RELIABILITY ASSESSMENT OF A MULTI-STATE SYSTEM USING DISCRETE TIME MARKOV CHAIN

By

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(9179)

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CERTIFICATION OF APPROVAL

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Approved by,

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

MOHAMAD ZAMRI BIN HASAN

ABSTRACT

Reliability assessment of a working system is performed to identify the most likely failures and time-to-failure so that appropriate actions can be planned to diminish the effects of the failures. Traditional model of reliability assessment assumes every working system would have two states which are the state during perfectly working and the complete failure state. However, systems that exhibit multi-state behavior may have finite number of failure rate which are called multi-state system (MSS). The MSS system will degrade from a perfect working system to certain minor failure states before it completely fail. Hence, new approach is needed to predict the reliability of such system. This report contains the selected MSS analysis as the new approach to assess the reliability of a MSS system and the findings about the selected method which is Discrete-Time Markov Chain (DTMC) analysis. The data was taken from UTP Gas District Cooling (GDC) production report focusing on the performance of a gas turbine in terms of kW. The performance data was clustered into some performance states and the state transition probabilities were estimated. From the estimation, reliability function and distribution parameter were obtained to be used to calculate the Mean Time Between Failure (MTBF) of the system. At the end of the project, the reliability of the MSS that predicted using DTMC analysis was compared to the reliability predicted using traditional method which was the exponential distribution method. The analysis shows that DTMC analysis has better prediction than the exponential distribution method. Moreover, by exploiting the state transition probabilities estimation process, the change of operation demand as well as Preventive Maintenance planning could be included in the analysis. Briefly, MSS analysis gave better reliability prediction of the MSS and the behavior of the system could be analyzed.

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CHAPTER 1

INTRODUCTION

1.1. PROJECT BACKGROUND

Reliability is the probability of a system to perform its required function, performance or demand without failure under stated operating condition for a stated period of time. Reliability prediction of working systems is essential to assess the probability of the failure thus appropriate maintenance actions can be planned to mitigate the effects of the failures.

In predicting reliability of working systems, traditional method assumes that the systems only have two states which are perfectly working state and complete failure state. The changes in states may happen from the working state to failure state and from failure state to working state after repair or maintenance. However, complex systems may go through a finite number of degradation states prior to complete failure and it is known as multi-state system (MSS). Therefore, Multi-State System Analysis is to be preferred in order to assess the reliability of MSS.

This project focuses on applying MSS method to analyze the reliability of performance of a selected MSS which is UTP District Gas Cooling gas turbine.

1.2 PROBLEM STATEMENT

Traditional reliability model might not be applicable to predict MSS's reliability. Complex systems such as turbine and steam absorption chillers exhibit multi-state behavior and thus new approach is needed to predict performance reliability of a MSS.

1.3 OBJECTIVE AND SCOPE OF STUDY

The objective of the project is to establish a model to assess the reliability of UTP Gas District Cooling (GDC) Plant gas turbine using Discrete Time Markov Chain (DTMC) approach and compare with the reliability prediction using traditional method.

Scope of project covers the following:

1. The MSS system; UTP GDC gas turbine.
2. Performance data in term of kWh of MSS system from production report.
3. MATLAB to perform K-mean Cluster.
4. Discrete-Time Markov Chain (DTMC) analysis using Microsoft Excel.

CHAPTER 2

LITERATURE REVIEW / THEORY

2.1 WHAT IS RELIABILITY

Reliability is a broad term that focuses on the ability of a product to perform its intended function. A practical definition of reliability is the probability that a piece of equipment operating under specified conditions shall perform satisfactorily for a given period of time. Mathematically speaking, assuming that an item is performing its intended function at time equals zero, reliability can be defined as the probability that an item will continue to perform its intended function without failure for a specified period of time under stated conditions.

2.2 NEED FOR RELIABILITY

The reliability of engineering systems has become an important issue during their design because of the increasing dependence of our day lives and schedules on the satisfactory functioning of these systems [P.O Otasowie, S.O Omoruyi (2010)]. Some examples of these systems are aircraft, trains, computers, automobiles, space satellites, and nuclear power–generating reactors. Many of these systems have become highly complex and sophisticated. For example, today a typical Boeing 747 jumbo airplane is made of approximately 4.5 million parts, including fasteners. Most of these parts must function normally for the aircraft to fly successfully. [P.O Otasowie, S.O Omoruyi (2010)].

2.3 RELIABILITY PREDICTION MODEL

In reliability prediction, models of the system have to be worked out. These models may be graphical or mathematical. A mathematical model is necessary in order to be able to bring in data and use mathematical and statistical methods to estimate reliability. To estimate the system reliability from a model, input data is needed. The data usually come from generic data sources and may also from the specific system. When establishing the system model, we have to consider the type, amount and quality of the available input data. [Marvin Rausand, Arnljot Hyland (2004)]

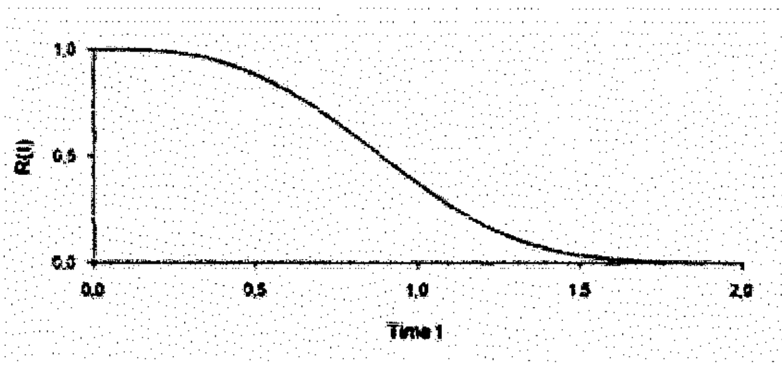


Figure 1 Reliability Function

Figure 1 shows reliability function of working systems vs time plot.
Or equivalently,

$$R(t) = 1 - \int_0^t f(u)du = \int_t^\infty f(u)du$$

Where *R(t)* is the reliability function, *f(u)* is the probability density function

Hence *R(t)* is the probability that the system does not fail in the time interval $(0,t]$ or the probability that the system survives the time interval $(0,t]$ and still functioning at time t.

2.4 MULTI-STATE SYSTEM (MSS)

All technical system are designed to perform their intended tasks in a given conditions. Some system can perform their tasks with various distinguished level of efficiency usually referred to as performance rates. A system that can have a finite number of performance rates is called a multi-state system (MSS) [Anatoly Lisnianski, Gregory Levitin (2003)]. There are many situations in which a system should be considered to be a MSS;

1. Any system consisting of different unites that have a cumulative effect on the entire system performance has to be considered as a MSS. Indeed, the performance rate of such a system depends on the availability of its units, as the different numbers of the available units can provide different levels of the task performance.

2. The performance rate of elements composing system can also vary as a result of their deterioration or partial failures. Element failures can lead to the degradation of the entire MSS performance.

2.4.1 Multi-State System Analysis

Multi-state system (MSS) reliability analysis relates to systems for which one cannot formulate an "all or nothing" type of failure criterion. Such systems are able to perform their task with partial performance (intensity of the task accomplishment). Failure of some system elements lead only to degradation of the system performance. The methods of MSS reliability assessment are based on some approaches such as an extension of Boolean models to the multi-valued case, the stochastic process and the universal generating function approach. [Anatoly Lisnianski, Gregory Levitin (2003)]

2.4.2 Markov Chain

A Markov chain is a mathematical system that undergoes transitions from one state to another (from a finite or countable number of possible states) in a chainlike manner. It is a random process characterized as memoryless as the next state depends only on the current state and not on the entire past. Figure 2 shows a Markov Model State-Space Diagram for 3 working states and a failure states. Markov chains have many applications as statistical models of real-world processes.

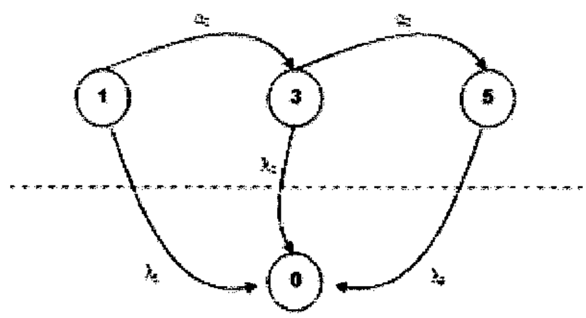


Figure 2 Markov Model State-Space Diagram

2.4.3 Discrete-Time Markov Chain (DTMC)

A discrete time stochastic process $\{X_n, n = 0, 1, 2, \ldots\}$ with discrete state space is a Markov chain if it satisfies the Markov property.

$P(X_n = i_n \mid X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i_{n-1}) = P(X_n = i_n \mid X_{n-1} = i_{n-1})$,

where i_k for all $k = 0, 1, \ldots, n$ are realized states of the stochastic process. [Taylor & Karlin (1998)] In the other word, DTMC is a markov process with discrete parameter T and discrete state space $X(t)$ which is time-homogeneous.

2.4.3.1 Transition Probabilities

Transition probabilities is defined as probability to jump from state i to state j. It is assumed to be stationary; independent of time. A simple transition probability matrix is written as $P = (p_{it})$

For two states Markov Chain, the transition probability is writtten as

$$P = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}$$

2.4.4 Stochastic Process

Stochastic or random process is, essentially, a set of random variables, where the variables are ordered in a given sequence. For example, the daily maximum temperatures at a weather station form a sequence of random variables, and this ordered sequence can be considered as a stochastic process. More formally, the sequence of random variables in a process can be denoted by $X(t)$ where t is the index of the process which usually represents time. [Anatoly Lisnianski, Gregory Levitin (2003)]

2.4.5 K-Mean Cluster Algorithm and Silhouette Plot

2.4.5.1 K-Mean Cluster Algorithm

K means clustering algorithm was developed by J. MacQueen (1967) and then by J. A. Hartigan and M. A. Wong around 1975. K-means is one of the simplest unsupervised learning algorithms that solve the well-known clustering problem. To simplify, k-means clustering is an algorithm to classify or to group your objects based on attributes into K number of group where K is positive integer number. The grouping is done by minimizing the sum of squares of distances between data and the corresponding cluster centroid. Thus the purpose of K-mean clustering is to classify the data.

The basic step of k-means clustering is by determining the number of cluster K and assume the centroid or center of these clusters as shown in Figure 3. Any random objects can be taken as the initial centroids or the first K objects in sequence can also serve as the initial centroids. Then the K means algorithm will do the three steps below until convergence or stable where no object move groups.

1. Determine the centroid coordinate
2. Determine the distance of each object to the centroids
3. Group the object based on minimum distance

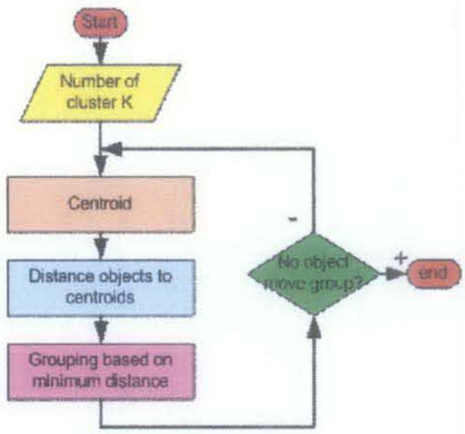


Figure 3 Steps in K means Algorithm

To get an idea of how well-separated the resulting clusters are, a silhouette plot can be made using the cluster indices output from K means.

2.4.5.2 Silhouette Plot

Silhouette refers to a method of interpretation and validation of clusters of data. The technique provides a brief graphical representation of how well each object lies within its cluster [Peter J. Rousseeuw (1986)]. The silhouette plot displays a measure of how close each point in one cluster is to the points in the neighboring clusters. This measure ranges from +1, indicating points that are very distant from neighboring clusters, through 0, indicating points that are not distinctly in one cluster or another, to -1, indicating points that are probably assigned to the wrong cluster. [R. Lleti, M.C. Ortiz, L.A. Sarabia, M.S. Sánchez (2004)]. Figure 4 below shows the example of Silhouette Plot.

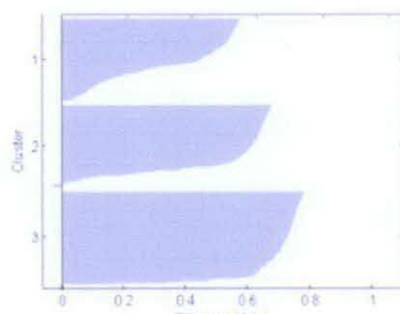


Figure 4 Example of Silhouette Plot

A more quantitative way to compare the solutions is to look at the average silhouette value for the cases where the highest value indicates the best amount of cluster, *k*.

2.5 TRADITIONAL MODEL

2.5.1 Exponential Distribution Method

The exponential distribution is a commonly used distribution in reliability engineering. Mathematically, it is a fairly simple distribution, which many times lead to its use in inappropriate situations. It is, in fact, a special case of the Weibull distribution where $\beta = 1$. The exponential distribution is used to model the behavior of units that have a constant failure rate (or units that do not degrade with time or wear out). [www.weibull.com]

The exponential distribution is often used to model the failure time of manufactured items in production. For example, if X denotes the time to failure of a light bulb of a particular make, with exponential distribution, then P(X>x) represent the survival of the light bulb. The larger the average rate of failure, the bigger will be the failure time. One of the most important properties of the exponential distribution is the memoryless property.

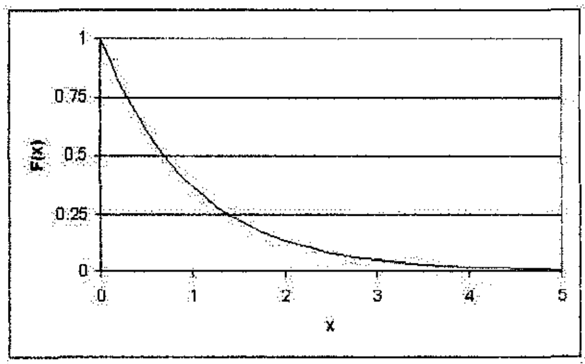


Figure 5 Example of Exponential Distribution Curve

Figure 5 shows the standard exponential distribution curve with parameter distribution, λ.

Figure 6 and Figure 7 show the plot of exponential probability density function with formula $pdf = \lambda e^{-\lambda t}$ and the plot of exponential cumulative density function with formula $cdf = 1 - e^{-\lambda t}$.

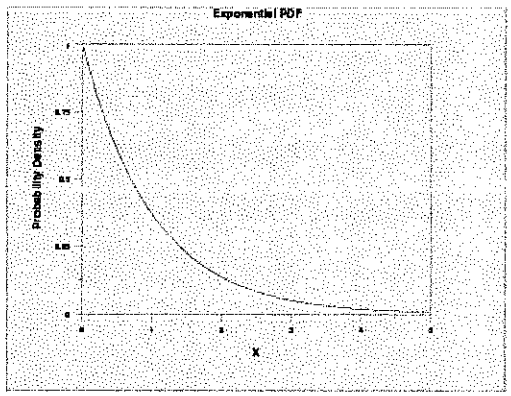


Figure 6 Example of Exponential Probability Density Function

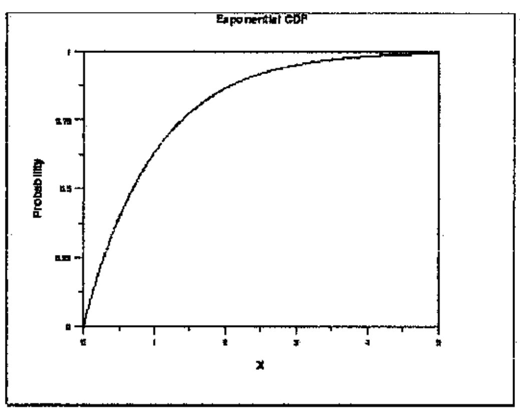


Figure 7 Example of Exponential Cumulative Density Function

2.5.2 Maximum Likelihood Estimation

The principle of maximum likelihood estimation (MLE), originally developed by R.A. Fisher in the 1920s, states that the desired probability distribution is the one that makes the observed data "most likely," which means that one must seek the value of the parameter vector that maximizes the likelihood function $L(w|y)$ The resulting parameter vector, which is sought by searching the multi-dimensional parameter space, is called the MLE estimate, and is denoted by $w_{MLE} = (w_{1,MLE}, .., w_{k,MLE})$. Briefly, maximum likelihood estimation is a method to seek the probability distribution that makes the observed data most likely.

2.6 PREVENTIVE MAINTENANCE

Preventive maintenance (PM) is a schedule of planned maintenance actions aimed at the prevention of breakdowns and failures. The primary goal of preventive maintenance is to prevent the failure of equipment before it actually occurs. It is designed to preserve and enhance equipment reliability by replacing worn components before they actually fail. Preventive maintenance activities include equipment checks, partial or complete overhauls at specified periods, oil changes, lubrication and so on. In addition, workers can record equipment deterioration so they know to replace or repair worn parts before they cause system failure. [www.weibull.com]

2.6.2 Minimum and Perfect PM

Minimum PM is done to restore the system to the next acceptable better working state. Contrary to minimum PM, perfect PM is done to restore the system to the perfectly working state which also known as good as new.

CHAPTER 3

METHODOLOGY / PROJECT WORK

3.1 METHODOLOGY

3.1.1 Data Collection

Data was taken from UTP Gas District Cooling (GDC) operation report. For the purpose of the project, daily operation report for gas turbine in terms of kWh was only collected. The data from the turbine output performance was collected in two different sets of resolutions which were day and hour resolutions that were required for the analysis stage.

3.1.2 Identify Performance States

Both sets of the data were analyzed using the same processes. To divide the data into some performance states, the collected performances data of the MSS were partitioned into several clusters. K-means cluster analysis was used to acquire the best amount of clusters.

The analysis was done using MATLAB. Silhouette plots were made for all the values of clusters tested, k from $k = 3, 4, 5, 6$, and 7. The selection of the best value of k was done by analyzing the silhouette plot and silhouette average value for each k value. After the best k value was obtained, the performances data were clustered into k amount of groups that indexed the MSS performance states.

3.1.3 Transition Probability Estimation

After the performance data were clustered accordingly into k amount of performance states, the transition from one state to another was calculated using Microsoft Excel functions. Prior to the project assumptions and later analysis stage, several sets of state transitions were calculated. Next, the state transition probability was estimated.

3.1.4 DTMC Governing Equation

The state transition probability is written in transition probability matrix as follow;

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & . & . & P_{1n} \\ P_{21} & P_{22} & P_{23} & . & . & P_{2n} \\ P_{31} & P_{32} & P_{33} & : & : & P_{3n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ P_{m1} & P_{m2} & P_{m3} & . & . & P_{mn} \end{bmatrix}$$

And the state probability matrix for day n is equal to $S^n = P^{n-1}.S^{n-1}$ where S^0 is the probability matrix at perfect state.

And $\sum S^n = R(t)$, the reliability of the system for day n.

3.1.5 Solving Equation and Reliability Prediction

By using the state probability, S^n state probability graph was plotted for each respective k states.

Finally, the reliability, $R(t)$ for the MSS was determined and the reliability graph was plotted. From the value of parameter, λ that obtained from the graph, Mean Time Between Failure (MTBF) was calculated.

3.1.6 Comparing MSS Method to the Traditional Method

The reliability of the MSS was calculated using the traditional method. From both sets of resolutions, times to failure (TTF) were calculated to be used as input in Weibull++ software to find the reliability of the MSS. The results were represented in graphs and the MTBF were calculated.

The MSS reliability calculated using traditional method was compared to the reliability calculated using MSS method.

3.2 METHODOLOGY FLOW CHART

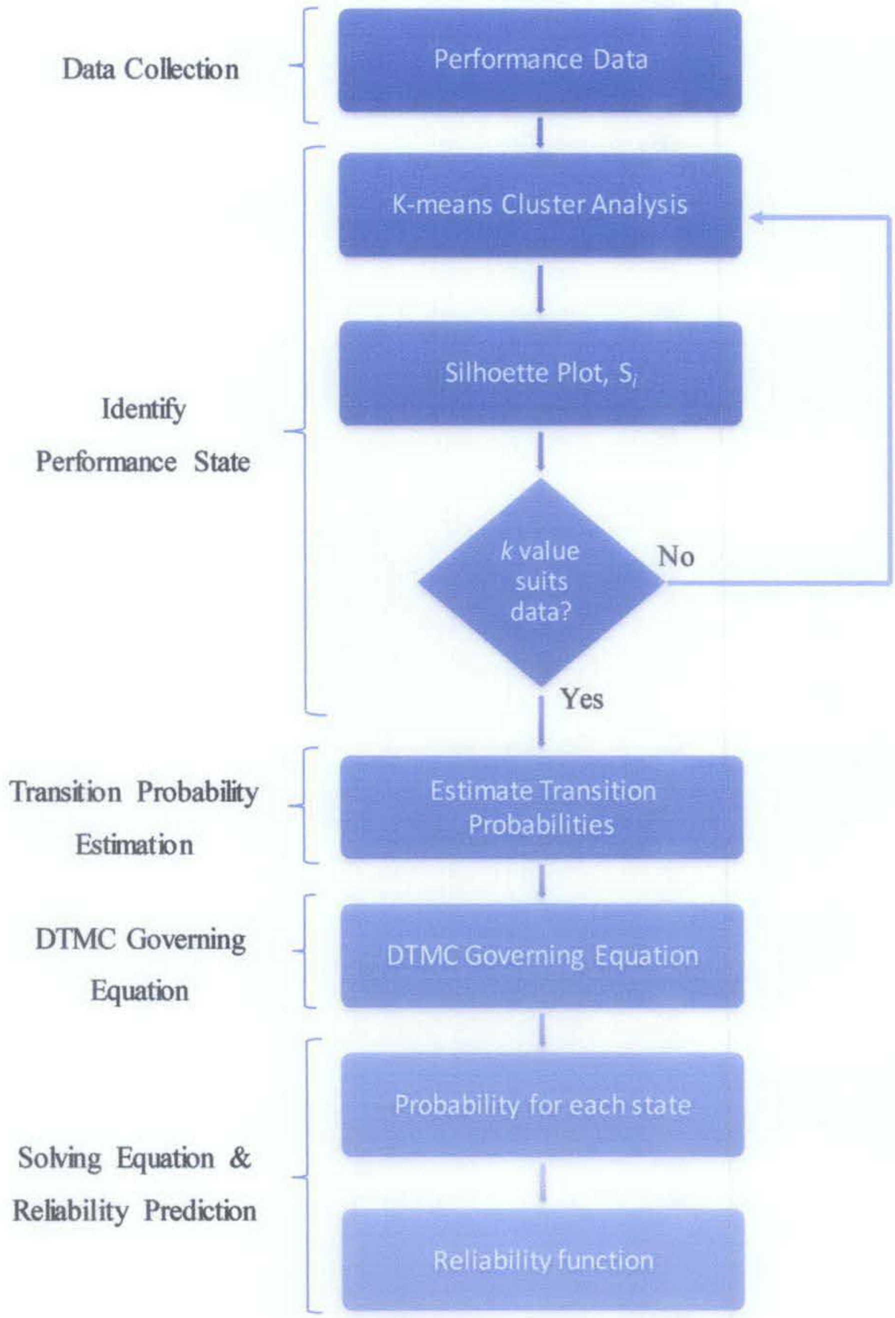


Figure 8 Project Flow Chart

3.3 PROJECT TIME LINE

No	Detail/Week	1	2	3	4	5	6	7		8	9	10	11	12	13	14	15
1	Literature Review	x	x	x	x	x	x	x	Mid-Sem break	x	x						
2	Understanding Project Objectives and Theories				x	x	x	x		x							
3	Data Collection			x	x	x											
4	Identify Performance States						x	x		x							
5	Estimate Transition Probability									x	x	x					
6	DTMC Governing Equation											x	x				
7	Solving Equation & Reliability Prediction											x	x	x			
8	Submission of Draft Report													x			
9	Submission of Dissertation (soft bound)														x		
10	Submission of Technical Paper														x		
11	Oral Presentation															x	
12	Submission of Project Dissertation (hard bound)																x

Figure 9 Project Time Line

CHAPTER 4

RESULT AND DISCUSSION

4.1 UTP GDC GAS TURBINE PRODUCTION REPORT

Data for analysis for this project was taken from the UTP GDC Production Report in Year 2009.

DAILY LOGGING DATA(AVG) FOR ELECTRICITY

	JI-PLANT1	JI-SUPPLY1		JI-IMPORT1	JI-EXPORT1	GTGA-POWERKW	GTGB-POWERKW
HOUR	PV	KW		PV	PV	PV	PV
0:00	1299	3490		0	2190	3492	0
1:00	1291	3391		0	2101	3400	0
2:00	1183	3240		0	2052	3241	0
3:00	692	2691		0	2006	2690	0
4:00	657	2745		0	2090	2746	0
5:00	609	3037		0	2433	3040	0
6:00	616	2735		0	2127	2733	0
7:00	1055	3831		0	2791	3134	694
8:00	1044	5498		0	4460	2854	2643
9:00	1317	5883		0	4577	2806	3081
10:00	1328	5836		0	4509	2750	3089
11:00	1366	6069		0	4712	2983	3084
12:00	1423	6140		0	4724	3048	3089
13:00	1431	6099		0	4673	3011	3085
14:00	1429	6039		0	4619	2955	3079
15:00	1433	6086		0	4659	2996	3086
16:00	1429	6025		0	4599	2936	3085
17:00	857	4336		170	3652	3127	1206
18:00	574	3354		469	3250	3354	0
19:00	698	3453		387	3142	3453	0
20:00	690	3436		321	3067	3436	0
21:00	687	3448		260	3022	3449	0
22:00	700	3499		53	2854	3503	0
23:00	714	3383		0	2668	3385	0

Figure 10 Piece of UTP GDC Operation Report

Figure 10 above shows piece of Day Production Report for January 2009 for 2 gas turbines (GTGA and GTGB) in terms of kW. Data that were collected prior to this project were the output performance of GTGA gas turbine during working days (Monday – Friday) and collected into two sets of data resolution;

1. Day resolution – The reading was taken at 0800 on every working day by assuming that the output represents the MSS performance for the whole day.
2. Hour resolution – The reading was taken from 0800 until 1700 for every working day.

Both of the resolutions were analyzed under an assumption that the data recorded in the production reports were the real output from the turbine and not manually set to certain desired outputs.

The same processes were done for both of the resolutions to get the MSS's reliability respectively.

4.2 RELIABILITY ANALYSIS IN DAY RESOLUTION

4.2.1 Identify Performance State

To apply MSS method, the performances of gas turbine GTGA were segregated into certain number of performance states. K-mean cluster analysis was performed using MATLAB and the number of suitable cluster could be determined.

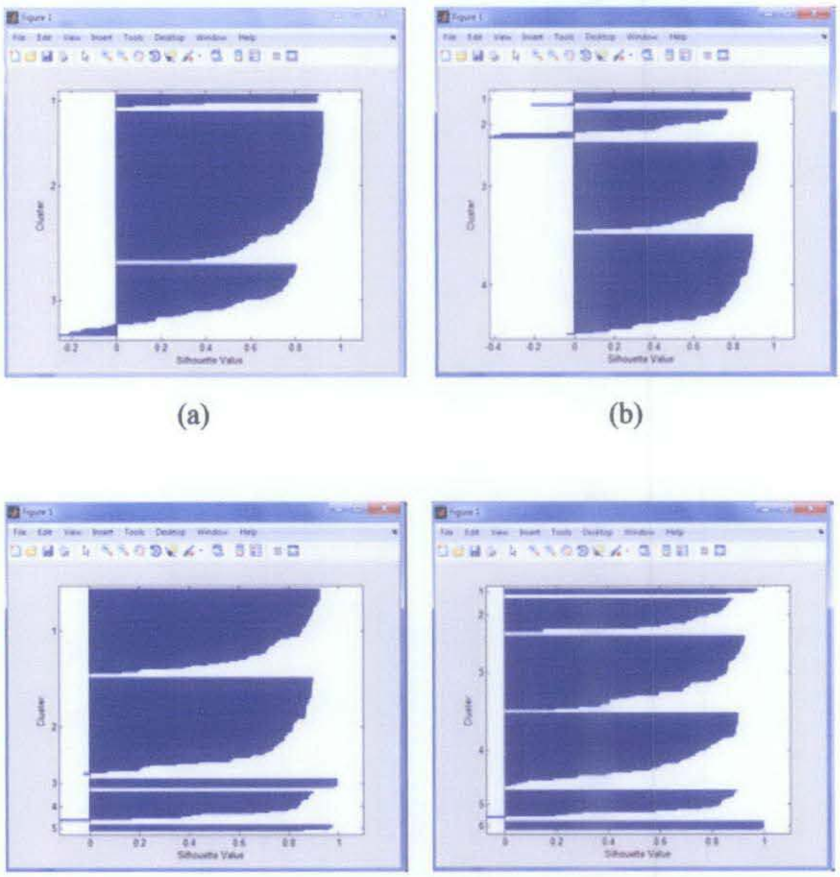
4.2.1.1 K-Mean Cluster Analysis and Silhouette Plot

Figure 11 shows command window in MATLAB to find the value of suitable k where the value tested is equal to 3. The k values were tested ranged from k=3,4,5,..8.

```
Command Window
>> idx3 = kmeans(X,3,'distance','city');
>> [silh3,h] = silhouette(X,idx3,'city');
>> set(get(gca,'Children'),'FaceColor',[.8 .8 1])
>> xlabel('Silhouette Value')
>> ylabel('Cluster')
>> idx3 = kmeans(X,3, 'dist','city', 'display','iter');
  iter   phase      num          sum
     1       1      560      2478.74
     2       1       15      2465.27
     3       1        6       2463.8
     4       1        2      2463.44
     5       1        2      2463.14
     6       1        1      2463.11
     7       2        0      2463.11
7 iterations, total sum of distances = 2463.11
```

Figure 11 Command Window in MATLAB for k=3

To get an idea of how well-separated the resulting clusters, silhouette plots were drawn using the cluster indices output from k-means values for each k value.



(a) (b) (c) (d)

Figure 12 Silhouette plot for each value of k, (a) k=3, (b) k=4, (c) k=5, and (d) k=6

The average silhouette value for each amount of k was calculated and shown in Table 1.

Table 1 Average Silhouette Values for every value of k for Day Resolution Data

k value	Average Silhouette Value
3	0.7112
4	0.7080
5	0.7543
6	0.7266

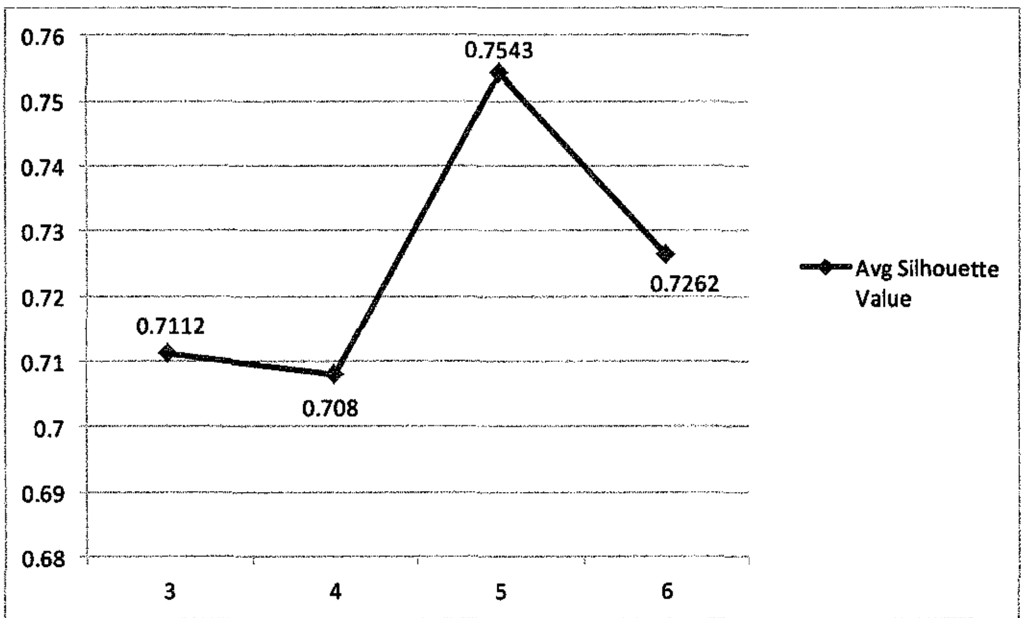


Figure 13 Comparison of Average Silhouette Values for every value of k for Day Resolution Data

Based on Figure 13, the average silhouette value for k=5 shows the highest value. Thus, the data for day resolution is divided into 5 performance states. A scatter plot was drawn to get a better view of how well the data were distributed as well as to determine the minimum value of each state as shown in Figure 14.

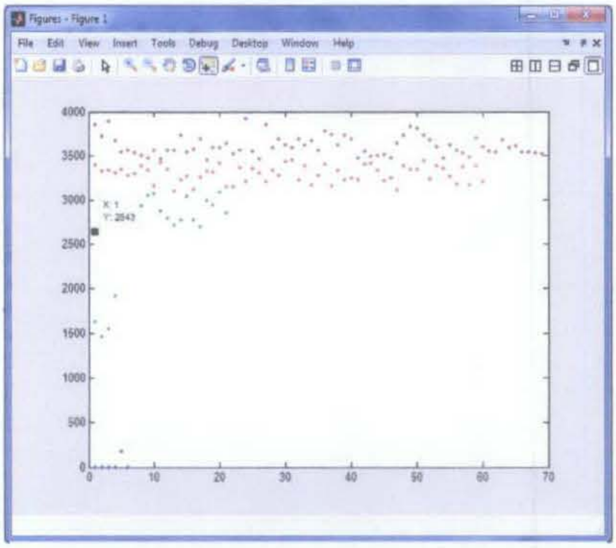


Figure 14 Scatter Plot for Day Resolution Data

From the plot, the minimum values for each state are 1462, 2643, 3089, and 3459 respectively.

4.2.2 Estimate State Transition Probability

After the performance data were clustered accordingly into *k* amount of performance states, the transition from one state to another was calculated using Microsoft Excel.

For day resolution, the performance data was clustered into 5 performance states as shown in Table 2 below.

Table 2 Performance States with Minimum Values for Day Resolution

State	Minimum Values
0	0
7	1462
5	2643
3	3089
1	3459

After grouping the data into respective performance states, the number of each state was counted as well as the state transitions as shown in Table 3.

Table 3 State Transition for Day Resolution Data

Day	Output,kW)	State	State Transition
1	2643	7	
2	3391	3	7-3
3	3324	3	3-3
4	3852	1	3-1
5	3719	1	1-1
6	3885	1	1-1
7	0	0	1-0
8	0	0	0-0
9	2703	5	0-5
10	3336	3	5-3
11	3304	3	3-3
12	3351	3	3-3
13	1624	7	3-7
14	3274	3	7-3
15	3294	3	3-3
16	2908	5	3-5
17	0	0	5-0

From the state transition count, the state transition probabilities were calculated and shown in Table 4 as follows;

Table 4 State Transition Probabilities for Day Resolution Data

State Transition	Count	Probability	State Transition	Count	Probability
1-0	2	0.030	5-0	2	0.100
1-1	39	0.582	5-1	8	0.400
1-3	20	0.299	5-3	5	0.250
1-5	4	0.060	5-5	5	0.250
1-7	2	0.030	5-7	0	0.000
3-0	1	0.016	7-0	0	0.000
3-1	16	0.262	7-1	2	0.400
3-3	33	0.541	7-3	3	0.600
3-5	9	0.148	7-5	0	0.000
3-7	2	0.033	7-7	0	0.000

4.2.3 Reliability Prediction Using MSS Method

The state transition probabilities are written in state transition probability, P where

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & . & . & P_{1n} \\ P_{21} & P_{22} & P_{23} & . & . & P_{2n} \\ P_{31} & P_{32} & P_{33} & . & . & P_{3n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ P_{m1} & P_{m2} & P_{m3} & . & . & P_{mn} \end{bmatrix}$$

For day resolution data, the state transition probabilities matrix is equal to

$$P = \begin{vmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.030 & 0.582 & 0.299 & 0.060 & 0.030 \\ 0.016 & 0.262 & 0.541 & 0.148 & 0.033 \\ 0.100 & 0.400 & 0.250 & 0.250 & 0.000 \\ 0.000 & 0.400 & 0.600 & 0.000 & 0.000 \end{vmatrix}$$

To find the state probability matrix, S^n for day *n*,

$S^n = P^{n-1}.S^{n-1}$. Table 5 shows the calculated state probability matrix for 5 working days.

Table 5 State Transition Probability Matrix for 5 Days for Day Resolution Data

Day/State	S0	S1	S3	S5	S7
0	0.000	1.000	0.000	0.000	0.000
1	0.030	0.582	0.299	0.060	0.030
2	0.058	0.453	0.368	0.094	0.027
3	0.087	0.409	0.374	0.105	0.026
4	0.116	0.388	0.366	0.106	0.024
5	0.144	0.374	0.355	0.104	0.024

The reliability of the MSS system, R_n for day *n* is equal to

$$\sum S^n = R_n(t)$$

And the reliability of the MSS for day *n* is calculated and shown in Table 6 below

Table 6 Reliability of GTGA Gas Turbine for 5 Days for Day Resolution

Day/State	S0	S1	S3	S5	S7	Reliability
0	0.000	1.000	0.000	0.000	0.000	1.000
1	0.030	0.582	0.299	0.060	0.030	0.970
2	0.058	0.453	0.368	0.094	0.027	0.942
3	0.087	0.409	0.374	0.105	0.026	0.913
4	0.116	0.388	0.366	0.106	0.024	0.884
5	0.144	0.374	0.355	0.104	0.024	0.856

The reliability of the GTGA gas turbine is plotted in a graph as shown in Figure 15.

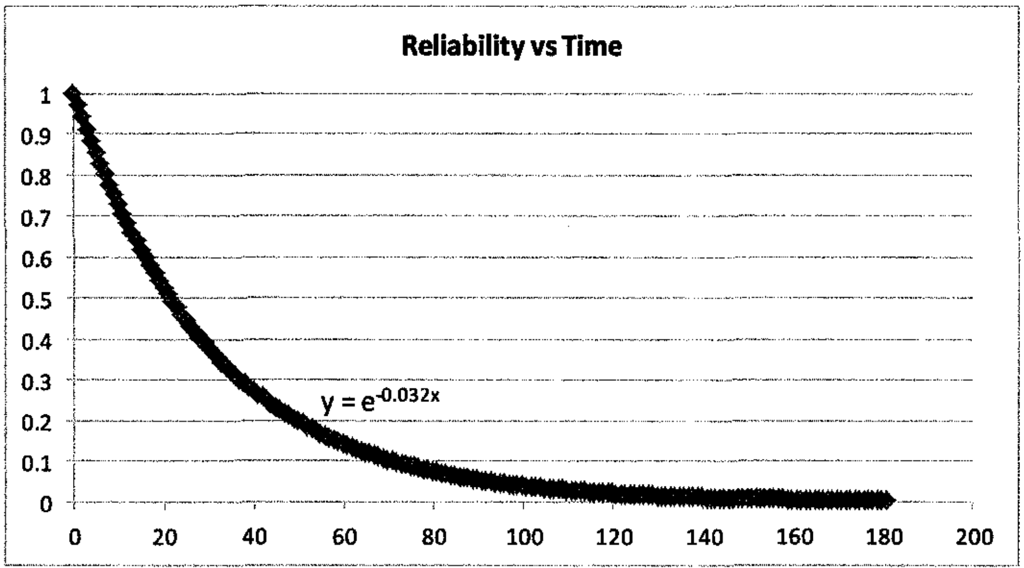


Figure 15 Reliability vs Time for Day Resolution Data

From the graph equation in Figure 15, the value of parameter distribution, λ is equal to 0.032. Using DTMC analysis, the Reliability function for GTGA gas turbine using day resolution data is equal to $R(t) = e^{-0.032t}$.

4.2.4 Reliability Prediction Using Traditional Method

The selected traditional method was the exponential distribution. The data of different resolutions were analyzed to calculate the time to failure (TTF) and cumulative time to failure (CTTF) of each set of data. The TTF for day resolution data was calculated and shown in Table 7 and graph as follows;

Table 7 TTF and CTTF for Day Resolution Data

T_n	TTF	CTTF
1	8	8
2	19	27
3	45	72
4	77	149

The TTF was used as input data in Weibull++ to find the reliability of the system and the result is shown in graph below;

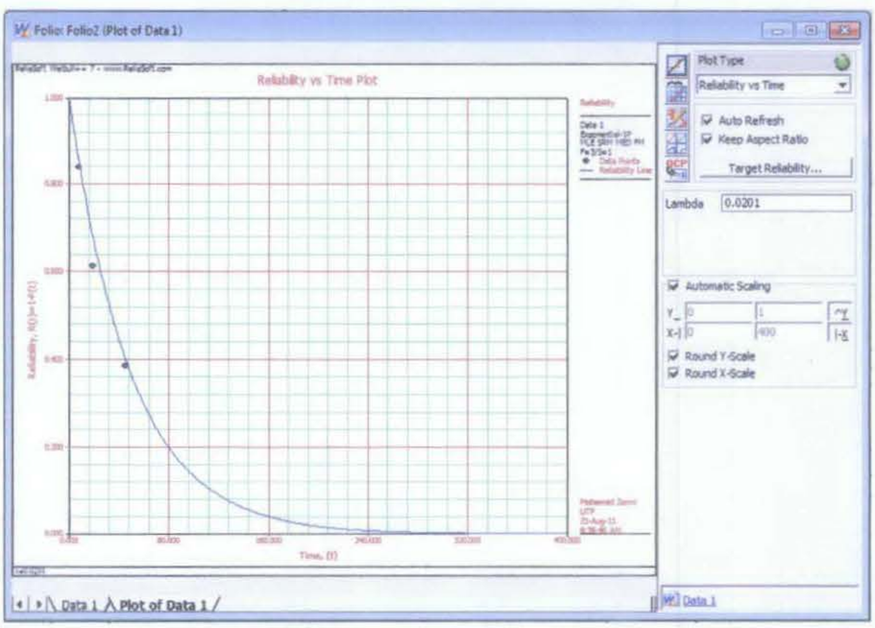


Figure 16 Reliability vs Time Plot for Day Resolution Data

From Figure 16, the value of λ for the exponential distribution is 0.021. By the equation $R(t) = e^{-\lambda t}$, and the value of λ acquired from Weibull++, the reliability of GTGA gas turbine was calculated. By exponential distribution, reliability function for the MSS using day resolution is $R(t) = e^{-0.021t}$

4.3 RELIABILITY ANALYSIS IN HOUR RESOLUTION

4.3.1 Identify Performance State

4.3.1.1 K-Mean Cluster Analysis and Silhouette Plot

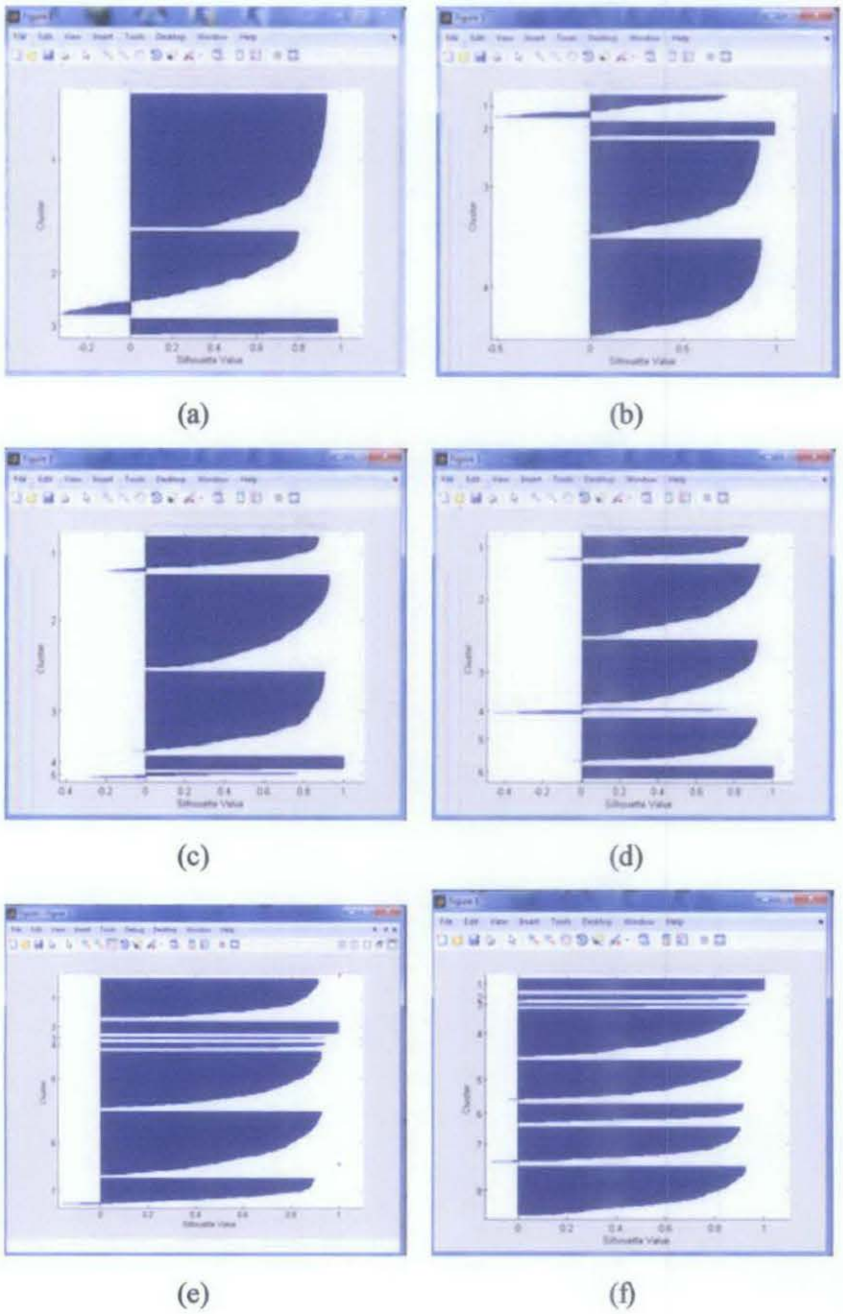


Figure 17 Silhouette plot for each value of k, (a) k=3, (b) k=4, (c) k=5, (d) k=6, (e) k=7 and (f) k=8

Figure 17 shows the silhouette plots for k=3, 4, 5, 6 and 7 for calculation in hour resolution. The Average Silhouette Values for every value of k was calculated and shown in Table 8.

Table 8 Average Silhouette Values for every value of k for Hour Resolution Data

k value	Average Silhouette Value
3	0.6976
4	0.7017
5	0.7110
6	0.7149
7	0.7402
8	0.7134

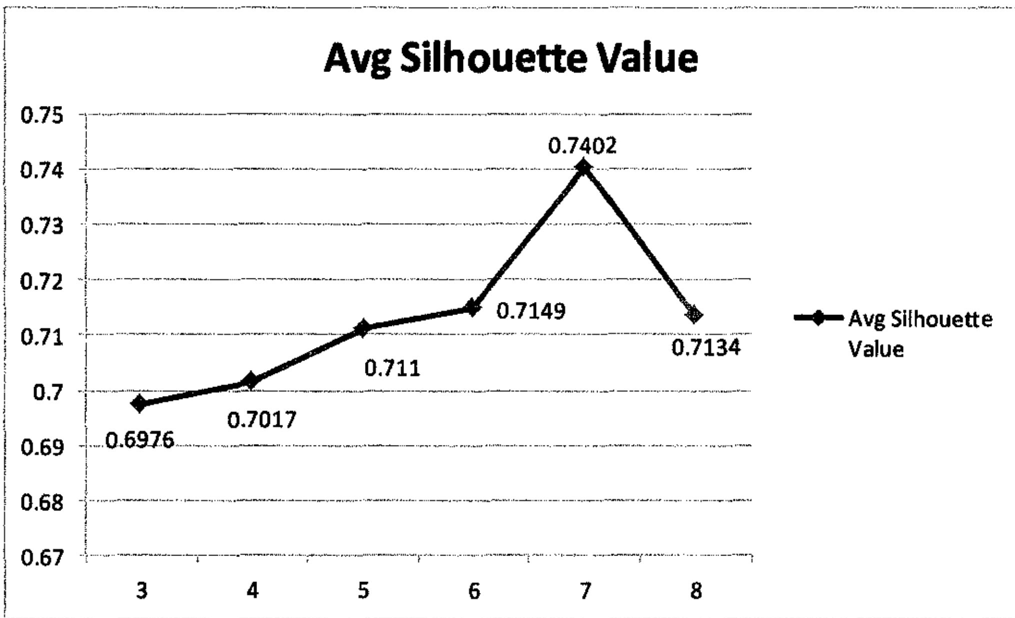


Figure 18 Average Silhouette Values for every value of k for Hour Resolution Data

The average silhouette value for each amount of k was calculated and shown in the figure above. Based on Figure 18, the average silhouette value for k=7 shows the highest value. Thus, the data for hour resolution was divided into 7 performance states. A scatter plot was drawn to get a better view of how well the data were distributed as well as to determine the minimum value of each state as shown in Figure 19.

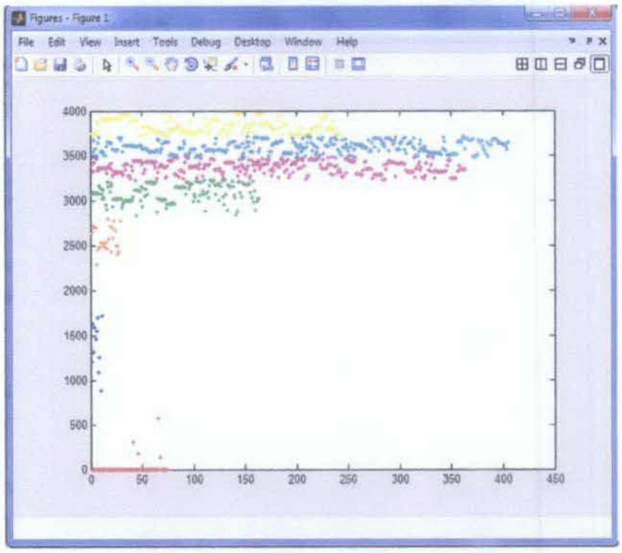


Figure 19 Scatter Plot for Hour Resolution Data

Based on Figure 19, the minimum values for each state are 881, 2285, 2832, 3233, 3484 and 3711 respectively.

After clustering the data into suitable performance states, the state transitions were calculated.

4.3.2 Estimate State Transition Probability

For hour resolution, the data were clustered in 7 performance states as shown in Table 9 below.

Table 9 Performance States with Minimum Values for Hour Resolution

State	Minimum Value, kW
0	0
11	881
9	2285
7	2832
5	3233
3	3484
1	3711

Table 10 below shows the day data that were clustered into performance states and the state transitions.

Table 10 State Transition for Hour Resolution Data

Day	Output, kW	State	State Transition
1	2643	9	
2	3081	7	9-7
3	3089	7	7-7
4	3084	7	7-7
5	3089	7	7-7
6	3085	7	7-7
7	3079	7	7-7
8	3086	7	7-7
9	3085	7	7-7
10	1206	11	7-11
11	3391	5	11-5
12	3536	3	5-3
13	3533	3	3-3
14	3541	3	3-3
15	3749	1	3-1
16	3751	1	1-1
17	3747	1	1-1

The state transitions were calculated and the state transition probabilities could be estimated as shown in Table 11 below.

Table 11 State Transition Probabilities for Hour Resolution Data

State Transition	Count	Probability	State Transition	Count	Probability	State Transition	Count	Probability
1-0	2	0.008	5-0	2	0.005	9-0	1	0.034
1-1	197	0.821	5-1	2	0.005	9-1	1	0.034
1-3	29	0.121	5-3	63	0.173	9-3	2	0.069
1-5	6	0.025	5-5	260	0.714	9-5	6	0.207
1-7	6	0.025	5-7	28	0.077	9-7	6	0.207
1-9	0	0.000	5-9	3	0.008	9-9	13	0.448
1-11	0	0.000	5-11	6	0.016	9-11	0	0.000
3-0	0	0.000	7-0	1	0.006	11-0	1	0.091
3-1	37	0.091	7-1	2	0.012	11-1	0	0.000
3-3	297	0.730	7-3	14	0.085	11-3	0	0.000
3-5	49	0.120	7-5	38	0.232	11-5	4	0.364
3-7	22	0.054	7-7	97	0.591	11-7	4	0.364
3-9	2	0.005	7-9	7	0.043	11-9	2	0.182
3-11	0	0.000	7-11	5	0.030	11-11	0	0.000

4.3.3 Reliability Prediction Using MSS

The state transition probabilities matrix for hour resolution data is equal to

$$P = \begin{vmatrix} 1.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.008 & 0.821 & 0.121 & 0.025 & 0.025 & 0.000 & 0.000 \\ 0.000 & 0.091 & 0.730 & 0.120 & 0.054 & 0.005 & 0.000 \\ 0.005 & 0.005 & 0.173 & 0.714 & 0.077 & 0.008 & 0.016 \\ 0.006 & 0.012 & 0.085 & 0.232 & 0.591 & 0.043 & 0.030 \\ 0.034 & 0.034 & 0.069 & 0.207 & 0.207 & 0.448 & 0.000 \\ 0.091 & 0.000 & 0.000 & 0.364 & 0.364 & 0.182 & 0.000 \end{vmatrix}$$

Since $S^n = P^{n-1}.S^{n-1}$

Thus, the state probability matrix, S^n for day n was calculated and tabulated in Table 12 below.

Table 12 State Transition Probability Matrix for 10 Days for Hour Resolution Data

Day	S0	S1	S3	S5	S7	S9	S11
0	0.000	1.000	0.000	0.000	0.000	0.000	0.000
1	0.008	0.821	0.121	0.025	0.025	0.000	0.000
2	0.015	0.685	0.194	0.059	0.044	0.002	0.001
3	0.022	0.581	0.238	0.093	0.059	0.004	0.002
4	0.028	0.500	0.266	0.125	0.071	0.007	0.003
5	0.034	0.436	0.282	0.153	0.081	0.009	0.004
6	0.040	0.386	0.293	0.176	0.089	0.011	0.005
7	0.045	0.346	0.299	0.196	0.096	0.013	0.006
8	0.051	0.314	0.303	0.211	0.101	0.014	0.006
9	0.056	0.288	0.305	0.224	0.105	0.015	0.007
10	0.061	0.267	0.306	0.234	0.109	0.016	0.007

The reliability of the MSS system, R_n for day n is equal to

$\sum Q^n = R_n(t)$, thus, reliability for every working day was calculated and the results shown in Table 13.

Table 13 Reliability of GTGA Gas Turbine for 10 Days for Hour Resolution

Day	S0	S1	S3	S5	S7	S9	S11	Reliability
0	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000
1	0.008	0.821	0.121	0.025	0.025	0.000	0.000	0.992
2	0.015	0.685	0.194	0.059	0.044	0.002	0.001	0.985
3	0.022	0.581	0.238	0.093	0.059	0.004	0.002	0.978
4	0.028	0.500	0.266	0.125	0.071	0.007	0.003	0.972
5	0.034	0.436	0.282	0.153	0.081	0.009	0.004	0.966
6	0.040	0.386	0.293	0.176	0.089	0.011	0.005	0.960
7	0.045	0.346	0.299	0.196	0.096	0.013	0.006	0.955
8	0.051	0.314	0.303	0.211	0.101	0.014	0.006	0.949
9	0.056	0.288	0.305	0.224	0.105	0.015	0.007	0.944
10	0.061	0.267	0.306	0.234	0.109	0.016	0.007	0.939

The reliability of the GTGA gas turbine for hour resolution data was plotted in a graph in Figure 20.

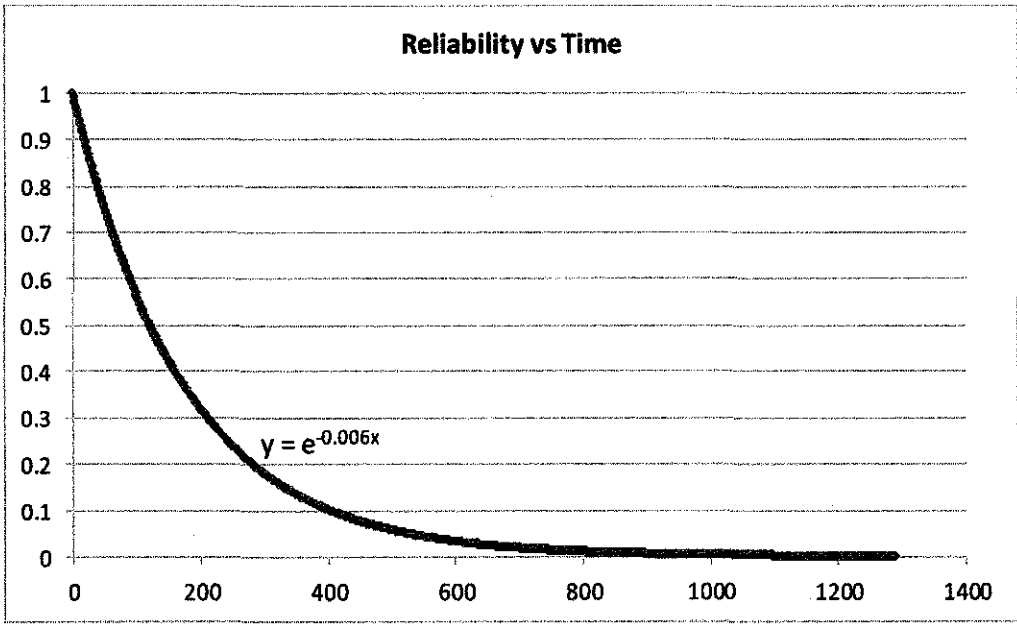


Figure 20 Reliability vs Time for Hour Resolution Data

Using DTMC analysis, the Reliability function for GTGA gas turbine using hour resolution data is equal to $R(t) = e^{-0.006t}$.

4.3.4 Reliability Prediction Using Traditional Method

For hour resolution, the TTF calculated are shown in Table 14.

Table 14 TTF and CTTF for Hour Resolution Data

T_n	TTF	CTTF
1	80	80
2	190	270
3	386	656
4	243	899
5	280	1179

Using Weibull++, exponential distribution for hour resolution data was obtained as follows;

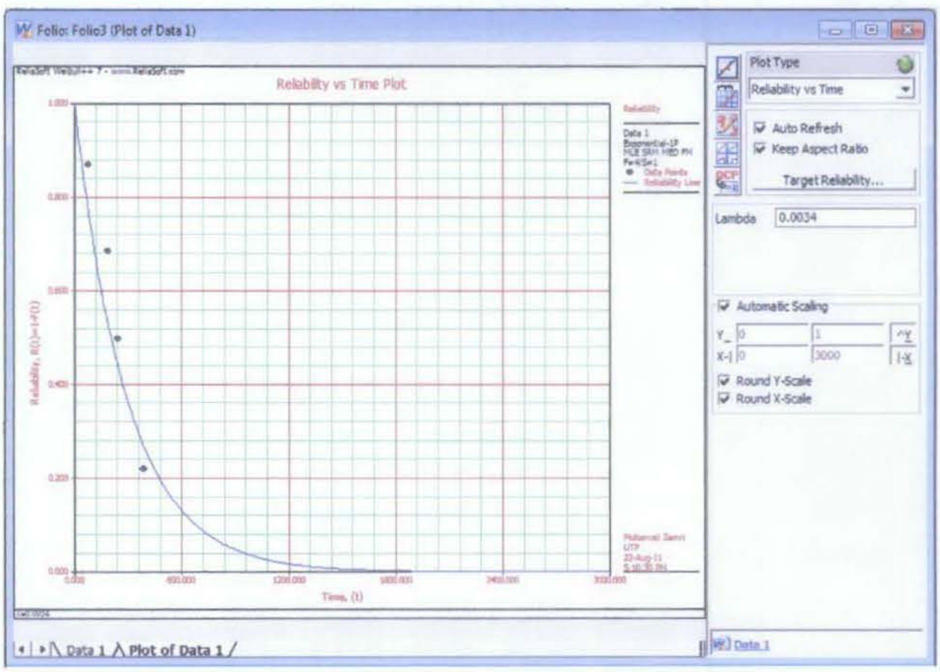


Figure 21 Reliability vs Time Plot for Hour Resolution Data

Based on Figure 21, the value of λ for the exponential distribution is 0.0034 and the reliability of the system was calculated. By exponential distribution, reliability function for the MSS using day resolution is $R(t) = e^{-0.0034t}$.

4.4 MSS METHOD AND TRADITIONAL METHOD COMPARISON

The reliability of GTGA gas turbine that was calculated using DTMC was compared to the reliability calculated using exponential distribution. The comparisons for both resolutions are shown in Figure 22 and Figure 23.

4.4.1 Reliability Graphs and Parameter Distributions Comparison

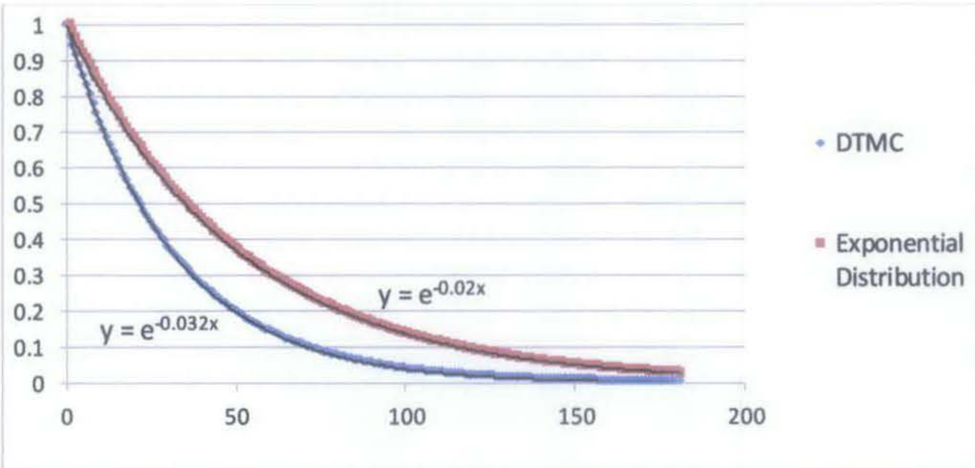


Figure 22 Reliability vs Time for Day Resolution Data

Figure 22 shows a graph of Reliability vs Time for day resolution data which displays reliability calculated using DTMC shows faster decrease than the reliability calculated using exponential distribution. The parameter distribution, λ for DTMC is higher than the exponential distribution.

Equally, the Reliability vs Time for hour resolution data that displayed in Figure 23 shows the same pattern where DTMC results with faster decrease than the exponential distribution. The parameter distribution, λ for DTMC is less by half of the value for the exponential distribution.

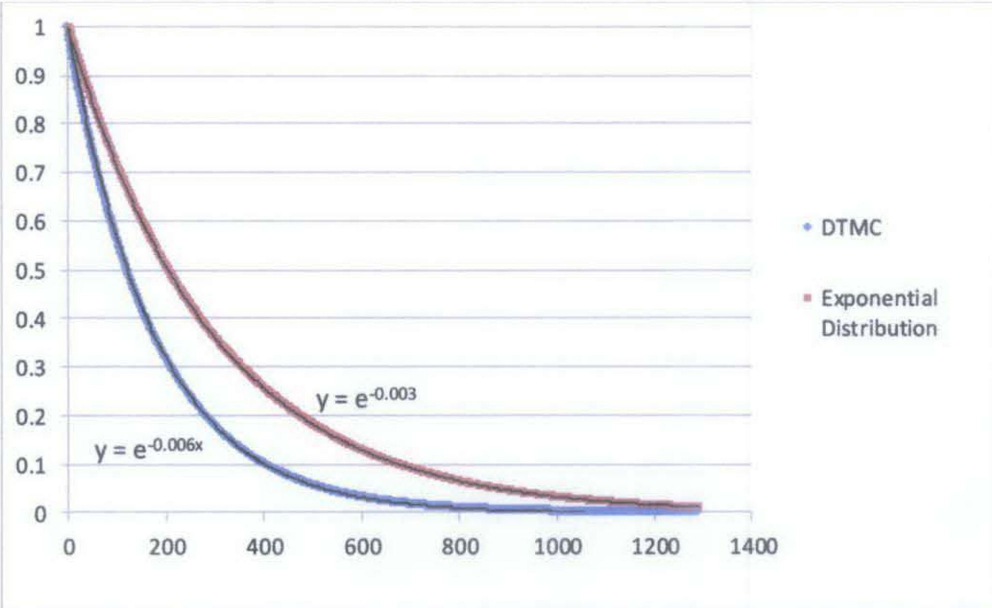


Figure 23 Reliability vs Time for Hour Resolution Data

4.4.2 Mean Time Between Failure (MTBF) Comparison

Another comparison is by calculating the MTBF of the results from both methods and compared against the actual data. From equation, $MTBF = 1/\lambda$, the comparisons are shown in Table 15 and Table 16.

Table 15 Comparison between DTMC and Exponential Distribution for Day Resolution Data

# Failure	Act	DTMC	Difference	EXP	Difference
1	8	31	23	50	42
2	27	63	36	100	73
3	72	94	22	150	78

Table 16 Comparison between DTMC and Exponential Distribution for Hour Resolution Data

# Failure	Actual	DTMC	Difference	EXP	Difference
1	80	167	87	333	253
2	190	333	143	667	477
3	386	500	114	1000	614
4	243	667	424	1333	1090
5	280	833	553	1667	1387

The differences are better viewed in Figure 24 and Figure 25 below.

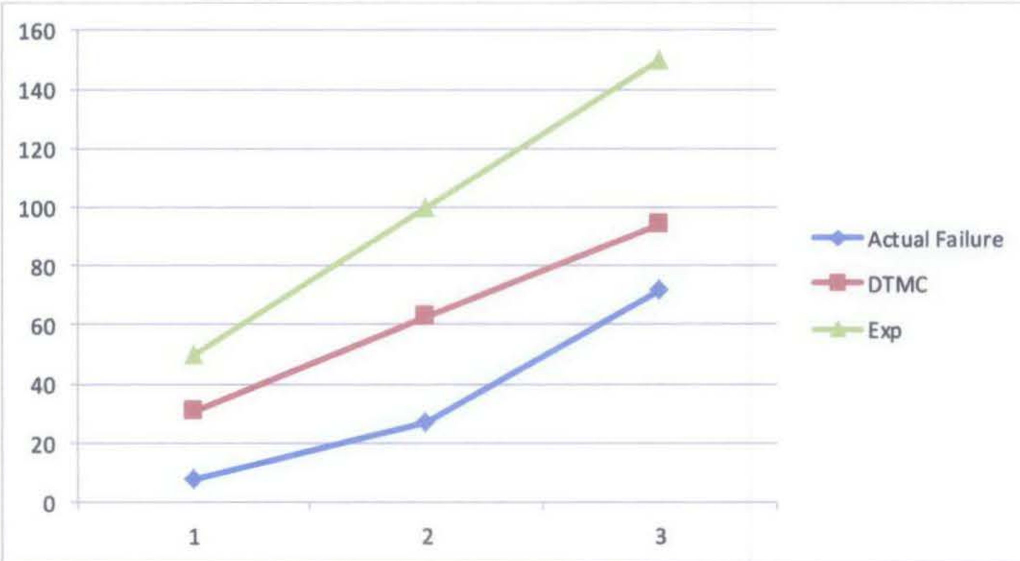


Figure 24 Comparison of Actual Failure Data to Predicted Failure Data For Hour Resolution Data

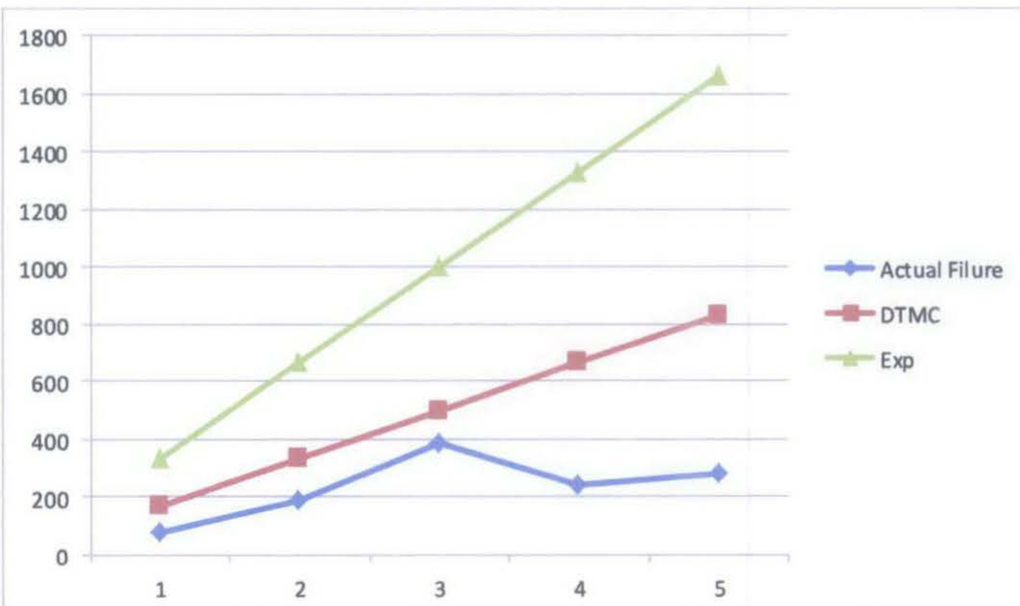


Figure 25 Comparison of Actual Failure Data to Predicted Failure Data For Day Resolution Data

For both set of resolutions in Figure 24 and Figure 25, MTBF predicted using DTMC have values that are closer to the actual failure time than the MTBF predicted using exponential distribution. From the above graphs, MTBF predicted using DTMC fit the actual failure data better than the exponential distribution.

4.5 DIFFERENT RESOLUTION COMPARISON

The two set of different resolutions are compared in graph below.

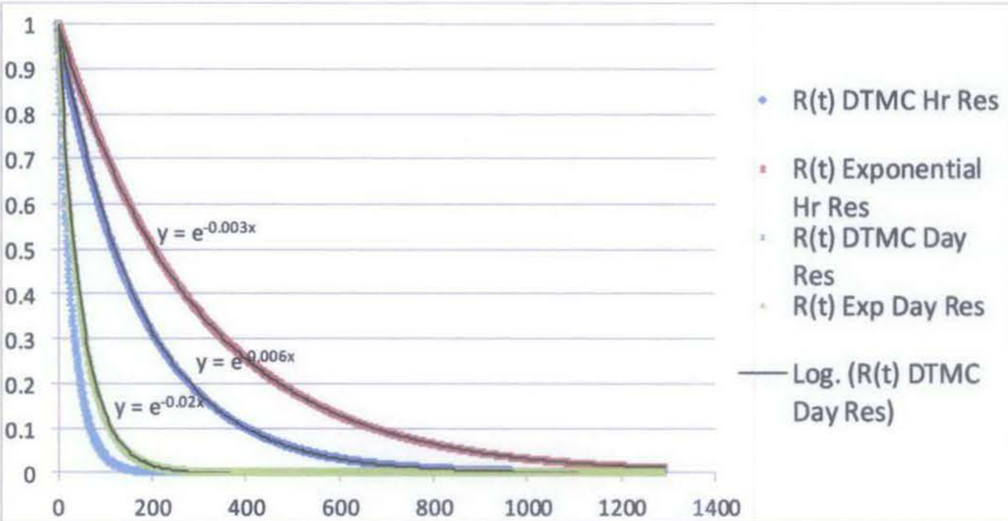


Figure 26 Reliability vs Time for Day and Hour Resolution Comparison

Based on Figure 26, both resolutions have bigger value of distribution parameter, λ for calculation using DTMC than using exponential distribution. However, calculation using day resolution has faster decrease than the calculation using hour resolution.

4.5.1 MTBF Comparison

For better analysis, the comparison was made based on the MTBF value for each resolution compared to the actual TTF data. The differences were converted into hour and day respectively for better comparison. The difference of MTBF value for day resolution is as shown in Table 17.

Table 17 MTBF difference to TFF for Day Resolution

# Failure	Actual	DTMC	Difference	Diff in Hrs
1	8	31	23	558
2	27	63	36	852
3	72	94	22	522

Meanwhile, the difference of MTBF comparison for hour resolution gives results as shown in Table 18.

Table 18 MTBF difference to TFF for Hour Resolution

# Failure	Actual	DTMC	Diff in Hrs	Diff in Days
1	80	167	87	4
2	190	333	143	6
3	386	500	114	5
4	243	667	424	18
5	280	833	553	23

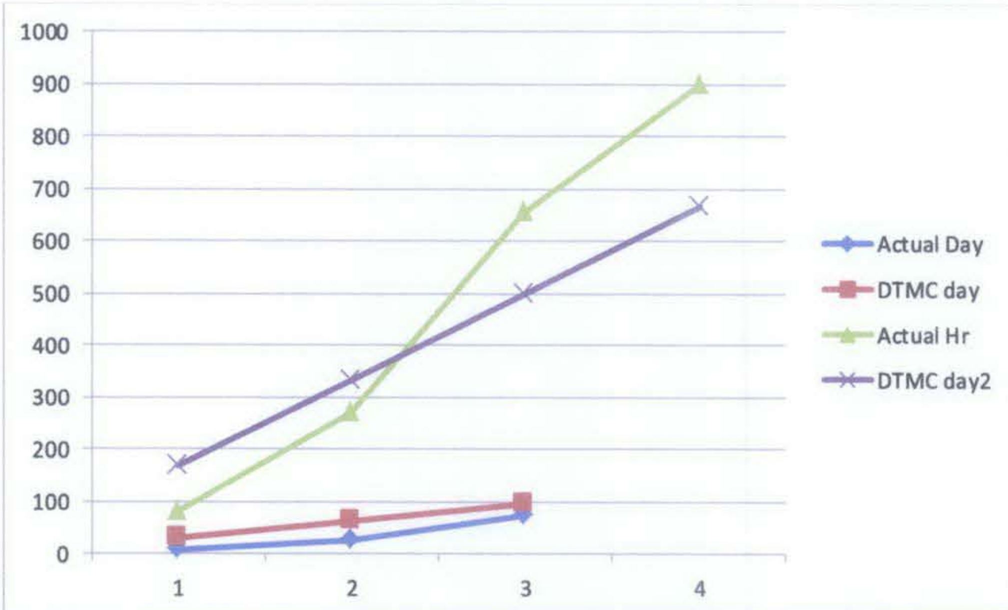


Figure 27 Comparison of DTMC and Actual Failure between Day and Hour Resolution

From Figure 27, by comparing the day and hour resolutions, difference between MTBF and the TTF for hour resolution is lower than the difference for day resolution data. Thus, it is safe to say that hour resolution has better result than the day resolution in predicting the reliability of MSS using DTMC.

4.6 ADVANTAGES OF RELIABILITY PREDICTION USING DTMC

The advantage of predicting reliability of MSS using MSS method is that the system can be partitioned into several states of performance where the working system will degrade from one system to another system before it completely fail. Plus, the failure state could be stated according to operating demand.

4.6.1 Predict Reliability of MSS with Variation of Demands.

In traditional method, output at value 0 only will be marked as failure state or complete failure. However, in MSS method, value of output that marks the unacceptable performance could be specified according to the operation demand. As the demand could be varied accordingly, the reliability of the MSS could still be estimated.

For instance, GTGA gas turbine for hour resolution analysis that has 7 performance states as show in Table 19 below is set to have operation demand at 2500kW. The output that falls under state 9 and 11 will be grouped under absorbing group that equal to state 0 (failure state).

Table 19 GTGA Gas Turbine State Performance for Hour Resolution

State	Minimum Value
0	0
11	881
9	2285
7	2832
5	3233
3	3484
1	3711

By applying DTMC, the reliability of GTGA gas turbine with new operating demand could be calculated.

The data was partitioned into 5 performance states and is shown in Table 20 below.

Table 20 New State Performance of GTGA Gas Turbine

State	Minimum Value
0	0
7	2500
5	3233
3	3484
1	3711

And the state probability transition matrix become

$$P = \begin{vmatrix} 0.008333 & 0.820833 & 0.120833 & 0.025 & 0.025 \\ 0 & 0.090909 & 0.72973 & 0.120393 & 0.058968 \\ 0.03022 & 0.005495 & 0.173077 & 0.714286 & 0.076923 \\ 0.04918 & 0.016393 & 0.076503 & 0.229508 & 0.628415 \end{vmatrix}$$

The reliability of GTGA gas turbines at 2500kW demand was calculated and graph Reliability vs Time was plotted.

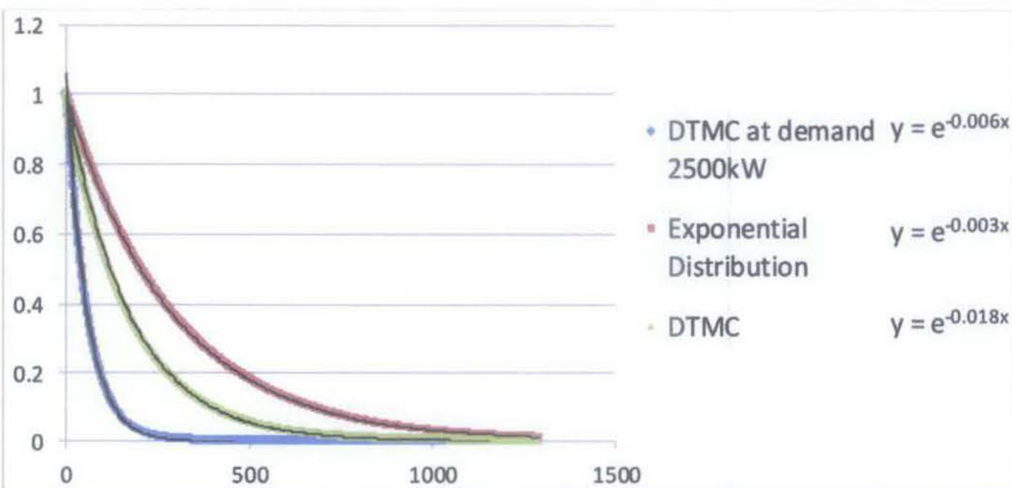


Figure 28 Reliability vs Time Graph of GTGA with New Demand

Figure 28 shows the reliability curve for new demand decreases faster than the reliability of the MSS without demand. The parameter distribution, λ is bigger than the parameter distribution for reliability analysis without demand. Therefore, DTMC analysis could be used to find MSS reliability with various operating demands.

4.6.2 MSS Method Enable Preventive Maintenance (PM) Planning

The ability of MSS method to access reliability of a working system that has multi state of performance enables Preventive Maintenance Planning to be drafted. MSS method includes calculation of state transition probabilities in accessing reliability of a MSS. To perform PM planning, the state transition could be altered conferring to the type of PM which is minimal PM or perfect PM.

4.6.2.1 Reliability Prediction for Minimal PM

When a system degrades into certain state, minimal PM is done to improve the performance of the system up until the next state of performance state.

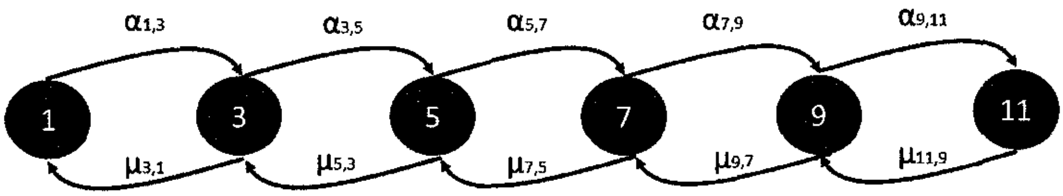


Figure 29 State Degradation of a 7 states MSS with minimal PM

Figure 29 shows state degradation of a MSS with α indicates the degradation rate from one state to another and μ indicates the repair rate. Note that the state moves from one state, k up to another state, $k+1$ since minimal PM is applied.

In the state transition probabilities estimation, the repair rate from one state to another state more than next state is considered equal to 0. Thus, the state transition probability matrix is

	1	0	0	0	0	0	0
	0.008333	0.820833	0.120833	0.025	0.025	0	0
	0	0.090909	0.72973	0.120393	0.054054	0.004914	0
P =	0.005525	0	0.174033	0.718232	0.077348	0.008287	0.016575
	0.006757	0	0	0.256757	0.655405	0.047297	0.033784
	0.05	0	0	0	0.3	0.65	0
	0.333333	0	0	0	0	0.666667	0

4.6.2.2 Reliability Prediction for Perfect PM

When perfect PM was performed, the performance of the system will return to the perfect working state which is state 1 as illustrated in Figure 30 below.

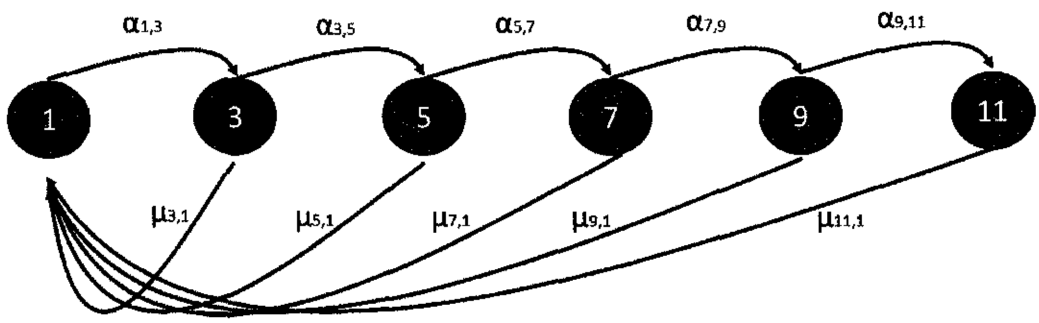


Figure 30 State Degradation of a 7 States MSS with Perfect PM

As only the repair rate, μ for all degraded state to state 1 are considered, the state transition probability matrix is written as

$$P = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.008333 & 0.820833 & 0.120833 & 0.025 & 0.025 & 0 & 0 \\ 0 & 0.090909 & 0.72973 & 0.120393 & 0.054054 & 0.004914 & 0 \\ 0.006645 & 0.006645 & 0 & 0.863787 & 0.093023 & 0.009967 & 0.019934 \\ 0.008929 & 0.017857 & 0 & 0 & 0.866071 & 0.0625 & 0.044643 \\ 0.066667 & 0.066667 & 0 & 0 & 0 & 0.866667 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Figure 31 shows the comparison of Reliability vs Time plot for Minimal PM and Perfect PM. Reliability plot for system that applied with perfect PM has faster decrease and bigger value of parameter distribution, λ. List of parameter distribution is as shown in Table 21.

Table 21 Parameter Distribution Comparison

Method	Distribution Parameter, λ
DTMC with Minimal+Perfect PM	0.006
DTMC with Minimal PM	0.032
DTMC with Perfect PM	0.011
Exponential Distribution	0.003

Using the parameter distribution, λ, from Table 21, reliability function, *R(t)* and MTBF of the MSS could be calculated.

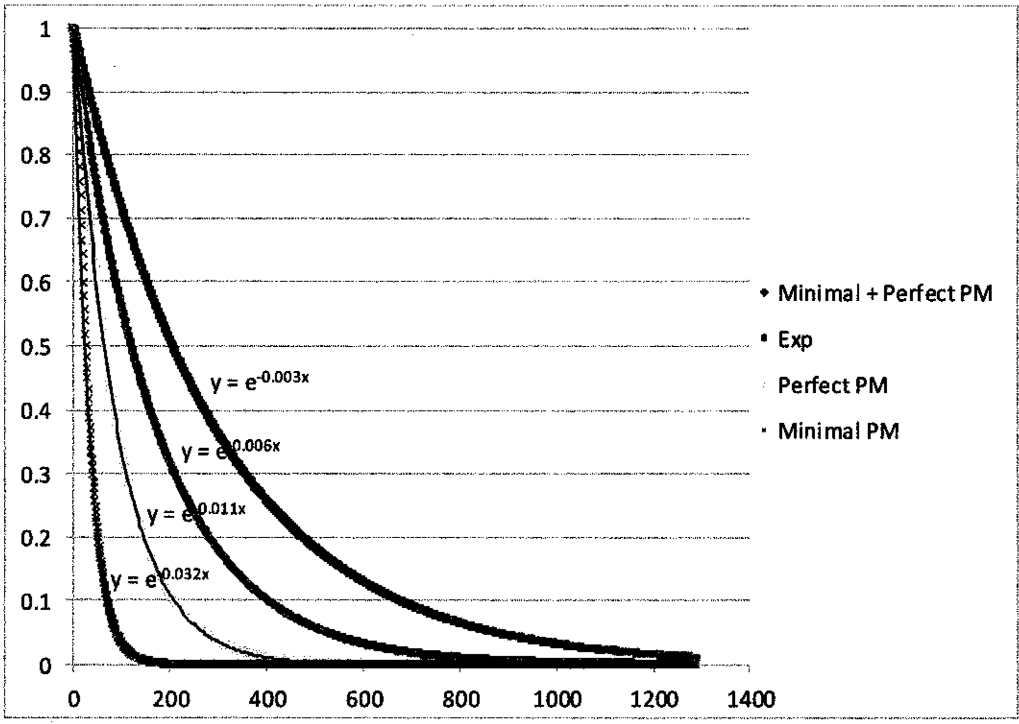


Figure 31 Reliability vs Plot Comparison Graph for Minimal and Perfect PM

Based on Figure 31, it was proven that DTMC Analysis could predict the reliability of MSS. Moreover, the reliability of the system could also be predicted if the system was running at certain operating demand. DTMC Analysis also allowed PM Planning to be drafted since the reliability of the system with PM planning also could be predicted using DTMC Analysis.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

Reliability of a MSS which is UTP GTGA gas turbine was calculated using DTMC approach and was compared to the reliability prediction using traditional method. From the data analysis, the calculated MTBF for MSS analysis gave results that were closer prediction to the actual TTF data than the traditional method. From the analysis, it could be concluded that hour resolution gave better prediction than the day resolution data since the prediction resulted from hour resolution data was closer to the actual TTF.

Since the MSS method included estimation of state transition probabilities, the performance data was partitioned into several performance states. By exploiting the performance states separation, reliability prediction of a MSS could be done for different operation demand by changing the minimum value of a particular performance states. Thus, the system behavior could be estimated. Other than that, PM planning could be drafted for the system once its behavior was learned. By applying DTMC method, reliability of a MSS could be predicted when minimal PM or perfect PM was included in the PM planning.

5.2 RECOMMENDATION

The application of Markov Chain in MSS method can be viewed from many directions and from lots of angle. Further studies are needed to explore the advantages of using MSS method in reliability prediction. Life data that used in this project analysis could be longer than 8 months to get better result and prediction. Therefore, more time and commitment is needed prior to that target.

From the results, it is proven that, when the resolution of data is increased, the closeness to the actual failure is increase. The resolution can be enlarged more and more and the analysis will be instead of DTMC, will be a CTMC analysis. CTMC analysis is believed to predict better prediction but is more complicated to be analyzed in short period of time.

A template or software could be built based on the finding from the analysis in order to predict the MSS reliability in the future. Last but not least, the DTMC analysis used in this project should be tested to other MSS life data i.e. KLIA GDC turbine system.

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