Stochastic Programming with Economic and Operational Risk Management in Petroleum Refinery Planning under Uncertainty

by

Nguyen Thi Huynh Nga

Dissertation submitted in partial fulfilment of
the requirements for the
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Universiti Teknologi PETRONAS
Bandar Seri Iskandar
31750 Tronoh
Perak Darul Ridzuan

CERTIFICATION OF APPROVAL

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Approved by,

S.Khok

(Cheng Seong Khor)

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

NGUYEN THI HUYNH NGA

ABSTRACT

Rising crude oil price and global energy concerns have revived great interests in the oil and gas industry, including the optimization of oil refinery operations. However, the economic environment of the refining industry is typically one of low margins with intense competition. This state of the industry calls for a continuous improvement in operating efficiency by reducing costs through engineering strategies. These strategies are derived based on an understanding of the world energy market and business processes, with the incorporation of advanced financial modeling and computational tools. Regard to the matter, this work proposes the application of the two-stage stochastic programming approach with fixed recourse to effectively account for both economic and operational risk management in the planning of oil refineries under uncertainty. The scenario planning and analysis approach is adopted to consider uncertainty in three parameters: prices of crude oil and commercial products, market demand for products, and production yields. However, a large number of scenarios are required to capture the probabilistic nature of the problem. Therefore, to circumvent the problem posed by the resulting large-scale model, a Monte Carlo simulation approach is implemented based on the sample average approximation (SAA) technique. The SAA technique enables the determination of the minimum number of scenarios required yet still able to compute the true optimal solution of the problem for a desired level of accuracy within the specified confidence intervals. Two measures of risk are considered, namely mean-absolute deviation (MAD) and Conditional Value-at-Risk (CVaR). A representative numerical example is presented to illustrate the proposed modeling approach using GAMS modeling language with the nonlinear solver CONOPT3.

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ABBREVIATIONS AND NOMENCLATURES

Sets and Indices

I	set of materials/product i
IP	set of materials/product i with price uncertainty
ID	set of materials/product i with demand uncertainty
IY	set of materials/product i with yield uncertainty
S	set of scenarios s
K	set of demand shortfall k_1 and demand surplus k_2
M	set of yield shortfall m_1 and yield surplus m_2
N	set of minimum number of scenario s

Parameters

$c_{s,i}$	unit price of raw materials and saleable products i with price uncertainty for scenario s
p_s	probability of each scenario s
$d_{i,k}$	demand penalty for material i with demand uncertainty for either demand shortfall or surplus k
$q_{i,m}$	yield penalty for product i with yield uncertainty for either yield shortfall or surplus m
θ	weight factor, θ_1, θ_2
α	confident level

Continuous Variables

x	material and saleable products flowrate
$CVaR$	conditional- value- at- risk
MAD	mean absolute deviation
$z_{i,s,k}$	demand penalty for material i in scenario s with demand uncertainty for either demand shortfall or surplus k
$y_{i,s,m}$	yield penalty for product i in scenario s with yield uncertainty for either yield shortfall or surplus m
z_o	deterministic profit
ξ	monetary value of demand and yield penalty
μ	expected deterministic profit $\mu(z_o)$ and expected demand and yield penalty $\mu(\xi)$
H	confident interval
$S(n)$	sampling variance estimator
z	confident interval

Non-convex variables

VaR	value-at-risk

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CHAPTER 1

INTRODUCTION

1. BACKGROUND OF STUDY

Stochastic Programming (SP) is a framework for modeling optimization that involves uncertainty. The following example of linear program helps to explain the concept of Stochastic:

$$\left.\begin{array}{l} \min\left\{c_1x_1 + c_2x_2 + \dots + c_nx_n\right\} \\ \text{Subject to} \\ a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le b_n \end{array}\right\} \qquad (1)$$

$$x_1, x_2, \dots, x_n > 0$$

By using matrix vector notation, formulation (1) can be written as:

$$\left.\begin{array}{l} \min c^T x \\ \text{s.t } Ax \le b \\ x \ge 0 \end{array}\right\} \qquad (2)$$

If b is a constant, the model is called *Deterministic model.* However, with b is uncertain, it is called *Stochastic Model.*

Two-stage Stochastic Programming with fixed recourse via scenario analysis is applied in Chemical process industry, oil and gas industry, water reservoir system, environment protection and control, etc... to predict planning under uncertainties and operational risk management by applying the concept of Variance, Mean Absolute Deviation (MAD) and Conditional-value-at-risk (CVaR)

Recourse is corrective action made after a random event has taken place. An example of two-stages Recourse is described as:

- Variables x: control the flow rate of Methane enters reactor to form Methanol.
- Random event: The control valve is malfunctioned and fully opened which results in the abundant amount of Methane fed into reactor, affecting the product quality.
- Recourse action y: To correct what has been messed up by the random event: fix the control valve, penalty for overproduction of Methanol.

The above example includes two stages: the data for the first stage (x) are known with certainty and some data for the second stage are stochastic or random which need corrective action. Two-stage Stochastic Programming aims to serve the optimization purpose of a process by minimizing uncertainties and maximizing profit.

2. PROBLEM STATEMENT

The midterm refinery production planning problem addressed in this paper can be stated as follows. Given the following information:

- The available process units and their capacities;
- Cost of crude oil and refinery products;
- Market demand of products

Our goal is to determine our goal is to determine the amount of materials that are processed in each process stream of each process unit by considering the following uncertain parameters whose stochastic data (including probabilities) are available or obtainable:

- Market demand for products, that is, production amounts of desired products;
- Prices of crude oil and the saleable products; and
- Product (or production) yields of crude oil from chemical reactions in the crude distillation unit

It is assumed that:

- The uncertain parameters of prices, costs, and demands are externally imposed, that is, they are exogenous uncertainties;
- Further, the uncertain parameters are random variables that exhibit the behavior and properties of discrete probability distribution functions; and
- The physical resources of the plant are fixed.

3. OBJECTIVES

Following are the objectives of the project:

- There are large numbers of scenarios that create difficulty to handle various circumstances. For example, there may be more than thousands of cases happening. It is hard to predict and control numerous scenarios. Therefore, it is necessary to find the minimum number of scenarios to capture all the circumstances. Monte Carlo simulation approach based on the sample average approximation (SAA) technique is applied in this thesis to generate the minimum number of scenarios which present for thousands cases.
- The risk terms in Khor (2007) are handled using the metric mean-absolute deviation. After obtaining the first model with MAD as risk measurement, the second model is developed in which the risk terms are performed by CVaR. A comparison between the two models to assess which of these two risk measures is superior, both computationally and conceptually, in capturing the economic and operating risk in the planning of a refinery. Khor model (2007) is expressed:

  $$\max z_4 = E[z_o] - \theta_1 V(z_o) - E_{s'} - \theta_3 W \quad (3)$$

  where:

  $E[z_o]$: Expected deterministic profit (crude oil and saleable products)

  θ_1, $\theta_3 \in (0, 1]$: weight the components of the objective function

  $V(z_o)$: Variance of price uncertainty

  $E_{s'}$: Expected penalty of demand and yield uncertainties

W : MAD of demand and yield penalty

Apply MAD as risk measure:

$$\max\ z = E[z_0] - \theta_1 \mathrm{MAD}_{z_0} - E[\xi] - \theta_3 \mathrm{MAD}_{z_0} \tag{4}$$

Apply CVaR as risk measure:

$$\max\ z = E[z_0] - \theta_1 CVaR_{\xi} - E[\xi] - \theta_3 CVaR_{\xi} \tag{5}$$

- Solving optimization problem to find the input flow rate raw materials and final product to maximize profit.

4. SCOPE OF STUDY

- Monte Carlo simulation approach based on Sample Average Approximation (SAA) to determine minimum number of scenario
- General formulation of stochastic refinery planning with risk measure expressed in terms of MAD
- General formulation of stochastic refinery planning with risk measure expressed in terms of CVaR.

CHAPTER 2
THEORY

1. GENERAL FORMULA OF TWO-STAGE-STOCHASTIC PROGRAMMING

The classical two-stage stochastic linear program (SLP) was originally proposed in the seminal works of Dantzig (1955) and Beale (1955) has the following general form:

$$\begin{aligned} \min \;& c^T x + E_\xi\left[Q(x,\xi(\omega))\right] \\ \text{s.t. to } & Ax = b \\ & x \in X \geq 0 \end{aligned} \tag{6}$$

where
$$\begin{aligned} Q(x,\xi(\omega)) &= \min\; q^T(\omega) y \\ \text{s.t. to } W(\omega)y &= h(\omega) - T(\omega)x \\ y &\geq 0 \end{aligned} \tag{7}$$

With the notation:

$x \in R^n$	: Vector of first-stage decision variables, size $(1\times n)$
c	: First-stage column vector of cost coefficient, sizes $(n \times 1)$
A	: First-stage coefficient matrix, size $(m \times n)$
b	: Corresponding right-hand side vectors, size $(m \times 1)$
$\omega \in \Omega$	: Random events or scenario
$\xi(\omega)$	: Random vector
$q(\omega)$	: Second stage vector of recourse cost coefficient vectors size $(k \times 1)$
$h(\omega)$	: Second stage right-hand side vectors, size $(l \times 1)$
$T(\omega)$	: Matrix that ties the two stages together, size $(l \times k)$
$W(\omega)$	: Random recourse coefficient matrix, size $(l \times k)$
y	: Vector of second-stage decision variables, size $(k\times 1)$

$c^T x$ is known as the first stage or "here and now" decision, x does not response to ω. In contrast, y presents second stage variable with $Q(x,\xi(\omega))$ is "wait and see" and is determined after observation regarding ω has been obtained. Model (4) is the second stage problem, the recourse subproblem.

2. TWO-STAGE-STOCHASTIC PROGRAMMING WITH SIMPLE RECOURSE SUBPROBLEM

Simple recourse model is a special case of recourse model when recourse coefficients in the second stage, W, form an identity matrix. In general, we have:

$$Q(x,\xi(\omega)) = \sum_{i \in I} Q_i(x,\xi(\omega))$$

Where:

$$\begin{aligned} Q_i(x,\xi(\omega)) = \min \quad & q^+_{\omega i} y^+_i + q^-_{\omega i} y^-_i \\ \text{s.t.} \quad & I\, y^+_i - I\, y^-_i = h(\omega_i) - (T(\omega)x)_i \\ & y^+_i,\, y^-_i \geq 0 \end{aligned} \tag{8}$$

$h(\omega) - T(\omega)x$, a feasible solution to (5) is easily determined by setting y^+ and y^- accordingly. Moreover, if the i^{th} component of q^+_ω - q^-_ω >0, this feasible solution is optimal.

Example of simple recourse is that when a target profit in one company is determined, the company will try to reduce the deviation from profit.

3. TWO-STAGE STOCHASTIC PROGRAMMING WITH FIXED RECOURSE SUBPROBLEM

Fixed recourse model is the model that the constraint matrix in the recourse subprolem is fixed (not subject to uncertainty). (8) is written as:

$$Q(x,\xi(\omega)) = \min q^T(\omega) y$$
$$\text{s.t. } Wy = h(\omega) - T(\omega)x \tag{9}$$
$$y \geq 0$$

When second stage objective coefficients are also fixed, the recourse subproblem can be written as:

$$Q(x,\xi(\omega)) = \min \quad \pi^T (h(\omega) - T(\omega)x) \tag{10}$$
$$\text{s.t.} \quad \pi^T W \leq q^T$$
$$\pi \geq 0$$

4. TWO-STAGE STOCHASTIC PROGRAMMING WITH COMPLETE RECOURSE SUBPROBLEM

A problem is said to have complete recourse if $Y(\omega,\chi) = \{y \mid W_\omega y \geq \chi\}$ is nonempty for any value of χ and the recourse function is necessary finite, $Q(x,\xi(\omega)) = \infty$. Moreover, relatively complete recourse results if $Y(\omega,\chi)$ is nonempty for all $\chi \in \{h(\omega) - T(\omega)x \mid (\omega,x) \in \Omega \times X\}$.

With the complete recourse problem, model (4) becomes:

$$Q(x,\xi(\omega)) = \text{Min } q^T(\omega)y + \text{M e}^T \text{z}$$
$$\text{s.t } Wy + z \geq h(\omega) - T(\omega)x \text{ , } y,z \geq 0 \tag{11}$$

With M: large constant and e: appropriately dimensioned vector of ones.

5. CONCEPTS FROM FINANCIAL MATHEMATICS ON RISK MEASUREMENT AND RISK MANAGEMENT

Three broad classes of risks are currently receiving wide attention and heavily studied in the literature related to financial mathematics, risk measurement, and risk management. The first, *market risk,* attempts to determine the uncertainty in the prices of an object that is traded in a liquid market. The second, *credit risk,* attempts to place a value on the uncertainty associated with an account receivable. For example, researchers are interested in answering the question of how should we account for the possibility that a debtor may default on an obligation. The third, *operational risk*, basically tries to handle everything else. It considers the full set of other risks that a business must typically face, including the risk of catastrophic political events, weather-related risk, and risk of criminal activity (Pulleyblank, 2000). Table 1 provides examples of risk metrics or measures according to the duration of application and nature of risk involved.

Table 1: Period, nature of risk and risk metric

Duration of application	Nature of risk	Risk metric
Short term (< 1 month)	Operational	Earnings
Intermediate/Medium/Midterm (1 month–1 year)	Financial/Trading	Value-at-risk, cash flow, earnings, credit risk
Long term (> 1 year)	Asset valuation	Equity

6. ECONOMIC RISK

Economic risk is perceived by business people in two ways. The first is risk of not achieving the targeted financial objective. The second is the risk of variation in the results (Park and Sharp-Bette, 1990). The first type of risk may be caused by a number of causes whether economic, political, technical, or the like (of it), and can be represented as the probability of not achieving the financial objective. This type of risk has been employed with planning activities by Barbaro and Bagajewicz (2003). The second type of risk can be well-handled by variance techniques such as the variance of Expected Monetary Value (EMV) (Bush and Johnson, 1998) or risk-adjusted return family of methods such as Sharpe ratio (Jones, 1998). Applequist et al. (2000) has adopted a risk premium defined as an increase in the expected return in exchange for a given amount of variance in order to evaluate risk and uncertainty for chemical manufacturing plants (Al-Sharrah, 2006).

7. VALUE-AT-RISK (VAR) AND CONDITIONAL-VALUE-AT-RISK (CVAR)

Figure 1 expresses the idea about VaR and CVaR.

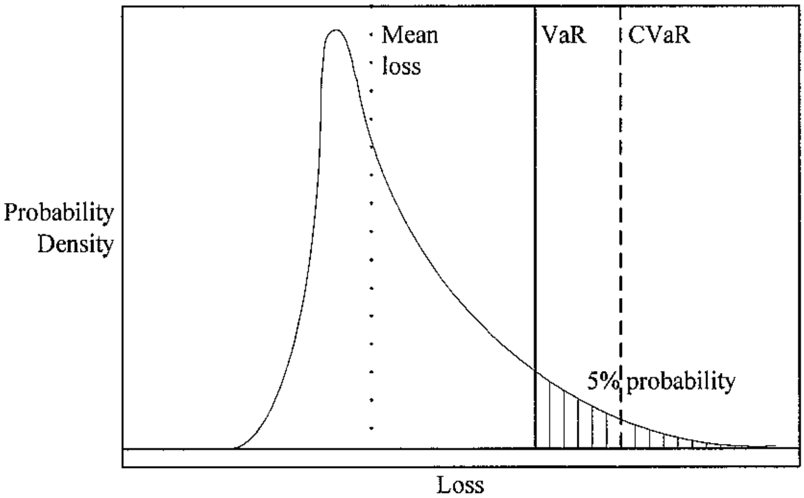

Figure 1: VaR and CVaR illustration

According to Rockafellar and Uryasev (2002), Value-atRisk (VaR) can informally be defined as a maximum loss in a specified period with some confidence level, except α (e.g., confidence level = 95%, period = 1 week). Formally, α-VaR is the α-percentile of the loss distribution. α-VaR is a smallest value such that probability that loss exceeds or equals to this value is bigger or equals to α. It suffers, however, from being unstable and difficult to work with numerically when losses are not normally distributed.

CVaR (Mean Excess Loss, Mean Shortfall or Tail VaR) is a risk assessment technique used to reduce the probability a portfolio will incur high losses. CVaR is performed by taking the likelihood (at a specific confidence level, example, 0.95 or 0.99, etc…) that a specific loss will exceed the value at risk (VaR). In mathematical point of view, CVaR is derived by taking a weighted average between the VaR and losses exceeding the VaR. CVaR maintains consistency with VaR by yielding the same results in limited settings where VaR computations are tractable, i.e., for normal distribution. Most importantly for applications, CVaR can be expressed by a remarkable minimization formula. This formula can readily be incorporated into problems of optimization with respect to the decision vector $x \in X$ that are designed to minimize risk or shape in within the bounds. Significant shortcuts are thereby achieved while preserving crucial problem features like convexity.

8. MONTE CARLO SIMULATION APPROACH BASED ON SAMPLE AVERAGE APPROXIMATION (SAA)

Two first step of SAA algorithm/steps proposed by Santoso et al. (2005):

Step1: Generate M independent samples each of size N. For each sample solve the corresponding SAA problem

$$\min_{y \in Y} \left\{ c^T y + \frac{1}{N} \sum_{n=1}^{N} Q(y, \xi_j^{\,n}) \right\} \tag{12}$$

Step 2: Compute minimum number of scenario

Step 3: Apply risk measure into the model

(Referred to chapter 3 for more detail about the mathematical equation)

9. PROCESS FLOW NETWORK

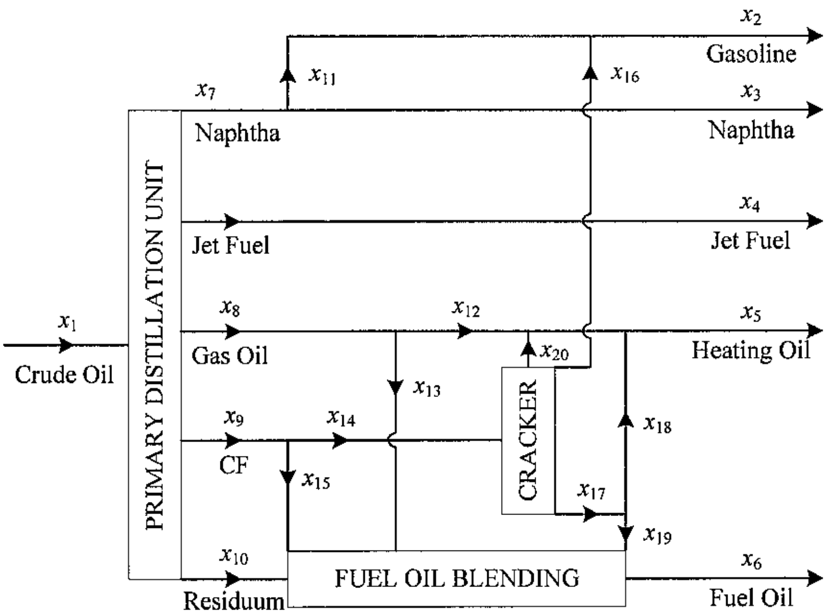

Figure 2: Process flow network

Table 2: Material balance around the units in the process flow network

Material balance	
$-0.40X_{14} + X_{16} = 0$	(13)
$-0.55X_{14} + X_{17} = 0$	(14)
$-0.05X_{14} + X_{20} = 0$	(15)
$0.5X_2 - X_{11} = 0$	(16)
$0.5X_2 - X_{16} = 0$	(17)
$0.75X_5 - X_{12} = 0$	(18)
$0.25X_5 - X_{18} = 0$	(19)
$-X_7 + X_3 + X_{11} = 0$	(20)
$-X_8 + X_{12} + X_{13} = 0$	(21)
$-X_9 + X_{14} + X_{15} = 0$	(22)
$-X_{17} + X_{18} + X_{19} = 0$	(23)
$-X_{10} - X_{13} - X_{15} - X_{19} + X_6 = 0$	(24)

Materials and saleable products are divided into three groups

- Demand uncertainty (X_{ID}): X_2, X_3, X_4, X_5, X_6

- Yield uncertainty (X_{IY}): X_4, X_7, X_8, X_9, X_{10}
- Price uncertainty (X_{IP}): X_1, X_2, X_3, X_4, X_5, X_6, X_{14}

CHAPTER 3

METHODOLOGY

1. OPTIMIZATION PROBLEM

a) Objective

MAD: $\max\ z = E\left[z_0\right] - \theta_1 \mathrm{MAD}_{z_0} - E[\xi] - \theta_3 \mathrm{MAD}_{z_0}$, or

CVaR : $\max\ z = E\left[z_0\right] - \theta_1 CVaR_\xi - E[\xi] - \theta_3 CVaR_\xi$

b) Variables

- X_i: material and saleable products
- Objective value: z ($/day)
- Penalty demand: Z(ID, S,K)
- Penalty yield: Y(IY,S, M)

c) Constrained equations of the model

$$X_1 \leq 15{,}000 \tag{25}$$

$$X_{14} \leq 2{,}500 \tag{26}$$

$$\text{-Yield(S,IY)} \times X_1 + X_{IY} + \text{Y(IY,S,'M1')} - \text{Y(IY,S,'M2')} = 0 \tag{27}$$

$$X_{ID} + \text{Z(IY,S,'K1')} - \text{Z(ID,S,'K2')} = 0 \tag{28}$$

And equation from eq.(13) to eq.(24)

Lower and upper bounds of all the materials and products flowrate:

Table 3: Lower bound and upper bound of all material and products flow rate

X_i	Lower bound	Upper bound
X_1	12,500	15000
X_2	2000	2700
X_3	625	1100
X_4	1875	2300
X_5	1700	1700
X_6	6175	9500
X_7	1625	1950
X_8	2750	3300
X_9	2500	3000
X_{10}	3750	3000
X_{11}	1000	1350
X_{12}	1275	1275
X_{13}	1475	3300
X_{14}	2500	2500
X_{15}	0	3000
X_{16}	1000	1200
X_{17}	1375	1650
X_{18}	425	425
X_{19}	950	1650
X_{20}	125	150

2 MONTE CARLO SIMULATION APPROACH BASED ON SAMPLE AVERAGE APPROXIMATION (SAA)

In this work, we adopt the Monte Carlo simulation approach for scenario generation based on the Sample Average Approximation (SAA) method (Shapiro, 2000; Shapiro and Homem-de-Mello , 1998; You and Grossmann, 2008) proposed by Santoso et al. (2005). The procedure involved is as follows:

Step 1:

A relatively small number of scenarios (for example, 50 scenarios) with their associated probabilities are randomly and independently generated for the uncertain parameters of prices, demands, and yields. (This data is otherwise obtained from plant historical data.) The resulting stochastic model (a linear program) with the objective function given in (4) or (5) is solved to determine the optimal stochastic profit with its corresponding material flowrates.

$$\text{max profit} = E[z] = \underbrace{\sum_{i\in I}\sum_{s\in S} p_s c_{i,s} x_i}_{E[z_o]} - \underbrace{\sum_{s\in S} p_s \left[\sum_{i\in I}\sum_{k\in K} d_{i,k} z_{i,s,k} + \sum_{i\in I}\sum_{m\in M} q_{i,m} y_{i,s,m}\right]}_{E[\xi]}$$

$$E[z] = E[z_o] - E[\xi] \tag{29}$$

Step 2:

- From the stochastic profit in step 1 and materials's flowrate in step 1, Monte Carlo sampling variance estimator is calculated:

$$S(n) = \sqrt{\frac{\sum_{s=1}^{S}\left(E[z] - z_s\right)^2}{S-1}} \tag{30}$$

Where $z_s = \sum_{i\in I} c_{i,s} x_i - \xi_s$

- Confidence interval H of 1-α is given as:

$$\left[E[z] - \frac{z_{\alpha/2} S(n)}{\sqrt{S}}, E[z] + \frac{z_{\alpha/2} S(n)}{\sqrt{S}}\right] \tag{31}$$

- The minimum number of scenarios N that is theoretically required to obtain an optimal solution is determined using the relation below:

$$N = \left[\frac{z_{\alpha/2} S(n)}{H}\right]^2 \tag{32}$$

Where:

- $z_{\alpha/2}$ = 1.96 at confidence interval (1-α) = 95% (Assuming $z_{\alpha/2}$ is standard normal deviate such that 1-$z_{\alpha/2}$ satisfies for a standard normal distributed variable z ~ N (0, 1), Pr (z $\leq z_{\alpha/2}$) = 1-α/2

Numerical experiments indicate that well controlled choice of the sample sizes can significantly reduce the computational time and improve the accuracy of obtained solutions.

Step 3:

Risk measure using the metrics of MAD and CVaR is incorporated in a new stochastic model with the scenarios given by the minimum number of scenarios N, in which the N number of scenarios are generated as a new set of independent random samples of the uncertain parameters.

A new stochastic model is formulated based on minimum number of scenarios N with the incorporation of the risk measure of MAD and CVaR, respectively,

3. GENERAL FORMULATION OF STOCHASTIC REFINERY PLANNING WITH RISK EXPRESSED IN TERMS OF MAD

Konno and Koshizuka (2005), Konno and Yamazaki (1991), L1 risk of mean-absolute deviation as a measure of deviation from the expected profit. Thus, the objective function of the model is reformulated as follow:

$$\mathrm{MAD}(x) = E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right] \tag{33}$$

In our case, the rate of return R in (33) is replaced by unit price or unit cost of materials (crude oil and refinery products) and x refers to the production flow rate of materials in a refinery. Therefore, the formulation of the MAD term becomes:

$$\begin{aligned}
\mathrm{MAD}(z_0) &= E\left[\left|z_{0,s} - E[z_0]\right|\right] \\
&= E\left[\left|z_{0,s} - \sum_{s\in S} p_s z_{0,s}\right|\right] \\
&= \sum_{s\in S} p_s \left|z_{0,s} - \sum_{s\in S} p_s z_{0,s}\right| \\
\mathrm{MAD}(z_0) &= \sum_{s\in S} p_s \left|\sum_{i\in I} c_{i,s} x_{i,s} - \sum_{i\in I}\sum_{s\in S} p_s c_{i,s} x_{i,s}\right|
\end{aligned} \tag{34}$$

The expectation operator or mean of a discrete probability of deterministic profit is given by:

$$E[z_0] = \sum_{i \in I} \sum_{s \in S} p_s \times c_{i,s} X_{IP_{i,s}} \tag{35}$$

Where $z_o = \sum_{i \in IP} c_{i,s} x_{i,s}$ (36)

Khor et al. 2007, $\text{MAD}_\xi = \sum_{s \in S} p_s \times \left| \xi_s - \sum_{s \in S} p_s \xi_s \right|$ (37)

where $\xi_s = \underbrace{\sum_{i \in I} \sum_{k \in K} d_{i,k} z_{i,s,k}}_{\text{demand uncertainty penalty}} + \underbrace{\sum_{i \in I} \sum_{m \in M} q_{i,m} y_{i,s,m}}_{\text{yield uncertainty penalty}} \quad \forall s \in S$

4. GENERAL FORMULATION OF STOCHASTIC REFINERY PLANNING WITH RISK EXPRESSED IN TERMS OF CVAR

Rockafellar and Uryasev (2002) define Conditional Value-at-risk for continuous distribution function:

$$F_\alpha(x, VaR) = VaR + \frac{1}{1-\alpha} \frac{1}{S} \sum_{i \in I} \sum_{s \in S} (f(x, y_{i,s}) - VaR) \tag{38}$$

With discrete probability distribution in y, CVaR is written as:

$$F_\alpha(x, VaR) = VaR + \frac{1}{1-\alpha} \sum_{i \in I} \sum_{s \in S} p_s (f(x, y_{i,s}) - VaR) \tag{39}$$

<u>Applying the concept of CVaR into the recourse terms:</u>

$\max z = E[z_o] - \theta_1 \text{CVaR}_{z_o} - E[\xi] - \theta_3 \text{CVaR}_\xi$

a) $CVaR_{z_o}$: Risk measure for uncertainty in price of crude oil and refinery products

$$\text{CVaR}(z_o) = \text{VaR}_1 + \frac{1}{1-\alpha} \sum_s \sum_i p_s \left(c_{i,s} x_{i,s} - \text{VaR}_1\right) \tag{40}$$

b) $CVaR_\xi$: Risk measure for uncertainty in market demand and production yield.

$$\text{CVaR}_\xi = \text{VaR}_2 + \frac{1}{1+\alpha}\sum_{s\in S} p_s\left[\sum_{i\in I}\sum_{k\in K} d_{i,k} z_{i,s,k} + \sum_{i\in I}\sum_{m\in M} q_{i,m} y_{i,s,m} - \text{VaR}_2\right] \tag{41}$$

Put eq. (40) and (41) into (5), we achieve the stochastic model in which risk elements are expressed in term of CVaR.

CHAPTER 4

NUMERICAL RESULTS AND DISCUSSIONS

1. NUMERICAL RESULT

1.1 Determining minimum number of scenarios by Monte Carlo simulation approach based on the sample average approximation (SAA) technique to generate the scenarios

We illustrate the risk modeling approach proposed in this paper on the numerical example taken from Khor et al. (2008) and provide major details on the implementation using GAMS/CONOPT3.

Table 4: Flowrates of crude oil and saleable products

X_1	7574	X_{11}	1000
X_2	2000	X_{12}	1274
X_3	950	X_{13}	1877
X_4	2300	X_{14}	2500
X_5	1698	X_{15}	500
X_6	6327	X_{16}	1000
X_7	1950	X_{17}	1375
X_8	3151	X_{18}	424.6
X_9	3000	X_{19}	950.4
X_{10}	3000	X_{20}	125

Table 5: Summary of computational results

Monte Carlo sampling variance estimator $S(n)$	489.4
Lower bound of confidence interval H	965.3
Upper bound of confidence interval H	1237
Range of confidence interval H	271.3
Minimum number of scenarios N	13

1.2 Solving stochastic programming model to maximize profit with MAD as risk measurement

$$\begin{aligned}
z_1 &= -8.02x_1 + 18.63x_2 + 8x_3 + 12.46x_4 + 14.56x_5 + 6.05x_6 - 1.49x_{14} \\
z_2 &= -7.97x_1 + 18.39x_2 + 8.06x_3 + 12.46x_4 + 14.4x_5 + 5.96x_6 - 1.5x_{14} \\
&\cdots \\
z_{13} &= -8.01x_1 + 18.65x_2 + 8.06x_3 + 12.59x_4 + 14.48x_5 + 6.01x_6 - 1.49x_{14}
\end{aligned} \tag{42}$$

with probability: $p_1 = 0.0038$, $p_2 = 0.0986$, …, $p_{13} = 0.000752$

The expectation of deterministic profit:

$$\begin{aligned}
E[z_0] = {} & (0.0038)(-8.02x_1 + 18.63x_2 + 8x_3 + 12.46x_4 + 14.56x_5 + 6.05x_6 - 1.49x_{14}) \\
& + (0.0986)(-7.97x_1 + 18.39x_2 + 8.06x_3 + 12.46x_4 + 14.4x_5 + 5.96x_6 - 1.5x_{14}) \\
& + \dots\dots\dots \\
& + (0.000752)(-8.01x_1 + 18.65x_2 + 8.06x_3 + 12.59x_4 + 14.48x_5 + 6.01x_6 - 1.49x_{14})
\end{aligned} \tag{43}$$

$$\begin{aligned}
\mathrm{MAD}(z_0) = {} & \underbrace{(0.0038)\left|\begin{array}{l} (-8.02x_1 + 18.63x_2 + 8x_3 + 12.46x_4 + 14.56x_5 + 6.05x_6 - 1.49x_{14}) \\ - \left[\begin{array}{l} (0.0038)(-8.02x_1 + 18.63x_2 + 8x_3 + 12.46x_4 + 14.56x_5 + 6.05x_6 - 1.49x_{14}) \\ + (0.0986)(-7.97x_1 + 18.39x_2 + 8.06x_3 + 12.46x_4 + 14.4x_5 + 5.96x_6 - 1.5x_{14}) \\ + \dots + (0.000752)(-8.01x_1 + 18.65x_2 + 8.06x_3 + 12.59x_4 + 14.48x_5 + 6.01x_6 - 1.49x_{14}) \end{array}\right] \end{array}\right|}_{\text{Scenario 1}} \\
& \underbrace{+ (0.0986)\left|\begin{array}{l} (-7.97x_1 + 18.39x_2 + 8.06x_3 + 12.46x_4 + 14.4x_5 + 5.96x_6 - 1.5x_{14}) \\ - \left[\begin{array}{l} (0.0038)(-8.02x_1 + 18.63x_2 + 8x_3 + 12.46x_4 + 14.56x_5 + 6.05x_6 - 1.49x_{14}) \\ + (0.0986)(-7.97x_1 + 18.39x_2 + 8.06x_3 + 12.46x_4 + 14.4x_5 + 5.96x_6 - 1.5x_{14}) \\ + \dots + (0.000752)(-8.01x_1 + 18.65x_2 + 8.06x_3 + 12.59x_4 + 14.48x_5 + 6.01x_6 - 1.49x_{14}) \end{array}\right] \end{array}\right|}_{\text{Scenario 2}} \\
& + \dots\dots\dots \\
& \underbrace{+ (0.000752)\left|\begin{array}{l} (-8.01x_1 + 18.65x_2 + 8.06x_3 + 12.59x_4 + 14.48x_5 + 6.01x_6 - 1.49x_{14}) \\ - \left[\begin{array}{l} (0.0038)(-8.02x_1 + 18.63x_2 + 8x_3 + 12.46x_4 + 14.56x_5 + 6.05x_6 - 1.49x_{14}) \\ + (0.0986)(-7.97x_1 + 18.39x_2 + 8.06x_3 + 12.46x_4 + 14.4x_5 + 5.96x_6 - 1.5x_{14}) \\ + \dots + (0.000752)\begin{pmatrix} -8.01x_1 + 18.65x_2 + 8.06x_3 + 12.59x_4 + 14.48x_5 + 6.01x_6 \\ -1.49x_{14} \end{pmatrix} \end{array}\right] \end{array}\right|}_{\text{Scenario 13}}
\end{aligned} \tag{44}$$

The expectation of the objective function value is given by the original objective function itself: The corresponding expression for expected profit is formulated for the 13 scenarios that have been randomly generated. Finally, computational results obtained from using GAMS/CONOPT3 gives a maximum profit of $681.95/day.

1.3 Solving stochastic programming to maximize profit with CVaR as risk measurement

The following is the procedure for developing a loss distribution in order to determine the value for the parameter VaR.

- Determining VaR_1 from loss distribution of deterministic profit

The value-at-risk for uncertainty in prices of products and raw materials is named VaR_1

  - For each scenario, computing the deterministic profit by :

    $$z_s = \sum_{i \in IP} c_{i,s} x$$

  - The probability of each scenario is randomly generated using the Monte Carlo simulation method based on pseudorandom number generation.
  - The computed values z_s are sorted in ascending order

Table 6: Deterministic profit and cumulative distribution function

Scenario	Net price	P(S)	Cumulative Distribution function
S7	70555.504	0.00	0.00
S8	70871.458	0.05	0.05
S9	71084.146	0.15	0.21
S2	71164.304	0.10	0.30
S10	71228.051	0.15	0.46
S4	71260.214	0.06	0.52
S12	71449.532	0.11	0.62
S11	71467.178	0.10	0.72
S3	71942.881	0.08	0.80
S5	72060.121	0.03	0.84
S1	72066.503	0.00	0.84
S13	72165.129	0.00	0.84
S6	72443.891	0.16	1.00

- The plot of cumulative distribution function against the sorted deterministic profit values is developed to obtain a representation of the loss distribution. At confidence interval of $(1-\alpha) = 0.95$, we can read off the value of VaR_1 from the loss distribution plot, as depicted in Figures 3, which represents the penalty for uncertainty in prices and in both demands and yields, respectively

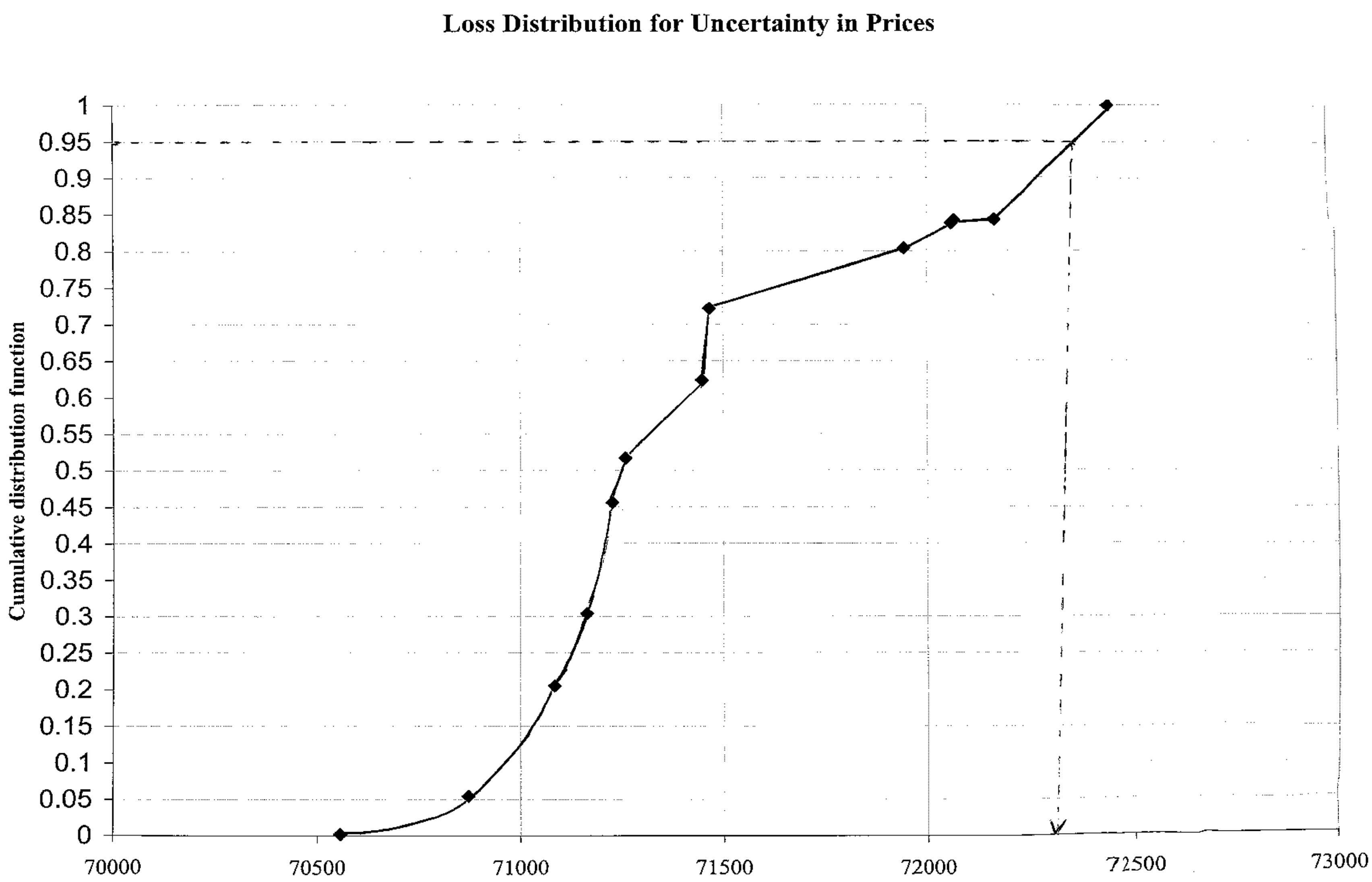

Figure 3: **Deterministic profit due to uncertainty in prices**

From Figure 3, at $(1-\alpha) = 0.95$, $VaR_1 = 7.235E+4$

- Determining VaR_2 from loss distribution by demand and yield penalty.

Similar to the procedure for determining VaR_1, the following is the procedure for developing a loss distribution in order to determine the value for the parameter VaR_2:

- For each scenario, penalty of market demand for products and production yield is calculated:

$$\xi_s = \sum_{i \in I} \sum_{k \in K} d_{i,k} z_{i,s,k} + \sum_{i \in I} \sum_{m \in M} q_{i,m} y_{i,s,m} \quad \forall s \in S$$

- The computed values of ξ_S are sorted in ascending order. The cumulative distribution function for each scenario is developed:

Table 7: Penalty of yield and demand and cumulative distribution function

Scenario	Yield+demand penalty	P(S)	Cumulative probability distribution
S3	165329.67	0.034	0.03
S4	165499.64	0.004	0.04
S5	165726.18	0.061	0.10
S12	166547.84	0.151	0.25
S1	166691.09	0.106	0.36
S10	167287.10	0.082	0.44
S8	167367.18	0.002	0.44
S2	169472.01	0.152	0.59
S7	169630.23	0.098	0.69
S9	170024.86	0.099	0.79
S11	173081.81	0.157	0.95
S13	173398.64	0.052	1.00
S6	173912.20	0.001	1.00

- The cumulative distribution function is plotted against the sorted computed values to obtain a representation of the loss distribution. At confidence interval of $(1-\alpha) = 0.95$, we can read off the value of VaR_2 from the loss distribution plot, as depicted in Figures 4, which represents the penalty for uncertainty in prices and in both demands and yields, respectively.

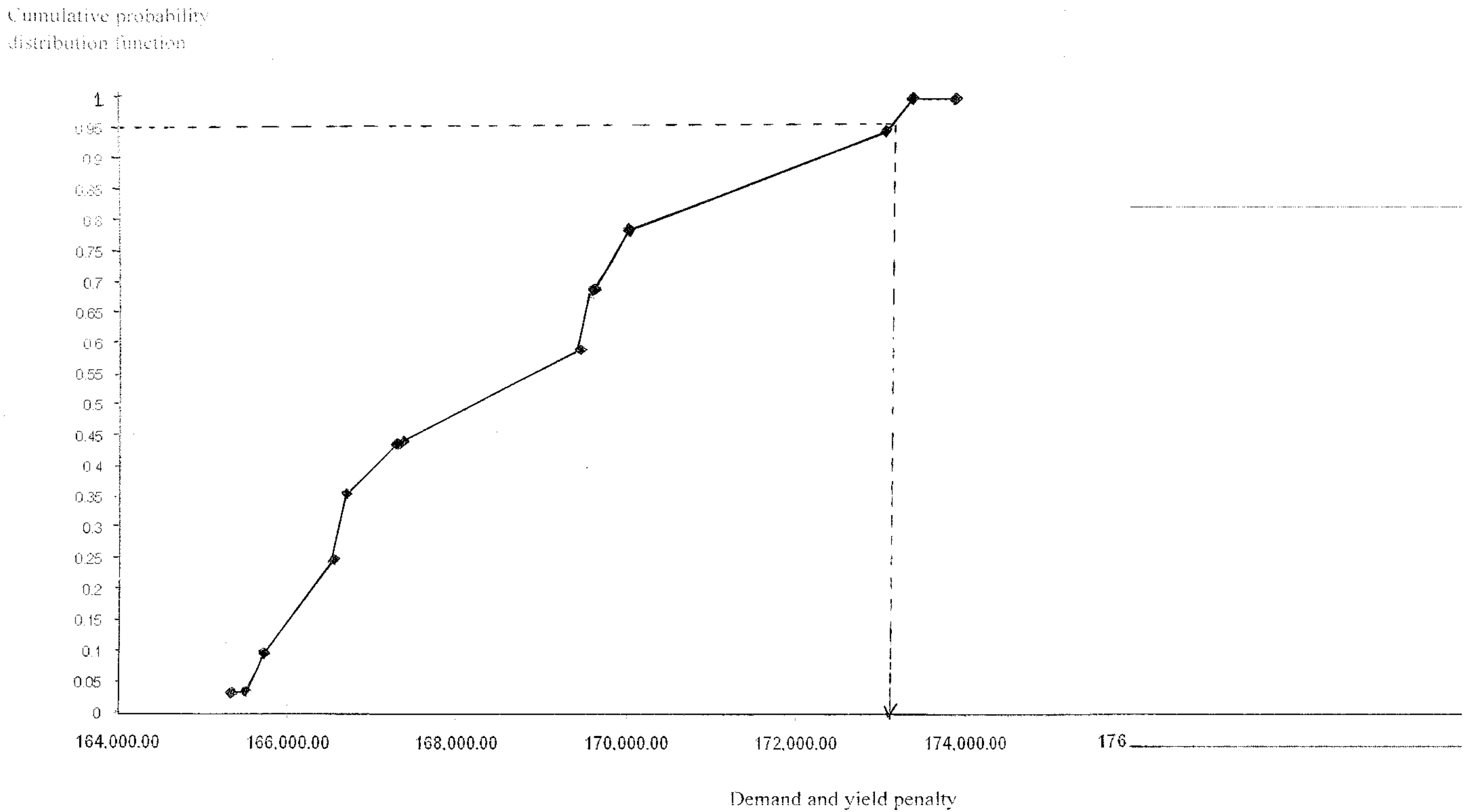

Figure 4: Cumulative probability distribution function versus demand and yield penalty

From Figure 4, at $(1-\alpha) = 0.95$, $Var_2 = 1.731E+5$

From VaR_1 and vaR_2 values, apply model in eq. (5) in Gams, $(Profit)_{max} =$ \$20,820.665/day

Summary of the numerical result

Table 8: (Values of) Parameters

Goal programming weight for risk measure on price uncertainty	0.0001
Goal programming weight for risk measure on demand and yield uncertainty	0.01
Confidence level β	0.95

Table 9: Computational results

Optimal solution for model with MAD	\$681.95/day
VaR_1	7.235E+4
VaR_2	1.731E+5
Optimal solution for model with CVaR	\$20 800.66/day
Expected profit $E[z_0]$ for model with CVaR	\$23343.776/day

Table 10: Computational statistics of GAMS implementation for determining optimal solutions of MAD- and CVaR-based mean–risk stochastic program

Solver	GAMS/CONOPT3
Number of continuous variables	281
Number of constraints	145
CPU time/resource usage	(trivial)
Number of iterations	20 (using MAD)
	101 (using CVaR)

Table 11: Model statistics

Number of continuous variables	281
Number of constraints	145

Table 12: Model performance

Solver	GAMS/CONOPT3
CPU time/resource usage	(trivial)
Number of iterations	20 (using MAD) 101 (using CVaR)

2. DISCUSSION

Table 13: Comparison between MAD and CVaR

Factor	MAD	CVaR
1. Considers confidence level (95%, 99%, etc...)	No	Yes
2. Differentiates distribution of objective function (whether discrete or continuous)	No	Yes
3. Computational property	Non-linear	Linear. Applicable to large portfolios & large number of scenarios with little computational resources

From Table 13, we see that CVaR is more advantageous than MAD because the former takes into account the confidence level and the class of distribution of the objective function (whether it is a discrete or continuous distribution). Moreover, the actual form of MAD is nonlinear (although it can be linearized, for example, as proposed by Papahristodoulou and Dotzauer (2004)) while CVaR is a linear function. Furthermore, in the case of a large number of scenarios, but with the availability of limited computational resource, CVaR is more useful than MAD.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

1. CONCLUSIONS

Stochastic programming is one of the ultimate operation research models for optimization that involves uncertainties. The input values such as materials flowrate, shortfall and surplus of demand and yield penalty are determined maximize the profit.

Monte Carlo simulation approach based on Sample Average Approximation (SAA) is a powerful method to estimate the minimum number of scenarios required to compute the optimal solution, because it can capture all the possible scenarios and is thus representative of all scenarios. This procedure saves time and is convenient particularly if a large number of scenarios have been sampled. In this work, for the risk metric of MAD, we have verified that the same optimal solution as for the mean–risk stochastic model with 13 scenarios is obtained if larger numbers of scenarios are considered, for instance, for 100 and 300 scenarios. A similar experiment on CVaR is currently ongoing.

This work attempts to consider the use of the risk metrics of MAD and CVaR for the explicit handling of economic and operational risk management in refinery planning problems under uncertainty in prices, demands, and yields.

The risk is expressed in form of Mean Absolute Deviation - a measure of operational risk provides the computational non-linear property. However, the non-linear property can be linearised.

Conditional Value-at Risk used in conjunction with Value- at Risk is convenient for large portfolios and a large number of scenarios with relatively small computation resources. Moreover, CVaR is a convex function. In petroleum refinery planning, the recourse term is written in form of CVaR.

Conditional Value-at Risk is more advantageous than Mean Absolute Deviation because Conditional Value-at Risk consider the confidence interval, the distribution of objective function (discrete or continuous)

2. RECOMMENDATIONS

The recommendations for future studies on the work presented here include the following:

- To develop a systematic approach for considering the weight factors θ_1 and θ_2 in the stochastic programming framework.
- To develop an approach for obtaining the Pareto-optimal curve for a mean–risk stochastic program with MAD and CVaR as risk measures.

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APPENDIX

1. PROGRAM CODE

```
*$TITTLE: FINDING THE NUMBER OF SCENARIO

SETS
I material /1*20/
M  YEILD SHORFALL OR SURPLUS        /M1 , M2/
K DEMAND SHORT FALL OR SURPLUS     /K1, K2/
;
$onecho >taskin.txt
dset=S rng=Sheet4!A16:A28 rdim=1
dset=ID rng=Sheet4!B15:F15 cdim=1
dset=IY rng=Sheet4!H15:L15 cdim=1
dset=IP rng=Sheet4!N15:T15 cdim=1
par=D rng=Sheet4!A15:F28 cdim=1 rdim=1
par=Yield rng=Sheet4!G15:L28  cdim=1 rdim=1
par=Price rng=Sheet4!M15:T28 cdim=1 rdim=1
par=P rng=Sheet4!U15:V28 rdim=1
$offecho

$call gdxxrw.exe CVaR.xls @taskin.txt

$gdxin CVaR.gdx

Sets
S(*) scenario
ID(I) demand
IY(I) yeild
IP(I) price;

$load S ID IY IP
display S, ID, IY, IP;

Parameters
D(S,ID) demand
Yield(S,IY) yield
Price(S,IP) price
P(S) probability
$load D Yield Price  P
display D, Yield, Price , P;
$gdxin

Table Penalty_Demand(ID,K) TABLE OF PENALTY DEMAND
            K1     K2
2           25     20
```

```
3           17      13
4           5       4
5           6       5
6           10      8;

Table Penalty_Yield(IY,M)  TABLE OF PENALTY YIELD
            M1      M2
7           5       3
4           5       4
8           5       3
9           5       3
10          5       3 ;

Variables
OBJ   Maximize Profit for Z (objective function value)
Var1
Var2
X(I) ;

Positive Variables
Y(IY,S,M)  SHORT FALL OR SURPLUS FOR YEILD
Z(ID,S,K)  SHORT FALL OR SURPLUS FOR DEMAND
;
Equations
OBJFNC              Objective function
DEMAND(ID,S)        Demand,
YIELD_CON(IY,S)     Yield,
Feed1,
Feed14,
PDU_14_16,
PDU_14_17,
PDU_14_20,
FB_2_11,
FB_2_16,
FB_5_12,
FB_5_18,
UB_8,
UB_14,
UB_17,
UB_18,
UB_6
;

OBJFNC.. OBJ =e= SUM((S,IP),P(S)*PRICE(S,IP)*X(IP)) -0.0001*(72400+ (1/(1-
0.95))*SUM((S,IP),P(S)*(PRICE(S,IP)*X(IP)-72400))) -
SUM(S,P(S)*(SUM((ID,K),Penalty_Demand(ID,K)*Z(ID,S,K))+SUM((IY,M), Penalty_Yield(IY,M)*Y(IY,S,M)))) -
0.01*(173200
+ (1/(1 0.95))*SUM(S,P(S)*(SUM((ID,K), Penalty_Demand(ID,K)*Z(ID,S,K))
+SUM((IY,M),Penalty_Yield(IY,M)*Y(IY,S,M))-173200)));
```

```
**LIMITATIONS OF PLANT CAPACITY
Feed1..    X('1') =L= 15000;
Feed14..   X('14') =L= 2500;

*Reformulated stochastic constraints to account for uncertain yield coefficient
YIELD_CON(IY,S).. -YIELD(S,IY)*X('1') + X(IY) + Y(IY,S,'M1') - Y(IY,S,'M2') =E= 0;

**CONSTRAINTS ON PRODUCTION DEMANDS
DEMAND(ID,S).. X(ID) + Z(ID,S,'K1')-Z(ID,S,'K2') =E= D(S,ID);


PDU_14_16.. -0.40*X('14') + X('16') =E= 0;
PDU_14_17.. -0.55*X('14') + X('17') =E= 0;
PDU_14_20.. -0.05*X('14') + X('20') =E= 0;
FB_2_11..   0.5*X('2') - X('11') =E= 0;
FB_2_16..   0.5*X('2') - X('16') =E= 0;
FB_5_12..   0.75*X('5') - X('12') =E= 0;
FB_5_18..   0.25*X('5')- X('18') =E= 0;
UB_8..      -X('7') + X('3') + X('11') =E= 0;
UB_14..     -X('8') + X('12') + X('13') =E= 0;
UB_17..     -X('9') + X('14') + X('15') =E= 0;
UB_18..     -X('17') + X('18') +X('19') =E= 0;
UB_6..      -X('10') - X('13') - X('15') - X('19') + X('6') =E= 0;

*DECISION  VARIABLE BOUNDS
Y.UP(IY,S,M) = 1500;

*Initial values
X.L('1') = 12500;
X.L('2') = 2000;
X.L('3') = 625;
X.L('4') = 1875;
X.L('5') = 1700;
X.L('6') = 6175;
X.L('7') = 1625;
X.L('8') = 2750;
X.L('9') = 2500;
X.L('10') = 3750;
X.L('11') = 1000;
X.L('12') = 1275;
X.L('13') = 1475;
X.L('14') = 2500;
X.L('15') = 0;
X.L('16') = 1000;
X.L('17') = 1375;
X.L('18') = 425;
X.L('19') = 950;
X.L('20') = 125;

* Upper bounds of variables
```

```
X.UP('1') = 15000;
X.UP('2') = 2700;
X.UP('3') = 1100;
X.UP('4') = 2300;
X.UP('5') = 1700;
X.UP('6') = 9500;
X.UP('7') = 1950;
X.UP('8') = 3300;
X.UP('9') = 3000;
X.UP('10') = 3000;
X.UP('11') = 1350;
X.UP('12') = 1275;
X.UP('13') = 3300;
X.UP('14') = 2500;
X.UP('15') = 3000;
X.UP('16') = 1200;
X.UP('17') = 1650;
X.UP('18') = 425;
X.UP('19') = 1650;
X.UP('20') = 150;

MODEL CVaR /all/;

SOLVE CVaR USING LP MAXIMIZING OBJ;
DISPLAY OBJ.L , X.L, Y.L, Z.L ;
EXECUTE_UNLOAD 'CVaR.GDX',OBJ;
EXECUTE 'GDXXRW.EXE CVaR.GDX  VAR=OBJ RNG=SHEET4!A68';
```

2. EXPECTED PROFIT

```
               S O L V E      S U M M A R Y

     MODEL    FindEprofit         OBJECTIVE  E_profit
     TYPE     LP                  DIRECTION  MAXIMIZE
     SOLVER   CPLEX               FROM LINE  118

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL
**** OBJECTIVE VALUE             1100.9109
```

3. VARIANCE ESTIMATOR

```
                    S O L V E      S U M M A R Y

          MODEL    FindSn_Sn            OBJECTIVE  Sn
          TYPE     DNLP                 DIRECTION  MINIMIZE
          SOLVER   CONOPT               FROM LINE  148

     *** SOLVER STATUS      1 NORMAL COMPLETION
     *** MODEL STATUS       2 LOCALLY OPTIMAL
     *** OBJECTIVE VALUE              503.0991
```

4. MAXIMUM PROFIT WITH MAD AS RISK MEASUREMENT

```
**** REPORT SUMMARY :         0     NONOPT
                              0 INFEASIBLE
                              0  UNBOUNDED
                              0     ERRORS
GAMS Rev 146  x86/MS Windows                           02/19/05 10:07:35 Page 8
G e n e r a l   A l g e b r a i c   M o d e l i n g   S y s t e m
E x e c u t i o n


----     200 VARIABLE OBJ.L                 =      681.954  Maximize Profit for Z
                                                            (objective function v
                                                            alue)
```

5. MAXIMUM PROFIT WITH CVAR AS RISK MEASUREMENT

```
**** REPORT SUMMARY :         0     NONOPT
                              0 INFEASIBLE
                              0  UNBOUNDED
GAMS Rev 146  x86/MS Windows                           03/28/05 15:35:02 Page 8
G e n e r a l   A l g e b r a i c   M o d e l i n g   S y s t e m
E x e c u t i o n


----     213 VARIABLE OBJ.L                 =    20820.665  Maximize Profit for Z
                                                            (objective function v
                                                            alue)
```

6. MATERIALS' FLOWRATE

```
----     204 VARIABLE X.L

1  7574.363,     2  2000.000,     3   950.000,     4  2300.000,     5  1689.393
6  6336.419,     7  1950.000,     8  3150.813,     9  3000.000,     10 3000.000
11 1000.000,     12 1267.045,     13 1883.768,     14 2500.000,     15  500.000
16 1000.000,     17 1375.000,     18  422.348,     19  952.652,     20  125.000
```