

**SYSTEM IDENTIFICATION APPLIED ON VARIOUS FORMS OF
CONTROL TANK SYSTEM**

By

ALWIN PRAKASH MICHAEL

FINAL REPORT

Submitted to the Department of Electrical & Electronics Engineering
in Partial Fulfillment of the Requirements
for the Degree
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Universiti Teknologi Petronas
Bandar Seri Iskandar
31750 Tronoh
Perak Darul Ridzuan

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CERTIFICATION OF APPROVAL

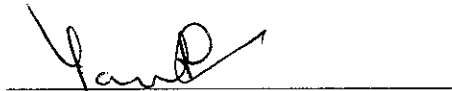
SYSTEM IDENTIFICATION APPLIED ON VARIOUS FORMS OF
CONTROL-TANK SYSTEM

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Alwin Prakash Michael

A project dissertation submitted to the
Electrical & Electronics Engineering Programme
Universiti Teknologi PETRONAS
in partial fulfilment of the requirement for the
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Approved by,



Dr. Vijanth Sagayan Asirvadam
Project Supervisor

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

December 2007

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.



Alwin Prakash s/o Michael Thevadass

ABSTRACT

Control Tanks are widely used and are considered as general equipment in the industry today. Therefore, it is required to have a good control system or control analysis specifically for the system. For any control analysis, it is desirable to be able to obtain a model of the tank system to allow complete off-line analysis with minimum interference to the process. This paper concerns the system identification of various form of control tank system used in the industry today. It simplifies the approach to identification and presents the use of analysis in a more readable, understandable and general manner. Through mathematical modelling, MATLAB simulation and technical computing the whole system can be characterized based on the relevant parameters for a variety of tank formations. This method presents the user to identify and know the system characteristic of the control tank system through simulation study and hence, help to design the best possible operating method to be used in real industry application. Through this simulation study, each type of system can be characterized individually and independently based on the flow rate and the height of the tank. Initially through mathematical derivation, the modelling is then performed and simulated using the Simulink application in MATLAB. An important stage in system identification process is the selection of the type and characteristics of each type of the tank system and this is the subject of this paper.

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CHAPTER 1

INTRODUCTION

A general overview of control tank system is introduced in this chapter and also the background of system identification and its history. This chapter also discusses about the difficulty in determining the system identification model based on the specification that is being normally used in the industry . To overcome the problem several solutions have been discussed in this chapter and most of the constant are made based on mere assumption. And lastly, this chapter discusses the project objective and scope.

1.1 Background of Study

Tanks or vessels are considered as a storage area or equipment, whereby, the products, normally fluid are collected in or being processed in. It can be said that in each and every industry, storage tanks can be found in their respective plant location for various purpose. In the industry today, there are various different formation of tank system, or, in this case, control-tank system, that can be found. Each formation and characteristic hook-up has different purpose and gives different feedback results based on the control element and the installed architecture.

Therefore, a tedious and tiring trip to the field to check upon the current operating condition is not necessary if a person or an engineer wants to know the characteristic and the response of the tank system that is being employed. The whole system can actually be identified through simple modelling methods. This is the main aim and objective of this project, which is to identify the characteristic applied on various forms of control-tank system through modelling and simulations or in other words off-line modelling.

Initially it involves deriving differential equations for the each type of the tank formation that is being studied upon. The second stage involves designing these various nonlinear mathematical process tank models into Matlab Simulink diagram. In addition to that, each formation of tanks will also be modelled using the model structure of ARX. This study will look at both controlling fluid level at the specific tank structure and also the liquid flow rate in and out of the tank.

1.1.1 Overview of System Identification

System identification is a general term to describe mathematical tools and algorithms that build dynamical models from measured data. A dynamical mathematical model in this context is a mathematical description of the dynamic behaviour of a system or process [4]. Examples include: physical processes such as the movement of a falling body under the influence of gravity; economic processes such as stock markets that react to external influences. [5]

1.1.1.1 History of System Identification

The development of identification theory in the control literature followed on the Heels of the development of model-based control design around 1960. Up until the late 1950's, much of control design relied on Bode, Nyquist and Nichols charts, or on step response analyses. These techniques were limited to control design for single-input single-output (SISO) systems. Around 1960, Kalman introduced the state-space representation and laid the foundations for state-space based optimal filtering and optimal control theory, with Linear Quadratic (LQ) optimal control as a cornerstone for model-based control design [7].

1.1.1.2 Case Study on Lennart Ljung

Lennart Ljung is a person who has contributed vastly to the development of system identification and produced some creative moves in this field. Ljung has been responsible for some key and judicious trajectory changes in the development of system identification theory, which have profoundly altered our ways of thinking about the topic. These are the following contributions that he had made to this field [12].

- Introduction of a solid identification framework with a clear separation between the parametric model, seen as a way of computing predictions, and a criterion, whose minimization yields an estimated model.
- Changes from properties of parameter estimates to properties of transfer function estimates, with the introduction of the concept of bias and variance of transfer function estimates. A critical feature here was the introduction of θ^* , Ljung's baby [5].
- Formulation of system identification as a design problem, with the introduction of the objective as a weighting in the identification criterion.
- The parallel development of the theory and of the identification toolbox. The impact of this effort on the spread of system identification in academia and industry has been immense[5].

1.2 Problem Statement

Generally, in process plants, the control-tank system are operated in the optimum or ideal range through various parameters. This optimum or ideal range refers to the safe or normal operating range. Normally, this range will be so small that the system

itself is not used to its full capacity. This is because to avoid any unwanted consequences and afraid that it may lead to further damage.

Through this simulation and modelling study, it can be seen clearly what will happen to the overall system when the relevant parameters are changed accordingly with respect to time. Through this, we can clearly see the total operating range and then see the effects of each tank formation under various operating condition. We can also be able to see the consequence of having the system in abnormal condition.

This will then be helpful to the engineers to have a clearer picture on the process flow and then determine the best operating condition or range that the system can overcome without any damage detected.

1.3 Objectives and Scope of Project

The main objective of this project is to be able to characterize the control-tank system in simulation using the Simulink application. Therefore, in event that the flow rate in and out of the system varies, based on mathematical modelling and derivation, the height of the tank can be easily identified. Using the results obtained, it benefits for the better monitoring of the particular tank-system.

The deliverables of this project includes a simulative study that requires the understanding of process control. This project may require the knowledge in both MATLAB and S-function. As for the initial stage, the mathematical expression that is involved in the project duration has been identified and verified. For this purpose, knowledge and understanding in Differential Equations were required, which involves the Laplace Function and Time Differentiation.

In short, it can be said that this project involves more modelling the overall system in Simulink MATLAB based on the given parameters.

Basically in real life application, where control-tank system is being described there are plenty of information / parameters that have to be taken into account, which are as below:

1. Height of the Tank
2. Flow Rate; In & Out
3. Process Fluid Characteristic
4. Temperature
5. Pressure
6. Maintenance/Reliability

But as for modelling the system in this project, we will only be dealing with two parameters, which are:

1. Height of the Tank
2. Flow Rate; In & Out

The other parameters will be assumed constant or fixed with a variable k , for instance. The modelling will only be done based on the two variable characteristic as above. The modelling applies to all the different types of control-tank system hook-up.

CHAPTER 2

LITERATURE REVIEW

2.1 What is System Identification?

A fundamental concept in science and technology is that of mathematical modelling. A mathematical model is very useful, as well as very compact way, of describing the knowledge we have about a process or system [8]. The determination of a mathematical model of a process or system is known as system identification. In control systems, a mathematical model of a process or system is in most cases necessary for the design of a controller. Most of the controller design based methods are based on the model of the system under control, which is known through system identification method.[5]

This system identification method reduces the development of methods which gives a good estimate of the parameters of the system model. This system refers to the mathematical expression that appropriately relates the physical system quantities to the system components. This mathematical relation constitutes the mathematical model of the whole system and further used for the modelling application. [9]

2.1.1 Model Structures in System Identification

The model-based control design process involves modelling the plant to be controlled, analyzing and synthesizing a controller for the plant, simulating the plant and controller, and deploying the controller [7]. Unlike modelling from first principles, which requires an in-depth knowledge of the system under

consideration, system identification methods can handle a wide range of system dynamics without knowledge of the actual system physics.

A variety of model structures are available to assist in modelling a system. The choice of model structure is based upon an understanding of the system identification method and insight and understanding into the system undergoing identification. Below are the most common methods used and which will be used for this project:

2.1.1.1 Parametric Model Structures

Parametric models describe systems in terms of differential equations and transfer functions. This provides insight into the system physics and a compact model structure. Generally, a system can be described using the following equation, which is known as the general-linear polynomial model or the general-linear model [4].

$$y(n) = q^{-k}G(q^{-1}, \theta)u(n) + H(q^{-1}, \theta)e(n)$$

where,

$u(n)$ and $y(n)$ are the input and output of the system respectively, and $e(n)$ is zero-mean white noise, or the disturbance of the system.

The general-linear model structure, shown in Figure 1, provides flexibility for both the system dynamics and stochastic dynamics. However, a nonlinear optimization method computes the estimation of the general-linear model. This method requires intensive computation with no guarantee of global convergence. [3]

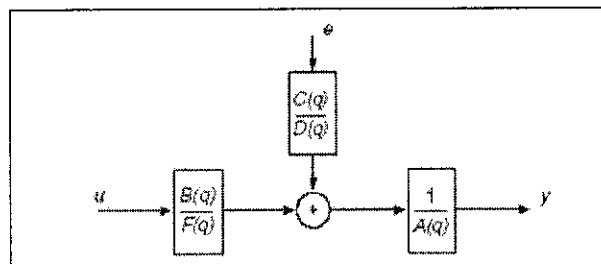


Figure 1 General Linear Model Structure

2.1.1.2 AR Model

The AR model structure is a process model used in the generation of models where outputs are only dependent on previous outputs. No system inputs or disturbances are used in the modelling. This is a very simple model that is limited in the class of problems it can solve. Strictly speaking this means that the AR model structure is the model for a signal, not a system. Time series analyses, such as linear prediction coding commonly use the AR model. [3]

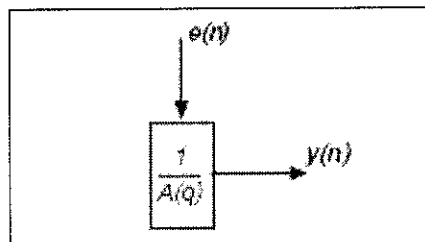


Figure 2 AR Model Structure

2.1.1.3 ARX Model

The ARX model, shown in Figure 3, is the simplest model incorporating the stimulus signal. The estimation of the ARX model is the most efficient of the polynomial estimation methods because it is the result of solving linear regression equations in analytic form. Moreover, the solution is unique. In other words, the solution always satisfies the global minimum of the loss function. The ARX model therefore is preferable, especially when the model order is high [3].

The disadvantage of the ARX model is that disturbances are part of the system dynamics. [2]

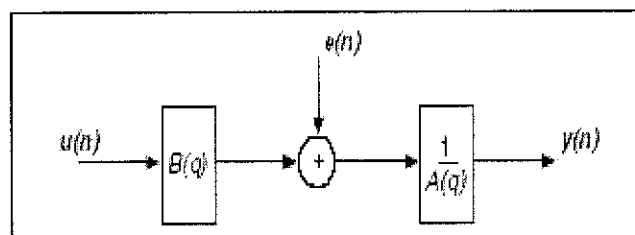


Figure 3 ARX Model Structure

2.1.1.4 ARMAX Model

The ARMAX model structure includes disturbance dynamics. ARMAX models are useful when there are dominating disturbances that enter early in the process, such as at the input. For example, a wind gust affecting an aircraft is a dominating disturbance early in the process. It has better flexibility in handling disturbances than the ARX Model. [3]

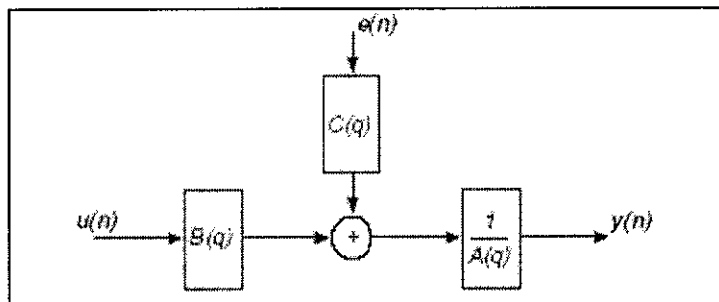


Figure 4 ARMAX Model Structure

2.2 What are Control Tanks?

Generally, tanks are characterized as vessels, where large process related fluids are being stored. This is not false, but partially true. However, there are various types of tanks and vessels in the market today, which are used for various purpose. These tanks also have different working condition depending on the application in the industry. Some tanks are custom made to withstand high temperature and pressure.

Normally, in industry these control tanks are not a single independent system. There will be a few control tanks which are connected to each other in one system. Therefore a single change in one tank will change the overall characteristic of the system.

2.2.1 Types of Control Tanks

- *Storage Tanks, Vessels, Silos (non-pressurized)*

These types of tanks are commonly used to store economically large quantities of liquid and pasty products on a long-term basis utilizing vessels made of corrosion resistant materials.

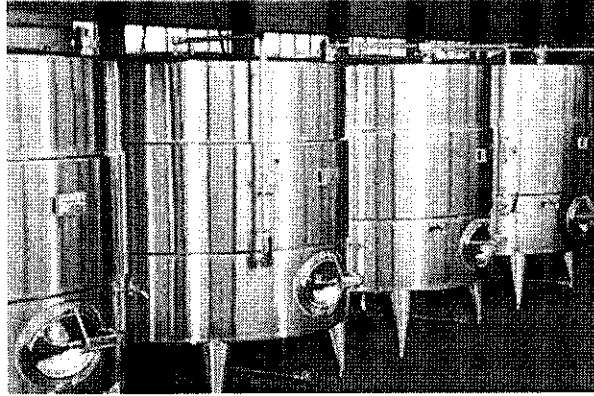


Figure 5 Storage Tanks

- *Pressure Tanks/Vessels*

Products, which due to biological or chemical processes and emissions cannot be stored in open storage vessels, or only for a short time, are affected under pressure, partly with overlay of inert gases, even during a longer period.

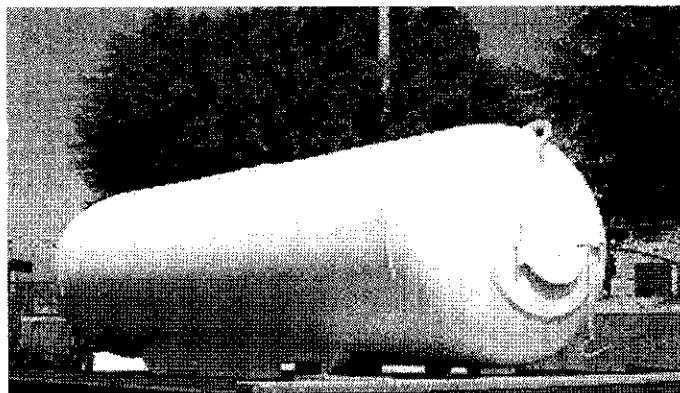


Figure 6 Pressure Tank

- *Process Tanks/Vessels*

Pressurized and non-pressurized tanks or vacuum vessels are utilized to process products according to the application required. Some process tanks

are called reactors and some are just neutralizing basins. It differs based on the application, requirement, operational conditions and sequences.

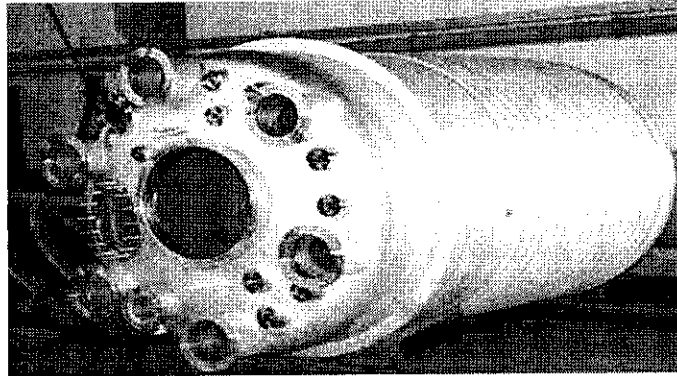


Figure 7 Process Tanks

- *Mixing Tank/Vessels*

The tanks/vessels are of type that used for special purpose like horizontal and vertical mixing, and special apparatus for the processing of liquid and free flowing products

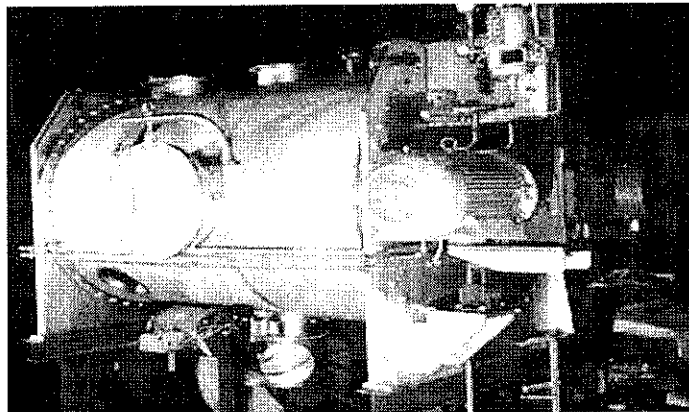


Figure 8 Mixing Tank/Vessels

CHAPTER 3

METHODOLOGY

This chapter describes the methodologies used to implement this project, problem encountered in the design, and solution to those problems. This chapter starts with the defining the goal of the project or project preparation, engineering tools used. Every steps used in designing the project is explained. The detail of chapter 3 is as below:

3.1 Procedures Identification

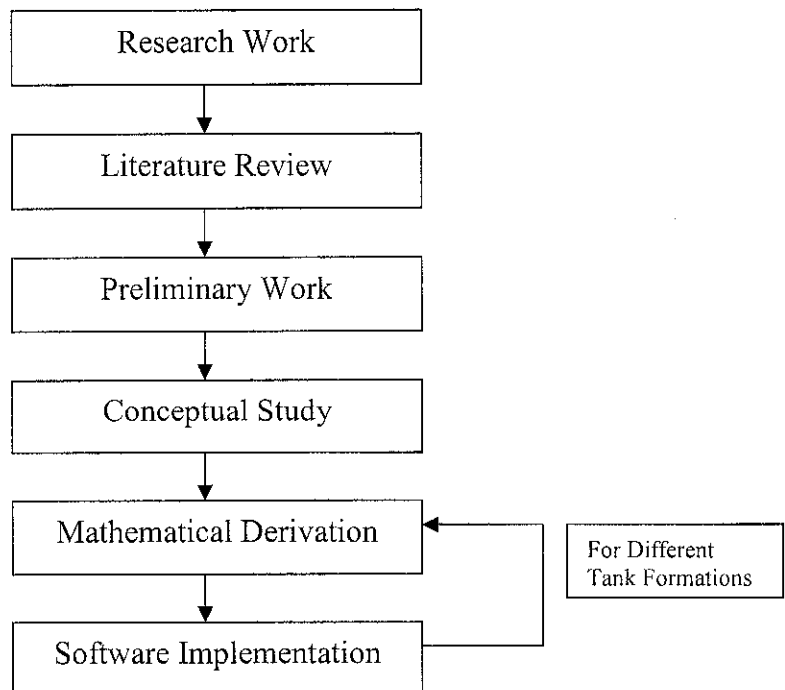


Figure 9 Project Work Flow

The main if not the only software that is involved in this project is MATLAB and the Simulink feature. Through this particular method, we will be able to do all the off-line modelling with respect the actual physical world. I will be using the Simulink Toolbox to come with the modelling architecture and then once all the tank mathematical model is derived, a simple Matlab file will be created to obtain the ARX model structure depending on the user preferences, whereby user can input the model order

“MATLAB is an acronym that stands for MATrix LABoratory. It is an inactive environment for performing technical computations and is considered as a standard engineering tool for most universities and industries, particularly.” [3]

Meanwhile, “ Simulink features a graphical interface to numerical integration routines, where a lot of standard operations both on discrete and continuous time signals are implemented as blocks.” [3]

In addition to that, Simulink has also gained a particular impact in industry for simulating dynamical systems and designing control system.

3.1.1 Understanding MATLAB and Simulink components

Since this project is all about modelling the different formation of tank system, it is required to fully understand MATLAB Programming and the use of SIMULINK component. In this project, MATLAB is used to design and simulate the control based model structures; (ARX).

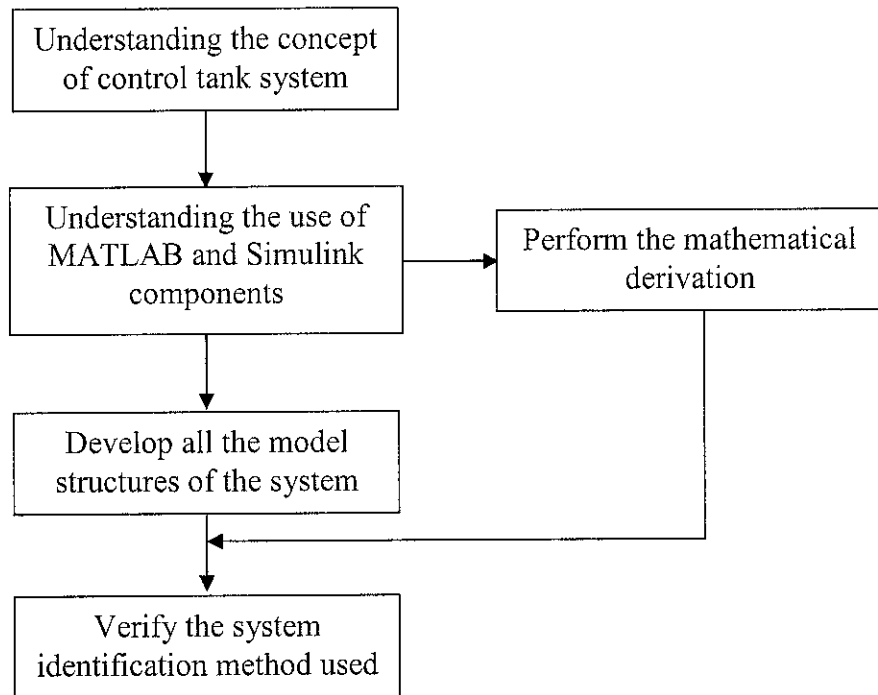


Figure 10 Project Design Flow

3.2 Tools

This project is more toward the simulation, therefore, the tools that will be heavily used for this project is MATLAB and its application, which is Simulink. The MATLAB will be used for the simulation works and will be carried out once all the initial progress is completed.

CHAPTER 4

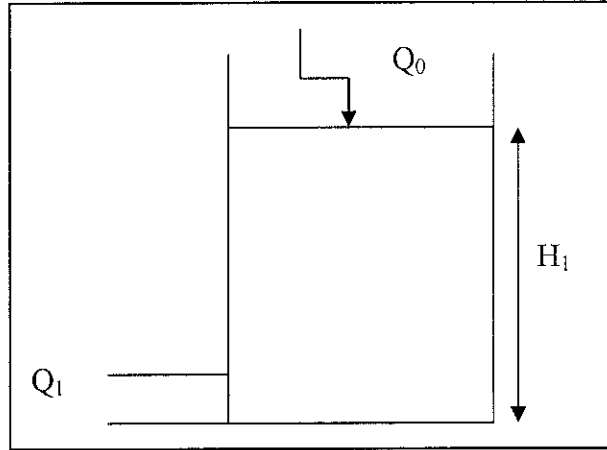
RESULTS & DISCUSSION

4.1 Mathematical Derivation

There has been some study conducted regarding this system. This study involves more into deriving the mathematical expression that involves in determining the characteristic of the control tank system. It is found that to find the mathematical derivation of the system, we need only to determine the energy balance equations that governs the formation of the tank system. After detail analysis, below are the method to find the mathematical derivation for the single tank and the two tank system. Based on these two method and concept the mathematical derivation for the other two system can be computed.

In actual derivation, there are many factors that need to be considered for the mathematical derivation of the forces acting on the control-tank system. The main forces acting on the system is the kinetic energy and the gravitational force. The other forces which comes from the effect of pressure, temperature and physical of the tanks are considered very small compared to the main forces and are assumed constant later on.

4.1.1 Mathematical Derivation for Single Tank System



For the mathematical expression,

Energy Storage = Change in Kinetic Energy + Change in Potential Energy

$$m \times g \times h = \frac{1}{2} \times m \times v^2$$

$$v^2 = 2 \times g \times h$$

$$v = \sqrt{2 \times g \times h}$$

$$v = k\sqrt{h}$$

where, k is

- i) Gravitational Force
- ii) Resistance
- iii) Density of the fluid

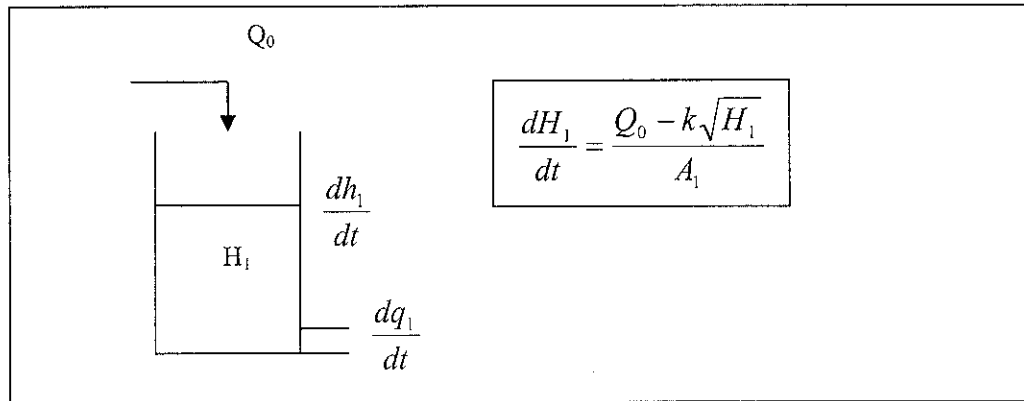
This is the method of how the equations are derived based on simple application theory. This equation is then further derived to solve for the volume of the tank system, for example (in this case a single tank system) [1]:

$$\frac{(dV)}{(dt)} = Q_1 = k\sqrt{h}$$

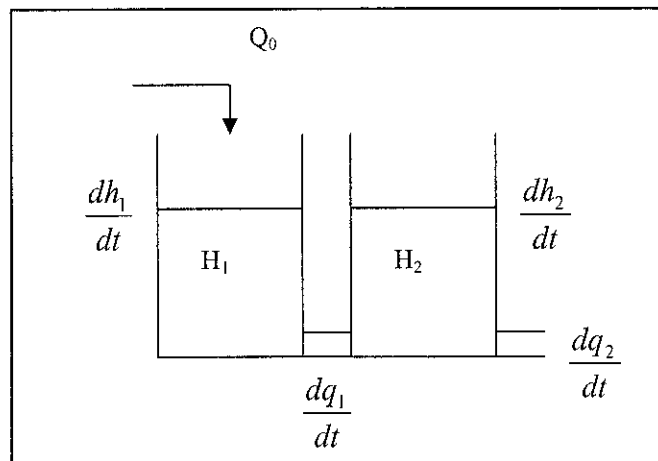
$$\frac{(dH_1)}{(dt)} = \left(\frac{Q_0 - Q_1}{A_1} \right)$$

$$\frac{(dH_1)}{(dt)} = \left(\frac{Q_0 - k\sqrt{h}}{A_1} \right)$$

From this method, we can see that the height of the tank can be identified based on the flow in and out of it.



4.1.2 Mathematical Derivation for Two Tank System



As per the single tank, the two tank modeling can also be performed in this project. But, initially the mathematical derivation has to be done based on the model desired. The mathematical derivation for each part/each control tank will follow as per below. Assume:

$$\frac{dq_1}{dt} = V_1 \quad \frac{dq_2}{dt} = V_2$$

For Control Tank 1,

$$(m \times g \times H_1) - (m \times g \times H_2) = \frac{1}{2} \times m \times V_1^2$$

$$V_1 \propto \sqrt{H_1 - H_2}$$

$$V_1 = K_1 \sqrt{H_1 - H_2}$$

For Control Tank 2,

$$(m \times g \times H_2) = \frac{1}{2} \times m \times V_2^2$$

$$V_2 \propto \sqrt{H_2}$$

$$V_2 = K_2 \sqrt{H_2}$$

Based on the derivation above, we can clearly see that the velocity, V for each tank flow is determined by the transfer of energy; potential energy into kinetic energy. The velocity is found to be proportional with the height of the tank. The other parameters are then categorized into a proportional constant, K.

Using the derivation as above, the height of each tank is identified based on the given volume for each tank individually.

- *Volume for Tank 1 [1]*

$$\begin{aligned} A_1 H_1 &= Q_o - V_1 \\ &= Q_o - K_1 \sqrt{H_1 - H_2} \end{aligned}$$

At time t,

$$A_1 \frac{dH_1}{dt} = Q_o - K_1 \sqrt{H_1 - H_2}$$

Therefore,

$$\frac{dH_1}{dt} = \frac{1}{A_1} [Q_o - K_1 \sqrt{H_1 - H_2}]$$

- *Volume for Tank 2 [1]*

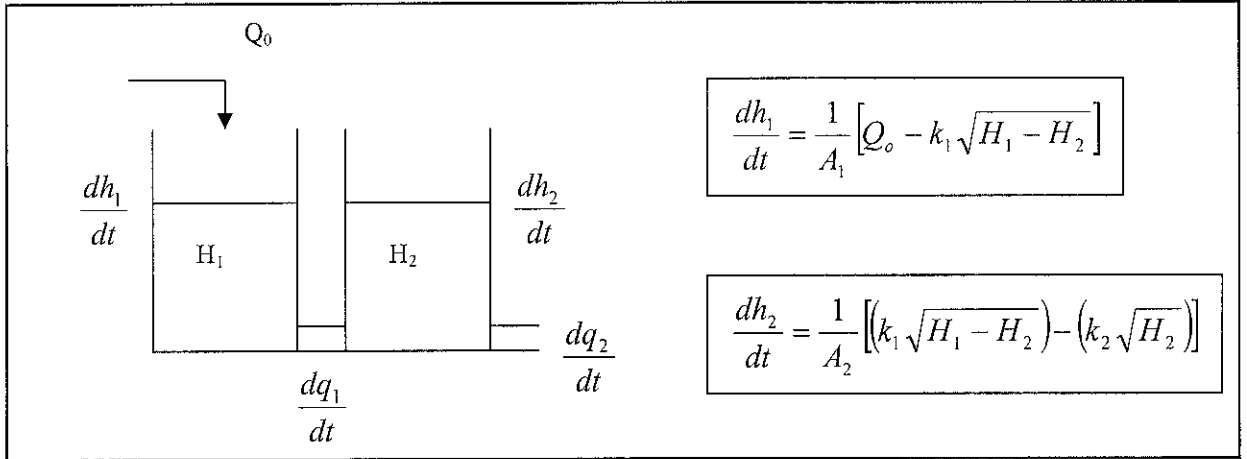
$$\begin{aligned} A_2 H_2 &= V_1 - V_2 \\ &= K_1 \sqrt{H_1 - H_2} - K_2 \sqrt{H_2} \end{aligned}$$

At time t,

$$A_2 \frac{dH_2}{dt} = K_1 \sqrt{H_1 - H_2} - K_2 \sqrt{H_2}$$

Therefore,

$$\frac{dH_2}{dt} = \frac{1}{A_{21}} \left[K_1 \sqrt{H_1 - H_2} - K_2 \sqrt{H_2} \right]$$



4.2 Tank Formations

For the formation of the tank to be applied for the system identification, 6 various formation has been discussed and identified for this project, based on the common industry utilization and preferences. Therefore, this project will only deal with the modelling of these tank formations. The tank formations identified are as per below:

- Single Tank
- Couple Tank
- Three Tank
- Split Tank
- Ladder Tank
- Custom Tank

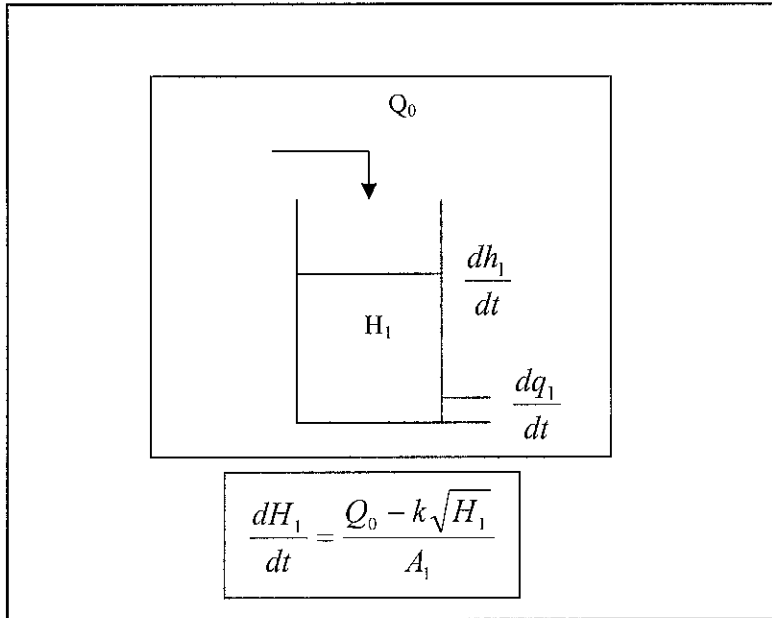


Figure 11 Diagram of a Single Tank System

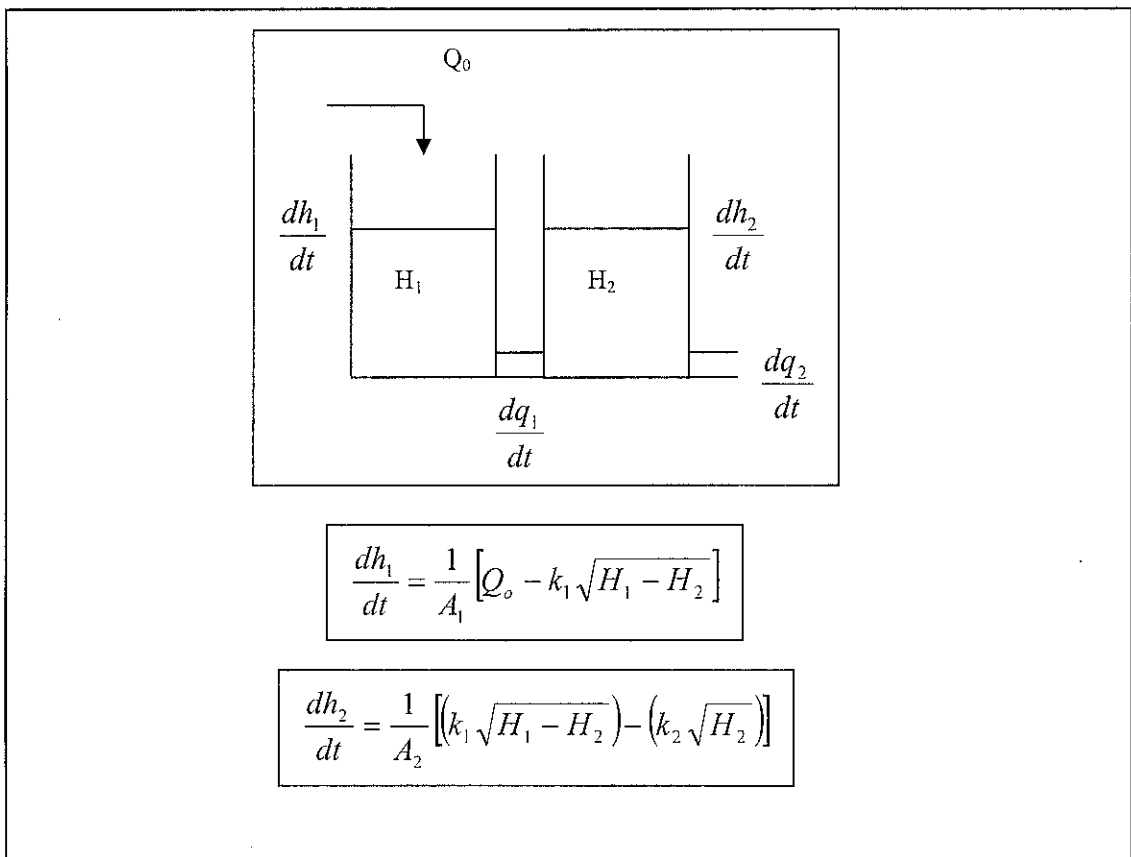


Figure 12 Diagram of a Couple Tank System

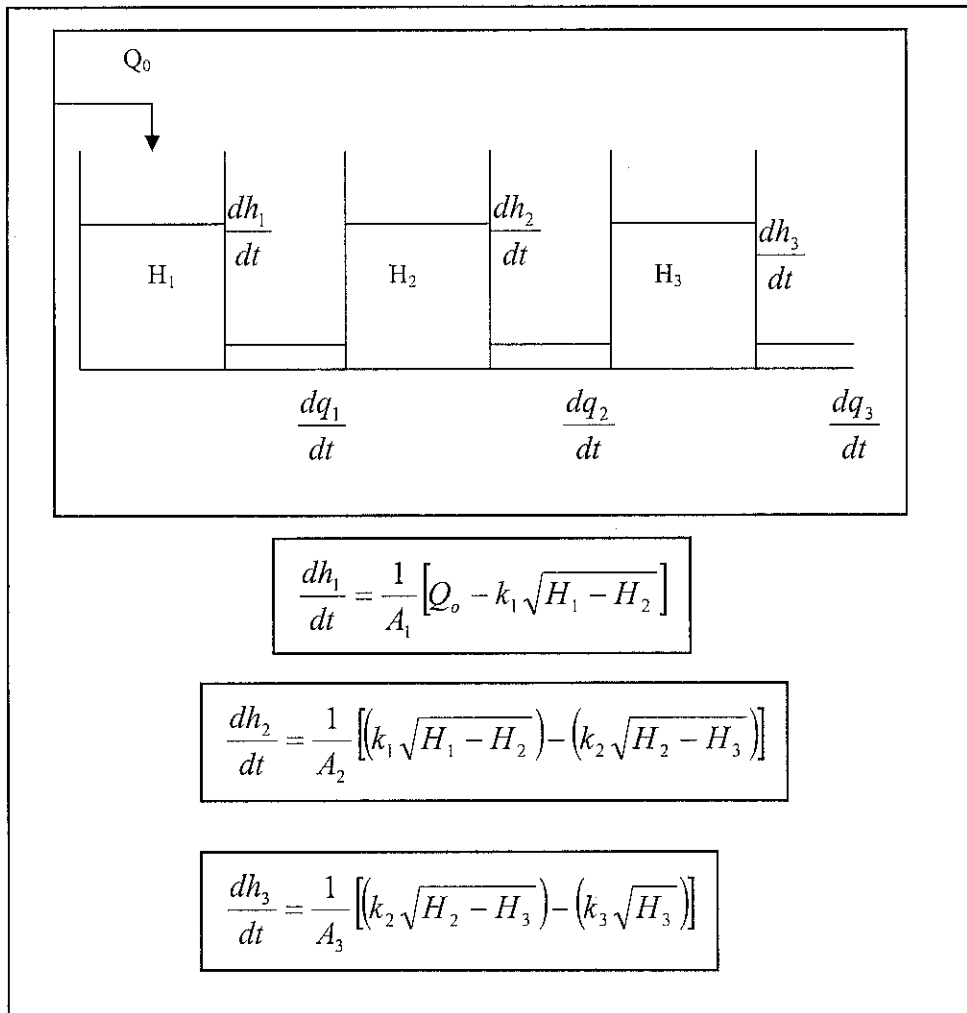


Figure 13 Diagram of a Three Tank System

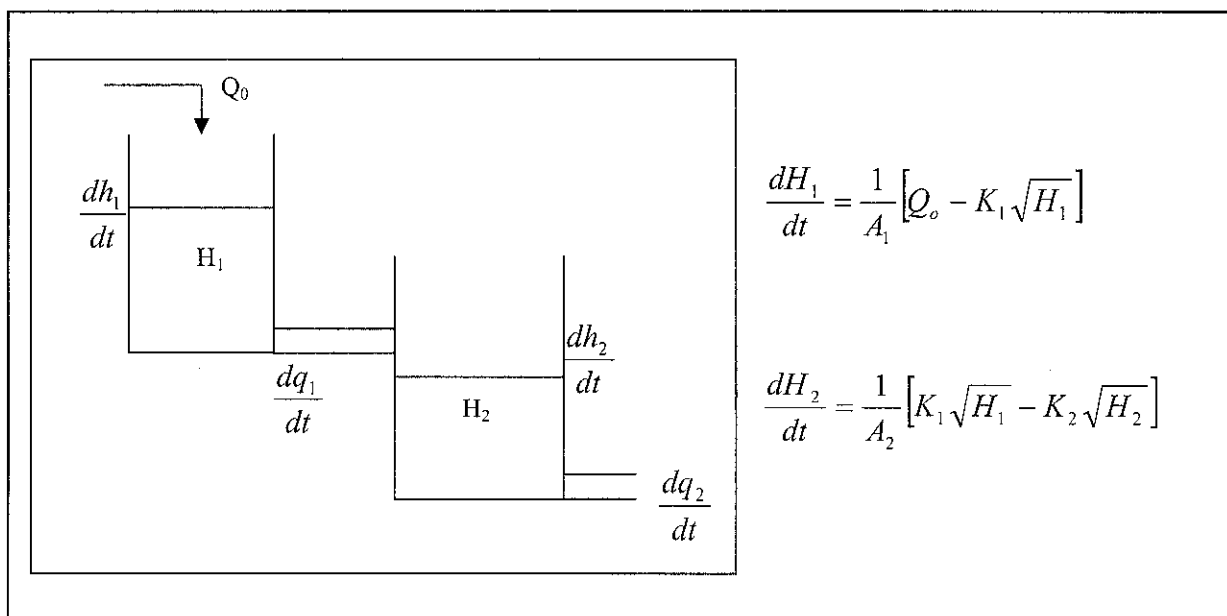


Figure 14 Diagram of a Split Tank System

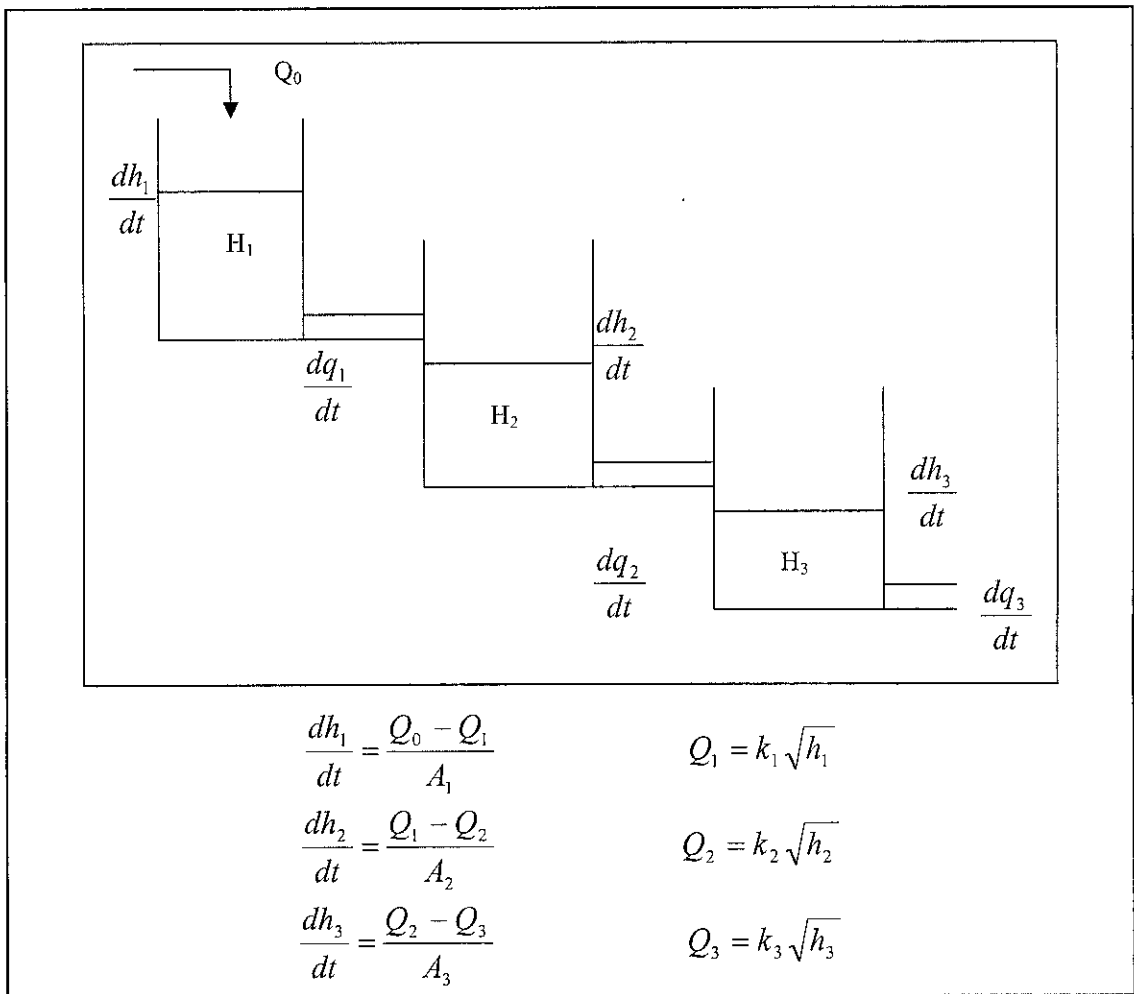


Figure 15 Diagram of a Ladder Tank System

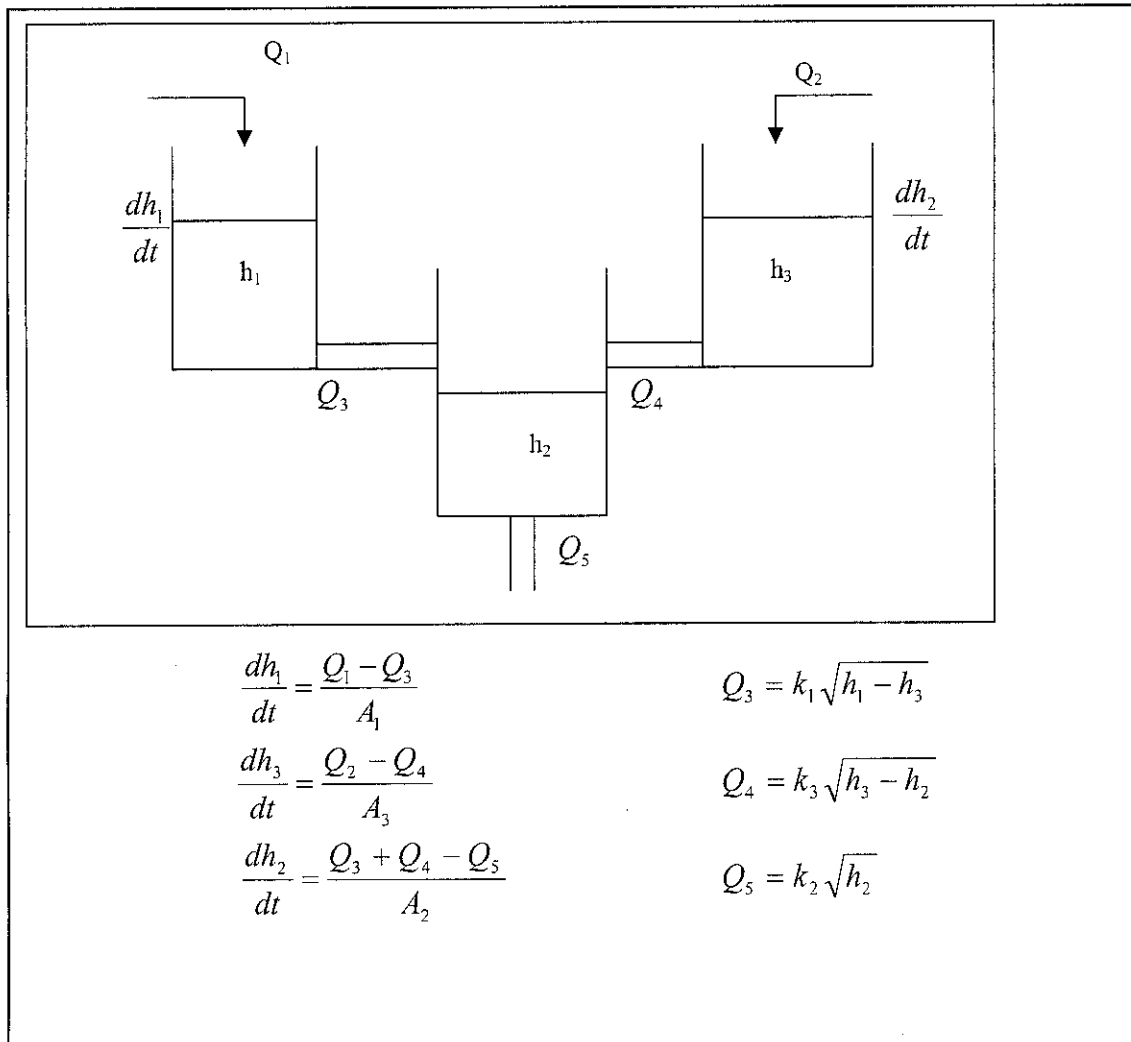


Figure 14 Diagram of a Custom Tank System

4.3 Simulation Results

Below are the results obtained from the modelling of the mathematical expression obtained from the initial derivation of the tank formations. This are the direct modelling without any model structures.

4.3.1 Single Tank System

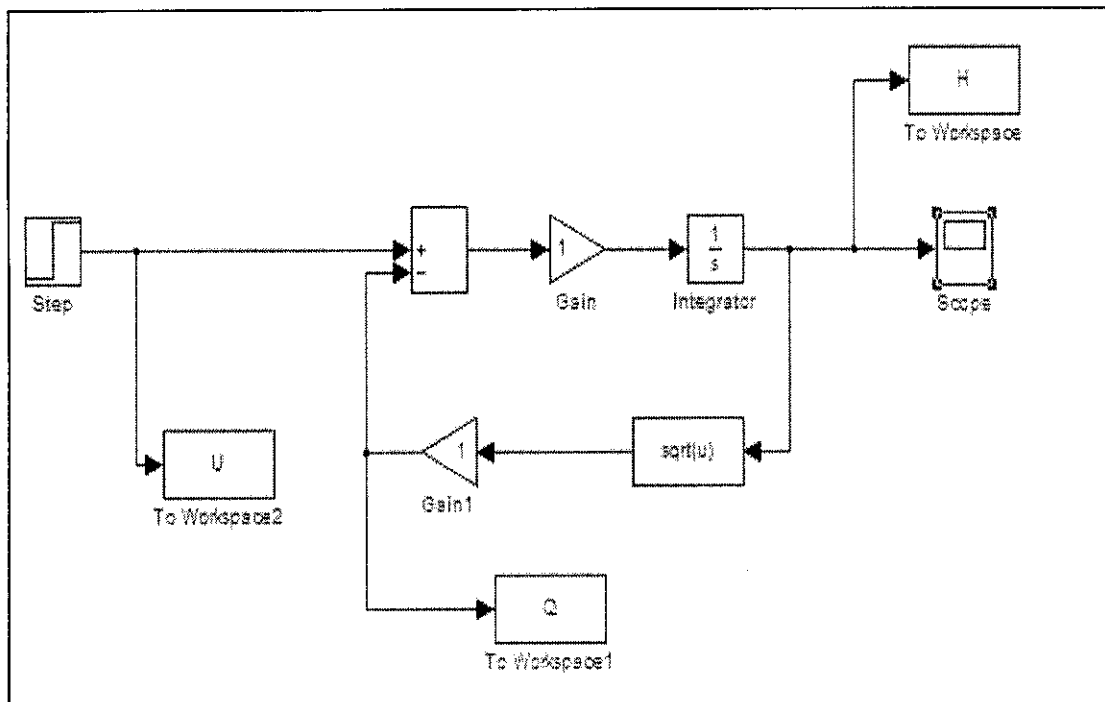


Figure 16 Simulink Block Diagram for Single Tank System

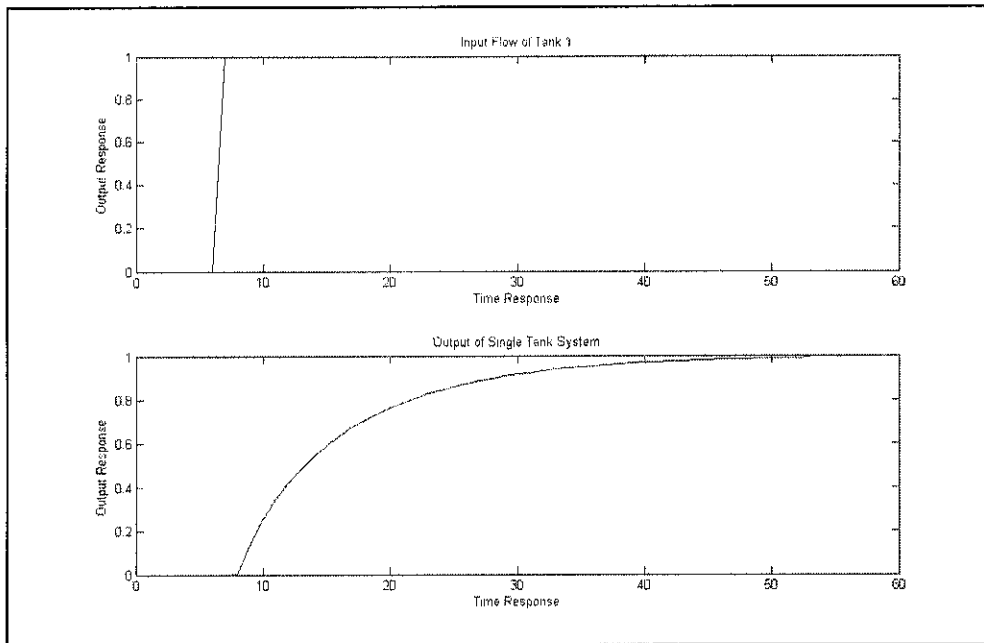


Figure 17 Output Waveform for Single Tank System

For the single tank formation, we can see that the changes or response in the first tank changes some time after the input is applied. As can be seen in the waveform, the input flow in this case a step flow of $1\text{m}^3/\text{hr}$ is applied to the system as the flow in. The initial height of the fluid is zero, meaning there is no any fluid in the tank initially. As the flow is increased in a step manner, the liquid level tends to increase slowly. The response of the tank is slow due to the dynamics of the tank itself. This is the basis for all the tank that will be discussed hereafter.

4.3.2 Two Tank System

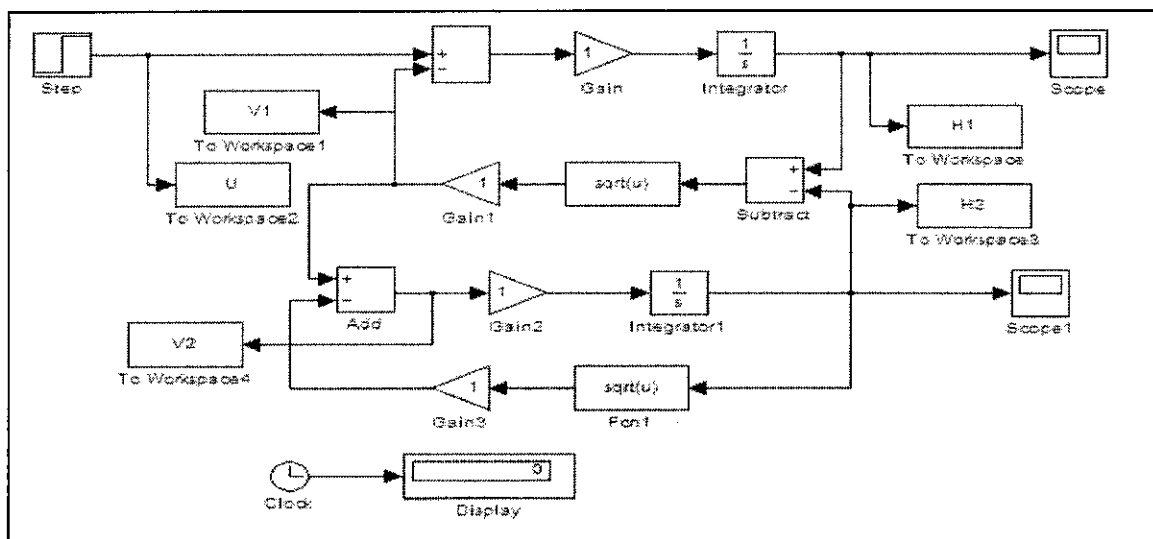


Figure 18 Simulink Block Diagram for Two Tank System

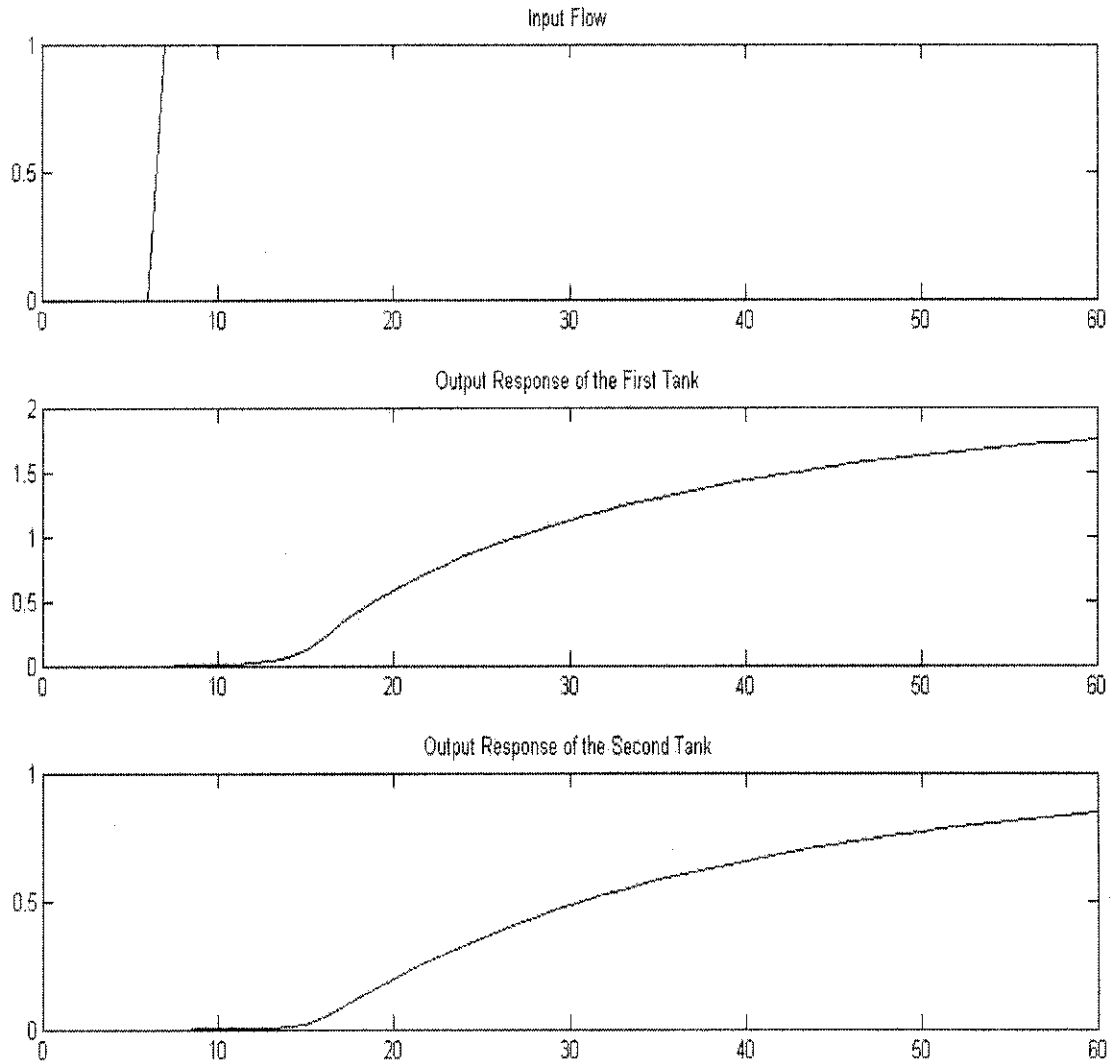


Figure 19 Output Waveform of Two Tank System

This is the output response obtained for the two tank system is as per above. We can see the output response for the two tank system is slower compared to the first tank. This is because the input to the second tank is controlled by the outflow of the first tank. The flow of the first tank is then controlled by the input flow rate that is applied to the tank system. In this case, the input flow rate is 1 m³/hr. As time increase we can the changes in tank one starts to increase higher than tank 2. It reaches around up to 1.8 meter and tank 2 reaches up to 0.8 meter. It is to be noted that the initial height in the tank is 0, meaning there is no fluid in the tank initially.

4.3.3 Three Tank System

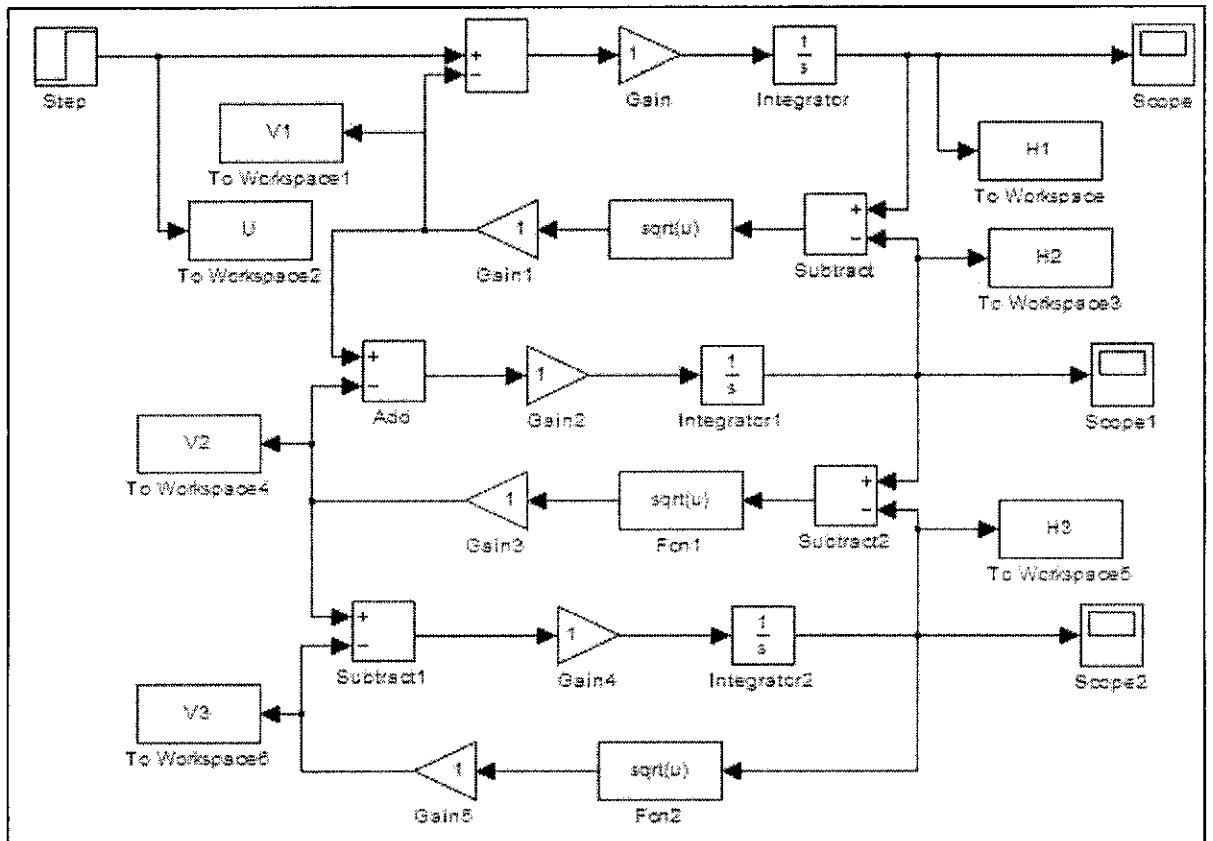


Figure 20 Simulink Block Diagram for Three Tank System

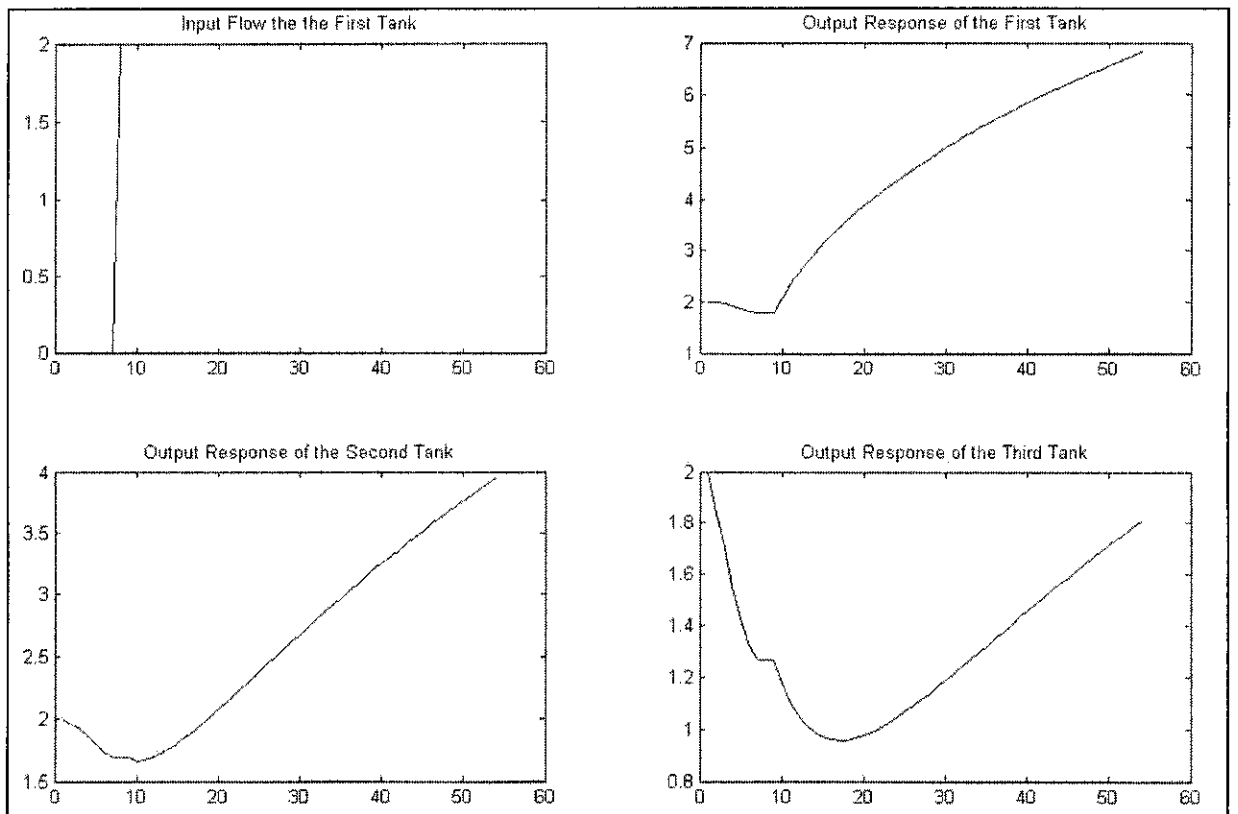


Figure 21 Output Waveform for Three Tank System

For the three tank system, the response for the third tank is much more slower. The tanks are a interacting series where the flow to the subsequent tanks is controlled by successive tank. Therefore, flow of the third tank is controlled by the second tank and flow of the second tank is controlled by the first tank. Meanwhile flow of the first tank is controlled by the input flow rate. As can be seen in the output response, the input flow rate is 2m³/hr. as time increases, the height of the tank also increases slowly. But initially, there is a slight drop in the height of the tank. It is to be noted that the initial height in the tank is at 2 meters. When the system is put to run ,the fluid in the first tank decreases and so does the response in the second and third tank. This is due to the dynamics of the tank, where before there is a flow in, there is drain out of the tank before there is a flow in and the height begins to increase.

4.3.4 Split Tank System

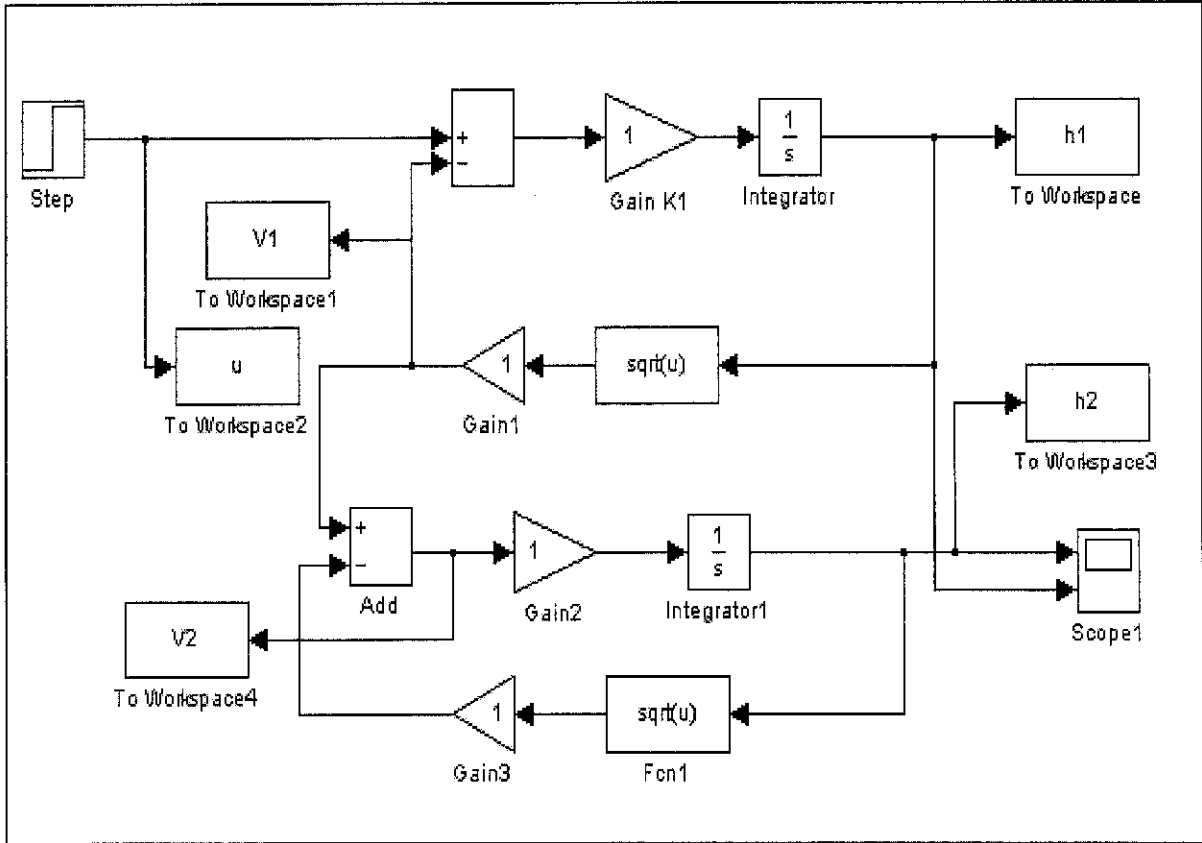


Figure 22 Simulink Block Diagram of a Split Tank System

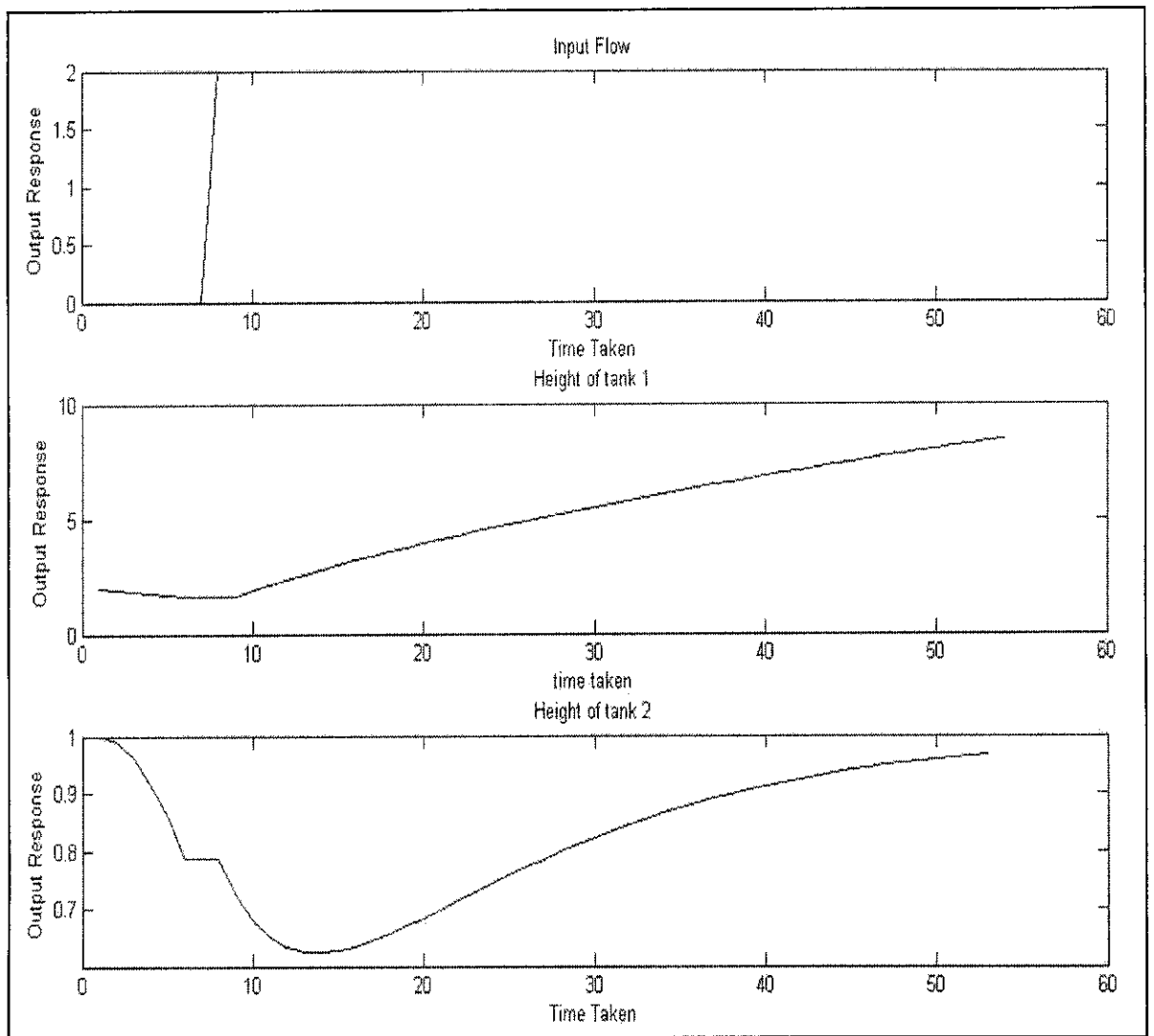


Figure 23 Output Waveform of Split Tank System

For the split tank system, it is a non-interacting series. The two tanks responses are independent with each other but the flow in for the second tank still depends on the first tank. Initially, the height in both the tank is 1 meter. Before there is a flow into the first tank, the fluid in the tanks dried out and later when there is a flow in of 2 m³/hr, the height response in both the tank tends to increase slowly, with much slower dynamics. However, we can see that the changes in the height for the first tank is very abrupt and high due to the non-interacting series formation.

4.3.5 Ladder Tank System

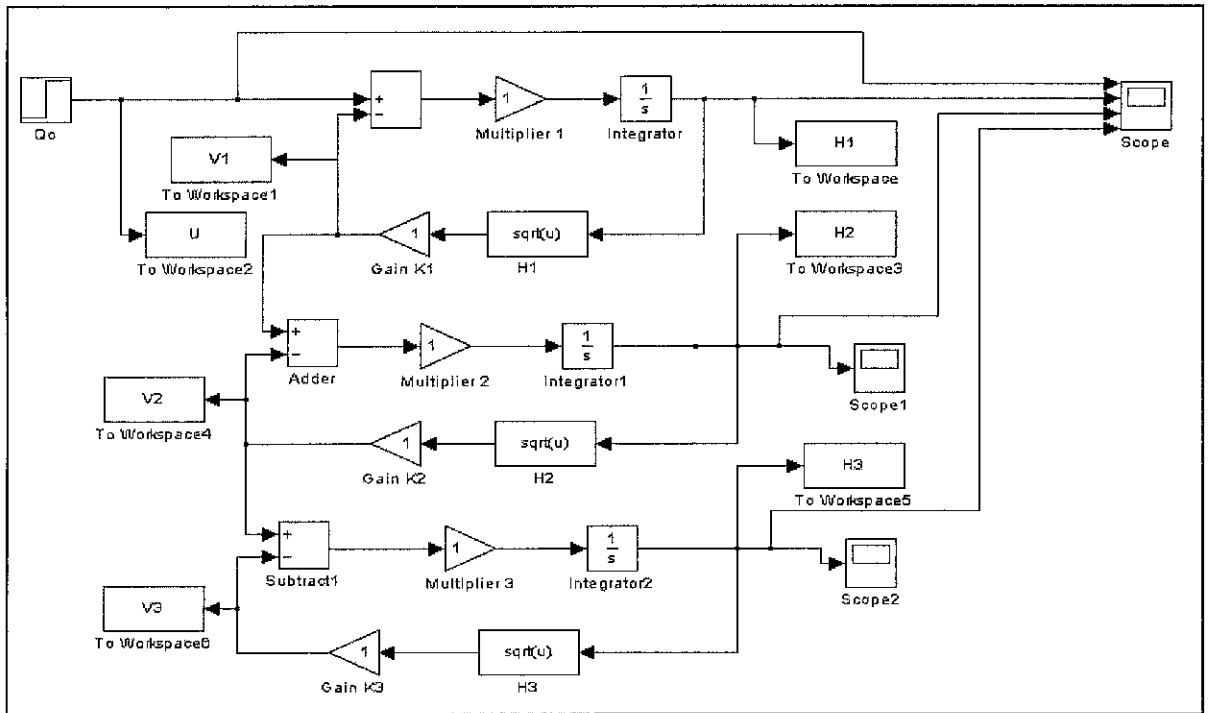


Figure 24 Simulink Block Diagram of a Ladder Tank System

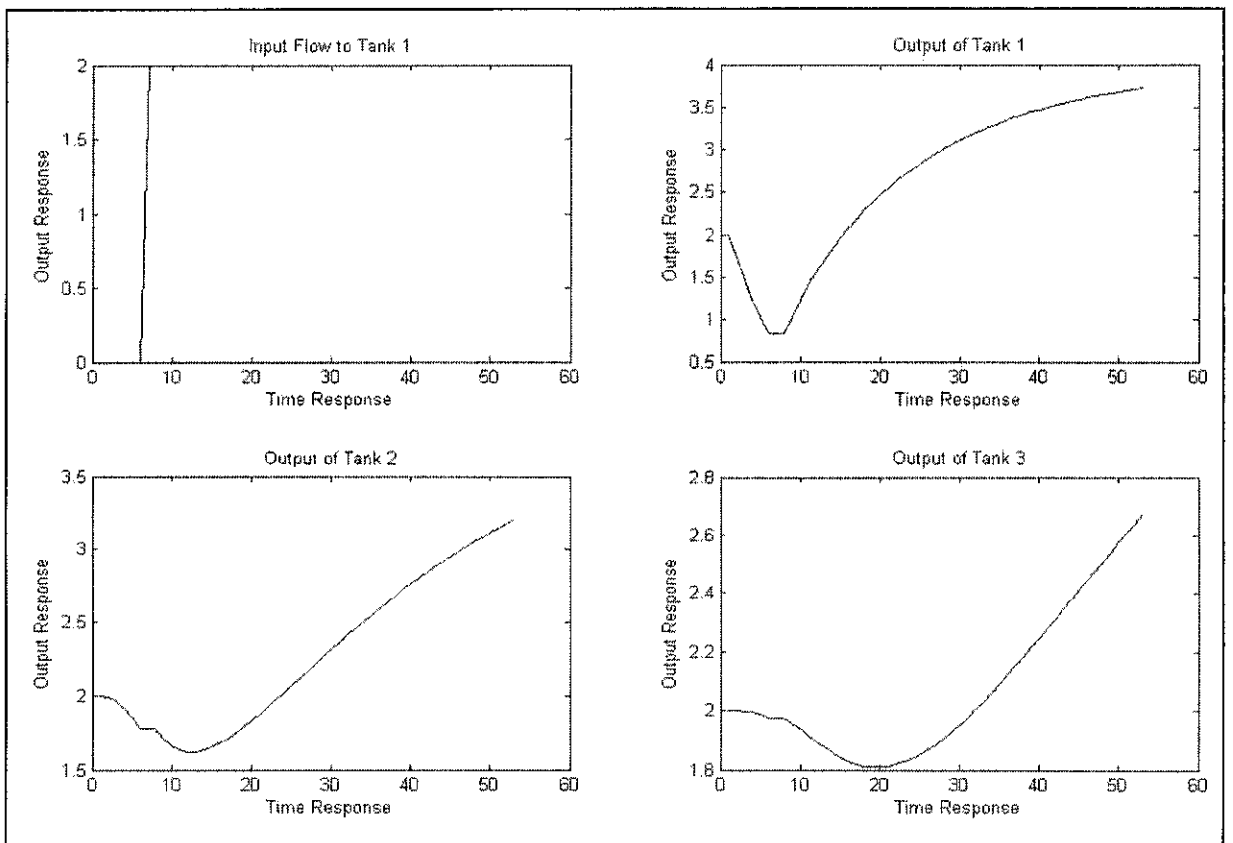


Figure 25 Output Waveform of Ladder Tank System

For the ladder tank formation, it is basically similar to the split tank system, whereby it is a non-interacting series and the tanks are independent to each other. The flow out from first tank is the flow in to the second tank and the flow out of the second tank is the flow in of the third tank and the flow out of the third tank is final flow out. The height of the tank depends on the flow in and out of the tank. Based on the output response obtained, we can say that the response of the tanks are much slower if the number of tanks increases and so is the dynamics of the whole system. Using this basis, we can also model more tanks with the same formation. As can be seen in the output response, the flow rate in of 2m³/hr to the first tank gives a very slower response to the third tank as the dynamics are very much slower. Initially, the height of the tanks tend to decrease before it increases because of the flow dependency of the tank formations. The output response are non-linear as can be seen in the output waveforms.

4.3.6 Custom Tank System (V-Shape)

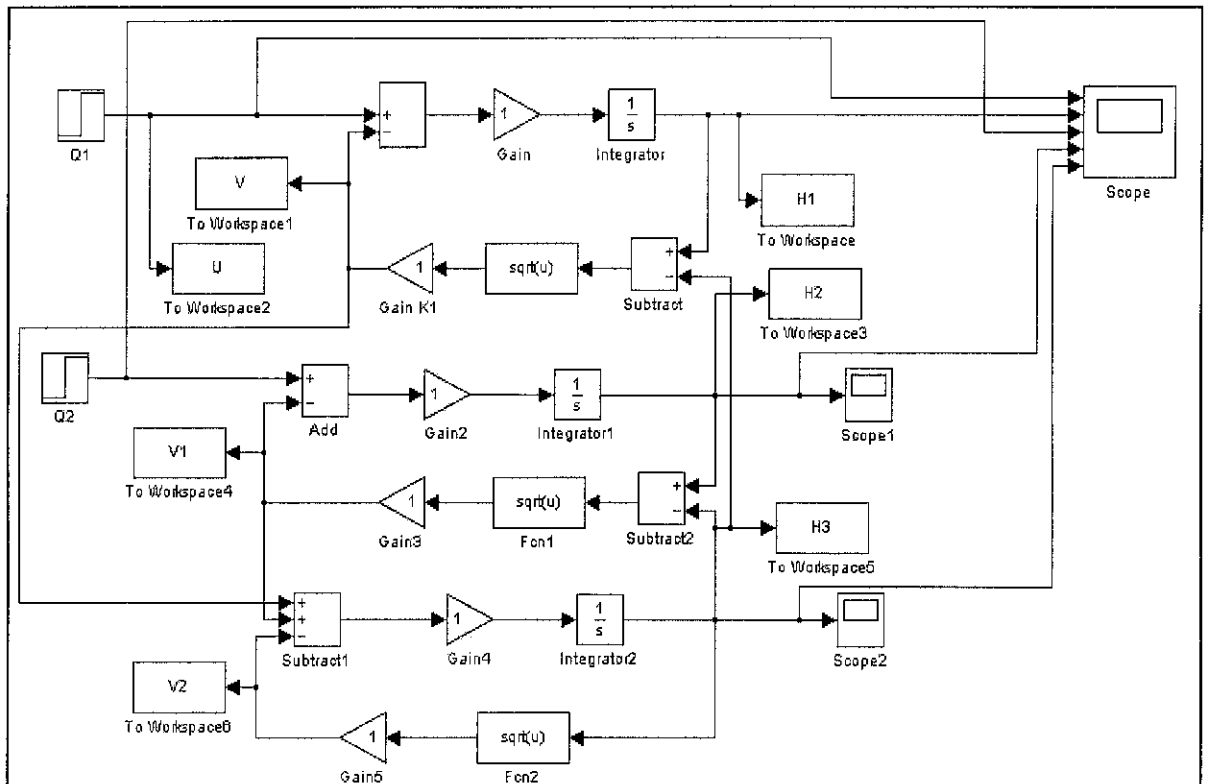


Figure 26 Simulink Block Diagram of the Custom Tank Formation (V-Shape)

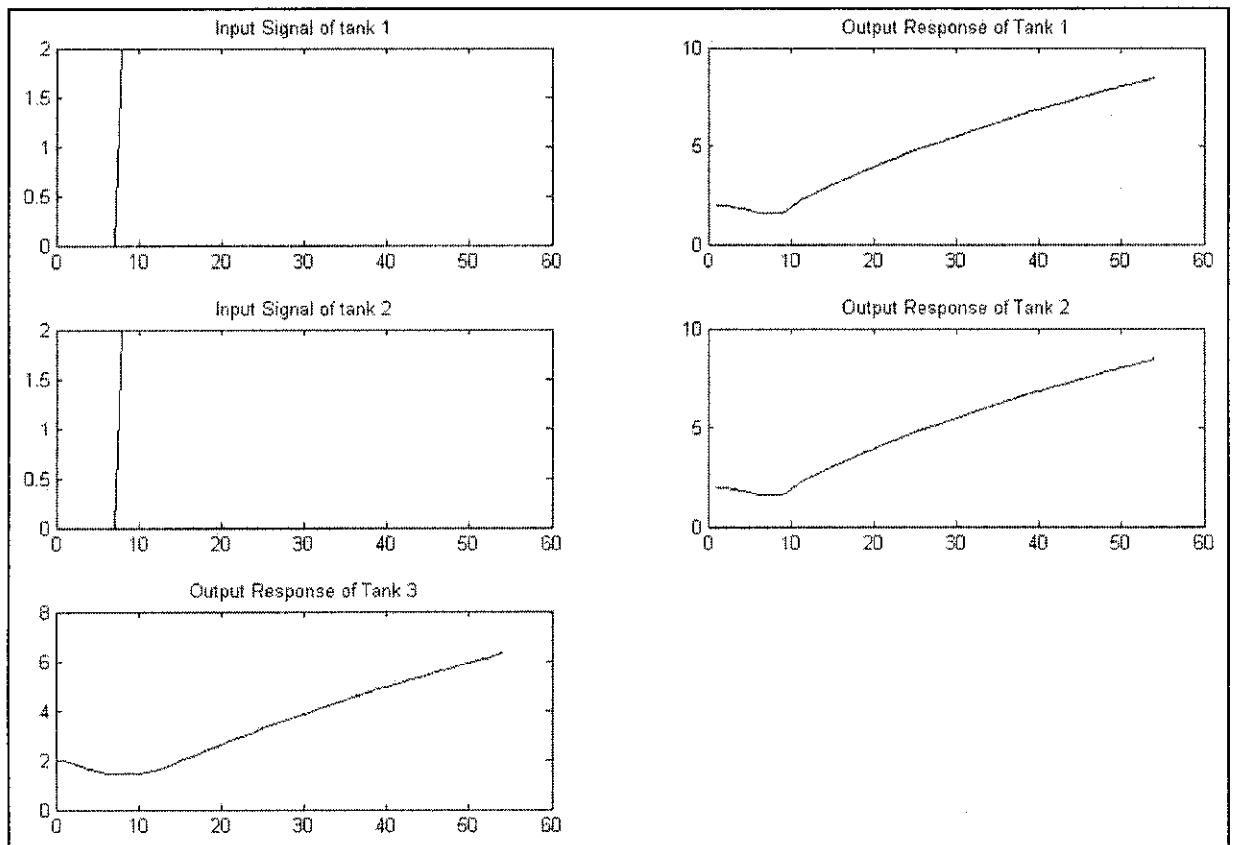


Figure 27 Output Waveforms Custom Tank Formation (V-Shape)

For the custom tank formation, the type of formation that is being studied upon is a V-shape tank, with more than one input flow. This type of tank system is normally used in the reactor for the industry. It is used to mix two type of different reactants and give a desired composition. Therefore it has two flow into two different tanks. The output flow for both the tank is then fed into the final tank that we are interested in. the dynamics of this type of tank is much faster as there is less interaction between the tanks. From the output waveform we can see that the flow rate in of $2\text{m}^3/\text{hr}$ is applied to both the tank 1 and 2. Then the output flow of both the tank 1 and 2 is then applied to the third tank. Therefore there is two in inflow to the third tank. Therefore the response of the height of fluid in the third tank is much more faster compared to the previous formations.

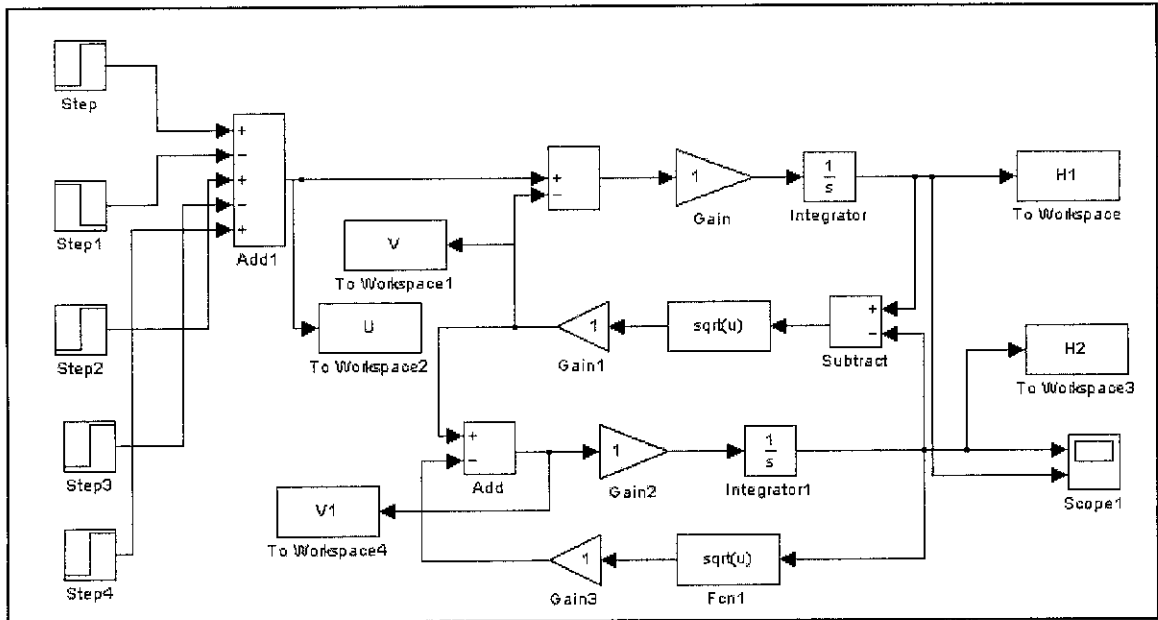


Figure 28 SimulinkBlock Diagram of a Multi-Step Input (Couple Tank System)

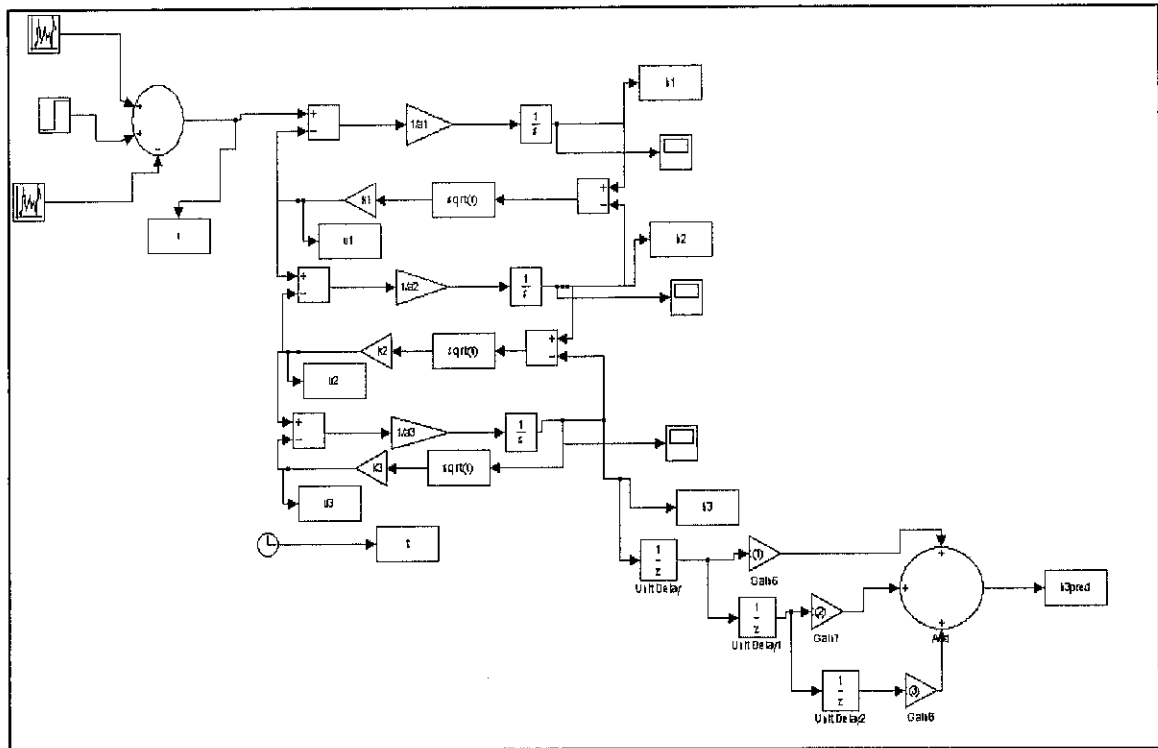


Figure 29 Simulink Block Diagram of a Multi-Input (Three Tank System)

Based on basic step flow response that we have looked at previously, it can be said that the multistep flow and unstable flow response can also be used to compute the height response of each tank. The formation of block diagrams for the multi-step input and multi-input (unstable flow) is as per above. Through this method, we can see the output response of the overall system.

4.4 ARX Model Structure

Based on the mathematical model obtained as per above for all the tank formations, the ARX model is built upon derived and comparison is done through the predicted tank height and actual tank height. The equation for the ARX model structure is as per below:

ARX (1st order)

$$y(t) = a_{11}y(t-1) + b_{11}x(t-1)$$

ARX (2nd order)

$$y(t) = a_{12}y(t-1) + a_{22}y(t-2) + b_{12}x(t-1) + b_{22}x(t-2)$$

ARX (3rd order)

$$y(t) = a_{13}y(t-1) + a_{23}y(t-2) + a_{33}y(t-3) + b_{13}x(t-1) + b_{23}x(t-2) + b_{33}x(t-3)$$

The coefficients a refers to the constant values that is associated to each model order. As the order increases, the number of constants also increases to properly identify and track each tank values. From the equations above, we can clearly see that the subsequent output response $y(t)$ is dependent on the previous output values, $y(t-i)$ and input values $x(t-i)$.

However, the determination of model constants, a , for each different tank formations is also dependent on the sampling time that is chosen to be. The results vary for different tank formations, depending upon the sampling time that is chosen to be.

From the tank formation obtained, we can clearly see that as the model order increases the amount of constant values associated with the order increases as well. And from the simulation study obtained, we can see that the constants are more or less around 1. This means that the output is tracking the previous outputs.

Attached in the appendices is the m-file source code for the determination of the ARX model parameters. This source code incorporates the model order preferred by the user and the sampling time that is interested in. User can set both the parameter values to suit the required need and observe the characteristic of the response obtained. Normally, the ideal values that will be obtained is the tracking values of the output. This implies that the output is tracking the input which is also based on the equation obtained. The table below shows the different parameters (constants) obtained for all six different tank formations under different sampling time intervals:

Sampling Time = 5 units

	Single	Couple	Three	Split	Ladder	Custom
ARX(1st Order)	a ₁₁ = 1.0218 a ₁₂ = -0.3814	a ₁₁ = 0.4435 a ₁₂ = -0.0616	a ₁₁ = 0.3150 a ₁₂ = 0.0870	a ₁₁ = 0.9169 a ₁₂ = -0.1075	a ₁₁ = 0.6700 a ₁₂ = -0.0631	a ₁₁ = 0.2047 a ₁₂ = 0.1172
ARX(2nd Order)	1.0e+003 * a ₁₂ = -2.2098 a ₂₂ = 0.0457 b ₁₂ = 0.7974 b ₂₂ = 1.3674	a ₁₂ = -0.5823 a ₂₂ = 0.0960 b ₁₂ = 0.0137 b ₂₂ = 0.4858	a ₁₂ = 0.5944 a ₂₂ = -0.2452 b ₁₂ = 0.8619 b ₂₂ = -0.9297	1.0e+009 * a ₁₂ = 6.3871 a ₂₂ = 0.0000 b ₁₂ = -5.226 b ₂₂ = -1.1608	1.0e+007 * a ₁₂ = 1.9784 a ₂₂ = -0.0002 b ₁₂ = -2.9048 b ₂₂ = -1.0517	a ₁₂ = -0.1869 a ₂₂ = 0.0526 b ₁₂ = 0.5277 b ₂₂ = 0.3812
ARX(3rd Order)	a ₁₃ = 0.0349 a ₂₃ = -0.1060 a ₃₃ = 11.272 b ₁₃ = 0.0349 b ₂₃ = 0.0349 b ₃₃ = -10.910	a ₁₃ = 24.384 a ₂₃ = -1.0042 a ₃₃ = 0.1264 b ₁₃ = -0.4945 b ₂₃ = -10.494 b ₃₃ = -2.5123	a ₁₃ = -0.4155 a ₂₃ = -0.3155 a ₃₃ = 0.0498 b ₁₃ = 0.7883 b ₂₃ = 0.7883 b ₃₃ = 0.4183	1.0e+010* a ₁₃ = -0.3484 a ₂₃ = 1.2774 a ₃₃ = 0.0000 b ₁₃ = -0.3484 b ₂₃ = -0.3484 b ₃₃ = -0.2322	a ₁₃ = 0.2588 a ₂₃ = 0.1766 a ₃₃ = -0.1469 b ₁₃ = 0.1294 b ₂₃ = 0.1294 b ₃₃ = -0.0251	a ₁₃ = -193.93 a ₂₃ = 48.0431 a ₃₃ = -0.8010 b ₁₃ = 273.27 b ₂₃ = 273.27 b ₃₃ = 40.8409

Table 1 Table of ARX Model Parameters and Constants

Sampling Time = 10 units

	Single	Couple	Three	Split	Ladder	Custom
ARX(1st Order)	$a_{11}= 14.9404$ $a_{12}= -14.748$	$a_{11}= 0.0120$ $a_{12}= -0.0016$	$a_{11}= 0.2255$ $a_{12}= 0.3355$	$a_{11}= 0.9537$ $a_{12}= -0.1446$	$a_{11}= 0.8085$ $a_{12}= -0.6402$	$a_{11}= 0.0084$ $a_{12}= 0.3541$
ARX(2nd Order)	$1.0e+003 *$ $a_{12}= -0.1441$ $a_{22}= 22.410$ $b_{12}= 0.1423$ $b_{22}= -22.120$	$a_{12}= -0.4345$ $a_{22}= 0.7827$ $b_{12}= 0.0001$ $b_{22}= 0.0601$	$a_{12}= 1.4895$ $a_{22}= -0.6274$ $b_{12}= 0.4757$ $b_{22}= -1.5462$	$a_{12}= 0.0203$ $a_{22}= -0.1209$ $b_{12}= 0.9414$ $b_{22}= -0.0747$	$a_{12}= -0.8114$ $a_{22}= 0.7881$ $b_{12}= 0.8640$ $b_{22}= 0.4749$	$a_{12}= -0.4472$ $a_{22}= 0.5613$ $b_{12}= 0.3099$ $b_{22}= 0.2574$
ARX(3rd Order)	$1.0e+011 *$ $a_{13}= 5.8258$ $a_{23}= -0.0030$ $a_{33}= -0.0000$ $b_{13}= -2.8749$ $b_{23}= -2.8749$ $b_{33}= -0.0730$	$a_{13}= 0.6863$ $a_{23}= -1.0663$ $a_{33}= 1.0354$ $b_{13}= -0.0079$ $b_{23}= -0.0079$ $b_{33}= -0.2046$	$a_{13}= 28.1734$ $a_{23}= -11.843$ $a_{33}= 0.5232$ $b_{13}= -5.7297$ $b_{23}= -18.834$ $b_{33}= -8.5155$	$1.0e+007 *$ $a_{13}= 3.0828$ $a_{23}= -0.0059$ $a_{33}= 0.0000$ $b_{13}= -1.258$ $b_{23}= -1.2589$ $b_{33}= -0.5590$	$1.0e+005 *$ $a_{13}= 5.3866$ $a_{23}= -0.0929$ $a_{33}= 0.0011$ $b_{13}= -3.9115$ $b_{23}= -3.9115$ $b_{33}= -2.7665$	$a_{13}= 8.3937$ $a_{23}= -7.2321$ $a_{33}= 2.1078$ $b_{13}= -1.8501$ $b_{23}= -1.8501$ $b_{33}= -8.2321$

Table 2 Table of ARX Model Parameters and Constants

4.5 DISCUSSION

In this simulation study, we would only look into the parameter estimation for linear models. From the ARX model parameters and constants obtained, we can say that if the results are close to 1.000, then the output is tracking the input whereby the increasing height of the tank corresponds directly to the flow into the tank. Besides that, the response of each tank also depends on the rate of flow in and out of the tank and the initial values that are assigned to each open tank formation. In this simulation study all constants are assumed to be 1, and the initial height of the tank is 10 m. From this assumption, all the above mentioned graphs are obtained and analysis is then performed.

For the multi-step and multi-input block diagram, the output waveform is obtained and then compared with the mathematical model observation. Then, the corresponding error is obtained and observed.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

System identification method is considered as very useful tool to conduct an off-line diagnostic of the overall tank system and to observe the output of the system under various parameters assigned. This topic involves plenty of mathematical derivation and the need to understand the concept of modelling a system with respect to the disturbances and harmonics that the system may experience in real time.

However, there some noticeable achievement within the project scope. First of all, a basic modelling has been done to identify all the six different type of tank formation to identify the basic system characteristic. Then, the model structure of ARX comprising of only linear models are built upon and the resolving parameters are observed for the identification purposes.

In conclusion, it can be said that this project is to able to be completed within the given time frame to achieve the required objectives. With proper planning and guidance from the Supervisor and cooperation and hard work from the student, the planned milestone and successful completion of this project can be achieved.

5.2 Recommendation

Through this project, the control-tank system can be easily monitored for its characteristics based on the parameters identified. For further expansion of this project, this simulation study can be used as a collaboration project with any host company to improve the existing system applied in the industry today. Moreover the model estimation can also be extended for other model structures such as AR and ARMAX models. This will give better error estimation when compared to the actual system response.

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APPENDICES

APPENDIX A

Below is the m-file code for ARX model architecture that is done to obtain the model constants that will be used for the mathematical equation of that particular tank formation

```
fprintf('Enter the Tank Design Form...\n\n');
fprintf('\n1.Single Tank');
fprintf('\n2.Two Tank');
fprintf('\n3.Three Tank');
fprintf('\n4.Split Couple Tank');
fprintf('\n5.Staircase Three Tank');
fprintf('\n6.V-Shape Three Tank System');
Selc = input('\n Enter Your Selection ---- ');
ModO = input('\nWhat is your model Order ----');

    if (Selc == 1)
        sim('singletanksystem.mdl');
        % get the length of vector - U
        SamT = input('Enter The sampling Time ----');
        nLt = length(U);
        US = U(1:SamT:nLt);
        %US1 = U1(1:SamT:nLt);
        YS = H(1:SamT:nLt);
        nL = length(US);
        Yy = [];
        Uu = [];
        %Uu1 = [];
        Nk = 1;
        for i = ModO:-1:1

            uu = US(i:nL-Nk);
            Uu = [Uu uu];
            %uu1 = US1(i:nL-Nk);
            %Uu1 = [Uu1 uu1];
            yy = YS(i:nL-Nk);
            Yy = [Yy yy];
            Nk = Nk+1;
        end
        Yt = H(ModO+1:nL);

        INPv = [Yy Uu ];
        paraX = pinv(INPv)*Yt
    end

    if (Selc == 2)
        sim('twotanksystemMathModel.mdl');
        % get the length of vector - U
```

```

SamT = input('Enter The sampling Time ----');
nLt = length(U);
US = U(1:SamT:nLt);
%US1 = U1(1:SamT:nLt);
YS = H2(1:SamT:nLt);
nL = length(US);
Yy = [];
Uu = [];
%Uu1 = [];
Nk = 1;
for i = ModO:-1:1

    uu = US(i:nL-Nk);
    Uu = [Uu uu];
    %uu1 = US1(i:nL-Nk);
    %Uu1 = [Uu1 uu1];
    yy = YS(i:nL-Nk);
    Yy = [Yy yy];
    Nk = Nk+1;
end
    Yt = H2(ModO+1:nL);

INPv = [Yy Uu ];
paraX = pinv(INPv)*Yt
end

if (Selc == 3)
sim('threetankssystem.mdl');
% get the length of vector - U
SamT = input('Enter The sampling Time ----');
nLt = length(U);
US = U(1:SamT:nLt);
%US1 = U1(1:SamT:nLt);
YS = H3(1:SamT:nLt);
nL = length(US);
Yy = [];
Uu = [];
%Uu1 = [];
Nk = 1;
for i = ModO:-1:1

    uu = US(i:nL-Nk);
    Uu = [Uu uu];
    %uu1 = US1(i:nL-Nk);
    %Uu1 = [Uu1 uu1];
    yy = YS(i:nL-Nk);
    Yy = [Yy yy];
    Nk = Nk+1;
end
    Yt = H3(ModO+1:nL);

INPv = [Yy Uu ];
paraX = pinv(INPv)*Yt
end

if (Selc == 4)
sim('coupletankssystem.mdl');

```

```

% get the length of vector - U
SamT = input('Enter The sampling Time ----');
nLt = length(U);
US = U(1:SamT:nLt);
%US1 = U1(1:SamT:nLt);
YS = H2(1:SamT:nLt);
nL = length(US);
Yy = [];
Uu = [];
%Uu1 = [];
Nk = 1;
for i = ModO:-1:1

    uu = US(i:nL-Nk);
    Uu = [Uu uu];
    %uu1 = US1(i:nL-Nk);
    %Uu1 = [Uu1 uu1];
    yy = YS(i:nL-Nk);
    Yy = [Yy yy];
    Nk = Nk+1;
end
Yt = H2(ModO+1:nL);

INPv = [Yy Uu ];
paraX = pinv(INPv)*Yt
end

if (Selc == 5)
sim('staircasethreetankssystem.mdl');
% get the length of vector - U
SamT = input('Enter The sampling Time ----');
nLt = length(U);
US = U(1:SamT:nLt);
%US1 = U1(1:SamT:nLt);
YS = H3(1:SamT:nLt);
nL = length(US);
Yy = [];
Uu = [];
%Uu1 = [];
Nk = 1;
for i = ModO:-1:1

    uu = US(i:nL-Nk);
    Uu = [Uu uu];
    %uu1 = US1(i:nL-Nk);
    %Uu1 = [Uu1 uu1];
    yy = YS(i:nL-Nk);
    Yy = [Yy yy];
    Nk = Nk+1;
end
Yt = H2(ModO+1:nL);

INPv = [Yy Uu ];
paraX = pinv(INPv)*Yt
end

```

```

if (Selc == 6)
    sim('threetankVshape2.mdl');
    % get the length of vector - U
    SamT = input('Enter The sampling Time ----');
    nLt = length(U);
    US = U(1:SamT:nLt);
    US1 = U1(1:SamT:nLt);
    YS = H3(1:SamT:nLt);
    nL = length(US);
    Yy = [];
    Uu = [];
    Uu1 = [];
    Nk = 1;
    for i = ModO:-1:1

        uu = US(i:nL-Nk);
        Uu = [Uu uu];
        uu1 = US1(i:nL-Nk);
        Uu1 = [Uu1 uu1];
        yy = YS(i:nL-Nk);
        Yy = [Yy yy];
        Nk = Nk+1;
    end
    Yt = H3(ModO+1:nL);

    INPv = [Yy Uu Uu1];
    paraX = pinv(INPv)*Yt
end

```