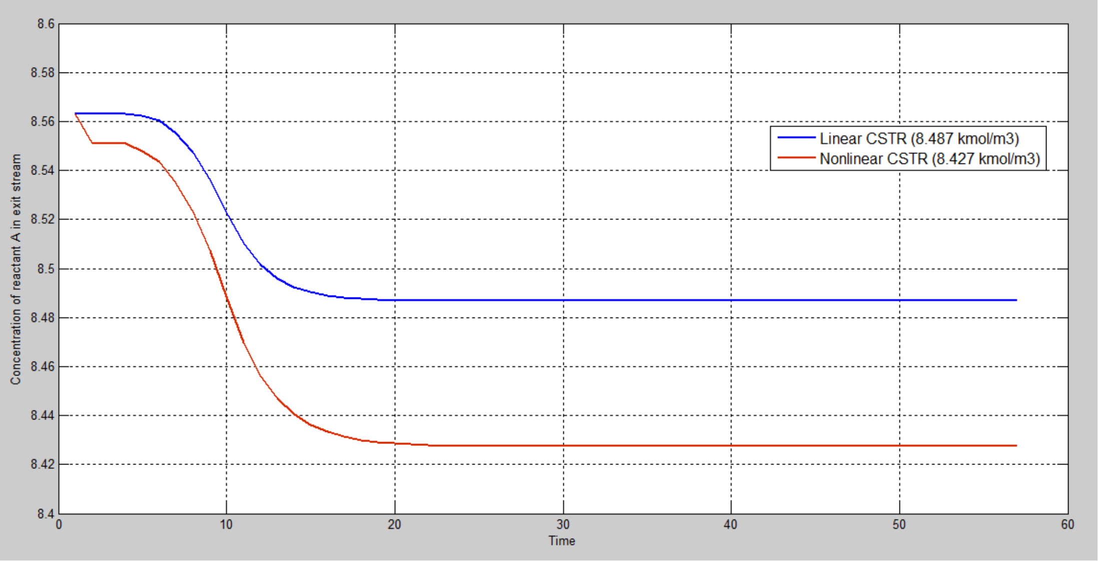


Figure 19: Changes on concentration of reactant A in exit stream when step change of cooling water flow rate was done far from the steady state value

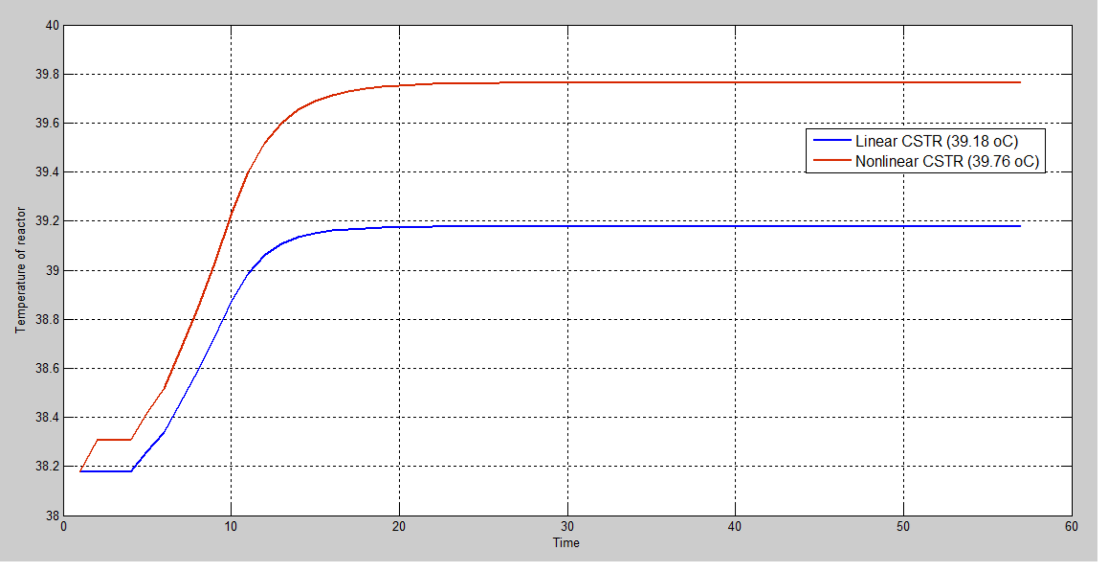


Figure 20: Changes on temperature of reactor when step change of cooling water flow rate was done far from the steady state value

Based on the response of water level (h), concentration of reactant A in the exit stream (CA) and reactor temperature (T) obtained from SIMULINK when step tests were carried out,, it was indicates that the CSTR shows nonlinear characteristics. CSTR exhibits nonlinearities behavior even when the input only deviates 3.0% from its steady state and the output become worse by oscillating fiercely when the system far away from steady state. Therefore, deduction that can be made is linearization systems of the nonlinear system behave quite differently and it cannot be used to represent nonlinear system such as CSTR. Next, linear controller will be developed to investigate either it can be used to stabilize the nonlinear system or not.

4.3 LINEAR CONTROL OF NONLINEAR SYSTEM

Based on the open loop test, dynamic behavior indicates that CSTR possess strong nonlinear characteristics and linear control supposedly is inadequate. Therefore, the next step is to design a linear controller for the nonlinear system in order to prove the proclamation. The controller design procedure should satisfy two criteria. Firstly, the controller design for a linear system should be much easier than a full nonlinear controller design. Secondly, the design procedure should guarantee stability of the system. For this study, there are two control objectives that need to be achieved which are the liquid level inside the reactor and the temperature of the reactor. Feedback control system will be used as the control strategy for this reactor.

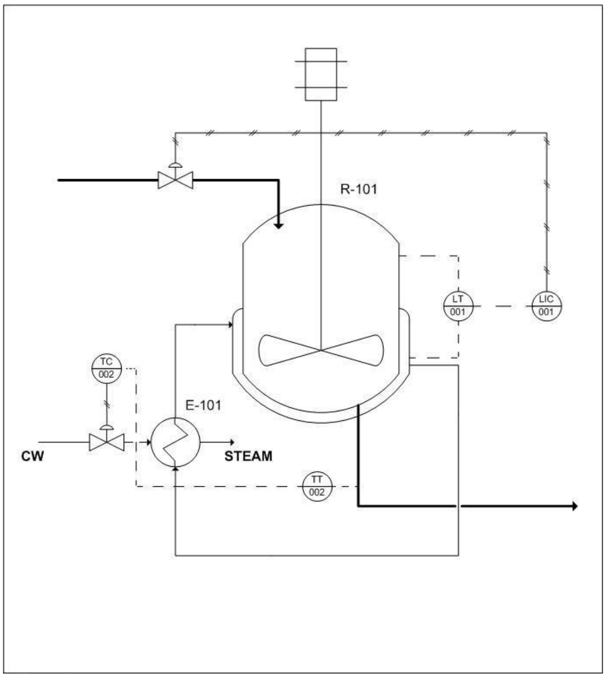


Figure 21: Control strateiesy for CSTR

Controlled variable	Manipulated variable
Liquid level inside the reactor	Volumetric feed flow rate
Reactor temperature	Cooling water feed flow rate

Table 8: Process variables for CSTR

4.3.1 Liquid level inside reactor

Proportional Integral Differential (PID) controller will be applied on the CSTR to control the liquid level inside the reactor. PID controller is set in a parallel from.

$$P(s)/E(s) = K_c(1 + 1/\tau_I s + \tau_D s) \quad (18)$$

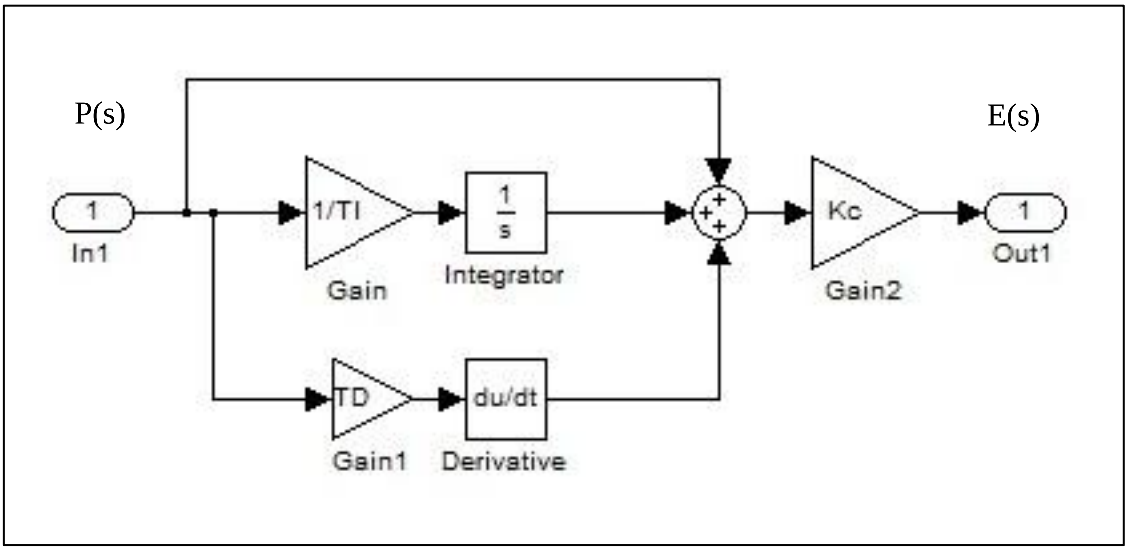


Figure 22: PID controllers in parallel form.

For the tuning purpose, Cohen Coon tuning method will be used to determine the tuning parameters; K_C, T_I and T_D. The reason why Cohen Coon method is used for this project is because the Cohen Coon tuning rules are suited to a wider variety od processes than the Ziegler-Nichols tuning rules. The Cohen Coon method of controller tuning corrects the slow, steady-state response given by the Ziegler-Nichols method when there is a large dead time or process delay relative to the open loop time constant. A large process delay is necessary to make this method practical because otherwise unreasonably large controller gains will be predicted.

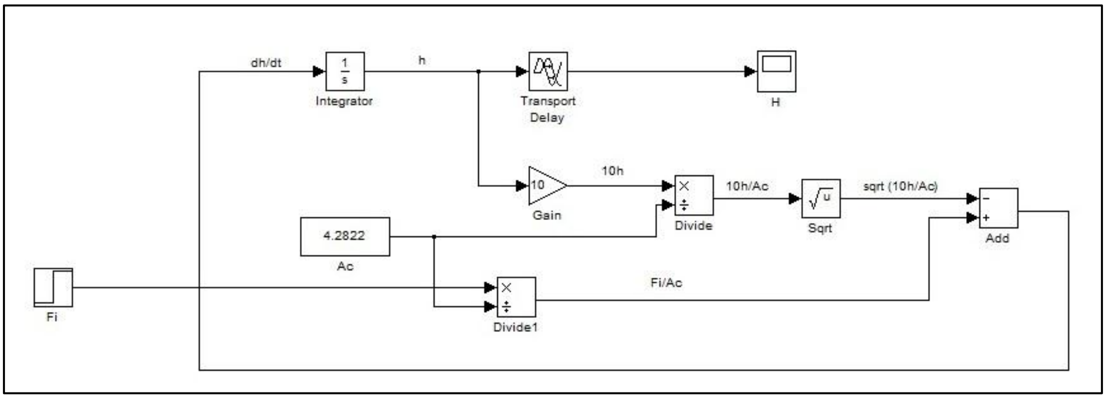


Figure 23: Cohen Coon tuning method for liquid level PID controller

The process in Cohen-Coon turning method is the following:

i. Wait until the process reaches steady state.

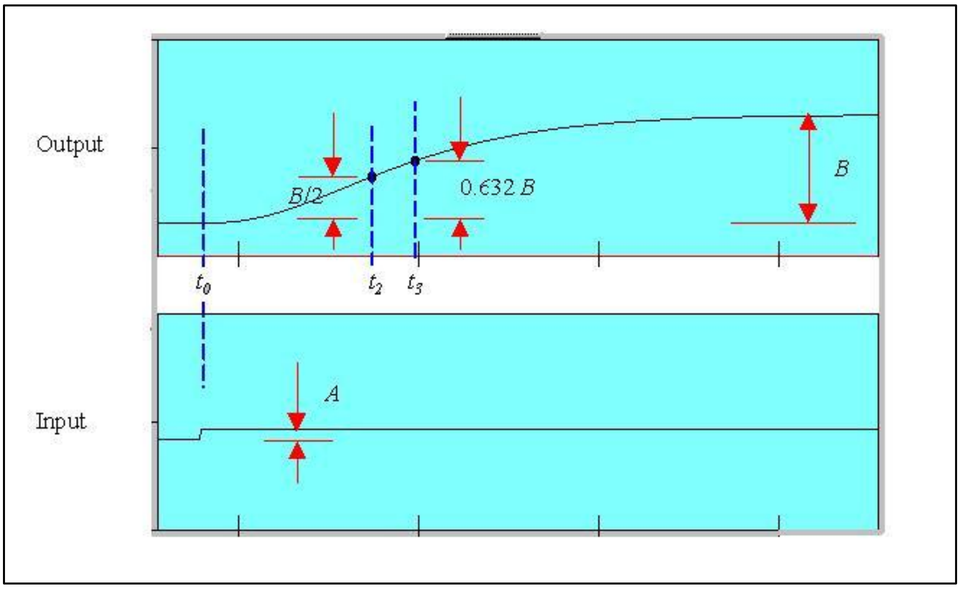
ii. Introduce a step change in the input.

iii. Based on the output, obtain an approximate first order process with a time constant τ delayed by τ_{dead} units from when the input step was introduced. The values of τ and τ_{dead} can be obtained by first recording the following time instances:

t0 = time at input step start point

t2 = time when reaches half point

t3 = time when reaches 63.2% point



iv. Using the measurements at t0, t2, t3, A and B, evaluate the process parameters τ, τ_{dead}, and K_o.

v. Find the controller parameters based on τ, τ_{dead}, and K_o.

$t_1 = (t_2 - \ln(2)\, t_3)/(1 - \ln(2))$

$\tau = t_3 - t_1$

$\tau_{DEL} = t_1 - t_0$

$K = B/A$

$r = \tau_{DEL}/\tau$

Tuning Parameter	Kc	TI	TD
PID	$\frac{1}{K.r}(\frac{4}{3} + \frac{r}{4})$	$\tau_{DEL}(\frac{32+6r}{13+8r})$	$\tau_{DEL}(\frac{4}{11+2r})$
	2.8576	0.7254	0.1113

Table 9: Tuning formula for PID controller

Performance of PID controller is observed by using these tuning parameter values.

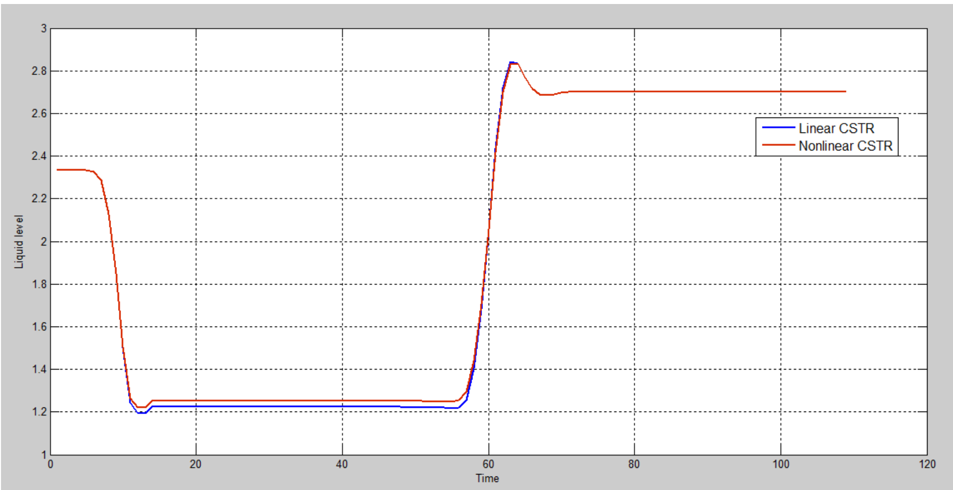


Figure 24: Performance of PID controller through set point tracking of liquid level inside reactor

From the graph, it shows that linear controller (PID) can be used to control the liquid level inside the reactor. This is because the liquid level inside the reactor is not much affected by nonlinearities as no disturbance variables that can ruin the system. That is why linear controller can be used to control liquid level by adjusting the volumetric feed flow rate into the reactor.

4.3.2 Temperature of reactor

Proportional Integral (PI) controller will be applied on the CSTR to control temperature of the reactor. PI controller is set in a parallel from.

$$P(s)/E(s) = K_c(1 + 1/\tau_I s) \tag{19}$$

Below is the block diagram in SIMULINK to perform Cohen Coon tuning method.

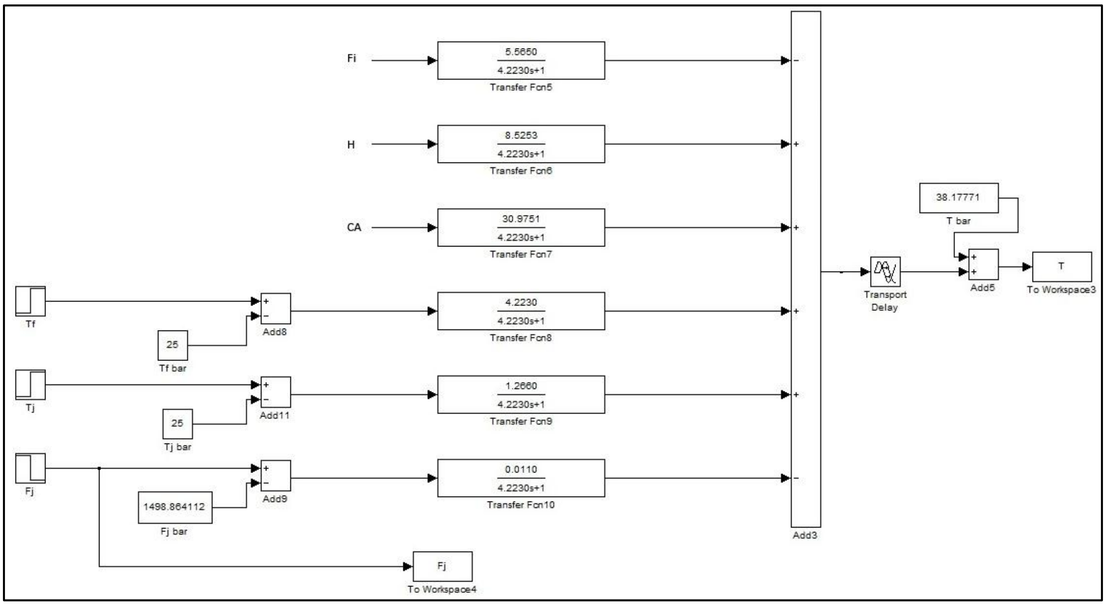


Figure 25: Cohen Coon tuning method for reactor temperature PI controller

Based on graph obtained from Cohen Coon tuning method, all the tuning parameters can be calculated using the formula below.

Tuning Parameter	Kc	TI
PI	$\frac{1}{K.r}(\frac{9}{10}+\frac{r}{12})$	$\tau_{DEL}(\frac{30+3r}{9+20r})$
	-146.5870	2.9904

Table 10: Tuning formula for PI controller

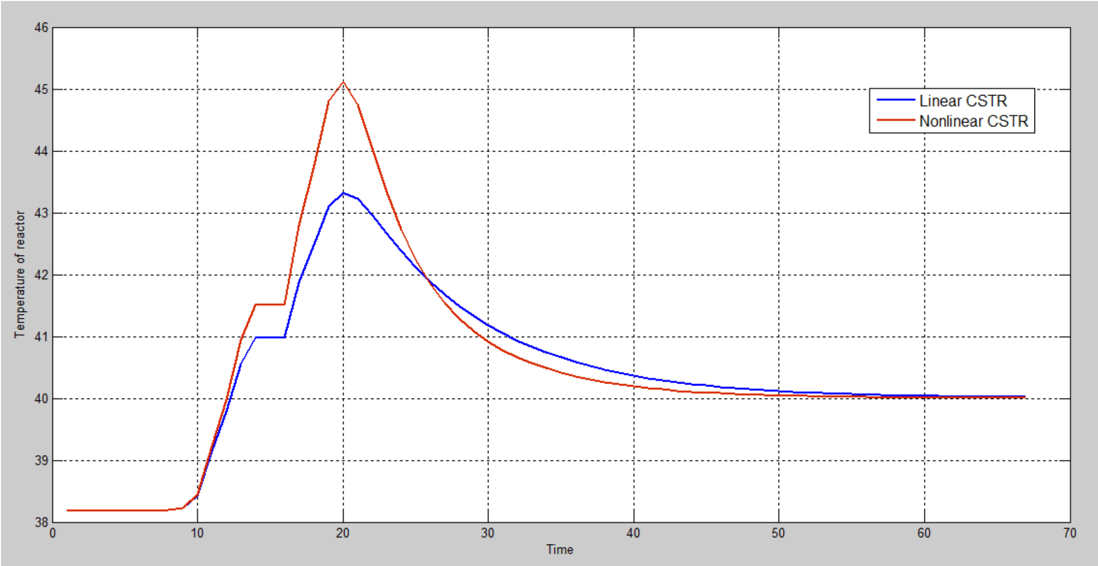


Figure 26: Performance of PI controller through set point tracking of temperature of reactor

In order to perceive the performance of PI controller against nonlinear CSTR, set point tracking was carried out. As shown in the Figure 24, it indicates that the performance of linear controller against nonlinear system is quite poor. This is because reactor temperature is very much affected by nonlinearities and many disturbance variables that can destabilize the system. That is why linear controller cannot be used to control highly nonlinear system.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

As a conclusion to this progress report, all the tasks that need to be done to develop a CSTR system had been accomplished successfully. There are two types of CSTR system, first is nonlinear model and the other one is linear model. Differential equations were derived for nonlinear CSTR while transfer functions were used to develop linear CSTR. The equations then translated into MATLAB SIMULINK. In order to observe the nonlinearity characteristics of the reactor, step input for feed flow rate and cooling water flow rate hds been carried out using simulations to illustrate how large the differences of outputs between linear and nonlinear models. The results shows that the reactor inherent nonlinearities especially when the system deviates far from its steady state.

There are many approaches that can used to quantify the level of nonlinearities. By using several nonlinearity assessments, CSTR has been proven highly nonlinear system. It was expected that single linear controller is not enough to stabilize the system because the CSTR was highly nonlinear, and it has be proven using PI controller to control the reactor temperature.

For this project, I personally recommend that this project need further improvements because this project only using basic linear PID controller to control highly nonlinear system which is CSTR. Therefore, decision to not use linear controller at all for CSTR still cannot be made because maybe more advance linear controller such as model predictive controller able to stabilize the nonlinear system compare to PID controller.

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