ESTIMATION OF RESERVOIR PARAMETERS USING MATERIAL BALANCE METHOD

By

Chin Pui Yee

Dissertation submitted in partial fulfillment of the requirements for the Bachelor of Engineering (Hons) of Petroleum Engineering

May 2011

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CERTIFICATION OF APPROVAL

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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ABSTRACT

Material Balance Equation (MBE) is introduced to understand the inventory of materials entering, leaving and accumulating in a reservoir which results in a better understanding of reservoir development planning as well as for the prediction of water influx. The linearized MBE introduced by Havlena & Odeh is designed in a manner whereby from plotting one variable group against another group, initial hydrocarbon in place can be subsequently obtained. Without detailed knowledge, trial and error approach is necessary and the calculation could be tedious and time consuming. Uncertainties in aquifer properties add up more complications. A simplified approach suggested by El-Khatib to estimate aquifer parameters is reviewed and applied to actual fields, focusing on the saturated oil reservoirs under simultaneous drives. By providing a reservoir's PVT and production history, estimation of initial hydrocarbon in place, ratio of initial hydrocarbon pore volume of gas to oil and water influx parameters could be solved simultaneously. By assuming the time adjustment factor, c in dimensionless time, t_D and dimensionless aquifer size, R_{eD} in sensitivity analysis, numerical inversion of Laplace transform is used to obtain the Van-Everdingen-Hurst (VEH) solution with respect to aquifer parameters. With that, the original oil in place N, gas cap ratio m, and water influx constant B, can be obtained simultaneously with their linear relations in MBE via multiple-regression. Sum of squares of residuals are then computed and mapped for different sets of c and R_{eD} to determine the regions of minima. The non-uniqueness of the map can be countered by understanding of the reservoir and aquifer characteristics. Finally, the approach is outlined to quantify the possibility in N, m and B. Results have shown convergence to the correct solutions suggested in literature. This project presents an innovative approach as a more robust approach for reservoir preliminary understanding.

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CHAPTER 1

INTRODUCTION

1.1 Background of Study

Material Balance Equation (MBE) wide applications cover from estimating initial hydrocarbon in place independent of geological interpretation as well as to assert the volumetric estimation. It is equally applicable in predicting aquifer performance and determining the drive mechanisms in a reservoir. And hence, it is a general equation used by reservoir engineers in oil and gas industry. [1]

MBE is a simple application of the law of conservation of matter to the hydrocarbon reservoirs which primary principles lay in a volumetric balance. It states that since the volume as defined by its initial limit of a reservoir is a constant, the algebraic sum of the volume changes of the oil, free gas, water and rock volumes in a reservoir must be zero. For instance, if both the oil and gas reservoir volume decreases, the sum of these two decreases must be balanced by some changes of equal magnitude. With an assumption that a complete equilibrium is attained at all times in a reservoir between the oil and its solution gas, a generalized material balance equation could be expressed in the terms of quantities of oil, gas, and water produced, average reservoir pressure, volume of water encroaching from the aquifer and finally derived into the initial oil and gas volume of the reservoir. [2]

The physical situation occur in a reservoir is that when an oil and gas reservoir is drilled with well, oil and gas, and often some water, is produced, hence reducing the reservoir pressure and causing the remaining oil and gas to expand to fill the space vacated by the fluids removed. When the oil or gas bearing formation is connected with an aquifer, water encroached into the reservoir as the pressure declines due to production. Water encroachment will retard the decline in reservoir pressure and thus decrease the extent of expansion of oil and water. By having bottom-hole samples, it is possible to predict how fluids behave in a reservoir when reservoir pressure declines. [2]

Since the connate water and formation compressibility are small, it can be concluded that their compressibility are less significant than of the gas and gas cap reservoirs as well as the undersaturated reservoirs below bubble point. And therefore, for the means of simplicity, they could be neglected for circumstances under consideration. [2]

Generally, necessary conditions would have to be fulfilled for a successful solution of the MBE: [3]

(1) An unspecified consistency of results

(2) Agreement between MBE results and those computed volumetrically

This criterion is usually overemphasized as the MBE initial hydrocarbon in place contributes to the pressure-production history while the volumetric initial hydrocarbon refers to the total hydrocarbon in place, which some portion of it may not contribute to the production history.

(3) Straight line of MBE Interpretation

Straight line method as proposed by Havlena and Odeh requires the plotting of a variable group versus another variable group according to the drive mechanisms of a reservoir. The most important aspect of this method is attached with the sequence of the plotted points and the resulting shape of the plot.

Another area should be highly highlighted in a MBE solution method is the information on water influx if there is any. Water-bearing rocks - aquifers surround almost all hydrocarbon reservoirs. These aquifers maybe so much larger than the reservoirs they adjoin appearing infinite in size, or they could be so small that they are negligible in their effect on reservoir performance. When reservoir fluids are produced and reservoir pressure declines, a pressure difference develops between the surrounding aquifer and the reservoir, hence following by aquifer water encroachment. [4]

Mathematical models have been introduced to estimate water influx based on some assumptions that describe the characteristics of the aquifer. It plays an important role in a MBE solution and yet very little information is obtained during the exploration-development period concerning on the presence or characteristics of an aquifer. Due to the massive uncertainties in the aquifer characteristics, all proposed models hence require historical reservoir performance data to evaluate aquifer property parameters. [4]

By applying the compressibility definition to the said aquifer, the total water influx is directly proportional with the product of aquifer compressibility, initial volume of water and pressure drop. Since the compressibility factors are usually very small, unless the initial volume of water is very large, or else the aquifer function as a drive mechanism is negligible. Though, if the aquifer is large enough, this assumption is inadequate to be implied in general practices as the pressure drop at the reservoir boundary is not instantaneously transmitted throughout the aquifer. There will be a time lag between the pressure change in the reservoir and the full response of the aquifer itself. Henceforth, the water drive is time dependent in this context. [5]

The van-Everdingen-Hurst (VEH) solution developed from the radial diffusivity equation is one of the most rigorous aquifer influx models to date for the context of unsteady state aquifer behaviours. The flow equations for oil flowing into a wellbore from the reservoir are identical in form of the equations describing flow from an aquifer into a cylindrical reservoir, only at a different radial scale. There is a greater interest lying in calculating water influx rate rather than the pressure and leading to the determination of water influx as a function of given pressure drop at the inner boundary of the reservoir-aquifer system. Van-Everdingen and Hurst had solved the radial diffusivity equation for the aquifer-reservoir system by applying the Laplace transformation to the equation, expressed in terms of dimensionless variables in which dimensionless radius refers to the ratio of radius of reservoir to aquifer and with all parameters referring to the aquifer instead of reservoir properties. [5]

The dimensionless water influx, W_D is generally expressed in tabular form or as a set of polynomial expressions providing that W_D as a function of dimensionless time, t_D for a ratios of the aquifer to reservoir radius, r_{eD}. Each table provides different resolution of the dimensionless time scale and that the graphs are valid for all values of t_D and hence are equally applicable for calculating the early, unstable influx (infinite-acting) and for the influx occurring at which the aquifer boundary effects are felt providing a convenient approach for calculating water influx. In practical cases of history matching, theory is extended to calculate the cumulative water influx corresponding to a continuous pressure decline at the reservoir-aquifer boundary. Conventional practices are to divide the continuous decline into a series of discrete pressure steps and with the superposition of the water influxes with respect of time, the answers give the cumulative water influx. [5]

1.2 Problem Statement

The linearized MBE introduced by Havlena & Odeh is arranged in manner such that trial and error approach is often used to estimate the parameters of reservoirs, e.g. initial hydrocarbon in place, the ratio of initial hydrocarbon pore volume of gas to oil, water influx etc depending on the known and unknown variables for different circumstances. By plotting one variable group against another group, these unknowns can be subsequently obtained. Though, without any prior knowledge on reservoir parameters, the calculation via the Havlena and Odeh method could be very tedious and time consuming as several guesses of are made via trial and error method till a straight line is obtained. [6]

More problems arise when there are uncertainties attached with the subject of water influx more than any other. This is because there would be a rare chance that companies choose to drill deep wells into an aquifer to collect the data on porosity, permeability, thickness, fluid properties and etc. Instead, the properties are usually inferred from the reservoir itself with unknown certainty. More uncertainties are revolving on the areal continuity and geometry of the aquifer itself. [4]

Since that the knowledge on reservoirs provide a better understanding on future development planning, a simplified approach has to be proposed to provide an efficient solution of the said problems both to prevent the hassles of estimating initial hydrocarbon in place and to estimate the future water influx if any better.

1.3 Objectives & Scope of Study

This study is focused on the saturated oil reservoirs with the presence of water influx and gas cap under simultaneous drive mechanisms. By providing a reservoir's PVT data and production history, estimation of initial hydrocarbon in place and water influx parameters are made possible by multi-regression method via programming.

The objective of this research is to determine the initial hydrocarbon in place, ratio of initial hydrocarbon pore volume of gas to oil and water influx parameters in a reservoir via Material Balance Equation (MBE) through programming by calculating the inventory of all materials entering, leaving and accumulating in a reservoir. From the results obtained, knowledge on the reservoirs enables us to grasp a more accurate idea on future development planning and to predict future water influx.

Specifically, the objective is to determine original oil in place, N, ratio of gas-cap volume to oil volume, m and water influx constant, B, and uncertainties in each, resulting from a combination of water influx parameters. The uncertainties in c and R_{eD} in water influx is considered and the effect of correlation between parameters is investigated in the prior distribution on the OHIP estimated. The analysis is deliberately limited to a 2-parameter problem so that the parameter relationship can be visualized in 2D plots.

In this project, I would first provide a mathematical background of relevant Material Balance theory as applied to the integration of van-Everdingen-Hurst (VEH) solution using numerical inversion of Laplace transform. Next, I would outline the approach to quantify uncertainties in time adjustment factor, c in dimensionless time, t_D and dimensionless aquifer size, R_{eD}. Finally, I will demonstrate the concept using examples reported in the literature.

CHAPTER 2

THEORIES & LITERATURE REVIEW

Material Balance in a Straight Line

The general form of Material Balance Equation (MBE) is first introduced by Schilthuis as an application of volumetric balance whereby the cumulative production, defined as underground withdrawal is equal to the expansion of the fluids in a reservoir resulting from a finite pressure drop. [5]

Underground withdrawal	=	Expansion of oil + originally dissolved gas	
		+	Expansion of gas cap gas
		+	Reduction of HCPV due to connate water expansion and decrease in pore volume

The zero dimensional approach is then derived and subsequently widely applied using mainly the interpretative technique of Havlena and Odeh, expressing the MBE in a straight line, to provide an invaluable insight of a reservoir drive mechanisms. The equations are then further developed by sophisticated numerical simulators into multi-dimensional, multi-phases, dynamic material balance programs. Still, a review on classical approach is of immense importance to illustrate the behavior of hydrocarbon reservoirs. [5]

One of the most popular MBE methods is proposed by Havlena and Odeh (1936) requires the plotting of a variable group versus another variable group, depending on the drive mechanisms in a reservoir. The most important aspect of this method of solution is that it attaches significance to the sequence of the plotted points and to the shape of the resulting plots. [7]

Underground Withdrawal:

$$F = N_p \left(B_o + B_g\left(R_p - R_s\right)\right) + W_p B_w \qquad (1)$$

Expansion of Oil and Originally Dissolved Gas:

$$E_o = (B_o - B_{oi}) + B_g(R_{si} - R_s) \tag{2}$$

Expansion of Gas-Cap Gas:

$$E_g = B_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) \tag{3}$$

Expansion of connate water and reduction in pore volume:

$$E_{fw} = (1 + m)B_{oi}\left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\Delta P \tag{4}$$

Hence,

$$N_p\left[B_o + B_g\left(R_p - R_s\right)\right] + W_p B_w = N\left(E_o + mE_g + E_{fw}\right) + W_e \tag{5}$$

For simplicity, engineers may usually neglect the effect of rock and water expansion in saturated reservoirs, whereby MBE is reduced to:

$$N_p\left[B_o + B_g\left(R_p - R_s\right)\right] + W_p B_w = N\left(E_o + mE_g\right) + W_e \tag{6}$$

Due to the inherent uncertainties related on the subject of water influx, it is often evaluated independently based on assumptions that best describe the characteristics of an aquifer. Several mathematical models are developed and proposed to evaluate constants representing aquifer properties based on reservoir historical performance data since the aquifer properties are rarely known from appraisal-development stage. The following describes some common mathematical models used in water influx interpretation.

Pot Aquifer

The simplest model indicates that a drop in the reservoir pressure due to the production of fluids causes the aquifer water to expand and flow into the reservoir. [4]

This model is only applicable for small aquifers as it assumes that a pressure drop in reservoir is instantaneously transmitted throughout the reservoir-aquifer system. Time dependence factor has to be considered for larger aquifer as it takes time for the aquifer to respond to a pressure change in reservoir. [4]

$$W_e = (c_w + c_f)W_i f(P_i - P) \tag{7}$$

$$W_i = \left[\frac{\pi(r_a{}^2 - r_e{}^2)h\varphi}{5.615}\right] \tag{8}$$

$$f = \frac{(\text{encroachment angle})^\circ}{360^\circ} = \frac{\theta}{360^\circ} \tag{9}$$

Schilthuis's Steady State Model

Schilthuis (1936) [8] proposed that once an aquifer enters steady state flow regime, the flow behavior could be explained by Darcy's Equation.

$$\frac{dW_e}{dt} = e_w = \left[\frac{0.00708\,kh}{\mu_w \ln\left(\frac{r_a}{r_e}\right)}\right](P_i - P) \tag{10}$$

C is expressed in bbl/day/psi and could be calculated from reservoir historical production data over time intervals.

Hurst's Modified Steady State Model

Hurst (1943) [9] proposed that apparent aquifer drainage area would increase with time, and dimensionless radius r_a/r_e should be a time dependent function:

$$\frac{r_a}{r_e} = at \tag{11}$$

$$W_e = C\int_0^t \left[\frac{P_i - P}{\ln at}\right] dt \tag{12}$$

Two unknowns, C and a must be determined from reservoir-aquifer pressure and water influx historical data.

Van Everdingen-Hurst Unsteady State Model

When a well surrounded by a large aquifer is brought on production, the flow of crude oil into a wellbore are identical with the flow of water from an aquifer into a cylindrical reservoir and in which the pressure behavior is behaving in transient/unsteady state condition. [10]

Van Everdingen and Hurst (1949) [10] proposed a Laplace transformation to solve the diffusivity constant for the aquifer-reservoir system which could be applied for both edge-water and bottom-water drive reservoirs. By providing an exact solution to the radial diffusivity equation, this method is considered the most accurate technique to calculate water influx.

(i) Edge-Water Drive

An idealized radial flow system represents an edge-water drive reservoir in which the inner boundary is defined as the interface between the reservoir and aquifer. By applying the constant terminal pressure boundary conditions, dimensionless diffusivity equation is served to solve the dimensionless water influx as a function of dimensionless time and dimensionless radius.

$$W_e = B\Delta P W_{eD} \tag{13}$$

$$B = \text{water influx constant } \left(\frac{\text{bbl}}{\text{psi}}\right) = 1.119\varphi c_t {r_e}^2 fh \tag{14}$$

$$f = \frac{(\text{encroachment angle})^\circ}{360^\circ} = \frac{\theta}{360^\circ} \tag{15}$$

(ii) Bottom-Water Drive

Due to the limitation of Van Everdingen-Hurst Unsteady State Model which could not account for the vertical water encroachment in bottom-water driven reservoirs, Allard and Chen (1988) [11] tabulate the new set of values of W_{eD} as a function of vertical permeability.

Carter-Tracy Water Influx Model

Carter-Tracy (1960) [12] proposed a calculation that does not require superposition as in Van Everdingen-Hurst Unsteady State Model and allows a direct calculation of water influx by assuming constant water influx rate over finite water interval.

$$(W_e)_n = (W_e)_{n-1} + [(t_D)_n - (t_D)_{n-1}]\left[\frac{B\Delta P_n - (W_e)_{n-1}(P'_D)_{n-1}}{(P_D)_n - (t_D)_{n-1}(P'_D)_n}\right] \tag{16}$$

Since Carter-Tray Model does not provide an exact solution for diffusivity equation, it is less accurate than Van Everdingen-Hurst Unsteady State Model and should be treated as an approximation.

Fetkovich's Method

Fetkovich (1971) [13] proposed a method of estimating water influx behavior of a finite aquifer. Fetkovich's Model applies the productivity index concept to describe the water flowing from aquifer to reservoir whereby it assumes that the water influx rate is proportional with the pressure drop happened at the reservoir-aquifer interface.

$$(\Delta W_e)_n = \frac{W_{ei}}{P_i}[(\bar{P}_a)_{n-1} - (\bar{P}_r)_n]\left[1 - \exp\left(-\frac{JP_i\Delta t_n}{W_{ei}}\right)\right] \tag{17}$$

By assuming that water influx rate is proportional to pressure drop directly without taking consideration of the time dependence factor, Fetkovich's model is only sufficiently accounted for finite reservoirs as it neglects the unsteady state behavior of an aquifer.

Statistical Method of History Matching and Simultaneous Solution of N and m

Omole and Ojo (1993) [6] has proposed a statistical model which involves the rearrangement of Havlena & Odeh method which removes the "m" from gas-cap gas and rock plus connate water expansion terms. Hence, it reduces the tediousness of trial and error approach when a prior knowledge of "m" is lacking. The estimation from the correlation and regression analysis gives way to N and m using computer programming.

$$\frac{F-W_e}{E_o+E_{fw}} = N + mN\left(\frac{E_g+E_{fw}}{E_o+E_{fw}}\right) \tag{18}$$

Material Balance Regression Analysis of Water-driven Oil and Gas Reservoirs Using Aquifer-Reservoir Expansion Term (CARET)

Sills (1996) [14] proposed the usage of CARET combines Tehrani's voidage minimization approach with straight line method by Havlena and Odeh. It is developed for van Everdingen and Hurst (VEH) unsteady state radial aquifer model. This method applies the concept of water influx function, S as a function of pressure and time as shown:

$$E_{CARET} = \left[2c_eS\left(\frac{1}{1-S_{wo}} + \frac{m}{1-S_{og}-S_{wg}}\right)\left(\frac{h_A}{h_R}\right) + m\left(\frac{E_g+E_{fwg}}{B_{gi}}\right)\right]B_{oi} + E_o + E_{fwo} \tag{19}$$

$$F = NE_{CARET} \tag{20}$$

A Polynomial Approach to the Van-Everdingen-Hurst Dimensionless Variables for Water Encroachment

Klins, Bouchard and Cable (1988) [15] have presented four sets of simplified polynomials to obtain P_D or Q_D for either infinite or finite aquifers, for constant terminal rate and constant terminal pressure respectively. This proposed method counters the several drawbacks of van-Everdingen-Hurts or Carter Tracy table look-up and interpolation methods. Table look-up is tedious, time consuming and is limited to $r_e/r_o < 10$ for finite aquifers. Besides, if the Carter-Tracy water influx model is used, the values of Pd derivatives are needed. These equations use up to 15 times less computation time than traditional table look-up and because r_D and t_D are implicit in the equation, there is no requirement for interpolation. Though, problems occur when there are uncertainties on r_D and t_D. Therefore, these equations are not suitable for this project. This approach distinguishes between finite and infinite aquifers by the calculation of t_{cross} as shown below:

Constant Terminal Rate Case, P_D:

$$t_{cross} = 0.0980958(r_D - 1) + 0.100683(r_D - 1)^{2.03863} \quad (21)$$

Infinite Aquifers:

1. $t_D \leq 0.01$

$$P_D' = 1/\sqrt{\pi t_D} \quad (22)$$

2. $0.01 \leq t_D \leq 500$

$$P_D' = \frac{b_0 + b_1(t_D)^{b_6} + b_2(t_D)^{b_7} + b_3(t_D)^{b_8} + b_4(t_D)^{b_9} + b_5(t_D)^{b_{10}}}{\left[b_{11} + b_{12}(t_D)^{b_7} + b_{13}(t_D) + t_D{}^{b_9}\right]^2} \quad (23)$$

$b_0 = 3577.752441$	$b_7 = 0.5003552$
$b_1 = 5121.404179$	$b_8 = 0.838834$
$b_2 = 552.462473$	$b_9 = 1.338479$
$b_3 = 364.062209$	$b_{10} = 0.338479$
$b_4 = 26.908805$	$b_{11} = 95.13748$
$b_5 = 896.239475$	$b_{12} = 77.0034$
$b_6 = -0.499645$	$b_{13} = 16.63856$

3. $500 \leq t_D$

$$P_D' = \frac{1}{2t_D}\left[1 - \frac{\ln t_D}{2t_D} + \frac{0.09546}{t_D}\right] \quad (24)$$

Finite Aquifers:

1. $t_{cross} \leq t_D$

$$P_D = \frac{2}{r_D^2 - 1} - \frac{2e^{-\beta_1^2 t_D} J_1^2(\beta_1 r_D)}{\left[J_1^2(\beta_1 r_D) - J_1^2(\beta_1)\right]} - \frac{2e^{-\beta_2^2 t_D} J_1^2(\beta_2 r_D)}{\left[J_1^2(\beta_2 r_D) - J_1^2(\beta_2)\right]} \quad (25)$$

Constant Terminal Pressure Case, Q_D:

$$t_{cross} = -1.767 - 0.606(r_D) + 0.12368(r_D)^{2.25} + 3.02[\ln(r_D)]^{0.50}$$

Finite Aquifers

1. $t_{cross} \leq t_D$

$$\alpha_1 = -0.00222107 - 0.627638[\mathrm{csch}(r_D)] + 6.277915(r_D)^{-2.734405} + 1.2708(r_D)^{-1.100417} \quad (26)$$

$$\alpha_2 = -0.00796608 - 1.85408[\mathrm{csch}(r_D)] + 18.71169(r_D)^{-2.758326} + 4.829162(r_D)^{-1.009021} \quad (27)$$

$$\mathrm{csch}(r_D) = \frac{2}{e^{r_D} - e^{-r_D}} \quad (28)$$

$$q_D = \frac{r_D^2 - 1}{2} - \frac{2e^{-\alpha_1^2 t_D} J_1^2(\alpha_1 r_D)}{\alpha_1^2[J_0^2(\alpha_1) - J_1^2(\alpha_1 r_D)]} - \frac{2e^{-\alpha_2^2 t_D} J_1^2(\alpha_2 r_D)}{\alpha_2^2[J_0^2(\alpha_2) - J_1^2(\alpha_2 r_D)]} \quad (29)$$

Infinite Aquifers

1. $t_D <= 0.01$

$$q_D = \left(\frac{2}{\sqrt{\pi}}\right)(\sqrt{t_D}) \quad (30)$$

2. $0.01 < t_D < 200$

$$q_D = \frac{1.129552\,(t_D)^{0.5002034} + 1.160436\,(t_D) + 0.2642821\,(t_D)^{1.5} + 0.01131791\,(t_D)^{1.979139}}{0.5900113\,(t_D)^{0.5002034} + 0.04589742\,(t_D) + 1} \quad (31)$$

3. $200 <= t_D < 2 \times 10^{12}$

$$q_D = 10^{\{4.3989 + 0.43693\ln(t_D) - 4.16078[\ln t_D]^{0.09}\}} \quad (32)$$

Estimation of Aquifer Parameters Using the Numerical Inversion of Laplace Transform

El-Khatib (2003) [16] has presented a new method to estimate parameters of a circular aquifer by non-linear regression analysis using numerical inversion of Laplace transform. Using the method of least squares, water influx data are fitted in the van-Everdingen-Hurst unsteady state model to estimate relative aquifer size (R_{eD}), storativity ($h\phi c_t$) and transmissibility (kh/μ). Due to the simpler solution in Laplace space, numerical inversion of Laplace transform is used to compute the partial derivatives of the VEH solution with respect to aquifer parameters needed for least square method. Besides, the Levenberg method is used for parameter estimation to promise convergence. For variable pressure history, two approaches are implemented and compared: step pressure (SP) and linear pressure (LP) methods. By comparing both methods, LP method is found to yield more accurate results.

SP method:

$$W_e(k) = B\sum_{j=1}^{k} \Delta P_j Q[t_D(k) - t_D(j-1)] \qquad (33)$$

LP method:

$$W_e(k) = B\sum_{j=1}^{k} \Delta m_j \tilde{Q}[t_D(k) - t_D(j-1)] \qquad (34)$$

Laplace transform of dimensionless water influx, $\overline{Q}(s)$:

$$\overline{Q}(s) = \frac{I(\sqrt{s}R_{eD})K_1(\sqrt{s}) - K_1(\sqrt{s}R_{eD})I_1(\sqrt{s})}{s^{3/2}[K_1(\sqrt{s}R_{eD})I_0(\sqrt{s}) + I_1(\sqrt{s}R_{eD})K_o(\sqrt{s})]} \qquad (35)$$

Inverse of $\overline{Q}(s)$ by Stehfest algorithm:

$$Q(t_D) = L^{-1}[\overline{Q}(s)] = \frac{\ln 2}{t_D}\sum_{i=1}^{N} V_i \overline{Q}\left(s = \frac{i\ln 2}{t_D}\right) \quad \text{where} \quad s = \frac{i\ln 2}{t_D} \qquad (36)$$

Simultaneous Estimation of Aquifer Parameters and OHIP using Numerical Inversion of Laplace Transform

El-Khatib (2007) [17] presented a simultaneous estimation of aquifer parameters and OHIP using least square method applied to van-Everdingen and Hurst solution by Laplace Transform, a continuation from his previous study. Pressure history is approximated by a series of linear segments instead of stair-like pressure steps which proven to be of higher accuracy. Sthefest algorithm for numerical inversion of Laplace transform is used to evaluate water influx as well as the first and second derivatives of objective function for B, C and R_{eD} along with the usage of Levenberg-Marquardt method to achieve convergence. The model is linear with respect to original hydrocarbon in place N, G_i and water influx constant, B, as shown in (37) but is non-linear with respect to the dimensionless aquifer size, R_{eD} and time adjustment factor, c used to convert real time, t to t_D. Assumptions on c and R_{eD} allow the calculation of N, G_i and B. Maps are generated to generate the regions of maxima and minima for aquifer parameters.

$$Y_k = NX_{1k} - G_jX_{2k} - B\sum_{j=1}^{k} \Delta m_j \frac{\ln 2}{c^2[t_k|-t_{j-1}]} \sum_{i=1}^{N} V_i \bar{\bar{Q}} \frac{i \ln 2}{c[t_k - t_{j-1}]} \quad (37)$$

Where:

$$Y = N_p(B_o - R_sB_g) + G_pB_g + W_pB_w \quad (38)$$

$$X_1 = B_o - B_{oi} + (R_{si} - R_s)B_g \quad (39)$$

$$X_2 = B_g - B_{gi} \quad (40)$$

Integration of Volumetric and Material Balance Analyses Using a Bayesian Framework to Estimate OHIP and Quantify Uncertainty

Ogele, Daoud, McVay and Lee (2006) [18] had presented a paper on the application of Bayesian formalism used with reservoir simulation to reconcile estimation of OHIP from both volumetric and material balance analyses to quantify the uncertainty in the combined OHIP estimate. Uncertainties in the observed pressure data as well as the volumetric data are considered and the effect of correlation between parameters is investigated in the prior distribution of OHIP estimates with analyses on 2-parameter problem so that parameter relationship could be visualized in 2D plots. A joint prior probability function of N and m is built using the mean and covariance matrix obtained from volumetric analysis assuming Gaussian distribution of the variables (41). Likelihood function is then calculated using the combination of observed pressures and Havlena and Odeh material balance model that predicts pressure for a given set of N and m (41, 42-45). Bayes Rule is then applied for the combination of prior distribution and the likelihood function to obtain posterior distribution, which quantifies the uncertainty in the model parameters given both the prior information and the measured data (46). The mode of the posterior distribution which is in this case, the maximum a posteriori (MAP) solution is selected as the most probable (N, m) set. Finally, the uncertainties in N and m are determined from the posterior distribution either analytically by approximating the covariance matrix (47-48) or numerically by using standard statistical equations (49-53).

Eqn. 41 is the multi-dimensional Gaussian probability distribution of the uncertainties in the model parameters, the prior distribution. It assumes that the prior distribution is multi-variate and normally distributed and therefore can be represented by the means and covariance of the variables. [18]

$$f(x) = \frac{1}{(2\pi)^{n_x/2}[\det(C_x)]^{1/2}} \exp\left\{-\frac{1}{2}\left[\left(x - x_{prior}\right)^T C_x^{-1}(x - x_{prior})\right]\right\} \quad (41)$$

Where:

n_x = number of model parameters

x_{prior} = vector of mean, or most likely

C_x = prior parameter covariance matrix

det() = determinant

Havelena & Odeh Formulations for Gas Cap Driven Reservoirs:

$$F = N(E_o + mE_g) \tag{42}$$

$$E_o = (B_o - B_{oi}) + (R_{si} - R_s)B_g \tag{43}$$

$$E_g = B_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) \tag{44}$$

$$F = N_p\left(B_o + \left(R_p - R_s\right)B_g\right) \tag{45}$$

Bayes Theorem:

$$f\left(x|d^{obs}\right) = f(x) \cdot \frac{f(d^{obs}|x)}{\int_{-\infty}^{+\infty} f(d^{obs}|x)f(x)dx} \tag{46}$$

Where:

x = vector of model parameters

d^{obs} = vector of observed pressure data

f(x) = prior probability distribution function of the model parameters

$f(d^{obs}|x)$= likelihood probability distribution of the observed pressure data given parameters, x

$f(x|d^{obs})$= posterior probability distribution of the model parameters given observed data

In analytical method, observed data and model parameters are assumed to quasi-linear around MAP estimate and covariance of posterior distribution is related to covariance of the observed data and prior by:

$$C_{x(posterior\ )} = (G_{MAP}^{T} \cdot C_{D}^{-1} \cdot G_{MAP} + C_{x(prior\ )}^{-1})^{-1} \tag{47}$$

$$G_{MAP} = \begin{bmatrix} \frac{\partial P_1}{\partial N} & \frac{\partial P_2}{\partial N} & \cdots & \frac{\partial P_{nd}}{\partial N} \\ \frac{\partial P_1}{\partial m} & \frac{\partial P_2}{m} & \cdots & \frac{\partial P_{nd}}{\partial m} \end{bmatrix}^{T} \tag{48}$$

Where:

$C_{x(posterior)}$= covariance matrix approximated at MAP

C_D = covariance matrix

$C_{x(prior)}$ = prior covariance matrix

G_{MAP} = sensitivity matrix at MAP of forward model with respect to N and m

Numerical method uses basic laws of joint probability function for discrete random variable to calculate covariance matrix for posterior probability distribution as follows:

$$C_{x(posterior\ )} = \begin{bmatrix} cov(N, N) & cov(N, m) \\ cov(m, N) & cov(m, m) \end{bmatrix} \tag{49}$$

$$cov(N, N) = E(N^2) - E(N) \cdot E(N) \tag{50}$$

$$E(N^2) = \sum_N \sum_m N^2 \cdot f(N, m|d^{obs}) \tag{51}$$

Another example is,

$$cov(N, m) = cov(m, N) = E(N \cdot m) - E(N) \cdot E(m) \tag{52}$$

$$E(N \cdot m) = \sum_N \sum_m N \cdot m \cdot f(N, m|d^{obs}) \tag{53}$$

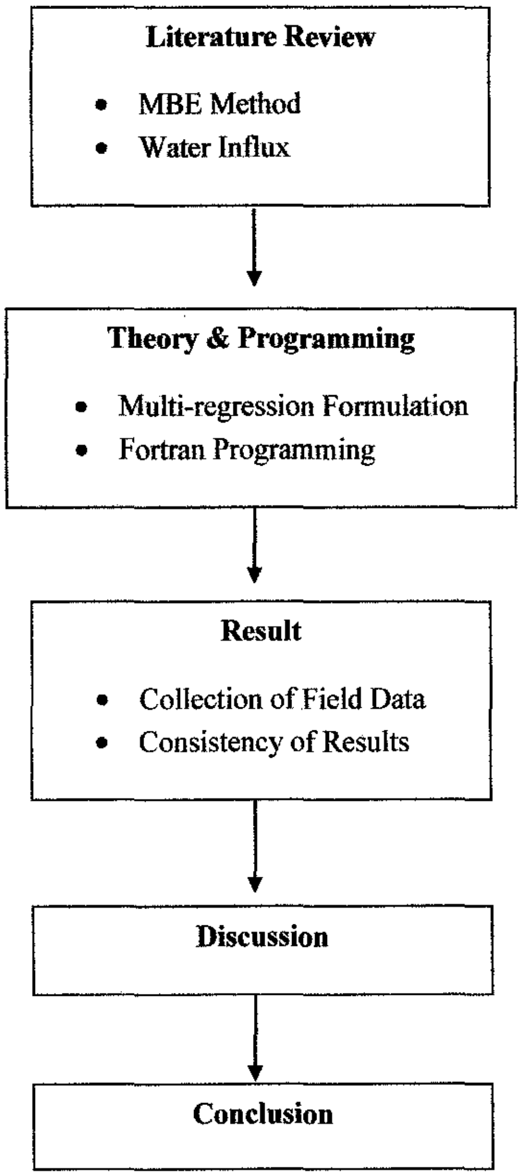
Where:

$f(N,m|d^{obs})$ = posterior joint probability function obtained from Eqn. 46

CHAPTER 3

PROJECT PLANNING

3.1 Basic Methodology



(a) Literature Review

In order to have a thorough idea on the topic involved, studies were conducted for project development ahead, mainly revolving on MBE computation and water influx models. Relevant studies were also carried out on statistics computation for multi-regression method in order to achieve the study objective.

(b) Theory & Programming

After the review of past studies, formulas were developed to obtain a multi-regression solution for N, m and water influx parameters, B provided with field production and PVT data in a simultaneous drive mechanisms oil reservoir. Below briefly describes the methodology involved. More details will be discussed in Chapter 3.2.

A generalized MBE is as below [7]:

$$N_p\left(B_o - R_s B_g\right) + G_p B_g + W_p B_w$$

$$= N\left[B_o - B_{oi} + B_g(R_{si} - R_s)\right] + G_i\left(B_g - B_{gi}\right) + W_e \qquad (54)$$

In simple model, whereby water influx is assumed under steady state, Schilthuis's model [8] demonstrates that:

$$W_e = k\int_0^t (P_i - P)dt \qquad (55)$$

Since the steady state aquifer model could not usually accommodate the actual behaviour of aquifer encroachment, an exact solution of diffusivity equation proposed by van-Everdingen and Hurst [10] for radial flow system of constant terminal pressure gives:

$$W_e = B\Delta PQ(t_D) \qquad (56)$$

A linear equation is resulted with the parameters of N, Gi and B take place as below [16]:

$$Y_k = NX_{1k} - G_i X_{2k} - B\sum_{j=1}^{k} \Delta P_j Q[t_D(k) - t_D(j-1)] \qquad (57)$$

Where:

$$Y = N_p(B_o - R_s B_g) + G_p B_g + W_p B_w$$

$$X_1 = B_o - B_{oi} + (R_{si} - R_s)B_g$$

$$X_2 = B_g - B_{gi}$$

$$X_3 = \sum_{j=1}^{k} \Delta P_j Q[t_D(k) - t_D(j-1)]$$

This displays a hyper plane relationship and multiple regression analysis can be used to estimate the three parameters N, G_i and B from reservoir production and PVT data.

Multiple regressions are a method used to examine the relationship between one dependent variable Y and one or more independent variables X_i. The regression parameters or coefficients (N, m and B) in the regression equation are estimated using the method of least squares or matrices calculation.

Program coding is taking place by using Microsoft FOTRAN PowerStation version 4.0 to combine all the relevant equations to give way to a MBE solution simultaneously.

(c) Result

After completed the program coding and formulae development, the functionality of both were verified with real field data. Results were tabulated and recorded for further analyzes.

(d) Discussion

Discussions were conducted to analyze the results obtained from the formulas and coding and to verify its validity.

3.2 Formulations

The basis of this project lies within the application of Material Balance Method [7] as in eqn. (58) via programming to solve initial oil in place, N, initial gas in place, m and water influx parameters, B, c and R_{eD} in a saturated oil reservoirs with the presence of water influx and gas cap under simultaneous drive mechanisms. By providing a reservoir's PVT data as well as production history, estimation of N, m, B, c and R_{eD} are obtained by multiple regression method with van-Everdingen and Hurst (VEH) unsteady-state model.

General Material Balance Equation:

$$N_p(B_o - R_s B_g) + G_p B_g + W_p B_w$$

$$= N[B_o - B_{oi} + B_g(R_{si} - R_s)] + G_i(B_g - B_{gi}) + W_e \tag{58}$$

By assuming that PVT and production data are readily available, VEH model [10] which accounts for exact analytical solution for circular aquifers with homogeneous properties is applied for water influx, W_e calculation. As stated in the previous section, by providing an exact solution to the radial diffusivity equation, this method is considered the most accurate technique to calculate water influx and are equally applicable for calculating the early, unstable influx (infinite-acting) and for the influx occurring at which the aquifer boundary effects are felt providing a convenient approach for calculating water influx at all time steps.

VEH Unsteady State Water Influx Model:

$$W_e = B\Delta P Q(t_D) \tag{59}$$

$$B = 1.119 h \emptyset c_t {r_w}^2 \tag{60}$$

$$t_D = ct,\ c = \frac{0.00634k}{\mu c_t \emptyset {r_w}^2} \tag{61}$$

$$Q(t_D) = \frac{{R_{eD}}^2 - 1}{2} - 2\sum_{n=1}^{\infty} \frac{J_1^2(\alpha_n R_{eD}) e^{-\alpha_n^2 t_D}}{\alpha_n^2 [J_0^2(\alpha_n) - J_1^2(\alpha_n R_{eD})]} \tag{62}$$

Where α_n are roots of equation

$$J_1(\alpha_n R_{eD})Y_0(\alpha_n) - Y_1(\alpha_n R_{eD})J_0(\alpha_n) = 0 \tag{63}$$

In the more practical cases of history matching the reservoir pressures observed at the oil-water contact, VEH model is extended to calculate the cumulative water influx corresponding to a continuous pressure decline at the reservoir-aquifer boundary. In order to perform these calculations, the pressure history is approximated into a number of constant pressure steps with discontinuous jumps at the data points, named as Step Pressure (SP) method as in eqn. (64). Vogt and Wang (1990) [19] approximated the pressure behavior by a series of linear segments connecting successive data points named as Linear Pressure (LP) method and the basis of this method is to replace ΔP' in eqn. (65) by the slope m and integrate by part. According to El-Khatib [17], results show that LP method is more accurate than SP method. Though, due to simplicity of computation, SP method would be used exclusively for this project.

SP Method:

$$W_e(k) = B\sum_{j=1}^{k} \Delta P_j Q[t_D(k) - t_D(j-1)] \tag{64}$$

LP Method:

$$W_e(k) = B\sum_{j=1}^{k} \Delta m_j \widetilde{Q}[t_D(k) - t_D(j-1)] \tag{65}$$

The complexity of solving (62) and (63) are apparent. Firstly, eqn. (63) have to be solved iteratively for enough numbers of successive roots, α_n followed by the summation term in eqn. (62) has to be continued until convergence of the infinite series is obtained. These complications prompt the application of Stehfest algorithm for the numerical inversion of Laplace transform as the solution in Laplace space is simpler than the solution in real time domain [16].

Laplace Transform of Dimensionless Water Influx $\overline{Q}(s)$:

$$\overline{Q}(s) = \frac{I_1(\sqrt{s}R_{eD})K_1(\sqrt{s}) - K_1(\sqrt{s}R_{eD})I_1(\sqrt{s})}{s^{3/2}[K_1(\sqrt{s}R_{eD})I_0(\sqrt{s}) + I_1(\sqrt{s}R_{eD})K_o(\sqrt{s})]} \tag{66}$$

Inverse of $\overline{Q}(s)$ by Stehfest algorithm $Q(t_D)$:

$$Q(t_D) = l^{-1}[\overline{Q}(s)] = \frac{\ln 2}{t_D}\sum_{i=1}^{N} V_i\overline{Q}\left(s = \frac{i \ln 2}{t_D}\right) \quad (67)$$

More complications arise when there are more uncertainties attached with the subject of water influx more than any other revolving on the areal continuity and geometry of the aquifer itself, including c and R_{eD} which are the must-know parameters in the computation of dimensionless water influx rates. To simplify the calculations involved, assumptions are first made on ranges of c and R_{eD} values in order to allow the calculation of N, G_i and B. Contour maps are then generated to generate the regions of maxima and minima for aquifer parameters.

By assuming c and R_{eD} values, the only left unknown variables in eqn. (58) are initial oil in place, N, initial gas in place, G_i and constant B in water influx term. A simplified form of eqn. (58) is presented in eqn. (68) and multiple regressions using matrix solution can be used to solve the 3 unknowns: N, G_i and B simultaneously by assuming that all the data (X_i, Y_i) are equally reliable.

Multiple Regression Formula:

$$Y_k = N_i X_{1k} - N_i m X_{2k} - B\sum_{j=1}^{k} \Delta P_j Q[t_D(k) - t_D(j-1)] \quad (68)$$

Where:

$$Y = N_p(B_o - R_s B_g) + G_p B_g + W_p B_w \quad (69)$$

$$X_1 = B_o - B_{oi} + (R_{si} - R_s)B_g \quad (70)$$

$$X_2 = B_g - B_{gi} \quad (71)$$

$$X_3 = \sum_{j=1}^{k} \Delta P_j Q[t_D(k) - t_D(j-1)] \quad (72)$$

Matrix Solution:

$$\begin{bmatrix} Y_{(1)} \\ Y_{(2)} \\ \vdots \\ Y_{(k)} \end{bmatrix} = \begin{bmatrix} X_{1(1)} & X_{2(1)} & X_{3(1)} \\ X_{1(2)} & X_{2(2)} & X_{3(2)} \\ \vdots & \vdots & \vdots \\ X_{1(k)} & X_{2(k)} & X_{3(k)} \end{bmatrix} \begin{bmatrix} N \\ G_i \\ B \end{bmatrix} \quad (73)$$

$$\begin{bmatrix} N \\ G_i \\ B \end{bmatrix} = [X^T X]^{-1} X^T Y \quad (74)$$

To select the best fitted R_{eD} and c values, 2-dimensional and 3-dimensional square of residual error map are generated for different combination of R_{eD} and c parameters. Each combination will result in unique N, m and B values. By calculating the difference between Y term and X term, residual error is obtained. The difference is first divided by each Y team at each point and then squared for absolute positive results for computation simplicity. The square of residual error is then totalled up for all points in a particular set of c and R_{eD}. The combination of R_{eD} and c which displays area of minima in map is selected for refinement to determine the best fitted R_{eD} and c values. In order to calculate square of residual error for each combination set,

$$\text{Total Error} = \sum_{j=1}^{k} \left[\left[Y_j - \left(N_i X_{1,j} + N_{i,j} m X_{2,j} + B X_{3,j} \right) \right] / Y_j \right]^2 \quad (75)$$

CHAPTER 4

RESULTS & DISCUSSIONS

4.1 Verification of Numerical Inversion of Laplace Transform Method

There is greater interest in calculating the influx rate compared with the pressure drop in the description of water influx encroaching from the aquifer into a reservoir which prompts the determination of influx as a function of a given pressure drop at the inner boundary of the system. Hurst and Everdingen proposed VEH model by solving the radial diffusivity equation for the aquifer-reservoir system by applying the Laplace transformation to the equation, as expressed in terms of dimensionless variables as follows in which all the parameters refer to aquifer rather than reservoir properties [5]:

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial P_D}{\partial t_D}\right)=\frac{\partial P_D}{\partial t_D} \qquad (76)$$

Where:

$$r_D=\frac{r}{r_e} \qquad (77)$$

$$t_D=\frac{kt}{\emptyset\mu c r_e^2} \qquad (78)$$

Hurst and Everdingen had derived constant terminal pressure solution and as it is more convenient to express the solution in terms of cumulative water influx, thus integrating with respect to time gives [5]:

$$W_e=2\pi\emptyset h\bar{c}r_e^2\Delta P W_{eD}(t_D) \qquad (79)$$

For the mean of simplicity, dimensionless water influx, $W_{eD}(t_D)$ is often presented in tabular form or as a set of polynomial expression given W_{eD} as a function of dimensionless time, t_D for different ratios of R_{eD} for radial aquifers. The plots of W_{eD} versus t_D for both radial and linear geometry are included in the published solution by Hurst and Everdingen where the graphs are valid for all values of t_D and hence are both applicable for calculating both the early, unsteady influx and for the influx occurring when the aquifer boundary effects are felt. Though, there are differences in the way in calculation depending on the geometry. [5]

Irrespective of the geometry there is a value of t_D for which the W_{eD} will arrive at a maximum value as follows: [5]

Radial: $W_{eD}(max) = 0.5\ (R_{eD}^2 - 1)$ (80)

Linear: $W_{eD}(max) = 1$ (81)

Assuming the aquifer is in radial geometry, calculations of dimensionless water influx W_{eD} are done on different R_{eD} values for ranges of t_D using Numerical Inversion of Laplace Transform method as shown in Table 1 and Figure 1. The results show close convergence to the solution proposed by VEH model and arrive at the same conclusion at every R_{eD} which justifies the applicability of this method.

Source code is as attached in Appendix A.

t_D	$Q_D(t_D)$				
	R_{eD}=2	R_{eD}=2.5	R_{eD}=3	R_{eD}=5	R_{eD}=10
1	1.29	1.53	1.56	1.57	1.57
2	1.47	2.11	2.36	2.44	2.45
3	1.50	2.38	2.89	3.20	3.20
5	1.50	2.57	3.49	4.50	4.53
10	1.50	2.63	3.92	6.98	7.40
15	1.50	2.63	3.99	8.62	9.94
20	1.50	2.63	4.00	9.73	12.29
30	1.50	2.62	4.00	10.96	16.57
50	1.50	2.62	4.00	11.77	23.69
100	1.50	2.62	4.00	12.00	35.45

Table 1 Dimensionless Water Influx versus Dimensionless Time & Dimensionless Radius using Numerical Inversion of Laplace Transform

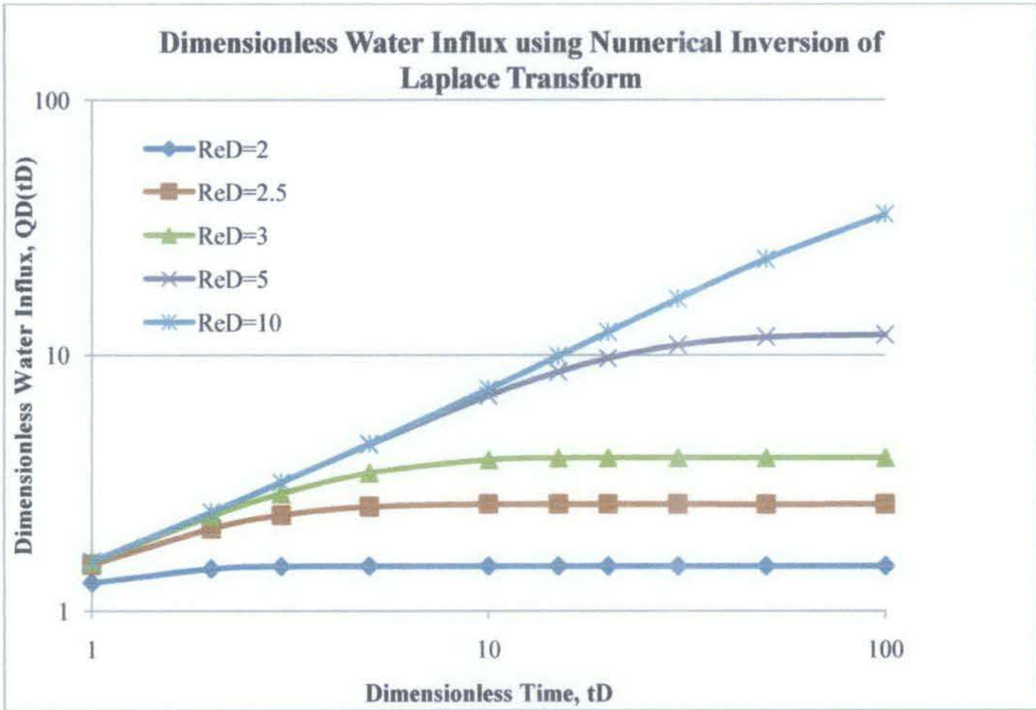


Figure 1 Dimensionless Water Influx versus Dimensionless Time & Dimensionless Radius using Numerical Inversion of Laplace Transform in Plot

4.2 Project Main Frame Source Codes & Results

4.2.1 Louisiana Reservoir

Test data of a Louisiana water drive reservoir with a small gas cap from literature [20] is used in order to test on the validity of programming coding. Table 2 and 3 listed out the said reservoir production and PVT data.

T (DAY)	P (PSIA)	N_P (MMSTB)	G_P (MMSCF)	W_P (MMSTB)
349	5479	0.635	480.12	0.002
417	5335	1.000	850.00	0.002
526	5223	1.338	1150.01	0.002
830	4923	2.429	2268.93	0.000
936	4870	2.759	2643.12	0.002
1299	4650	3.979	3990.94	0.002
1660	4375	5.201	5507.86	0.003
2020	4080	6.491	7094.66	0.003
2378	3750	7.922	9340.04	0.105

Table 2 Louisiana Reservoir Production History

P (PSIA)	B_O (RB/STB)	B_G (RB/SCF)	R_S (SCF/STB)
5479	1.3609	0.0006586	609.5
5335	1.3548	0.0006701	592.0
5223	1.3501	0.0006794	578.4
4923	1.3376	0.0007070	542.2
4870	1.3353	0.0007125	535.7
4650	1.3262	0.0007362	509.3
4375	1.3148	0.0007697	476.5
4080	1.3027	0.0008122	441.4
3750	1.2893	0.0008694	402.5

Table 3 Louisiana Reservoir PVT Data

As discussed in section 3.2, formulations are programmed via FORTRAN to achieve the objectives on solving N, m and B simultaneously in a saturated oil reservoirs under simultaneous drives built on the basis of Material Balance equations. Complications of VEH model as described prompt the application of Stehfest algorithm for the numerical inversion of Laplace transform to compute dimensionless water influx (64, 66-67).

To simplify the calculations involved, assumptions are first made on ranges of c and R_{eD} values which are the must-know parameters in the computation of dimensionless water influx rates. In this field example, range of c estimated is in logarithmic scale: 0.1, 1, 10, and 100 and R_{eD}: 10 – 80.

Examining the Material Balance equation (58), the only left unknown variables are initial oil in place, N, initial gas in place, G_i and constant B in water influx term. A simplified form of eqn. (58) is presented in eqn. (68) and multiple regressions using matrix solution is programmed to solve the 3 unknowns: N, G_i and B simultaneously by assuming that all the data (X_i, Y_i) are equally reliable.

2-dimensional and 3-dimensional square of residual error map (75) are generated in 3DField graph plotting software for different combination of R_{eD} and c parameters as displayed in Figure 2 and Figure 3. The combination of R_{eD} and c which displays area of minima in map (as circled) is selected for refinement to determine the best fitted R_{eD} and c values. The non-uniqueness of solution is countered with the preliminary understanding of the reservoir-aquifer system, in which this case has used the volumetric estimation as the basis of reference.

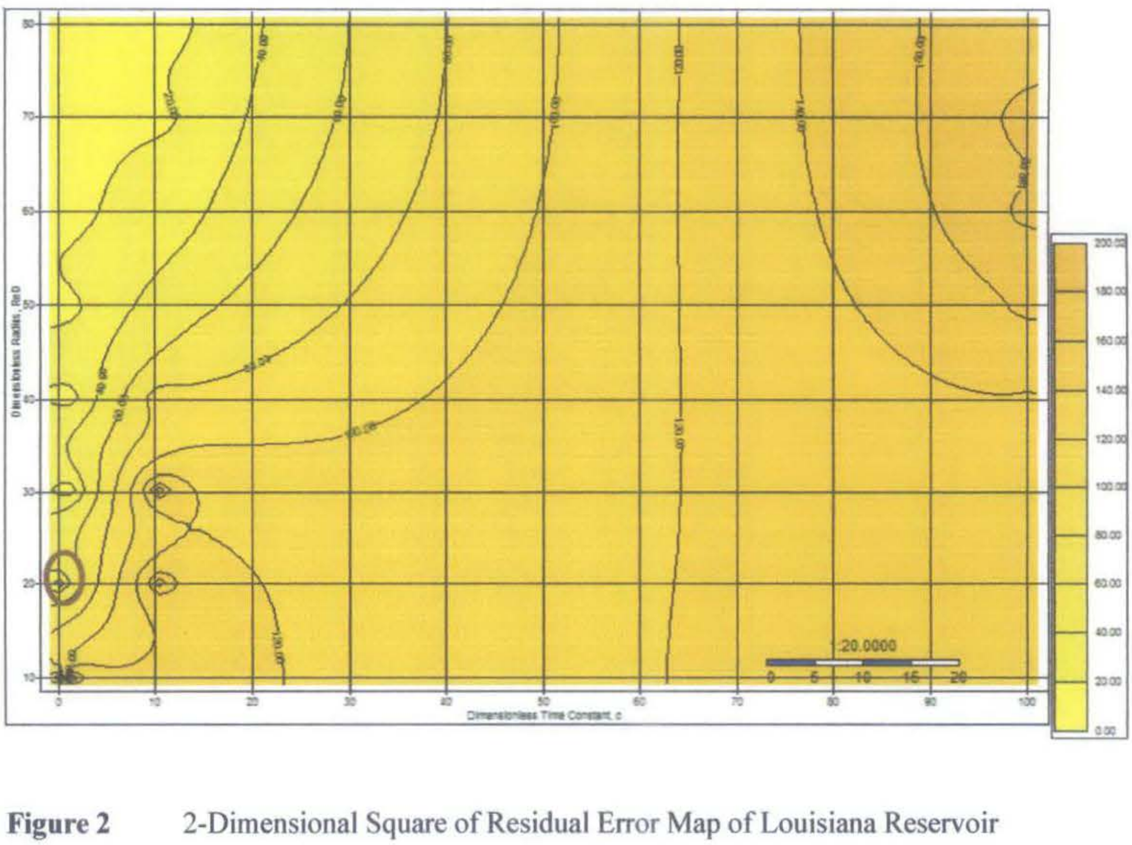


Figure 2 2-Dimensional Square of Residual Error Map of Louisiana Reservoir

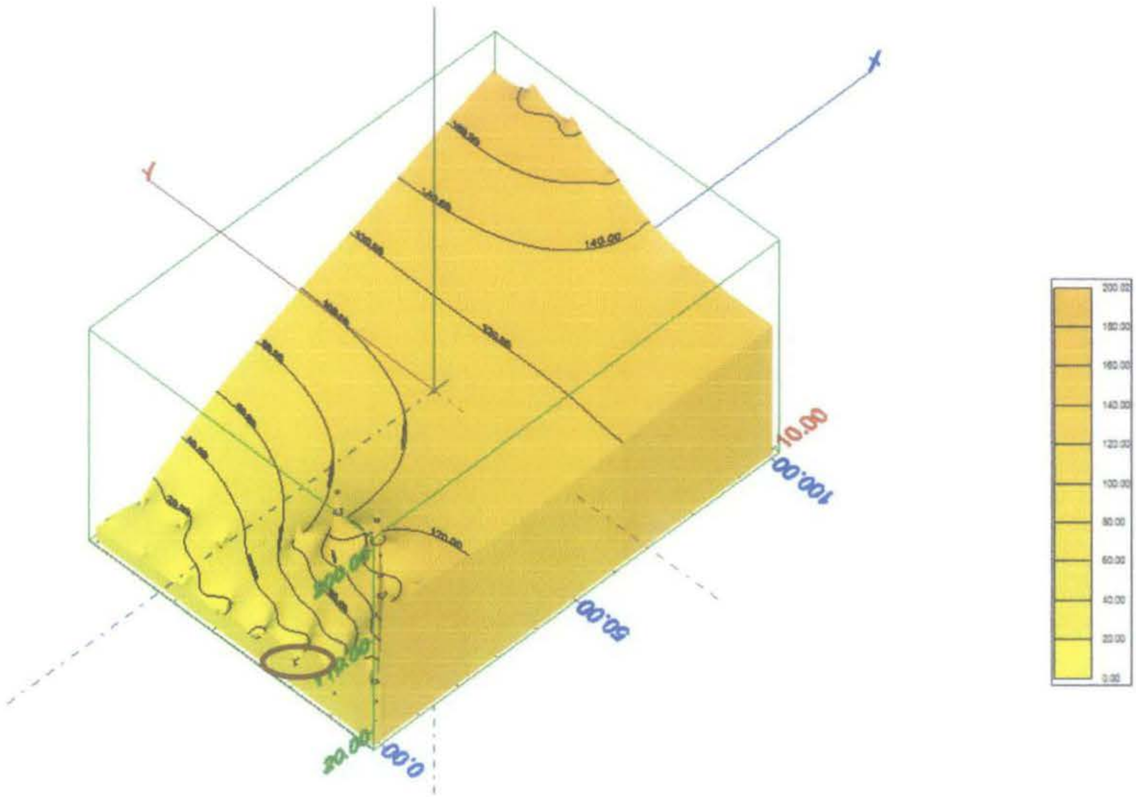


Figure 3 3-Dimensional Square of Residual Error Map of Louisiana Reservoir

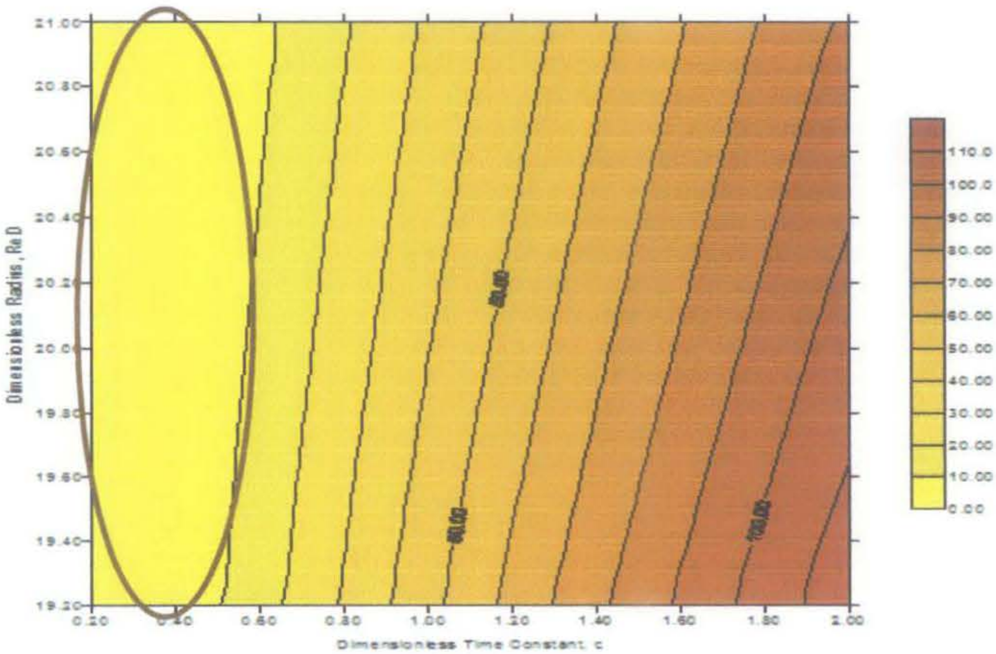


Figure 4 2-Dimensional Square of Refined Scale Residual Error Map of Louisiana Reservoir

Refinement approach is conducted in areas of minima as circled in accordance to the preliminary understanding of volumetric estimation to assure the best fitted R_{eD} and c values. For example, Figure 4 displays the refined scale map of c: 0.2 - 2.0 and ReD: 20.1 - 21.0. The observed area of minima is again refined till the results obtained are in accordance to what is described in volumetric estimation.

The best fitted results are obtained at:

R_{eD} = 20.27

c = 0.28

N = 22.26 MMstb

m = 0.26

B = 45.34 rb/psi

These results converge closely with the suggested reservoir data of an N of 22.36 MMSTB and m of 0.169 in the literature with the difference of N being less than 1%.

$$\text{Difference of N} = \frac{22.36-22.26}{22.36} \text{x}100\% = 0.45\%$$

Results demonstrate that this reservoir is under simultaneous drive mechanisms of moderate water influx and small gas cap as in accordance with the preliminary understanding of reservoir-aquifer system. In other words, it justifies the reliability and consistency of the formulations and coding to achieve the project objectives which is to determine the initial hydrocarbon in place, ratio of initial hydrocarbon pore volume of gas to oil and water influx parameters in a reservoir via Material Balance Equation (MBE) in the saturated oil reservoirs with the presence of water influx and gas cap under simultaneous drive mechanisms.

Please refer to Appendix B for program main frame source codes.

CHAPTER 5

CONCLUSIONS & RECOMMENDATIONS

5.1 Conclusions

1. A more robust reservoir preliminary understanding method is presented for simultaneous estimations of aquifer parameters and original hydrocarbon in place applied to unsteady-state van-Everdingen-Hurst (VEH) solution in Laplace domain.

2. Numerical inversion of Laplace transform provides a consistent yet reliable estimation of aquifer parameters with the incorporation of reservoir pressure history.

3. Stehfest algorithm is used in numerical inversion of Laplace transform to evaluate water influx which subsequently solves the complications in VEH solution.

4. Map for square of residual error is constructed to identify the area of minima that achieve convergence to correct solution.

5. Solutions to material balance problems may be highly non-unique, even for 2-parameter problems. Additional of geological and engineering knowledge is prior to counter the non-uniqueness of solution.

5.2 Recommendations

As stated in Chapter 2, the application of Bayesian formalism used with reservoir simulation to reconcile estimation of OHIP from both volumetric and material balance analyses presented by Ogele, Daoud, McVay and Lee (2006) [18] could be used to quantify the uncertainty in the combined OHIP estimate. Uncertainties in the observed pressure data as well as the volumetric data are both considered and the effect of correlation between these parameters can be investigated in the prior distribution of OHIP estimates with analyses on 2-parameter which quantifies the uncertainty in the model parameters given both the prior information and the measured data.

NOMENCLATURE

B	=	aquifer constant, bbl/psi
B_o	=	oil formation volume factor, rbbl/stb
B_g	=	gas formation volume factor, rbbl/scf
B_w	=	water formation volume factor, rbbl/stb
C_e	=	effective aquifer compressibility, psi^{-1}
C_f	=	formation compressibility, psi^{-1}
C_w	=	water compressibility, psi^{-1}
E_o	=	oil expansion term
E_g	=	gas expansion term
E_{fw}	=	connate water expansion and reduction in pore volume
F	=	underground withdrawal
G_i	=	initial gas in place, scf
h	=	thickness, ft
h_A	=	aquifer thickness, ft
h_R	=	reservoir thickness, ft
I_0	=	modified Bessel function of the first kind of order zero
I_1	=	modified Bessel function of the first kind of order one
J	=	productivity index for aquifer, bbl/d/psi
J_0	=	Bessel function of the first kind of order zero
J_1	=	Bessel function of the first kind of order one
K	=	absolute permeability, md
K_0	=	modified Bessel function of the third kind of order zero
K_1	=	modified Bessel function of the third kind of order one
m	=	gas cap ratio
N	=	initial oil in place, stb
N_p	=	cumulative oil production, stb

P	=	pressure, psi
Pa	=	average aquifer pressure, psi
Pr	=	average reservoir boundary pressure, psi
P_D	=	dimensionless pressure
P'_D	=	dimensionless pressure derivative
Q	=	flow rate, bbl/d
$Q(t_D)$	=	dimensionless water influx, or Q_D
r_a	=	radius of aquifer, ft
r_e	=	reservoir radius, ft
R_{eD}	=	dimensionless aquifer radius
R_s	=	gas solubility in oil, scf/stb
S	=	saturation, fraction
S_{wi}	=	initial water saturation, fraction
S_{og}	=	intial gas cap oil saturation, fraction
S_{wg}	=	initial gas cap water saturation, fraction
S_{wo}	=	initial oil zone water saturation, fraction
t	=	time, day
t_D	=	dimensionless time
W_e	=	water influx, bbl
W_{eD}	=	dimensionless water influx
W_i	=	initial volume of water, bbl
W_p	=	water production, bbl
Y_0	=	Bessel function of the second kind of order zero
Y_1	=	Bessel function of the second kind of order one
μ	=	viscosity, cp
φ	=	porosity, fraction

SUBSCRIPTS

g	=	gas
i	=	initial
o	=	oil
w	=	water

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Appendix A: Verification of Numerical Inversion of Laplace Transform Method

```
REAL FUNCTION QS(s, a, PROD)

        USE MSIMSL

        IMPLICIT NONE

        ! Declare calling arguments
        REAL, INTENT (IN) :: s
        REAL, INTENT (IN) :: a
        REAL, INTENT (IN) :: PROD

        QS = ((BSI1(PROD)*BSK1(a))-
        (BSK1(PROD)*BSI1(a)))/((s**(1.5))*((BSK1(PROD)*BSI0(a))+(BSI1(PROD)*BSK0(a))))

RETURN

END FUNCTION


PROGRAM VERIFICATION
! VERIFICATION OF NUMERICAL INVERSION OF LAPLACE TRANSFORM METHOD

USE MSIMSL

IMPLICIT NONE

INTEGER :: N, I, J, K, NH, FF, K1, KF, S1, G

REAL :: s, QS, DTD, QDTD, H, V, StoreQS, StoreA, TotalA, a, PROD, ReD

DIMENSION :: G(12), H(12), V(12)

REAL, DIMENSION(10) :: TD = (/1., 2., 3., 5., 10., 15., 20., 30., 50., 100./)

REAL, DIMENSION(5) :: RD = (/2., 2.5, 3., 5., 10./)

N=8

G(0) = 1

DO I = 1 ,N

        G(I) = I * G(I - 1)

ENDDO

NH = N / 2
H(1) = 2.0 / G(NH - 1)

DO I = 2 ,NH

        H(I) = I** NH * G(2 * I) /( G(I) * G(I - 1) * G(NH - I))

ENDDO

FF = NH - INT(NH / 2) * 2
```

```
S1=1

IF(FF .EQ. 0)  S1=-1
! S1 = 2 * ISIGN(1,F) - 1

DO I= 1 , N

        V(I) = 0
        KF = NH

        IF (I .LE. NH ) KF=I

        K1 = INT((I + 1) / 2)

        DO K = K1 , KF

                V(I) = V(I) + H(K) / G(I - K) / G(2 * K - I)

        ENDDO

        V(I) = S1 * V(I)
        S1 = -S1

ENDDO


DO I = 1, 5

        ReD = RD(I)
        WRITE(*,*) 'ReD: ', ReD

        DO J = 1, 10

                TotalA=0
                DTD = TD(J)
                WRITE (*,*) 'TD: ', DTD

                        DO K = 1,8

                                s = K * LOG(2.0) / DTD
                                a = SQRT(s)
                                PROD = a * ReD
                                StoreQS = QS(s, a, PROD)
                                StoreA = V(K) * storeQS
                                TotalA = TotalA + StoreA

                        END DO

                QDTD   = LOG(2.0) / DTD * TotalA
                WRITE (*,*) 'QDTD: ',QDTD

        END DO

END DO


STOP

END PROGRAM VERIFICATION
```

Appendix B: Project Main Frame Source Codes – Louisiana Reservoir

```
REAL FUNCTION QS(s, a, PROD)
        USE MSIMSL

        IMPLICIT NONE

        ! Declare calling arguments

        REAL, INTENT (IN) :: s

        REAL, INTENT (IN) :: a

        REAL, INTENT (IN) :: PROD

        QS=((BSI1(PROD)*BSK1(a))-
(BSK1(PROD)*BSI1(a)))/((s**(1.5))*((BSK1(PROD)*BSI0(a))+(BSI1(PROD)*BSK0(a))))

RETURN

END FUNCTION



PROGRAM FYP_MBE

! ESTIMATION OF RESERVOIR PARAMETERS USING MATERIAL BALANCE METHOD

USE MSIMSL

IMPLICIT NONE

! VARIABLES DECLARATION

INTEGdfcER :: status1, status2, status3, status4, Z, N, I, J, K, NH, FF, K1, KF, KK, LL, JJ, S1, G,
LDA, LDAINV, Num, test

        ! status1 & status 2         : Status of file opening
        ! Z                          : No. of data set
        ! N                          : No. of loop for V calculation

REAL :: Pi, Boi, Bgi, Rsi, s, QS, DTD, QDTD, H, V, StoreQS, StoreA, TotalA, StoreB, TotalB, a,
PROD, TEMP_SERR

        ! subscript i     : Initial condition at time = 0
        ! Pi              : Initial pressue, psia
        ! Boi             : Initial oil formation volume factor, rbbl/stb
        ! Bgi             : Initial gas formation volume factor, rbbl/scf
        ! Rsi             : Initial solution gas-oil ratio, scf/stb
        ! QS              : Dimensionless water influx in Laplace transform
        ! DTD             : Delta dimensionless time (TD)
        ! QDTD            : Dimensionless water influx rate as a function of delta dimensionless time

DIMENSION :: G(12), H(12), V(12)

REAL, ALLOCATABLE, DIMENSION (:) :: T, P, Np, Gp, Wp, Bo, Bg, Rs, DP, TD, Y, X1, X2, X3
```

```
        ! T            : Time, day
        ! P            : Pressure, psia
        ! Np           : Cumulative oil production, MMstb
        ! Gp           : Cumulative gas production, MMscf
        ! Wp           : Cumulative water production, MMstb
        ! Bo           : Oil formation volume factor, rbbl/stb
        ! Bg           : Gas formation volume factor, rbbl/scf
        ! Rs           : Solution gas-oil Ratio, scf/stb
        ! DP           : Delta Pressure, psia

REAL, DIMENSION(3,3) :: Mathold, Mat1

REAL, ALLOCATABLE, DIMENSION (:,:) :: MatX, MatY, MatB, MatXT, Mat2, Ni, Gi, m, B,
SERR

        ! Ni           : OIIP, MMrb     (Unknown Variables)
        ! Gi           : GIIP, MMrb     (Unknown Variables)
        ! m            : Gas Cap Ratio  (Unknown Variables)
        ! B            : Constant B in water influx, rb/psia(Unknown Variables)
        ! SERR         : Square of Residual Error (Unknown Variables)

REAL :: c, ReD   !Unknown Variables

        ! c   : Constant c in dimensionless time in water influx
        ! ReD : Dimensionless radius of aquifer


! DATA ACQUISITION

! FILE 1: INITIAL CONDITIONS

OPEN (UNIT=1, FILE='INITIAL.DAT', STATUS='OLD', ACTION ='READ', IOSTAT=status1)

IF (status1==0) THEN

READ (1,*) Pi, Boi, Bgi, Rsi

READ (1,*) Z,N

ALLOCATE (T(0:Z),P(0:Z),Np(1:Z),Gp(1:Z),Wp(1:Z),Bo(1:Z),Bg(1:Z),Rs(1:Z))

! NN=8

! TWO=2.0

G(0) = 1

DO I = 1 ,N

        G(I) = I * G(I - 1)

ENDDO

NH = N / 2

H(1) = 2.0 / G(NH - 1)
```

```
DO I = 2 ,NH
        H(I) = I** NH * G(2 * I) /( G(I) * G(I - 1) * G(NH - I))
        ENDDO
                FF = NH - INT(NH / 2) * 2
                S1=1
                IF(FF .EQ. 0)  S1=-1
                ! S1 = 2 * ISIGN(1,F) - 1
        DO I= 1 , N
            V(I) = 0
            KF = NH
            IF (I .LE. NH ) KF=I
            K1 = INT((I + 1) / 2)
            DO K = K1 , KF
                V(I) = V(I) + H(K) / G(I - K) / G(2 * K - I)
            ENDDO
            V(I) = S1 * V(I)
            S1 = -S1
        ENDDO
ELSE
        WRITE (*,*)     'An error occured opening file 1.'
END IF


! FILE 2: PRODUCTION & PVT DATA
OPEN (UNIT=2, FILE='INPUT.DAT', STATUS='OLD', ACTION ='READ', IOSTAT=status2)
OPEN(UNIT=3,FILE='RESULT_1.DAT',STATUS='REPLACE',ACTION='READWRITE',IOSTAT
=status3)
OPEN(UNIT=4,FILE='RESULT_2.DAT',STATUS='REPLACE',ACTION='READWRITE',IOSTAT
=status4)
IF (status2==0) THEN
        T(0) = 0.0
        P(0) = Pi
```

```
DO I = 1, Z

        READ (2,*) T(I),P(I),Np(I),Gp(I),Wp(I),Bo(I),Bg(I),Rs(I)

END DO

ALLOCATE (Y(1:Z),X1(1:Z), X2(1:Z),X3(1:Z))

DO J = 1, Z

        Y(J)  = (Np(J) * (Bo(J) - (Rs(J) * Bg(J)))) + (Gp(J) * Bg(J)) + Wp(J)

        X1(J) = Bo(J) - Boi + (Bg(J) * (Rsi - Rs(J)))

        X2(J) = Bg(J) - Bgi

END DO

ALLOCATE (DP(1:Z))

DP(1) = (P(0)-P(1))/2

DO I = 2, Z

        DP(I)=(P(I-2)-P(I))/2

END DO

WRITE (3,100)

100 FORMAT ('ESTIMATION OF RESERVOIR PARAMETERS USING MATERIAL BALANCE
METHOD')

WRITE (3,110)

110 FORMAT ('cReD      Ni(MMrb)        m        B(rb/psi)')

ALLOCATE (Ni(10,10),Gi(10,10),m(10,10),B(10,10),SERR(10,10))

DO LL = 1, 4

        c = 10.0**(LL-2)

        ALLOCATE (TD(0:Z))

        DO I = 0, Z

                TD(I)= c * T(I)

        END DO

        DO KK = 1, 8

                ReD = 10 * KK

                DO I = 1, Z
```

```
        TotalB = 0.0
DO J = 1, I
        test = J-1
        ! IF (test>=I)   EXIT
        TotalA = 0.0
        DTD = TD(I) - TD(J-1)
                DO K = 1,8
                        s = K * LOG(2.0) / DTD
                        a = SQRT(s)
                        PROD = a * ReD
                        StoreQS = QS(s, a, PROD)
                        StoreA = V(K) * storeQS
                        TotalA = TotalA + StoreA
                END DO
                QDTD   = LOG(2.0) / DTD * TotalA
                StoreB = DP(J) * QDTD
                TotalB = TotalB + StoreB
        END DO
        X3(I) = TotalB
END DO
ALLOCATE (MatX(Z,3), MatY(Z,1), MatB(3,1), MatXT(3,Z), Mat2(3,1))
DO I = 1 ,Z
        MatX(I,1:3) = (/X1(I), X2(I), X3(I)/)
        MatY(I,1) = Y(I)
END DO
LDA = 3
LDAINV = 3
Num = 3
MatXT = TRANSPOSE(MatX)
Mathold = MATMUL (MatXT, MatX)
```

```
        CALL LINDS(3,Mathold,3,Mat1,3)

        Mat2 = MATMUL (MatXT, MatY)

        MatB = MATMUL (Mat1, Mat2)

        Ni(LL,KK) = MatB (1,1)

        Gi(LL,KK) = MatB (2,1)

        m(LL,KK) = Gi(LL,KK) * Bgi / (Ni(LL,KK) * Boi)

        B(LL,KK) = MatB (3,1) * 1000000.0

        DO JJ = 1, Z

            TEMP_SERR = 0

            SERR(LL,KK) = TEMP_SERR + (((Y(JJ) - (Ni(LL,KK)*X1(JJ)) -
(Ni(LL,KK)*m(LL,KK)*X2(JJ))- ((B(LL,KK)/1000000.0)*X3(JJ)) )/Y(JJ))**2.0)

        END DO

        WRITE (3,*) c, RED, Ni(LL,KK), m(LL,KK), B(LL,KK)

        WRITE (4,*) c, RED, SERR(LL,KK)

        DEALLOCATE (MatX, MatY, MatB, MatXT, Mat2)

    END DO

    DEALLOCATE (TD)

END DO

ELSE

    WRITE (*,*)     'An error occured opening file 2.'

END IF

CLOSE (1)

CLOSE (2)

CLOSE (3)

CLOSE (4)


STOP

END PROGRAM FYP_MBE
```