CERTIFICATION OF APPROVAL

POWER SYSTEM STATE ESTIMATION

by

Noor Ain Binti Ahmad

Final Report submitted to the
Electrical & Electronics Engineering Programme
Universiti Teknologi PETRONAS
in partial fulfilment of the requirement for the
Bachelor of Engineering (Hons)
(Electrical & Electronics Engineering)

Approved by,

(Ir. N Perumal)

(Mr. Nursyarizal Mohd. Nor)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

December 2006

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

(NOOR AIN BINTI AHMAD)

ABSTRACT

This project proposes a method of state estimation using the Weight Least Squares, tested on three bus systems. It gives an overview of state estimation and finds the best power system network parameters, in terms of voltage, real and reactive power injections, and real and reactive power flow and phase angle by estimating the state variables, so that the security of the power system is maintained. This project provides coding and results of power system state estimation, of test bus system using the MATLAB programme. The result, obtained in the 2 - Bus System show that the networks have bad data measurements, while no bad data measured in 3 -Bus System and IEEE 14 -Bus System. The bad data have to be eliminated to get new and precise estimated state variable value. Deduction of bad data means reducing the error in network. In conclusion, minimizing the errors in State Estimation using the proposed method is achieved and the final power system network parameters are obtained.

ACKNOWLEDGEMENTS

First of all, I would like to thank God, The Almighty because without His love and grace none of this would have been possible. My warmest appreciation especially to my Supervisor, Ir. N Perumal and also Mr. Nursyarizal Mohd Nor for continuous supporting, discussions and guidance. As the advisors, they are very keen in facilitating the author to complete this project. Heartfelt appreciation is next extended to the Final Year Projects (FYP) coordinator who always give advises and prepare a lots of FYP seminars to make sure the project go smoothly. Special appreciation goes to the lecturers and the lab technician for their effort and assistance. Last but definitively not least, I would like to thank my family and friends for their encouragement, effort and time in giving the moral support. The projects would have been possible without their supports.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

IEEE	Institute of Electrical and Electronics Engineers
SCADA	Supervisory control and data acquisition
P_i	Real Power at bus i
Q_i	Reactive Power at bus i
P_{ij}	Real Power Flow from bus I to bus j
Q_{ij}	Reactive Power Flow from bus I to bus j
V_i	Voltage at bus i
∂_n	Phase angle at bus n
Z	Bus Impedance
WLS	Weight Least Squares
PSAT	Power System Analysis Toolbox

CHAPTER 1
INTRODUCTION

1.1 Background

Now days, interconnected power systems have become more and more complex. To prevent the insecure operation (major systems failures and regional power blackout) electric network utilities have been installed by SCADA systems throughout the network. It contains application for service quality, economic operation, secure operation, system regulation, and operational quality. Before taking any control action, the reliable estimation of existing state of the system must be determined first. The reliable estimations include the topology studies, observability analysis, state estimation, and the bad data processing.

The real power system is an online operating system where the data measurement is collected while the plant or power system is running. Monitoring the online system is not easy as compare to statistical power system hence the development of online power systems now days are quite complex and complicated. Most of data are collected online because the power system cannot be rapidly stopped just for analysis the healthy of the system. To ensure the healthy of economic system operation and less failure of equipment in power systems, the security assessment are needed.

1.2 Problem Statement

Before any security assessment or control action can be made or an action taken, a system state must be estimated, so the state estimation comes to solve this crucial power system problem as explained before. However, since there are so many data measurements are available in the network, in duration of time, the estimation processes should make sure that the data collected are free from the errors. It sounds like impossible to eliminate all the errors, but state estimation will try to reduce as much as possible the error in the measurement with the Weight Least Square method so that the estimated values are precise.

1.3 Objectives and Scope of Project

The objectives of this project are;

a) To use the Weight Least Squares to estimate the state variables (voltage magnitudes and phase angles) of Bus System.

b) To write programme in estimating the state variable by using the MATLAB programming.

c) To execute the MATLAB Program, to estimate the state variable of Bus System.

CHAPTER 2

LITERATURE REVIEW

2.1 State Estimation

The concerns of the state estimation problems were first recognized and addressed by Fred Scheweppe, who proposed the idea of the state estimation in power systems [1, 2, and 3]. The state estimation is one of the most important functions for real time monitoring and control of power networks. It is a process for determining the node voltage magnitudes and angles from a set of measurements. The measurements consist of real and reactive line followers, real and reactive node injection powers and node voltage magnitudes. Depending on the number and distribution of the available measurements, state estimation may or may not be able to provide the reliable estimates. In the positive case the network is said to be observable. An observability test should be executed prior to performing the state estimation [4]. State estimation is also can be defined as an estimating process of a system variable using the measurement made on the systems to know the systems variables. State estimation in a power system involves the measurement of the various bus voltages and power injections, and the bus angles are the estimated quantity [5]. The basic first step of estimating the state relate, is the measured value to the unknown state variables [6].

The main advantage of state estimation compare to the conventional power flow analysis is the number of physical measurements is not only restricted to only those quantities required to support conventional power flow calculations. In conventional power flow program, it requires all the input, which is P, Q injections at the load bus and P, V values at the voltage-controlled bus. If one or more of the values is unavailable, the conventional power flow analysis cannot be determined. Moreover, the gross error occurred in the one or more input quantities, the power-flow solution become useless [7]. In the other hand, the state estimation power analysis is sufficient where it make use all the others conveniently measured quantities such as P_{ij}, Q_{ij}, P_{ji} and Q_{ji} that are available in all power flow calculations.

2.2 Basic Power Flow Analysis

Power flow analysis is the most fundamental study to be performed in a power system both during the Planning and Operational phases. It constitutes the major portion of electric utility. The study is concerned with the normal steady state operation of power system and involves the determination of bus voltages and power flows for a given network configuration and loading condition.

The results of power flow analysis help to know

i. The present status of the power system, required for continuous monitoring system.
ii. Alternative plans for system expansion to meet the ever increasing demand.

The mathematical formulation of the power flow problem results in a system of non-linear algebraic equations and hence calls for an iterative technique for obtaining the solution. Gauss-Seidel method and Newton Raphson (N.R.) method are commonly used to get the power flow solution. Decoupled N.R method, that uses two sub-matrices of Jacobian matrix, is significant improvement over Newton Raphson method. The coefficient matrices are constant in Fast Decoupled Power Flow (FDPF) method. Since factorization is done only once, FDPF method is well suited for large scale power systems. [8]

2.3 Classification of Busses

There are four quantities associated with each bus. They are P, Q, $|V|$ and δ.

Here

P the real power injected into the bus

Q the reactive power injected into the bus

$|V|$ The magnitude of the bus voltage

δ The phase angle of the bus voltage

Any two of these four may be treated as independent variables (i.e. specified) while the other two may be computed by solving the power flow equations. Depending on which of the two variables are specified, buses are classified into three types. Three types of bus classification based on practical requirements are given in Figure 2.1 below.

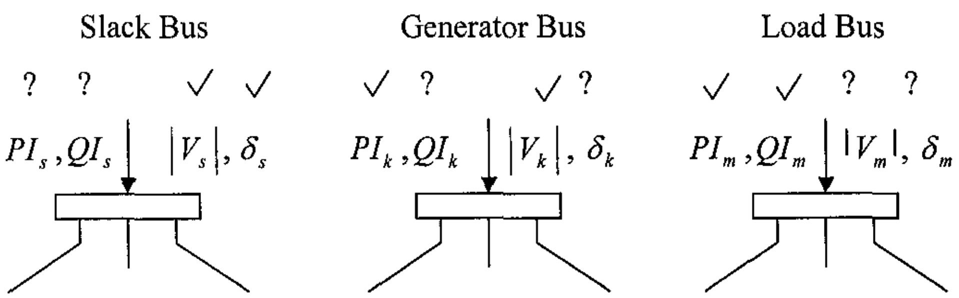

Figure 2.1: Type of Buses

2.3.1 Slack bus

In a power system with N buses, power flow problem is primarily concerned with determining the $2N$ bus voltage variables, namely the voltage magnitude and phase angles. These can be obtained by solving the $2N$ power flow equations provided there are $2N$ power specifications. However, the Slack Bus cannot be specified beforehand. This leaves us with no other alternative but to specify two variables

$|V_s|$ and δ_s arbitrarily for the slack bus so that 2(N-1) variables can be solved from 2N known power specifications.

Incidentally, the specification of $|V_s|$ helps us to fix the voltage level of the system and the specification of δ_s serves as the phase angle reference for the system.

Thus for the slack bus, both $|V|$ and δ are specified and P and Q are to be determined. P and Q can be computed at the end, when all the $|V_s|$ and δ_s are solved .

2.3.2 Generator bus

In a generator bus, it is customary to maintain the bus voltage magnitude at a desired level which can be achieved in practice by proper reactive power injection. Such buses are termed as Voltage Controlled Buses or P – V buses. At these buses P and $|V|$ are specified and Q and δ are to be solved.

2.3.3 Load bus

The buses where there is no controllable generation are called as Load Buses or P – Q buses. At the load buses, both P and Q are specified and $|V|$ and δ are to be solved.

Of the N total number of buses in the power system, let the number of P-Q buses be N_1, P-V buses be N_2. Then $N = N_1 + N_2 + 1$. Basic problem is to find the

i. Unknown phase angles δ at the $N_1 + N_2$ number of P-Q and P-V buses and

ii. Unknown voltage magnitudes $|V|$ at the N_1 number of P-Q buses.

Thus total number of unknown variables = $2N_1 + N_2$ (2.1)

2.4 Power System State Estimation

The overview process of state estimation is involving three stages. The first stage is state variable – This stage of estimation is carried out by using the Newton Raphson Method. In this stage, the measurement value and the variable value are defined. The measurement values (*Nm*) are obtained form the series of online network measured data. It can be in *P* and *Q*, P_{ij} and Q_{ij}, P_{ji} and Q_{ji}. While the variable values, *Ns* is the variable that aimed to be estimated. The variable, *Ns* values are the voltage magnitude and angle at the busses, or in other words, *Ns* is the total number of unknown variable (from equation 2.1) .From the estimated value, the others parameters of the network can be calculated using the standard equations as follow;

$$P_i = |V_i|^2 G_{ii} + \sum_{\substack{n=1 \\ n \neq i}}^{N} |V_i||V_n||Y_{in}|\cos(\theta_{in} + \partial_n - \partial_i) \quad (2.2)$$

$$Q_i = -|V_i|^2 B_{ii} - \sum_{\substack{n=1 \\ n \neq i}}^{N} |V_i||V_n||Y_{in}|\sin(\theta_{in} + \partial_n - \partial_i) \quad (2.3)$$

$$P_{ij} = -|V_i|^2 G_{ij} + |V_i||V_n||Y_{ij}|\cos(\theta_{ij} + \partial_j - \partial_i) \quad (2.4)$$

$$Q_{ij} = |V_i|^2 B_{ij} - |V_i||V_n||Y_{ij}|\sin(\theta_{ij} + \partial_j - \partial_i) \quad (2.5)$$

The following equations and step are needed to calculate the state variables.

i. Find the Jacobian Matrix of equation

$$H = \begin{bmatrix} \frac{\partial V}{\partial \delta} & \frac{\partial V}{\partial |V|} \\ \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|} \\ \frac{\partial P_{flow}}{\partial \delta} & \frac{\partial P_{flow}}{\partial |V|} \\ \frac{\partial Q_{flow}}{\partial \delta} & \frac{\partial Q_{flow}}{\partial |V|} \end{bmatrix} \quad (2.6)$$

Where the column is for state variable, $Ns=2N-1$ (2.7)

And the row if for number of measurement, $Nm=3N+4B$ (2.8)

N = Number of busses

B = Number of branches of lines

ii. Estimate the new estimated value of state variables

$$G(x^k) = H_X{}^T R^{-1} H_X \quad (2.9)$$

G is built and stores as a sparse matrix for computational efficiency and memory consideration. It is built by processing one measurement at a time. Consider the measurement Jacobian H and the covariance matrix for a set of m measurement, each one corresponding to one row, as shown in the equation 2.6.

R stands for variance. In obtaining the state variables, the variances of the measurement base on the Gaussian probability density function, which will be discussed in the methodology part under Weight Least Squares Method.

Then, we can write;

$$G = \sum_{i=1}^{m} H_i^T R_{ii}^{-1} H_i \quad (2.10)$$

$$x^{(k+1)} = x^{(k)} + G(x^{k})\text{x}\, H_x^T R^{-1} \begin{bmatrix} z_1 - h_1(x_1^{(k)}, x_2^{(k)}, \cdots, x_{Ns}^{(k)}) \\ z_2 - h_2(x_1^{(k)}, x_2^{(k)}, \cdots, x_{Ns}^{(k)}) \\ \vdots \\ z_{Ns} - h_{Nm}(x_1^{(k)}, x_2^{(k)}, \cdots, x_{Ns}^{(k)}) \end{bmatrix} \quad (2.11)$$

Where $x^{(k+1)}$ = new estimated value,

$x^{(k)}$ = flat start value or previous estimated value.

Second stage is checking for the Bad Data – after obtaining the estimation value, the measurement value, *Nm* are tested to see the existence of the bad data. The weight sum squares of the final errors are calculated us the equation 2.13, then will be compared with the standard Chi-squares distribution. Chi-square $\hat{f} > X_{k,\alpha}^2$ must be satisfied unless the measurements are treated as having the bad data.

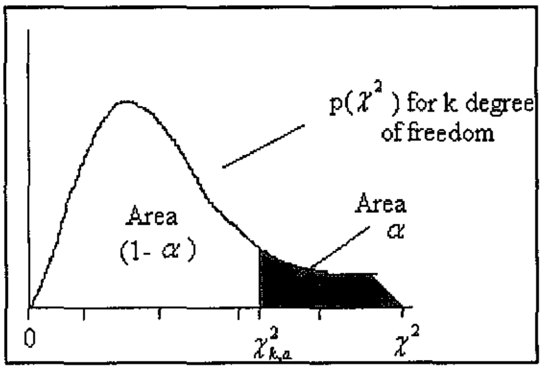

Figure 2.2: Chi-square curve

$$k = Nm - Ns \quad (2.13)$$

'*k*' is the degree of freedom, '*Nm*' is measurement value, '*Ns*' is variable value and 'α' is the confidence interval. The area which is not shaded is the probability (1-α) that the calculated value of the weighted sum of squares $\hat{f}$, with *k* degrees of freedom, will take on a value less than; $\chi_{k,\alpha}^2$, $\Pr(\hat{f} < \chi_{k,\alpha}^2) = (1-\alpha)$

$$\hat{f} = \sum_{j=1}^{m} \left(\frac{\hat{e}_j}{\sigma_j} \right)^2 \quad (2.14)$$

Table 2.1: Values of area α to the right $\chi^2 = \chi^2_{k,\alpha}$

	α				
k	0.1	0.05	0.025	0.01	0.005
1	2.71	3.84	5.02	6.64	7.88
2	4.61	5.99	7.38	9.21	10.60
3	6.25	7.82	9.35	11.35	12.84
4	7.78	9.49	11.14	13.28	14.86
5	9.23	11.07	12.83	15.09	16.75
6	10.64	12.59	14.45	16.81	18.55
7	12.02	14.07	16.01	18.48	20.28
8	13.36	15.51	17.54	20.09	21.96
9	14.68	16.92	19.02	21.67	23.59
10	15.99	18.31	20.48	23.21	25.19
11	17.28	19.68	21.92	24.73	26.76
12	18.55	21.03	23.34	26.22	28.30
13	19.81	22.36	24.74	27.69	29.82
14	21.06	23.69	26.12	29.14	31.32
15	22.31	25.00	27.49	30.58	32.80
16	23.54	26.30	28.85	32.00	34.27
17	24.77	27.59	30.19	33.41	35.72
18	25.99	28.87	31.53	34.81	37.16
19	27.20	30.14	32.85	36.19	38.58
20	28.41	31.41	34.17	37.57	40.00

The third stage is deduction the bad data. This process is done by removing the largest standardized measurement value, and then recalculates the estimation process to the get new estimated value. The largest standardized measurement values are represented by the following equation;

$$\text{Standardized error} = \frac{\hat{e}_1}{\sqrt{R_{11}}} \tag{2.15}$$

If the sum of squares is less than the standard value of Chi-Squares, we cannot accept the calculated values of the state variable as being accurate. Bad data might be more than one. Noted here, state estimation can only apply on observable network. The network is observable when the number of variable, *Ns* is lesser than the number of measurement, *Nm* or *(Nm > Ns*)

The following diagram represents the three stages of power system state estimation procedures;

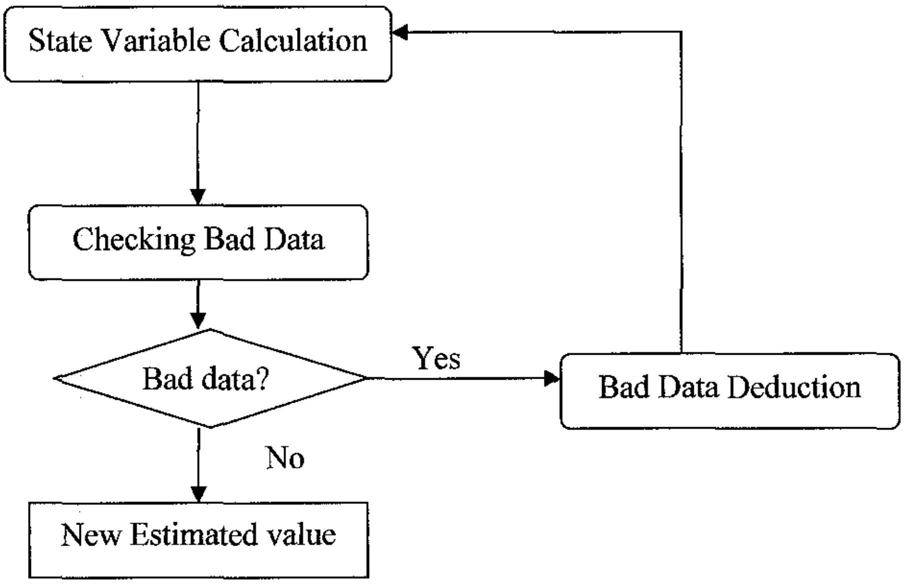

Figure 2.3: Power system state estimation stage

2.5 Functionality Diagram

Operating state of power system includes three difference changes as shown in the Figure 2.4. The three states are Normal State, Emergency State, and Restorative State. The state is normal when all the loads supplied by existing generators without violating any operational constraints. In normal state, there are two conditions-

a) Secure normal state which is it is remaining in normal state following the occurrence of each contingency from a list of critical contingencies.

b) and insecure normal state , where the power balance at each bus and all operating inequality constraint are still satisfied, respect to some considered contingencies.

In this state, the preventive maintenance should be done unless the normal state will move to the emergency state. Emergency state requires immediate corrective action to be taken to bring the system back to normal state. Corrective control measures may be able to avoid system collapse.

The power system state estimator is core of the on-line security analysis function. The estimator functions as a filter between a raw measurement and all the applications function, which require the most reliable data for current state of the system. The application is shown in the Figure 2.4.

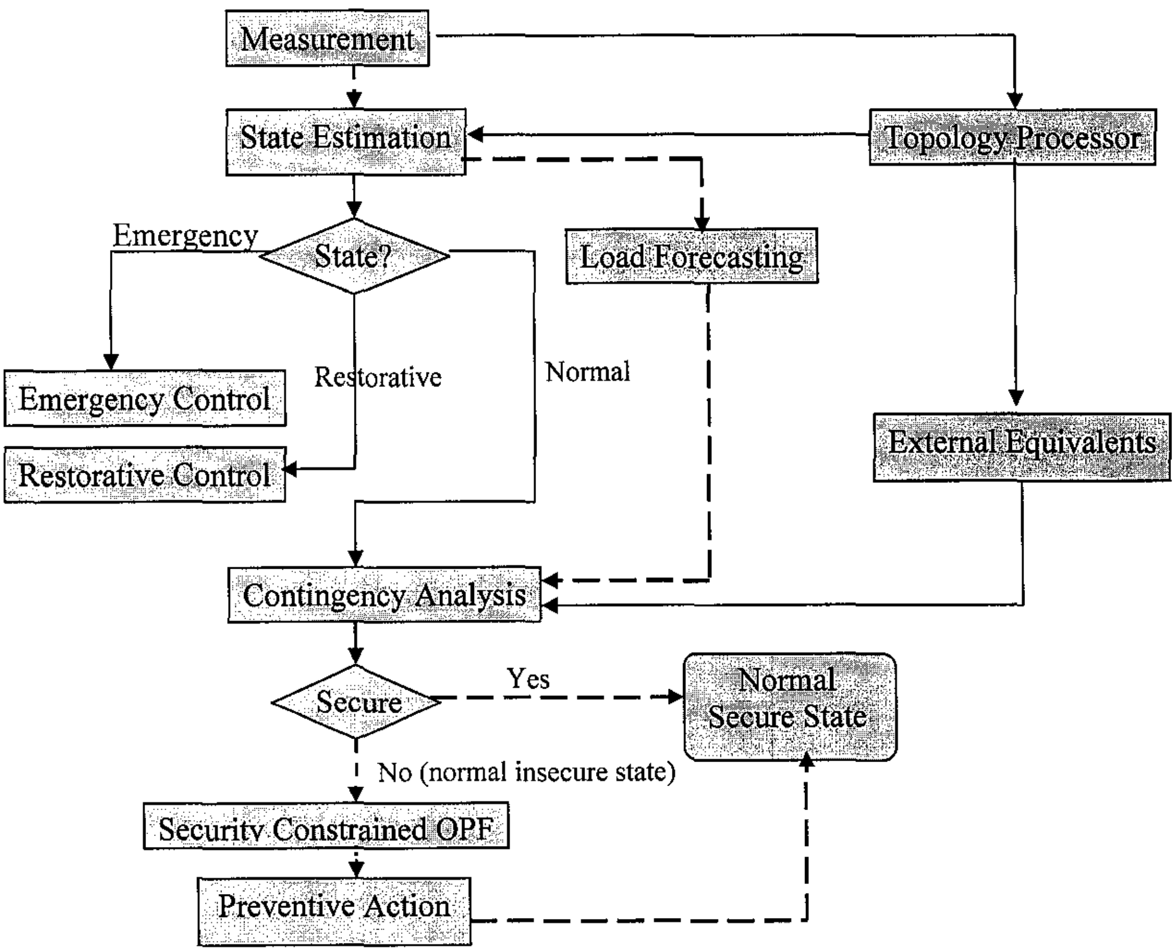

Figure 2.4: Functional Diagram of State Estimator

The dotted line shows the flow of the estimation process for this project. The raw measurement is obtained from the SCADA system, will be estimated either via the online system (topology processor) or directly estimated. The result obtained by the state estimator will be checked in order to classify the system into three possible states (normal, emergency and restorative). It also analyses the contingency of existing power system by using the load forecasting to see whether the existing system in secure condition or not. If not, the optimal power flow process will be taken for prevention action by adding some controller so that the state will be secured.

CHAPTER 3
METHODOLOGY

3.1 State Estimator Methodology

In order to get the desire output to analysis the state estimation power /load performance, the following step method will be used: The entire 5 steps are needed to develop security power analysis

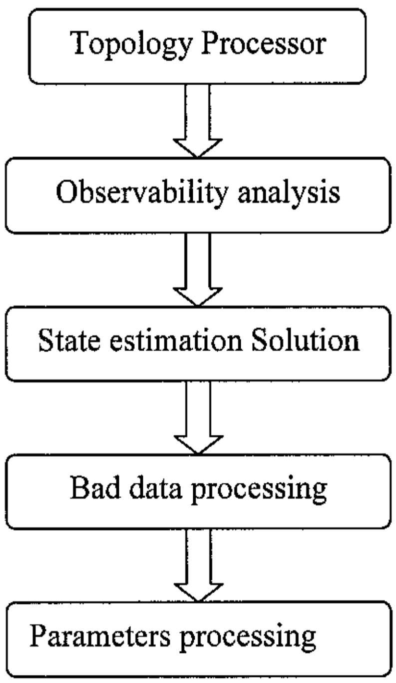

Figure 3.1: Developing the security power analysis

The "Topology Processor" is the estimation level to gather the status data about the circuit breakers and switches, and configures the one-line diagram of the system. For the static state estimation or offline measurement, the topology processor is put aside since the measurements are collected from the historical data. Observability analysis is to determine if a state estimation solution for the entire system can be obtained using the available set of measurement. After observability of the system are analysed, the best solution of the sate estimation are found in the "State estimation solution steps". For this time, determination of the optimal estimate for the system state, based on the network model and the gathered measurements from the system. In the "Bad data processing" step, the existence of gross error in the measurement set will be detected to identify and eliminate bad data. And finally, the various networks will be processed in the "Parameters processing" step [9].

3.2 Method of Least Squares

The analog quantities are monitored by current and potential transformers installed on the lines and on transformers and buses of the power plants and substations of the system. The analog quantities pass through transducers and analog to digital converters, and the digital outputs are then taken telemetered to the energy control center over various communication links. Because of noise, the true values of the physical quantities are never known and we have to consider how to calculate the best possible estimates of the unknown quantities. The method of least square is often used to "best fit" measured data relating two or more quantities. We can represent the measured quantities as sum of true values and errors.

$$z = z_{true} + e \tag{3.1}$$

$$e = z - z_{true} = z - Hx \tag{3.2}$$

'e' represents the errors between the actual measurements z and the true values z_{true} (Hx) of the measured quantities. 'x' is the true value of the state variables. The true values cannot be determined, but the estimates $\hat{x}$ can be determined. The estimated error is given by

$$\hat{e} = z - \hat{z} = z - H\hat{x} = e - H(\hat{x} - x) \tag{3.3}$$

To compute the estimates, we have to minimize the errors and it is not desirable to choose the algebraic sum of errors since positive and negative errors could then offset one another. It is preferable to minimize the direct sum of squares of the errors. However, to ensure that measurements from meters of known greater accuracy are treated more favorably than less accurate measurements, each term in the sum of squares is multiplied by an appropriate weighting factor 'w' to give the objective function

$$f = \sum_{j=1}^{N} w_j e_j^2 \tag{3.4}$$

'w' is the weighting factor for the respective measurement. The best estimate is the one which makes the objective function 'f' to take on its minimum value.

$$\frac{\partial f}{\partial x} = \sum_{j=1}^{N} 2 w_j e_j \frac{\partial e_j}{\partial x_j} = 0 \tag{3.5}$$

The partial derivates with respect to state variables are the elements of the Jacobian matrix 'H'. Using the compact notations of equation (3.2) in equation (3.5) yields

$$H^T W \hat{e} = H^T W (z - H\hat{x}) = 0 \tag{3.6}$$

Solving for $\hat{x}$ given by

$$\hat{x} = (H^T W H)^{-1} H^T W z = G^{-1} H^T W z \tag{3.7}$$

Where G = $H^T WH$, is called the gain matrix.

$$\hat{x} = G^{-1} H^T W (Hx + e) = G^{-1} (H^T W H) x + G^{-1} H^T W e \tag{3.8}$$

$$\hat{x} = G^{-1} H^T W e \tag{3.9}$$

So;

$$\hat{e} = z - \hat{z} = e - H G^{-1} H^T W e = [I - H G^{-1} H \hat{T} W] e \tag{3.10}$$

The actual errors cannot be determined, but the estimated errors can be used if we introduce the statistical properties of the errors.

3.3 MATLAB Algorithm

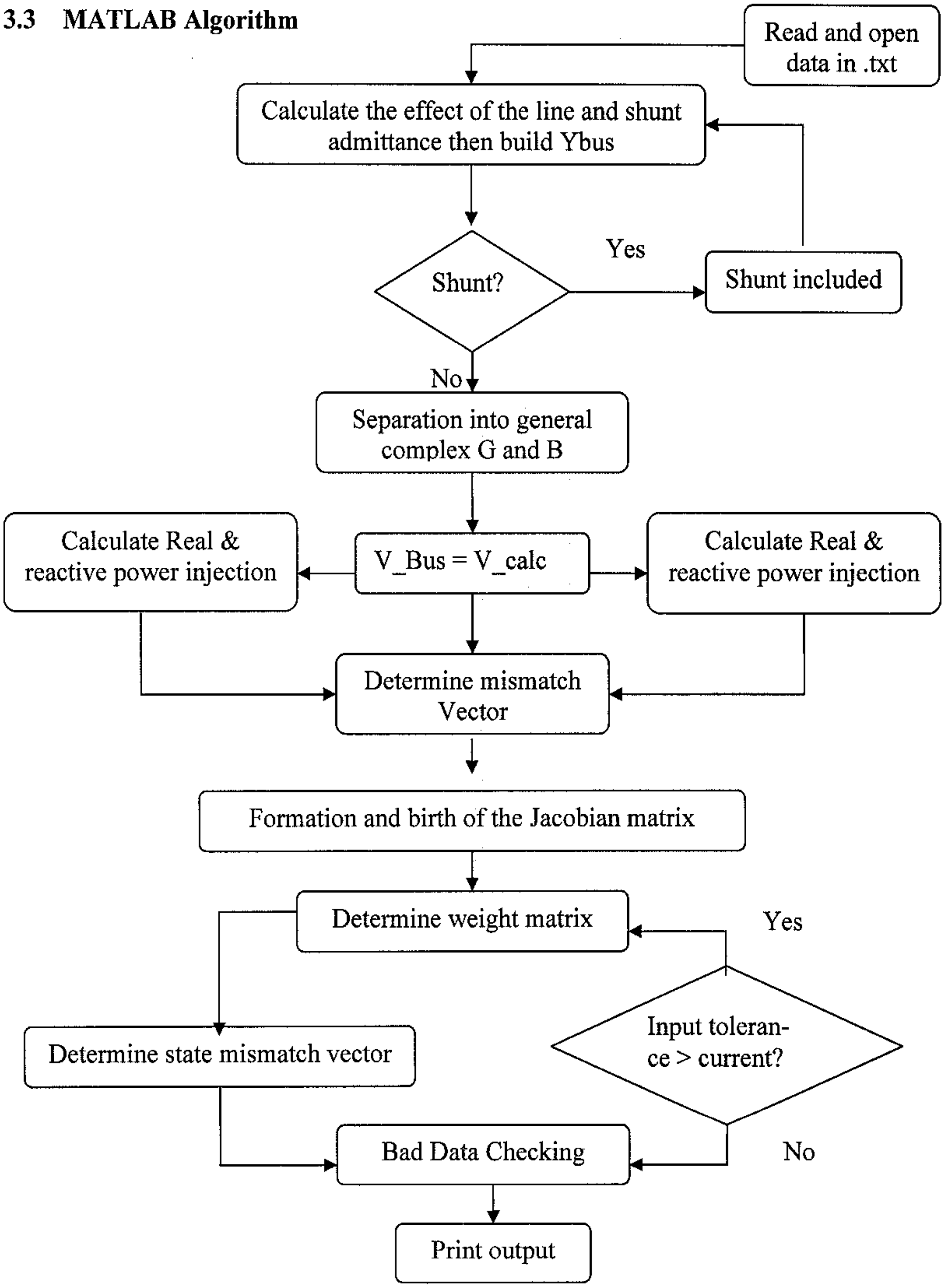

Figure 3.2: Flow process on MATLAB script

To make sure that the program is successfully run, the few files should be included. They are;

i. busdata.txt. Should has six columns as following: bus status, initial value of voltage, initial value of angle, measured value of voltage, measured real power injection, measured reactive power injection.

ii. Busweightsdata.txt. Should has a row vector. This is a row vector. Each bus has three elements corresponding to it, the weights for the measured voltage, the weights for the measured value of real power injection, weights for the measured reactive power injection.

iii. Linedata.txt. Should has ten columns. The originating bus of the line, the terminating bus, the resistance of the line (p.u), the reactance (p.u), line charging (p.u.), tap magnitude, real power flow from originating to terminating, reactive power flow, real power flow from terminating to originating, reactive power flow from terminating to originating.

iv. Lineweightsdata.txt. should has a row vector, contains data corresponding to each of the lines, real power flow weights and reactive power weights.

v. Shuntdata.txt. should contain shunt data i.e. bus number and the value of the shunt.

vi. Chisquare.txt - contains of chi-square data for purpose of bad data detection, consist of five columns, which is degree of freedoms, confidence 0.05, confidence 0.025, confidence 0.01, confidence 0.005.

3.4 Description of the MATLAB Script

The flow chart of the MATLAB work in the Figure 3.2 shows the sequence flow of the estimation process. The MATLAB procedures actually broke down from the three stages of power system as discussed in the Chapter 2 section 2.4. The step involve in the MATLAB programming are discussed here

3.4.1 Reading the Data

For this first step, the required data as mentioned in section 3.3 are needed. All the data should be saved as .txt file and save together with the mainprog.m and progengine.m (provided in the Appendix) in the MATLAB work folder. Only one type of bus system data can be saved in a folder under the 'MATLAB work' folder so that the program will not be confused with other system data. The MATLAB programming will open the text file and read when the data are necessarily needed.

3.4.2 State estimation program

The second step is starting with formation of the *Ybus*. The effect of the line impedance and shunt admittance is important to form the *Ybus* matrix of the system. The following equation is show the line impedance formation in the MATLAB calculation process.

$$Tap\_effect_{11} = \frac{\dfrac{1}{Zreal(i) + Zimag(i) * j}}{tap(i)^\wedge 2} \tag{3.11}$$

$$Tap\_effect_{12} = \frac{\dfrac{-1}{Zreal(i) + Zimag(i) * j}}{tap(i)} \tag{3.12}$$

$$Tap\_effect_{21} = \frac{\dfrac{-1}{Zreal(i) + Zimag(i) * j}}{tap(i)} \tag{3.13}$$

$$Tap\_effect_{22} = \frac{-1}{Zreal(i) + Zimag(i) * j} \tag{3.14}$$

$$\begin{aligned} Y_{bus1} &= Y_{bus11} + Tap\_effect_{11} \\ Y_{bus11} &= Y_{bus11} + \frac{\dfrac{1}{Zreal(i) + Zimag(i) * j}}{tap(i)^\wedge 2} \end{aligned} \tag{3.15}$$

$$Y_{bus12} = Y_{bus12} + \frac{\dfrac{-1}{Zreal(i) + Zimag(i) * j}}{tap(i)} \tag{3.16}$$

$$Y_{bus21} = Y_{bus21} + \frac{\frac{-1}{Zreal(i) + Zimag(i) * j}}{tap(i)} \tag{3.17}$$

$$Y_{bus22} = Y_{bus22} \frac{-1}{Zreal(i) + Zimag(i) * j} \tag{3.18}$$

After the *Ybus* matrix is formed, then it is converted to general complex G and B. where;

G = real of Ybus

B = imaginary of Ybus

When the matrix is formed into the real and imaginary of complex number, the calculation of the injection real and reactive power, and real and reactive power flow of the system are calculated using the following equation [2.2] to equation [2.5].

3.4.3 Determining mismatch vector

For this part, MATLAB firstly will identify the measurement value that available or set in the data file then calculate for the mismatch. The advantage of the state estimation compare to the conventional power flow analysis is actually here, since state estimation no needs all data at the buses. Compare to the conventional power flow analysis, all buses should have the measured value, *m* or the power flow cannot be analysed. Determining of the mismatch vector for all vectors is calculated to get the difference between the measured value, *m* and the calculated value, *c*. If there are differences here, the estimation process will be proceed until all the data converge.

The series of equation below show the equation for the mismatching parameters.

Voltage mismatch, mmv = measured voltage – calculated voltage

Real power mismatch, mmp= measured Real power – calculated Real power

Reactive power mismatch, mmq = Reactive power measured– Reactive power calculated

Real power flow mismatch, mmpij = Real power flow measured– Real power flow calculated

Reactive power flow mismatch, mmqij = Reactive power flow measured– calculated Reactive power flow.

Real power flow mismatch, mmpji = Real power flow measured– Real power flow calculated

Reactive power flow mismatch, mmqji = Reactive power flow measured– calculated Reactive power flow.

Then, the Jacobian matrix is formed using the equation (2.6)

Where, the elements corresponding to the power injection measurements are;

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{n=1 \\ n \neq i}}^{N} V_i V_n Y_{in} \sin(\theta_{in} + \delta_n - \delta_i) \tag{3.19}$$

$$\frac{\partial P_i}{\partial \delta_j} = -V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i) \tag{3.20}$$

$$\frac{\partial P_i}{\partial |V_i|} = 2V_i G_{ii} + \sum_{\substack{n=1 \\ n \neq i}}^{N} V_n Y_{in} \cos(\theta_{In} + \delta_n - \delta_i) \tag{3.21}$$

$$\frac{\partial P_i}{\partial |V_j|} = V_i Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) \tag{3.22}$$

The following elements for reactive power injection measurement;

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{n=1 \\ n \neq i}}^{N} V_i V_n Y_{in} \cos(\theta_{in} + \delta_n - \delta_i) \tag{3.23}$$

$$\frac{\partial Q_i}{\partial \delta_j} = -V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) \tag{3.24}$$

$$\frac{\partial Q_i}{\partial |V_i|} = -2V_i B_{ii} - \sum_{\substack{n=1 \\ n \neq i}}^{N} V_n Y_{in} \sin(\theta_{in} + \delta_n - \delta_i) \tag{3.25}$$

$$\frac{\partial P_i}{\partial |V_j|} = -V_i Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i) \tag{3.26}$$

The following elements for corresponding power flow measurements;

$$\frac{\partial P_{ij}}{\partial \delta_i} = V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i) \tag{3.27}$$

$$\frac{\partial P_{ij}}{\partial \delta_j} = -V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i) \tag{3.28}$$

$$\frac{\partial P_{ij}}{\partial |V_i|} = 2V_i G_{ii} + V_j Y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) \tag{3.29}$$

$$\frac{\partial P_{ij}}{\partial |V_j|} = V_i Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) \tag{3.30}$$

$$\frac{\partial P_{ji}}{\partial \delta_i} = -V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) \tag{3.31}$$

$$\frac{\partial P_{ji}}{\partial \delta_j} = V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) \tag{3.32}$$

$$\frac{\partial P_{ji}}{\partial |V_j|} = 2V_j G_{ij} + V_i Y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) \tag{3.33}$$

$$\frac{\partial P_{ji}}{\partial |V_i|} = V_i Y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) \tag{3.34}$$

The following elements for corresponding reactive flow measurements;

$$\frac{\partial Q_{ij}}{\partial \delta_i} = -V_i V_n Y_{in} \cos(\theta_{ij} + \delta_j - \delta_i) \tag{3.35}$$

$$\frac{\partial Q_{ij}}{\partial \delta_j} = -V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) \tag{3.36}$$

$$\frac{\partial Q_{ij}}{\partial |V_i|} = -2V_i B_{ii} - V_j Y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) \tag{3.37}$$

$$\frac{\partial Q_{ij}}{\partial |V_j|} = -V_i Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i) \tag{3.38}$$

$$\frac{\partial Q_{ji}}{\partial \delta_i} = V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) \tag{3.39}$$

$$\frac{\partial Q_{ji}}{\partial \delta_j} = V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_i - \delta_j) \tag{3.40}$$

$$\frac{\partial Q_{ji}}{\partial |V_j|} = -2V_j B_{ij} - V_i Y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) \tag{3.41}$$

$$\frac{\partial Q_{ji}}{\partial |V_i|} = -V_i Y_{ij} \sin(\theta_{ij} + \delta_i - \delta_j) \tag{3.42}$$

Then, the process of the state estimation procedure will determine the state mismatch vector with the Gain matrix. Gain matrix is actually formed using the Jacobian, H and the measurement of error covariance matrix, R

$$G(x^k) = H^T R^{-1} H \tag{3.43}$$

G is built and stores as a sparse matrix for computational efficiency and memory consideration. It is built by processing one measurement at a time. Consider the measurement Jacobian, H and the covariance matrix for a set of m measurement, each one corresponding to one row, as shown below;

$$H = \begin{pmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{pmatrix}, R = \begin{pmatrix} R_{11} & 0 & \cdots & 0 \\ 0 & R_{22} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & R_{mm} \end{pmatrix} \tag{3.44}$$

Then, the G can be written as follow, and finally the state estimation part settled.

$$G = \sum_{i=1}^{m} H_i^T R_{ii}^{-1} H_i \tag{3.45}$$

3.4.4 Checking for presence of bad data

The last step in this program is checking the presence of the bad data. As discussed in the section 2.5,equation 2.5 and onwards, MATLAB will count the freedom value then the weight sum squares of the final errors calculated will compare to the standard Chi-Square Distribution value.

CHAPTER 4

RESULT AND DISCUSSION

The state estimation method was studied thoroughly before produced a state estimation MATLAB script. The first phase of study started with the research to get the understanding of the Weight Least Squares Method and followed by the bad data detection process. The MATLAB stimulation was executed on few Bus System.

In the second phase, all the idea of studies were transformed to the mathematical approach. Here, the state variables were estimated and the other network parameters were calculated mathematically so that they can be compared with the MATLAB result. MATLAB software start to be used since the iteration are needed to get the converge value of parameters. The mathematical calculation is used to verify the MATLAB programmed is correct by comparing the Jacobian Matrix result from calculation and MATLAB approach. The confidence used in this case study is 99%. One bad data found in fifth measurement for 2-Bus System. While for 3-Test Bus System and IEEE 14 – Bus System, no bad data are measured. All result can be found in Appendix B, C and D

The case study results are compared to the conventional power flow result from PSAT (Power System Analysis Toolbox). PSAT is an open code MATLAB based toolbox for electric power system analysis and control. The comparison function is to see the differences of the state variables estimated value between the state estimation process and PSAT estimation

4.1 Study Case 1: 2-Bus System

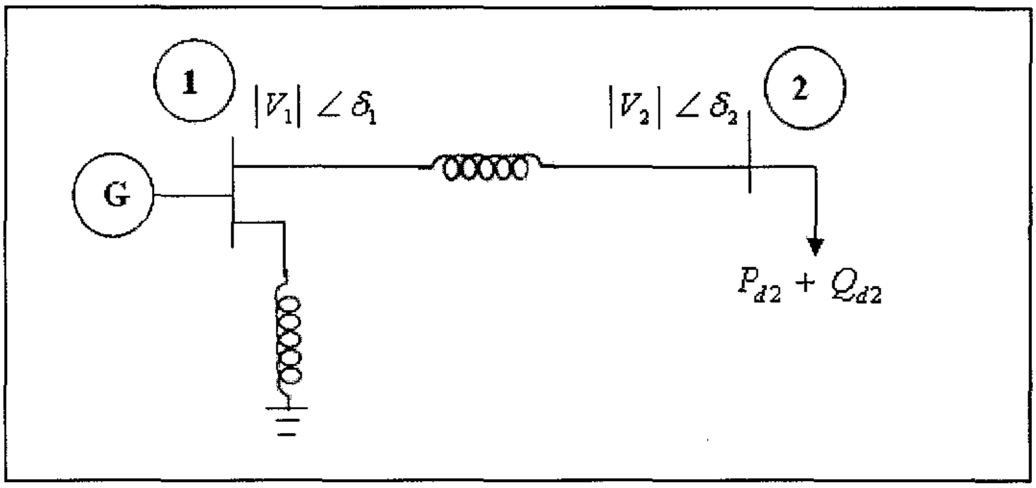

Figure 4.1: 2 -Test Bus System Network

Refer to Table 4.1 below, this first case study illustrates that 5 measurements data are available, converge at 4^{th} iteration and presence bad data at measurement number 5, reactive power form Bus Two to Bus One. The value of weight sum of squares of the errors around 543.7, which is greater than the standards Chi-square distributed for 99 % (alpha =1- 0.99 = 0.1), three unknown variables is 9.21 as refer to Table 2.1. Therefore, the data should be eliminating form the measurement value.

Table 4.1: Measurement data available for the 2-bus system

Without Bad Data Deduction		After Bad data detection	
Measurement	Type	Measure ment	Type
1	V_2	1	V_2
2	V_1	2	V_1
3	Q_1	3	Q_1
4	P_{12}	4	P_{12}
5	Q_{21}	Bad data	

Table 4.2: Estimated value for 2-Test Bus System

Initial, flat start		Without Bad data detection	After bad data deduction
δ_1	0	0	0
δ_2	0	-0.1762	-0. 16049
V_1	1	0.98464	1.01736
V_2	1	0.95826	0.92227

The flat start for the estimation as written, as all voltages set to all angles are zero except to slack bus. No errors for phase angle at Bus 1 because it was set as slack Bus. It is clear from Table 4.2, a single bad data in the measurement values leads to very bad estimations and it is mandatory to eliminate the bad data before the final estimation. By using the estimated value, other parameters can also be estimated. The Table 4.3 and Figure 4.2 below show the new parameters value for 2 Test Bus System, before and after Bad Data elimination

Table 4.3: Parameters for 2 Test Bus Systems

Estimation	Without Bad data detection	After bad data deduction
P_1	0.66163	0.59976
P_2	-0.66163	-0.59976
Q_1	0.32390	0.60769
Q_2	-0.04264	-0.30256
P_{12}	0.66163	0.59976
P_{21}	-0.66163	-0.59976
Q_{12}	0.16231	0.43519
Q_{21}	-0.04264	-0.30256

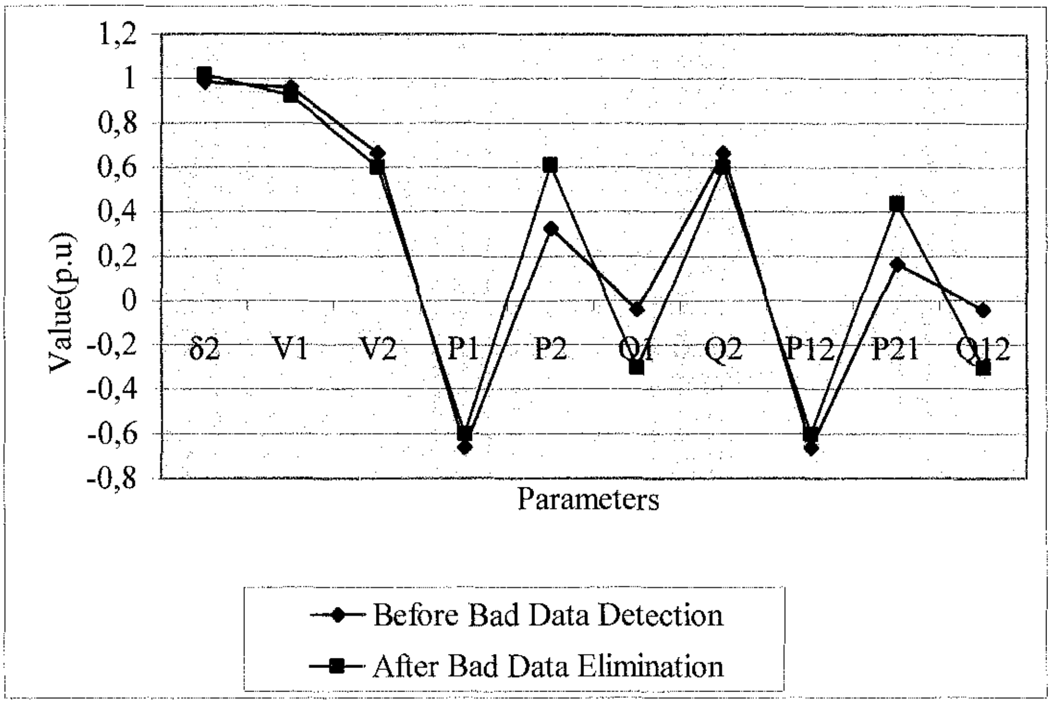

Figure 4.2: Comparison State Estimation Network Parameters

4.1.1 Mathematical Approach

To verify the MATLAB programming correct or not, the mathematical approach are used. The Jacobian Matrix from the mathematical calculation will be compare with the MATLAB result. If they close to each other mean the programmed are correct.

i. Find the *Ybus* matrix

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$Y = \begin{bmatrix} -j4.1667 & j4 \\ j4 & -j4 \end{bmatrix}$$

$$Y = G + jB$$

$$Y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -4.1667 & 4 \\ 4 & -4 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -4.1667 & 4 \\ 4 & -4 \end{bmatrix}$$

$$Y = Y\angle\theta^{\circ} = \begin{bmatrix} 4.1667 & 4 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} -90 & 90 \\ 90 & -90 \end{bmatrix}$$

$$|Y| = \begin{bmatrix} -4.1667 & 4 \\ 4 & -4 \end{bmatrix}$$

$$\theta^{\circ} = \begin{bmatrix} -90 & 90 \\ 90 & -90 \end{bmatrix}$$

ii. Define the unknown variable;

$$\partial_1 = 0^{\circ}$$

$$x_1 = \partial_2$$

$$x_2 = |V_2|$$

$$x_3 = |V_1|$$

The first two measurement m1 and m2 can be simply convert as

$$P_i = |V_i|^2 G_{ii} + \sum_{\substack{n=1 \\ n\neq i}}^{N} |V_i||V_n||Y_{in}|\cos(\theta_{in} + \partial_n - \partial_i)$$

$$Q_i = -|V_i|^2 B_{ii} - \sum_{\substack{n=1 \\ n\neq i}}^{N} |V_i||V_n||Y_{in}|\sin(\theta_{in} + \partial_n - \partial_i)$$

$$Q_1 = -|V_1|^2 B_{11} - \sum_{\substack{n=1 \\ n\neq i}}^{N} |V_2||V_1||Y_{21}|\sin(\theta_{21} + \partial_1 - \partial_2)$$

$$B = \begin{bmatrix} -4.1667 & 4 \\ 4 & -4 \end{bmatrix}$$

$$B_{11} = -4.1667$$

$$Q_1 = 4.1667 x_3^{\ 2} - 4 x_3 x_2 \sin(90 - x_1)$$

$$Q_1 = 4.1667 x_3^{\ 2} - 4 x_3 x_2 \cos x_1$$

$$P_{12} = -P_2$$

$$P_{12} = -|V_1|^2 G_{11} - \sum_{\substack{n=1 \\ n\neq i}}^{N} |V_2||V_1||Y_{21}|\cos(\theta_{21} + \partial_1 - \partial_2)$$

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$P_{12} = -4x_2x_3\cos\left(90^\circ - x_1\right)$$
$$P_{12} = -4x_2x_3\sin x_1$$
$$Q_2 = -|V_2|^2 B_{22} - \sum_{\substack{n=1 \\ n\neq i}}^{N} |V_2||V_1||Y_{21}|\sin\left(\theta_{21} + \partial_1 - \partial_2\right)$$
$$B = \begin{bmatrix} -4.1667 & 4 \\ 4 & -4 \end{bmatrix}$$
$$B_{22} = -4$$
$$Q_1 = 4x_3^{\ 2} - 4x_3x_2\sin\left(90 - x_1\right)$$
$$Q_1 = 4x_3^{\ 2} - 4x_3x_2\cos x_1$$

iii. Equation for measurement error

$$e_1^{(k)} = z_1 - h_1(x_1^{(k)}, x_2^{(k)}, x_3^{(k)})$$
$$e_1^{(k)} = z_1 - |V_2|$$
$$e_1^{(k)} = z_1 - x_2^{(k)}$$
$$e_1^{(0)} = z_1 - x_2^{(0)}$$
$$e_1^{(0)} = 0.92 - 1.00$$
$$e_1^{(0)} = -0.08$$
$$e_2^{(k)} = z_2 - h_2(x_1^{(k)}, x_2^{(k)}, x_3^{(k)})$$
$$e_2^{(k)} = z_2 - |V_1|$$
$$e_2^{(k)} = z_2 - x_3^{(k)}$$
$$e_2^{(0)} = z_2 - x_3^{(0)}$$
$$e_2^{(0)} = 1.02 - 1.00$$
$$e_2^{(0)} = 0.02$$
$$e_3^{(k)} = z_3 - h_2(x_1^{(k)}, x_2^{(k)}, x_3^{(k)})$$
$$e_3^{(k)} = z_3 - |Q_1|$$
$$e_3^{(k)} = z_3 - 4.1667x_3^{(k)2} + 4x_3^{(k)}x_2^{(k)}\cos x_1^{(k)}$$

$$e_3^{(0)} = z_3 - 4.1667x_3^{(0)2} + 4x_3^{(0)}x_2^{(0)}\cos x_1^{(0)}$$
$$e_3^{(0)} = 0.605 - 4.1667(1) + 4(1)(1)$$
$$e_3^{(0)} = 0.4383$$

$$e_4^{(k)} = z_4 - h_4(x_1^{(k)}, x_2^{(k)}, x_3^{(k)})$$
$$e_4^{(k)} = z_4 - |P_{12}|$$
$$e_4^{(k)} = z_4 - 4x_2^{(k)}x_3\sin^{(k)} x_1^{(k)}$$

$$e_4^{(0)} = z_4 - 4x_2^{(0)}x_3\sin^{(0)} x_1^{(0)}$$
$$e_4^{(0)} = 0.598 - [(-4)(1)(1)(0)]$$
$$e_4^{(0)} = 0.598$$

$$e_5^{(k)} = z_5^{(k)} - h_5(x_1^{(k)}, x_2^{(k)}, x_3^{(k)})$$
$$e_5^{(k)} = z_5 - |Q_{21}|$$
$$e_5^{(k)} = z_5 - 4x_2^{(k)2} + 4x_3^{(k)}x_2^{(k)}\cos x_1^{(k)}$$

$$e_5^{(0)} = z_5 - 4x_2^{(0)2} + 4x_3^{(0)}x_2^{(0)}\cos x_1^{(0)}$$
$$e_5^{(0)} = 0.305 - [(4)(1) - 4]$$
$$e_5^{(0)} = 0.305$$

So;

$$\begin{bmatrix} e_1^{(0)} \\ e_2^{(0)} \\ e_3^{(0)} \\ e_4^{(0)} \\ e_5^{(0)} \end{bmatrix} = \begin{bmatrix} -0.080 \\ 0.0200 \\ 0.4383 \\ 0.5980 \\ 0.3050 \end{bmatrix}$$

iv. find partial derivatives by using the following general matrix equations;

$$H_x^{(k)} = \begin{bmatrix} \left.\frac{\partial h_1}{\partial x_1}\right|^{(k)} & \left.\frac{\partial h_1}{\partial x_2}\right|^{(k)} & \left.\frac{\partial h_1}{\partial x_3}\right|^{(k)} \\ \left.\frac{\partial h_2}{\partial x_1}\right|^{(k)} & \left.\frac{\partial h_2}{\partial x_2}\right|^{(k)} & \left.\frac{\partial h_2}{\partial x_3}\right|^{(k)} \\ \left.\frac{\partial h_3}{\partial x_1}\right|^{(k)} & \left.\frac{\partial h_3}{\partial x_2}\right|^{(k)} & \left.\frac{\partial h_3}{\partial x_3}\right|^{(k)} \\ \left.\frac{\partial h_4}{\partial x_1}\right|^{(k)} & \left.\frac{\partial h_4}{\partial x_2}\right|^{(k)} & \left.\frac{\partial h_4}{\partial x_3}\right|^{(k)} \\ \left.\frac{\partial h_5}{\partial x_1}\right|^{(k)} & \left.\frac{\partial h_5}{\partial x_2}\right|^{(k)} & \left.\frac{\partial h_5}{\partial x_3}\right|^{(k)} \end{bmatrix}$$

$$H_x^{(k)} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4x_3^{(k)}x_2^{(k)} \sin x_1^{(k)} & -4x_3^{(k)} \cos x_1^{(k)} & \frac{25}{3}x_3^{(k)} - 4x_2^{(k)} \cos x_1^{(k)} \\ -4x_3^{(k)}x_2^{(k)} \cos x_1^{(k)} & -4x_3^{(k)} \sin x_1^{(k)} & -4x_2^{(k)} \sin x_1^{(k)} \\ 4x_3^{(k)}x_2^{(k)} \sin x_1^{(k)} & 8x_2 - 4x_3^{(k)} \cos x_1^{(k)} & -4x_2^{(k)} \cos x_1^{(k)} \end{bmatrix}$$

$$H_x^{(0)} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -4 & \frac{13}{3} \\ -4 & 0 & 0 \\ 0 & 4 & -4 \end{bmatrix} \qquad (4.1)$$

$$x^{(k+1)} - x^{(k)} = G^{(k)-1} H_x^T R^{-1}$$

Where;

$$G^{(k)} = (H_x^{(k)T} R^{-1} H_x^{(k)})$$

$$H_x^{(k)T}R^{-1} = \begin{bmatrix} 0 & 0 & 1 & -4 & 0 \\ 1 & 0 & -4 & 0 & 4 \\ 0 & 1 & \frac{13}{3} & 0 & -4 \end{bmatrix} \text{x} \begin{bmatrix} \frac{1}{(0.01)^2} & . & . & . & . \\ . & \frac{1}{(0.01)^2} & . & . & . \\ . & . & \frac{1}{(0.02)^2} & . & . \\ . & . & . & \frac{1}{(0.015)^2} & . \\ . & . & . & . & \frac{1}{(0.02)^2} \end{bmatrix}$$

$$H_x^{(k)T}R^{-1} = \begin{bmatrix} 0 & 0 & 0 & -1.7778 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 1.0833 & 0 & -1 \end{bmatrix} \text{x}10^4$$

$$G^{(0)} = 10^4 \text{x} \begin{bmatrix} 0 & 0 & 0 & -1.7778 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 1.0833 & 0 & -1 \end{bmatrix} \text{x} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -4 & \frac{13}{3} \\ -4 & 0 & 0 \\ 0 & 4 & -4 \end{bmatrix}$$

$$G^{(0)} = \begin{bmatrix} 7.1111 & 0 & 0 \\ 0 & 9.0000 & -8.3333 \\ 0 & -8.3333 & 9.6944 \end{bmatrix} \text{x}10^4$$

so;

$$x^{(k+1)} - x^{(k)} = G^{(k)-1}H_x^{(k)T}R^{-1} \qquad x^{(k+1)} - x^{(k)} = G^{(k)-1}H_x^{(k)T}R^{-1}$$

$$\begin{bmatrix} x_1{}^{(k+1)} \\ x_2{}^{(k+1)} \\ x_3{}^{(k+1)} \end{bmatrix} - \begin{bmatrix} x_1{}^{(k)} \\ x_2{}^{(k)} \\ x_3{}^{(k)} \end{bmatrix} = G^{(k)-1}H_x^T R^{-1} \begin{bmatrix} e_1^{(k)} \\ e_2^{(k)} \\ e_3^{(k)} \\ e_4^{(k)} \\ e_5^{(k)} \end{bmatrix} \begin{bmatrix} x_1{}^{(k+1)} \\ x_2{}^{(k+1)} \\ x_3{}^{(k+1)} \end{bmatrix} - \begin{bmatrix} x_1{}^{(k)} \\ x_2{}^{(k)} \\ x_3{}^{(k)} \end{bmatrix} = G^{(k)-1}H_x^T R^{-1} \begin{bmatrix} e_1^{(k)} \\ e_2^{(k)} \\ e_3^{(k)} \\ e_4^{(k)} \\ e_5^{(k)} \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ x_3^{(1)} \end{bmatrix} = \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \\ x_3^{(0)} \end{bmatrix} + G^{(0)-1} H_x^T R^{-1} \begin{bmatrix} e_1^{(0)} \\ e_2^{(0)} \\ e_3^{(0)} \\ e_4^{(0)} \\ e_5^{(0)} \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ x_3^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 7.1111 & 0 & 0 \\ 0 & 9.0000 & -8.3333 \\ 0 & -8.3333 & 9.6944 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & -1.7778 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 1.0833 & 0 & -1 \end{bmatrix} \text{x}10^4 \begin{bmatrix} -0.080 \\ 0.0200 \\ 0.4383 \\ 0.5980 \\ 0.3050 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ x_3^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 7.1111 & 0 & 0 \\ 0 & 9.0000 & -8.3333 \\ 0 & -8.3333 & 9.6944 \end{bmatrix}^{-1} \begin{bmatrix} -1.0631 \\ -0.2133 \\ 0.1898 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ x_3^{(1)} \end{bmatrix} = \begin{bmatrix} -0.1495 \\ 0.9727 \\ 0.9961 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \\ x_3^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ x_3^{(1)} \end{bmatrix} = \begin{bmatrix} -0.1495 \\ 0.9727 \\ 0.9961 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = \begin{bmatrix} -0.1721 \\ 0.9610 \\ 0.9870 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(3)} \\ x_2^{(3)} \\ x_3^{(3)} \end{bmatrix} = \begin{bmatrix} -0.1756 \\ 0.9583 \\ 0.9847 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(4)} \\ x_2^{(4)} \\ x_3^{(4)} \end{bmatrix} = \begin{bmatrix} -0.1761 \\ 0.9579 \\ 0.9843 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(5)} \\ x_2^{(5)} \\ x_3^{(5)} \end{bmatrix} = \begin{bmatrix} -0.1762 \\ 0.9578 \\ 0.9843 \end{bmatrix}$$

$$\begin{bmatrix} x_1^{(6)} \\ x_2^{(6)} \\ x_3^{(6)} \end{bmatrix} = \begin{bmatrix} -0.1762 \\ 0.9578 \\ 0.9843 \end{bmatrix}$$

v. Test for bad data

Chi-square $= \hat{f} = \sum_{j=1}^{m} \left(\frac{\hat{e}_j}{\sigma_j} \right)^2 = 544.8149$ greater than 9.21 standard Chi-square value (refer to Table 2.1). So, we eliminate the largest standardized error.

Standardized error $= \frac{\hat{e}_1}{\sqrt{R_{11}}} = \begin{bmatrix} 5.7213 \\ 5.0407 \\ 20.5086 \\ 22.6473 \\ 23.3403 \end{bmatrix}$, measurement 5 will be discarded.

So second time checking for bad data, the Chi-square = 0.1355 is less than the standard Chi-squares value. So the process estimation use only four measurements to estimate new parameters.

4.1.2 MATLAB Approach

The Jacobian of the five measurements is;

$$G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -4.16666666666667 & 4.00000000000000 \\ 4.00000000000000 & -4.00000000000000 \end{bmatrix}$$

Jacobian Matrix at the first iteration is same as in calculated (equation

$$Hx = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 4.33333333333333 & -4.00000000000000 \\ -4.00000000000000 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -4.0000000000000 & 4.000000000000000 \end{bmatrix} \quad (4.2)$$

Then the estimation is converge at the 5^{th} iteration

$$Hx = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.65192811330384 & 4.43772818706490 & -3.88903086936647 \\ -3.73730573763515 & 0.66055556487062 & 0.67839474081956 \\ -0.65192811330384 & -3.78676429539243 & 3.79886025450926 \end{bmatrix} \quad (4.3)$$

By comparing the mathematical Jacobian Matrix calculation, showed by equation 4.1 and MATLAB approach, showed by equation 4.2, we can see that the MATLAB was correctly programmed sine the MATLAB value are approximately to the mathematical calculation value. So, the others bus system also can be estimated using the MATLAB since the programmed are correct.

Result for the MATLAB work can be found in Appendix B

4.2 Study Case2: 3-Bus System

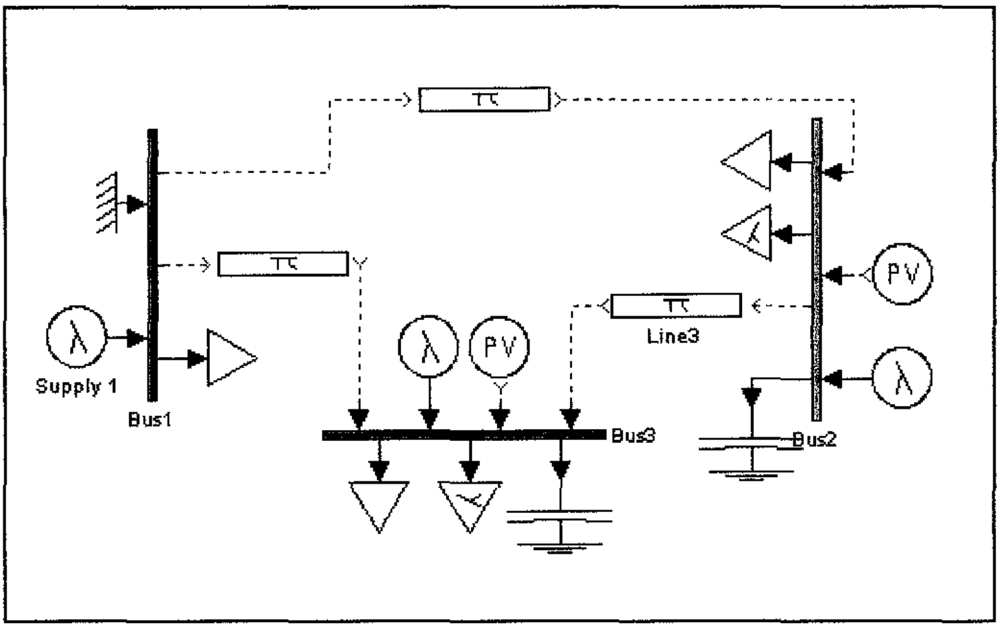

Figure 4.3: 3 - Bus System Network

Figure 4.3 was established from the PSAT software. Refer to Table 4.4 below, this second case study illustrates that 7 measurements data are available, converge also at 8th iteration. This time of case study there are no presence of bad data since the value of weight sum of squares of the errors around 0.01, which less than the standards Chi-square distributed for 99% (alpha=1-0.99=0.1), 5 variables is 9.21 as refer to Table 2.1. Therefore, the data should not be eliminated form the measurement because there are no bad data in the measurement.

Table 4.4: Measurement data available for the 3-Bus system

Without Bad Data Deduction	
Measurement	Type
1	V_1
2	V_2
3	V_3
4	P_1

5	P_2
6	P_{13}
7	P_{23}

Table 4.5: Estimated Value for 3- Bus Systems

Estimation No Bad data Presence	Value(p.u) of Network Parameters	Estimation No Bad data Presence	Value(p.u) of Network Parameters
δ_1	0	P_{12}	0.12434
δ_2	-0.01229	P_{13}	0.40961
δ_3	-0.03182	P_{23}	0.38035
V_1	1.03435	Q_{12}	0.58322
V_2	0.97804	Q_{13}	0.50379
V_3	0.99589	Q_{23}	-0.34546
P_1	0.53395	P_{21}	-0.12343
P_2	0.25601	P_{31}	-0.40961
P_3	-0.178996	P_{32}	-0.38035
Q_1	1.26533	Q_{21}	-0.54998
Q_2	-0.59544	Q_{31}	-0.47227
Q_3	-0.11301	Q_{32}	0.35926

Since this 3 test Bus System is free from the bad data, the estimated value from the Table 4.4 can be used as the new estimated parameters value for the network.

The following result show the comparison between State Estimation result and PSAT result.

Table 4.6: State Variable for 3- Bus Systems

Flat start		State Estimation	PSAT Stimulation
δ_1	0	0	0
δ_2	0	-0.01229	-0.08709
δ_3	0	-0.03182	-0.22266
V_1	1	1.03435	1.06
V_2	1	0.97804	1.045
V_3	1	0.99589	1.01

Table 4.7: Real and Reactive Power Injection for 3- Bus Systems

Measurement	State Estimation	PSAT Stimulation
P_1	0.53395	0.2
P_2	0.25601	0.4
P_3	-0.178996	0.3
Q_1	1.26533	0.003779
Q_2	-0.59544	0.005779
Q_3	-0.11301	0.609113

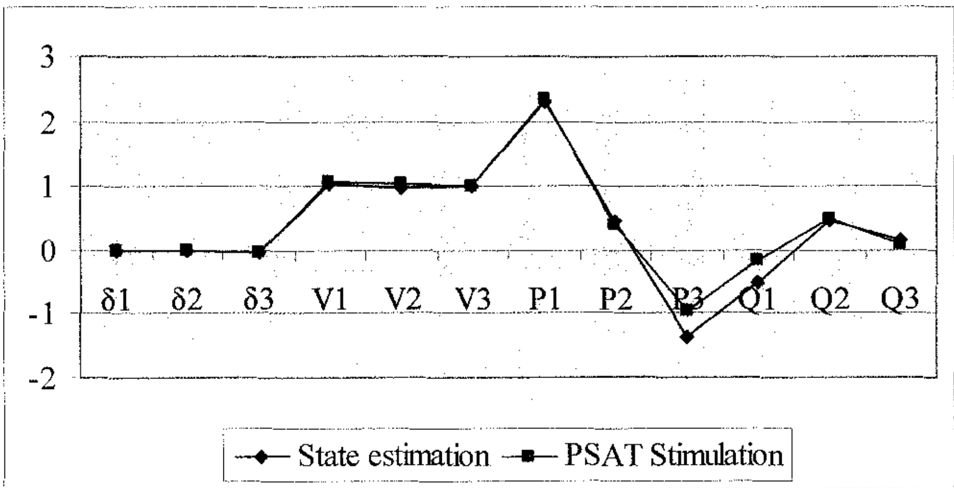

Figure 4.4: Comparison State Variable, Real, and Reactive Power Injection for 3- Bus System

Table 4.8: Real and Reactive Power Flow for 3-Test Bus Systems

Measurement	State Estimation	PSAT Stimulation
P_{12}	0.12434	-0.06668
P_{13}	0.40961	0.26677
P_{23}	0.38035	0.3333
Q_{12}	0.58322	0.000445
Q_{13}	0.50379	0.007113
Q_{23}	-0.34546	0.01111
P_{21}	-0.12343	0.06667
P_{31}	-0.40961	-0.266668
P_{32}	-0.38035	-0.33332
Q_{21}	-0.54998	0.000222
Q_{31}	-0.47227	0.003556
Q_{32}	0.35926	0.005557

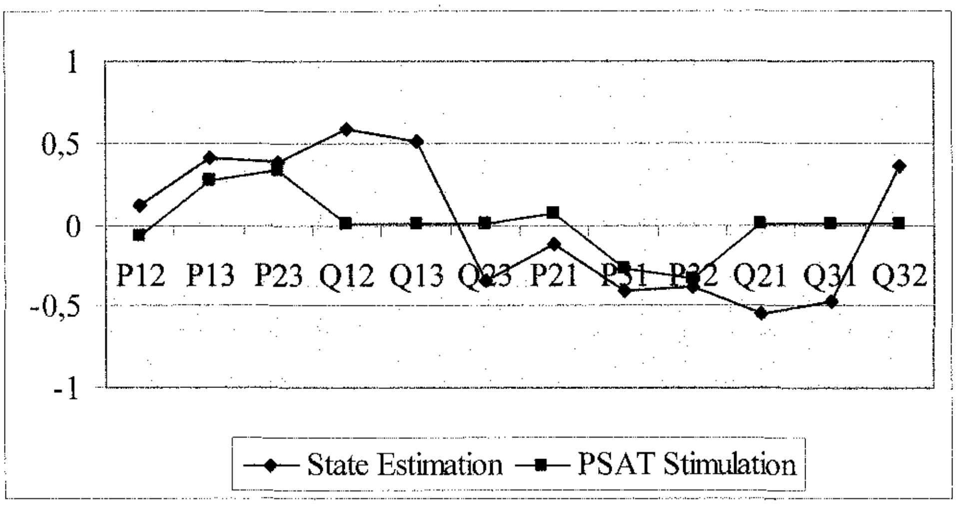

Figure 4.4: Real and Reactive Power Flow for 3-Bus System

4.3 Study Case 3: IEEE 14-Bus System

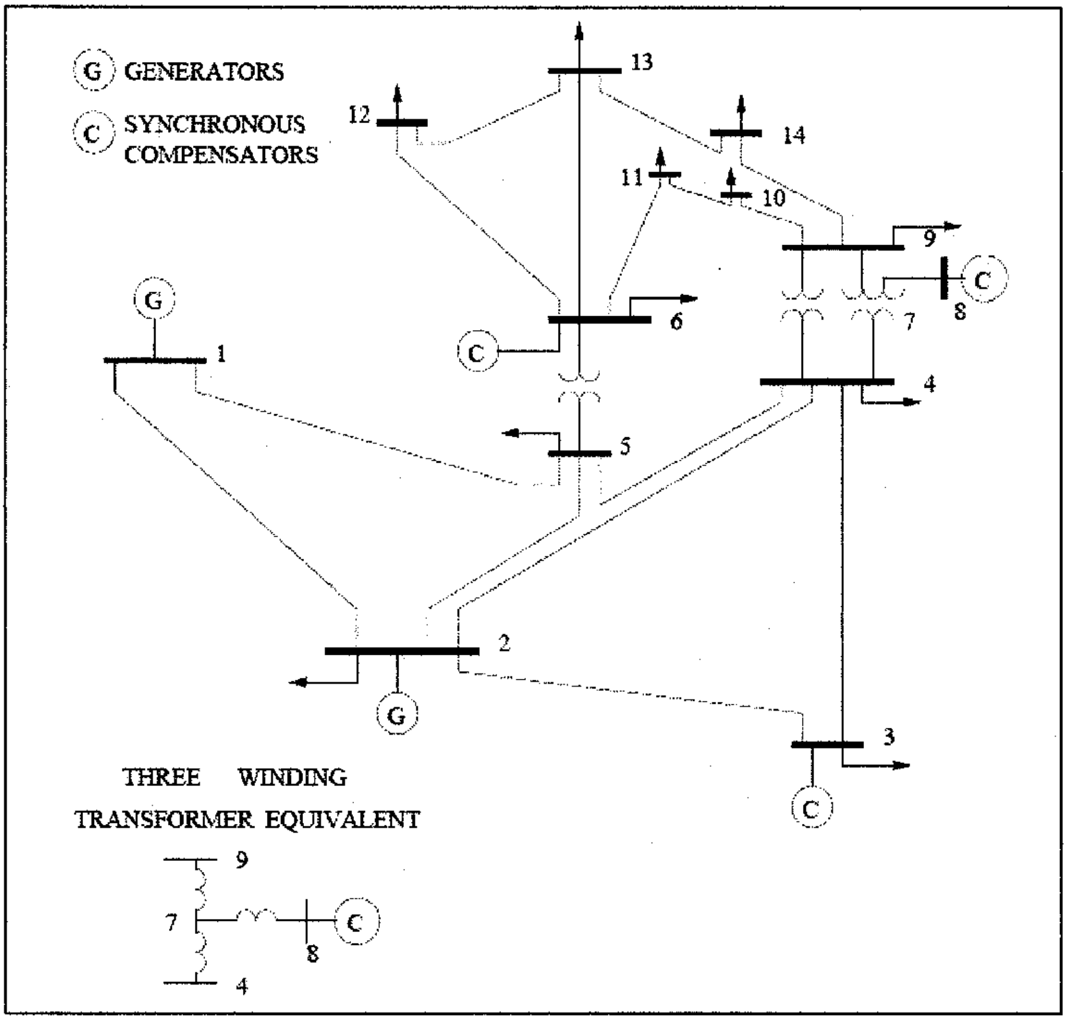

Figure 4.5: IEEE 14 - Bus System Network

For IEEE 14-Bus System, no bad data are measured and convergence occurred at sixth iteration. The detail result for IEEE 14-Bus System can be found in Appendix D. The comparison value for state variable and network parameters are shown in following result

Table 4.9: Phase Angle, δ_s Comparison for IEEE 14-Bus System

Flat start		State Estimation	PSAT Stimulation
δ_1	0	0.0000	0
δ_2	0	-0.10269	-0.08709
δ_3	0	-0.30553	-0.22266
δ_4	0	-0.21292	-0.17850
δ_5	0	-0.17667	-0.15273
δ_6	0	-0.27935	-0.25158
δ_7	0	-0.26556	-0.23091
δ_8	0	-0.23235	-0.23091
δ_9	0	-0.29824	-0.25852
δ_{10}	0	-0.30660	-0.26222
δ_{11}	0	-0.31106	-0.25900
δ_{12}	0	-0.32427	-0.26642
δ_{13}	0	-0.31789	-0.26708
δ_{14}	0	-0.32854	-0.28016

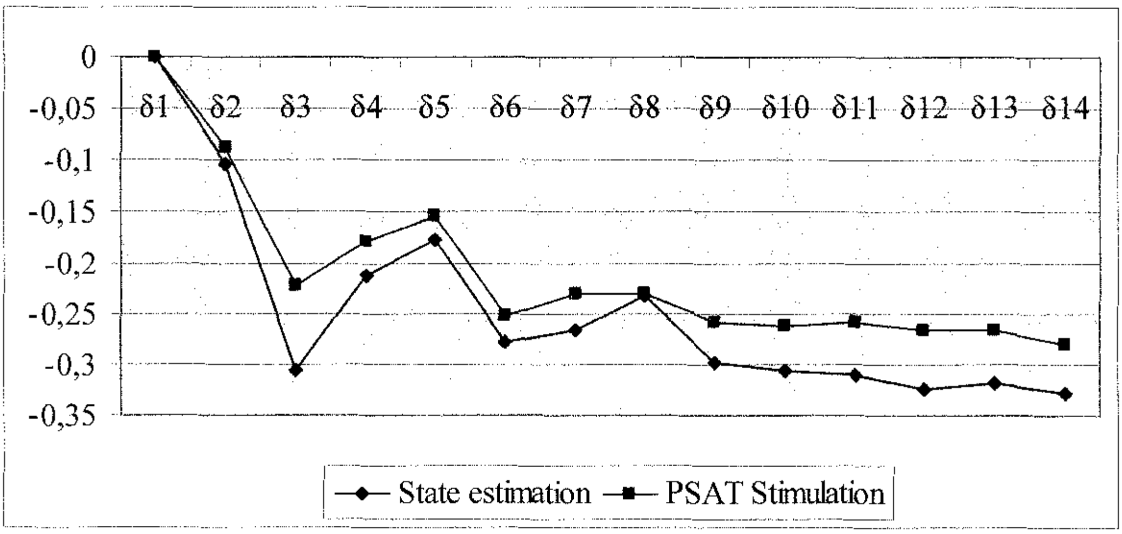

Figure 4.6: Phase Angle, δ_s Comparison for IEEE 14-Bus System

Table 4.10: Voltage Magnitude, V_s Comparison for IEEE 14-Bus System

Flat start		State Estimation	PSAT Stimulation
V_1	1	0.99309	1.06
V_2	1	0.99262	1.045
V_3	1	0.94981	1.01
V_4	1	0.97054	1.012011
V_5	1	0.96498	1.016031
V_6	1	0.99611	1.07
V_7	1	0.95312	1.049465
V_8	1	0.95542	1.09
V_9	1	0.95850	1.032845
V_{10}	1	0.96620	1.031843
V_{11}	1	0.99835	1.047119
V_{12}	1	1.01390	1.053435
V_{13}	1	0.99653	1.046978
V_{14}	1	0.95891	1.020759

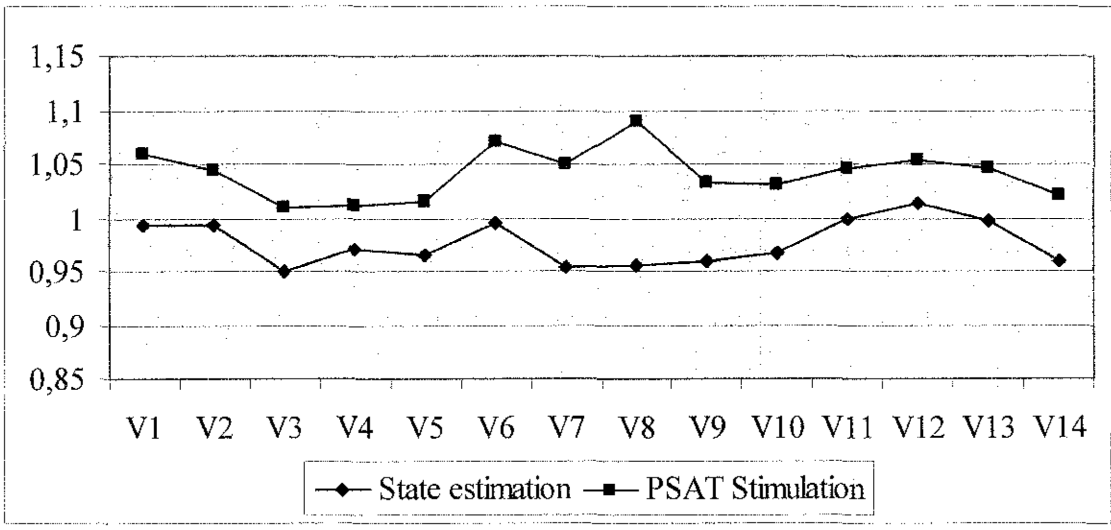

Figure 4.7: Voltage Magnitude, V_s Comparison for IEEE 14-Bus System

Table 4.11: Real and Reactive Power Injection Comparison for IEEE-14 Bus System

	State estimation	PSAT Stimulation		State Estimation	PSAT stimulation
P_1	2.31414	2.32581	Q_1	-0.49930	-0.14985
P_2	0.47637	0.183	Q_2	0.47481	0.36093
P_3	-1.38027	-0.942	Q_3	0.15724	0.083573
P_4	-0.45817	-0.478	Q_4	0.57236	-0.04
P_5	0.03984	-0.076	Q_5	-0.08391	-0.016
P_6	0.05253	0.112	Q_6	-0.44992	0.14969
P_7	-0.13315	0	Q_7	-0.21429	0
P_8	0.16801	0	Q_8	0.01499	0
P_9	-0.27883	-0.295	Q_9	-0.16701	-0.166
P_{10}	-0.09344	-0.09	Q_{10}	-0.03324	-0.058
P_{11}	-0.08471	-0.035	Q_{11}	0.22200	-0.018
P_{12}	-0.08661	-0.061	Q_{12}	0.18272	-0.016
P_{13}	-0.18718	-0.135	Q_{13}	0.14987	-0.058
P_{14}	-0.14649	-0.149	Q_{14}	-0.02909	-0.05

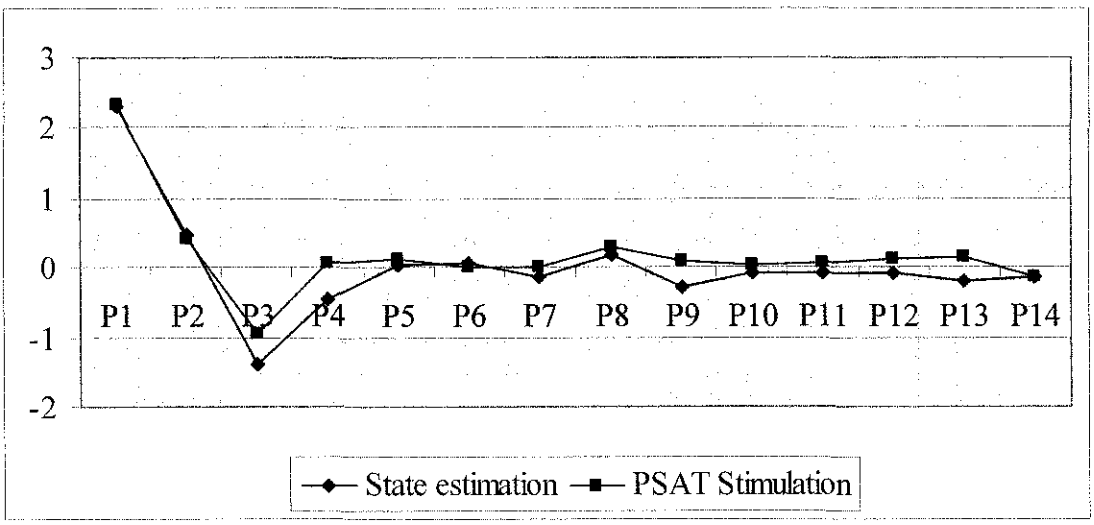

Figure 4.8: Real Power Injection Comparison for IEEE-14 Bus System

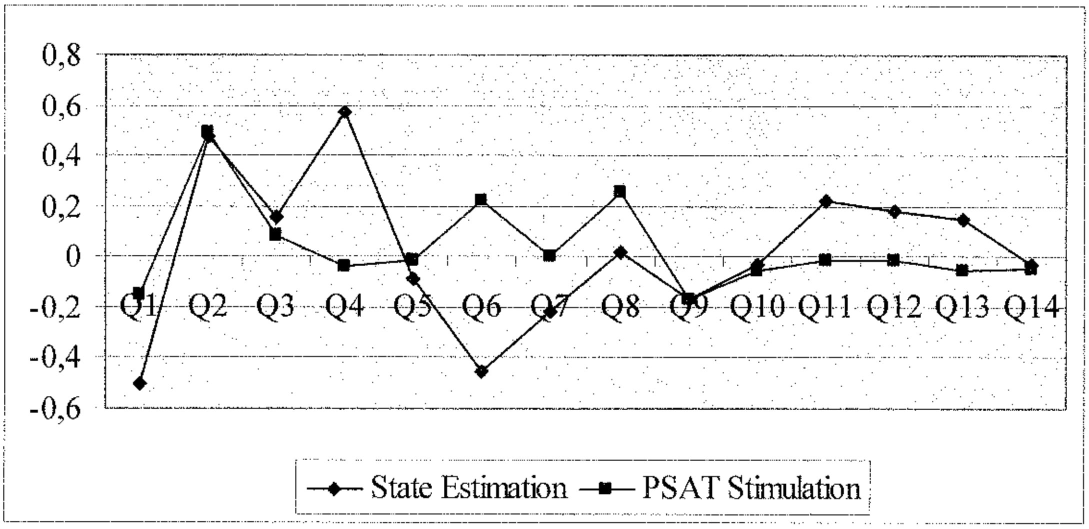

Figure 4.9: Reactive Power Injection Comparison for IEEE-14 Bus System

Table 4.12: Real and Reactive Power Flow Comparison for IEEE-14 Bus System

	State Estimation	PSAT Stimulation		State estimation	PSAT Stimulation
P_{1-2}	1.54409	1.57111	Q_{1-2}	-0.46970	-0.20458
P_{1-5}	0.77005	0.75470	Q_{1-5}	-0.02960	0.054723
P_{2-3}	0.96653	0.71116	Q_{2-3}	0.02811	0.035397
P_{2-4}	0.57618	0.55945	Q_{2-4}	-0.07722	0.015758
P_{2-5}	0.42582	0.417027	Q_{2-5}	-0.0300	0.03209
P_{3-4}	-0.46137	-0.23084	Q_{3-4}	0.07030	0.066763
P_{4-5}	-0.76334	-0.601984	Q_{4-5}	0.33187	0.09235
P_{4-7}	0.23649	0.27208	Q_{4-7}	0.08839	-0.06556
P_{4-9}	0.14593	0.155039	Q_{4-9}	0.02775	0.027879
P_{5-6}	0.42378	0.456823	Q_{5-6}	-0.10745	0.10982
P_{6-11}	0.12783	0.081763	Q_{6-11}	-0.06618	0.084201
P_{6-12}	0.11936	0.080368	Q_{6-12}	-0.11931	0.031165
P_{6-13}	0.22912	0.182692	Q_{6-13}	-0.12087	0.097337
P_{7-8}	-0.16801	-0.22E-14	Q_{7-8}	-0.0938	-0.252624
P_{7-9}	0.27135	0.27208	Q_{7-9}	-0.04217	0.162313

P_{9-10}	0.05470	0.044901	Q_{9-10}	-0.11234	-0.00457
P_{9-14}	0.08375	0.087219	Q_{9-14}	-0.04021	0.006082
P_{10-11}	-0.03926	-0.04516	Q_{10-11}	-0.14693	-0.063803
P_{12-13}	0.02930	0.01857	Q_{12-13}	0.05594	0.013505
P_{13-14}	0.06566	0.06269	Q_{13-14}	0.07537	0.04586

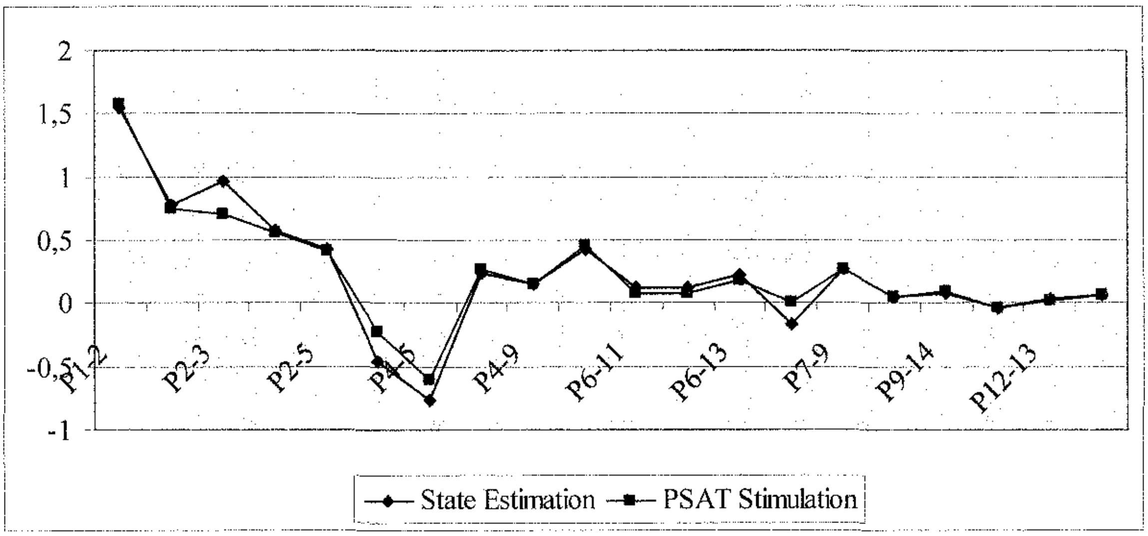

Figure 4.10: Real Power Flow Comparison for IEEE-14 Bus System

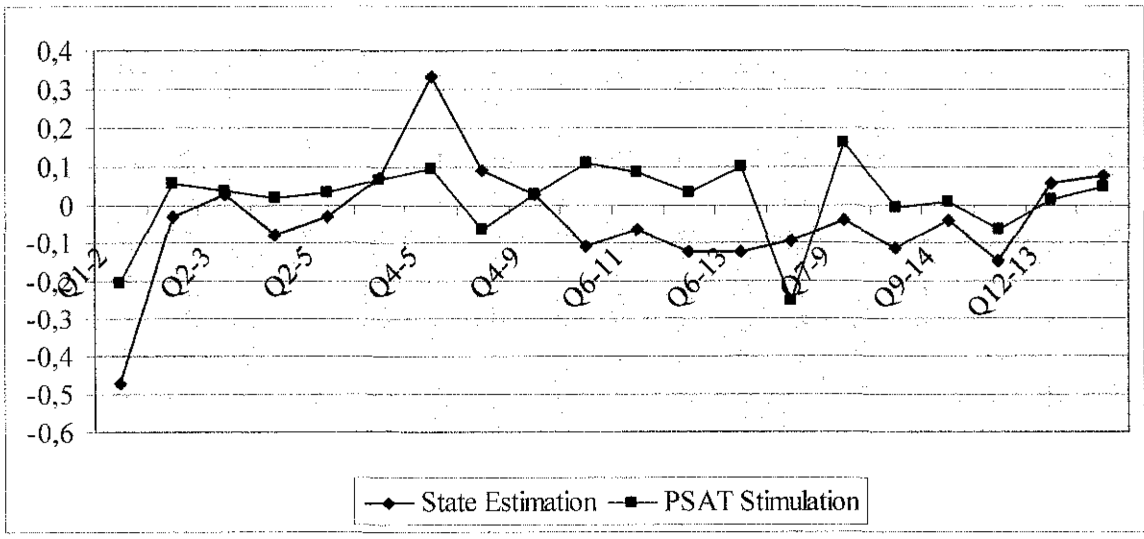

Figure 4.11: Reactive Power Flow Comparison for IEEE-14 Bus System

CHAPTER 5
CONCLUSION

The state estimation project for three case studies has been successfully completed. Explanation on all the steps involved in estimating the state variables using Weight Least Squares method, are discussed in details in this report. The bad data among measured values in 2-Bus System, influencing the performance of state estimation. MATLAB program that has been developed able to identify and remove the bad data as discuss in result and discussion. However, there are still rooms to be improved. This program will be more beneficial and proven to be generalized if it was tested on 24-Bus System, 30-Bus System or introduced more cases that have bad data measurements. The level of confident to use this program is more if all the proposed cases are execute successful by this program

CHAPTER 6
RECOMMENDATION

The selected 'weight' value for gain matrix for this project is one, so further researches on selecting the best weighting value should be done. Further studies on implementing state estimation in a large scale network also recommended. To stabilize the large scale network, the usage of Unified Power Flow Controller (UPFC) is needed. The state estimation should be altered or modified so that it can meet the large scale network system, together with the UPFC.

CHAPTER 7
REFERENCES

[1] Schweppe F.C. and Wildes J. ," Power System Static-State Estimation, Part I : Exact Model", *IEEE Transactions on Power Apparatus and Systems, Vol..PAS-89, January 1970, pp.120-125.*

[2] Schweppe F.C and Rom D.B. ," Power System Static-State Estimation, Part III : Implementation", *IEEE Transactions on Power Apparatus and Systems, Vol..PAS-89, January 1970, pp.130-135*

[3] Schweppe F.C and Rom D.B. ," Power System Static-State Estimation, Part II : Approximate Model", *IEEE Transactions on Power Apparatus and Systems, Vol..PAS-89, January 1970, pp.125 - 130.*

[4] Amit Jain, Yoshiyuki Kawakazoe, "Network Observability: A Solution Techniques Using Neural Networks", *Institue or Material research (IMR),Tohoku University,Japan pp1*

[5] Chen-Chin Liu, Kevin Schhneider,Harold Kirkham and Bruce Howe " State Estimation for the NEPTUNE Power System", *IEEE Transactions on Power Systems.*

[6] E.Handscin, F.C.Schwepe, J. Kohlas, and A.Fiecher, " Bad Data Analysis for Power System State estimation", *IEEE Transactions on Power Systems, PAS 94. no2.pp.329-336,1975*

[7]Noor Ain Ahmad, "Final Year Project 1 Weekly Report", *Week 3, 4, 5, 6, 7, all pages.*

[8] Dr. Ramiah Jegathesan, "Power Flow Analysis", *Lecture Notes Power Sstem II, University Technology PETRONAS.*

[9] Ali Abur, "Power System State Estimation", pp.5 .

[10]Ali Abur, "Power System State Estimation", pp.21-23

CHAPTER 8

APPENDICES

APPENDIX A

MATLAB coding for State Estimation

```
 the main state estimate program
1;

bus;
lines;
shunts;
olerance;
onfidence;

ut('Please enter the number of buses...');
nput('Please enter the number of lines...');
input('Please enter the number of shunts...');
e=input('Please enter the tolerance...');
ease enter the level of confidence for');
ce=input('the purpose of bad data detection...');
ease select the followings:');
Input data');
Run program');
nput('to continue the programming.........');
e==1
=input('Please enter the number of buses...');
es=input('Please enter the number of lines...');
nts=input('Please enter the number of shunts...');
rance=input('Please enter the tolerance...');
('Please enter the level of confidence for');
idence=input('the purpose of bad data detection...');
entry(choice,nbus,nlines,nshunts,tolerance,confidence);
s=x(1);
nes=x(2);
unts=x(3);
erance=x(4);

e==2
ramengine(choice,nbus,nlines,nshunts,tolerance,confidence);

ce==3
woutput(choice);
```

```
 x1=programengine(choice,nbus,nlines,nshunts,tolerance,confidence);
s;
lines;
nshunts;
e=tolerance;

 data related to buses
n('busdata.txt','r');
ad('busdata.txt');
id);

 data pertaining to lines
n('linedata.txt','r');
ad('linedata.txt');
id);

 data pertaining to shunts
n('shuntdata.txt','r');
ad('shuntdata.txt');
id);

 data pertaining to weights
n('busweightsdata.txt','r');
extread('busweightsdata.txt');
id);

n('lineweightsdata.txt','r');
=textread('lineweightsdata.txt');
id);

 data pertaining to chi-square distribution
n('chisquare.txt','r');
tread('chisquare.txt');
id);

swts(:,1:3:3*nbus-2);
swts(:,2:3:3*nbus-1);
swts(:,3:3:3*nbus);
=linewts(:,1:4:4*nlines-3);
=linewts(:,2:4:4*nlines-2);
=linewts(:,3:4:4*nlines-1);
=linewts(:,4:4:4*nlines);

ESTIMATION PROGRAM

ion of Ybus
ros(nbus);
,6);
);
);
(:,3);
(:,4);
5)*sqrt(-1);
:nlines
x(i,:);
y(i,:);
e_charging(a1,b1)=y1(i,:);
```

```
rging;
al(line_charging);
ag(line_charging);
bs(line_charging);
c)~=0
_bus=c(:,1);
_val=c(:,2);
n=(-1/shun_val)*sqrt(-1);


of line impedances and shunt admittances
:length(x)
effect11=(1/(zreal(i)+zimag(i)*sqrt(-1)))/(tap(i)^2);
effect12=(-1/(zreal(i)+zimag(i)*sqrt(-1)))/(tap(i));
effect21=(-1/(zreal(i)+zimag(i)*sqrt(-1)))/(tap(i));
effect22=(1/(zreal(i)+zimag(i)*sqrt(-1)));
(x(i),x(i))=Ybus(x(i),x(i))+tap_effect11+y1(i);
(y(i),y(i))=Ybus(y(i),y(i))+tap_effect22+y1(i);
(x(i),y(i))=Ybus(x(i),y(i))+tap_effect12;
(y(i),x(i))=Ybus(y(i),x(i))+tap_effect21;


on of shunts
c)~=0
i=1:length(shun_bus)
Ybus(shun_bus(i),shun_bus(i))=Ybus(shun_bus(i),shun_bus(i))+Yshun(i);


ong,Ybus;
ngle(Ybus);
eta1*180/pi;


Ybus);
ion into G and B
bus)
bus)

tion of measurement mismatch
surement mismatch vector is of the form [V Pi Qi Pij Qij Pji Qji]' where each
presents a subvector

ismatch vector
ismatch vector is of the form [theta V] where term represents a
or and the theta subvector excludes the slack bus angle


:,2)
=a(:,3);
(:,4);
(:,5);
(:,6);
=b(:,7);
=b(:,8);
=b(:,9);
=b(:,10);
```

```
ag>0 & count<6
t=count+1;
v_bus;
ang_busr*180/pi;

d reactive power injection calculation
];

];
nbus
lc1(gogo,:)=0;
rmp(gogo,:)=v_bus(i,:)*v_bus(i,:)*G(i,i);
lc1(gogo,:)=0;
rmq(gogo,:)=v_bus(i,:)*v_bus(i,:)*B(i,i);
j=1:nbus
if j~=i
  p_calc(i)=p_calc(i)+v_bus(i)*v_bus(j)*(G(i,j)*cos(ang_bus(i)-ang_bus(j))+B(i,j) 
_bus(i)-ang_bus(j)));
lc1(gogo,:)=p_calc1(gogo,:)+v_bus(i,:)*v_bus(j,:)*mag(i,j)*cosd(theta(i,j)+ang_bu 
ng_bus(i,:));
lc1(gogo,:)=q_calc1(gogo,:)+v_bus(i)*v_bus(j)*mag(i,j)*sind(theta(i,j)+ang_bus(j) 
(i));
end

lc3(gogo,:)=f_termp(gogo,:)+p_calc1(gogo,:);
lc3(gogo,:)=-(f_termq(gogo,:)+q_calc1(gogo,:));
=gogo+1;

_calc3';
lc;
_calc3'
lc;

s=x;
y;

real and reactive flows calculation
=[];
=[];
);
length(x)
art_bus(i);
d_bus(i);
_calc(i)=-((v_bus(q))^2)*G(q,r)+v_bus(q)*v_bus(r)*(cos(ang_bus(q)-ang_bus(r))*G(q 
ang_bus(q)-ang_bus(r))*B(q,r));
calc(i)=-(v_bus(q)*v_bus(q))*G(q,r)+v_bus(q)*v_bus(r)*mag(q,r)*cosd(theta(q,r)+an 
-ang_bus(q));
calc(i)=-(v_bus(q)*v_bus(q)*(Bline(q,r)-B(q,r))+v_bus(q)*v_bus(r)*mag(q,r)*sind(t 
)+ang_bus(r)-ang_bus(q)));

alc';
calc';

real and reactive flows calculation
```

```
:=[];
:=[];
length(x)
.rt_bus(i);
._bus(i);
calc(i)=-((v_bus(r))^2)*G(q,r)+v_bus(q)*v_bus(r)*(cos(ang_bus(r)-ang_bus(q))*G(q,
.ng_bus(r)-ang_bus(q))*B(q,r));
:alc(i)=-(v_bus(r)*v_bus(r))*G(q,r)+v_bus(q)*v_bus(r)*mag(q,r)*cosd(theta(q,r)+ang
ang_bus(r));
:alc(i)=-(v_bus(r)*v_bus(r)*(Bline(q,r)-B(q,r))+v_bus(q)*v_bus(r)*mag(q,r)*sind(th
+ang_bus(q)-ang_bus(r)));

calc';
calc';
NING MISMATCH VECTORS
 mismatch




nbus;
_meas(i,:)==0
mmv(ov,:)=v_meas(i,:);

v_meas(i,:)~=0
mv(ov,:)=v_meas(i,:)-v_calc(i,:);
.f mmv(ov,:)==0
   mmv(ov,:)=3.5;
:nd

:ov+1;


ijections




:as';
nbus;
p_meas(i,:)==0
    mmp(:,op)=p_meas(i,:);

p_meas(i,:)~=0
mp(:,op)=kjh(:,i)-p_calc(:,i);
.f mmp(:,op)==0
   mmp(:,op)=3.5;
:nd

:op+1;


7e injections
```

```
as';
nbus;
_meas(i,:)~=0
mq(:,oq)=kju(:,i)-q_calc(:,i);
f mmq(:,oq)==0
   mmq(:,oq)=3.5;
nd

if q_meas(i,:)==0
    mmq(:,oq)=q_meas(i,:);
end
oq=oq+1;



*******create A indices**************************
nlines
i);
i);
a)=1;
j=1:nbus
if b==j
    A(i,b)=1;
end
   end


*****************************************************
 flows
w
c
;

j_flow';
nlines;
ij_flow(i,:)~=0
    mmpij(:,opij)=kupij(:,i)-pij_calc(:,i);
    if mmpij(:,opij)==0
        mmpij(:,opij)=3.5;
    end
end
if pij_flow(i,:)==0
    mmpij(:,opij)=pij_flow(i,:);
end
opij=opij+1;

ij;
e ij flows
w
c
;

j_flow';
nlines;
ij_flow(i,:)==0
    mmqij(:,oqij)=qij_flow(i,:);
```

```
if qij_flow(i,:)~=0
    mmqij(:,oqij)=kuqij(:,i)-qij_calc(:,i);
    if mmqij(:,oqij)==0
        mmqij(:,oqij)=3.5;
    end
end
oqij=oqij+1;

[ij;
 flows
'w
c
;

i_flow';
nlines;
'ji_flow(i,:)~=0
mmpji(:,opji)=kupji(:,i)-pji_calc(:,i);
if mmpji(:,opji)==0
        mmpji(:,opji)=3.5;
end

if pji_flow(i,:)==0
    mmpji(:,opji)=pji_flow(i,:);
end
opji=opji+1;

pji;
'e ji flows
w
c
;

i_flow';
nlines;
ji_flow(i,:)==0
    mmqji(:,oqji)=qji_flow(i,:);

if qji_flow(i,:)~=0
    mmqji(:,oqji)=kuqji(:,i)-qji_calc(:,i);
    if mmqji(:,oqji)==0
        mmqji(:,oqji)=3.5;
    end
end
oqji=oqji+1;

qji;

ISMATCH VECTOR
 mmp mmq mmpij mmpji mmqij mmqji]';
' mmp mmq mmpij mmqij]';
ength(mm)

******mmvf**********************
```

```
:length(mmv);
 mmv(i)~=0
ply(rtrt)=mmv(i);
trt=rtrt+1;
.d


;

:length(ply)
ply(i)==3.5
mmvf(ro1)=0;

 ply(i)~=3.5
mmvf(ro1)=ply(i);
d
1=ro1+1;


*****mmpf**********************


:length(mmp);
 mmp(i)~=0
puy(Lrt)=mmp(i);
rt=Lrt+1;
d


;

:length(puy)
puy(i)==3.5
mmpf(ro2)=0;

 puy(i)~=3.5
mmpf(ro2)=puy(i);
d
2=ro2+1;


********mmqf*******************


length(mmq)
 mmq(i)~=0
piy(trew)=mmq(i);
ew=trew+1;
d


;

:length(piy)
piy(i)==3.5
mmqf(ro3)=0;
```

```
: piy(i)~=3.5
mmqf(ro3)=piy(i);
ıd
›3=ro3+1;


*********mmpijf********************


:length(mmpij);
 mmpij(i)~=0
pty(wyet)=mmpij(i);
yet=wyet+1;
d


[];

:length(pty)
pty(i)==3.5
mmpijf(ro4)=0;

 pty(i)~=3.5
mmpijf(ro4)=pty(i);
d
4=ro4+1;


*********mmqijf********************


length(mmqij);
 mmqij(i)~=0
psy(srup)=mmqij(i);
rup=srup+1;
d


[];

:length(psy)
psy(i)==3.5
mmqijf(ro5)=0;

 psy(i)~=3.5
mmqijf(ro5)=psy(i);
d
5=ro5+1;


***********mmpjif*****************


:length(mmpji);
 mmpji(i)~=0
ptyr(wrf)=mmpji(i);
rf=wrf+1;
```

```
.d


[];

:length(ptyr)
ptyr(i)==3.5
mmpjif(ro6)=0;

 ptyr(i)~=3.5
mmpjif(ro6)=ptyr(i);
d
6=ro6+1;


***********mmqjif******************


length(mmqji);
 mmqji(i)~=0
psyr(srf)=mmqji(i);
rf=srf+1;
d


];

:length(psyr)
psyr(i)==3.5
mmqjif(ro7)=0;
 ro7=ro7+1;

 psyr(i)~=3.5
mmqjif(ro7)=psyr(i);
 ro7=ro7+1;
d
7=ro7+1;



f mmpf mmqf mmpijf mmpjif mmqijf mmqjif]'
vf mmpf mmqf mmpijf mmqijf]'
ngth(mmf)



length(v_meas)
_meas(i,:)~=0
ote(rope,:)=(i);
ope=rope+1;



;

length(p_meas)
```

```
 _meas(i,:)~=0
ote1(rope1,:)=(i);
ope1=rope1+1;




;

length(q_meas)
[_meas(i,:)~=0
ote2(rope2,:)=(i);
ope2=rope2+1;





;
];

length(pij_flow)
ij_flow(i,:)~=0
ote3(rope3,:)=x(i);
ote31(rope3,:)=y(i);
ope3=rope3+1;






;
];

length(qij_flow)
ij_flow(i,:)~=0
ote4(rope4,:)=x(i);
ote41(rope4,:)=y(i);
ope4=rope4+1;






;
];

length(pji_flow)
ji_flow(i,:)~=0
ote5(rope5,:)=x(i);
ote51(rope5,:)=y(i);
ope5=rope5+1;





|;
```

```
];

length(qji_flow)
ji_flow(i,:)~=0
ote6(rope6,:)=x(i);
ote61(rope6,:)=y(i);
ope6=rope6+1;




ote',vote1',vote2',vote3',vote4',vote5',vote6']';

ate=zeros(4*nlines+3*nbus,2*nbus-1);
atrixstate(1,:))';
ength(state);

on of the Jacobian
(nbus,2*nbus-1);
(nbus,2*nbus-1);
(nbus,2*nbus-1);
os(nlines,2*nbus-1);
os(nlines,2*nbus-1);
os(nlines,2*nbus-1);
os(nlines,2*nbus-1);

er=[];
nbus
number=[bus_number i];


plete Jacobian H will be an agglomeration of these submatrices
cker=[x';y'];
er;

on of Hv submatrix corresponding to voltage mismatches

nbus
j=1:nstate
 j~=(nbus+i-1)
  Hv(i,j)=0;
end
if j==(nbus+i-1)
    av=v_meas(i,:);
    if av~=0
    Hv(i,j)=1;
    end
    if av==0
    Hv(i,j)=0;
    end
end
d



on of Hp submatrix corresponding to real power injection mismatches
```

```
length(mmpf)
j=1:nbus
if i==j
    Hpd(i,j)=-q_calc3(i)-v_bus(i)*v_bus(i)*B(i,i);
    Hpv(i,j)=(p_calc3(i)+ v_bus(i)*v_bus(i)*G(i,i))/v_bus(i);
end
if j~=i
    Hpd(i,j)=-v_bus(i)*v_bus(j)*mag(i,j)*sind(theta(i,j)+ang_bus(j)-ang_bus(i));
    Hpv(i,j)=v_bus(i)*mag(i,j)*cosd(theta(i,j)+ang_bus(j)-ang_bus(i));
    %Hpd(i,j)=-v_bus(i)*v_bus(j)*(B(i,j)*cos(ang_bus(j)-ang_bus(i))+G(i,j)*sin(an
-ang_bus(i)));
end




l    Hpv];

:Hpt);
/ul);

sum(ar1)==0


ar1)~=0
(:,1)=[];




ion of Hq submatrix corresponding to reactive power injection
:hes


:length(mmqf)
j=1:nbus
if i==j
    Hqd(i,j)=p_calc3(i)- v_bus(i)*v_bus(i)*G(i,i);
    Hqv(i,j)=(q_calc3(i)-v_bus(i)*v_bus(i)*B(i,i))/v_bus(i);
end
if j~=i
    Hqd(i,j)=-v_bus(i)*v_bus(j)*mag(i,j)*cosd(theta(i,j)+ang_bus(j)-ang_bus(i));
    Hqv(i,j)=-v_bus(i)*mag(i,j)*sind(theta(i,j)+ang_bus(j)-ang_bus(i));
end




d    Hqv];

Hqt);
.(yu);
e';
are1)==0
```

```
rel)~=0
:,i)=[];



*****************************create Hpij*******************
ros(nlines,nbus);
nlines
ij_flow(i,:)~=0
Subuh(i,:)=A(i,:);

ij_flow(i,:)==0
Subuh(i,:)=0;




nlines
(i);
(i);
j=1:nbus
if Subuh(i,j)~=0 & j==a2
  Hot(i,j)= v_bus(a2)*v_bus(b2)*mag(a2,b2)*sind(theta(a2,b2)+ang_bus(b2)-ang_bus(

  Hit(i,j)= -2*v_bus(a2)*G(a2,b2)+v_bus(b2)*mag(a2,b2)*cosd(theta(a2,b2)+ang_bus(
bus(a2));
end
if Subuh(i,j)~=0 & j==b2
    Hot(i,j)= -(v_bus(a2)*v_bus(b2)*mag(a2,b2)*sind(theta(a2,b2)+ang_bus(b2)-ang_
);
    Hit(i,j)= v_bus(a2)*mag(a2,b2)*cosd(theta(a2,b2)+ang_bus(b2)-ang_bus(a2));
end
if Subuh(i,j)==0
    Hot(i,j)=0;
    Hit(i,j)=0;
end



=[];

t Hit];

********************create Hpji************************************
ros(nlines,nbus);
nlines
ji_flow(i,:)~=0
r(i,:)=A(i,:);

ji_flow(i,:)==0
Zohor(i,:)=0;




nlines
```

```
:(i);
'(i);
j=1:nbus
if Zohor(i,j)~=0 & j==b4
  Hotr(i,j)= v_bus(a4)*v_bus(b4)*mag(a4,b4)*sind(theta(a4,b4)+ang_bus(a4)-ang_bus ↵

  Hitr(i,j)= -2*v_bus(b4)*G(a4,b4)+v_bus(a4)*mag(a4,b4)*cosd(theta(a4,b4)+ang_bus ↵
_bus(b4));
end
if Zohor(i,j)~=0 & j==a4
    Hotr(i,j)= -(v_bus(a4)*v_bus(b4)*mag(a4,b4)*sind(theta(a4,b4)+ang_bus(a4)-ang ↵
));
    Hitr(i,j)= v_bus(b4)*mag(a4,b4)*cosd(theta(a4,b4)+ang_bus(a4)-ang_bus(b4));
end
if Zohor(i,j)==0
    Hotr(i,j)=0;
    Hitr(i,j)=0;
end



)=[];

tr Hitr];

**********************create Hqij***************************************
os(nlines,nbus);
nlines
ij_flow(i,:)~=0
Asar(i,:)=A(i,:);

ij_flow(i,:)==0
Asar(i,:)=0;




nlines
.(i);
'(i);
j=1:nbus
if Asar(i,j)~=0 & j==a6
  Hotq(i,j)= v_bus(a6)*v_bus(b6)*mag(a6,b6)*cosd(theta(a6,b6)+ang_bus(b6)-ang_bus ↵

  Hitq(i,j)= -2*v_bus(a6)*(Bline(a6,b6)-B(a6,b6))-v_bus(b6)*mag(a6,b6)*sind(theta ↵
ang_bus(b6)-ang_bus(a6));
end
if Asar(i,j)~=0 & j==b6
    Hotq(i,j)= -(v_bus(a6)*v_bus(b6)*mag(a6,b6)*cosd(theta(a6,b6)+ang_bus(b6)-ang ↵
));
    Hitq(i,j)= -v_bus(a6)*mag(a6,b6)*sind(theta(a6,b6)+ang_bus(b6)-ang_bus(a6));
end
if Asar(i,j)==0
    Hotq(i,j)=0;
    Hitq(i,j)=0;
end
```

```
.)=[];

ıtq Hitq];
*******************create Hqji*****************************
:zeros(nlines,nbus);
nlines
:ji_flow(i,:)~=0
Maghrib(i,:)=A(i,:);

:ji_flow(i,:)==0
Maghrib(i,:)=0;




nlines
(i);
·(i);
j=1:nbus
if Maghrib(i,j)~=0 & j==b8
  Hotqr(i,j)= v_bus(a8)*v_bus(b8)*mag(a8,b8)*cosd(theta(a8,b8)+ang_bus(a8)-ang_bu ↵

  Hitqr(i,j)= -2*v_bus(b8)*(Bline(a8,b8)-B(a8,b8))-v_bus(a8)*mag(a8,b8)*sind(thet ↵
+ang_bus(a8)-ang_bus(b8));
end
if Maghrib(i,j)~=0 & j==a8
    Hotqr(i,j)= -(v_bus(a8)*v_bus(b8)*mag(a8,b8)*cosd(theta(a8,b8)+ang_bus(a8)-an ↵
)));
    Hitqr(i,j)= -v_bus(b8)*mag(a8,b8)*sind(theta(a8,b8)+ang_bus(a8)-ang_bus(b8));
end
if Maghrib(i,j)==0
    Hotqr(i,j)=0;
    Hitqr(i,j)=0;
end




1)=[];

tqr Hitqr];

f Jacobian
Hp; Hq; Hpij; Hpji; Hqij; Hqji]
 Hp; Hq; Hpij; Hqij]
1);

length (H1)
(i,:);
1(i,:);
um(h)~=0
Jac(phD,:)=yo;
phD=phD+1;
```

```
**************************************************************************
ze(H);
==1
open('output.txt','w');
tf(fid,'\t\tRESULTS OF POWER STATE ESTIMATION FOR %d-BUS SYSTEMS\n\n',nbus);
tf(fid,'Number of busses = %d.\n',nbus);
tf(fid,'Number of lines = %d.\n',nlines);
tf(fid,'Number of available measurements = %d.\n',nmeas);
tf(fid,'List of available measurements:\n');
=1:length(vote)
ote(i,:)~=0
fprintf(fid,'          V%d\n',vote(i));


length(vote1)
ote1(i,:)~=0
fprintf(fid,'          P%d\n',vote1(i));


length(vote2)
ote2(i,:)~=0
fprintf(fid,'          Q%d\n',vote2(i));


length(vote3)
ote3(i,:)~=0
fprintf(fid,'          P%d-%d\n',vote3(i),vote31(i));


length(vote4)
ote4(i,:)~=0
fprintf(fid,'          Q%d-%d\n',vote4(i),vote41(i));


length(vote5)
ote5(i,:)~=0
fprintf(fid,'          P%d-%d\n',vote51(i),vote5(i));


length(vote6)
ote6(i,:)~=0
fprintf(fid,'          Q%d-%d\n',vote61(i),vote6(i));


fid,'\n\nIteration number %d.\n\n',count);
tf(fid,'State variables\n');
tf(fid,'Bus number  Voltage   Angle(rad)\n');
=1:nbus
tf(fid,'%6.2f      %6.5f   %6.5f\n',bus_number(i),v_bus(i),ang_busr(i));

intf(fid,'The Jacobian\n\n');
r i=1:hsize(1)
rintf(fid,'%10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f\ ↵
));

e(fid);
```

```
>1
fopen('output.txt','a');
ntf(fid,'\nIteration number %d.\n\n',count);
ntf(fid,'State variables\n');
ntf(fid,'Bus number  Voltage   Angle(rad)\n');
i=1:nbus
ntf(fid,'%6.2f      %6.5f   %6.5f\n',bus_number(i),v_bus(i),ang_busr(i));

rintf(fid,'The Jacobian\n\n');
r i=1:hsize(1)
  fprintf(fid,'%10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10
(i,:));
d
se(fid);

nation of weight matrix
wts';
wts';
wts';
pij_wts';
qij_wts';
pji_wts';
qji_wts';



length(wtsvu)
tsvu(i,:)~=0
wtsv(w11,:)=wtsvu(i,:);
w11=w11+1;






nbus
tspu(i,:)~=0
wtsp(w12,:)=wtspu(i,:);
w12=w12+1;






nbus
tsqu(i,:)~=0
wtsq(w13,:)=wtsqu(i,:);
w13=w13+1;




];

nlines
```

```
'tspiju(i,:)~=0
wtspij(w14,:)=wtspiju(i,:);
w14=w14+1;




];

nlines
tsqiju(i,:)~=0
wtsqij(w15,:)=wtsqiju(i,:);
w15=w15+1;




];

nlines
tspjiu(i,:)~=0
wtspji(w16,:)=wtspjiu(i,:);
w16=w16+1;




];

nlines
tsqjiu(i,:)~=0
wtsqji(w17,:)=wtsqjiu(i,:);
w17=w17+1;



v' wtsp' wtsq' wtspij' wtspji' wtsqij' wtsqji']';
length(wts));
length(wts)
j=1:length(wts)
if i==j
    W(i,j)=wts(i,:);
end



nation of state mismatch vector
W*H;
ze(Gain);
pen('output.txt','a');
f(fid,'\n\nThe following is the Gain matrix.\n\n');
1:gsize(1);
rintf(fid,'%10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f %10.2f\
i,:));

(fid);
Gain)*H'*W*mmf
```

```
count+1;

s;
sr=ang_bus*pi/180;
(1:nbus-1,:)

nbus-1
(gaban,:)=smm1(i,:);
n=gaban+1;


(nbus:2*nbus-1,:)
ng_busr;v_bus];
:)=[];
s+smm
(nbus:2*nbus-1);
nbus,v_meas,'c+:',1:nbus,est2,'y-');
nation of tolerance
ta=[];

e;
bs(smm);
length(smm)
mmabs(i,:)>tolerance
plz_giveta(attt,:)=1;
attt=attt+1;


ta;

th(plz_giveta)>0
1;
=v_bus+smmv;
us=1+smmv;
usr=ang_busr+smma;
usr(1)=0;
usr;
us=ang_busr*180/pi;


th(plz_giveta)==0
=0;
fopen('output.txt','a');
ntf(fid,'\n********************************************************************** ↙

ntf(fid,'\n\nCONVERGENCE has occured in %d iterations at tolerance of %d.\n',coun ↙
nce);
ntf(fid,'\nFINAL STATE ESTIMATION WITHOUT BAD DATA DETECTION.\n');
ntf(fid,'\nESTIMATED STATE VARIABLES\n');
ntf(fid,'Bus number  Voltage   Angle(rad) Angle(degree)\n');
i=1:nbus
ntf(fid,'%6.2f      %6.5f     %6.5f     %6.5f\n',bus_number(i),v_bus(i),ang_busr(i) ↙
(i));

s=x;
y;
fid,'\nACTIVE AND REACTIVE POWER ESTIMATED\n');
```

```
fid,'  Bus number        P          Q \n');
i=1:nbus
fid,'   %6.2f        %6.5f     %6.5f\n',bus_number(i),joy(i),jay(i));

fid,'\nACTIVE AND REACTIVE LINE FLOW ESTIMATED\n');
fid,'  From     To       Pij        Qij         Pji         Qji\n');
i=1:length(start_bus)
fid,'%6.2f %6.2f   %6.5f   %6.5f   %6.5f   %6.5f\n',start_bus(i),end_bus(i),et(i)
et2(i),et3(i));

******to check the presences of bad data*******************
i=1:length(mmf)
,:)=mmf(i,:)^2*wts(i,:);

m(error1)
fid,'\n\nWeighted sum of squares of the errors equal to %d.\n',error);
=length(mmf)-nstate;
onfidence==0.1
a=2;

onfidence==0.05
a=3;

onfidence==0.025
a=4;

onfidence==0.01
a=5;

onfidence==0.005
a=6;

2(deg_free,alpha)
fid,'\nChi-square distributed equal to %d.\n',comp);
id);
rror>comp
v(W)-H*inv(Gain)*H';
=1;
for i=1:length(R)
    stand_error(ipoh,:)=mmf(i,:)/sqrt(abs(R(ipoh,ipoh)));
    ipoh=ipoh+1;
end
d_error;
_stand_error=max(stand_error);
ur=1;
for i=1:length(stand_error)
    if stand_error(i,:)==lgst_stand_error;
       nstand_error(syukur,:)=0;
       syukur=syukur+1;
    end
    if stand_error(i,:)~=lgst_stand_error
       nstand_error(syukur,:)=stand_error(i,:);
       syukur=syukur+1;
    end
end
nstand_error;
no_measurement=find(nstand_error==0);
```

```
if nmeas<=nstate
   fid=fopen('output.txt','a');
   fprintf(fid,'\n****************************************************************
***');
   fprintf(fid,'\n\nTo preserve redundancy, bata data will be stop at this level'

   fprintf(fid,'\nsince %d (no of measurement) is less than or equal to %d (no of
.\n',nmeas,nstate);
   fprintf(fid,'\nFINAL STATE ESTIMATION AFTER BAD DATA DETECTION\n');
   fprintf(fid,'\nESTIMATED STATE VARIABLES\n');
   fprintf(fid,'Bus number  Voltage   Angle(rad) Angle(degree)\n');
    for i=1:nbus
   fprintf(fid,'%6.2f      %6.5f    %6.5f    %6.5f\n',bus_number(i),v_bus(i),ang_
ng_busd(i));
    end
   start_bus=x;
   end_bus=y;
   fprintf(fid,'\nACTIVE AND REACTIVE POWER ESTIMATED\n');
   fprintf(fid,'  Bus number       P          Q \n');
    for i=1:nbus
   fprintf(fid,'   %6.2f       %6.5f    %6.5f\n',bus_number(i),joy(i),jay(i));
    end
   fprintf(fid,'\nACTIVE AND REACTIVE LINE FLOW ESTIMATED\n');
   fprintf(fid,'  From    To      Pij       Qij        Pji        Qji\n');
    for i=1:length(start_bus)
   fprintf(fid,'%6.2f %6.2f   %6.5f   %6.5f   %6.5f   %6.5f\n',start_bus(i),end_b
.(i),et1(i),et2(i),et3(i));
    end
   fclose(fid);
end
if nmeas>nstate
   no_meas_to_delete=deg_free-1;
   fid=fopen('output.txt','a');
   fprintf(fid,'\n****************************************************************
·***');
   fprintf(fid,'\n\nBAD MEASUREMENTS HAVE BEEN DETECTED AT THE RAW MEASUREMENTS.\

   fprintf(fid,'\n\nAfter completed the bata data detection process, the total nu
);
   fprintf (fid,'\nmeasurement that possible to discard is %d.\n',no_meas_to_dele

   fprintf(fid,'\n\nAs detected the large standardized error(%d)is associated wit
_stand_error);
   fprintf (fid,'\nmeasurement no %d.\n',no_measurement);
   fclose(fid);
end

?rror<=comp
:id=fopen('output.txt','a');
:printf(fid,'\n****************************************************************
);
:printf(fid,'\nNO BAD DATA WAS DETECTED.\n');
:printf(fid,'\nFINAL STATE ESTIMATION AFTER BAD DATA DETECTION.\n');
:printf(fid,'\nESTIMATED STATE VARIABLES\n');
:printf(fid,'Bus number  Voltage   Angle(rad) Angle(degree)\n');
:or i=1:nbus
:printf(fid,'%6.2f      %6.5f    %6.5f    %6.5f\n',bus_number(i),v_bus(i),ang_busr
```

```
bus(i));
nd
tart_bus=x;
nd_bus=y;
printf(fid,'\nACTIVE AND REACTIVE POWER ESTIMATED\n');
printf(fid,'  Bus number       P          Q \n');
or i=1:nbus
printf(fid,'   %6.2f        %6.5f    %6.5f\n',bus_number(i),joy(i),jay(i));
nd
printf(fid,'\nACTIVE AND REACTIVE LINE FLOW ESTIMATED\n');
printf(fid,'  From    To      Pij       Qij        Pji        Qji\n');
or i=1:length(start_bus)
printf(fid,'%6.2f %6.2f   %6.5f   %6.5f   %6.5f   %6.5f\n',start_bus(i),end_bus(i ↵
et1(i),et2(i),et3(i));
nd
close(fid);
reak;
```

APPENDIX B

MATLAB Result for 2 - Bus System

```
            RESULTS OF POWER STATE ESTIMATION FOR 2-BUS SYSTEMS

Number of busses = 2.
Number of lines = 1.
Number of available measurements = 5.

Measurements   Parameters
     1
     2
     3
     4
     5


Iteration number 0.

State variables
Bus number  Voltage   Angle(rad)
  1.00      1.00000   0.00000
  2.00      1.00000   0.00000

Iteration number 1.

State variables
Bus number  Voltage   Angle(rad)
  1.00      0.99612   0.00000
  2.00      0.97271   -0.15000

Iteration number 2.

State variables
Bus number  Voltage   Angle(rad)
  1.00      0.98999   0.00000
  2.00      0.96468   -0.17167

Iteration number 3.

State variables
Bus number  Voltage   Angle(rad)
  1.00      0.98779   0.00000
  2.00      0.96246   -0.17479

Iteration number 4.

State variables
Bus number  Voltage   Angle(rad)
  1.00      0.98737   0.00000
  2.00      0.96211   -0.17523

************************************************************************
**

CONVERGENCE has occured in 5 iterations at tolerance of 1.000000e-004.

FINAL STATE ESTIMATION WITHOUT BAD DATA DETECTION.

ESTIMATED STATE VARIABLES
Bus number  Voltage   Angle(rad) Angle(degree)
  1.00      0.98737    0.00000    0.00000
  2.00      0.96211    -0.17523    -10.03986
```

ACTIVE AND REACTIVE POWER ESTIMATED

From	To	P	Q
1.00	2.00	0.66244	0.32046

ACTIVE AND REACTIVE LINE FLOW ESTIMATED

From	To	Pij	Qij	Pji	Qji
1.00	2.00	0.66244	0.15798	-0.66244	-0.03905

Weighted sum of squares of the errors equal to 5.440291e+002.

Chi-square distributed equal to 9.210000e+000.

****

BAD MEASUREMENTS HAVE BEEN DETECTED AT THE RAW MEASUREMENTS.

After completed the bata data detection process, the total of measurement
that possible to discard is 1.

As detected the large standardized error(2.333090e+001)is associated with
measurement no 5.

```
             RESULTS OF POWER STATE ESTIMATION FOR 2-BUS SYSTEMS

Number of busses = 2.
Number of lines = 1.
Number of available measurements = 4.

Measurements   Parameters
     1
     2
     3
     4


Iteration number 0.

State variables
Bus number  Voltage    Angle(rad)
  1.00      1.00000    0.00000
  2.00      1.00000    0.00000

Iteration number 1.

State variables
Bus number  Voltage    Angle(rad)
  1.00      1.02354    0.00000
  2.00      0.91673    -0.15000

Iteration number 2.

State variables
Bus number  Voltage    Angle(rad)
  1.00      1.01712    0.00000
  2.00      0.92222    -0.16053

************************************************************************
**

CONVERGENCE has occured in 3 iterations at tolerance of 1.000000e-004.

FINAL STATE ESTIMATION WITHOUT BAD DATA DETECTION.

ESTIMATED STATE VARIABLES
Bus number  Voltage    Angle(rad) Angle(degree)
  1.00      1.01712     0.00000    0.00000
  2.00      0.92222    -0.16053    -9.19779

ACTIVE AND REACTIVE POWER ESTIMATED
  From     To        P         Q
  1.00    2.00    0.59974    0.60674

ACTIVE AND REACTIVE LINE FLOW ESTIMATED
  From     To       Pij        Qij         Pji         Qji
  1.00    2.00    0.59974    0.43432    -0.59974    -0.30182


Weighted sum of squares of the errors equal to 1.403678e-001.

Chi-square distributed equal to 6.640000e+000.

************************************************************************
**
```

NO BAD DATA WAS DETECTED.

FINAL STATE ESTIMATION AFTER BAD DATA DETECTION.

ESTIMATED STATE VARIABLES

Bus number	Voltage	Angle(rad)	Angle(degree)
1.00	1.01712	0.00000	0.00000
2.00	0.92222	-0.16053	-9.19779

ACTIVE AND REACTIVE POWER ESTIMATED

From	To	P	Q
1.00	2.00	0.59974	0.60674

ACTIVE AND REACTIVE LINE FLOW ESTIMATED

From	To	Pij	Qij	Pji	Qji
1.00	2.00	0.59974	0.43432	-0.59974	-0.30182

APPENDIX C

MATLAB Result for 3 - Bus System

RESULTS OF POWER STATE ESTIMATION FOR 3-BUS SYSTEMS

Number of busses = 3.
Number of lines = 3.
Number of available measurements = 7.

Measurements	Parameters
1	
2	
3	
4	
5	
6	
7	

Iteration number 0.

State variables

Bus number	Voltage	Angle(rad)
1.00	1.00000	0.00000
2.00	1.00000	0.00000
3.00	1.00000	0.00000

Iteration number 1.

State variables

Bus number	Voltage	Angle(rad)
1.00	1.01856	0.00000
2.00	1.00533	-0.00952
3.00	0.98610	-0.03078

Iteration number 2.

State variables

Bus number	Voltage	Angle(rad)
1.00	1.01800	0.00000
2.00	1.00543	-0.00927
3.00	0.98654	-0.03071

****
**

CONVERGENCE has occured in 3 iterations at tolerance of 1.000000e-004.

FINAL STATE ESTIMATION WITHOUT BAD DATA DETECTION.

ESTIMATED STATE VARIABLES

Bus number	Voltage	Angle(rad)	Angle(degree)
1.00	1.01800	0.00000	0.00000
2.00	1.00543	-0.00927	-0.53124
3.00	0.98654	-0.03071	-1.75976

ACTIVE AND REACTIVE POWER ESTIMATED

From	To	P	Q
1.00	2.00	0.48041	0.70727
1.00	3.00	0.33043	0.25855
2.00	3.00	-0.81084	-0.75018

ACTIVE AND REACTIVE LINE FLOW ESTIMATED

From	To	Pij	Qij	Pji	Qji

1.00	2.00	0.09490	0.12835	-0.09490	-0.12589
1.00	3.00	0.38551	0.40620	-0.38551	-0.38199
2.00	3.00	0.42533	0.38444	-0.42533	-0.36818

Weighted sum of squares of the errors equal to 8.111939e-001.

Chi-square distributed equal to 9.210000e+000.

****
**

NO BAD DATA WAS DETECTED.

FINAL STATE ESTIMATION AFTER BAD DATA DETECTION.

ESTIMATED STATE VARIABLES

Bus number	Voltage	Angle(rad)	Angle(degree)
1.00	1.01800	0.00000	0.00000
2.00	1.00543	-0.00927	-0.53124
3.00	0.98654	-0.03071	-1.75976

ACTIVE AND REACTIVE POWER ESTIMATED

From	To	P	Q
1.00	2.00	0.48041	0.70727
1.00	3.00	0.33043	0.25855
2.00	3.00	-0.81084	-0.75018

ACTIVE AND REACTIVE LINE FLOW ESTIMATED

From	To	Pij	Qij	Pji	Qji
1.00	2.00	0.09490	0.12835	-0.09490	-0.12589
1.00	3.00	0.38551	0.40620	-0.38551	-0.38199
2.00	3.00	0.42533	0.38444	-0.42533	-0.36818

APPENDIX D

MATLAB Result for IEEE 14 - Bus System

RESULTS OF POWER STATE ESTIMATION FOR 14-BUS SYSTEMS

```
Number of busses = 14.
Number of lines = 20.
Number of available measurements = 75.
List of available measurements:
          V1
          V2
          V3
          V4
          V5
          V6
          V7
          V8
          V9
          V10
          V11
          V12
          V13
          V14
          P1
          P2
          P3
          P4
          P5
          P6
          P9
          P10
          P11
          P12
          P13
          P14
          Q1
          Q2
          Q3
          Q5
          Q6
          Q8
          Q9
          Q10
          Q11
          Q12
          Q13
          Q14
          P1-2
          P1-5
          P2-3
          P2-4
          P2-5
          P3-4
          P4-5
          P4-7
          P4-9
          P5-6
          P6-11
          P6-12
          P6-13
          P7-9
          P9-10
          P9-14
          P10-11
```

```
          P12-13
          P13-14
          Q1-2
          Q1-5
          Q2-3
          Q2-4
          Q2-5
          Q3-4
          Q4-5
          Q4-7
          Q5-6
          Q6-11
          Q6-12
          Q6-13
          Q7-8
          Q7-9
          Q9-14
          Q10-11
          Q12-13
          Q13-14

Iteration number 1.

State variables
Bus number  Voltage   Angle(rad)
  1.00      1.03000   0.00000
  2.00      1.03000   0.00000
  3.00      1.03000   0.00000
  4.00      1.00000   0.00000
  5.00      1.00000   0.00000
  6.00      1.03000   0.00000
  7.00      1.00000   0.00000
  8.00      1.03000   0.00000
  9.00      1.00000   0.00000
 10.00      1.00000   0.00000
 11.00      1.00000   0.00000
 12.00      1.00000   0.00000
 13.00      1.00000   0.00000
 14.00      1.00000   0.00000

Iteration number 2.

State variables
Bus number  Voltage   Angle(rad)
  1.00      1.04318   0.00000
  2.00      1.04061   -0.09461
  3.00      1.01309   -0.23272
  4.00      0.99492   -0.18048
  5.00      0.99305   -0.15508
  6.00      1.02765   -0.26233
  7.00      1.00152   -0.25086
  8.00      1.03094   -0.25258
  9.00      1.00661   -0.26430
 10.00      1.00729   -0.27411
 11.00      1.00963   -0.27524
 12.00      0.98876   -0.27788
 13.00      0.99639   -0.27667
 14.00      0.98075   -0.29410

************************************************************************
```

**

CONVERGENCE has occured in 2 iterations at tolerance of 1.000000e-002.

FINAL STATE ESTIMATION WITHOUT BAD DATA DETECTION.

ESTIMATED STATE VARIABLES

Bus number	Voltage	Angle(rad)	Angle(degree)
1.00	1.04318	0.00000	0.00000
2.00	1.04061	-0.09461	-5.42064
3.00	1.01309	-0.23272	-13.33404
4.00	0.99492	-0.18048	-10.34090
5.00	0.99305	-0.15508	-8.88538
6.00	1.02765	-0.26233	-15.03031
7.00	1.00152	-0.25086	-14.37299
8.00	1.03094	-0.25258	-14.47183
9.00	1.00661	-0.26430	-15.14320
10.00	1.00729	-0.27411	-15.70521
11.00	1.00963	-0.27524	-15.77020
12.00	0.98876	-0.27788	-15.92131
13.00	0.99639	-0.27667	-15.85178
14.00	0.98075	-0.29410	-16.85068

ACTIVE AND REACTIVE POWER ESTIMATED

Bus number	P	Q
1.00	2.33107	-0.38589
2.00	0.15230	0.64561
3.00	-0.92775	0.22344
4.00	-0.36613	-0.01375
5.00	-0.08736	-0.20207
6.00	-0.07953	0.16155
7.00	-0.20735	-0.26265
8.00	-0.00990	0.16851
9.00	-0.04438	0.00887
10.00	-0.10528	0.03614
11.00	-0.09029	-0.03746
12.00	-0.12765	-0.11248
13.00	-0.10857	-0.10655
14.00	-0.18097	-0.04835

ACTIVE AND REACTIVE LINE FLOW ESTIMATED

From	To	Pij	Qij	Pji	Qji
1.00	2.00	1.57590	-0.45407	-1.52733	0.49125
1.00	5.00	0.75517	0.06818	-0.72827	-0.05357
2.00	3.00	0.72852	-0.03207	-0.70401	0.04575
2.00	4.00	0.52959	0.06547	-0.51339	-0.09979
2.00	5.00	0.42152	0.12097	-0.41037	-0.15145
3.00	4.00	-0.22375	0.17769	0.23013	-0.22269
4.00	5.00	-0.57767	0.18906	0.58144	-0.19373
4.00	7.00	0.34045	-0.01990	-0.34045	0.04408
4.00	9.00	0.15435	-0.01494	-0.15435	0.02814
5.00	6.00	0.46984	-0.12258	-0.46984	0.17817
6.00	11.00	0.09052	0.05228	-0.08959	-0.05021
6.00	12.00	0.10876	0.10398	-0.10619	-0.09840
6.00	13.00	0.19104	0.14508	-0.18723	-0.13799
7.00	8.00	0.00990	-0.16368	-0.00990	0.16851
7.00	9.00	0.12320	-0.04558	-0.12320	0.04747
9.00	10.00	0.10640	-0.04780	-0.10599	0.04887
9.00	14.00	0.12677	0.03701	-0.12453	-0.03236
10.00	11.00	0.00071	-0.01273	-0.00070	0.01276
12.00	13.00	-0.02146	-0.01408	0.02161	0.01421

13.00 14.00 0.05705 0.01724 -0.05644 -0.01598

Weighted sum of squares of the errors equal to 1.305077e+002.

Chi-square distributed equal to 7.368260e+001.

****
**

BAD MEASUREMENTS HAVE BEEN DETECTED AT THE RAW MEASUREMENTS.

After completed the bata data detection process, the total number of measurement that possible to discard is 47.

As detected the large standardized error(3.136220e-001)is associated with
measurement no 30.

RESULTS OF POWER STATE ESTIMATION FOR 14-BUS SYSTEMS

```
Number of busses = 14.
Number of lines = 20.
Number of available measurements = 66.
List of available measurements:
          V1
          V2
          V3
          V4
          V5
          V6
          V7
          V8
          V9
          V10
          V11
          V12
          V13
          V14
          P1
          P3
          P4
          P5
          P9
          P13
          P14
          Q1
          Q2
          Q3
          Q6
          Q8
          Q9
          Q10
          Q12
          Q13
          Q14
          P1-2
          P1-5
          P2-3
          P2-4
          P2-5
          P3-4
          P4-5
          P4-7
          P4-9
          P5-6
          P6-11
          P6-12
          P6-13
          P7-9
          P9-14
          P10-11
          P12-13
          P13-14
          Q1-2
          Q1-5
          Q2-3
          Q2-4
          Q2-5
          Q3-4
```

```
        Q4-5
        Q4-7
        Q5-6
        Q6-11
        Q6-12
        Q6-13
        Q7-8
        Q9-14
        Q10-11
        Q12-13
        Q13-14

Iteration number 1.

State variables
Bus number   Voltage    Angle(rad)
  1.00       1.03000    0.00000
  2.00       1.03000    0.00000
  3.00       1.03000    0.00000
  4.00       1.00000    0.00000
  5.00       1.00000    0.00000
  6.00       1.03000    0.00000
  7.00       1.00000    0.00000
  8.00       1.03000    0.00000
  9.00       1.00000    0.00000
 10.00       1.00000    0.00000
 11.00       1.00000    0.00000
 12.00       1.00000    0.00000
 13.00       1.00000    0.00000
 14.00       1.00000    0.00000

Iteration number 2.

State variables
Bus number   Voltage    Angle(rad)
  1.00       1.03390    0.00000
  2.00       1.02743    -0.10489
  3.00       1.01818    -0.18447
  4.00       0.98658    -0.15835
  5.00       0.98812    -0.14295
  6.00       1.03797    -0.24026
  7.00       1.00198    -0.24037
  8.00       1.03787    -0.23680
  9.00       1.00883    -0.26996
 10.00       1.01173    -0.25726
 11.00       1.01991    -0.25174
 12.00       1.01822    -0.25661
 13.00       1.01325    -0.25845
 14.00       0.99506    -0.28707

Iteration number 3.

State variables
Bus number   Voltage    Angle(rad)
  1.00       1.01515    0.00000
  2.00       1.01235    -0.11133
  3.00       0.99956    -0.22695
  4.00       0.98370    -0.15995
  5.00       0.98353    -0.13772
  6.00       1.03261    -0.25560
```

7.00	1.00790	-0.24088
8.00	1.04075	-0.23903
9.00	1.01864	-0.27015
10.00	1.01529	-0.27598
11.00	1.02722	-0.27198
12.00	1.02871	-0.27756
13.00	1.01936	-0.27720
14.00	1.00364	-0.29381

Iteration number 4.

State variables

Bus number	Voltage	Angle(rad)
1.00	1.00147	0.00000
2.00	0.99899	-0.11198
3.00	0.97253	-0.26807
4.00	0.97388	-0.17486
5.00	0.97260	-0.14582
6.00	1.01786	-0.26536
7.00	0.99232	-0.24864
8.00	1.01653	-0.23856
9.00	1.00451	-0.27928
10.00	1.00124	-0.29093
11.00	1.01973	-0.28962
12.00	1.02695	-0.29815
13.00	1.01345	-0.29470
14.00	0.99186	-0.30589

Iteration number 5.

State variables

Bus number	Voltage	Angle(rad)
1.00	0.99348	0.00000
2.00	0.99192	-0.10842
3.00	0.95442	-0.29665
4.00	0.96855	-0.19504
5.00	0.96530	-0.16091
6.00	1.00401	-0.27399
7.00	0.97186	-0.25777
8.00	0.98431	-0.23519
9.00	0.98250	-0.28974
10.00	0.98319	-0.30242
11.00	1.00903	-0.30412
12.00	1.02141	-0.31535
13.00	1.00488	-0.30971
14.00	0.97536	-0.31879

Iteration number 6.

State variables

Bus number	Voltage	Angle(rad)
1.00	0.99309	0.00000
2.00	0.99262	-0.10269
3.00	0.94981	-0.30553
4.00	0.97054	-0.21292
5.00	0.96498	-0.17667
6.00	0.99611	-0.27935
7.00	0.95312	-0.26556
8.00	0.95542	-0.23235
9.00	0.95850	-0.29824
10.00	0.96620	-0.30660

11.00	0.99835	-0.31106
12.00	1.01390	-0.32427
13.00	0.99653	-0.31789
14.00	0.95891	-0.32854

****
**

CONVERGENCE has occured in 6 iterations at tolerance of 1.000000e-002.

FINAL STATE ESTIMATION WITHOUT BAD DATA DETECTION.

ESTIMATED STATE VARIABLES

Bus number	Voltage	Angle(rad)	Angle(degree)
1.00	0.99309	0.00000	0.00000
2.00	0.99262	-0.10269	-5.88383
3.00	0.94981	-0.30553	-17.50578
4.00	0.97054	-0.21292	-12.19964
5.00	0.96498	-0.17667	-10.12243
6.00	0.99611	-0.27935	-16.00582
7.00	0.95312	-0.26556	-15.21554
8.00	0.95542	-0.23235	-13.31242
9.00	0.95850	-0.29824	-17.08789
10.00	0.96620	-0.30660	-17.56711
11.00	0.99835	-0.31106	-17.82228
12.00	1.01390	-0.32427	-18.57948
13.00	0.99653	-0.31789	-18.21397
14.00	0.95891	-0.32854	-18.82374

ACTIVE AND REACTIVE POWER ESTIMATED

Bus number	P	Q
1.00	2.31414	-0.49930
2.00	0.47637	0.47481
3.00	-1.38027	0.15724
4.00	-0.45817	0.57236
5.00	0.03984	-0.08391
6.00	0.05253	-0.44992
7.00	-0.13315	-0.21429
8.00	0.16801	0.01499
9.00	-0.27883	-0.16701
10.00	-0.09344	-0.03324
11.00	-0.08471	0.22200
12.00	-0.08661	0.18272
13.00	-0.18718	0.14987
14.00	-0.14649	-0.02909

ACTIVE AND REACTIVE LINE FLOW ESTIMATED

From	To	Pij	Qij	Pji	Qji
1.00	2.00	1.54409	-0.46970	-1.49215	0.52692
1.00	5.00	0.77005	-0.02960	-0.73997	0.06609
2.00	3.00	0.96653	0.02811	-0.91890	0.08695
2.00	4.00	0.57618	-0.07722	-0.55588	0.06104
2.00	5.00	0.42582	-0.00300	-0.41473	-0.02309
3.00	4.00	-0.46137	0.07030	0.47862	-0.08372
4.00	5.00	-0.76334	0.33187	0.77076	-0.32092
4.00	7.00	0.23649	0.08839	-0.23649	-0.07446
4.00	9.00	0.14593	0.02775	-0.14593	-0.01502
5.00	6.00	0.42378	-0.10745	-0.42378	0.15518
6.00	11.00	0.12783	-0.06618	-0.12595	0.07036
6.00	12.00	0.11936	-0.11931	-0.11591	0.12678
6.00	13.00	0.22912	-0.12087	-0.22439	0.12966

7.00	8.00	-0.16801	-0.00938	0.16801	0.01499
7.00	9.00	0.27135	-0.04217	-0.27135	0.05130
9.00	10.00	0.05470	-0.11234	-0.05419	0.11370
9.00	14.00	0.08375	-0.04021	-0.08253	0.04275
10.00	11.00	-0.03926	-0.14693	0.04124	0.15164
12.00	13.00	0.02930	0.05594	-0.02845	-0.05516
13.00	14.00	0.06566	0.07537	-0.06395	-0.07185

Weighted sum of squares of the errors equal to 9.541354e-001.

Chi-square distributed equal to 6.242810e+001.

****
**
NO BAD DATA WAS DETECTED.

FINAL STATE ESTIMATION AFTER BAD DATA DETECTION.

ESTIMATED STATE VARIABLES

Bus number	Voltage	Angle(rad)	Angle(degree)
1.00	0.99309	0.00000	0.00000
2.00	0.99262	-0.10269	-5.88383
3.00	0.94981	-0.30553	-17.50578
4.00	0.97054	-0.21292	-12.19964
5.00	0.96498	-0.17667	-10.12243
6.00	0.99611	-0.27935	-16.00582
7.00	0.95312	-0.26556	-15.21554
8.00	0.95542	-0.23235	-13.31242
9.00	0.95850	-0.29824	-17.08789
10.00	0.96620	-0.30660	-17.56711
11.00	0.99835	-0.31106	-17.82228
12.00	1.01390	-0.32427	-18.57948
13.00	0.99653	-0.31789	-18.21397
14.00	0.95891	-0.32854	-18.82374

ACTIVE AND REACTIVE POWER ESTIMATED

Bus number	P	Q
1.00	2.31414	-0.49930
2.00	0.47637	0.47481
3.00	-1.38027	0.15724
4.00	-0.45817	0.57236
5.00	0.03984	-0.08391
6.00	0.05253	-0.44992
7.00	-0.13315	-0.21429
8.00	0.16801	0.01499
9.00	-0.27883	-0.16701
10.00	-0.09344	-0.03324
11.00	-0.08471	0.22200
12.00	-0.08661	0.18272
13.00	-0.18718	0.14987
14.00	-0.14649	-0.02909

ACTIVE AND REACTIVE LINE FLOW ESTIMATED

From	To	Pij	Qij	Pji	Qji
1.00	2.00	1.54409	-0.46970	-1.49215	0.52692
1.00	5.00	0.77005	-0.02960	-0.73997	0.06609
2.00	3.00	0.96653	0.02811	-0.91890	0.08695
2.00	4.00	0.57618	-0.07722	-0.55588	0.06104
2.00	5.00	0.42582	-0.00300	-0.41473	-0.02309
3.00	4.00	-0.46137	0.07030	0.47862	-0.08372

4.00	5.00	-0.76334	0.33187	0.77076	-0.32092
4.00	7.00	0.23649	0.08839	-0.23649	-0.07446
4.00	9.00	0.14593	0.02775	-0.14593	-0.01502
5.00	6.00	0.42378	-0.10745	-0.42378	0.15518
6.00	11.00	0.12783	-0.06618	-0.12595	0.07036
6.00	12.00	0.11936	-0.11931	-0.11591	0.12678
6.00	13.00	0.22912	-0.12087	-0.22439	0.12966
7.00	8.00	-0.16801	-0.00938	0.16801	0.01499
7.00	9.00	0.27135	-0.04217	-0.27135	0.05130
9.00	10.00	0.05470	-0.11234	-0.05419	0.11370
9.00	14.00	0.08375	-0.04021	-0.08253	0.04275
10.00	11.00	-0.03926	-0.14693	0.04124	0.15164
12.00	13.00	0.02930	0.05594	-0.02845	-0.05516
13.00	14.00	0.06566	0.07537	-0.06395	-0.07185

APPENDIX E

State Estimation Data for 2 - Bus System

1. Busdata.txt

```
0.00   1.0   0.00    1.02   0.00   0.605
1.00   1.0   0.00    0.92   0.00   0.000
```

2. Busweightsdata.txt

```
10000 0 2500   10000   0  0
```

3. linedata.txt

```
1.00   2.00   0.00   0.25   0.00   1.00   0.60   0.00 0.00
0.305
```

4. lineweightsdata.txt

```
4444.444 0 0 2500
```

5. shuntdata.txt

```
no shunt data
```

APPENDIX F

State Estimation Data for 3 Test Bus System

1. Busdata.txt

```
0.00    1.0    0.00  1.01    0.48    0.000
1.00    1.0    0.00  1.02    0.33    0.000
1.00    1.0    0.00  0.98    0.00    0.000
```

2. Busweightsdata.txt

```
1  1  0   1  1  0  1  0  0
```

3. linedata.txt

```
1.00    2.00    0.00    0.1    0.00    1.00    0.00    0.00 0.00 0.00
1.00    3.00    0.00    0.08   0.00    1.00    0.41    0.00 0.00 0.00
2.00    3.00    0.00    0.05   0.00    1.00    0.38    0.00 0.00 0.00
```

4. lineweightsdata.txt

```
0 0 0 0 400 0 0 0 400 0 0 0
```

5. shuntdata.txt

```
no shunt data
```

APPENDIX G

Standard Chi-Square Distributed

1	2.7055	3.8415	5.0239	6.6349	7.8794
2	4.6052	5.9915	7.3778	9.2104	10.5965
3	6.2514	7.8147	9.3484	11.3449	12.8381
4	7.7794	9.4877	11.1433	13.2767	14.8602
5	9.2363	11.0705	12.8325	15.0863	16.7496
6	10.6446	12.5916	14.4494	16.8119	18.5475
7	12.017	14.0671	16.0128	18.4753	20.2777
8	13.3616	15.5073	17.5345	20.0902	21.9549
9	14.6837	16.919	19.0228	21.666	23.5893
10	15.9872	18.307	20.4832	23.2093	25.1881
11	17.275	19.6752	21.92	24.725	26.7569
12	18.5493	21.0261	23.3367	26.217	28.2997
13	19.8119	22.362	24.7356	27.6882	29.8193
14	21.0641	23.6848	26.1189	29.1412	31.3194
15	22.3071	24.9958	27.4884	30.578	32.8015
16	23.5418	26.2962	28.8453	31.9999	34.2671
17	24.769	27.5871	30.191	33.4087	35.7184
18	25.9894	28.8693	31.5264	34.8052	37.1564
19	27.2036	30.1435	32.8523	36.1908	38.5821
20	28.412	31.4104	34.1696	37.5663	39.9969
21	29.6151	32.6706	35.4789	38.9322	41.4009
22	30.8133	33.9245	36.7807	40.2894	42.7957
23	32.0069	35.1725	38.0756	41.6383	44.1814
24	33.1962	36.415	39.3641	42.9798	45.5584
25	34.3816	37.6525	40.6465	44.314	46.928
26	35.5632	38.8851	41.9231	45.6416	48.2898
27	36.7412	40.1133	43.1945	46.9628	49.645
28	37.9159	41.3372	44.4608	48.2782	50.9936
29	39.0875	42.5569	45.7223	49.5878	52.3355
30	40.256	43.773	46.9792	50.8922	53.6719
31	41.4217	44.9853	48.2319	52.1914	55.0025
32	42.5847	46.1942	49.4804	53.4857	56.328
33	43.7452	47.3999	50.7251	54.7754	57.6483
34	44.9032	48.6024	51.966	56.0609	58.9637
35	46.0588	49.8018	53.2033	57.342	60.2746
36	47.2122	50.9985	54.4373	58.6192	61.5811
37	48.3634	52.1923	55.668	59.8926	62.8832
38	49.5126	53.3835	56.8955	61.162	64.1812
39	50.6598	54.5722	58.1201	62.4281	65.4753
40	51.805	55.7585	59.3417	63.6908	66.766
41	52.9485	56.9424	60.5606	64.95	68.0526
42	54.0902	58.124	61.7767	66.2063	69.336
43	55.2302	59.3035	62.9903	67.4593	70.6157
44	56.3685	60.4809	64.2014	68.7096	71.8923
45	57.5053	61.6562	65.4101	69.9569	73.166

46	58.6405	62.8296	66.6165	71.2015	74.4367
47	59.7743	64.0011	67.8206	72.4432	75.7039
48	60.9066	65.1708	69.0226	73.6826	76.9689
49	62.0375	66.3387	70.2224	74.9194	78.2306
50	63.1671	67.5048	71.4202	76.1538	79.4898
51	64.2954	68.6693	72.616	77.386	80.7465
52	65.4224	69.8322	73.8099	78.6156	82.0006
53	66.5482	70.9934	75.0019	79.8434	83.2525
54	67.6728	72.1532	76.1921	81.0688	84.5018
55	68.7962	73.3115	77.3804	82.292	85.7491
56	69.9185	74.4683	78.5671	83.5136	86.994
57	71.0397	75.6237	79.7522	84.7327	88.2366
58	72.1598	76.7778	80.9356	85.9501	89.477
59	73.2789	77.9305	82.1174	87.1658	90.7153
60	74.397	79.082	83.2977	88.3794	91.9518
61	75.5141	80.2321	84.4764	89.5912	93.1862
62	76.6302	81.381	85.6537	90.8015	94.4185
63	77.7454	82.5287	86.8296	92.0099	95.6492
64	78.8597	83.6752	88.004	93.2167	96.8779
65	79.973	84.8206	89.1772	94.422	98.1049
66	81.0855	85.9649	90.3488	95.6256	99.3303
67	82.1971	87.108	91.5193	96.8277	100.5538
68	83.3079	88.2502	92.6885	98.0283	101.7757
69	84.4179	89.3912	93.8565	99.2274	102.9961
70	85.527	90.5313	95.0231	100.4251	104.2148
71	86.6354	91.6703	96.1887	101.6214	105.4323
72	87.7431	92.8083	97.353	102.8163	106.6473
73	88.8499	93.9453	98.5162	104.0098	107.8619
74	89.9561	95.0815	99.6784	105.2019	109.0742
75	91.0615	96.2167	100.8393	106.3929	110.2854
76	92.1662	97.351	101.9992	107.5824	111.4954
77	93.2702	98.4844	103.1581	108.7709	112.7037
78	94.3735	99.617	104.3159	109.9582	113.9107
79	95.4762	100.7486	105.4727	111.144	115.1163
80	96.5782	101.8795	106.6285	112.3288	116.3209
81	97.6796	103.0095	107.7834	113.5123	117.524
82	98.7803	104.1387	108.9373	114.6948	118.7261
83	99.8805	105.2672	110.0902	115.8762	119.927
84	100.98	106.3949	111.2422	117.0566	121.1262
85	102.0789	107.5217	112.3933	118.2356	122.3244
86	103.1773	108.6479	113.5436	119.4137	123.5218
87	104.275	109.7733	114.6929	120.5909	124.7176
88	105.3723	110.898	115.8415	121.7672	125.9123
89	106.4689	112.022	116.989	122.9422	127.106
90	107.565	113.1452	118.1359	124.1162	128.2987
91	108.6606	114.2679	119.282	125.2893	129.4902
92	109.7556	115.3898	120.427	126.4616	130.6812
93	110.8501	116.511	121.5714	127.633	131.8705
94	111.9442	117.6317	122.7152	128.8032	133.0589
95	113.0377	118.7516	123.858	129.9725	134.2466
96	114.1307	119.8709	125.0001	131.1411	135.4327
97	115.2232	120.9897	126.1414	132.3089	136.6188

98	116.3153	122.1077	127.2821	133.4756	137.803
99	117.4069	123.2252	128.4219	134.6415	138.9869
100	118.498	124.3421	129.5613	135.8069	140.1697