

**DETERMINATION OF PRESSURE DROP IN A TWO-PHASE,
LIQUID-LIQUID SYSTEM IN A HORIZONTAL PIPELINE THROUGH
MATLAB SIMULATION**

By

Nor Hidayah binti Khalid

10915

Dissertation submitted in partial fulfilment of
the requirements for the
Bachelor of Engineering (Hons)
(Chemical Engineering)

SEPTEMBER 2011

Universiti Teknologi PETRONAS,
Bandar Seri Iskandar,
31750 Tronoh,
Perak Darul Ridzuan.

CERTIFICATION OF APPROVAL

**DETERMINATION OF PRESSURE DROP IN A TWO-PHASE,
LIQUID-LIQUID SYSTEM IN A HORIZONTAL PIPELINE THROUGH
MATLAB SIMULATION**

By

Nor Hidayah binti Khalid (10915)

A project dissertation submitted to the

Chemical Engineering Programme

Universiti Teknologi PETRONAS

in partial fulfilment of the requirement for the

BACHELOR OF ENGINEERING (Hons)

(CHEMICAL ENGINEERING)

Approved by,



(Mr Zamri Abdullah)

**UNIVERSITI TEKNOLOGI PETRONAS
TRONOH, PERAK
SEPTEMBER 2011**

ACKNOWLEDGEMENT

First and foremost, I give thanks and praise to God for His guidance and blessings throughout the entire course of my Final Year Project.

I wish to express my gratitude to my supervisor, Mr Zamri Abdullah, for his guidance and support throughout completing my Final Year Project as partial fulfilment of the requirement for the Bachelor of Engineering (Hons) of Chemical Engineering.

Many figures had provided immeasurable amount of guidance, ideas, assistance, support and advice. Without help from these people, this Final Year Project may not be that meaningful and successful. Greatest appreciation is expressed to them.

My acknowledgement would be incomplete without giving credit to Universiti Teknologi PETRONAS, especially Chemical Engineering Department which has equipped students with essential skills for self-learning.

Finally, I would like to thank my family. They have been a wonderful source of encouragement and joy to me and also not to forget the fellow colleagues. Their help was a tremendous gift to me. May God bless all of us and only He, the Almighty could repay all my debts to them.

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.



NOR HIDAYAH BINTI KHALID

ABSTRACT

The pressure drop of the flow inside the pipeline is an important parameter to be determined before proceeding with the design. This parameter is very important to pipeline size selection and the design of the downstream facilities. Underestimation of pressure drop will give a smaller pipe size than required, thus the transportation capacity will be restricted. In the other hand, overestimation of pressure drop will cause in oversized pipeline, worse sweeping characteristics, and possible solid dropout and corrosion issues. The wrong prediction of pressure drop is likely to occur in a liquid-liquid two phase system which false predictions of interface configurations are made. A flat interface is assumed between the phases which actually highly applicable for high-density differential system, such as gas-liquid system under earth condition. However, for liquid-liquid system with small density differences or in reduced gravity system, the factor of curvature interface must be considered. The interface configuration for liquid-liquid systems can either be flat, concave or convex. Hence, to overcome this problem, a model is developed to calculate pressure drop for liquid-liquid system that will consider the factor of curvature interface between the phases. In this modelling, two-fluid model is used for prediction of pressure drop and this model is derived to make it applicable for stratified flow system only. The model is developed by using MATLAB programming and it is tested with few sets of input data. The calculated pressure drop from this model is compared with experimental data to check for its reliability. As a conclusion, it is shown that flat-shape interface assumption is not the best assumption for this prediction. The percentage difference of prediction is very large when it was compared to experimental data. Curvature interfacial configuration is assumed to give best prediction, however, in this project, the curvature interface assumption not give an expected result. This is due to some ambiguity in cross sectional area and wetted perimeter derivation formula used in this model. Hence, modification in the correlated function has to be developed to prove that calculation using the curved interface will give better assumption of pressure drop.

TABLE OF CONTENTS

| | |
|--|----|
| CHAPTER 1 | 1 |
| INTRODUCTION..... | 1 |
| 1.1 Background of Study | 1 |
| 1.2 Problem Statement | 1 |
| 1.3 Objective and Scope of Study | 2 |
| CHAPTER 2..... | 3 |
| LITERATURE REVIEW | 3 |
| 2.1 General Description of Liquid-Liquid Flows: Flow Patterns..... | 3 |
| 2.2 Two-fluid Flow Experiments..... | 6 |
| 2.3 Two-fluid Flow Modelling | 9 |
| CHAPTER 3..... | 12 |
| METHODOLOGY | 12 |
| 3.1 Research Methodology..... | 12 |
| 3.2 The Two-Fluid Model (TFM)..... | 15 |
| 3.3 Calculation of Cross Sectional Area and Perimeter..... | 18 |
| 3.4 MATLAB Simulation | 20 |
| CHAPTER 4..... | 21 |
| RESULTS AND DISCUSSION | 21 |
| CHAPTER 5..... | 30 |
| CONCLUSION AND RECOMMENDATIONS..... | 30 |
| 4.1 Conclusion..... | 30 |
| 4.2 Recommendations..... | 30 |
| RERERENCES..... | 31 |
| Appendix I: List of Previous Researches | 32 |
| Appendix II: Gantt chart | 37 |
| Appendix III: List of Previous Experimental Works..... | 38 |
| Appendix IV: Area and Perimeter Calculation for Flat Interfacial Configuration..... | 39 |
| Appendix V: Area and Perimeter Calculation for Curvature Interfacial Configuration..... | 42 |
| Appendix VI: MATLAB Programming Code for Flat Interfacial Configuration by using Wire Probe Height..... | 49 |
| Appendix VII: MATLAB Programming Code for Flat Interfacial Configuration by using Ring Probe Height | 54 |
| Appendix VIII: MATLAB Programming Code for Curvature Interfacial Configuration..... | 59 |

LIST OF FIGURES

| | |
|--|----|
| Figure 1: Flow Patterns for Two- Phase, Liquid-Liquid Flow System (Hewitt et al. 2002) | 4 |
| Figure 2: Schematic Description of Stratified Flow Configuration (Bertola et al, 2003) | 5 |
| Figure 3: Flow pattern map for two-phase, oil-water flow in a 14.0 mm diameter pipe..... | 6 |
| Figure 4: Schematic of the parallel chromel wires (Liu et. al., 2008) | 7 |
| Figure 5: Schematic of the parallel ring probes (Liu et. al., 2008)..... | 8 |
| Figure 6: Methodology for the development of the programming code of the two-fluid model. | 14 |
| Figure 7: Cross sectional area of the pipeline for flat interface | 18 |
| Figure 8: The area of lune due to the curved interface in the pipeline. | 19 |
| Figure 9: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.55$ m/s | 21 |
| Figure 10: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.50$ m/s | 22 |
| Figure 11: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.45$ m/s | 22 |
| Figure 12: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.40$ m/s | 23 |
| Figure 13: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.35$ m/s | 23 |
| Figure 14: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.30$ m/s | 24 |
| Figure 15: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.25$ m/s | 24 |
| Figure 16: Interface shape at different contact angle (Lawrence, 2002) | 28 |
| Figure 17: Interfacial Shear Stress vs Superficial Velocity of oil (Ring Probe height).. | 29 |

LIST OF TABLES

| | |
|--|----|
| Table 1: Properties of Fluids used in Study | 12 |
| Table 2: Percentage difference between experimental data and theoretical data..... | 25 |

NOMENCLATURES

| | |
|---------|----------------------------|
| A | Cross sectional area |
| C | Constant |
| D | Diameter |
| f | Friction factor |
| g | Gravitational acceleration |
| n | Constant |
| P | Pressure |
| S | Perimeter of the tube |
| U | Velocity of a phase |
| z | Length of the tube |
| τ | Shear stress |
| ρ | Density |
| β | Contact Angle |
| μ | Viscosity |

CHAPTER 1

INTRODUCTION

1.1 Background of Study

The flows of two immiscible liquids are encountered in a diverse range of processes and equipments especially in the petroleum industry, where mixtures of oil and water are transported inside the pipeline over long distances. A lot of studies have been conducted to predict of oil-water flow characteristics, such as flow pattern, water holdup and pressure gradient of the flow (Bertola, 2003). In this project, the study will be specifically designed to determine the pressure drop for two-phase, liquid-liquid system inside the pipeline by using MATLAB as a programming tool.

The flow pattern of stratified flow is used in this study since it is considered as the basic flow configuration in horizontal and inclined two-phase systems of a finite density differential (Bertola, 2003) . The flow patterns are assumed to have three types of interface, which are convex, concave and plane-shaped interfaces. These types of interface are associated with a different contact area between the two fluids and between the fluids and the pipe wall. Depending on the physical system involved, these variations can have prominent effects on the pressure drop and transport phenomena in the system (Gorelik & Brauner, 1999).

1.2 Problem Statement

1.2.1 Problem Identification

Traditionally, the consideration of interface curvature is related to capillary and small scale systems, where the effect of surface tension becomes comparable with gravity. In a large scale system, the natural trend is to neglect the surface phenomena. This is justified in high-density differential systems, such as gas-liquid systems under earth conditions. However, in liquid-liquid systems with small density differences or in reduced gravity systems, surface phenomena maybe dominated which resulted in a curved interface configuration. This curved interface may significantly affect the local and integral two-phase flow characteristics (Brauner, Rovinsky, & Moalem

Maron, 1996). In order to do more precise pressure drop prediction for this system, the curvature effect is important and must take into account in the calculation.

1.2.2 Significance of the Project

Determination of the two phases, liquid-liquid system pressure drop is not as easy as gas-liquid system as it is a must to consider the curvature factor of the interface between the phases of liquid-liquid system. This modelling will assist engineers to obtain pressure drop value to be used as a main basis of their design. The closer the prediction, the better of size of pipeline can be made thus will be beneficial in terms of cost. As a conclusion, this modelling is very essential for engineers as they need to understand the characteristic of the flow system such as the pressure drop in order for them to design the pipeline with proper and safer size.

1.3 Objective and Scope of Study

1.3.1 Project Objective

The objective of this project is to develop a MATLAB programming code to simulate and predict the pressure drops in a two-phase, liquid-liquid system in a horizontal pipeline based upon various interfacial configurations. The calculated pressure drop from the simulation will be compared to the experimental data for validation.

1.3.2 Scope of Project

The project involves computer simulation work using MATLAB programming tool to predict the effect of interfacial curvature shapes towards the pressure drop in a two-phase, liquid-liquid flow in a horizontal pipe. Based upon two-fluid model and the experimental data of liquid heights in the pipeline system, a programming code will be developed that will calculate the differential pressure. The calculated pressure drops will be compared with the experimental pressure drop data previously found to validate the findings.

CHAPTER 2

LITERATURE REVIEW

2.1 General Description of Liquid-Liquid Flows: Flow Patterns

Flows of two immiscible liquids are encountered in a diverse range of processes and equipments such as petroleum industry, where mixtures of oil and water are transported in pipes over long distances. Lot of studies have been conducted to predict oil-water flow characteristics such as flow pattern, water hold-up and pressure gradient; and these characteristics are important in many engineering applications. However, despite their importance, liquid-liquid flows have not been explored to the same extent as gas-liquid flow (Brauner, 2003).

Diverse flow patterns were observed in liquid-liquid systems through their visual observation such as photographic/video techniques, or on abrupt changes in the average system pressure drop. Based on their observation, the flow patterns can be classified into four basic prototypes which includes stratified layers with either smooth or wavy interface; large slugs, elongated or spherical, of one liquid in the other; a dispersion of relatively fine drops of one liquid in the other; annular flow, where one of the liquids forms the core and the other liquid flows in the annulus. However, in many cases, the flow pattern is usually combination of these basic prototypes (Brauner, 2003).

Sketches of various possible flow patterns observed in horizontal systems are illustrated as in Figure 1. Stratified flow with a complete separation of the liquids may happen for some limited range of relatively low flow rates where the stabilizing gravity force due to a finite density difference is dominant. When the flow rates are increasing and exceed the upper limit of stratified flow, the interface will display a wavy character with possible entrainment of drops at one side or both sides of the interface.

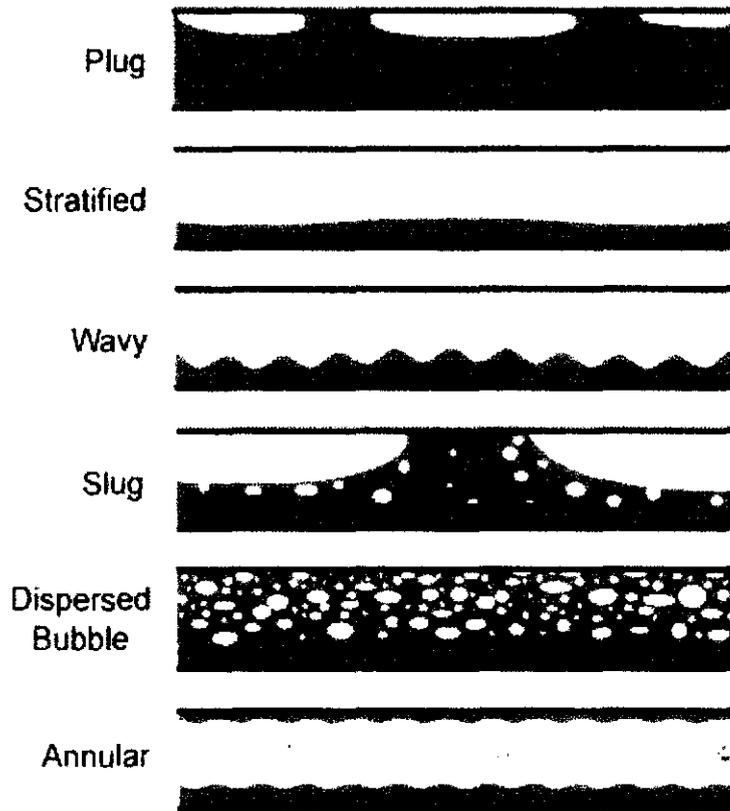


Figure 1: Flow Patterns for Two- Phase, Liquid-Liquid Flow System (Hewitt et al. 2002)

From the above figure, stratified flow is the simplest and basic flow. This type of flow is chosen as a case of study in this project. Further explanation of the stratified flow will be discussed in the following paragraph.

2.1.1 Stratified Flow

Stratified flow is considered as a basic flow pattern in horizontal configuration liquid-liquid systems of a finite density difference in some range of low flow rates. The two phases of liquids will segregate and form two layers in the pipeline. The modelling of liquid-liquid stratified flows phenomenon requires the consideration of additional aspects in comparison to gas-liquid stratified flow due to differences in their physical properties. The uncertainty in measuring an interfacial shear stress for liquid-liquid system is greater in comparison with gas-liquid system.

In liquid-liquid flow system which is having a relatively low density difference, surface tension and wetting effects become important, and the interface

shape (convex, concave, plane) is an additional field that has to be solved (Brauner, 2003). Figure 2 shows the basic interfacial configurations for a liquid-liquid system in a stratified flow.

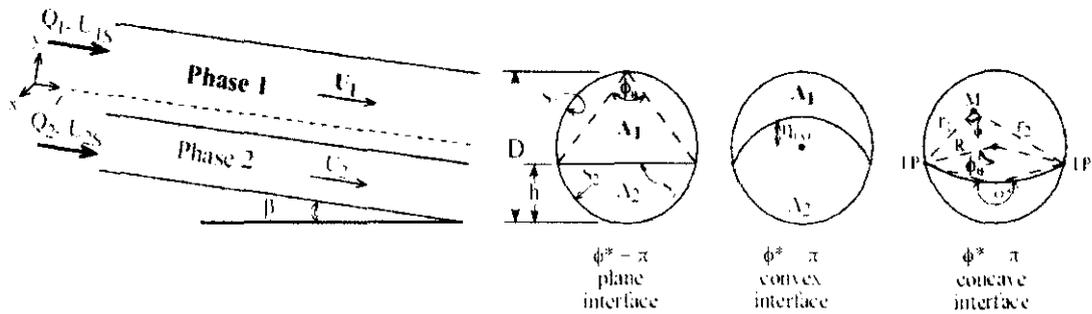


Figure 2: Schematic Description of Stratified Flow Configuration (Bertola et al, 2003)

A configuration of a curved interface is associated with a variation in the contact area between the two fluids, and between the fluids and the pipe wall. This variation will significantly affect on the pressure drop and transport phenomena depending on the physical properties involved.

Based on (Abdullah, 2008, 2009), at certain superficial velocity of water and superficial velocity of oil, stratified flow can be observed by using high-speed camera. Results from his observation are shown in Figure 3.

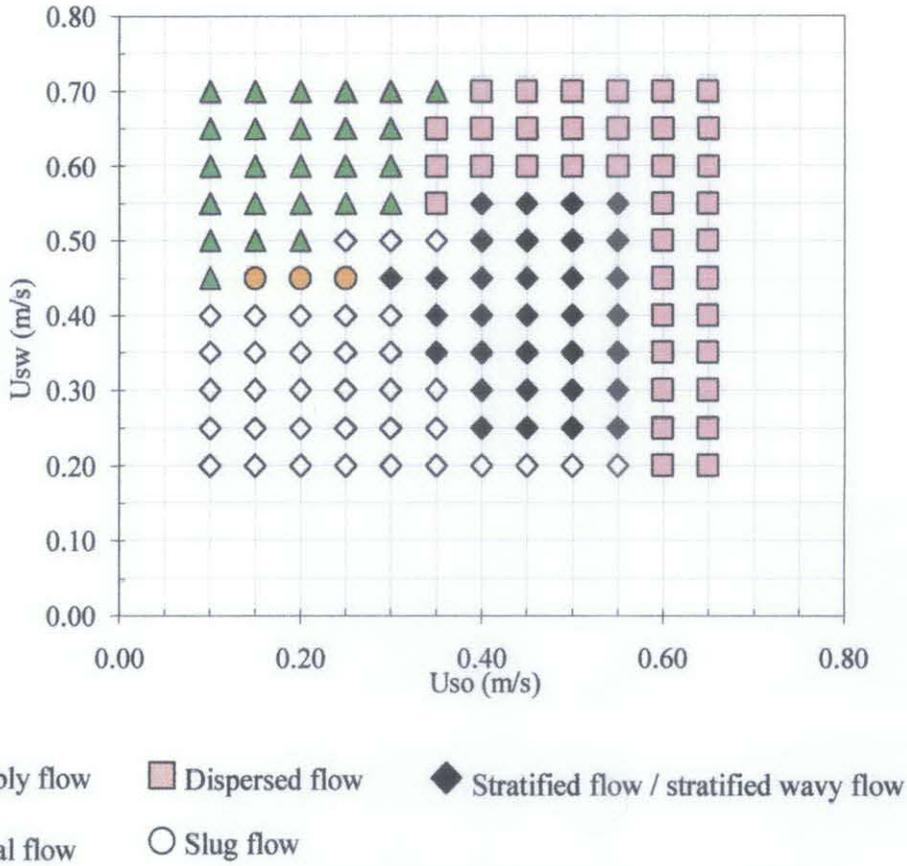


Figure 3: Flow pattern map for two-phase, oil-water flow in a 14.0 mm diameter pipe.

2.2 Two-fluid Flow Experiments

Many studies have been conducted to understand the characteristics of oil-water flow in horizontal pipelines. Some of the experimental results obtained are used to propose several flow pattern maps or correlations for horizontal oil-water flow. In addition to the experimental studies, models for predicting the flow pattern transitions have also been developed.

Most of the available experimental data are for small-diameter pipes and mineral oils. Even though these studies provide a considerable amount of information regarding oil-water flow patterns in horizontal pipes, several important aspects of this problem have yet to be considered (Arenas-Medina et al. 2000). The models for flow pattern transition prediction were validated using very limited experimental data. Thus, it is not clear whether the proposed criteria can be used to predict the flow pattern in real lines transporting liquid-liquid two phase system.

There are various methods has been applied in order to study the types of flow pattern in a two-phase, liquid-liquid system in pipeline. One of them is through the visualization technique using a high-speed video recording. This technique is very difficult to apply when studying flow patterns at high flow velocities where the interface may not have a clear shape. Moreover, flow visualization techniques require the use of pipes with transparent walls. The other method is using the photon attenuation technique, which however unsuitable for a system involving crude oil since its physical properties is almost similar to water.

The latest method is the utilization of conductivity probes, which requires the interpretation of the measured raw electrical signal into the local volume fractions of a phase. The highly fluctuating nature of two-phase flow often introduces large uncertainties in the signal processing. One example on the identification of flow patterns is carried out based on measurement of the transversal water fraction profile (Arenas-Medina et al., 2000).

Liu et. al, (2008) investigated the *in-situ* phase distribution of the two fluids in the pipeline by characterizing by the height of water climbing along the wall and the height of water layer of the vertical plane passing the pipe axis, which was measured by two sets of different conductance probes. Each set included parallel chromel wires and parallel ring probes with the spacing of 40 mm. A probe consisting of two chromel wires traversed the diameter of the pipe vertically as shown in Figure 4.

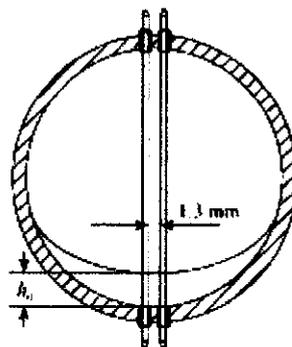


Figure 4: Schematic of the parallel chromel wires (Liu et. al., 2008)

The wire probe which consists of the parallel wires that behaved like a pair of parallel cylinders separated by a fixed distance of 1.3 mm. One of the wires was excited with a high-frequency alternating voltage inducing a current through the probe that was dependent on the height of water layer between the wires. On the other hand, the parallel ring probe is shown in Figure 5 was composed of a pair of brass rings with the thickness of 4 mm, and these rings were embedded flush with the inner surface of pipe covered by insullac. Nonconductive acrylic resin with the axial thickness of 10 mm was filled between the parallel rings. Both probes were statically calibrated by locating the depth of probes submerged by water.

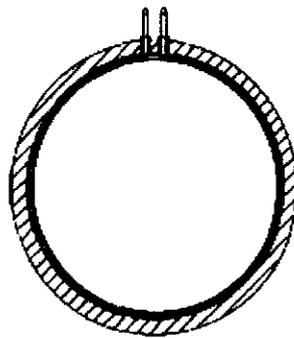


Figure 5: Schematic of the parallel ring probes (Liu et. al., 2008)

In order to get the image of the flow pattern, 30 frames per second digital video camera is placed at a position 0.5 m downstream of the inlet (Timmerhaus et. al, 2003). The image will be recorded through the acrylic viewing section by this camera and the data will be sent to a computer-based data acquisition for further analysis.

The list of other previous experimental works related to this study is shown in Appendix III.

2.3 Two-fluid Flow Modelling.

Two-fluid modelling has been studied by various researchers in both gas-liquid and liquid-liquid systems. A number of derived models based upon the two-fluid model have been developed. In this section, some of previous works on stratified flow were reviewed.

Brauner & Rovinsky (1996) explored that a configuration of a plane interface between two stratified layers is appropriate for two-phase system which are dominated by gravity, as is the case for large scale air-water system under the earth gravitation. However, for general two-fluid system the prescription of the characteristic interface curvature is required in order to initiate the solution of the flow problem and associated transport phenomena. Energy considerations are employed to predict the interface configuration. The effect of the fluid physical properties, *in situ* hold up, tube dimension and wall adhesion on the characteristic interface curvature are explored. The prediction of interface curvature provides the closure relation required for a complete solution of stratified flows with curved interfaces for a variety of two fluid systems.

The two-fluid model is also used to solve the momentum equations for a variable interface curvature (Brauner & Rovinsky, 1997). Energy considerations provide a closure relation for the interface curvature. The analysis identifies all the input dimensionless parameters which determine the solution for the stratified flow pattern. When these are given, a complete solution of the problem is obtained, including the interface shape, *in situ* hold-up and pressure drop. Two-fluid model provides a reasonable estimate of the *in situ* hold-up and pressure drop over a wide range of interfacial curvature and flow rates. The biggest error is obtained when the two-fluid model is applied for a configuration of a fully eccentric highly viscous core, in which case the two-fluid model significantly over predicts the lubrication effect of the less viscous phase.

Gorelik and Brauner (1999) found out that the analytical solutions for the interface shape between two immiscible fluids and for the capillary pressure in unidirectional axial laminar pipe flow is determined by three parameters. They are the holdup; the fluid or wall wettability angle, and the Eötvös number. The model of

constant characteristic curvature provides a good description of the interfacial shape and enables extending the parameter space where analytical solutions of stratified flow can be obtained.

The theory-based closure relations for the wall and interfacial shear stresses previously obtained for laminar stratified flow has been further expanded in order to be applied into turbulent flows in either or both of the phases (Brauner & Ullman, 2005). The closure relations are formulated in terms of the single phase-based expressions, which are augmented by two-phase interaction factors subjected to the flow of the two phases in the same channel. These closure relations were used as a platform for introducing necessary empirical corrections required in the stratified wavy flow regimes. They also had obtained new empirical correlation for the wave effect on the interface curvature, on the interfacial shear and on the liquid wall shear wear. The new closure relations are essentially representing correctly the interaction between the phases over a wide range of the stratified flow parameters space in the stratified smooth and stratified wavy regime.

In order to investigate flow pattern transition in horizontal pipelines carrying oil-water mixtures, full-scale experiments have been carried out by Arenas-Medina and the colleagues (2000). In the experiment, a 16-in pipeline conveying light crude oil was used and it was connected to freshwater network to control the input water volume fraction. A special device i.e. the multi-point sampling probe was designed and installed into the pipeline. Based on the water fraction data obtained from the experiment, a flow pattern map was constructed. The experimental stratified transition boundary was compared with the theoretical criteria obtained in the linear stability analysis of stratified two-phase liquid-liquid flow. It was found that the stratified transition can be predicted with reasonable accuracy based on the viscous Kelvin-Helmholtz analysis. The study also revealed that in stratified crude oil-water flow, complete phase separation does not occur. There is always a small amount of water dispersed almost uniformly in the oil layer.

A similar approach has also being employed by Chakrabarti et al. (2005) to study the pressure drop characteristics during the simultaneous flow of kerosene-water mixture. Using a horizontal pipeline facility of 0.025-m diameter pipe, measurements of pressure gradient were made for different combinations of phase

superficial velocities ranging from 0.03–2 m/s such that the regimes encountered includes the smooth stratified, wavy stratified, three layer flow, plug flow and oil dispersed in water flow patterns. A model was developed, which considered the energy minimization and pressure equalization of both phases.

On the other note, Fan and Wang (2007) proposed a new closure relationship of wetted-wall fraction: liquid-wall friction factor and interfacial-friction factor. An iterative calculation procedure was proposed to solve the two-fluid model for liquid hold up and pressure gradient. Comparison between model predictions and experimental data show that the proposed model agrees well with the data collected in the present study. As a result, the average percentage errors of liquid holdup and pressure-gradient prediction are 2.9% and 3.2%, respectively.

Liu et al. (2008) had performed experiments to study the segregated flow pattern in a 26.1–mm diameter, horizontal, stainless steel test section. The oil-water interfacial behaviour was observed carefully. Due to the dominant effect of interfacial tension and wall-wetting properties of liquids over the gravity especially for small Eötvös number system, the oil water interface exhibits a concave-down configuration. Two-fluid model has been used to calculate a pressure gradient in this system. Comparison between experimental data and theoretical data shows that the experimental data agrees well with the measurement after the conventional two-fluid model is extended to tackle segregated flow with curved interface. The full description of the researches will be shown in tabulated table in Appendix I.

A lot of benefits can be obtained from this study. As example, from the pressure drop profile prediction, engineers can predict the behaviour of the flow thus helping them to make a preliminary design in pipeline sizing. If the pressure drop is expected to be very high or too low, the engineer must take action and do modification in their design before finalizing their design and before it is ready to be manufactured.

CHAPTER 3

METHODOLOGY

3.1 Research Methodology

In this study, a computer simulation work involving the MATLAB programming will be carried out to determine the pressure drop of a two-phase, liquid-liquid system in a horizontal pipeline. Available data and fluid properties from the actual experiment such as the height of wire and ring probes; liquid densities, viscosities and flow rates; experimental pressure drops; are obtained from the study previously done by Abdullah (2008, 2009). The details and properties of fluids involved in this study are shown in Table 1.

Table 1: Properties of Fluids used in Study

| Properties | Oil | Water |
|--------------------------------|------------|--------------|
| Density, kg/m ³ | 828.00 | 1000.00 |
| Viscosity, mPa·s at 25 °C | 5.50 | 1.00 |
| Surface tension, mN/m at 25 °C | 39.6 | |

As stated in above table, two liquids that are used in the experimental works which are oil and water with properties as shown in Table 1. The pipeline used in experimental work has 14-mm inner diameter, and 50 cm in length for the pressure drop calculation. Other properties such as wire-probe height, ring-probe height, oil flow rate and water flow rates are obtained during experiment and will be used as input data for this model.

The calculation of pressure drop is divided into two main assumptions which are:

- Calculation of pressure drop in the pipeline system with assumption of flat interface by using wire probe and ring probe heights that were obtained from experiment.
- Calculation of pressure drop in the pipeline system with assumption of curvature interfacial shape by using a combination of both wire probe and ring probe heights that were obtained from experiment.

Previous input data are used to determine other parameters or variables required to be used in two-fluid model. A MATLAB programming code will be developed for calculating all parameters in the two-fluid model system. Figure 6 shows the methodology for the development of the programming code of the two-fluid model.

Pressure drop gained from this model is analysed and compared with experimental data. The variation between experimental and calculated data from the outcome of the simulation is determined, whereby the validity of the model is thus justified based upon the comparison.

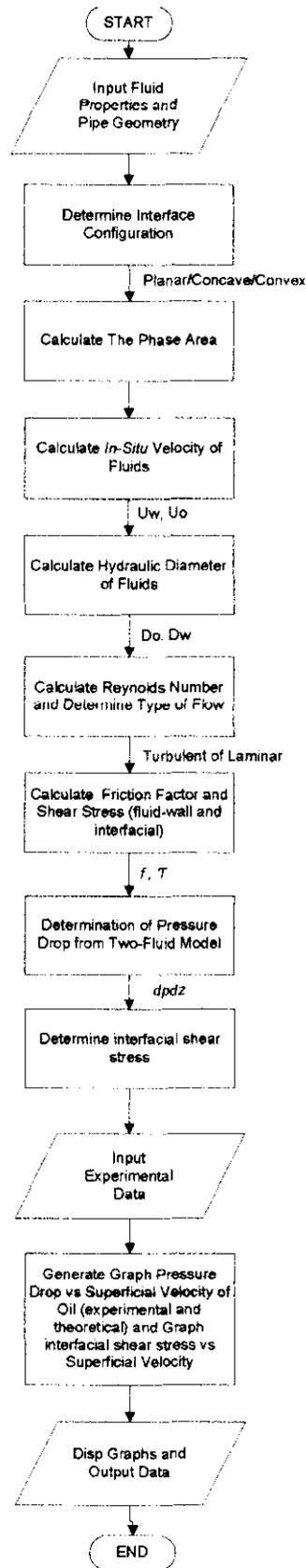


Figure 6: Methodology for the development of the programming code of the two-fluid model.

3.2 The Two-Fluid Model (TFM)

Two-fluid model is used in this study due to of its capability in handling laminar and turbulent flows in horizontal and inclined systems, both in co-current and counter-current stratified flows (Brauner, 2003). By assuming a fully developed flow, the integral forms of the momentum equations for the two fluids are shown as below.

$$-A_1 \left(\frac{dP}{dz} \right) - \tau_1 S_1 - \tau_i S_i + \rho_1 A_1 g \sin \beta = 0 \quad (1)$$

$$-A_2 \left(\frac{dP}{dz} \right) - \tau_2 S_2 + \tau_i S_i + \rho_2 A_2 g \sin \beta = 0 \quad (2)$$

Where,

τ_1, τ_2, τ_i = Shear stresses for phase 1, 2 and interfacial shear stress;

S_1, S_2, S_i = Liquid wetted-wall perimeter for phase 1, 2 and interfacial perimeter;

A_1, A_2 = Cross sectional area for phase 1 and 2;

$\left(\frac{dP}{dz} \right)$ = pressure drop in liquid phase;

β = Pipe inclination angle, degrees.

g = Gravitational acceleration.

If the pipeline is assumed horizontal (no inclination), the degree of inclination, β is equal to zero. Thus, equation (1) and (2) can be simplified into:

$$-A_1 \left(\frac{dP}{dz} \right) - \tau_1 S_1 - \tau_i S_i = 0 \quad (3)$$

$$-A_2 \left(\frac{dP}{dz} \right) - \tau_2 S_2 + \tau_i S_i = 0 \quad (4)$$

The perimeter (S) and the area (A) of the phase system can be calculated by performing trigonometric derivation equation. The Blasius equation is used to provide the closure laws required for the wall and interfacial shear stresses (τ_1, τ_2, τ_i) in terms of the average velocities, U_1, U_2 and friction factors f_1, f_2 and f_i .

$$\tau_1 = -\frac{1}{2}f_1\rho_1U_1|U_1|; \quad f_1 = C_1\left(\frac{\rho_1D_1|U_1|}{\mu_1}\right)^{-n_1} \quad (5)$$

$$\tau_2 = -\frac{1}{2}f_2\rho_2U_2|U_2|; \quad f_2 = C_2\left(\frac{\rho_2D_2|U_2|}{\mu_2}\right)^{-n_2} \quad (6)$$

$$\tau_i = -\frac{1}{2}f_i\rho_i(U_1 - U_2)|U_1 - U_2| \quad (7)$$

Where,

U_1, U_2 = velocity for phase 1 and 2;

f_1, f_2, f_i = friction factor for phase 1, 2 and interfacial friction factor;

ρ_1, ρ_2, ρ_i = density for phase 1, 2 and interfacial density;

μ_1, μ_2 = viscosity for phase 1 and 2;

C_1, C_2 = constants for phase 1 and 2, $C = 16$ for laminar flow and $C = 0.046$ for turbulent flow;

n_1, n_2 = constants for phase 1 and 2, $n = 1$ for laminar flow and $n = 0.2$ for turbulent flow.

Clearly, the two phases in stratified flow may result in laminar laminar (L L), laminar turbulent (L T), turbulent laminar (T L), or (turbulent turbulent (T T) regimes (Brauner, 1996).

The Reynolds numbers for the two fluids are based on the equivalent hydraulic diameters, which are defined according to the relative velocity of the phases. In co-current flow, the interface is considered as ‘free’ for the slower phase and as a “wall” for the faster phase.

When the velocities are of the same order, the interface is considered “free” with respect to both phases (Brauner, 2003).

$$D_1 = \frac{4A_1}{(s_1+s_i)}; \quad D_2 = \frac{4A_2}{s_2}; \quad \rho = \rho_1 \text{ and } f_i = F_1f_1 \quad \text{for } |U_1| > |U_2| \quad (8)$$

$$D_1 = \frac{4A_1}{s_1}; \quad D_2 = \frac{4A_2}{(s_2+s_i)}; \quad \rho = \rho_2 \text{ and } f_i = F_2f_2 \quad \text{for } |U_2| > |U_1| \quad (9)$$

$$D_1 = \frac{4A_1}{s_1}; \quad D_2 = \frac{4A_2}{s_2}; \quad \tau_i \cong 0 \quad \text{for } U_1 \cong U_2 \quad (10)$$

A value of $F_i > 1$ is introduced to account for a possible augmentation of f_i due to irregularities at the free interface. However, the interface appears less roughened compared to gas-liquid systems due to the lower density difference (hence velocity) and surface tension encountered in liquid-liquid systems.

Assuming that the pressure drops for both liquid phases are equal, equation (3) and (4) can be combined as the followings:

$$-A_1 \left(\frac{dP}{dz} \right) - \tau_1 S_1 - \tau_i S_i = 0 \quad (3)$$

$$-A_2 \left(\frac{dP}{dz} \right) - \tau_2 S_2 + \tau_i S_i = 0 \quad (4)$$

Considering $\tau_i S_i$ is the same, equation (3) + (4) will result,

$$-A_1 \left(\frac{dP}{dz} \right) - \tau_1 S_1 - \tau_i S_i - A_2 \left(\frac{dP}{dz} \right) - \tau_2 S_2 + \tau_i S_i = 0$$

$$-A_1 \left(\frac{dP}{dz} \right) - \tau_1 S_1 - A_2 \left(\frac{dP}{dz} \right) - \tau_2 S_2 = 0$$

$$-\left(\frac{dP}{dz} \right) (A_1 + A_2) - \tau_1 S_1 - \tau_2 S_2 = 0$$

$$-\left(\frac{dP}{dz} \right) = \frac{\tau_1 S_1 + \tau_2 S_2}{(A_1 + A_2)} \quad (11)$$

From this combined equation, pressure drop of the system can be simply obtained thus further calculation for interfacial stress can be done.

$$\tau_i = \frac{A_1 \left(\frac{dP}{dz} \right) + \tau_1 S_1}{-S_i} \quad (12)$$

3.3 Calculation of Cross Sectional Area and Perimeter

In order to calculate oil and water cross sectional area, wetted perimeter, and interface perimeter to be used later in two-fluid model (TFM), basic geometry is considered and calculations are made by using basic trigonometry. The drawing of cross sectional area of the horizontal pipeline is shown as in Figure 7.

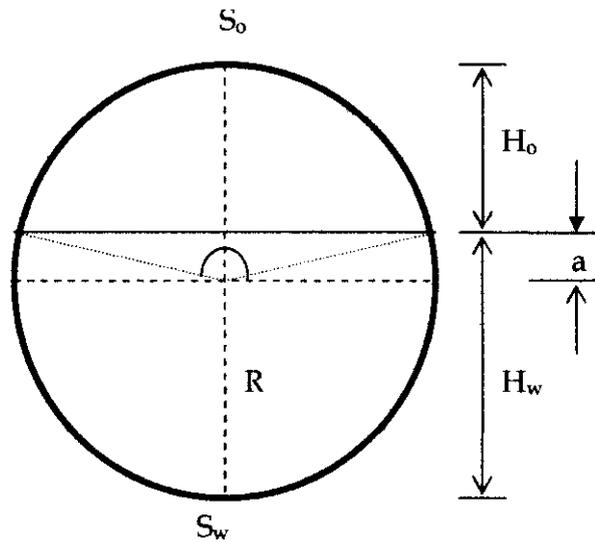
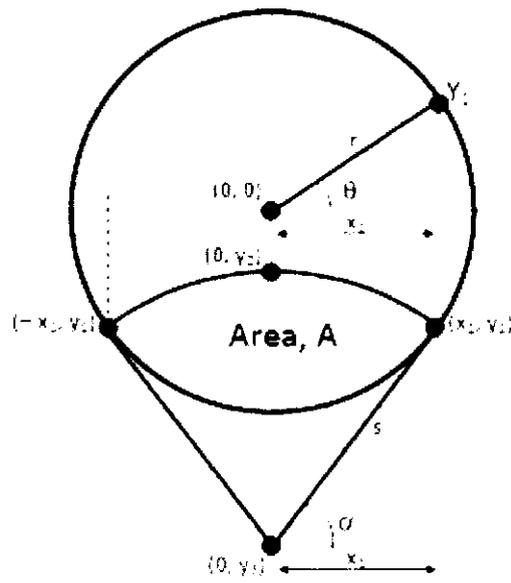


Figure 7: Cross sectional area of the pipeline for flat interface

Wire probe and ring probe height data that were obtained from experiment are important parameters to use in calculation of cross sectional area. The derivation of cross sectional area of water and oil, wetted perimeters of oil, water and interface perimeter will be shown in Appendix IV.



Y_2

Figure 8: The area of lune due to the curved interface in the pipeline.

As well as flat interface configuration, curvature interfacial configuration is also derived based upon wire and ring probes' heights obtained from experiment. However, the derivation is not as simple as flat interface configuration. The basic idea of this derivation is a combination of two circles, one circle is the true cross sectional area of the pipeline and another is an imaginary circle that makes the curvature interface line. These two circles are having two different centres of circle. The interception area of two circles is calculated as cross sectional area of water and the rest will be considered as cross sectional area of oil. The derivation of cross sectional area and perimeters for phases, oil and water will be shown in Appendix V.

3.4 MATLAB Simulation

The formula for prediction of pressure drop has been translated into MATLAB programming language. Hence, pressure drop prediction can be easily simulated in MATLAB. The MATLAB programming code for interface and curvature interfacial configuration will be shown in Appendix VI-VIII.

CHAPTER 4

RESULTS AND DISCUSSION

The comparison between the pressure drops of the oil-liquid, stratified flows in the horizontal pipeline are shown Figure 9 –Figure 15. The figures illustrated the results obtained through MATLAB simulation using the plane-shaped and curved interfaces, and the experimental data. The summary of the findings are tabulated in Table 2.

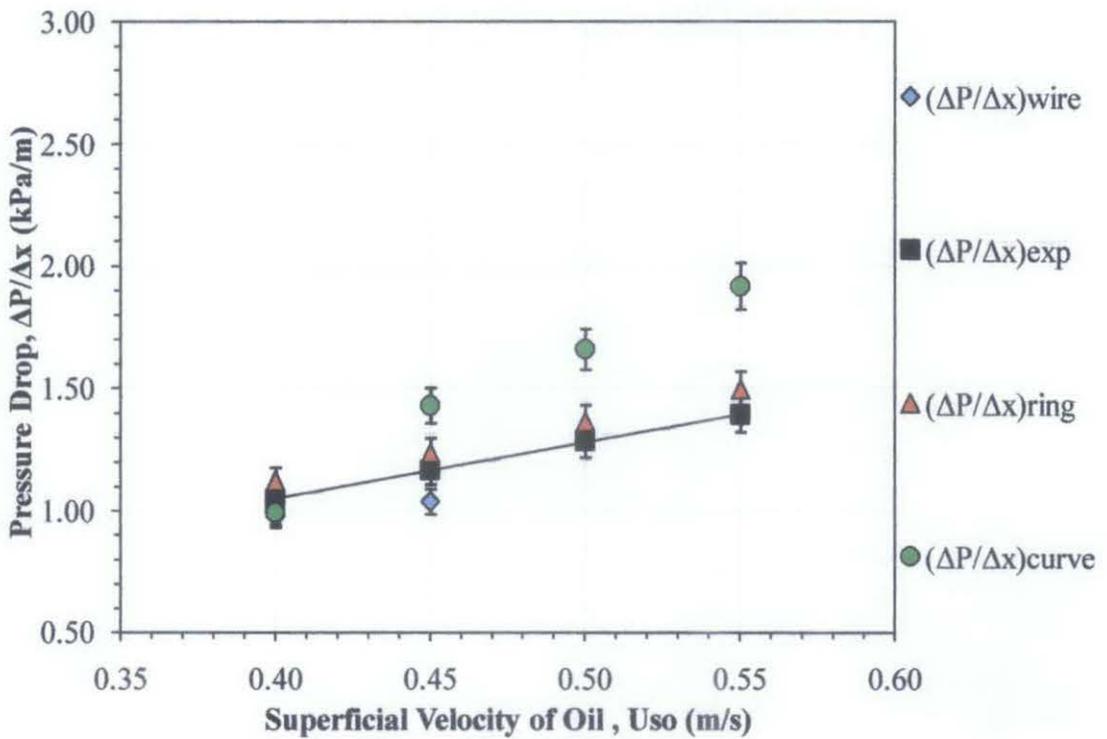


Figure 9: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.55$ m/s

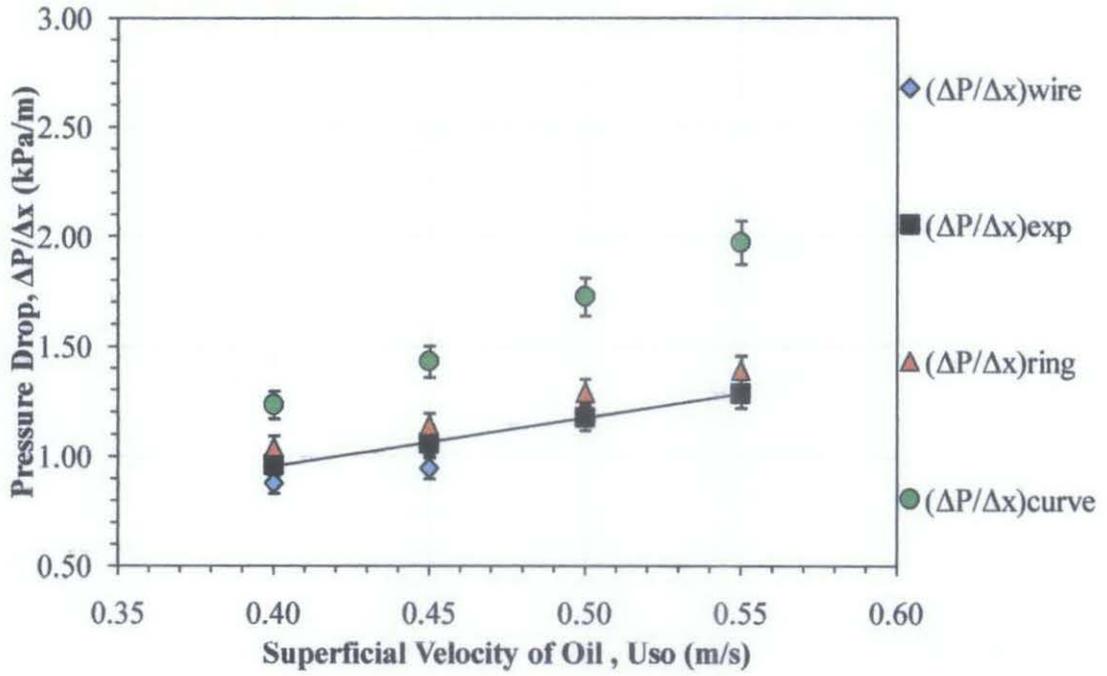


Figure 10: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.50$ m/s

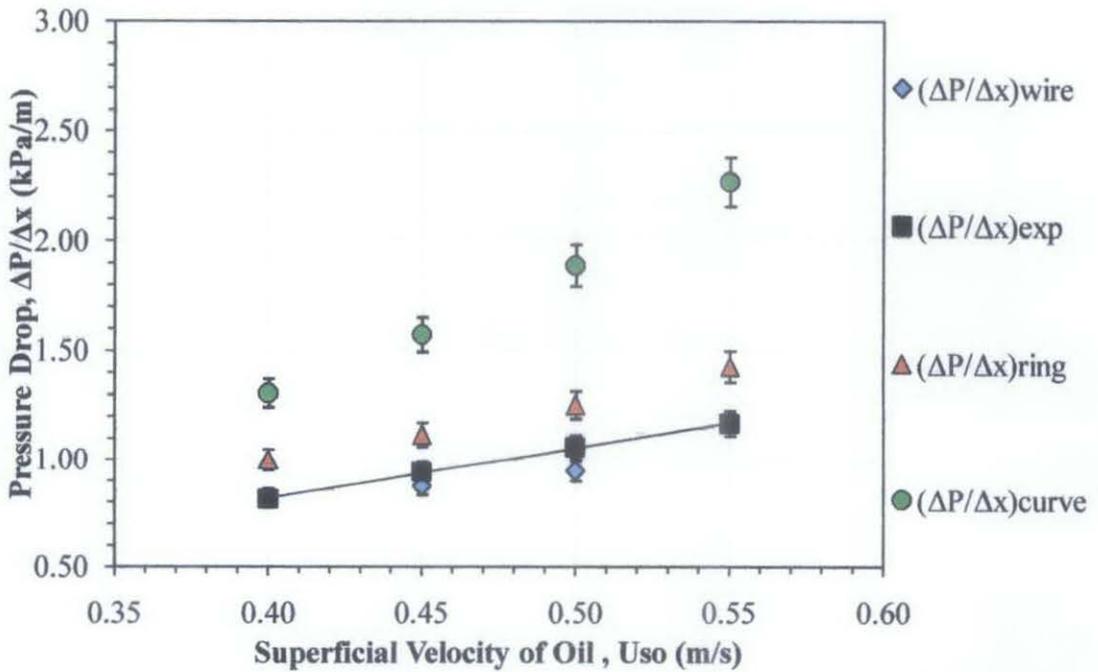


Figure 11: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.45$ m/s

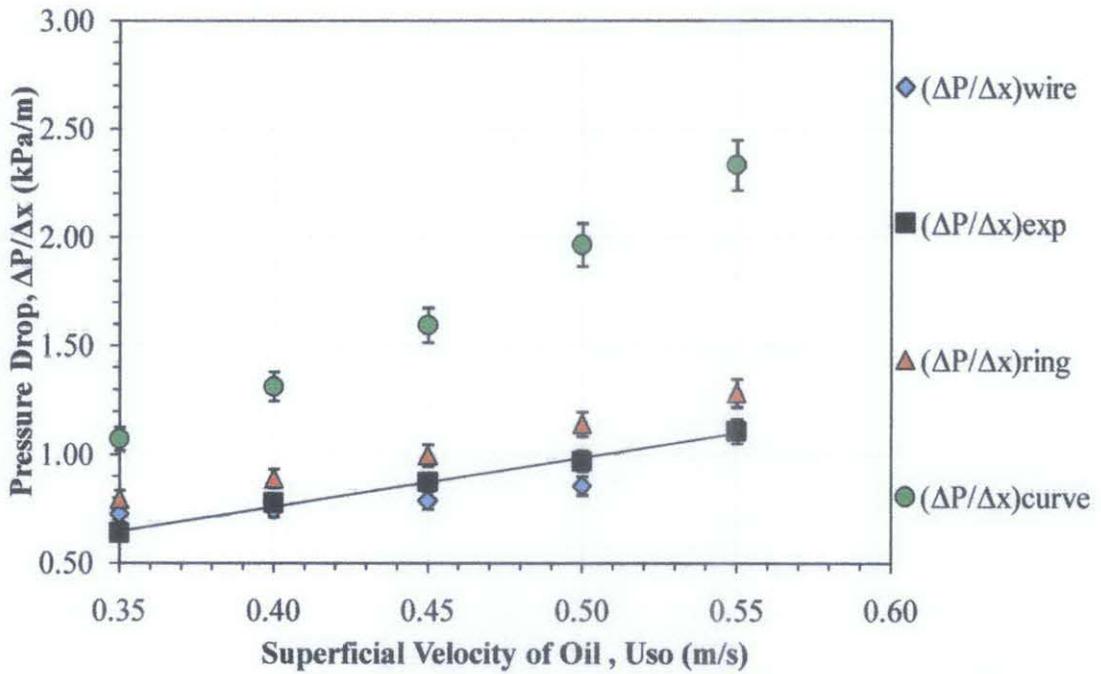


Figure 12: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.40$ m/s

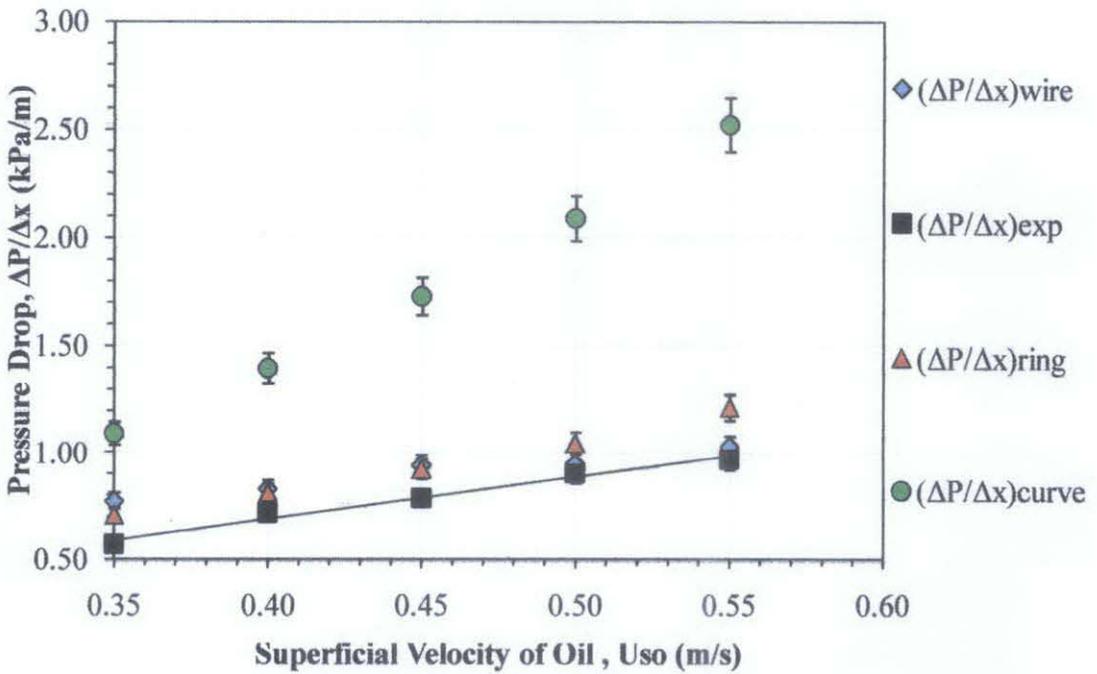


Figure 13: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.35$ m/s

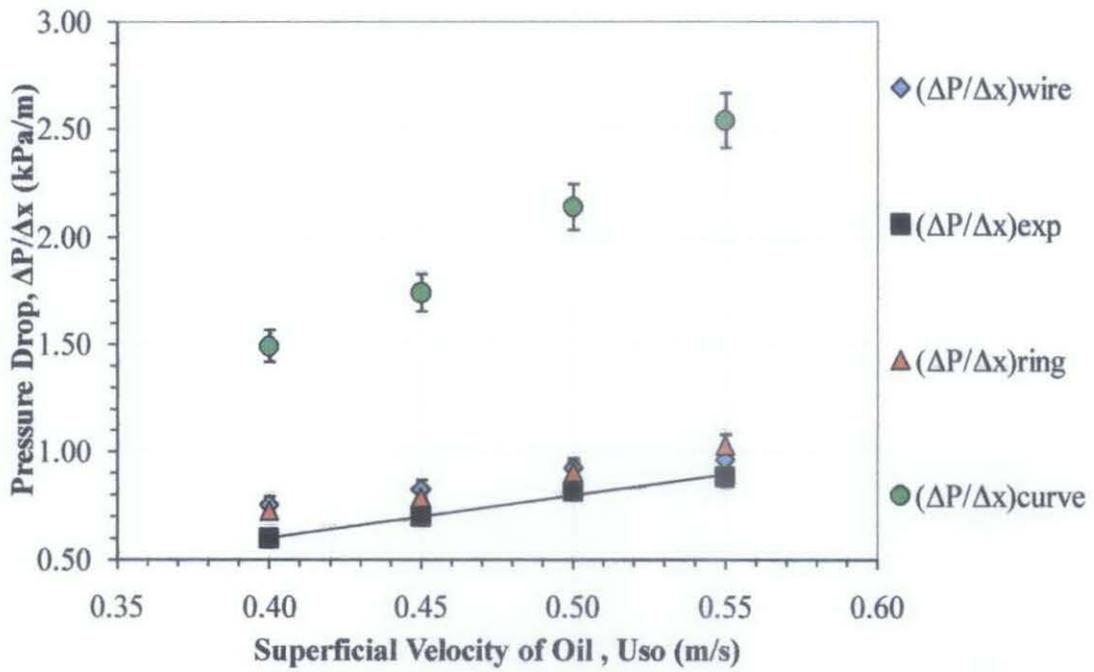


Figure 14: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.30$ m/s

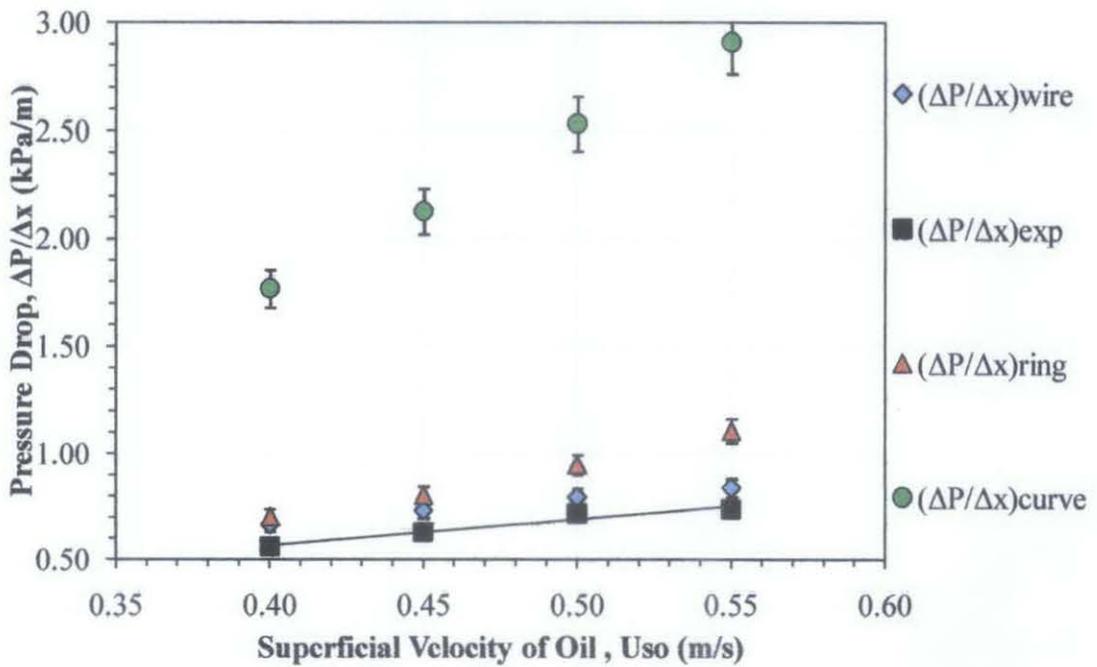


Figure 15: Comparison between the experimental pressure drop with the calculated pressure drop for water superficial velocity, $U_{sw} = 0.25$ m/s

Table 2: Percentage difference between experimental data and theoretical data

| U _{sw} | U _{so} | Wire Probe Height | Ring Probe Height | Experimental $\Delta P/\Delta x$ | Flowrate Oil | Flowrate water | Calculated $\Delta P/\Delta x$ (wire probe) | Calculated $\Delta P/\Delta x$ (ring probe) | Calculated $\Delta P/\Delta x$ (curvature) | Percentage difference (wire probe) | Percentage difference (ring probe) | Percentage difference (curvature) |
|-----------------|-----------------|-------------------|-------------------|----------------------------------|-----------------------|-----------------------|---|---|--|------------------------------------|------------------------------------|-----------------------------------|
| m/s | m/s | mm | mm | kPa/m | m ³ /s | m ³ /s | kPa/m | kPa/m | kPa/m | % | % | % |
| 0.55 | 0.40 | 7.19 | 6.17 | 1.05 | 6.16×10^{-5} | 8.47×10^{-5} | 0.98 | 1.12 | 1.00 | 6.65 | 7.01 | 5.15 |
| | 0.45 | 7.03 | 5.85 | 1.17 | 6.93×10^{-5} | 8.47×10^{-5} | 1.04 | 1.24 | 1.43 | 10.96 | 5.86 | 22.55 |
| | 0.50 | 6.66 | 5.53 | 1.29 | 7.70×10^{-5} | 8.47×10^{-5} | 1.12/1.24 | 1.36 | 1.66 | #N/A | 6.20 | 29.24 |
| | 0.55 | 6.52 | 5.26 | 1.39 | 8.47×10^{-5} | 8.47×10^{-5} | 1.18/1.35 | 1.50 | 1.92 | #N/A | 7.36 | 37.74 |
| 0.50 | 0.40 | 7.27 | 5.92 | 0.96 | 6.16×10^{-5} | 7.70×10^{-5} | 0.88 | 1.04 | 1.24 | 8.49 | 8.39 | 28.57 |
| | 0.45 | 6.89 | 5.65 | 1.06 | 6.93×10^{-5} | 7.70×10^{-5} | 0.95 | 1.14 | 1.43 | 10.38 | 7.57 | 35.21 |
| | 0.50 | 6.62 | 5.25 | 1.18 | 7.70×10^{-5} | 7.70×10^{-5} | 1.02/1.13 | 1.29 | 1.73 | #N/A | 9.47 | 46.71 |
| | 0.55 | 6.37 | 5.06 | 1.29 | 8.47×10^{-5} | 7.70×10^{-5} | 1.09/1.24 | 1.39 | 1.97 | #N/A | 8.12 | 53.61 |
| 0.45 | 0.40 | 6.69 | 5.48 | 0.81 | 6.16×10^{-5} | 6.93×10^{-5} | 0.82 | 1.00 | 1.31 | 1.17 | 22.74 | 60.70 |
| | 0.45 | 6.54 | 5.16 | 0.94 | 6.93×10^{-5} | 6.93×10^{-5} | 0.88 | 1.12 | 1.57 | 6.76 | 18.53 | 67.03 |
| | 0.50 | 6.25 | 4.84 | 1.06 | 7.70×10^{-5} | 6.93×10^{-5} | 0.95 | 1.25 | 1.89 | 10.61 | 18.29 | 78.27 |
| | 0.55 | 6.01 | 4.50 | 1.17 | 8.47×10^{-5} | 6.93×10^{-5} | 1.02/1.15 | 1.43 | 2.27 | #N/A | 22.21 | 94.27 |
| 0.40 | 0.35 | 6.21 | 5.68 | 0.64 | 5.39×10^{-5} | 6.16×10^{-5} | 0.73 | 0.79 | 1.07 | 13.98 | 24.48 | 68.19 |
| | 0.40 | 6.44 | 5.34 | 0.77 | 6.16×10^{-5} | 6.16×10^{-5} | 0.75 | 0.89 | 1.31 | 3.61 | 14.78 | 69.30 |
| | 0.45 | 6.43 | 5.02 | 0.87 | 6.93×10^{-5} | 6.16×10^{-5} | 0.79 | 1.00 | 1.60 | 9.68 | 14.06 | 82.70 |
| | 0.50 | 6.10 | 4.64 | 0.97 | 7.70×10^{-5} | 6.16×10^{-5} | 0.85 | 1.14 | 1.96 | 11.98 | 17.40 | 102.08 |
| | 0.55 | 5.80 | 4.35 | 1.11 | 8.47×10^{-5} | 6.16×10^{-5} | 0.93/1.05 | 1.28 | 2.33 | #N/A | 15.70 | 110.17 |
| 0.35 | 0.35 | 7.02 | 5.46 | 0.57 | 5.39×10^{-5} | 5.39×10^{-5} | 0.77 | 0.71 | 1.09 | 35.63 | 24.29 | 92.13 |
| | 0.40 | 6.70 | 5.03 | 0.72 | 6.16×10^{-5} | 5.39×10^{-5} | 0.83 | 0.81 | 1.40 | 15.80 | 13.50 | 94.95 |

| | | | | | | | | | | | | |
|------|------|------|------|------|-----------------------|-----------------------|------|------|------|-------|-------|--------|
| | 0.45 | 7.27 | 4.68 | 0.78 | 6.93×10^{-5} | 5.39×10^{-5} | 0.94 | 0.92 | 1.73 | 20.07 | 17.73 | 120.21 |
| | 0.50 | 6.33 | 4.37 | 0.90 | 7.70×10^{-5} | 5.39×10^{-5} | 0.95 | 1.05 | 2.09 | 5.24 | 15.87 | 131.43 |
| | 0.55 | 6.62 | 4.01 | 0.97 | 8.47×10^{-5} | 5.39×10^{-5} | 1.03 | 1.21 | 2.52 | 6.09 | 25.05 | 159.52 |
| 0.30 | 0.40 | 6.54 | 4.74 | 0.60 | 6.16×10^{-5} | 4.62×10^{-5} | 0.75 | 0.73 | 1.49 | 26.04 | 21.69 | 149.40 |
| | 0.45 | 6.63 | 4.60 | 0.70 | 6.93×10^{-5} | 4.62×10^{-5} | 0.83 | 0.79 | 1.74 | 18.66 | 13.10 | 149.93 |
| | 0.50 | 6.94 | 4.24 | 0.81 | 7.70×10^{-5} | 4.62×10^{-5} | 0.92 | 0.90 | 2.14 | 13.32 | 11.01 | 162.77 |
| | 0.55 | 6.62 | 3.94 | 0.88 | 8.47×10^{-5} | 4.62×10^{-5} | 0.96 | 1.03 | 2.54 | 8.92 | 16.46 | 187.71 |
| 0.25 | 0.40 | 5.78 | 4.11 | 0.56 | 6.16×10^{-5} | 3.85×10^{-5} | 0.67 | 0.70 | 1.77 | 19.03 | 25.74 | 216.03 |
| | 0.45 | 6.05 | 3.83 | 0.63 | 6.93×10^{-5} | 3.85×10^{-5} | 0.73 | 0.80 | 2.13 | 16.57 | 27.99 | 238.77 |
| | 0.50 | 6.09 | 3.49 | 0.72 | 7.70×10^{-5} | 3.85×10^{-5} | 0.79 | 0.95 | 2.53 | 11.01 | 32.18 | 253.66 |
| | 0.55 | 5.71 | 3.20 | 0.74 | 8.47×10^{-5} | 3.85×10^{-5} | 0.84 | 1.11 | 2.91 | 13.86 | 50.57 | 295.47 |

Based on Table 2, at water superficial velocity is 0.55 m/s, pressure drop calculated based on wire probe height by assuming flat interface, $(\Delta P/\Delta x)_{\text{wire}}$, is under-predicted the experimental value. In contrast, pressure drop calculated based on ring probe height by assuming flat interface, $(\Delta P/\Delta x)_{\text{ring}}$, is slightly over-predicted the experimental value with maximum percentage difference of 7.36%.

At point of oil superficial velocities are 0.50 m/s and 0.55 m/s, the pressure drop cannot be determined using this model since the phases are appeared to be in transitional flow. Some of parameters required by this model such as shear stress for both phases cannot be determined in a transitional flow system.

Based on the calculated Reynolds number, most of transitional phases are approaching laminar with range of Reynolds number from 2038 until 2221. Since, the exact value of pressure drop cannot be calculated in this phase, the phase is assumed to approach either in laminar or turbulent phase. Thus, the value of pressure drop is calculated based upon these two flow regimes and the pressure drop is assumed to be either one of the value. For example, as shown in Table 2 at superficial velocity of water is 0.55 m/s and superficial velocity of oil is 0.50 m/s, the oil phase is appeared in transitional phase. Thus, the pressure drop is assumed to be either 1.12 kPa/m or 1.23 kPa/m.

As the water superficial velocity decreases to 0.50 m/s, the predictions are not improved. When it is decrease further to 0.45 m/s, $(\Delta P/\Delta x)_{\text{ring}}$ is highly over-predicted the value with maximum percentage difference of 22.74%.

At water superficial velocity is 0.40 m/s, $(\Delta P/\Delta x)_{\text{wire}}$ is still under-predicted the value. However, when oil superficial velocity is 0.40 m/s, the prediction is very close to the experimental data with percentage difference of 3.61%. When water superficial velocity goes down to 0.35 m/s, both $(\Delta P/\Delta x)_{\text{wire}}$ and $(\Delta P/\Delta x)_{\text{ring}}$ are over-predicted the value. However, $(\Delta P/\Delta x)_{\text{wire}}$ are close to the experimental pressure drop, $(\Delta P/\Delta x)_{\text{exp}}$ at 0.50 m/s and 0.55 m/s oil superficial velocities with difference of 5 to 6%.

Water superficial velocity is decreased further to 0.30 m/s and finally to 0.25 m/s. As a result, both $(\Delta P/\Delta x)_{\text{wire}}$ and $(\Delta P/\Delta x)_{\text{ring}}$ are always over-predicted the value. As overall, it can be observed that $(\Delta P/\Delta x)_{\text{wire}}$ data gives closest prediction as compared to $(\Delta P/\Delta x)_{\text{ring}}$.

Prediction of pressure drop based upon wire and ring probe heights by assuming curvature interface, $(\Delta P/\Delta x)_{\text{curve}}$ between the phases should give better prediction as has been discussed in the previous section. However, through the approach carried out in this project, $(\Delta P/\Delta x)_{\text{curve}}$ deviates tremendously from the values determined when ring or wire probes data only being used, in the percentage of error ranges between 5% to 296%. It is foreseen that this deviation is attributed to the utilization of incorrect correlation in the formulae derived to determine the cross sectional area for oil/water.

Hence, modification in the correlated function has to be developed to prove that calculation using the curved interface will give better assumption of pressure drop. One way to achieve this is by redefining the terms of imaginary circle's radius, s used in the derivation steps.

Another reason of the error is might be due to interfacial configuration itself. The configuration may not be as assumed. It may not having a curvature or plane configuration as predicted but the interfacial configuration might having a flat interface with a slight curve at the middle point and near the wall as shown in the Figure 16. As mention before, the curve of the interface is highly depending on the contact angle between the phases and the wall. If there is a case, the curvature assumption may not be accurate, however, flat-interface assumption either using wire probe height or ring probe height may give closer prediction.

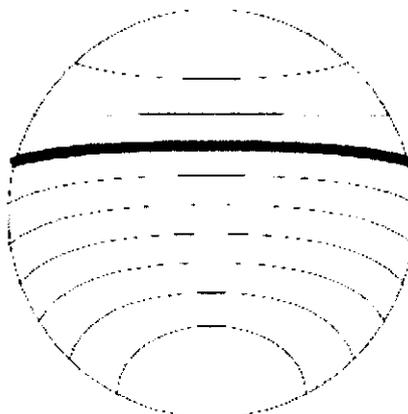


Figure 16: Interface shape at different contact angle (Lawrence, 2002)

From the above results, it is also can be observed that when superficial velocities of oil, U_{so} and water, U_{sw} increase, pressure drop also increases. This is because of the increases of shear stresses for both phases when the velocity of oil/water increases. As a result, pressure drop will also increase due to increases in resistance of flow inside the pipeline.

Interfacial shear stress also increases when superficial velocity increases. This can be observed in Figure 17. At constant superficial velocity of water, when superficial velocity of oil increases, the interfacial shear stress will also increase. Thus, this is also one of the factors that cause pressures to reduce as the superficial velocities increase.

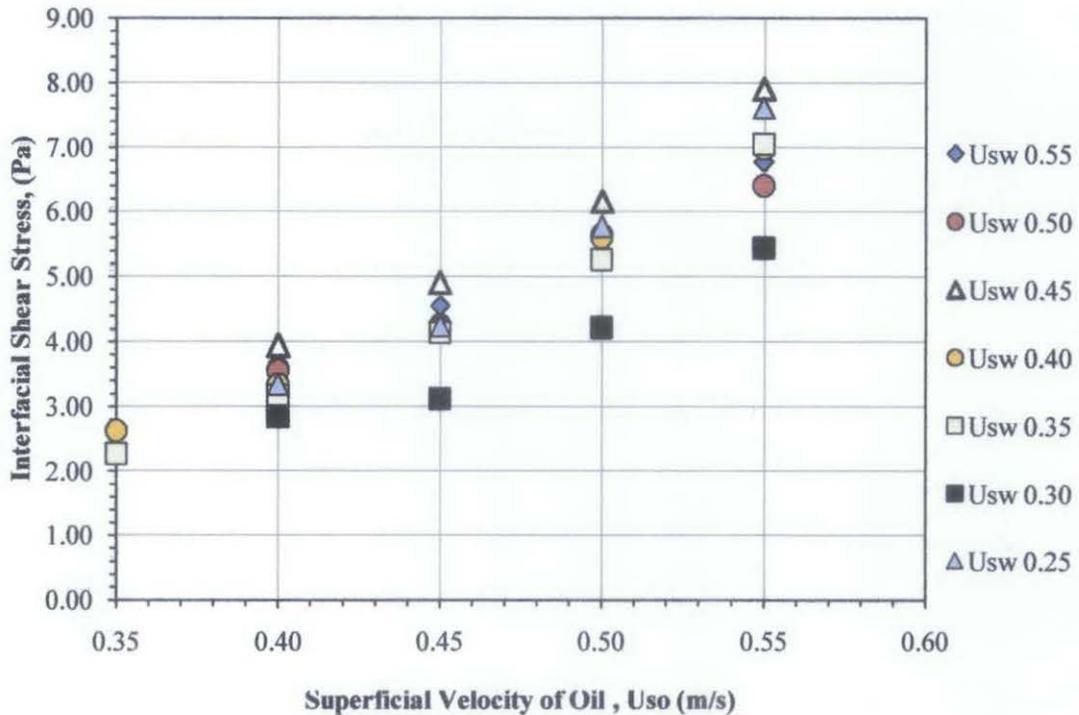


Figure 17: Interfacial Shear Stress vs Superficial Velocity of oil (Ring Probe height)

Pressure drop, $\Delta P/\Delta x$ calculated through ring probe height (H_R) is always greater than the experimental pressure drop, $(\Delta P/\Delta x)_{exp}$ due to lower estimation of cross sectional area of water, A_w , thus make the effects of shear stress of oil, τ_{oil} towards $\Delta P/\Delta x$ became superior.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

A MATLAB programming code developed in this study is able to closely predict the pressure drop of the two-phase, oil-water flow in a horizontal pipeline system. At different configurations of the interfacial curvature, a comparison between the calculated and experimental data is proven that the shape of the interface does significantly affect the measurement of pressure drops in an actual scenario. Based on the results obtained, it is shown that flat-shape interface assumption is not the best assumption for this prediction. The percentage difference of the prediction is very large when it was compared to experimental data. Curvature interfacial configuration is assumed to give best prediction, however, in this project, the curvature interface not give an expected result. This is due to some errors in cross sectional area and wetted perimeter derivation formula used in this model. The derived formulae for the calculation of the cross sectional area using curved interface poses ambiguity due to the Cartesian coordinates utilized in the data is on the opposite side of the curvature model. Modification should be done to improve the prediction and it is still believed that prediction of pressure drop based upon curvature interfacial configuration assumption will give the closest prediction.

4.2 Recommendations

Some recommendations are suggested in order to improve the reliability of the model. The recommendations are listed as below.

- Modifications in the determination of cross sectional area and wetted perimeter derived for the curved interface configuration need to be carried out to improve the calculation of $\Delta P/\Delta x$.

RERERENCES

- Arenas-Medina, P., Verdejo-Fierro, J., & Gonzalez-Islas, R. (2000). Flow pattern transitions in horizontal pipelines carrying oil-water mixtures: Full-scale experiments. *Journal of Energy Resources Technology*, 122, 169.
- Bertola, V. (2003). In Tasso C. (Ed.), *Modelling and experimentation in two-phase flow* (2003rd ed.). Udine: International Centre for Mechanical Sciences.
- Brauner, N. (2003). Liquid-liquid two-phase flow systems. *COURSES AND LECTURES-INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES*, , 221-280.
- Brauner, N., Moalem Maron, D., & Rovinsky, J. (1998). A two-fluid model for stratified flows with curved interfaces. *International Journal of Multiphase Flow*, 24(6), 975-1004.
- Brauner, N., Rovinsky, J., & Moalem Maron, D. (1996). Determination of the interface curvature in stratified two-phase systems by energy considerations. *International Journal of Multiphase Flow*, 22(6), 1167-1185.
- Chakrabarti, D., Das, G., & Ray, S. (2005). Pressure drop in Liquid-liquid two phase horizontal flow: Experiment and prediction. *Chemical Engineering & Technology*, 28(9), 1003-1009.
- Fan, Y., Wang, Q., Zhang, H. Q., Sarica, C., & Danielson, T. (2005). A model to predict liquid holdup and pressure gradient of near-horizontal wet-gas pipelines. *SPE Annual Technical Conference and Exhibition*,
- Gorelik, D., & Brauner, N. (1999). The interface configuration in two-phase stratified pipe flows. *International Journal of Multiphase Flow*, 25(6-7), 977-1007.
- Liu, Y., Zhang, H., Wang, S., & Wang, J. (2008). Prediction of pressure gradient and holdup in small eötvös number liquid-liquid segregated flow*. *Chinese Journal of Chemical Engineering*, 16(2), 184-191.
- Ng, T.S., Lawrence, C.J., Hewitt, G.F. (2002), Laminar Stratified Pipe Flow. *International Journal of Multiphase Flow*, 28(2002), 963-996.
- Peters, M. S., Timmerhaus, K. D., West, R. E., Timmerhaus, K., & West, R. (2003). *Plant design and economics for chemical engineers* McGraw-Hill New York.
- Ullmann, A., & Brauner, N. (2006). Closure relations for two-fluid models for two-phase stratified smooth and stratified wavy flows. *International Journal of Multiphase Flow*, 32(1), 82-105.

Appendix I: List of Previous Researches

| Author | Year | Title | Objective | Findings |
|------------------------|------|---|--|--|
| (Brauner et al., 1996) | 1996 | Determination of the Interface Curvature in Stratified Two-Phase Systems by Energy Considerations | Employ energy consideration to predict interface configuration Explore the effect of the fluid physical properties, <i>in situ</i> hold up, tube dimension, wall adhesion and gravitation on the characteristic interface curvature | <ul style="list-style-type: none"> • Explored the changes in the system potential energy and surface energies associated with the curving process of the interface • The characteristic interface curvature is predicted as a function of the fluids physical properties (<i>in situ</i> holdup, wall/phases wettability angle, tube dimensions and gravity conditions • Solution of laminar two-phase flows is shown to be dependent on the phase flow rates ratio, the phase viscosity ratio, density differential, surface tension effects, tube dimension or gravitation and is determined by four dimensional parameters: phases viscosity ratio, flow rates ratio, wall/phases wettability angle and Eotvos number. |
| (Brauner et al., 1998) | 1998 | A two-fluid model for stratified flows with curved interfaces | Develop a practical tool for predicting the interface shape in stratified flow of general two-fluid system To use a two-fluid model to solve momentum equations for a variable interface | <ul style="list-style-type: none"> • The solutions of the two-fluid model are used to construct 'flow monograms' which provide a relation between a specified interface curvature and the <i>in situ</i> hold-up and the associated pressure drop. • Construction of operational monograms for laminar, turbulent or mixed flow regimes in |

| | | | | |
|------------------------------|------|--|---|--|
| | | | curvature | <p>the two-phases, for horizontal and inclined systems.</p> <ul style="list-style-type: none"> • Two-fluid model provides a reasonable estimate of the in situ hold-up and pressure drop over a wide range of interfacial curvature and flow rates • The biggest error are obtained when the two-fluid model is applied for a configuration of a fully eccentric highly viscous core, in which case the two-fluid model significantly over predicts the lubrication effect of the less viscous phase |
| (Gorelik & Brauner, 1999) | 1999 | The interface configuration in two-phase stratified pipe flows | Obtain exact analytical solution for the interface shape between two immiscible fluids and for the capillary pressure in the case of unidirectional axial laminar pipe flow | <ul style="list-style-type: none"> • The solution is determined by three dimensionless parameters: the holdup, fluid/wall wettability angle and the Eotvos number. • The model of constant characteristic curvature provides a good description of the interfacial shape and enables extending the parameter space where analytical solutions of stratified flow can be obtained |
| (Arenas-Medina et al., 2000) | 2000 | Flow pattern transitions in horizontal pipelines carrying Oil-Water Mixtures: Full-Scale Experiments | To investigate flow pattern transitions in horizontal pipelines carrying oil-mixtures. | <ul style="list-style-type: none"> • The data obtained from this experiment were used to construct a simplified flow pattern map that shows the transition from stratified to nonstratified flow configuration • Found that the stratified transition can be predicted with reasonable accuracy based on the viscous Kelvin-Helmholtz analysis |

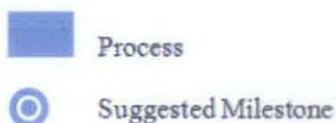
| | | | | |
|---------------------------------|------|--|---|--|
| | | | | <ul style="list-style-type: none"> • Revealed that in stratified crude oil-water flow, complete phase separation does not occur • There is always a small amount of water dispersed almost uniformly in the oil layer |
| (Chakrabarti, Das, & Ray, 2005) | 2005 | Pressure Drop in Liquid-Liquid Two-Phase Horizontal Flow: Experiment and Prediction | To investigate the pressure drop characteristic during the simultaneously flow of kerosene-water mixtures through a horizontal pipe of 0.025 m diameter | <ul style="list-style-type: none"> • Estimate of pressure drop could be obtained by the simultaneously consideration of <ol style="list-style-type: none"> a) The principle of minimization of total system energy b) The criteria of equal pressure drop of the system in both phases, where the total energy is comprised of the kinetic energy, potential energy, and surface energy of both phases • A flat interface has been used in this study and the result obtained from this model has yielded an accuracy of $\pm 10\%$ for regimes where fragmented droplets of one phase do not appear. • For smooth stratified (SS) and stratified wavy (SW) regimes the results agree closely with the experimental data. |
| (Ullmann & Brauner, 2006) | 2006 | Closure relations for two-fluid models for two-phase stratified smooth and stratified wavy flows | To extend the theory-based closure relations for the wall and interfacial shear stresses to be applicable also to turbulent flows in either or both of the phases | <ul style="list-style-type: none"> • The closure relations are formulated in terms of the single-phase-based expressions, which are augmented by two-phase interaction factors, due to the flow of the two phases in the same channel • These closure relations were used as a |

| | | | | |
|---|------|---|---|--|
| | | | | <p>platform for introducing necessary empirical corrections required in the stratified wavy flow regimes</p> <ul style="list-style-type: none"> • Obtained new empirical correlation for the wave effect on the interface curvature, on the interfacial shear and on the liquid wall shear wear obtained. • The new closure relations are essentially representing correctly the interaction between the phases over a wide range of the stratified flow parameters space in the stratified smooth and stratified wavy regime. |
| (Fan, Wang, Zhang, Sarica, & Danielson, 2005) | 2007 | A model to predict liquid holdup and pressure gradient of near-horizontal wet-gas pipelines | To predict liquid holdup and pressure gradient of stratified flow | <ul style="list-style-type: none"> • New closure relationship of wetted-wall fraction, liquid-wall friction factor and interfacial-friction factor were proposed. • An iterative calculation procedure was proposed to solve the two-fluid model for liquid hold up and pressure gradient. • Comparison between model predictions and experimental data show that the proposed model agrees well with the data collected in the present study. • The average percentage errors of liquid holdup and pressure-gradient prediction are 2.9 and 3.2%, respectively. |
| (Liu, Zhang, Wang, & Wang, 2008) | 2008 | Prediction of pressure gradient and holdup in small | To predict pressure gradient and holdup in small Eötvös | <ul style="list-style-type: none"> • Due to the dominant effect of interfacial tension and wall-wetting properties of the |

| | | | | |
|--|--|---|--------------------------------------|--|
| | | Eötvös Number liquid-liquid segregated flow | Number liquid-liquid segregated flow | <p>liquids over the gravity, especially $E_{OD} < 5$, the oil-water interface exhibits a concave-down configuration</p> <ul style="list-style-type: none"> • Comparison between experimental and theoretical data shows that experimental data agrees with the measurement after the conventional two-fluid model is extended to tackle segregated flow with curved interface |
|--|--|---|--------------------------------------|--|

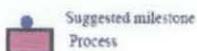
Appendix II: Gantt chart

| No. | Activities | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | |
|-----|---|---|---|---|---|---|---|---|--------------------|---|----|----|----|----|----|---|
| 1. | Selection of the topic project | | | | | | | | Mid Semester Break | | | | | | | |
| 2. | Preliminary Research Work | | | | | | | | | | | | | | | |
| 3. | Submission of Extended Proposal Defence | | | | | | ◎ | | | | | | | | | |
| 4. | Proposal Defence | | | | | | | | | | | | | | | |
| 5. | Project work continues | | | | | | | | | | | | | | | |
| 6. | Submission of Interim Draft Report | | | | | | | | | | | | | | ◎ | |
| 7. | Submission of Interim Report | | | | | | | | | | | | | | | ◎ |



Timelines for FYP 2

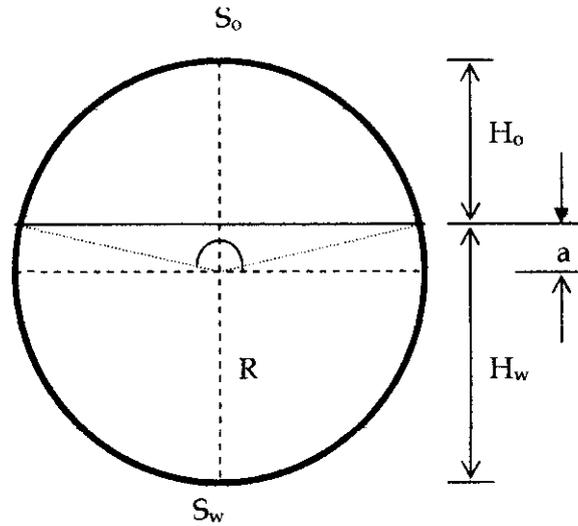
| No. | Detail/ Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | |
|-----|---|---|---|---|---|---|---|---|--------------------|---|----|----|----|----|----|----|---|
| 1 | Project Work Continues | | | | | | | | Mid-Semester Break | | | | | | | | |
| 2 | Submission of Progress Report | | | | | | | | | | ● | | | | | | |
| 3 | Project Work Continues | | | | | | | | | | | | | | | | |
| 4 | Pre-EDX | | | | | | | | | | | | ● | | | | |
| 5 | Submission of Draft Report | | | | | | | | | | | | | ● | | | |
| 6 | Submission of Dissertation (soft bound) | | | | | | | | | | | | | | ● | | |
| 7 | Submission of Technical Paper | | | | | | | | | | | | | | ● | | |
| 8 | Oral Presentation | | | | | | | | | | | | | | | ● | |
| 9 | Submission of Project Dissertation (Hard Bound) | | | | | | | | | | | | | | | | ● |



Appendix III: List of Previous Experimental Works

| Author | Year | Original Flow Pattern Names | Equivalent Flow Patterns for this Study | Flow Pattern Detection Method | Additional Measurements | Angle ° | v_{cr} m/s | WC % |
|-----------------------|-------|--|--|--|---|---------|--------------|----------|
| Trallero | 1995 | ST, ST & MI D O/W, D O/W&W, D W/O, D O/W&W/O | ST, ST & MI, D O/W, D O/W&W, D W/O, D O/W&W/O | Visual Observation Photographic Conductance Probe | Pressure Gradient Holdup | 0 | 0.25 - 3.0 | 0 - 100 |
| Angel | 1996 | SW, SWD, SM/Oil, 3 Layer, SM/Water, Mixed | ST, ST & MI, D W/O&O, D O/W&W/O, D O/W&W, Dispersed | Visual Observation High Speed Camera HF Impedance Probe | Pressure Gradient Phase Distribution | 0 | 0.3 - 4.0 | 0 - 100 |
| Nädler & Mewes | 1997 | ST, ST & MI, D W/O&W, D W/O, D O/W&W/O&W, D O/W&W, D O/W | ST, ST & MI, D W/O&W, D W/O, D O/W&W/O, D O/W&W, D O/W | Visual Observation Conductance Probe | Pressure Gradient | 0 | 0.3 - 1.6 | 20 - 100 |
| Solcimani | 1999 | SW, SWD, SM/Oil, 3 Layer, SM/Water, Mixed | ST, ST & MI, D W/O&O, D O/W&W/O, D O/W&W, Dispersed | Visual Observation HF Impedance Probe Gamma Densitometer | Pressure Gradient Phase Distribution | 0 | 0.5 - 1.5 | 15 - 80 |
| Alkaya <i>et al</i> | 2000 | ST, ST & MI, D O/W, D O/W&W, D W/O, D O/W&W/O | ST, ST & MI, D O/W, D O/W&W, D W/O, D O/W&W/O | Visual Observation Photographic Conductance Probe | Pressure Gradient Holdup | -2 - 2 | 0.05 - 2.75 | 1 - 99 |
| Angel & Hewitt | 2000b | SW, SWD, SM/Oil, 3 Layer, SM/Water, Mixed | ST, ST & MI, D W/O&O, D O/W&W/O, D O/W&W, Dispersed | Visual Observation High Speed Camera HF Impedance Probe | Pressure Gradient Phase Distribution | 0 | 0.3 - 1.0 | 0 - 100 |
| Fairuzov <i>et al</i> | 2000 | ST Dispersed | ST D W/O | Main Point Sample Probe | Local Water Fraction | 0 | 0.0 - 2.1 | 2 - 49 |
| Elserh | 2001 | SS, SW, D O-DF, D O-I, SM, D W-DF, D W-L, D W-H | ST, ST & MI, D W/O&O, D W/O, D O/W&W/O, D O/W&W, D O/W (L & H) | Visual Observation Gamma Densitometer | Pressure Gradient Local Oil Fraction Velocity Profile | 0 | 0.4 - 3.0 | 0 - 95 |
| Simmons & Azzopardi | 2001 | ST & MI, D W/O&W, D W/O | ST & MI, D W/O&W, D W/O | Visual Observation High Speed Camera | Droplet Size | 0 - 90 | 0.8 - 3.1.0 | 6 - 42 |
| Oddie <i>et al</i> | 2003 | S, Semi-S, Semi-Mixed, Mixed, Dispersed, Homogeneous | ST, ST & MI, D O/W&W/O, D O/W&W, Dispersed | Visual Observation Conductance Probe Gamma Densitometer | Holdup | -2 - 90 | 0.661 - 2.43 | 16 - 92 |
| Lovick & Angel | 2004 | SW, D W/O, D O/W DC | ST & MI, D W/O, D O/W, D O/W&W/O | Visual Observation Conductance Probe HF Impedance Probe | Pressure Gradient Phase Distribution | 0 | 0.8 - 3.0 | 10 - 90 |

Appendix IV: Area and Perimeter Calculation for Flat Interfacial Configuration



If $H_w > R$

$$a = H_w - R$$

$$H_o = 2R - H_w = R - a$$

$$\theta = 2 \cos^{-1} \left(\frac{a}{R} \right) = 2 \cos^{-1} \left(\frac{H_w - R}{R} \right)$$

$$S_o = R\theta = 2R \cos^{-1} \left(\frac{H_w - R}{R} \right)$$

$$\begin{aligned} S_w &= 2\pi R - R\theta \\ &= 2R \left[\pi - \cos^{-1} \left(\frac{a}{R} \right) \right] \\ &= 2R \left[\pi - \cos^{-1} \left(\frac{H_w - R}{R} \right) \right] \end{aligned}$$

$$\begin{aligned} S_i &= 2(R^2 - a^2)^{1/2} \\ &= 2 [R^2 - (H_w - R)^2]^{1/2} \end{aligned}$$

$$\begin{aligned}
A_o &= \frac{R^2 \theta}{2} - a(R^2 - a^2)^{1/2} \\
&= R^2 \cos^{-1} \left(\frac{H_w - R}{R} \right) - (H_w - R) \left[R^2 - (H_w - R)^2 \right]^{1/2}
\end{aligned}$$

$$\begin{aligned}
A_w &= \pi R^2 - A_o \\
&= \pi R^2 - \left[\frac{R^2 \theta}{2} - a(R^2 - a^2)^{1/2} \right] \\
&= \pi R^2 - R^2 \cos^{-1} \left(\frac{H_w - R}{R} \right) + (H_w - R) \left[R^2 - (H_w - R)^2 \right]^{1/2}
\end{aligned}$$

If $H_w < R$

$$a = R - H_w$$

$$H_o = 2R - H_w = R + a$$

$$\theta = 2 \cos^{-1} \left(\frac{a}{R} \right) = 2 \cos^{-1} \left(\frac{R - H_w}{R} \right)$$

$$S_w = R\theta = 2R \cos^{-1} \left(\frac{R - H_w}{R} \right)$$

$$\begin{aligned}
S_o &= 2\pi R - R\theta \\
&= 2R \left[\pi - \cos^{-1} \left(\frac{a}{R} \right) \right] \\
&= 2R \left[\pi - \cos^{-1} \left(\frac{R - H_w}{R} \right) \right]
\end{aligned}$$

$$\begin{aligned}
S_i &= 2(R^2 - a^2)^{1/2} \\
&= 2 \left[R^2 - (R - H_w)^2 \right]^{1/2}
\end{aligned}$$

$$\begin{aligned}
A_w &= \frac{R^2 \theta}{2} - a(R^2 - a^2)^{1/2} \\
&= R^2 \cos^{-1} \left(\frac{R - H_w}{R} \right) - (R - H_w) \left[R^2 - (R - H_w)^2 \right]^{1/2}
\end{aligned}$$

$$\begin{aligned}
A_o &= \pi R^2 - A_w \\
&= \pi R^2 - \left[\frac{R^2 \theta}{2} - a(R^2 - a^2)^{1/2} \right] \\
&= \pi R^2 - R^2 \cos^{-1} \left(\frac{R - H_w}{R} \right) - (R - H_w) \left[R^2 - (R - H_w)^2 \right]^{1/2}
\end{aligned}$$

Where,

H_w, H_o = Height of oil and water phase;

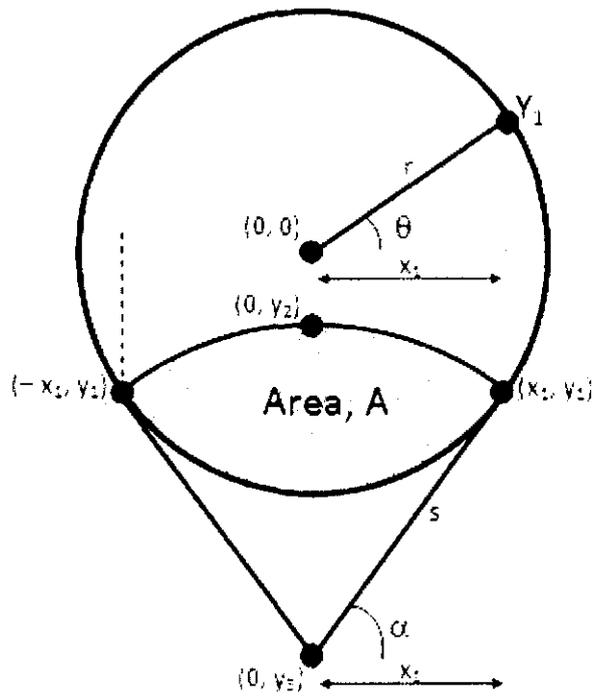
R = Radius of pipeline;

A_w, A_o = Cross sectional area for oil and water phase;

S_w, S_o, S_i = Perimeter for oil, water phases, and interfacial.

Appendix V: Area and Perimeter Calculation for Curvature Interfacial Configuration

Determination of cross sectional area in a pipe for a two-phase flow system subjected to interfacial curvature through lune approach.



$$\begin{aligned} \frac{X_2}{r} &= \cos \theta \\ X_2 &= r \cos \theta \\ Y_2 &= s \cos \alpha \\ r \cos \theta &= s \cos \alpha \\ \cos \theta &= \frac{s}{r} \cos \alpha \\ \theta &= \cos^{-1} \left(\frac{s}{r} \cos \alpha \right) \end{aligned}$$

Y_2

$$\text{Area, } A = \int_{-x_1}^{x_1} Y_1 - Y_2 \, dx = 2 \int_0^{x_1} Y_1 - Y_2 \, dx$$

$$\text{Area, } A = 2 \int_0^{x_1} Y_1 - Y_2 \, dx$$

$$= 2 \int_0^{x_1} \sqrt{r^2 - x^2} \, dx - 2 \int_0^{x_1} \sqrt{s^2 - x^2} + y_3 \, dx$$

$$= 2 \int_0^{x_1} \sqrt{r^2 - x^2} \, dx - 2 \int_0^{x_1} \sqrt{s^2 - x^2} + y_1 + \frac{x_1^2}{y_1} \, dx$$

$$= 2 \int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{r}\right)} \sqrt{r^2 - r^2 \cos^2 \theta} (-r \sin \theta) d\theta - 2 \int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} \sqrt{s^2 - s^2 \cos^2 \alpha} (-s \sin \alpha) d\alpha -$$

$$2 \int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} \left(y_1 + \frac{x_1^2}{y_1}\right) (-s \sin \alpha) d\alpha$$

$$= -2r^2 \int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{r}\right)} \sin \theta \sin \theta \, d\theta + 2s^2 \int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} \sin \alpha \sin \alpha \, d\alpha$$

$$+ 2s \int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} \left(y_1 + \frac{x_1^2}{y_1}\right) \sin \alpha \, d\alpha$$

$$= \frac{-2r^2}{2} \int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{r}\right)} (1 - \cos 2\theta) d\theta + \frac{2s^2}{2} \int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} (1 - \cos 2\alpha) d\alpha$$

$$+ 2s \left(y_1 + \frac{x_1^2}{y_1}\right) \int_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} \sin \alpha \, d\alpha$$

$$= -r^2 \left[\theta - \frac{\sin 2\theta}{2} \right]_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{r}\right)} + s^2 \left[\alpha - \frac{\sin 2\alpha}{2} \right]_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)} + 2s \left(y_1 + \frac{x_1^2}{y_1}\right) [-\cos \alpha]_{\frac{\pi}{2}}^{\cos^{-1}\left(\frac{x_1}{s}\right)}$$

$$A = -r^2 \left\{ \left[\cos^{-1}\left(\frac{x_1}{r}\right) - \frac{1}{2} \sin 2\left(\cos^{-1}\left(\frac{x_1}{r}\right)\right) \right] - \left[\frac{\pi}{2} - \frac{\sin \pi}{2} \right] \right\}$$

$$+ s^2 \left\{ \left[\cos^{-1}\left(\frac{x_1}{s}\right) - \frac{1}{2} \sin 2\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right) \right] - \left[\frac{\pi}{2} - \frac{\sin \pi}{2} \right] \right\}$$

$$+ 2s \left(y_1 + \frac{x_1^2}{y_1}\right) \left[-\cos\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right) + \cos \frac{\pi}{2} \right]$$

$$A = -r^2 \left\{ \cos^{-1}\left(\frac{x_1}{r}\right) - \frac{1}{2} 2 \sin\left(\cos^{-1}\left(\frac{x_1}{r}\right)\right) \cos\left(\cos^{-1}\left(\frac{x_1}{r}\right)\right) - \frac{\pi}{2} \right\}$$

$$+ s^2 \left\{ \cos^{-1}\left(\frac{x_1}{s}\right) - \frac{1}{2} 2 \sin\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right) \cos\left(\cos^{-1}\left(\frac{x_1}{s}\right)\right) - \frac{\pi}{2} \right\}$$

$$+ 2s \left(y_1 + \frac{x_1^2}{y_1}\right) \left[-\frac{x_1}{s} \right]$$

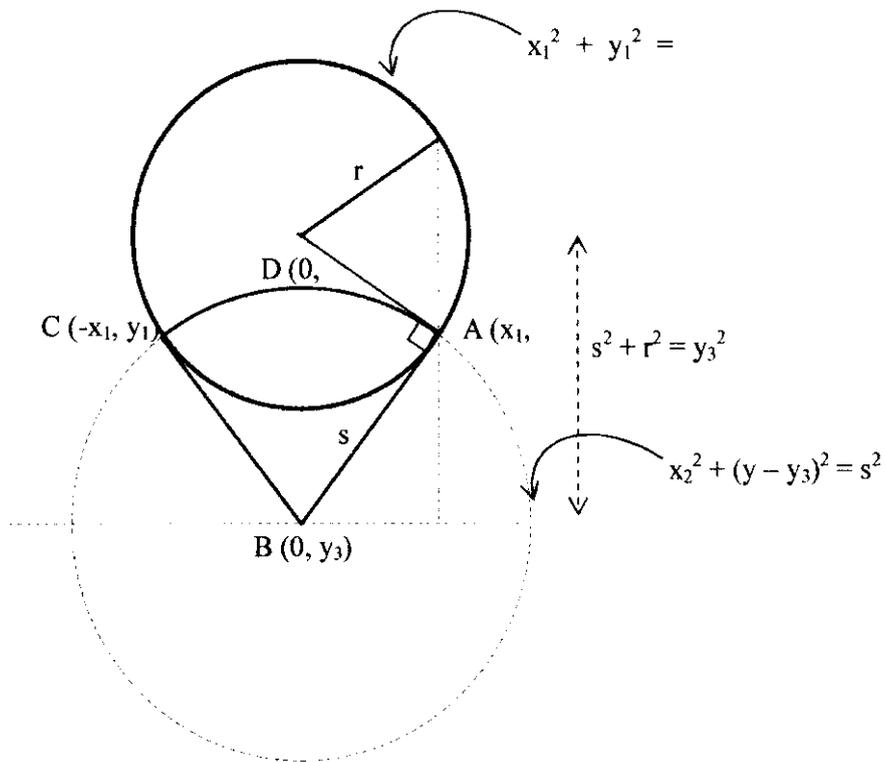
$$A = -r^2 \left\{ \cos^{-1} \left(\frac{x_1}{r} \right) - \sin \left(\cos^{-1} \left(\frac{x_1}{r} \right) \right) \left(\frac{x_1}{r} \right) - \frac{\pi}{2} \right\} \\ + s^2 \left\{ \cos^{-1} \left(\frac{x_1}{s} \right) - \sin \left(\cos^{-1} \left(\frac{x_1}{s} \right) \right) \left(\frac{x_1}{s} \right) - \frac{\pi}{2} \right\} - 2x_1 \left(y_1 + \frac{x_1^2}{y_1} \right)$$

$$A = -r^2 \cos^{-1} \left(\frac{x_1}{r} \right) + r^2 \left(\frac{x_1}{r} \right) \sin \left(\cos^{-1} \left(\frac{x_1}{r} \right) \right) + r^2 \frac{\pi}{2} \\ + s^2 \cos^{-1} \left(\frac{x_1}{s} \right) - s^2 \left(\frac{x_1}{s} \right) \sin \left(\cos^{-1} \left(\frac{x_1}{s} \right) \right) - s^2 \frac{\pi}{2} \\ - 2x_1 y_1 - 2 \frac{x_1^3}{y_1}$$

$$A = -r^2 \cos^{-1} \left(\frac{x_1}{r} \right) + r x_1 \sin \left(\cos^{-1} \left(\frac{x_1}{r} \right) \right) + r^2 \frac{\pi}{2} \\ + s^2 \cos^{-1} \left(\frac{x_1}{s} \right) - s x_1 \sin \left(\cos^{-1} \left(\frac{x_1}{s} \right) \right) - s^2 \frac{\pi}{2} \\ - 2x_1 y_1 - 2 \frac{x_1^3}{y_1}$$

$$A = -r^2 \cos^{-1} \left(\frac{x_1}{r} \right) + r x_1 \sin \left(\cos^{-1} \left(\frac{x_1}{r} \right) \right) + \frac{\pi}{2} (r^2 - s^2) \\ + s^2 \cos^{-1} \left(\frac{x_1}{s} \right) - s x_1 \sin \left(\cos^{-1} \left(\frac{x_1}{s} \right) \right) - 2x_1 y_1 - 2 \frac{x_1^3}{y_1}$$

Where, $s = \sqrt{x_1^2 + \frac{x_1^4}{y_1^2}}$



Distance AB; $x_1^2 + (y_1 - y_3)^2 = s^2$ -----(1)

Distance CB; $(-x_1)^2 + (y_1 - y_3)^2 = s^2$

Distance DB; $0^2 + (y_2 - y_3)^2 = s^2$ -----(2)

(1) - (2) \rightarrow $x_1^2 + (y_1 - y_3)^2 - (y_2 - y_3)^2 = 0$

$x_1^2 + y_1^2 - 2y_1y_3 + y_3^2 - y_2^2 + 2y_2y_3 - y_3^2 = 0$

$x_1^2 + y_1^2 - y_2^2 - 2y_3(y_1 - y_2) = 0$

$y_3 = \frac{x_1^2 + y_1^2 - y_2^2}{2(y_1 - y_2)}$

and so, $s^2 = (y_2 - y_3)^2$ from (2)

$s^2 = y_2 - \left(\frac{x_1^2 + y_1^2 - y_2^2}{2(y_1 - y_2)} \right)$

or:

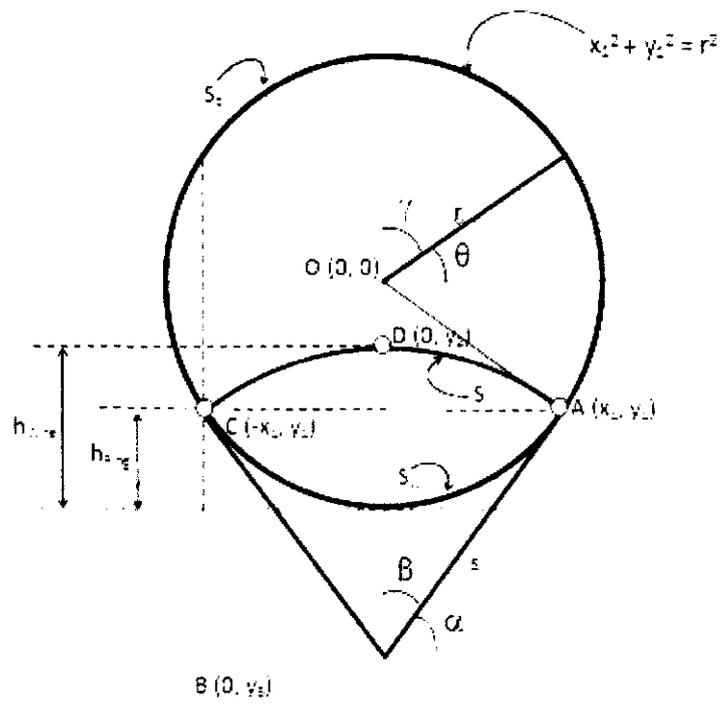
$$s^2 = y_2^2 - 2y_2y_3 + y_3^2$$
$$= y_2^2 - 2y_2\sqrt{(s^2 + r^2)} + (s^2 + r^2)$$

$$2y_2\sqrt{(s^2 + r^2)} - y_2^2 + r^2$$

$$4y_2^2(s^2 + r^2) = (y_2^2 + r^2)^2$$

$$s^2 = \frac{(y_2^2 + r^2)^2}{4y_2^2} - r^2$$

$$s = \sqrt{\frac{(y_2^2 + r^2)^2}{4y_2^2} - r^2}$$



$$x_2^2 + (y - y_2)^2 = s^2$$

$$y_1 = r - h_{\text{ring}}$$

$$y_2 = r - h_{\text{wire}}$$

$$x_1 = \sqrt{r^2 - y_1^2}$$

$$\begin{aligned} x_1 &= \sqrt{r^2 - (r - h_{\text{ring}})^2} \\ &= \sqrt{r^2 - (r^2 - 2rh_{\text{ring}} + (h_{\text{ring}})^2)} \\ &= \sqrt{r^2 - r^2 + 2rh_{\text{ring}} - (h_{\text{ring}})^2} \\ &= \sqrt{h_{\text{ring}}(2r - h_{\text{ring}})} \end{aligned}$$

$$\frac{x_1}{s} = \cos \alpha$$

$$\alpha = \cos^{-1} \left(\frac{x_1}{s} \right)$$

$$\beta = \frac{\pi}{2} - \alpha$$

$$2\beta = \pi - 2\alpha$$

$$S_i = s(2\beta) = s \left(\pi - 2\cos^{-1} \left(\frac{x_1}{s} \right) \right)$$

$$\frac{x_1}{r} = \cos \theta$$

$$\theta = \cos^{-1} \left(\frac{x_1}{r} \right)$$

$$\gamma = \frac{\pi}{2} - \theta$$

$$2\gamma = \pi - 2\theta$$

$$S_w = r(2\gamma) = r \left(\pi - 2\cos^{-1} \left(\frac{x_1}{r} \right) \right)$$

$$S_o = 2\pi r - S_w$$

Where,

H_w, H_o = Height of oil and water phase;

R = Radius of pipeline;

A_w, A_o = Cross sectional area for oil and water phase;

S_w, S_o = Perimeter for oil and water phase;

s = Radius for imaginary circle

Appendix VI: MATLAB Programming Code for Flat Interfacial Configuration by using Wire Probe Height

```

fprintf('\n\n');
disp('This program will determine the pressure drop inside the pipeline for
liquid-liquid two phase system by using wire probe height with assumption that
the flat interface between the phases');

% Prompt the user to enter a string character information
wireprobe_height = input('\nPlease enter the set of wire probe heights in
bracket(unit of mm){example = [1 2 3 4]}:')*10^(-3);
superficialvelocity_water = input('Please enter the superficial velocity of
water(unit of m/s):');
superficialvelocity_oil = input('Please enter the set of superficial velocity
of oil in bracket(unit of m/s){example = [1 2 3 4]}:');
pipeline_radius = input('Please enter the radius of pipeline (unit of
mm):')*10^(-3);
pipeline_length = input('Please enter the length of pipeline (unit of m):');
density_oil = input('Please enter the density of oil (unit of kg/m^3):');
density_water = input('Please enter the density of water (unit of kg/m^3):');
viscosity_oil = input('Please enter the viscosity of oil at 25 degree
Celcius(unit of kg/m.s or Pa.s):');
viscosity_water = input('Please enter the viscosity of water at 25 degree
Celcius(unit of kg/m.s or Pa.s):');
%surface_tension = input('Please enter the surface tension at 25 degree
Celcius(unit of kg/s^2 or N/m):');
flowrate_oil = input('Please enter the set of oil flowrate in bracket for the
corresponding wire probe heights(unit of 10^(-5)m^3/s){example = [1 2 3
4]}:')*10^(-5);
flowrate_water = input('Please enter the set of water flowrate in bracket for
the corresponding wire probe heights(unit of 10^(-5)m^3/s){example = [1 2 3
4]}:')*10^(-5);
fprintf('\n\n');

% Set first value to all variables
a = zeros(size(wireprobe_height));
theta = zeros(size(wireprobe_height));
height_water = zeros(size(wireprobe_height));
height_oil = zeros(size(wireprobe_height));
perimeter_oil = zeros(size(wireprobe_height));
perimeter_water = zeros(size(wireprobe_height));
perimeter_interface = zeros(size(wireprobe_height));
area_oil = zeros(size(wireprobe_height));
area_water = zeros(size(wireprobe_height));

k = 1;
while k <= length(wireprobe_height)

    if wireprobe_height(k) > pipeline_radius

        a(k) = wireprobe_height(k) - pipeline_radius;
        theta(k) = 2*(acos(a(k)/pipeline_radius)); %array

% Calculation of height of water and height of oil in the pipeline
        height_water(k) = wireprobe_height(k); %array
        height_oil(k) = pipeline_radius - a(k); %array
    end
    k = k + 1;
end

```

```

    % Calculation of oil phase perimeter, water phase perimeter and interface
    perimeter
    perimeter_oil(k) = pipeline_radius*theta(k); %array
    perimeter_water(k) = (2*pi*pipeline_radius) - (pipeline_radius*theta(k));
%array
    perimeter_interface(k) = 2*sqrt(pipeline_radius^2 - (a(k)).^2); %array

    % Calculation of oil phase area and water phase area
    area_oil(k) = ((pipeline_radius^2)*(theta(k))/2) -
((a(k)).*sqrt(pipeline_radius^2 - (a(k)).^2)); %array
    area_water(k) = (pi*pipeline_radius^2) - area_oil(k); %array

    elseif wireprobe_height(k) < pipeline_radius

    a(k) = pipeline_radius - wireprobe_height(k);
    theta(k) = 2*(acos(a(k)/pipeline_radius)); %array

    % Calculation of height of water and height of oil in the pipeline
    height_water(k) = wireprobe_height(k); %array
    height_oil(k) = pipeline_radius + a(k); %array

    % Calculation of oil phase perimeter, water phase perimeter and interface
    perimeter
    perimeter_oil(k) = (2*pi*pipeline_radius) - (pipeline_radius*(theta(k)));
%array
    perimeter_water(k) = pipeline_radius*(theta(k)); %array
    perimeter_interface(k) = 2*sqrt(pipeline_radius^2 - (a(k)).^2); %array

    % Calculation of oil phase area and water phase area
    area_water(k) = ((pipeline_radius^2)*(theta(k))/2) -
((a(k)).*sqrt(pipeline_radius^2 - (a(k)).^2)); %array
    area_oil(k) = (pi*pipeline_radius^2) - area_water(k); %array

    else

    disp(' Could not determine area and perimeter of the phases');

    end

k=k+1;

end

    % Calculation of in-situ velocity for water and oil
    velocity_oil = flowrate_oil./area_oil;
    velocity_water = flowrate_water./area_water;

    % Calculation to determine the hydraulic diameter of water and oil -
    % Conditions outlined by Moalem-Maron et. al. (1998)

    % Set first value to all variables
    diameter_oil = zeros(size(wireprobe_height));
    diameter_water = zeros(size(wireprobe_height));

k = 1;
while k <= length(wireprobe_height)

    if (velocity_oil(k) > velocity_water(k))

```

```

    diameter_oil(k) =
(4*area_oil(k))./(perimeter_oil(k)+perimeter_interface(k));
    diameter_water(k) = (4*area_water(k))./perimeter_water(k);

    elseif (velocity_oil(k) < velocity_water(k))

    diameter_oil(k) = (4*area_oil(k))./perimeter_oil(k);
    diameter_water(k) =
(4*area_water(k))./(perimeter_water(k)+perimeter_interface(k));

    else

    diameter_oil(k) = (4*area_oil(k))./perimeter_oil(k);
    diameter_water(k) = (4*area_water(k))./perimeter_water(k);

    end

k=k+1;

end

% Calculation of Reynolds number
Reynolds_oil = (density_oil*velocity_oil.*diameter_oil)/viscosity_oil;
Reynolds_water =
(density_water*velocity_water.*diameter_water)/viscosity_water;

% Determination of C and n constant for water phase.

% Set first value to all variables
C_water = zeros(size(wireprobe_height));
n_water = zeros(size(wireprobe_height));
flowtype_water = 'ERROR';

k = 1;
while k <= length(wireprobe_height)

    if Reynolds_water(k) < 2000

    C_water(k) = 16;
    n_water(k) = 1;
    flowtype_water = 'Laminar Flow';

    elseif Reynolds_water(k) > 4000

    C_water(k) = 0.046;
    n_water(k) = 0.2;
    flowtype_water = 'Turbulent Flow';

    else

    C_water(k) = NaN;
    n_water(k) = NaN;
    flowtype_water = 'Transitional Flow';

    end

k = k+1;

end

```

```

% Determination of C and n constant for oil phase.

% Set first value to all variables
C_oil = zeros(size(Reynolds_water));
n_oil = zeros(size(Reynolds_water));
flowtype_oil = 'ERROR';

k = 1;
while k <= length(Reynolds_water)

    if Reynolds_oil(k) < 2000
        C_oil(k) = 16;
        n_oil(k) = 1;
        flowtype_oil = 'Laminar Flow';

    elseif Reynolds_oil(k) > 4000

        C_oil(k) = 0.046;
        n_oil(k) = 0.2;
        flowtype_oil = 'Turbulent Flow';

    else

        C_oil(k) = NaN;
        n_oil(k) = NaN;
        flowtype_oil = 'Transitional Flow';

    end

    k = k+1;
end

% Calculation of friction factor for oil and water flow.
friction_oil =
C_oil.*((density_oil*velocity_oil.*diameter_oil/viscosity_oil).^(-n_oil));
friction_water =
C_water.*((density_water*velocity_water.*diameter_water/viscosity_water).^(-
n_water));

% Calculation of shear stress for oil and water flow.
shearstress_oil = 0.5*(friction_oil*density_oil).*(velocity_oil.^2);
shearstress_water = 0.5*(friction_water*density_water).*(velocity_water.^2);

% Calculation of pressure drop (final stage)
dpdz =
((shearstress_oil.*perimeter_oil)+(shearstress_water.*perimeter_water))./(-
1*(area_oil + area_water));
pressure_drop = abs(dpdz/1000);

% Calculation of shear stress of interface
shearstress_interface = (area_oil.*dpdz + shearstress_oil.*perimeter_oil)./(-
1*perimeter_interface);

% Shows parameters for both oil and water phases
fprintf('\n\n');
disp('By assuming a flat interface between phases, the parameters for both oil
and water phases are obtained');
fprintf('\n');

```

```

fprintf('Wire Probe Height\t\tPerimeter of Oil Phase\t\tPerimeter of Water
Phase\t\tPerimeter of Interface\t\tArea of Oil Phase\t\tArea of Water
Phase\n');
table_1 = [wireprobe_height; perimeter_oil; perimeter_water;
perimeter_interface; area_oil; area_water];
fprintf('%10.6f m %25.6f m %25.6f m %27.6f m %23.6f m^2 %20.6f m^2
\n',table_1);
fprintf('\n\n');
fprintf('Velocity of Oil\t\tVelocity of Water\t\tDiameter of Oil\t\tDiameter of
Water\t\tReynolds Number of Oil\t\tReynolds Number of Water\n');
table_2 = [velocity_oil; velocity_water; diameter_oil; diameter_water;
Reynolds_oil; Reynolds_water];
fprintf('%10.6f m/s %15.6f m/s %20.6f m %17.6f m %28.6f %25.6f \n',table_2);
fprintf('\n\n');
fprintf('Friction factor of Oil\t\tFriction factor of Water\t\tShear Stress of
Oil\t\tShear Stress of Water\t\tPressure Drop\t\tShear Stress of Interface\n');
table_3 = [friction_oil; friction_water; shearstress_oil; shearstress_water;
pressure_drop; shearstress_interface];
fprintf('%15.6f %25.6f %30.6f Pa %20.6f Pa %21.6f kPa/m %20.6f Pa\n',table_3);
fprintf('\n\n');

% Graph plot
disp('Graph Comparison of Theoretical Data vs Experimental Data');
experimental_pressuredrop = input('\nPlease enter the set of experimental data
in bracket(unit of kPa){example = [1 2 3 4]}:'); %array

subplot(2,1,1),plot(superficialvelocity_oil,pressure_drop,'-
or',superficialvelocity_oil, experimental_pressuredrop./pipeline_length,'-xk')
title('Graph of Pressure Drop vs Superficial Velocity of Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Pressure Drop, kPa/m')
legend('theoretical value (wire probe)', 'experimental value')
grid on

% Graph plot
fprintf('\n\n');
disp('Graph Shear Stress of Interface vs Superficial Velocity of Oil');

subplot(2,1,2),plot(superficialvelocity_oil,shearstress_interface,'-or')
title('Graph of Shear Stress of Interface vs Superficial Velocity of Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Shear Stress of Interface, Pa')
grid on

```

Appendix VII: MATLAB Programming Code for Flat Interfacial Configuration by using Ring Probe Height

```

fprintf('\n\n');
disp('This program will determine the pressure drop inside the pipeline for
liquid-liquid two phase system by using ring probe height with assumption that
the flat interface between the phases');
% Prompt the user to enter a string character information

ringprobe_height = input('\nPlease enter the set of ring probe heights in
bracket(unit of mm){example = [1 2 3 4]}:'); %array
superficialvelocity_water = input('Please enter the superficial velocity of
water(unit of m/s):'); %array
superficialvelocity_oil = input('Please enter the set of superficial velocity
of oil in bracket(unit of m/s){example = [1 2 3 4]}:'); %array
pipeline_radius = input('Please enter the radius of pipeline (unit of
mm):')*10^(-3); %single number
pipeline_length = input('Please enter the length of pipeline (unit of m):');
%single number
density_oil = input('Please enter the density of oil (unit of kg/m^3):');
%single number
density_water = input('Please enter the density of water (unit of kg/m^3):');
%single number
viscosity_oil = input('Please enter the viscosity of oil at 25 degree
Celcius(unit of kg/m.s or Pa.s):'); %single number
viscosity_water = input('Please enter the viscosity of water at 25 degree
Celcius(unit of kg/m.s or Pa.s):'); %single number
%surface_tension = input('Please enter the surface tension at 25 degree
Celcius(unit of kg/s^2 or N/m):'); %single number
flowrate_oil = input('Please enter the set of oil flowrate in bracket for the
corresponding ring probe heights(unit of 10^(-5)m^3/s){example = [1 2 3
4]}:'); %array
flowrate_water = input('Please enter the set of water flowrate in bracket for
the corresponding ring probe heights(unit of 10^(-5)m^3/s){example = [1 2 3
4]}:'); %array
fprintf('\n\n');

% Set first value to all variables
a = zeros(size(ringprobe_height));
theta = zeros(size(ringprobe_height));
height_water = zeros(size(ringprobe_height));
height_oil = zeros(size(ringprobe_height));
perimeter_oil = zeros(size(ringprobe_height));
perimeter_water = zeros(size(ringprobe_height));
perimeter_interface = zeros(size(ringprobe_height));
area_oil = zeros(size(ringprobe_height));
area_water = zeros(size(ringprobe_height));

k = 1;
while k <= length(ringprobe_height)

    if ringprobe_height(k) > pipeline_radius

        a(k) = ringprobe_height(k) - pipeline_radius;
        theta(k) = 2*(acos(a(k)/pipeline_radius)); %array

        % Calculation of height of water and height of oil in the pipeline
        height_water(k) = ringprobe_height(k); %array
        height_oil(k) = pipeline_radius - a(k); %array
    end
end

```

```

    % Calculation of oil phase perimeter, water phase perimeter and interface
    perimeter
    perimeter_oil(k) = pipeline_radius*theta(k); %array
    perimeter_water(k) = (2*pi*pipeline_radius) - (pipeline_radius*theta(k));
%array
    perimeter_interface(k) = 2*sqrt(pipeline_radius^2 - (a(k)).^2); %array

    % Calculation of oil phase area and water phase area
    area_oil(k) = ((pipeline_radius^2)*(theta(k))/2) -
((a(k)).*sqrt(pipeline_radius^2 - (a(k)).^2)); %array
    area_water(k) = (pi*pipeline_radius^2) - area_oil(k); %array

    elseif ringprobe_height(k) < pipeline_radius

    a(k) = pipeline_radius - ringprobe_height(k);
    theta(k) = 2*(acos(a(k)/pipeline_radius)); %array

    % Calculation of height of water and height of oil in the pipeline
    height_water(k) = ringprobe_height(k); %array
    height_oil(k) = pipeline_radius + a(k); %array

    % Calculation of oil phase perimeter, water phase perimeter and interface
    perimeter
    perimeter_oil(k) = (2*pi*pipeline_radius) - (pipeline_radius*(theta(k)));
%array
    perimeter_water(k) = pipeline_radius*(theta(k)); %array
    perimeter_interface(k) = 2*sqrt(pipeline_radius^2 - (a(k)).^2); %array

    % Calculation of oil phase area and water phase area
    area_water(k) = ((pipeline_radius^2)*(theta(k))/2) -
((a(k)).*sqrt(pipeline_radius^2 - (a(k)).^2)); %array
    area_oil(k) = (pi*pipeline_radius^2) - area_water(k); %array

    else

    disp(' Could not determine area and perimeter of the phases');

    end

k=k+1;

end

% Calculation of in-situ velocity for water and oil
velocity_oil = flowrate_oil./area_oil;
velocity_water = flowrate_water./area_water;

% Calculation to determine the hydraulic diameter of water and oil -
% Conditions outlined by Moalem-Maron et. al. (1998)

% Set first value to all variables
diameter_oil = zeros(size(ringprobe_height));
diameter_water = zeros(size(ringprobe_height));

k = 1;
while k <= length(ringprobe_height)

    if (velocity_oil(k) > velocity_water(k))

```

```

    diameter_oil(k) =
(4*area_oil(k))./(perimeter_oil(k)+perimeter_interface(k));
    diameter_water(k) = (4*area_water(k))./perimeter_water(k);

    elseif (velocity_oil(k) < velocity_water(k))

    diameter_oil(k) = (4*area_oil(k))./perimeter_oil(k);
    diameter_water(k) =
(4*area_water(k))./(perimeter_water(k)+perimeter_interface(k));

    else

    diameter_oil(k) = (4*area_oil(k))./perimeter_oil(k);
    diameter_water(k) = (4*area_water(k))./perimeter_water(k);

    end

k=k+1;

end

% Calculation of Reynolds number
Reynolds_oil = (density_oil*velocity_oil.*diameter_oil)/viscosity_oil;
Reynolds_water =
(density_water*velocity_water.*diameter_water)/viscosity_water;

% Determination of C and n constant for water phase.

% Set first value to all variables
C_water = zeros(size(ringprobe_height));
n_water = zeros(size(ringprobe_height));
flowtype_water = 'ERROR';

k = 1;
while k <= length(ringprobe_height)

    if Reynolds_water(k) < 2000

    C_water(k) = 16;
    n_water(k) = 1;
    flowtype_water = 'Laminar Flow';

    elseif Reynolds_water(k) > 4000

    C_water(k) = 0.046;
    n_water(k) = 0.2;
    flowtype_water = 'Turbulent Flow';

    else

    C_water(k) = NaN;
    n_water(k) = NaN;
    flowtype_water = 'Transitional Flow';

    end

k = k+1;

end

```

```

% Determination of C and n constant for oil phase.

% Set first value to all variables
C_oil = zeros(size(Reynolds_water));
n_oil = zeros(size(Reynolds_water));
flowtype_oil = 'ERROR';

k = 1;
while k <= length(Reynolds_water)

    if Reynolds_oil(k) < 2000
        C_oil(k) = 16;
        n_oil(k) = 1;
        flowtype_oil = 'Laminar Flow';

    elseif Reynolds_oil(k) > 4000

        C_oil(k) = 0.046;
        n_oil(k) = 0.2;
        flowtype_oil = 'Turbulent Flow';

    else

        C_oil(k) = NaN;
        n_oil(k) = NaN;
        flowtype_oil = 'Transitional Flow';

    end

    k = k+1;
end

% Calculation of friction factor for oil and water flow.
friction_oil =
C_oil.*((density_oil*velocity_oil.*diameter_oil/viscosity_oil).^(-n_oil));
friction_water =
C_water.*((density_water*velocity_water.*diameter_water/viscosity_water).^(-
n_water));

% Calculation of shear stress for oil and water flow.
shearstress_oil = 0.5*(friction_oil*density_oil).*(velocity_oil.^2);
shearstress_water = 0.5*(friction_water*density_water).*(velocity_water.^2);

% Calculation of pressure drop (final stage)
dpdz =
((shearstress_oil.*perimeter_oil)+(shearstress_water.*perimeter_water))./(-
1*(area_oil + area_water));
pressure_drop = abs(dpdz/1000);

% Calculation of shear stress of interface
shearstress_interface = (area_oil.*dpdz + shearstress_oil.*perimeter_oil)./(-
1*perimeter_interface);

% Shows parameters for both oil and water phases
fprintf('\n\n');
disp('By assuming a flat interface between phases, the parameters for both oil
and water phases are obtained');
fprintf('\n\n');

```

```

fprintf('Ring Probe Height\t\tPerimeter of Oil Phase\t\tPerimeter of Water
Phase\t\tPerimeter of Interface\t\tArea of Oil Phase\t\tArea of Water
Phase\n');
table_1 = [ringprobe_height; perimeter_oil; perimeter_water;
perimeter_interface; area_oil; area_water];
fprintf('%10.6f m %25.6f m %25.6f m %27.6f m %23.6f m^2 %20.6f m^2
\n',table_1);
fprintf('\n\n');
fprintf('Velocity of Oil\t\tVelocity of Water\t\tDiameter of Oil\t\tDiameter of
Water\t\tReynolds Number of Oil\t\tReynolds Number of Water\n');
table_2 = [velocity_oil; velocity_water; diameter_oil; diameter_water;
Reynolds_oil; Reynolds_water];
fprintf('%10.6f m/s %15.6f m/s %20.6f m %17.6f m %28.6f %25.6f \n',table_2);
fprintf('\n\n');
fprintf('Friction factor of Oil\t\tFriction factor of Water\t\tShear Stress of
Oil\t\tShear Stress of Water\t\tPressure Drop\t\tShear Stress of Interface\n');
table_3 = [friction_oil; friction_water; shearstress_oil; shearstress_water;
pressure_drop; shearstress_interface];
fprintf('%15.6f %25.6f %30.6f Pa %20.6f Pa %21.6f kPa/m %20.6f Pa\n',table_3);
fprintf('\n\n');

% Graph plot
disp('Graph Comparison of Theoretical Data vs Experimental Data');
experimental_pressuredrop = input('\nPlease enter the set of experimental data
in bracket(unit of kPa)(example = [1 2 3 4]):'); %array

subplot(2,1,1),plot(superficialvelocity_oil,pressure_drop,'-
or',superficialvelocity_oil, experimental_pressuredrop./pipeline_length,'-xk')
title('Graph of Pressure Drop vs Superficial Velocity of Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Pressure Drop, kPa/m')
legend('theoretical value (ring probe)', 'experimental value')
grid on

% Graph plot
fprintf('\n\n');
disp('Graph Shear Stress of Interface vs Superficial Velocity of Oil');

subplot(2,1,2),plot(superficialvelocity_oil,shearstress_interface,'-or')
title('Graph of Shear Stress of Interface vs Superficial Velocity of Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Shear Stress of Interface, Pa')
grid on

```

Appendix VIII: MATLAB Programming Code for Curvature Interfacial Configuration

```
fprintf('\n\n');
disp('This program will determine the pressure drop inside the pipeline for
liquid-liquid two phase system by using wire probe height and ring probe height
with assumption that the curve interface between the phases');

% Prompt the user to enter a string character information
wireprobe_height = input('\nPlease enter the set of wire probe heights in
bracket(unit of mm){example = [1 2 3 4]}:');*10^(-3); %array
ringprobe_height = input('\nPlease enter the set of ring probe heights in
bracket(unit of mm){example = [1 2 3 4]}:');*10^(-3); %array
superficialvelocity_water = input('Please enter the superficial velocity of
water(unit of m/s):'); %array
superficialvelocity_oil = input('Please enter the set of superficial velocity
of oil in bracket(unit of m/s){example = [1 2 3 4]}:'); %array
pipeline_radius = input('Please enter the radius of pipeline (unit of
mm):')*10^(-3); %single number
pipeline_length = input('Please enter the length of pipeline (unit of m):');
%single number
density_oil = input('Please enter the density of oil (unit of kg/m^3):');
%single number
density_water = input('Please enter the density of water (unit of kg/m^3):');
%single number
viscosity_oil = input('Please enter the viscosity of oil at 25 degree
Celcius(unit of kg/m.s or Pa.s):'); %single number
viscosity_water = input('Please enter the viscosity of water at 25 degree
Celcius(unit of kg/m.s or Pa.s):'); %single number
% surface_tension = input('Please enter the surface tension at 25 degree
Celcius(unit of kg/s^2 or N/m):'); %single number
flowrate_oil = input('Please enter the set of oil flowrate in bracket for the
corresponding wire probe heights(unit of 10^(-5)m^3/s){example = [1 2 3
4]}:');*10^(-5); %array
flowrate_water = input('Please enter the set of water flowrate in bracket for
the corresponding wire probe heights(unit of 10^(-5)m^3/s){example = [1 2 3
4]}:');*10^(-5); %array
fprintf('\n\n');

%zeros(s Set first value to all variables
y1 = zeros(size(ringprobe_height));
y2 = zeros(size(wireprobe_height));

% Calculation of oil phase area and water phase area

k = 1;
while k <= length(ringprobe_height)

    if (pipeline_radius > ringprobe_height(k))

        y1(k) = 0-(pipeline_radius - ringprobe_height(k));

    elseif (pipeline_radius < ringprobe_height(k))

        y1(k) = 0+(ringprobe_height(k) - pipeline_radius);

    else

        y1(k) = 0;
```

```

        end
k=k+1;
end
k = 1;
while k <= length(wireprobe_height)
    if (pipeline_radius > wireprobe_height(k))
        y2(k) = 0-(pipeline_radius - wireprobe_height(k));
    elseif (pipeline_radius < wireprobe_height(k))
        y2(k) = 0+(wireprobe_height(k) - pipeline_radius);
    else
        y2(k) = 0;
    end
k=k+1;
end

x1 = sqrt((pipeline_radius^2)-(y1.^2));
s = sqrt(x1.^2 + ((x1.^4)./(y1.^2)));
area_water = ((-1*(pipeline_radius^2))*(acos(x1/pipeline_radius))) +
(pipeline_radius*x1.*(sin(acos(x1/pipeline_radius)))) +
((pi/2)*((pipeline_radius^2)- (s.^2))) + ((s.^2).*acos(x1./s))-
(s.*x1.*sin(acos(x1./s))) - (2*x1.*y1) - (2*(x1.^3)./y1);
area_oil = (pi*pipeline_radius^2) - area_water;

theta = acos(x1/pipeline_radius);
ghama = (pi/2)- theta;
perimeter_water = pipeline_radius*(2*ghama);
perimeter_oil = (2*pi*pipeline_radius) - perimeter_water;
alpha = acos(x1./s);
betha = (pi/2)- alpha;
perimeter_interface = s.*(2*betha);

% Calculation of in-situ velocity for water and oil
velocity_oil = flowrate_oil./area_oil;
velocity_water = flowrate_water./area_water;

% Calculation to determine the hydraulic diameter of water and oil -t6
% Conditions outlined by Moalem-Maron et. al. (1998)

% Set first value to all variables
diameter_oil = zeros(size(wireprobe_height));
diameter_water = zeros(size(wireprobe_height));

k = 1;
while k <= length(wireprobe_height)
    if (velocity_oil(k) > velocity_water(k))

```

```

    diameter_oil(k) =
(4*area_oil(k))./(perimeter_oil(k)+perimeter_interface(k));
    diameter_water(k) = (4*area_water(k))./perimeter_water(k);

    elseif (velocity_oil(k) < velocity_water(k))

    diameter_oil(k) = (4*area_oil(k))./perimeter_oil(k);
    diameter_water(k) =
(4*area_water(k))./(perimeter_water(k)+perimeter_interface(k));

    else

    diameter_oil(k) = (4*area_oil(k))./perimeter_oil(k);
    diameter_water(k) = (4*area_water(k))./perimeter_water(k);

    end

k=k+1;

end

% Calculation of Reynolds number
Reynolds_oil = (density_oil*velocity_oil.*diameter_oil)/viscosity_oil;
Reynolds_water =
(density_water*velocity_water.*diameter_water)/viscosity_water;

% Determination of C and n constant for water phase.

% Set first value to all variables
C_water = zeros(size(wireprobe_height));
n_water = zeros(size(wireprobe_height));
flowtype_water = 'ERROR';

k = 1;
while k <= length(wireprobe_height)

    if Reynolds_water(k) < 2000

    C_water(k) = 16;
    n_water(k) = 1;
    flowtype_water = 'Laminar Flow';

    elseif Reynolds_water(k) > 4000

    C_water(k) = 0.046;
    n_water(k) = 0.2;
    flowtype_water = 'Turbulent Flow';

    else

    C_water(k) = NaN;
    n_water(k) = NaN;
    flowtype_water = 'Transitional Flow';

    end

k = k+1;

end

```

```

% Determination of C and n constant for oil phase.

% Set first value to all variables
C_oil = zeros(size(Reynolds_water));
n_oil = zeros(size(Reynolds_water));
flowtype_oil = 'ERROR';

k = 1;
while k <= length(Reynolds_water)

    if Reynolds_oil(k) < 2000
        C_oil(k) = 16;
        n_oil(k) = 1;
        flowtype_oil = 'Laminar Flow';

    elseif Reynolds_oil(k) > 4000

        C_oil(k) = 0.046;
        n_oil(k) = 0.2;
        flowtype_oil = 'Turbulent Flow';

    else

        C_oil(k) = NaN;
        n_oil(k) = NaN;
        flowtype_oil = 'Transitional Flow';

    end

    k = k+1;
end

% Calculation of friction factor for oil and water flow.
friction_oil =
C_oil.*(density_oil*velocity_oil.*diameter_oil/viscosity_oil).^(-n_oil));
friction_water =
C_water.*(density_water*velocity_water.*diameter_water/viscosity_water).^(-
n_water));

% Calculation of shear stress for oil and water flow.
shearstress_oil = 0.5*(friction_oil*density_oil).*(velocity_oil.^2);
shearstress_water = 0.5*(friction_water*density_water).*(velocity_water.^2);

% Calculation of pressure drop (final stage)
dpdz =
((shearstress_oil.*perimeter_oil)+(shearstress_water.*perimeter_water))./(-
1*(area_oil + area_water));
pressure_drop = abs(dpdz/1000);

% Calculation of shear stress of interface
shearstress_interface = (area_oil.*dpdz + shearstress_oil.*perimeter_oil)./(-
1*perimeter_interface);

% Shows parameters for both oil and water phases
fprintf('\n\n');
disp('By assuming a flat interface between phases, the parameters for both oil
and water phases are obtained');
fprintf('\n');

```

```

fprintf('Ring Probe Height\t\tWire Probe Height\t\tPerimeter of Oil
Phase\t\tPerimeter of Water Phase\t\tPerimeter of Interface\t\tArea of Oil
Phase\t\tArea of Water Phase\n');
table_1 = [ringprobe_height; wireprobe_height; perimeter_oil; perimeter_water;
perimeter_interface; area_oil; area_water];
fprintf('%10.6f m %25.6f m %25.6f m %25.6f m %27.6f m %23.6f m^2 %20.6f m^2
\n',table_1);
fprintf('\n\n');
fprintf('Velocity of Oil\t\tVelocity of Water\t\tDiameter of Oil\t\tDiameter of
Water\t\tReynolds Number of Oil\t\tReynolds Number of Water\n');
table_2 = [velocity_oil; velocity_water; diameter_oil; diameter_water;
Reynolds_oil; Reynolds_water];
fprintf('%10.6f m/s %15.6f m/s %20.6f m %17.6f m %28.6f %25.6f \n',table_2);
fprintf('\n\n');
fprintf('Friction factor of Oil\t\tFriction factor of Water\t\tShear Stress of
Oil\t\tShear Stress of Water\t\tPressure Drop\t\tShear Stress of Interface\n');
table_3 = [friction_oil; friction_water; shearstress_oil; shearstress_water;
pressure_drop; shearstress_interface];
fprintf('%15.6f %25.6f %30.6f Pa %20.6f Pa %21.6f kPa/m %20.6f Pa\n',table_3);
fprintf('\n\n');

% Graph plot
disp('Graph Comparison of Theoretical Data vs Experimental Data');
experimental_pressuredrop = input('\nPlease enter the set of experimental data
in bracket(unit of kPa){example = [1 2 3 4]}:'); %array

subplot(2,1,1),plot(superficialvelocity_oil,pressure_drop,'-
or',superficialvelocity_oil, experimental_pressuredrop./pipeline_length,'-xk')
title('Graph of Pressure Drop vs Superficial Velocity of Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Pressure Drop, kPa/m')
legend('theoretical value (wire probe)','experimental value')
grid on

% Graph plot
fprintf('\n\n');
disp('Graph Shear Stress of Interface vs Superficial Velocity of Oil');

subplot(2,1,2),plot(superficialvelocity_oil,shearstress_interface,'-or')
title('Graph of Shear Stress of Interface vs Superficial Velocity of Oil')
xlabel('Superficial Velocity of Oil, m/s')
ylabel('Shear Stress of Interface, Pa')
grid on

```