ESTABLISHING CORRELATION BETWEEN LEAF SPRING’s SPECIFICATIONS AND CAMBER DROP

by

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Dissertation submitted in partial fulfillment of
the requirements for the
Bachelor of Engineering (Hons)
(Mechanical Engineering)

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the

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BACHELOR OF ENGINEERING (Hons)

(MECHANICAL ENGINEERING)

Approved

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May 2013

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

(MOHAMAD ROZAIDI BIN MOHAMAD ALI)

ABSTRACT

Camber drop is the study of the differences between the actual camber and the desired camber (master camber). Camber is one of the importing specifications that need to be followed by manufacturer to avoid rejection of the products by the customers. However, the manufacturer is said to be having difficulty in producing leaf spring with a camber matches the desired specifications in a first attempt, which can cause them to the losses in term of time and money. The importance of this study is to find a correlation between the leaf spring specifications and camber drop, and to establish an equation that can be used to estimate the amount of camber drop. This project is to be conducted by collecting the data required, which are the Quenching Camber, Half-length of the spring, Spring Rate, and End-thickness of the spring. All the data collected then will be converted into a graphical presentation based on each specs to start the finding of the relation and also to govern the equation of estimation of the camber drop. It is expected to obtain the correlation between the leaf spring specifications and the camber drop, and also to be able governing the equation of estimation of the camber drop.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1: INTRODUCTION

1.1 SUSPENSION

Suspension is a type of system that present on every type of vehicles in the world, which has been invented by William Brush in 1904. Basically, suspension is the term refers to shock absorbers, springs, and linkage which connect the vehicle body to the wheel in order to allow relatives movement for both. Suspension refers to important system for a vehicle as the purposes for suspension are to contribute to the vehicles road holding/handling, and braking for good lively safety and driving satisfaction, and to keep the passengers of the vehicle comfortable and reasonably well isolated from road noise and vibration due to bumps and rough surface of road. Based on these functions, the suspension can ensure the safety of the conditions of the vehicle and also any cargo that has been load onto the vehicle. Besides, it can ensure the passenger from having an injury when over through a rough surface road or accidentally run onto a bump without slowing the speed of the vehicles. For the suspension to work on the desired output, some properties of the suspensions' material should be considered, for example the spring rate and weight transfer.

1.2 LEAF SPRING

1.2.2 TYPE OF LEAF SPRING

Leaf spring consists of various types. The variation of the type is depends on the usage of the spring. These various types consist of Multileaf Spring, and Parabolic Spring. The evolution of those two types of spring somehow was influence by time. Increasing competition and innovations in automobile sector tends to modify the existing products or replacing old products by new and advanced material products. More efforts are taken in order to increase the comfort of user [2]. And nowadays, a research on applying a sensor-probe with stable reflectivity and high density has been done [6].

i. **MULTILEAF SPRING**

Multileaf spring does have a multiple number of leaves that has been combined together. The thickness of each leaves is constant right from the centre of the leaves until the end of the leaves. The leaves fits each other closely when there is no load applied [7]. The advantage of this type of leaf spring is that it can withstand a heavy load. This spring can control the oscillation very well as the inter-surface friction is higher due to the constant thickness of each leaves. However, having constant thickness of the leaves will cause a noise from the springs and also will reduce the life-time of the spring.

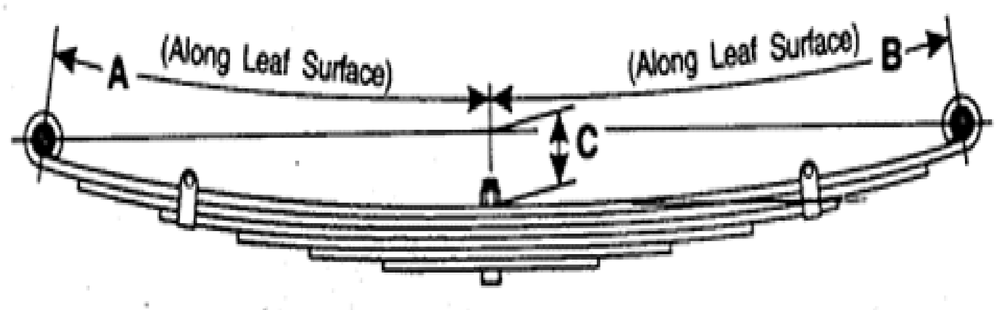


Figure 1: Sample of Multileaf Spring (C is notation for camber)

ii. **PARABOLIC SPRING**

Parabolic spring is the evolution of multileaf spring, which is the modest among the types of leaf spring. For parabolic spring, the inter-surface friction is unwanted, therefore the thickness of each leaves is vary from the center of the leaves to the end of the leaves. This result with only contact of each leaves is at the point of center of the leaves and at the end of the leaves. A spacer is used in order to reduce the contact point of each leaves. The main advantages of parabolic leaf spring are that they are lighter, cheaper, better fatigue life, and they isolate noise [2]. However, in sequence, the load that can be applied on the parabolic is reduced due to the inconstant of the thickness.

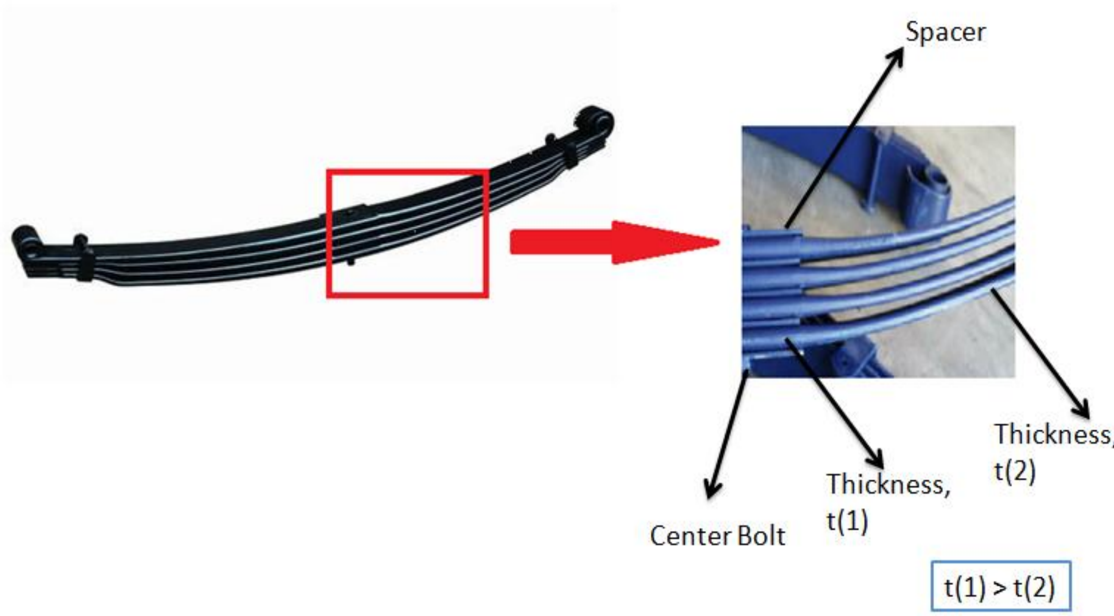


Figure 2: Sample of Parabolic Spring

Leaf spring basically use different types of materials, depends on the standard of a company. For example, Case-A company has used the material of JIS G 4801 SUP 9A. Different types of material used will have a different result on the properties of the spring. Glass fibre reinforced composite is one of the material used for leaf spring production. According to J.P. Hou [8], a 60% reduction in suspension weight can be obtained by replacing a steel spring with a composite spring of the same function. The fatigue life of composite springs is about five times that of steel springs and they have excellent corrosion resistance. These entire advantages make the glass fiber reinforced composite leaf springs for transportation an excellent substitution for steel springs. However, the application of composite materials is still limited by the design of the shackle which connects the leaf spring to the wagon.

1.3 MANUFACTURING PROCESS OF LEAF SPRING

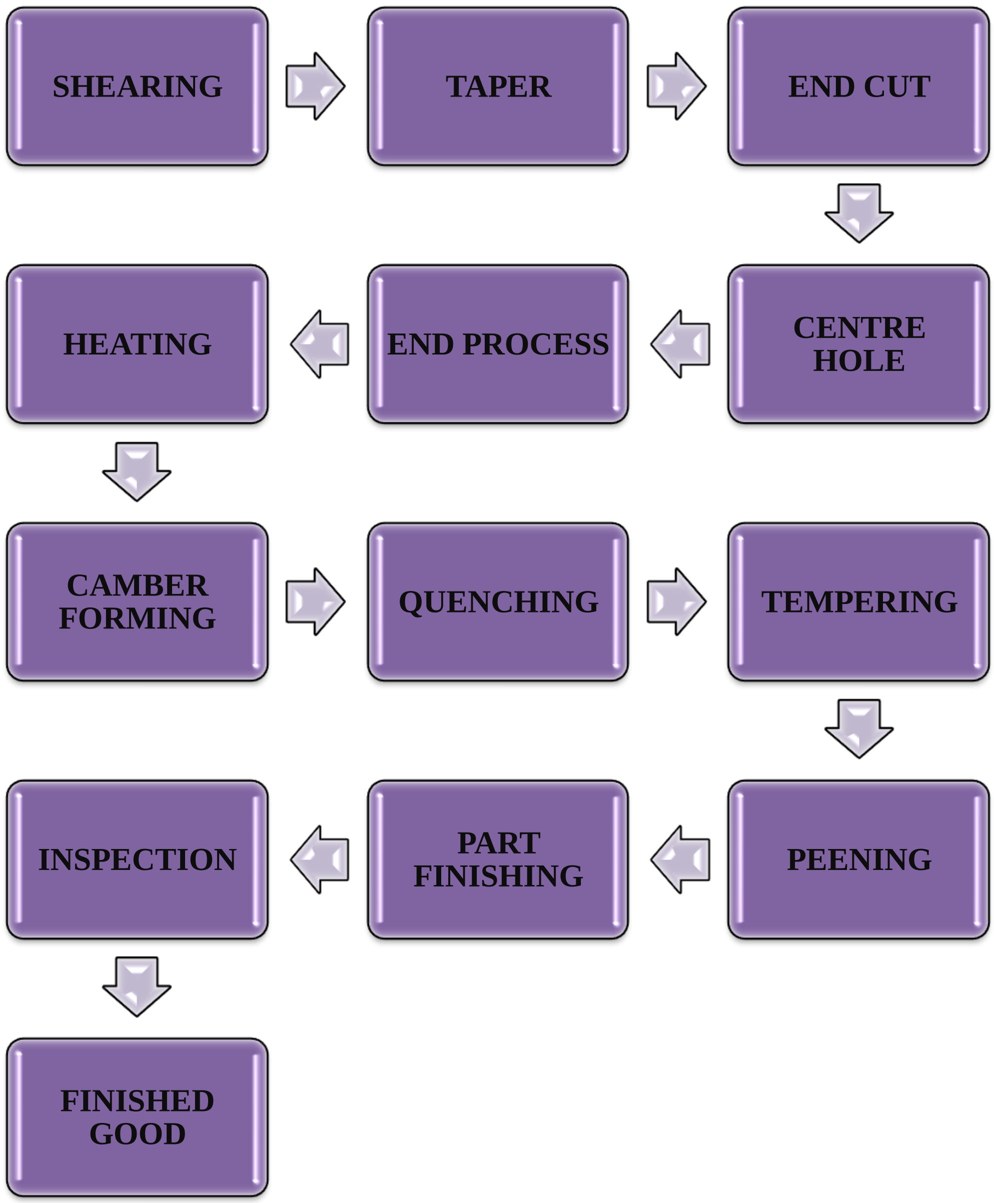


Figure 3: Process Flow of Leaf Spring Manufacturing

The manufacturing process of leaf spring is started with the shearing process. Raw material is being cut according to desired length, using a shear machine. For parabolic spring, it will undergo the process of tapering. This is to vary the thickness of the leaves. The excess of the leaves after tapering process will be cut off during the process of end cutting. Next, a center hole will be made on the leaves by using center hole punching machine. End process of the leaves covers a finishing process of the end of the leaves before undergo the camber forming process. This process covers the making of eye formation which using Eye Rolling machine (hand operated) [5], diamond cut, or end trim process. The camber formation process is applying the concepts of heat treatment of a metal. Annealing process is the type of metallurgy work that has been applied. The heat treatment process covers the process of heating, quenching, and tempering. To grant these elastic properties to the springs, necessary due to the high loads to which theses components are submitted during service, the manufacturing process must guarantee high yield strength and tensile strength through quenching and tempering [1]. After completing heat treatment process, the leaves will undergo the process of peening, in order to improve the surface structure of the leaves. Later, it will be touch up by the process of part finishing before it being inspected. If the leaves are good, then it will be ready for shipment out.

1.4 PROBLEM STATEMENT

Based on the process flow of leaf spring's manufacturing, the problem existed during the process of camber forming. The camber forming process is the process of bending the spring into a curvature shape by referring to its camber dimension. During this process, the spring is to be heated up to 950°C inside a furnace, in order to make the deformation of the spring to be much more easier by using the concept of metallurgy work, annealing. To perform the camber forming process, an NC Camber Forming machine is being used. The machine consists of upper die (moveable die) and lower die (static die), where the upper die will press the spring against the lower die and forced the spring to follow the shape that has been designed. NC stands for numerical control, as the machine is using the automatically controlled by the computer. Thus, this machine does not use the human power to setup the die; instead the engineer only need to copy the design of the spring into software and then the software then will send the design to the machine for itself auto setup.

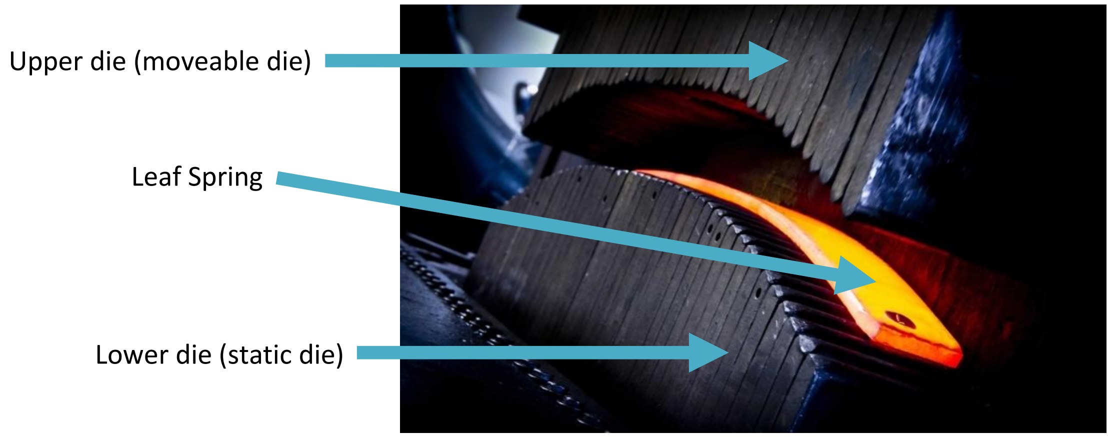
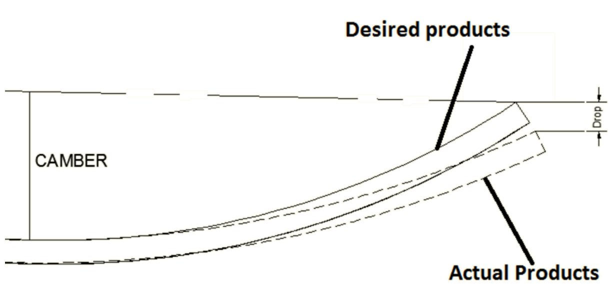


Figure 4: A view of Finger-Press Camber Forming Machine

However, the pressed spring will not completely follow the shape that has been design. Due to the properties of the material, the spring will rebound back at a certain perimeter as the spring will tend to remain its original shape. In term of state of matter, only liquid can completely follow the shape of its container/die, differently with solid state. In term of spring manufacturing, the spring is pressed according the designated height of camber. As the spring is rebound back, the camber of the spring will significantly drop. This phenomenon is referred to the term of springback, which is the tendency of the metal to remain at its original geometry.

The picture aside is showing of how the camber drop is being determined. The desired product is the designed product, while the actual product is the spring after going through the process of camber forming. Camber drop is the difference in height of both camber of



desired products and the actual products. The presence of camber drop has somehow giving a trouble for the engineer to accurately designing the spring. Due to this problem, the engineers need to use the methods of try and error in order to come out with the finalized design of the product, which can cost them in term of time and money.

Therefore, the problem that needs to be overcome by performing this project is the springback of the material when undergone the process of camber forming.

1.5 OBJECTIVE

In this project, specifications of the product have been taken into account as one of the way to overcome the problem that has been countered during the camber forming process. Those specs are Quenching Camber, Half length of the spring, Spring Rate, and the End Thickness of the leaf spring.

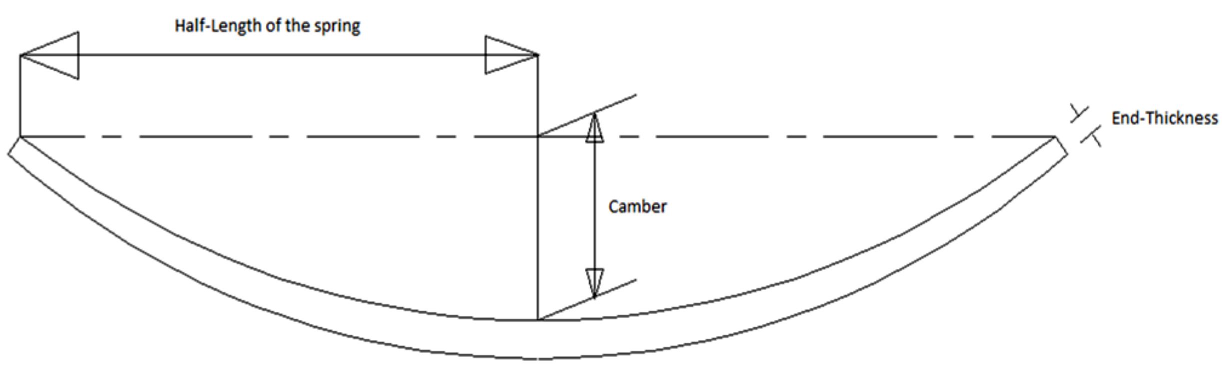


Figure 5: Specifications of Leaf Spring's Design

Figure above shows the definition of each specification that has been mentioned earlier. Camber in the picture also can be replaced by Quenching Camber, where Quenching

Camber is the value of the height of the camber in designing process which means the camber's value after the Leaf Spring has undergone the final process of heat treatment, which is quenching process, in other words Quenching Camber also can be defined as the camber of the desired products. Thus, one of the objectives of this project is to correlate between the leaf spring specifications and camber drop.

The other important objective of this project is to obtain a best fit model of 60% by performing the regression analysis.

CHAPTER 2: LITERATURE REVIEW AND THEORY

2.1 HEAT TREATMENT PROCESS

2.1.1 ANNEALING

Annealing is the process of heating up a material to a temperature slightly above the eutectoid temperature of the material, basically 50°C and then it will be maintain at the temperature for a period of before it being cooled to a room temperature. This process is in purpose to soften the material by increasing the ductility of the material, relieves the stress inside the material, and reduces the hardness in order to make it easier to deform or reshaping to a desired shape. This process has much helped Case-A company in producing their main products, which is the leaf spring by softens the material before forming the desired camber.

Full annealing is one of the specialized cycles of annealing. This cycle will form the microstructure inside the material to fully austenite or austenite-cementite grain structure. By fully obtaining the austenite grain structures, it will lead to a fine product of spring. This will be explaining completely in the next theory, which is martensite and tempered martensite.

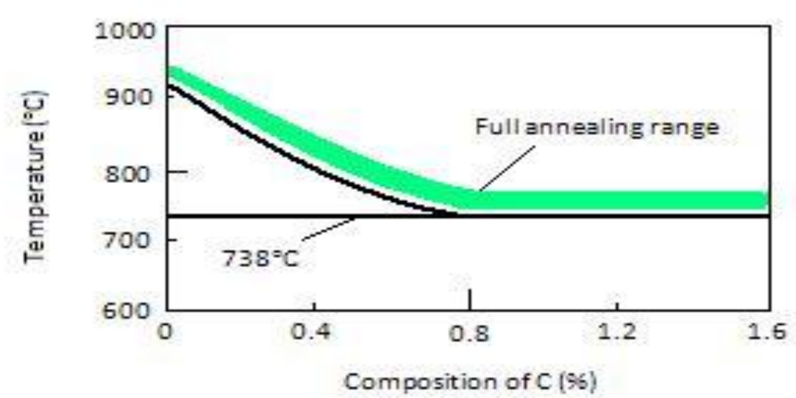


Figure 6: Full annealing range

2.1.2 MARTENSITE AND TEMPERED MARTENSITE

Martensite is a type of microstructure produced when a material is being heat up to form completely austenite microstructure and being rapidly cooled to a relatively lower temperature. This microstructure is very hard and very strongest, yet very brittle and the ductility of the product can be negligible. This is due to the effectiveness of the of the interstitial carbon atoms in hindering dislocation motion and to the relatively few slip systems of the BCT structure.

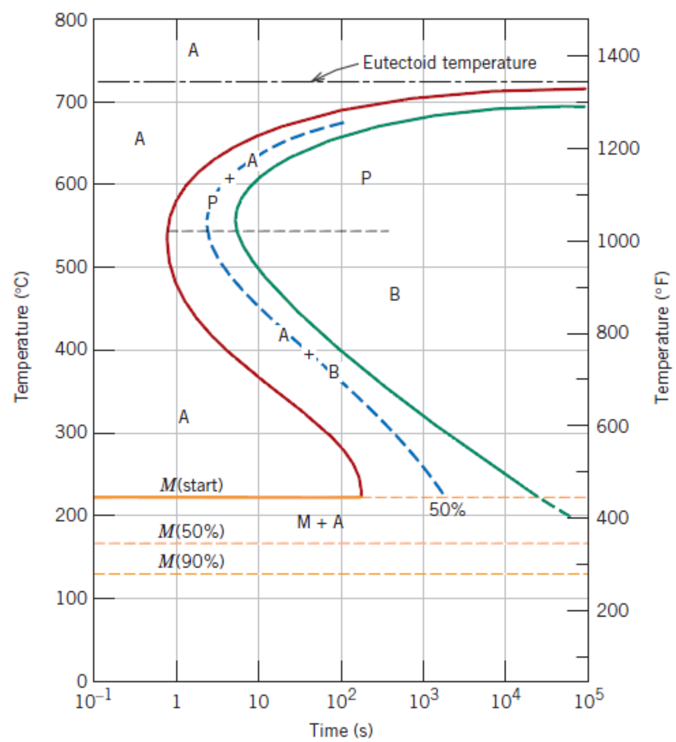


Figure 7: Continues Cooling Transformation Diagram

In diagram above, shows the types of microstructures that can be formed by applying the heat treatment process on a material. Letter ‘A’ represent Austenite, ‘B’ represents Bainite, and ‘P’ represents Pearlite, while ‘M’ represents Martensite. For APM, they prefer to achieve 90% martensite without the presence of any other microstructure excepts austenite. To obtain that kind of structure, the material needs to be rapidly cooled to a temperature below than approximately 125°C by applying the quenching method. In APM, they are using oil as the quenching base.

Martensite is extremely hard and brittle. It is must be tempered to mitigate internal stresses to enhance in toughness with some decrease in final hardness. If the martensite is heated to a temperature where the carbon atoms have mobility, the carbon atoms diffuse from the octahedral sit to form carbides, as a result the tetragonality is relived, and martensite is replaced by a mixture of (α + C) [3]. A spring must not be too brittle, otherwise it can be easily broken due to the bounce of the load that being applied to the spring. To overcome this situation, tempering is being introduced into the process flow. Tempering is the process of heating the material to a lower critical temperature which is normally ranging from 350°C to 400°C and then being keeping to that temperature for a while, lastly will be cooled to a desired temperature. This process is to form the microstructure of tempered martensite. The mechanical properties of tempered martensite are it still strongest and hardest, but the material has been induced to increase the ductility and toughness. This is the perfect properties that is needed in the production of the springs where it can withstand the bouncing effects from the load applied on the spring thus will increase the quality of the spring produced. Below is the summary of producing the suitable material for a spring.

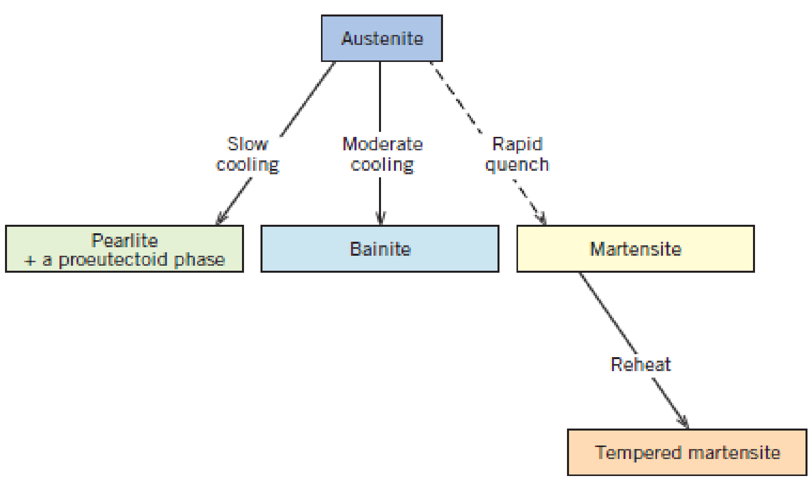


Figure 8: Flow of producing Tempered Martensite

2.2 SPRINGBACK

Springback is a type of phenomenon that occurs when a solid is being deformed from its original shape. This phenomenon occurs due to the mechanical properties of a material, which one most noticeable is the stress that presence inside the material. According to S.K. Panthi, the springback is one of the effects of the considerable elastic recovery during unloading [1]. As accuracy in dimensions is always remains a major concern in sheet metal bending process, the presence of springback has become a major problem in achieving a product with an accurate dimension.

In the image, it has 4 segment which showing the condition of the sheet metal when it is 45% of total punch displacement, 75% of the total punch displacement, 100% of the total punch displacement, and sheet after the release of the forming load. From 45% until 100% of the total punch displacement, the sheet metal looks to follow the shape both of the punch die, and it is completely deformed as a new shape that has been designed. However, when the forming load being removes, we can see radius of the curvature of the sheet metal is slightly bigger when compared to the radius of curvature of the punch die. Because of springback, we cannot get the product which is exactly as what we are desired of.

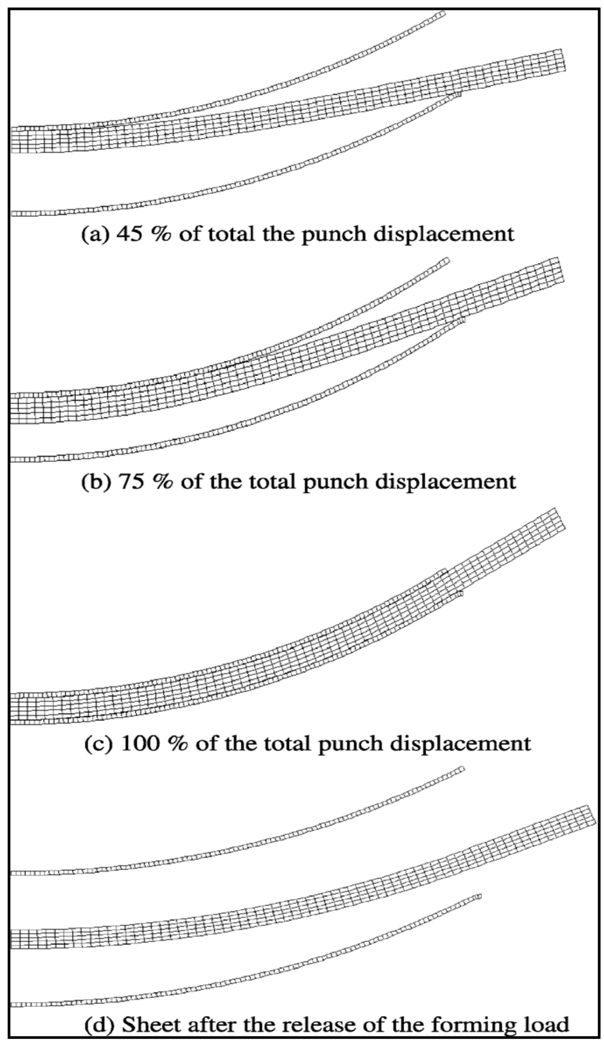


Figure 9: The Geometry of the Sheet Metal by Percent of Forming Load

2.2.1 RESEARCH RELATED ON SPRINGBACK

Author	Journals	Description
Gisario A., et. al. [4]	Springback control in sheet metal bending by laser-assisted bending: Experimental analysis, empirical and neural network modelling	Implementing laser treatment on the tension surface of the sheet metal during bending formation process taken place to reduced value of springback
Panthi S.K., et. al. [1]	Finite Element Analysis of sheet metal bending process to predict the springback	Applying the Finite Element Analysis software in order to predict the springback, and to investigate the influence of parameters on springback.
Fei F., et. al. [9]	Springback prediction for incremental sheet forming based on FEM-PSONN technology	➢ A 3D elasto-plastic FEM was developed to simulate the process and to simulated results (to compare with the experiment) ➢ Artificial Neural Networks (ANN) and finite element technique was developed to predict the spring backs. ➢ A particle swarm optimization (PSO) algorithm was used to optimize the weights and thresholds of the neural network model.

Table 1: Research Related on Springback

2.3 REGRESSION ANALYSIS

Regression Analysis is a type of statistical tools which has been used in order to investigate or to correlate of relationships between variables. To explain specifically, the regression analysis is a medium that helps people to understand on how the characteristic value of the dependant variable changes when some of the variables are fixed while the rest is being varied. Basically, regression analysis is commonly used with objective to predicting and forecasting. According to Skyes O.A [15], usually, the investigator seeks to ascertain the causal effect of one variable upon another—the effect of a price increase upon demand, for example, or the effect of changes in the money supply upon the inflation rate. To explore such issues, the investigator assembles data on the underlying variables of interest and employs regression to estimate the quantitative effect of the causal variables upon the variable that they influence. The investigator also typically assesses the “statistical significance” of the estimated relationships, that is, the degree of confidence that the true relationship is close to the estimated relationship.

There are two types of regression analysis, which are Simple Regression and Multiple Regression. Basically, these types of regression are being classified based on the number of independent variables which has been included in the analysis. Simple Regression consist only one independent variable being tested in the analysis. On the other hand, Multiple Regression consists of more than two variables being tested in the analysis. Thus, the hypothesized relationship for the regression analysis would be [15]:

i. For Simple Regression: $y = \beta_0 + \beta_1 X_1 + \varepsilon$

ii. For Multiple Regression: $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n + \varepsilon$

For the value of ε, it is refers to the noise terms reflecting the other terms that influence the dependent variable, and the value of β is refers to the slop of the regression line.

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2.3.1 REGRESSION ANALYSIS INTERPRETATION FROM MICROSOFT EXCEL

Basically, the regression analysis is going to be performed by using the software of Microsoft Excel. Below show the output that will be obtained from the analysis.

SUMMARY OUTPUT

Regression Statistics		
Multiple R		(1)
R Square		(2)
Adjusted R Square		(3)
Standard Error		(4)
Observations		(5)

ANOVA	(6)	(7)	(8)	(9)	(10)
	df	*SS*	*MS*	*F*	*Significance F*
Regression					
Residual					
Total					

	(11)	*(12)*	*(13)*	*(14)*	*(15)*	*(16)*
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept						
X Variable 1						
X Variable 2						
X Variable 3						
X Variable 4						
X Variable 5						

Table 2: Summary Output from the Microsoft Excel

No	Items	Description
1	*Multiple R*	The square root of R square.
2	*R Square*	Proportion of the variation in y which is explained by variation in x.
3	*Adjusted R Square*	Modification of R square that adjusts for the number of explanatory terms in a mode.
4	*Standard Error*	Variance of the actual y value with predicted y value.
5	*Observations*	The total number of observation in the analysis.
6	*df*	Degree of freedom of the regression analysis.
7	*SS*	Sum of Square of the regression analysis.
8	*MS*	Mean Square (the ratio of SS to the df).
9	*F*	Test Statistics (the ratio of MS Regression to the MS Residual).
10	*Significance F*	Test Statistic associated with P-value.
11	*Coefficients*	The slope of the regression line of each plot.
12	*Standard Error*	Standard Error of the coefficient.
13	*t Stat*	Ratio of Coefficient and Standard error of the coefficient.
14	*P-value*	Probability value of the hypothesis test (ß=0 or ß≠0).
15	*Lower 95%*	Limits of a confidence interval for the slope of the regression line
16	*Upper 95%*	Limits of a confidence interval for the slope of the regression line

Table 3: Abbreviation of the Summary Output from Microsoft Excel.

CHAPTER 3: METHODOLOGY

3.1 PROJECT'S PROCESS FLOW `

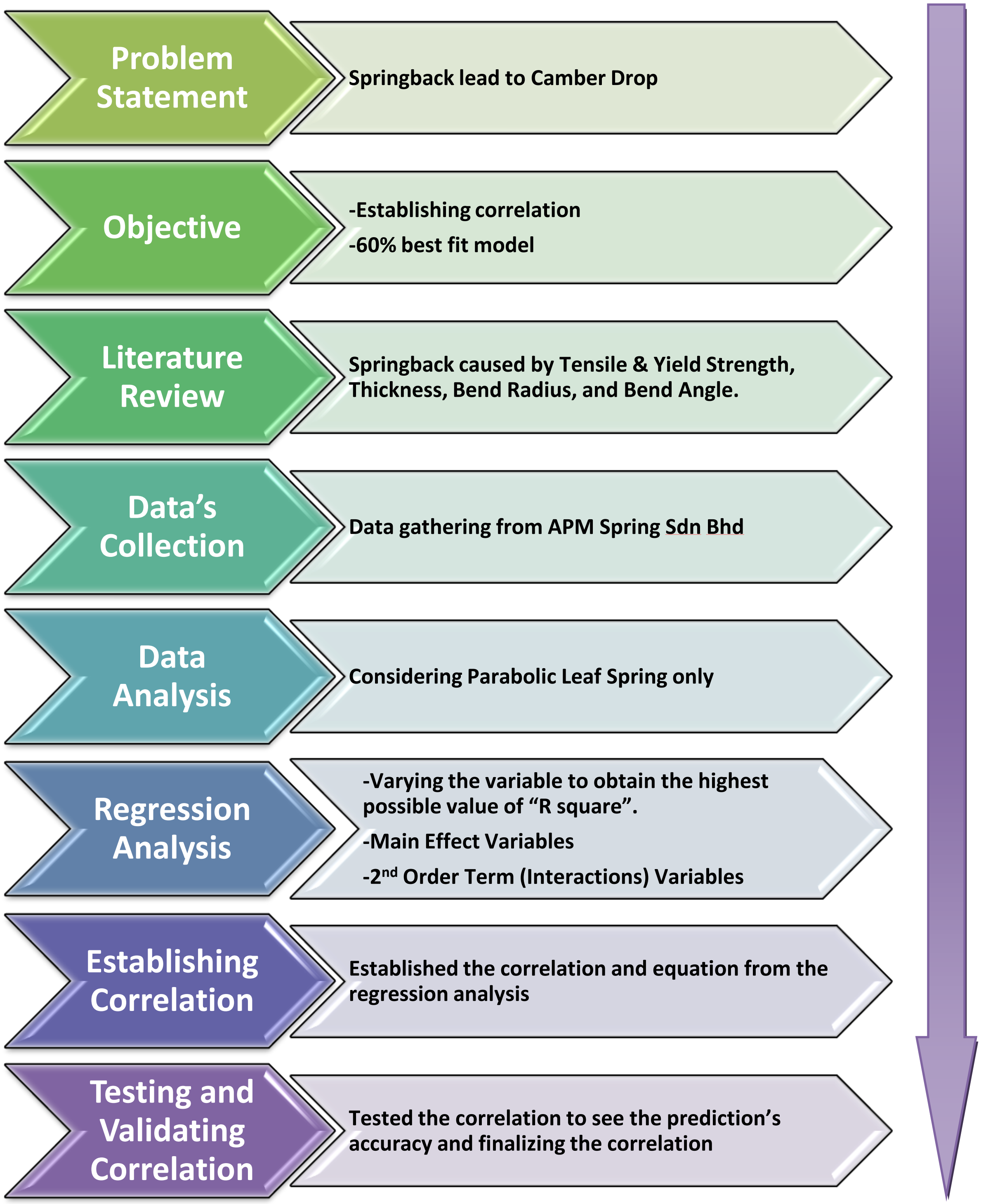


Figure 10: Projects Process Flow

3.2 SETTING VARIABLES OF REGRESSION ANALYSIS

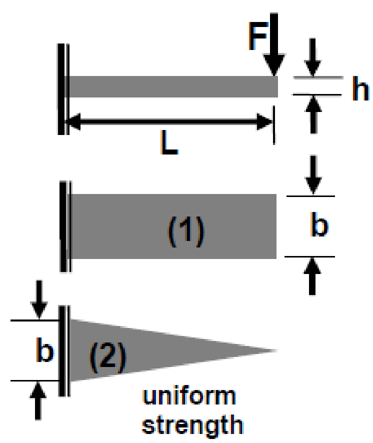
Variables are the main important factor in contributing a best regression analysis model. It is impossible to generate a regression analysis without having a single variable. This is because to estimate an output, there must be an indicator for the estimation to be made. So in this case, having a variable is a must.

For the analysis, the variables are transformed into the logarithmic function, with each of the log variables is in the form of polynomial, which is raised to the power of two (2). This condition is decided after considering the basic formula in designing the Leaf Spring:

$$\sigma_{max} = \frac{6FL}{bh^2}$$

$$\delta_{max} = \frac{4FL^3}{Ebh^3}$$

Where:



And E = Young's Modulus of the material

As the equation in designing the basic Leaf Spring is in multiplication form, thus it is more possible to come up a model which is much more accurate by transform the equation from normal function into Logarithmic function. For the pattern of the graph plotted using the raw data of camber drop against the variables, it is observed that the line is in curved form which is a polynomial form, thus the possible relation to the function is by raising the equation to higher order, which is 2^{nd} order terms.

The best fit of the regression model is determined by the number of variables which is used in the regression analysis. The higher the number of variables, the model of regression is said to be best fit. However, if having too much of variables' number, it will make the model too complicated. Example of graph below explained the reason of limiting the number of variables.

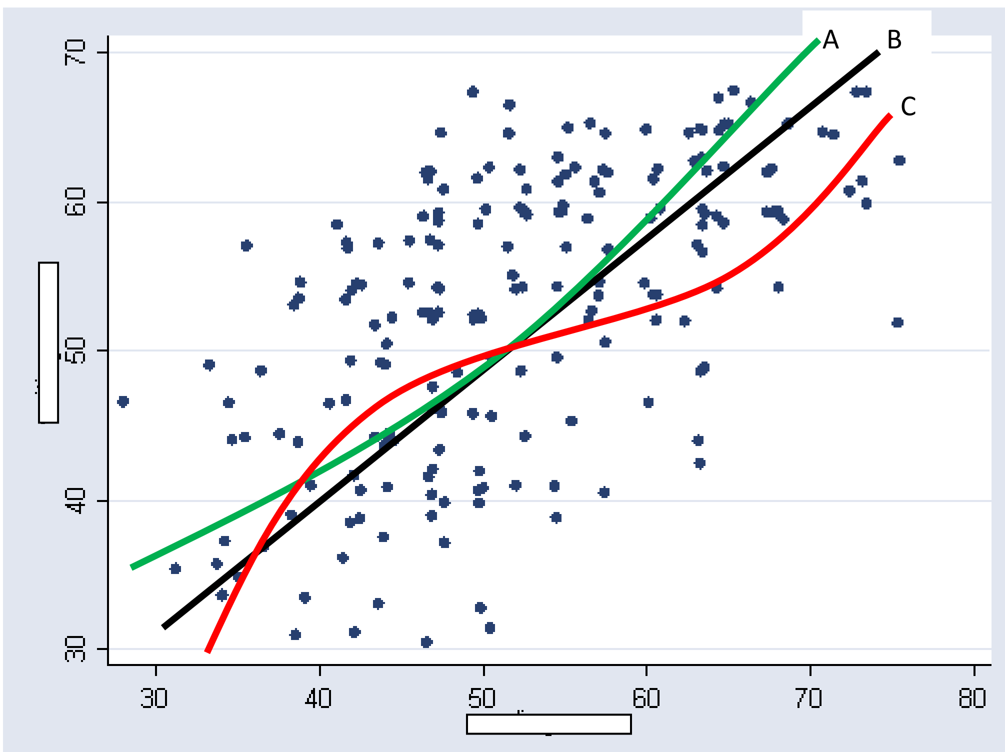


Figure 11: Example of scatter plot

Considering the example of graph plotted above, it is showing a graph which having 3 different best fit line. Best fit A is the variables in 2^{nd} Order Terms, best fit B is in 1^{st} Order Terms, and best fir C is in 3^{rd} Order Terms. With different number of variables being considered inside the model, the best fit line is different, but still it is in the same region of the data. Therefore, to have such higher order terms will only making the model to be complicated. Thus, to avoid this situation, the higher orders of variables in this project are only to be considered in 2nd Order Terms.

In this project, the variables are being set up in two groups, which is Main Effects or the 1st Order Terms, and 2nd Order Terms (Interactions). The lists of variables are shown below:

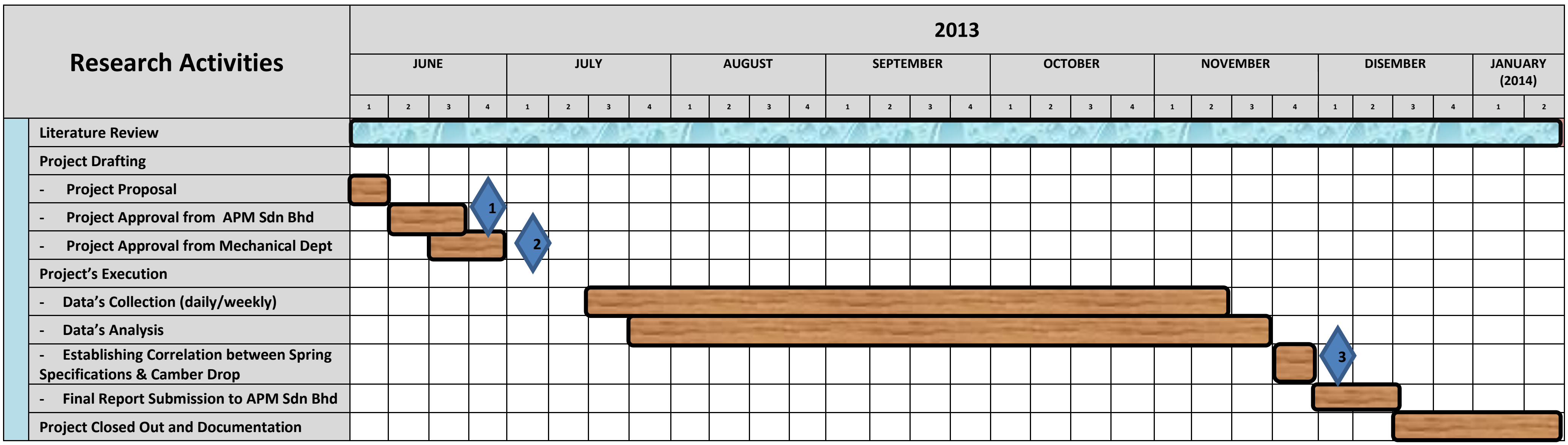
Main Effects:

1) X_1 = Log of Spring Rate
2) X_2 = Log of Half Length
3) X_3 = Log of End Thickness
4) X_4 = Log of Quenching Camber

2nd Order Terms:

5) X_5 = (Log of Spring Rate) ^{2}
6) X_6 = (Log of Half Length) ^{2}
7) X_7 = (Log of End Thickness) ^{2}
8) X_8 = (Log of Quenching Camber) ^{2}
9) $X_9 = X_1.X_2$
10) $X_{10} = X_1.X_3$
11) $X_{11} = X_1.X_4$
12) $X_{12} = X_2.X_3$
13) $X_{13} = X_2.X_4$
14) $X_{14} = X_3.X_4$

3.3 GANTT CHART AND RESEARCH ACTIVITIES



Extension required maximizing research

1. Obtaining approval from APM Sdn Bhd to conduct the project in APM Sdn Bhd, and the approval to use the data from APM Sdn Bhd.

2. Obtaining approval from Mechanical Engineering Department of Universiti Teknologi PETRONAS to conduct the project as Final Year Project.

3. Objective of finding estimation of camber drop for each factor consists of Master Camber, Spring Thickness, Spring Half-length, and Spring Rate, and to establish correlation among the four factors are expected to be achieved.

CHAPTER 4: RESULTS AND DISCUSSION

In this chapter, the current result is being discussed. For the set of data, data gathered during the FYP period and Internship period is being combined together in order to increase the number of data made available for this project, which in objective of to increase the accuracy of the result. A total set of 95 data's has been analyzed to correlate the Leaf Spring's specifications.

4.1 SET OF DATA

PART NO.	TYPE	LEAF NO.	SPRING RATE (kg/mm)	HALF LENGTH (mm)	END THICKNESS (mm)	QENCHING CAMBER (mm)	Camber drop(mm)
SH63XXXX	P	1	5.3	750	13	142	24.4
540100XXXX	P	4	9.84	790	12	200	24
540100XXXX	P	3	9.84	790	12	185	21.3
54010JXXXX	P	1	10.32	775	12	96	42.4
54010JXXXX	P	2	9.84	790	10	96	38.8
540100XXXX	P	1	5.7	775	13	125	23.7
54010JXXXX	P	4	9.08	790	10	116	35.4
54010JXXXX	P	3	9.08	790	10	111	33.9
ER137XXXX	P	3	10.36	930	13.6	229	20.7
MI610XXXX	P	1	4.58	825	10	131	33
APM431 XXXX	P	1	4.1	675	12	102	16.2
MI610 XXXX	P	1	4.58	825	10	131	31
ER137 XXXX	P	1	11.32	910	13.6	172	24.3
15 XXXX	P	1	9.3	910	14	130	24.3
MI404 XXXX	P	3	6.4	990	10.2	175	36.2
2171 XXXX	P	1	6.06	775	16	108	19.7
1 XXXX	P	3	10.36	930	13.6	229	19.5
APM43 XXXX	P	3	7.78	685	12	137	24.8
13 XXXX 7	P	1	13.74	910	14	122	24
1 XXXX	P	1	6.22	910	13.6	172	19.5
2171 XXXX	P	1	6.06	775	16	108	22.83
SH63 XXXX	P	1	5.3	750	13	142	24.2
15 XXXX	P	1	5.84	850	14	154	28.3
1 XXXX	P	3	10.28	870	12.5	203	37.2
1 XXXX	P	1	5.84	850	12.5	154	30
MK3190 XXXX	P	4	11.02	770	11	203	17.1
SH63143 XXXX	P	1	10.08	765	13	145	21.7

CT8 XXXX	P	1	37.24	810	20	40	11.9
CT91 XXXX	P	1	36.84	810	20	54	17.8
1377 XXXX	p	1	44.82	770	13.4	78	8.9
MK30 XXXX	P	H1	6.62	595	9	120	21.1
MK30 XXXX	P	H2	6.62	595	9	140	19
MK30 XXXX	P	H3	6.62	595	9	150	17.2
SH6314 XXXX	P	1	10.08	765	13	145	25.4
481503 XXXX	P	1	5.14	750	13.5	119	27.5
481503 XXXX	P	3	9.14	780	11	172	46.4
ERA161341 XXXX A	P	2	7.43	663	11.35	172	26.25
ERA161341 XXXX B	P	2	7.43	757	11.35	232	77.25
ERA161341 XXXX A	P	1	6.02	663	11.5	151	12.83
ERA161341 XXXX B	P	1	6.02	737	11.5	178	29.33
CW530 XXXX A	P	2	7.03	775	12	152	22.8
CW530 XXXX B	P	2	7.03	790	12	156	19.3
CW53 XXXX	P	1	5.5	775	13	131	23.6
CW530 XXXX	P	1	5.5	775	13	131	23.2
SH6314 XXXX	P	1	10.08	765	13	145	24
SH6314 XXXX	P	1	10.08	765	13	145	26.3
SH6314 XXXX	P	1	10.08	765	13	145	22.4
APMUSA XXXX	P	2	12.54	674	10.5	156	19
APMUSA XXXX	P	3	12.54	674	10.5	183	23
55020J XXXX	P	2	11.22	800	10.75	126	28
MK3190 XXXX	P	2	12.24	750	11	137	15
MK3190 XXXX	P	3	12.24	750	11	173	16
MK3190 XXXX	P	4	12.24	750	11	203	17
48150-3 XXXX	P	2	13.78	750	12	121	20
MA XXXX	P	2	10.48	943	12	174	47
ER353 XXXX	P	2	17.58	910	13	154	22
790 XXXX	P	1	32.16	615	13	87	10
790 XXXX	P	2	32.16	615	13	94	19
790 XXXX	P	3	38.92	570	13	101	12
MK3190 XXXX	P	1	12.86	750	13	105	10
MA XXXX	P	1	10.54	943	13.5	133	46
APMUSA XXXX	P	1	12.73	674	14	138	25
9020 XXXX	P	1	11.34	910	14	158	23
46 XXXX	P	1	23.14	655	14	74	11
46 XXXX	P	2	23.14	655	14	100	11
46 XXXX	P	3	38.94	575	14	109	9
APMUSA XXXX	P	H1	42.39	674	15	70	9
ER8143402 XXXX	P	H1	79.02	725	16	80	8
ER8143402 XXXX	P	2	39.78	725	16.5	90	14
ER8143402 XXXX	P	1	40.28	725	19.5	69	15

CT XXXX	P	3	37.06	810	20	83	13
APMUSA XXXX	P	2	12.54	640	10.5	156	23
APMUSA XXXX	P	3	12.54	640	10.5	183	24
55020 XXXX	P	2	11.22	800	10.75	126	20
MK3190 XXXX	P	2	12.24	750	11	141	15
MK3190 XXXX	P	3	12.24	750	11	173	16
MK3190 XXXX	P	4	12.24	750	11	203	17
48150-3 XXXX	P	2	13.78	750	12	128	33
MA XXXX	P	2	10.48	810	12	132	37
ER35 XXXX	P	2	17.58	910	13	154	18
79 XXXX	P	1	32.16	585	13	91	25
79 XXXX	P	2	32.16	585	13	98	21
790 XXXX	P	3	38.92	570	13	105	3
MK3190 XXXX	P	1	12.86	750	13	105	10
MA XXXX	P	1	10.54	810	13.5	95	27
APMUSA 75-218	P	1	12.73	640	14	115	17
9020 XXXX	P	1	11.34	910	14	158	23
46 XXXX	P	1	23.14	655	14	89	16
46 XXXX	P	2	23.14	655	14	107	27
46 XXXX	P	3	38.94	575	14	109	9
APMUSA XXXX	P	H1	42.39	640	16	55	13
ER8143402 XXXX	P	H1	79.02	725	16	80	8
ER8143402 XXXX	P	2	39.78	725	16.5	90	14
ER8143402 XXXX	P	1	40.28	725	19.5	69	15
CT XXXX	P	3	37.06	810	20	83	13.3

Table 4: Finalized set of Data's

4.2 DEVELOPING REGRESSION MODELS

4.2.1 REGRESSION ANALYSIS USING MAIN EFFECT VARIABLES [MODEL 0]

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.69546262
R Square	0.483668256
Adjusted R Square	0.460720179
Standard Error	0.151734947
Observations	95

ANOVA (Analysis of Variance)

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	1.941031146	0.485257786	21.07663514	2.74831E-12
Residual	90	2.072114476	0.023023494		
Total	94	4.013145622			

	Coefficients	***Standard Error***	***t Stat***	***P-value***	***Lower 95%***	***Upper 95%***
Intercept	-1.2760	0.8877	-1.4374	0.1541	-3.0395	0.4875
X Variable 1	-0.2834	0.0755	-3.7507	0.0003	-0.4335	-0.1333
X Variable 2	1.2117	0.3369	3.5963	0.0005	0.5423	1.8810
X Variable 3	-0.4848	0.3159	-1.5346	0.1284	-1.1123	0.1428
X Variable 4	-0.0246	0.1612	-0.1529	0.8788	-0.3449	0.2956

Table 5: Summary Output of Regression Analysis Using Main Effect Variables

Table above shows the summary of the regression analysis by using Main Effect Variables. From that analysis, it is stated that the value of “R square” is 0.483668256 which means that the model established from this analysis by using the variables is fit for only 48.37% from the set of the given data. To come up a model based on this regression analysis, the things that need to be included into the function are Coefficients and Intercept which can be obtained directly from the table. The statistical model that can be established from this analysis is as follow:

$$(Log\ Camber\ Drop) = -1.276 - 0.2834X_1 + 1.2117X_2 - 0.4848X_3 + 0.0246X_4$$

As this function is in Logarithm function, to make it simpler then the function is transformed in Antilog function. Below is the Antilog function for this analysis:

$$Camber\ Drop = 0.0529\ (\frac{HL^{1.2117}}{SR^{0.2834} \times ET^{0.4848} \times QC^{0.0246}})$$

Where: *SR = Spring Rate*

HL = Half Length

ET = End Thickness

QC = Quenching Camber

4.2.2 REGRESSION ANALYSIS USING COMBINATION OF MAIN EFFECT VARIABLES AND 2ND ORDER TERMS (INTERACTIONS) [MODEL 1]

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.742935153
R Square	0.551952642
Adjusted R Square	0.473544355
Standard Error	0.149919949
Observations	95

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	14	2.215066	0.158219	7.039468	3.38E-09
Residual	80	1.798079	0.022476		
Total	94	4.013146			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-26.1023	41.7782	-0.6247	0.5338	-109.2440	57.0391
X Variable 1	4.8551	5.9082	0.8217	0.4136	-6.9026	16.6129
X Variable 2	2.3756	32.4530	0.0732	0.9418	-62.2079	66.9592
X Variable 3	14.2238	22.3083	0.6376	0.5255	-30.1711	58.6188
X Variable 4	11.9198	11.0268	1.0809	0.2829	-10.0242	33.8639
X Variable 5	-0.7555	0.2684	-2.8150	0.0061	-1.2897	-0.2214
X Variable 6	4.3247	7.0478	0.6136	0.5412	-9.7008	18.3504
X Variable 7	3.8191	5.4089	0.7060	0.4821	-6.9449	14.5833
X Variable 8	0.2687	1.3240	0.2029	0.8396	-2.3662	2.9036
X Variable 9	-0.4299	2.1760	-0.1975	0.8438	-4.7603	3.9004
X Variable 10	1.1254	2.0110	0.5596	0.5773	-2.8767	5.1275
X Variable 11	-1.6915	1.1604	-1.4577	0.1488	-4.0009	0.6177
X Variable 12	-12.2597	9.0932	-1.3482	0.1813	-30.3559	5.8362
X Variable 13	-5.9291	4.6151	-1.2847	0.2025	-15.1136	3.2552
X Variable 14	5.3753	4.3166	1.2452	0.2166	-3.2151	13.9658

Table 6: Summary Output of Regression Analysis Using Combination of Main Effect Variables and 2nd Order Terms (Interactions)

In order to increase the value of “R square”, the number of variables or parameters that is being included into the analysis has been increased. For this type of analysis, the Main Effects variables are being added with the variables of 2nd Order Terms, which make it all 14 variables for this analysis.

The table above shows the summary output of the regression by using the combination of the Main Effect Variables and 2nd Order Terms Variables. From the table, the value of “R square” obtained is a bit higher from the previous regression, which is 0.551952642 or 55.2%. Thus, the Logarithm model established for this analysis is as follow:

$$
\begin{aligned}
(Log\ Camber\ Drop) &= -26.1 + 4.86X_1 + 2.38X_2 + 14.22X_3 + 11.92X_4 - 0.76X_5 + 4.32X_6 \\
&+ 3.82X_7 + 0.27X_8 - 0.43X_9 + 1.13X_{10} - 1.69X_{11} - 12.26X_{12} \\
&- 5.93X_{13} + 5.38X_{14}
\end{aligned}
$$

Transforming the model into Antilog function for a simpler model, the model is as follow:

$$
\begin{aligned}
&Camber\ drop \\
&= 7.94 \\
&\times 10^{-27} \left(\frac{SR^{4.86} \times HL^{2.38} \times ET^{14.22} \times QC^{11.92} \times HL^{4.32(Log\ HL)} \times ET^{3.82(Log\ ET)} \times QC^{0.27(Log\ QC)} \times SR^{1.13(Log\ ET)}}{SR^{0.76(Log\ SR)} \times SR^{0.43(Log\ HL)} \times SR^{1.69(Log\ QC)} \times HL^{12.26(Log\ ET)} \times HL^{5.93(Log\ QC)} \times ET^{-5.38(Log\ QC)}}\right)
\end{aligned}
$$

Where: *SR = Spring Rate*

HL = Half Length

ET = End Thickness

QC = Quenching Camber

4.3 REGRESSION ANALYSIS AND INTERPRETATION (DISCUSSION)

4.3.1 TESTING THE SIGNIFICANCE OF THE MODEL

To test the significance of the model, the value of $F_{Observations}$ is being compared to the value of $F_{Theoretical}$. The value of $F_{Theoretical}$ can be obtained by computing using Matlab software, while the value of $F_{Observation}$ is being calculated by using the equation below:

$$F = \frac{\frac{SSR_0 - SSR_1}{dfR_0 - dfR_1}}{\frac{SSR_1}{dfR_1}}$$

Where:

SSR = Sum of Square of Residual

dfR = Degree of Freedom of Residual

And:

Model 0 = Regression Analysis using Main Effects

Model 1 = Regression Analysis using Main Effects and 2nd Order Terms

The value of SSR and dfR both can be obtained from the table of ANOVA for both models. Thus, the values for the notations are as follows:

$SSR_0 = 2.072114476$

$SSR_1 = 1.798079$

$dfR_0 = 90$

$dfR_1 = 80$

Substituting the value into equation,

$$F = \frac{\frac{2.072114476 - 1.798079}{90 - 80}}{\frac{1.798079}{80}}$$

$F = 1.219237$

Computing the $F_{Theoretical}$ by using Matlab software would obtain the value of:

$F_{Theoretical}$,

$$F_{10,80,0.05} = 1.95$$

The condition of accepting the significance of the Model 0 and Model 1 is to accept if the value of the $F_{Observation}$ is lower than the value of $F_{Theoretical}$. Since the value of $F_{Observation}$, 1.219237 is lower than the value of $F_{Theoretical}$, 1.95, thus these model is signified.

4.3.2 REMOVING UNIMPORTANT VARIABLES

The total number of variables in the regression analysis using Main Effect variables and 2nd Order Terms are 14 variables. If the final model of this project is to be taken from this type of regression, the final model would be too complicated and making the engineers taking a bit long time in order to set up the camber drop estimation. Thus, it is necessary to remove the variables which are not important from the final model of the regression analysis.

The regression model is said to be at best when the model has a low value of Error or Residual. Increasing the number of variables will improve the best fit of the model, by reducing the Error or Residual of the regression model. Therefore, to justify this statement, the Residual's Sum of Square (SS) of both models is being considered. The value of the Residual's SS is obtained from the table of ANOVA from each model. Below shows on how to justify the statement above:

$$\Delta SS = SS_0 - SS_1$$

From the ANOVA table,

$$SS_0 = 2.072114476$$

$$SS_1 = 1.798079$$

Computing the difference of Error of the model:

$$\Delta SS = 2.072114476 - 1.798079$$

$$\Delta SS = 0.273324476$$

From what has been computed above, by increasing the total number of variables from 4 to 14 will result in reduction of the value of error of the model, with a difference of 0.273324476. The purpose of increasing the number of variables has successfully justified with the computational above. However, the difference of ΔSS between Model 0 and Model 1 is not that high or convincing after the additional of 10 more variables between these two models. Each variable added to the model will only result in 0.0273324476, which is small. Thus, to consider all 14 variables in a model is not convinced. Some of the variables need to be reconsidered and removed from the final in order to make the final model simpler.

To remove a variable, the component that needs to be considered is the P-value of each variable in the final model. For removing a variable, the condition below must be met. The condition is:

$$P\ value > 0.05$$

P-value represents the test hypothesis of the variable in the model, after testing the important of the variable. A variable with P-value of lower than 0.05 is said to be very important and should be keep in the model, while a variable with P-value of higher than 0.05 is said to be not important and should be considered to be rejected from the model.

Below shows the P-value of each variable:

Variable	P-value	Variable	P-value
Intercept	0.5338	X Variable 8	0.8396
X Variable 1	0.4136	X Variable 9	0.8438
X Variable 2	0.9418	X Variable 10	0.5773
X Variable 3	0.5255	X Variable 11	0.1488
X Variable 4	0.2829	X Variable 12	0.1813
X Variable 5	0.0061	X Variable 13	0.2025
X Variable 6	0.5412	X Variable 14	0.2166
X Variable 7	0.4821		

Table 7: P-value of each variable

From the table above, the highlighted variable is the only variable that having the P-value of lower than 0.05 and this will make the variable to be kept in the final model. However, for the variable of 1 – 4 will still be included in the final model because it is the Main Effects variables which related to the 2nd Order Terms variable. Besides, the objective of this project is to correlate the four main variables to the Camber Drop, thus it will not be removed even though it has high P-value.

Therefore, the only variables to be included in the final model are Variable 1, Variable 2, Variable 3, Variable 4, and Variable 5.

4.4 DEVELOPING THE FINAL MODEL OF REGRESSION ANALYSIS

After deciding the variable that need to be included, the final model of regression analysis can be developed. Below shows the summary output of the final model:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.710978
R Square	0.50549
Adjusted R Square	0.477709
Standard Error	0.149326
Observations	95

ANOVA (ANALYSIS OF VARIANCE)

	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	2.028606	0.405721	18.19524	2.12E-12
Residual	89	1.98454	0.022298		
Total	94	4.013146			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-1.35043	0.874415	-1.54438	0.126045	-3.08787	0.387017
X Variable 1	0.49995	0.402231	1.242942	0.217154	-0.29927	1.299175
X Variable 2	1.11837	0.334904	3.339379	0.001228	0.452924	1.783816
X Variable 3	-0.38558	0.314867	-1.22457	0.22397	-1.01121	0.240058
X Variable 4	-0.11234	0.164697	-0.68208	0.496962	-0.43959	0.214914
X Variable 5	-0.34173	0.172435	-1.98178	0.05059	-0.68435	0.000897

Table 8: Summary Output of Final Model of Regression Analysis

From the summary output of this final regression analysis, the value of “R square” is a bit lower when compared to regression analysis using the combination of Main Effect variables and 2^{nd} Order Term variables, which is 0.50549. This means that the final regression model is to be fit with 50.55% of the statistic data.

To establish the correlation of the variables to the respondent variable of Camber Drop, the Logarithm function is being established by using the coefficient obtained from the summary output above. The Logarithm function of this regression model is as follows:

$$(\boldsymbol{Log\ Camber\ Drop}) = \mathbf{-1.35 + 0.5}\boldsymbol{X_1} \mathbf{+ 1.12}\boldsymbol{X_2} \mathbf{- 0.39}\boldsymbol{X_3} \mathbf{- 0.11}\boldsymbol{X_4} \mathbf{- 0.34}\boldsymbol{X_5}$$

From the Logarithm function, the final correlation in term of Antilog function can be obtained.

Transforming into Antilog function:

$$Camber\ Drop = (0.045)\left(\frac{SR^{0.5} \times HL^{1.12}}{ET^{0.39} \times QC^{0.11} \times SR^{0.24(Log\ SR)}}\right)$$

Where: *SR = Spring Rate*

HL = Half Length

ET = End Thickness

QC = Quenching Camber

The function above is the final model of the regression analysis, and it is the correlation between Leaf Spring's Specifications and Camber Drop.

4.5 TESTING THE FINAL MODEL OF REGRESSION ANALYSIS

In order to test the final model of the regression analysis, the first 20 part number has been taken to see the difference between the actual Camber Drop and the predicted Camber Drop. An equation representing the model has been set up in Microsoft Excel, as the testing process has been performed by using Microsoft Excel. The equation that has been set up is as follows:

= (0.045)*((SR^0.5)*(HL^1.12))/((ET^0.39)*(QC^0.11)*(SR^(0.24*(LOG10((SR))))))

The result of the testing is being shown below:

PART NO.	*TYPE*	*LEAF NO.*	*Predicted Camber Drop (mm)*	*Actual Camber drop(mm)*	*Percentage Difference (%)*
SH63XXXX	P	1	27.45	24.4	11.68
540100XXXX	P	4	30.51	24	27.13
540100XXXX	P	3	30.77	21.3	44.46
54010JXXXX	P	1	32.39	42.4	23.61
54010JXXXX	P	2	35.51	38.8	8.99
540100XXXX	P	1	29.17	23.7	23.08
54010JXXXX	P	4	34.69	35.4	2.01
54010JXXXX	P	3	34.86	33.9	2.83
ER137XXXX	P	3	34.39	20.7	66.14
MI610XXXX	P	1	33.29	33	0.88
APM431 XXXX	P	1	25.04	16.2	54.57
MI610 XXXX	P	1	33.29	31	7.39
ER137 XXXX	P	1	34.65	24.3	42.59
15 XXXX	P	1	35.23	24.3	44.98
MI404 XXXX	P	3	41.24	36.2	13.93
2171 XXXX	P	1	27.56	19.7	39.89
1 XXXX	P	3	34.39	19.5	76.36
APM43 XXXX	P	3	26.81	24.8	8.10
13 XXXX 7	P	1	35.40	24	47.50
1 XXXX	P	1	33.49	19.5	71.74

Table 9: Regression model test's results

From the result, it shows that 8 out of 20 part numbers are having the percentage difference of below 20%, and 4 out of 10 part numbers are having the percentage difference of above 50%. From the result also it shows that 3 out of part numbers are having the Predicted Camber Drop lower than the Actual Camber Drop. More details are as follows:

PART NO.	*TYPE*	*LEAF NO.*	*Predicted Camber Drop (mm)*	*Actual Camber drop(mm)*	*Percentage Difference (%)*
SH63XXXX	P	1	27.45	24.4	11.68
54010JXXXX	P	2	35.51	38.8	8.99
54010JXXXX	P	4	34.69	35.4	2.01
54010JXXXX	P	3	34.86	33.9	2.83
MI610XXXX	P	1	33.29	33	0.88
MI610 XXXX	P	1	33.29	31	7.39
MI404 XXXX	P	3	41.24	36.2	13.93
APM43 XXXX	P	3	26.81	24.8	8.10

Table 10: Part Number having Percentage difference of below 20%

PART NO.	*TYPE*	*LEAF NO.*	*Predicted Camber Drop (mm)*	*Actual Camber drop(mm)*	*Percentage Difference (%)*
ER137XXXX	P	3	34.39	20.7	66.14
APM431 XXXX	P	1	25.04	16.2	54.57
1 XXXX	P	3	34.39	19.5	76.36
1 XXXX	P	1	33.49	19.5	71.74

Table 11: Part Number having difference of above 50%

PART NO.	*TYPE*	*LEAF NO.*	*Predicted Camber Drop (mm)*	*Actual Camber drop(mm)*	*Percentage Difference (%)*
54010JXXXX	P	1	32.39	42.4	23.61
54010JXXXX	P	2	35.51	38.8	8.99
54010JXXXX	P	4	34.69	35.4	2.01

Table 12: Part Number having the Predicted Camber Drop lowers than Actual Camber Drop

CHAPTER 5: CONCLUSION

As a conclusion, the establishment of final model of Regression Analysis means that the objective of this project, which is to establish correlation between Leaf Spring's Specifications and Camber Drop has been able to achieve. The correlation between Leaf Spring Specifications and Camber Drop is represented by the following equation:

$$Camber\ Drop = (0.045)\left(\frac{SR^{0.5} \times HL^{1.12}}{ET^{0.39} \times QC^{0.11} \times SR^{0.24(Log\ SR)}}\right)$$

Where: *SR = Spring Rate*

HL = Half Length

ET = End Thickness

QC = Quenching Camber

The regression model that has been established is best fit of 50.55%, which is determined by the value of "R square" from the regression's summary output, and it does not meet the objective of targeting 60% best fit. However, the best fit of the regression model ranging from 50% and above is more likely to be acceptable. This is because in real life, it is impossible to achieve 100% best fit due fluctuation of the data caused by constrain and also the external factors which affecting the production process, such as external temperature and also the uncertainty of the machine. Therefore, by having best fit 50.55%, the regression model is said to be valid. A test has been conducted to see the difference between Predicted Camber Drop and Actual Camber Drop. The statistic of the result is based on 20 part numbers. The result has been classified as follows:

PART NO.	TYPE	LEAF NO.	Predicted Camber Drop (mm)	Actual Camber drop(mm)	Percentage Difference (%)
SH63XXXX	P	1	27.45	24.4	11.68
54010JXXXX	P	2	35.51	38.8	8.99
54010JXXXX	P	4	34.69	35.4	2.01
54010JXXXX	P	3	34.86	33.9	2.83
MI610XXXX	P	1	33.29	33	0.88
MI610 XXXX	P	1	33.29	31	7.39
MI404 XXXX	P	3	41.24	36.2	13.93
APM43 XXXX	P	3	26.81	24.8	8.10

Table 13: Part Number having Percentage difference of below 20%

PART NO.	TYPE	LEAF NO.	Predicted Camber Drop (mm)	Actual Camber drop(mm)	Percentage Difference (%)
ER137XXXX	P	3	34.39	20.7	66.14
APM431 XXXX	P	1	25.04	16.2	54.57
1 XXXX	P	3	34.39	19.5	76.36
1 XXXX	P	1	33.49	19.5	71.74

Table 14: Part Number having difference of above 50%

PART NO.	TYPE	LEAF NO.	Predicted Camber Drop (mm)	Actual Camber drop(mm)	Percentage Difference (%)
54010JXXXX	P	1	32.39	42.4	23.61
54010JXXXX	P	2	35.51	38.8	8.99
54010JXXXX	P	4	34.69	35.4	2.01

Table 15: Part Number having the Predicted Camber Drop lowers than Actual Camber Drop

REFERENCES

[1] Panthi S.K., Ramakrishnan N., et.al. Finite Element Analysis of Sheet Metal Bending Process To Predict the Springback. 2010.

[2] Kumar K., Aggarwal M.L. Computer Aided FEA Simulation of EN45A Parabolic Leaf Spring. 2013

[3] Gebril M.A., Aldlemey M.S., Kablan A.F. Effect of Tempering on Mechanical Properties and Corrosion Rate of Medium and High Carbon Steel. 2013. 685: pp. 81-85

[4] Gisario A., Barletta M., Conti C., Guarino S. Springback Control In Sheet Metal Bending By Laser-Assisted Bending; Experimental Analysis, Empirical, and Neural Network Modeling. 2011

[5] MSME-Development Institute. Project Profile on Auto Leaf Spring. March (2011)

[6] Lee Y.G., Kim D.H., Kim C.G. Design of Patterned Leaf Spring for Sensor-Probe with Stable Reflectivity and High Sensitivity. January (2012)

[7] Osipenko M.A., Nyashin Y.I., Rudakov R.N. A Contact Problem in the Theory of Leaf Spring Bending. 2003

[8] Hou J.P., Cherruault J.Y., Nairne I., Jeronimidis G., Mayer R.M. Evolution of the Eye-end Design of a Composite Leaf Spring for Heavy Axle Loads. April (2006)

[9] Fei H., Jian M.H., Hong Q.W., et.al.Springback Prediction for Incremental Sheet Forming Based on FEM-PSONN Technology.2013

[10] Engineers Edge.com, Leaf Spring Design and Engineering, 2013 [Cited on 22.06.2013], available from http://www.engineersedge.com/material_science/leaf-spring-design.htm

[11] Roberts Automotive & Industrial Parts Manufacturing Corporation, Leaf Spring FAQ's, 2013 [Cited on 22.06.2013], available from http://www.roberts.com.ph/faq-leafspring.php?nav=nav/nav_product

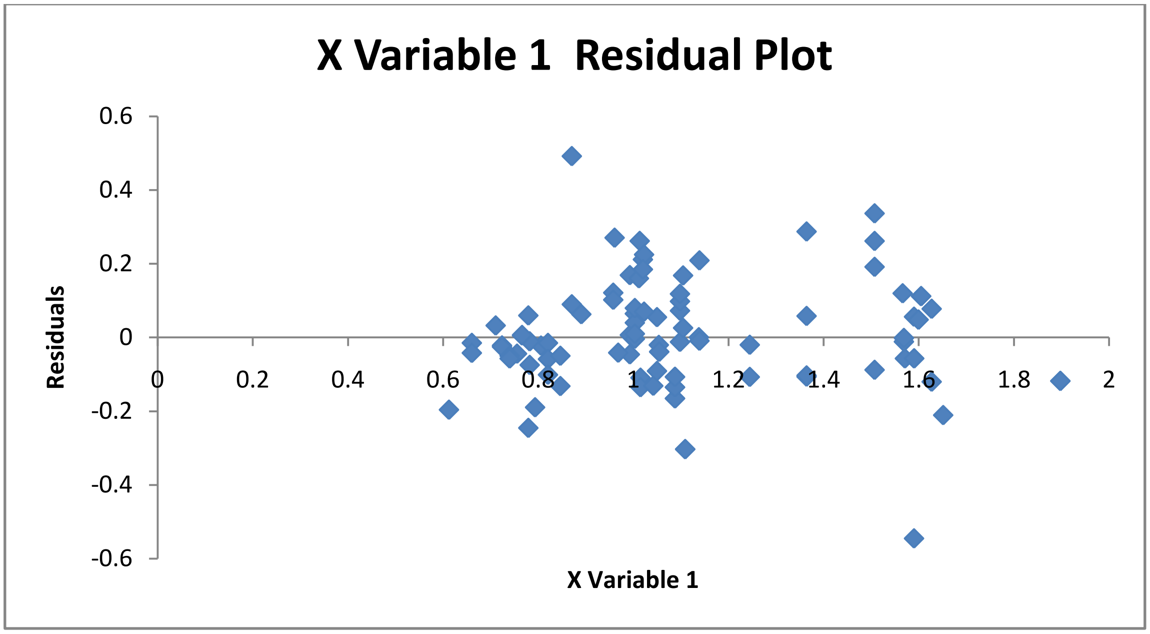
[12] EastCoastRover.com, Parabolic Springs, 2013 [Cited on 23.06.2013], available from http://www.eastcoastrover.com/Parabolics.html

[13] Callister, W.D., & Rethwisch, D.G. (2011). Material Science and Engineering 8th Edition, Asia, John Wiley & Sons (Asia) Pte Ltd

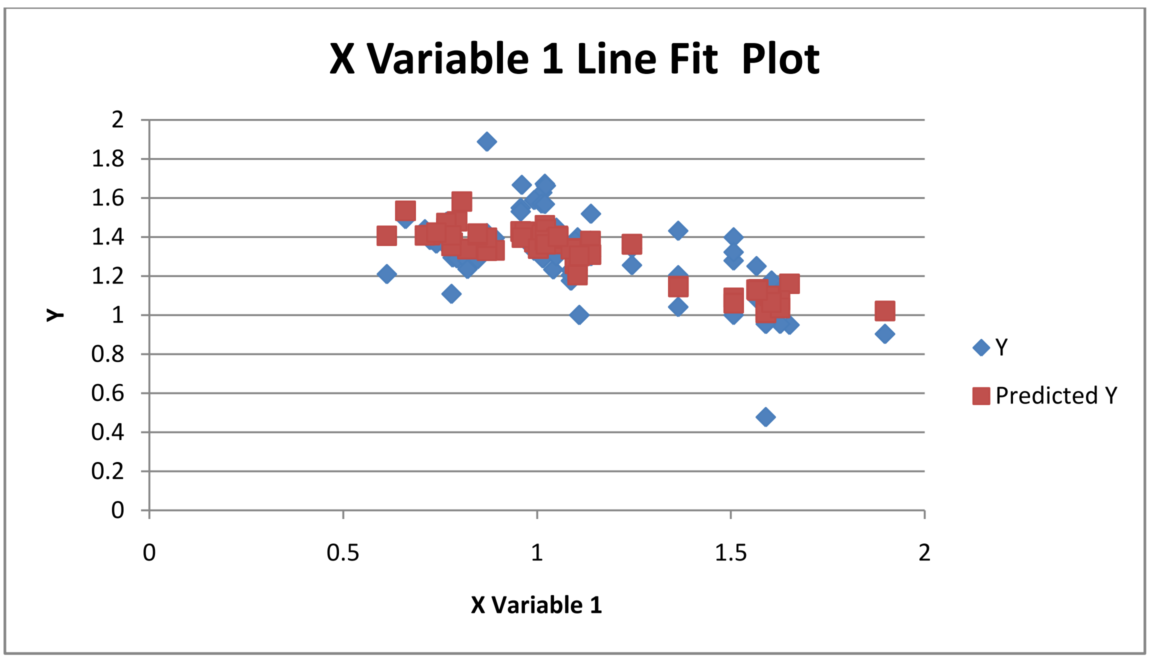
[14] Materials from course of Manufacturing Systems & Operation, 2012

[15] Skyes O.A., *The Inaugural Coase Lecture*, An Introduction to Regression Anlysis, 2003

APPENDICES



Residual plot of Variable 1 for Regression Analysis Using Main Effect Variables



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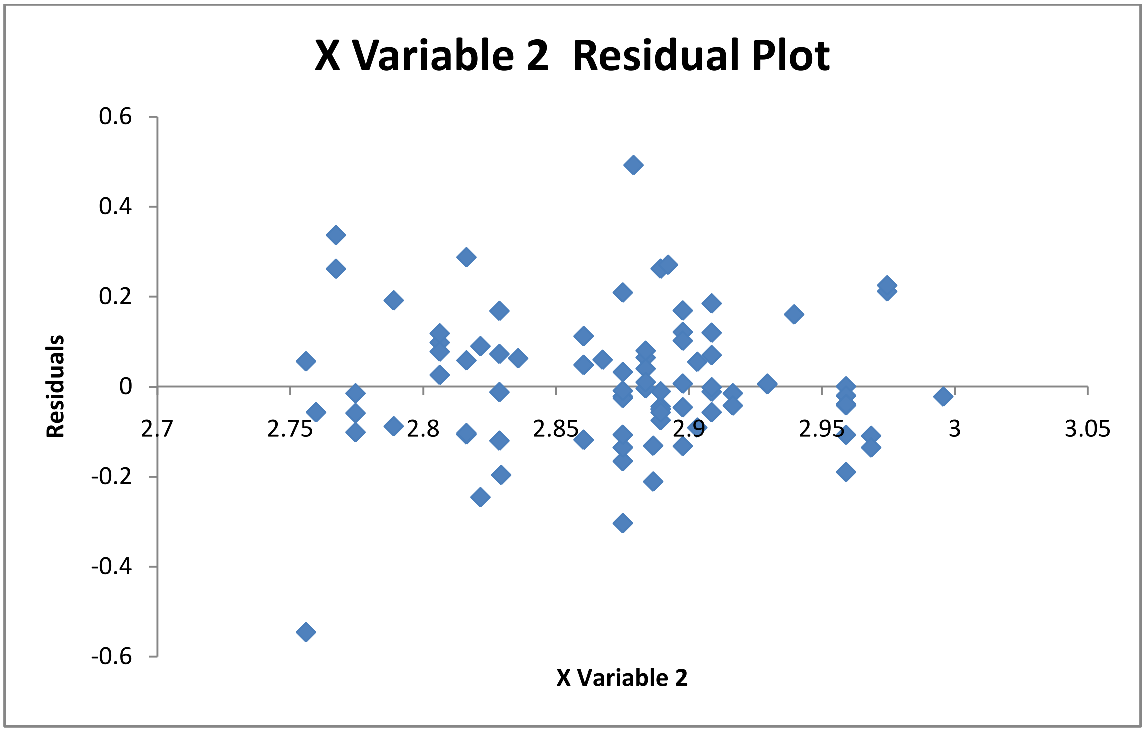

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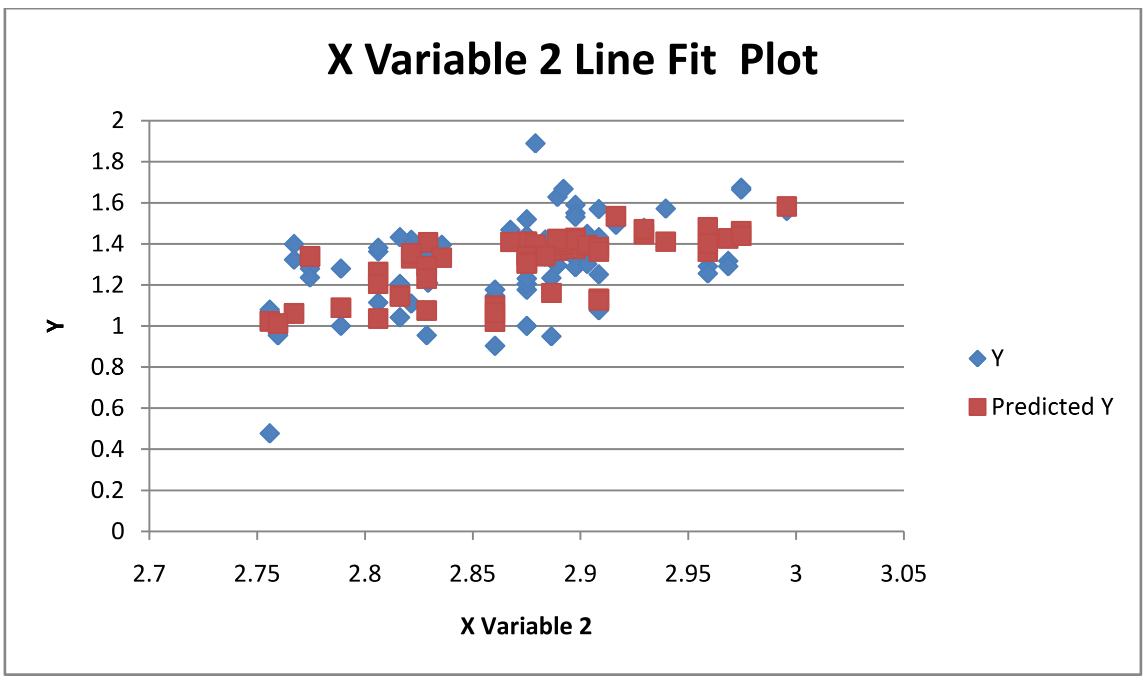

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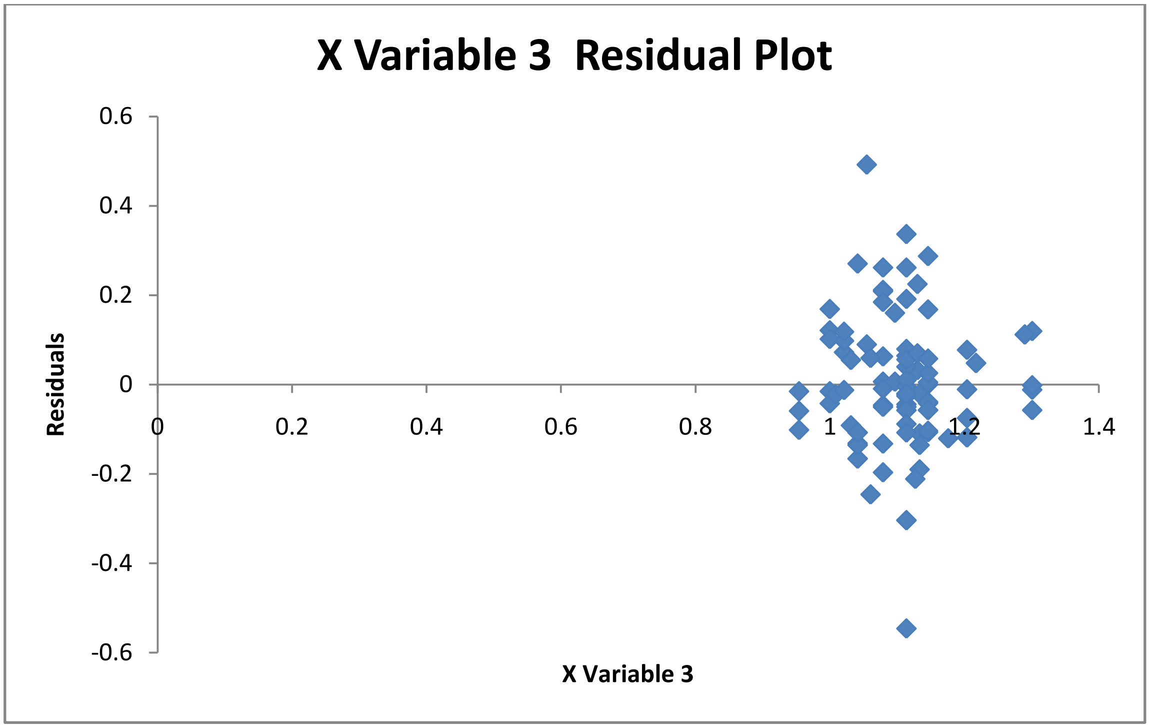

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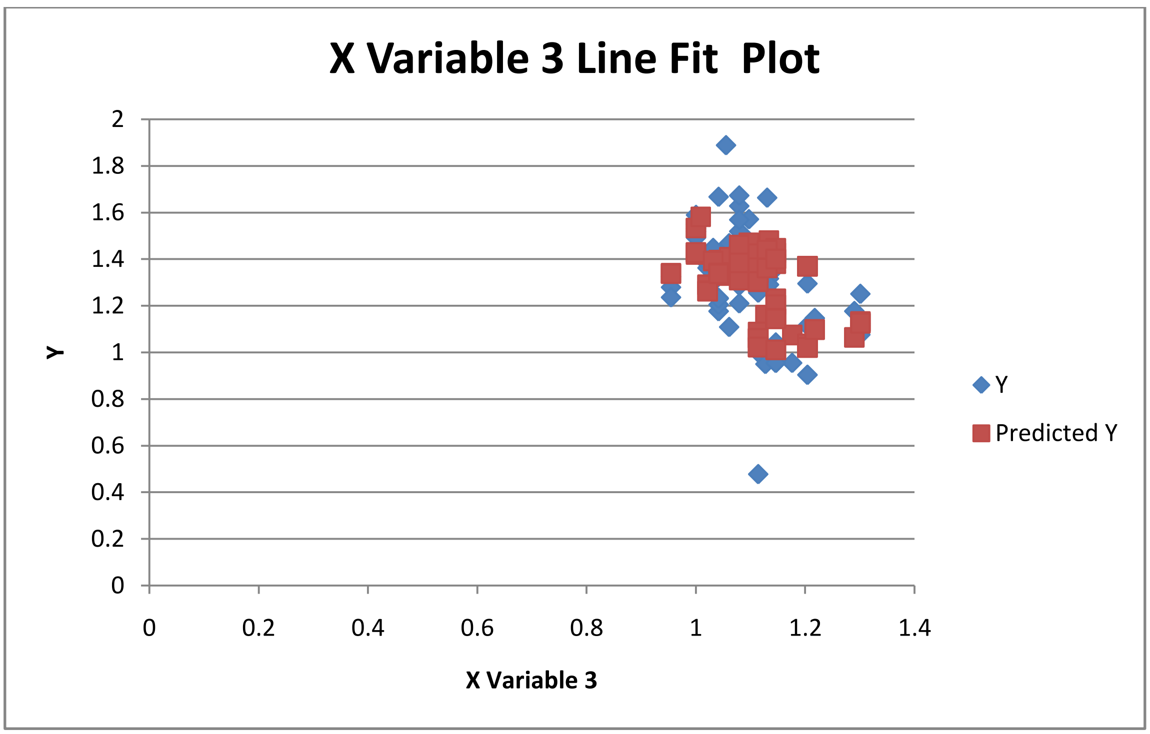

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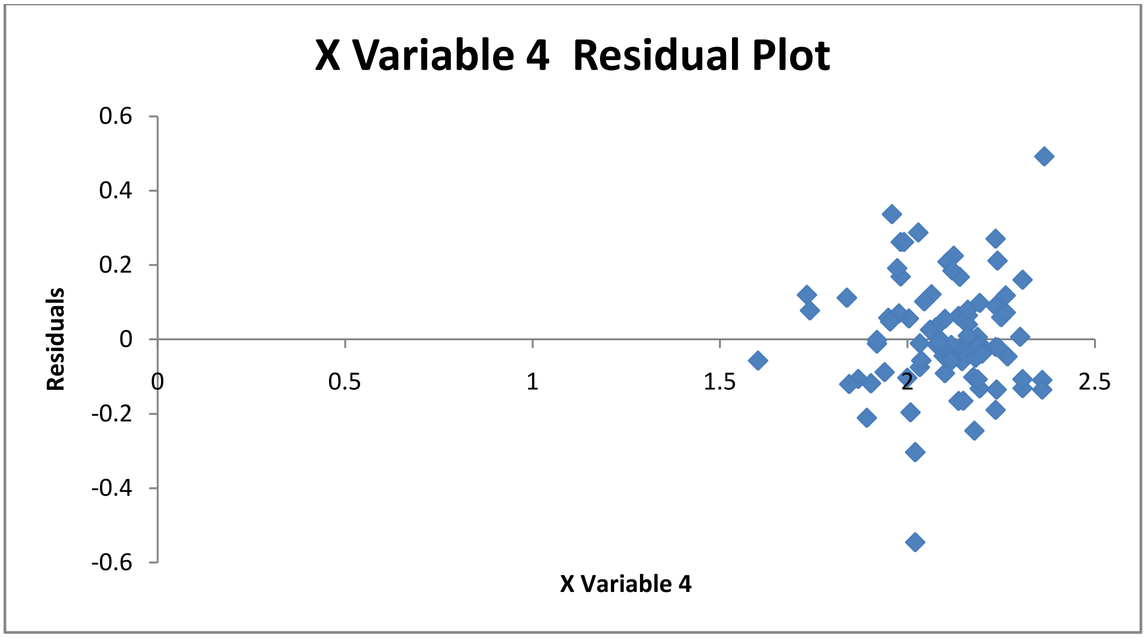

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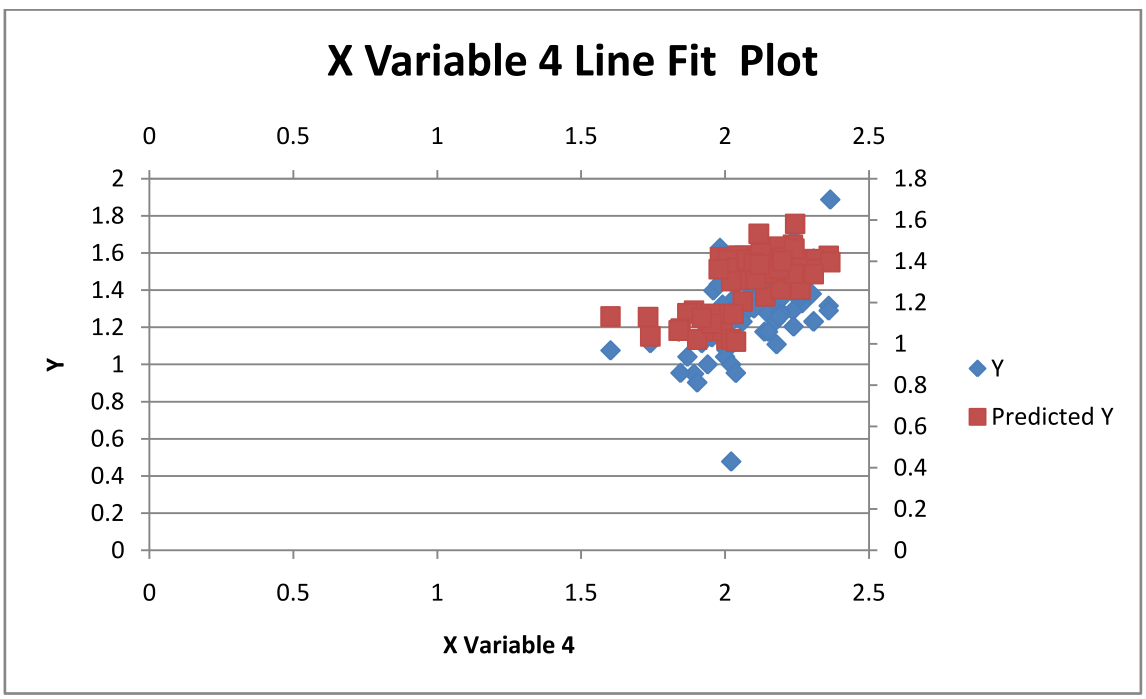

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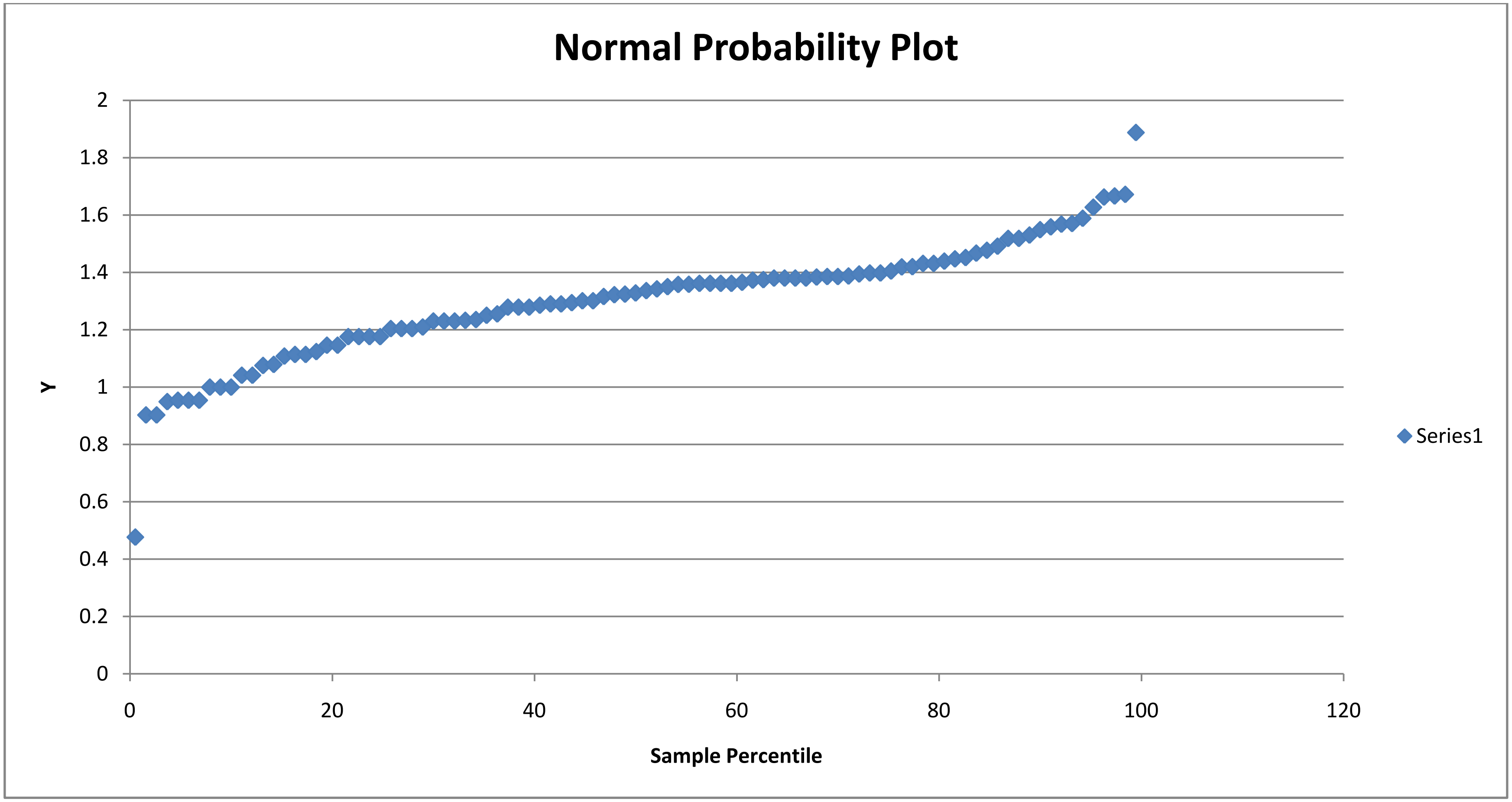

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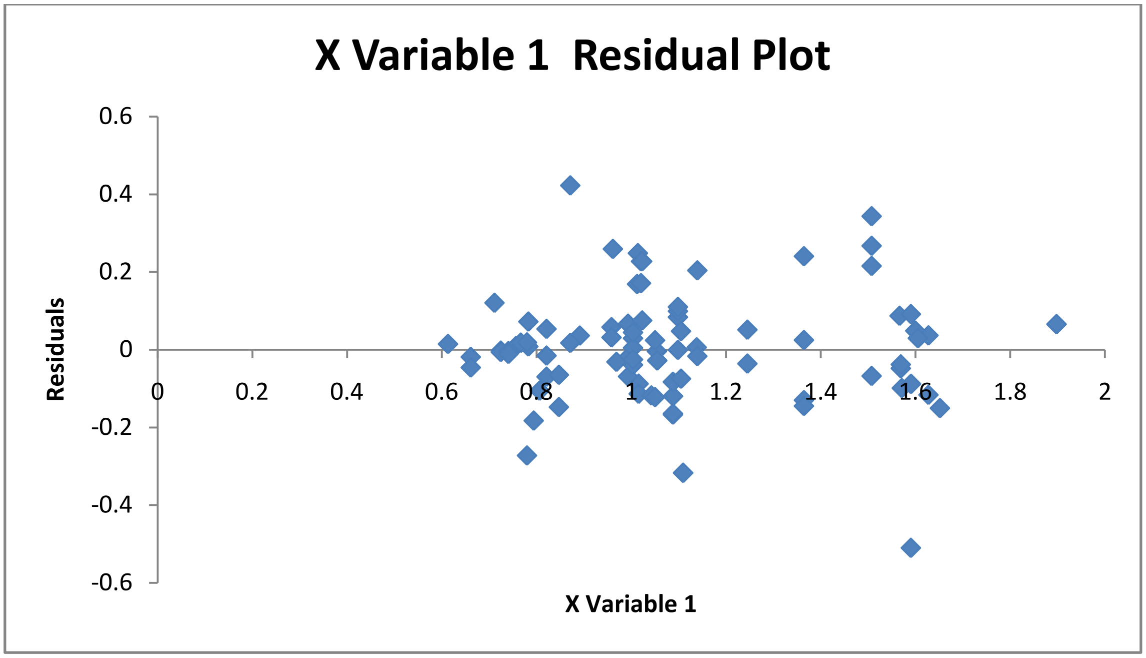

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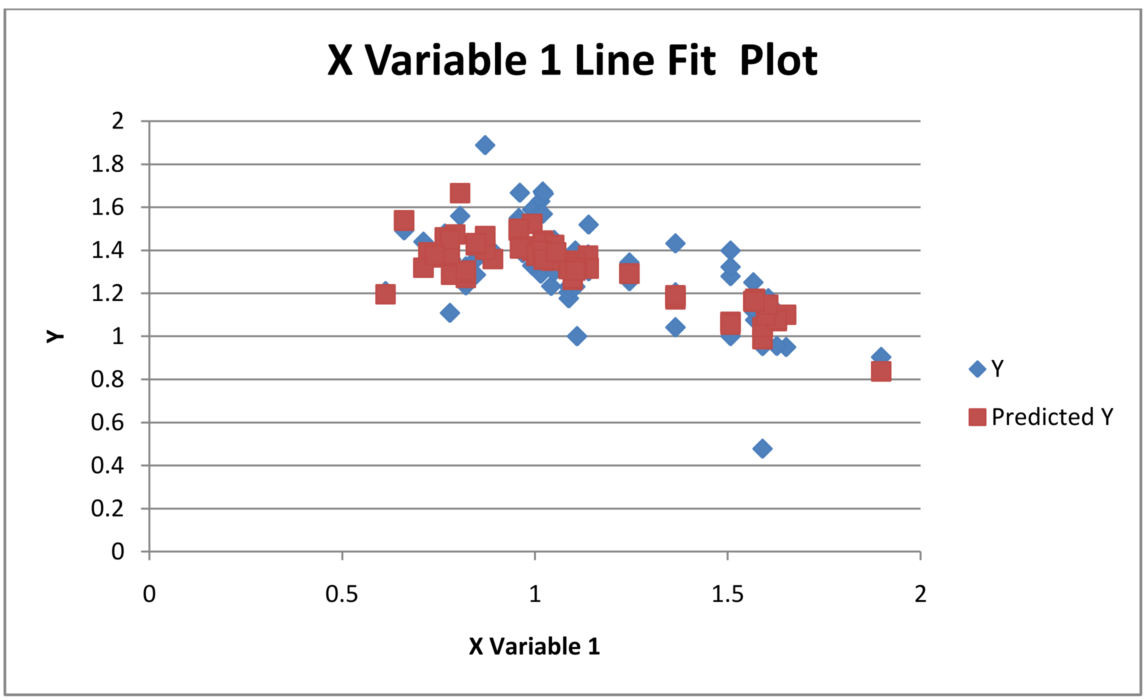

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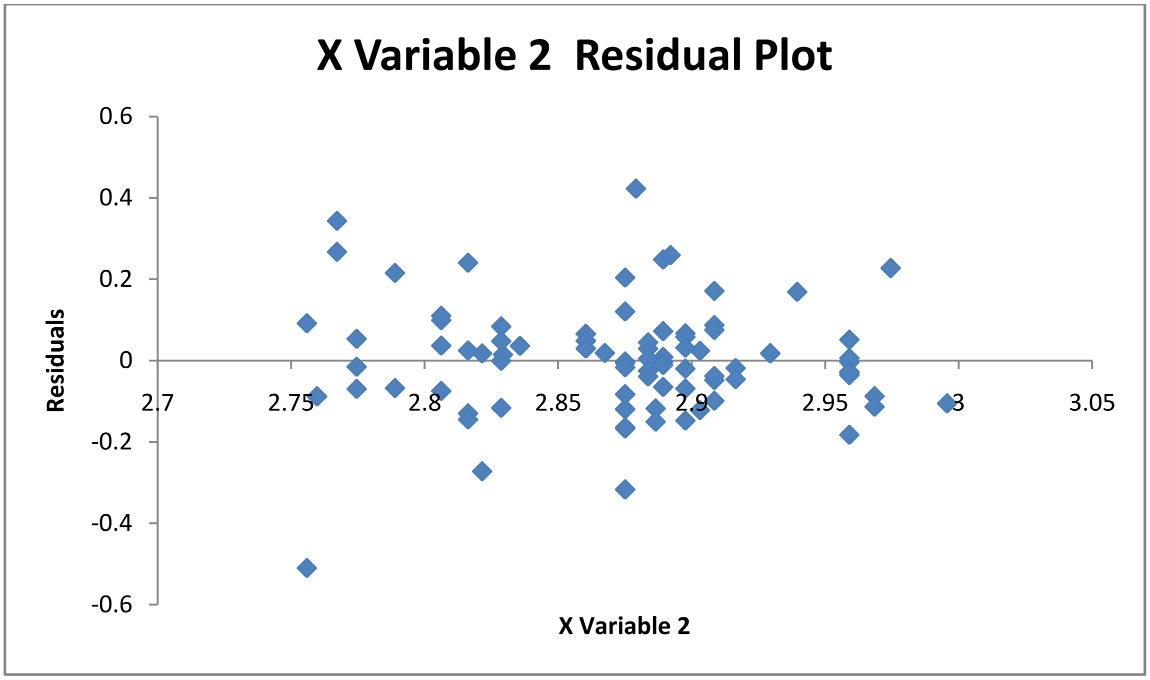

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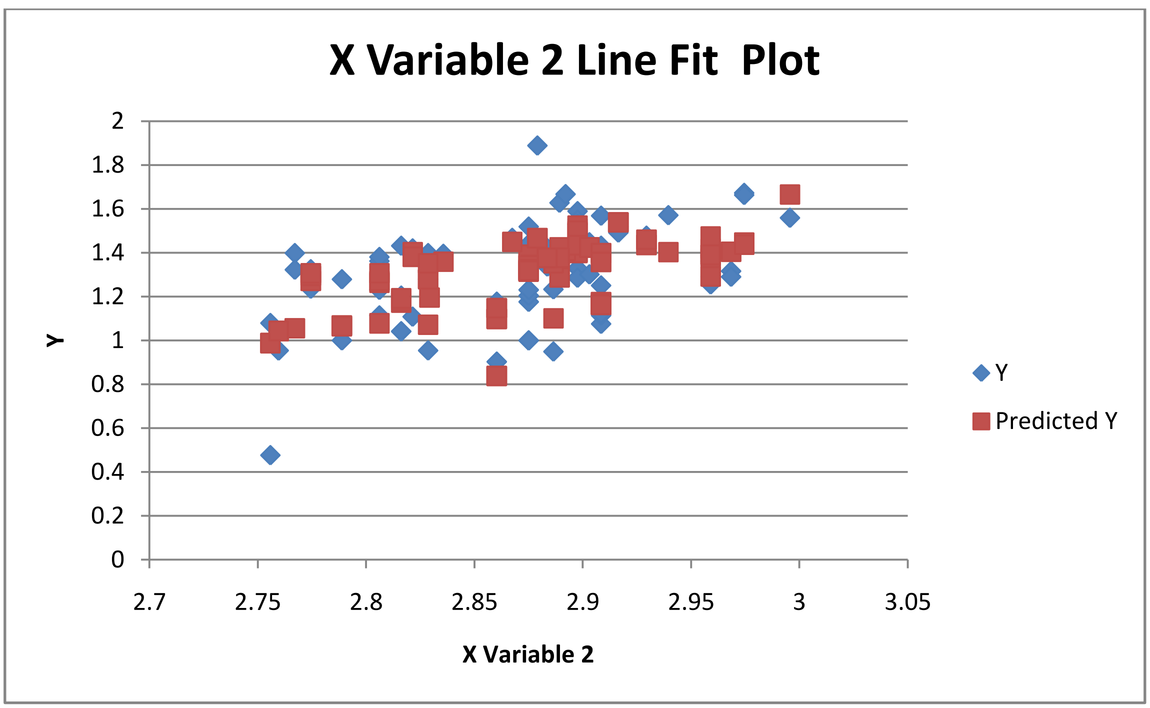

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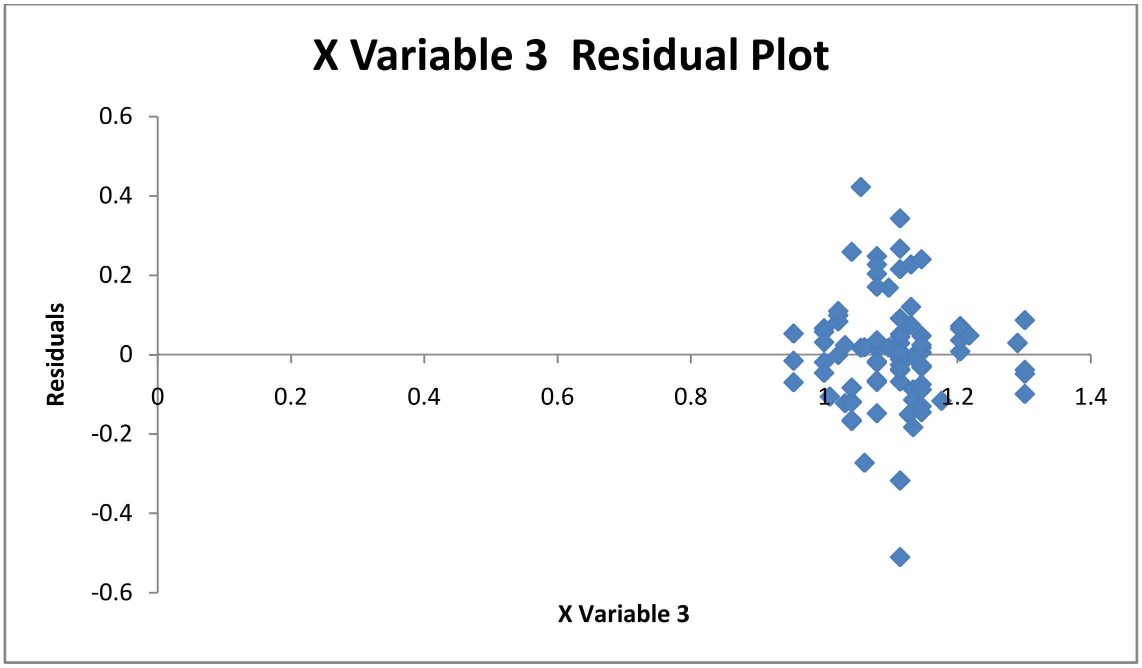

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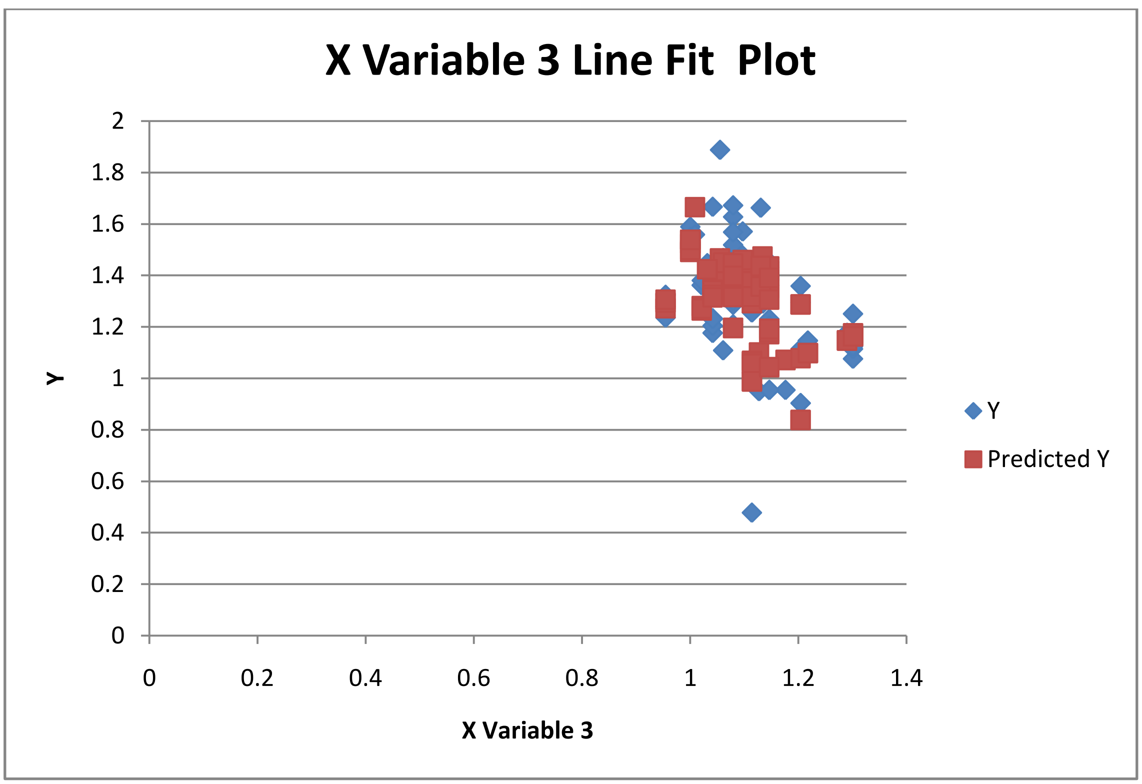

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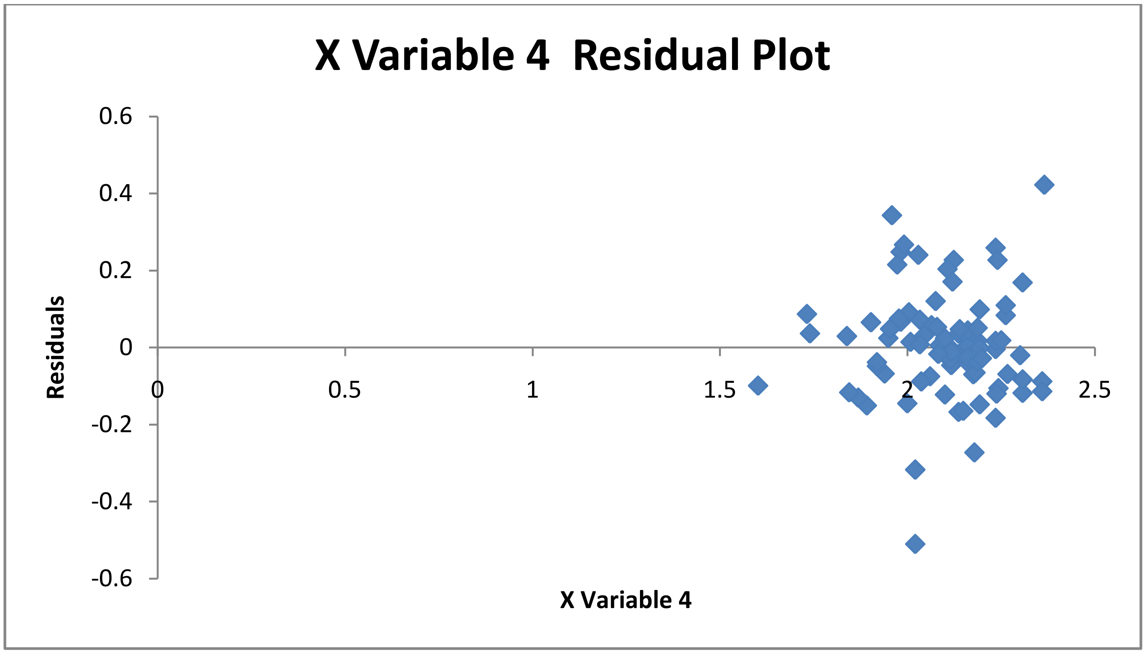

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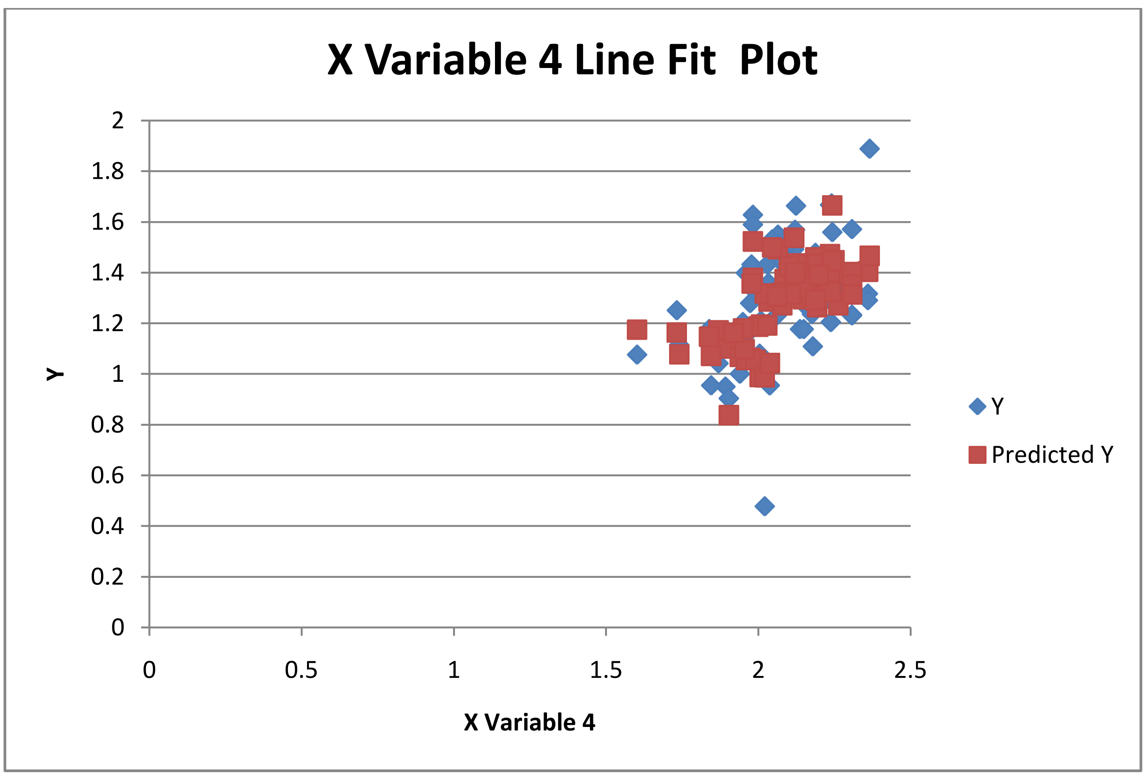

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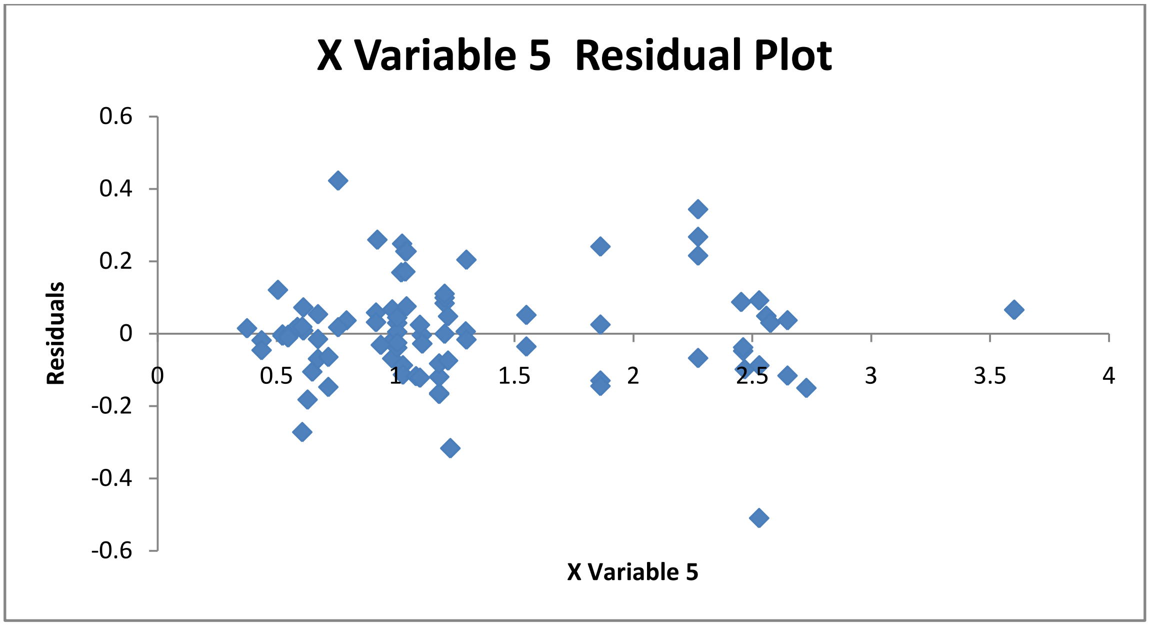

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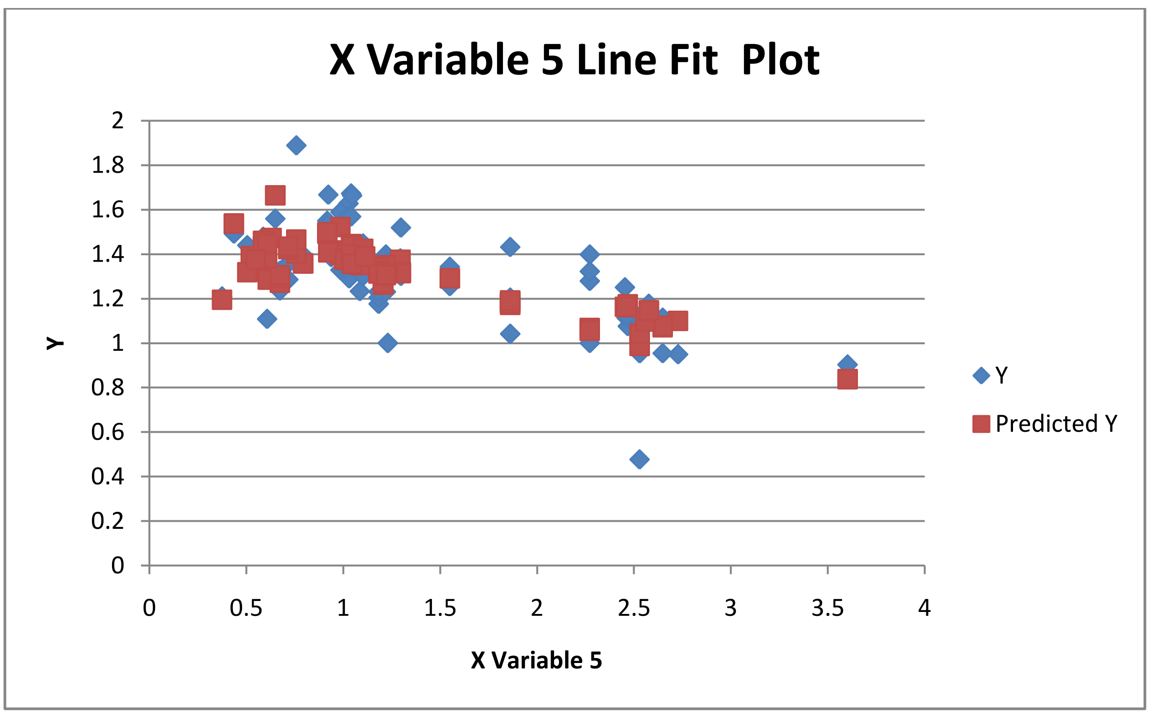

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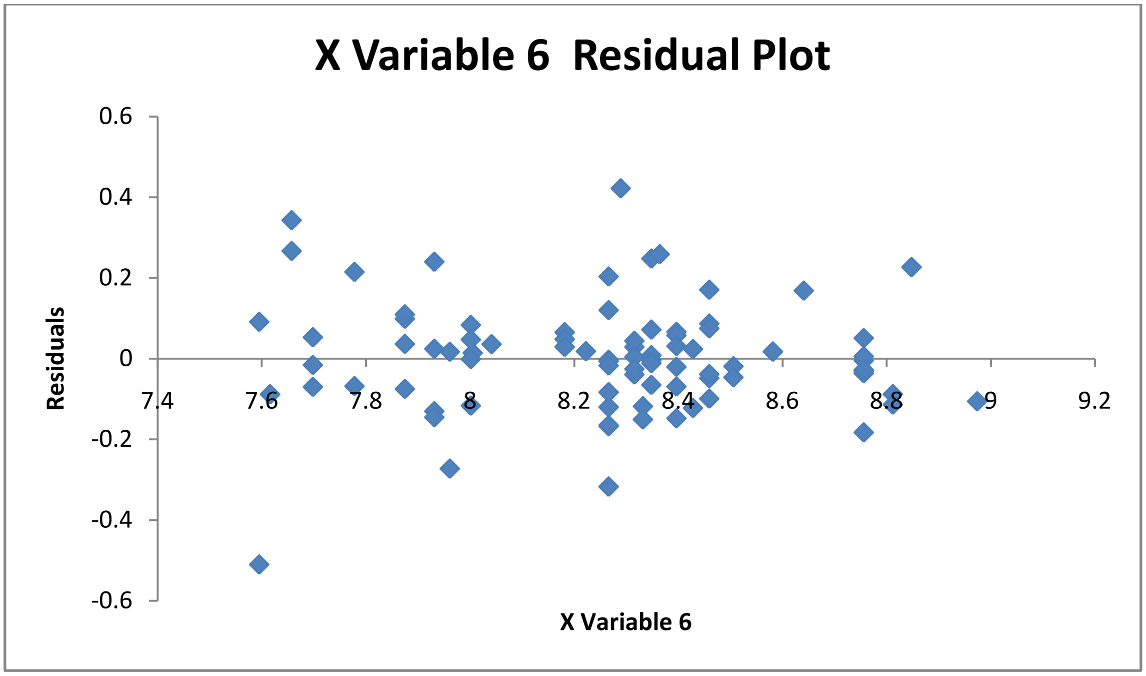

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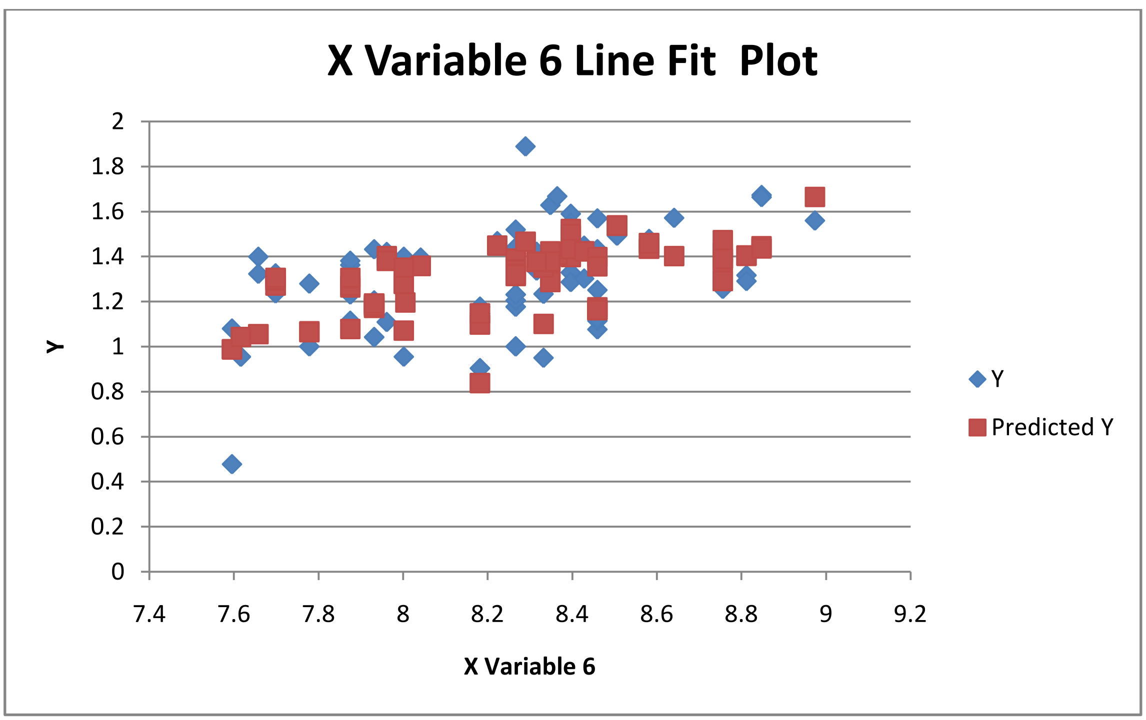

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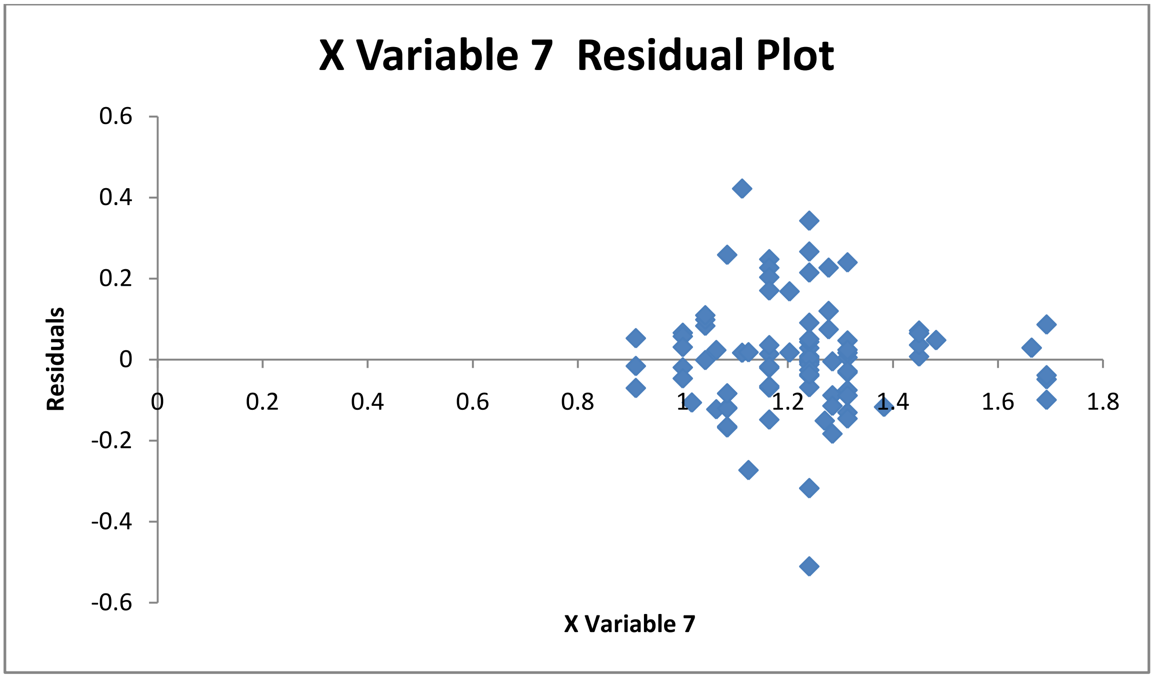

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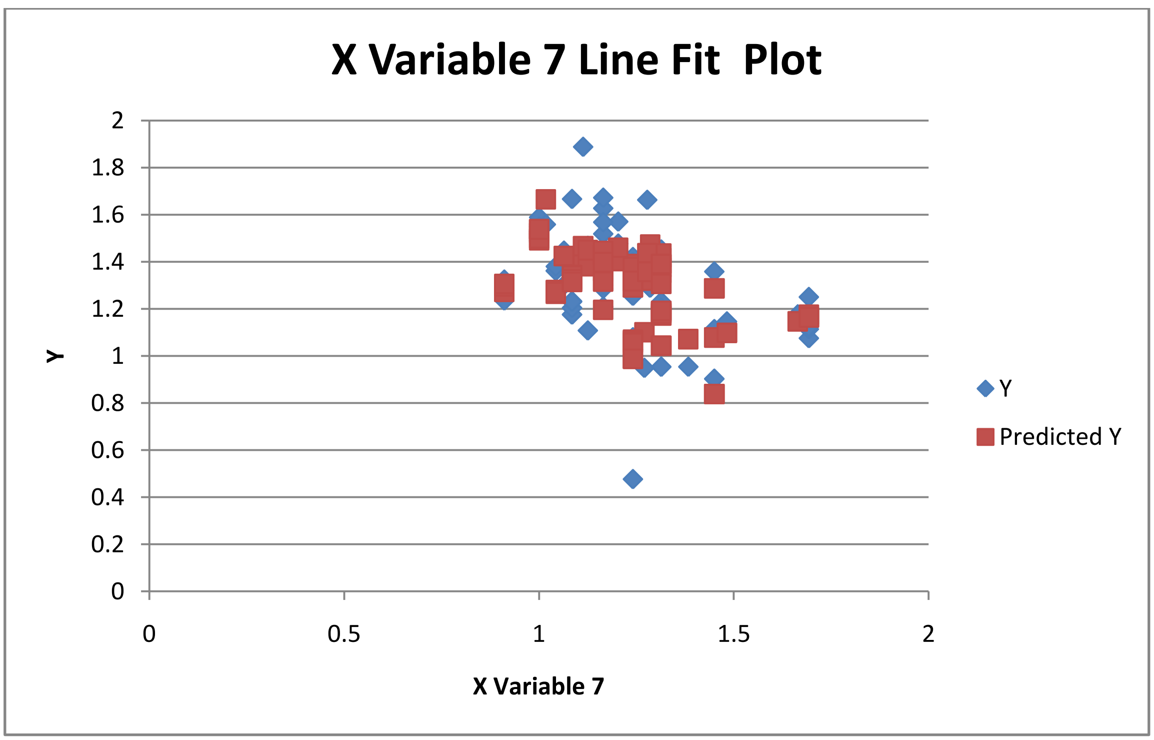

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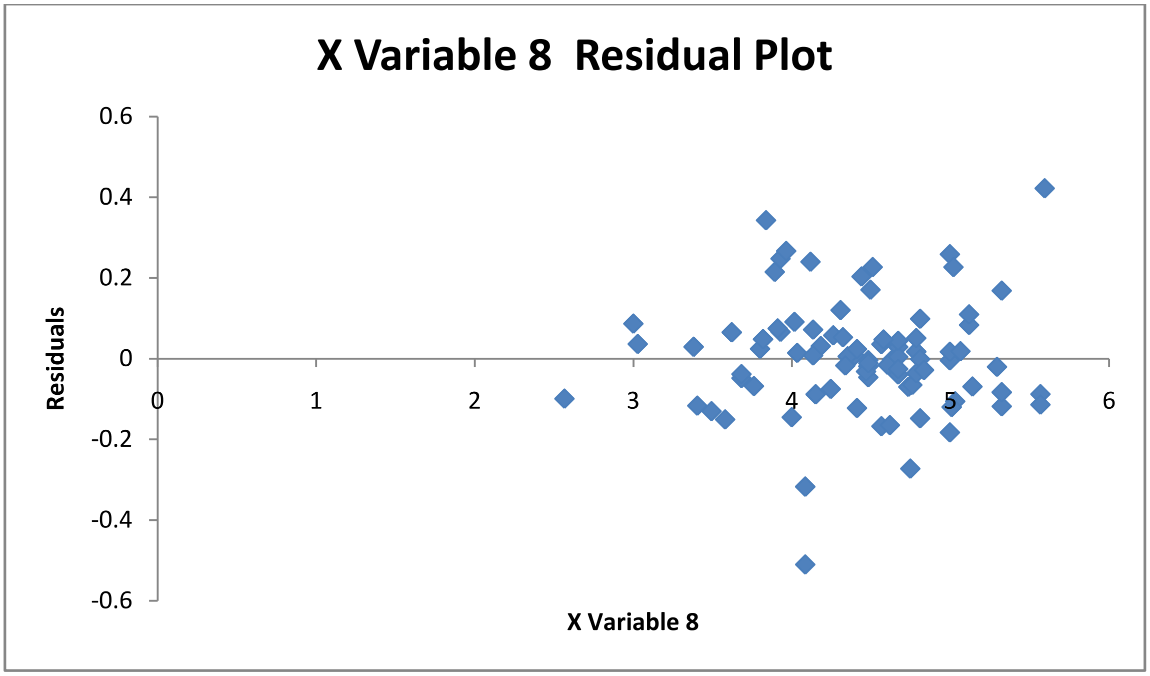

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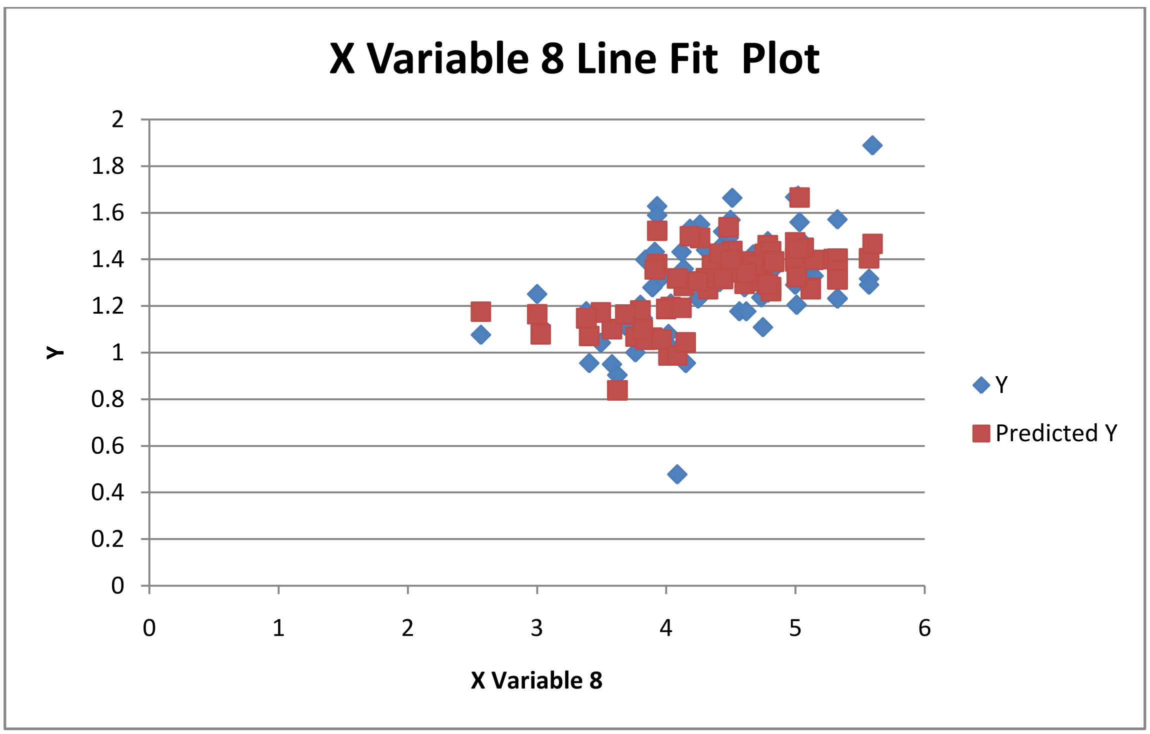

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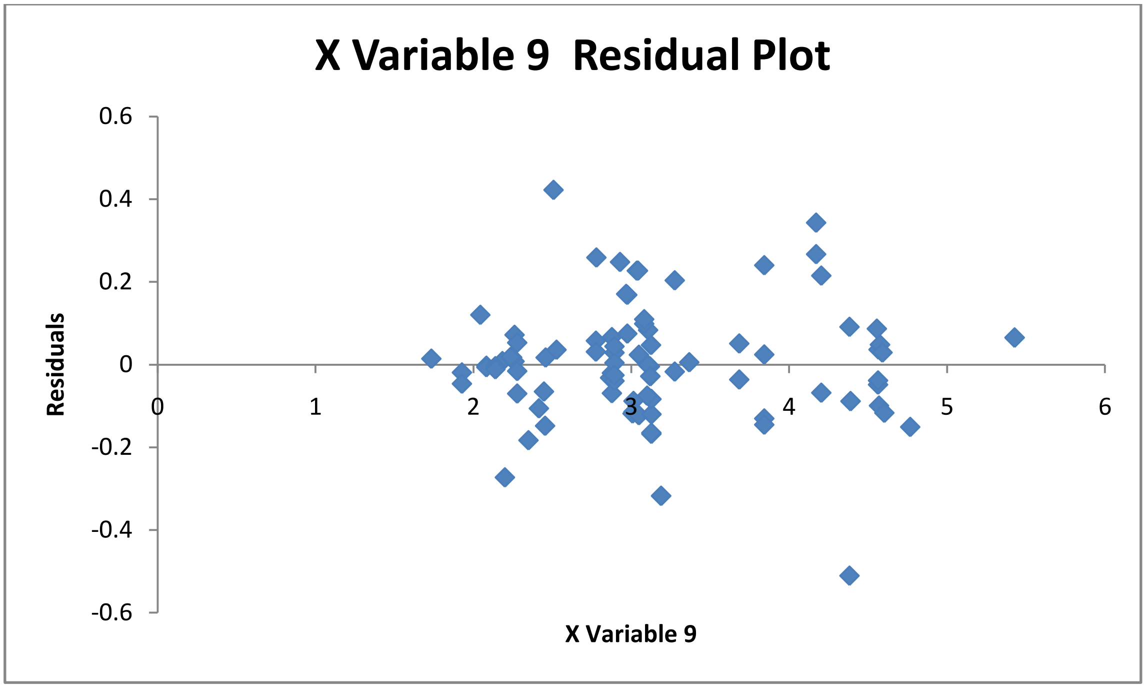

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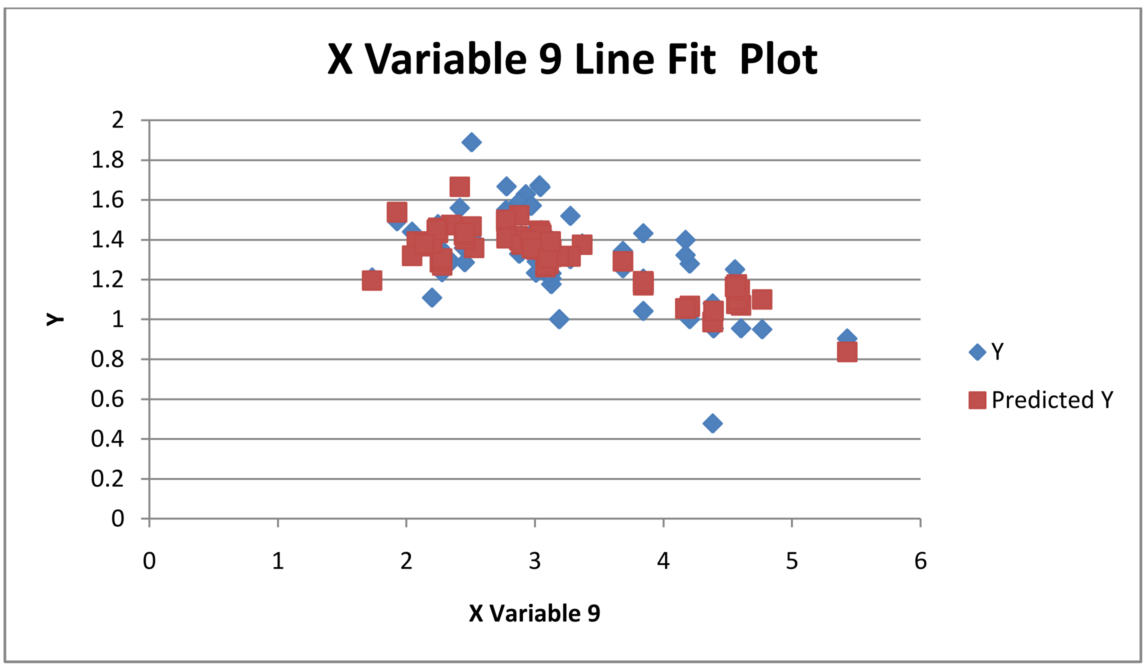

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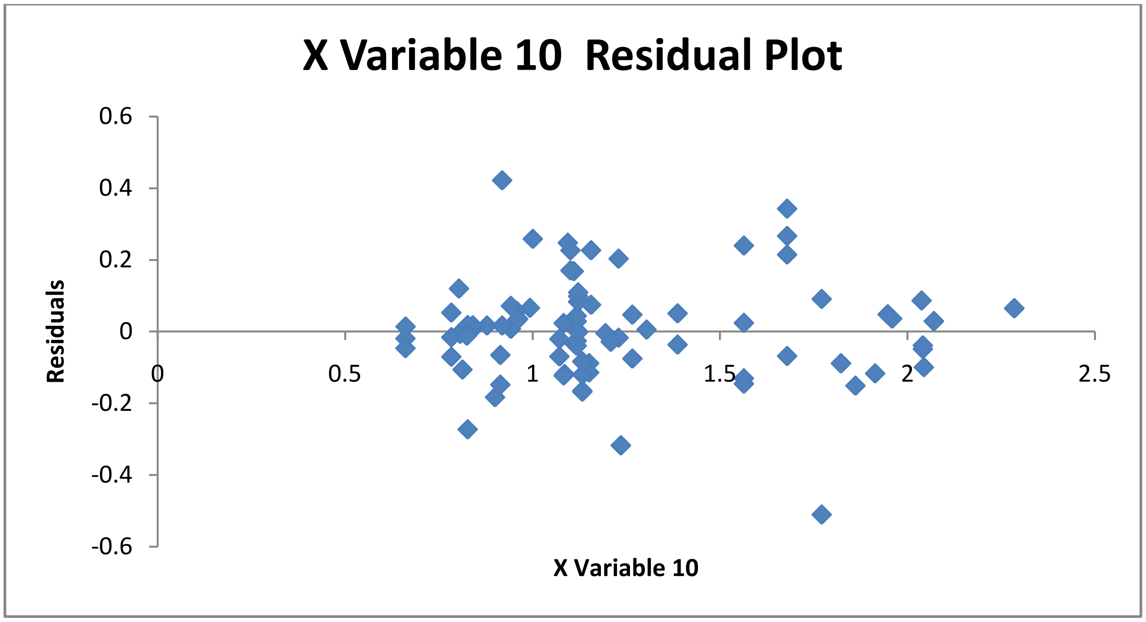

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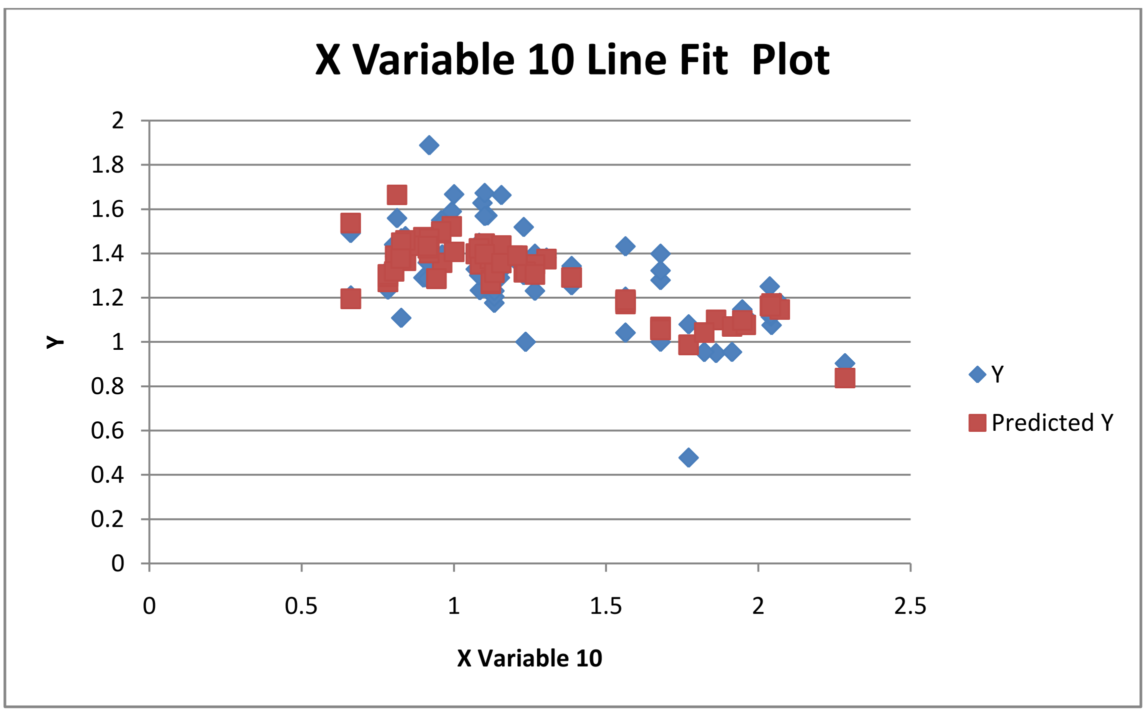

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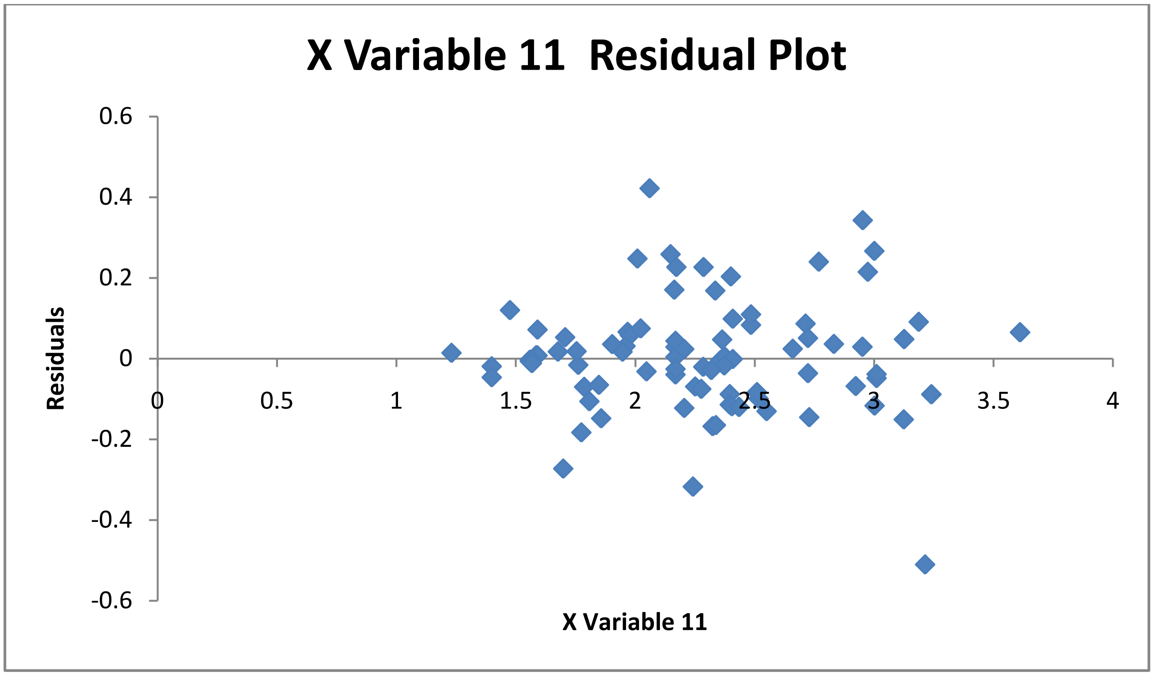

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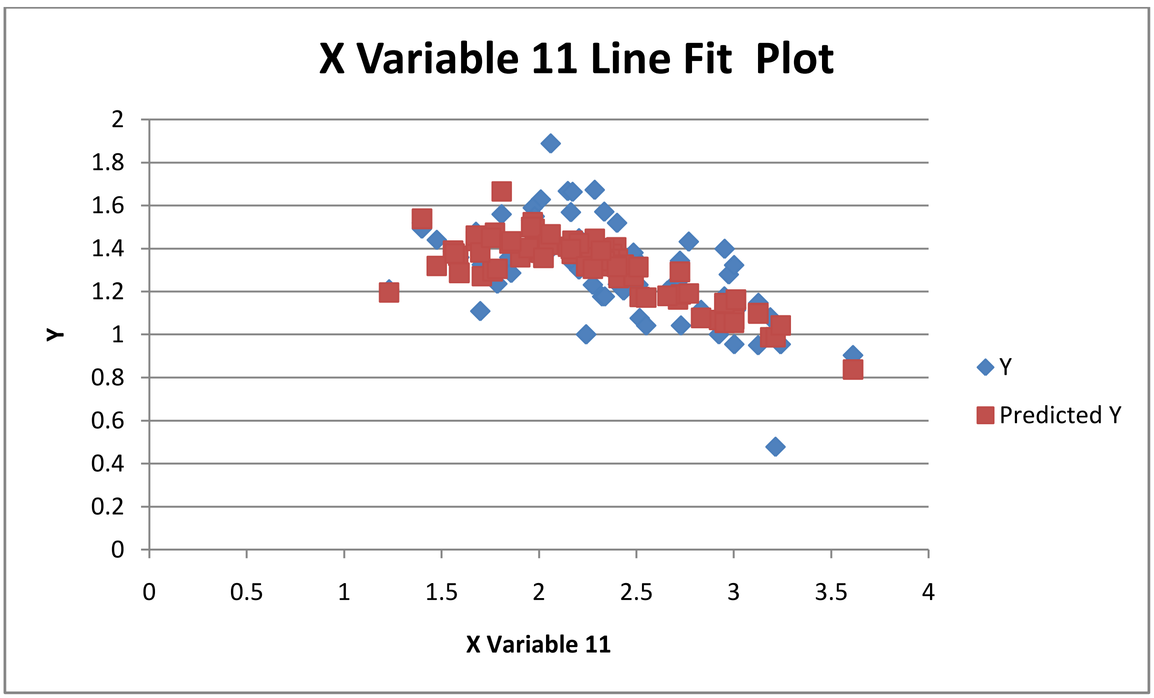

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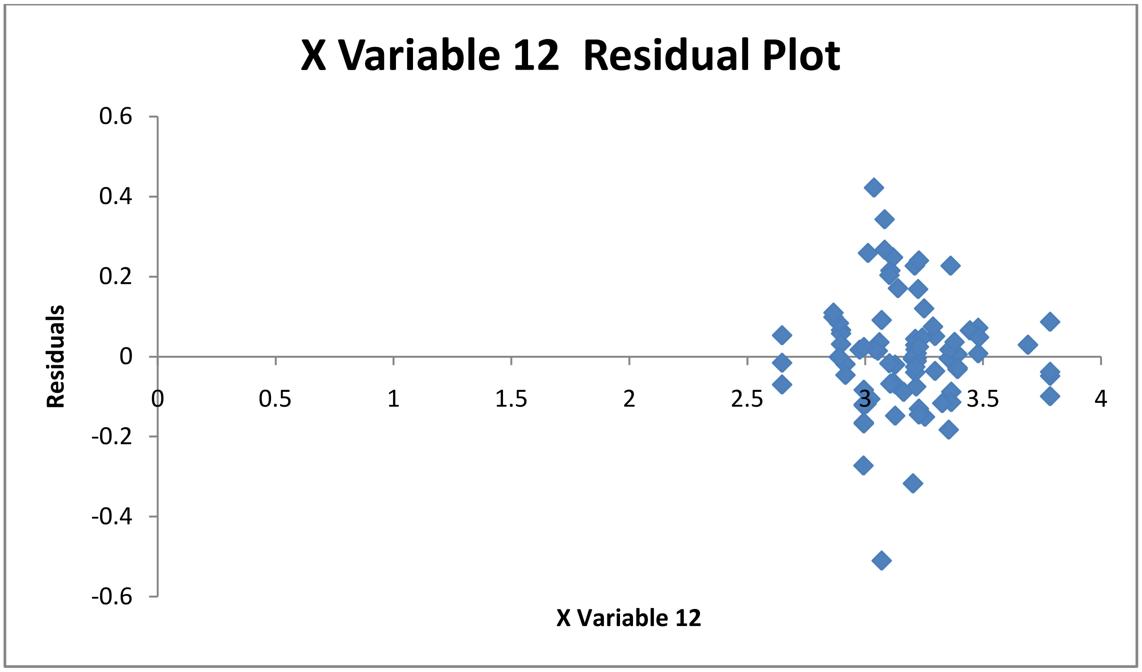

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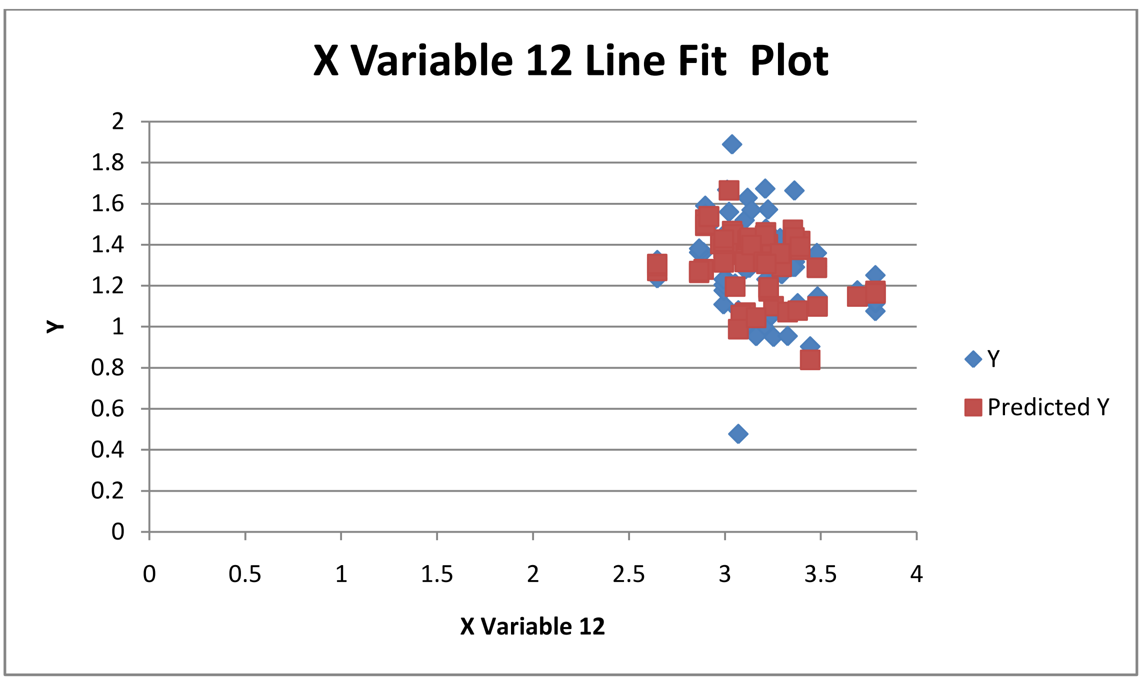

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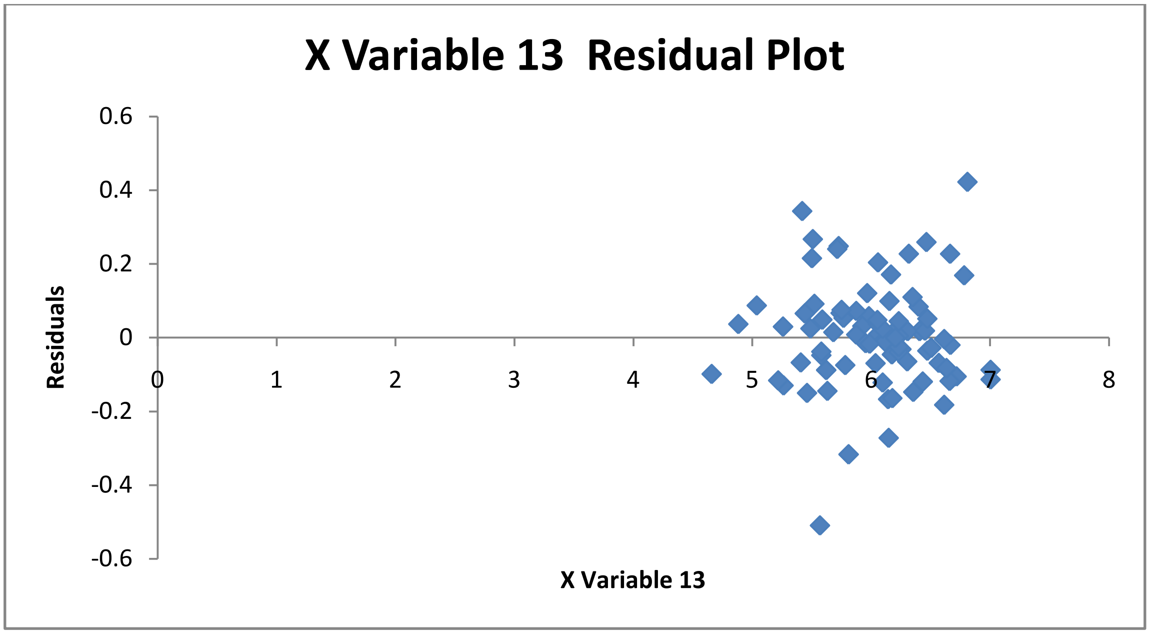

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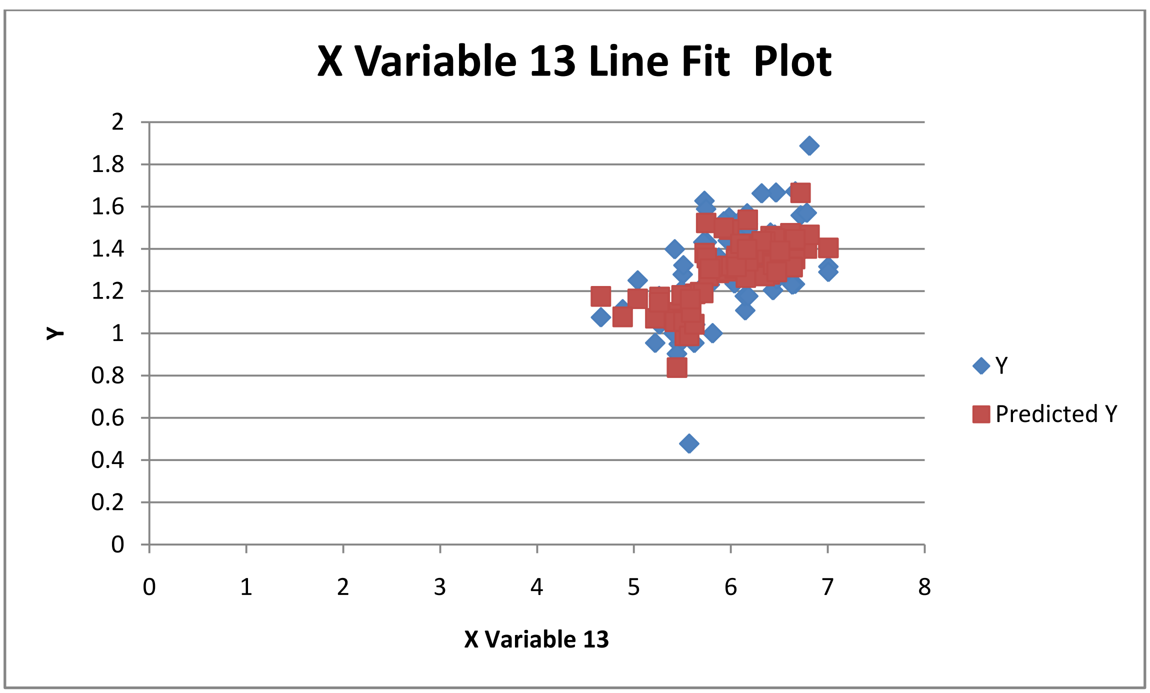

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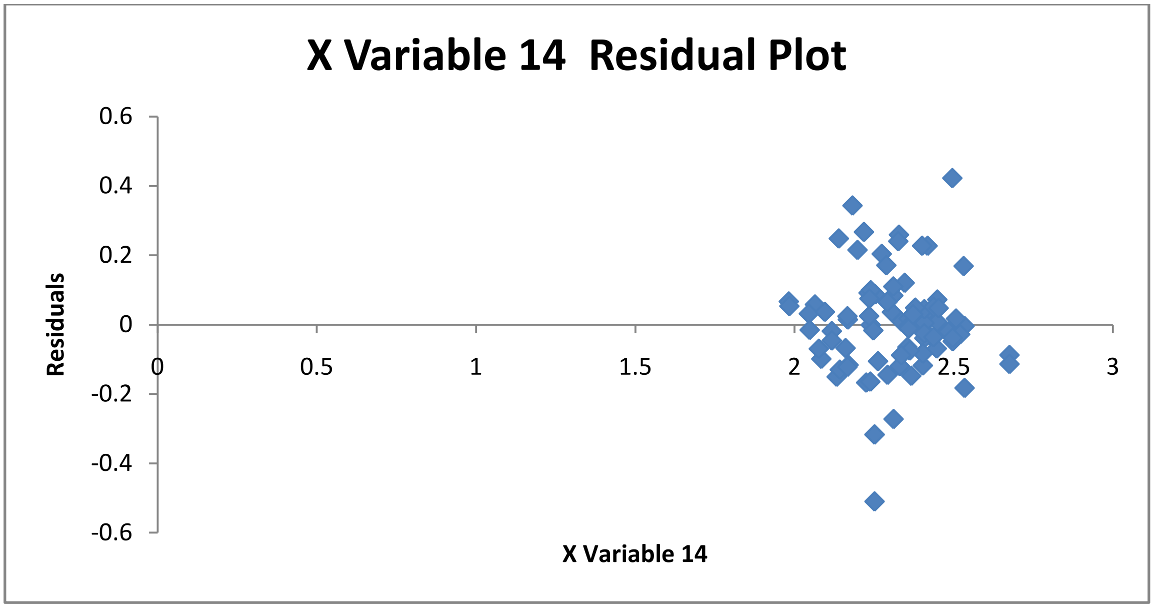

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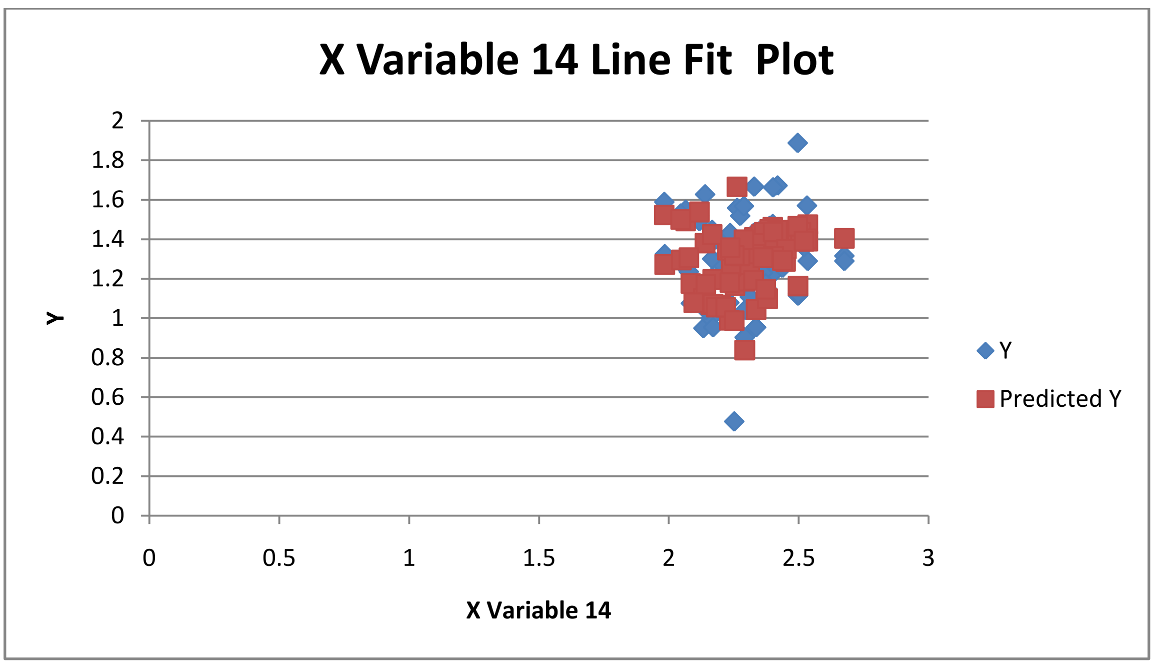

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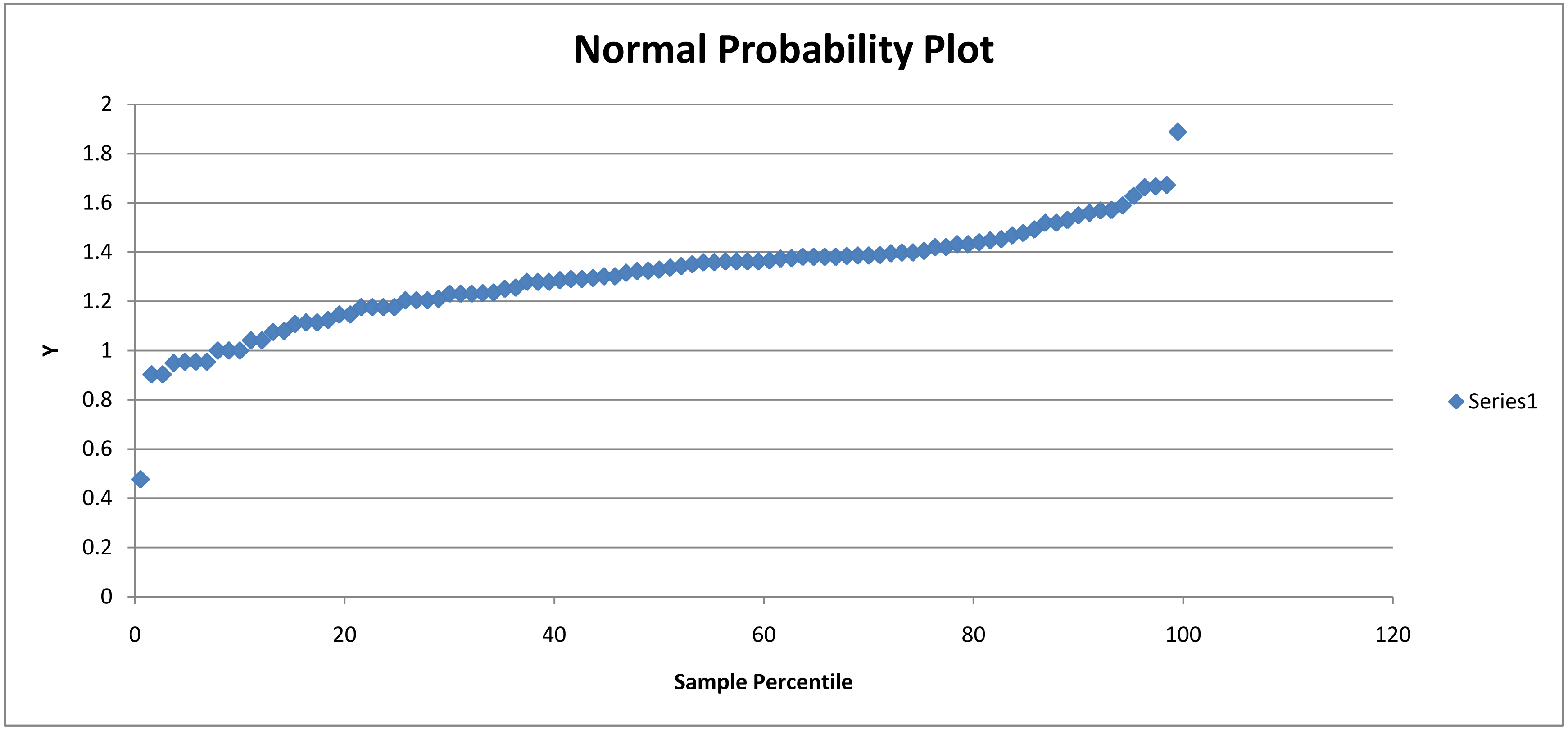

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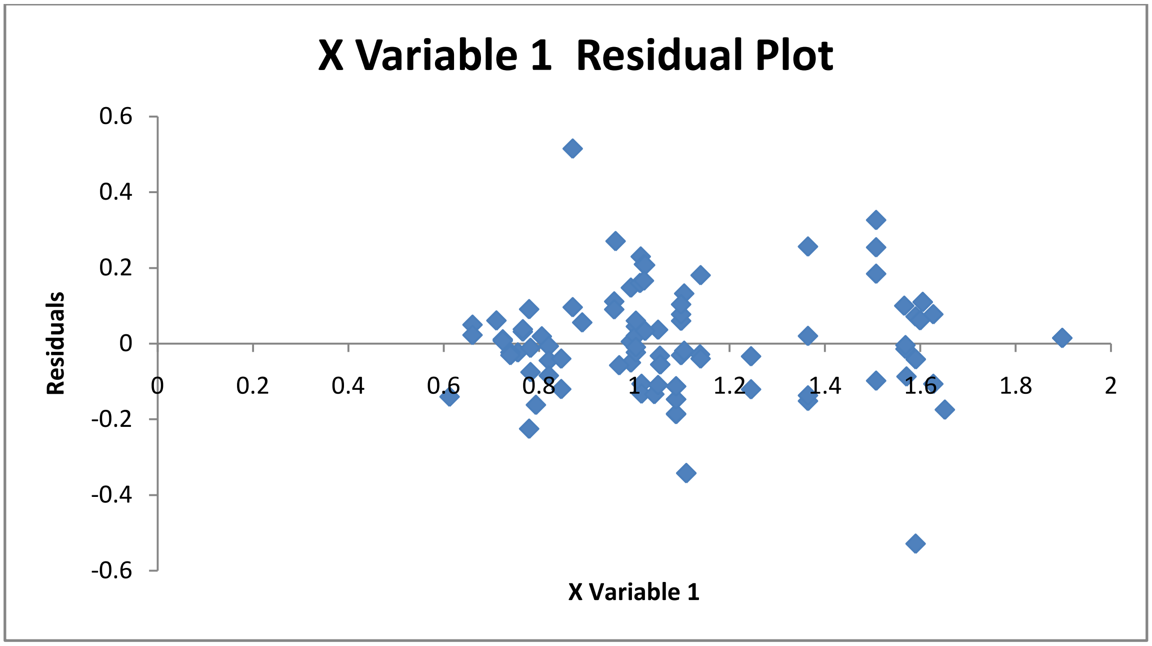

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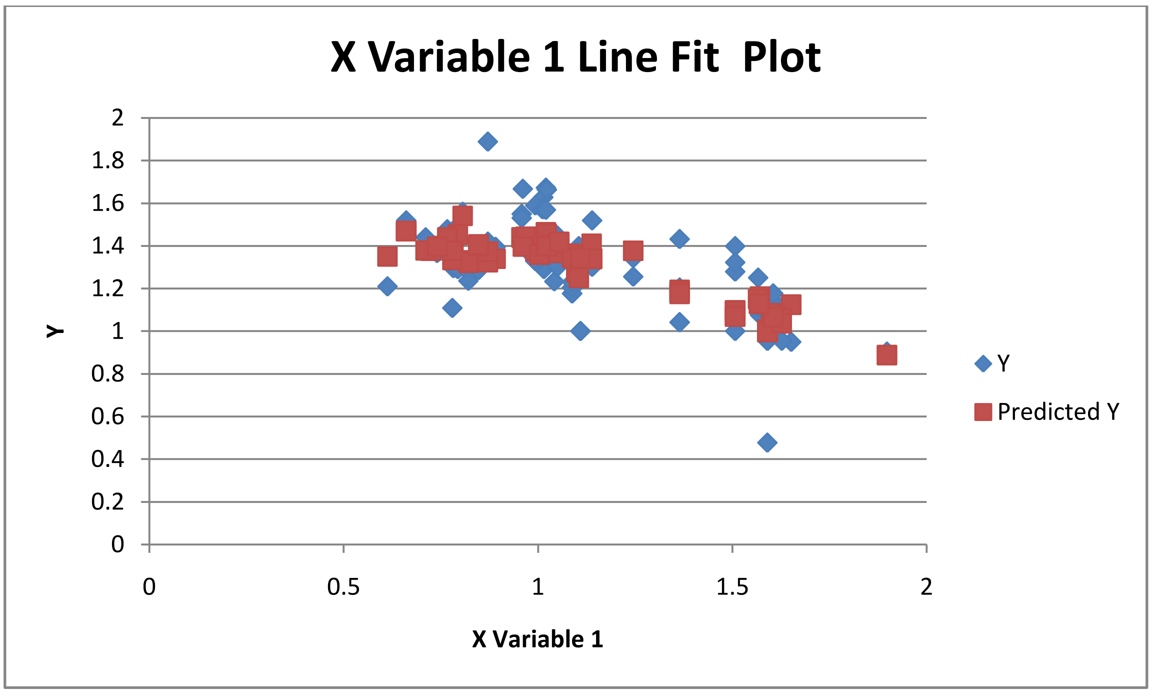

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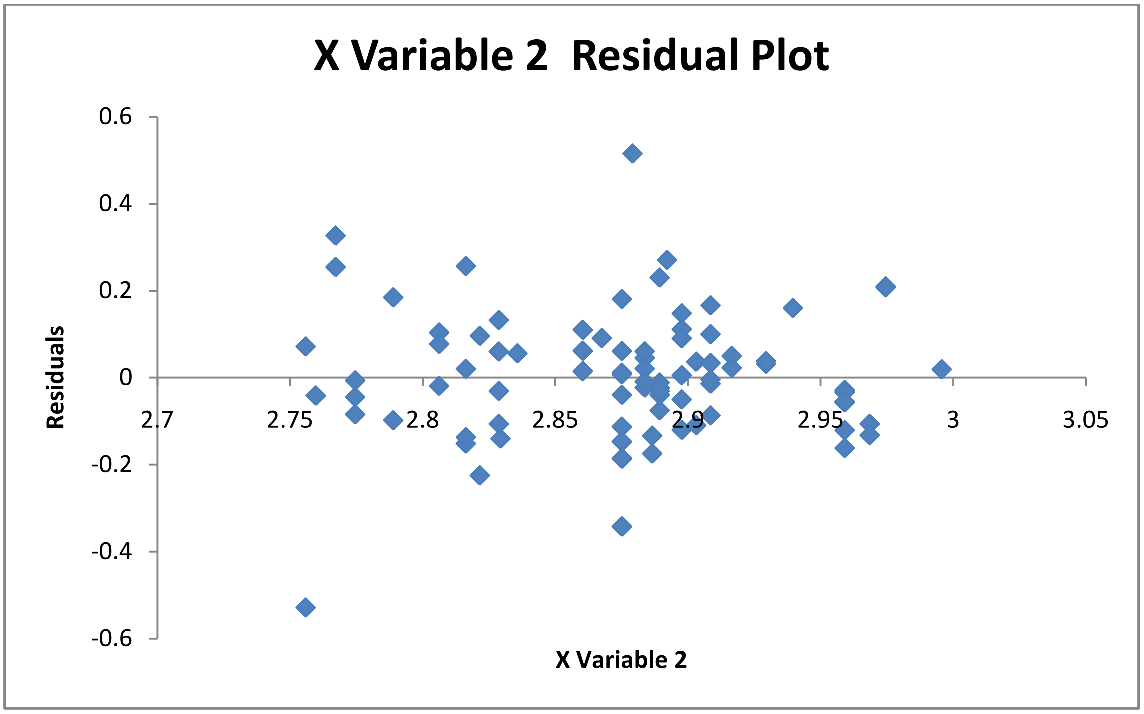

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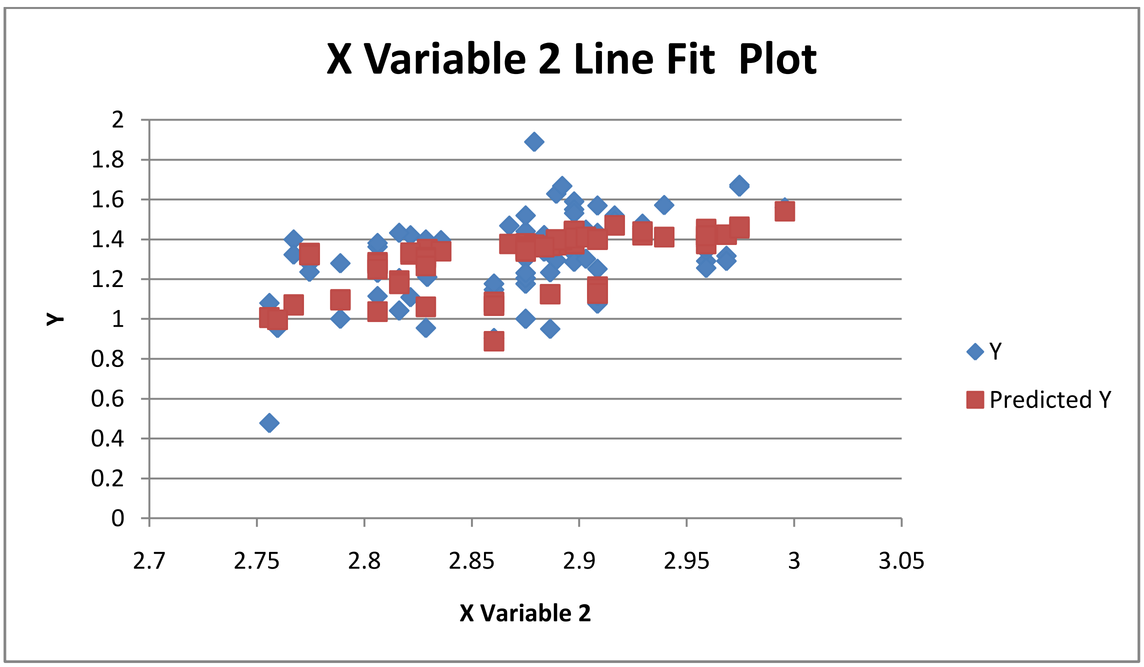

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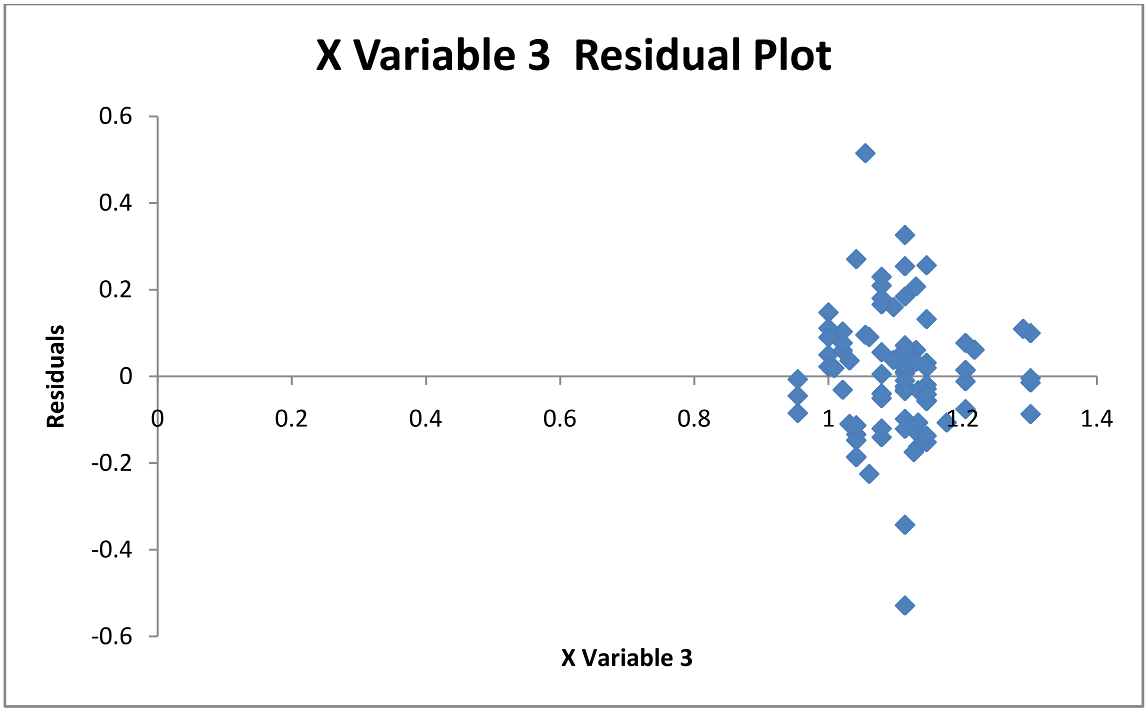

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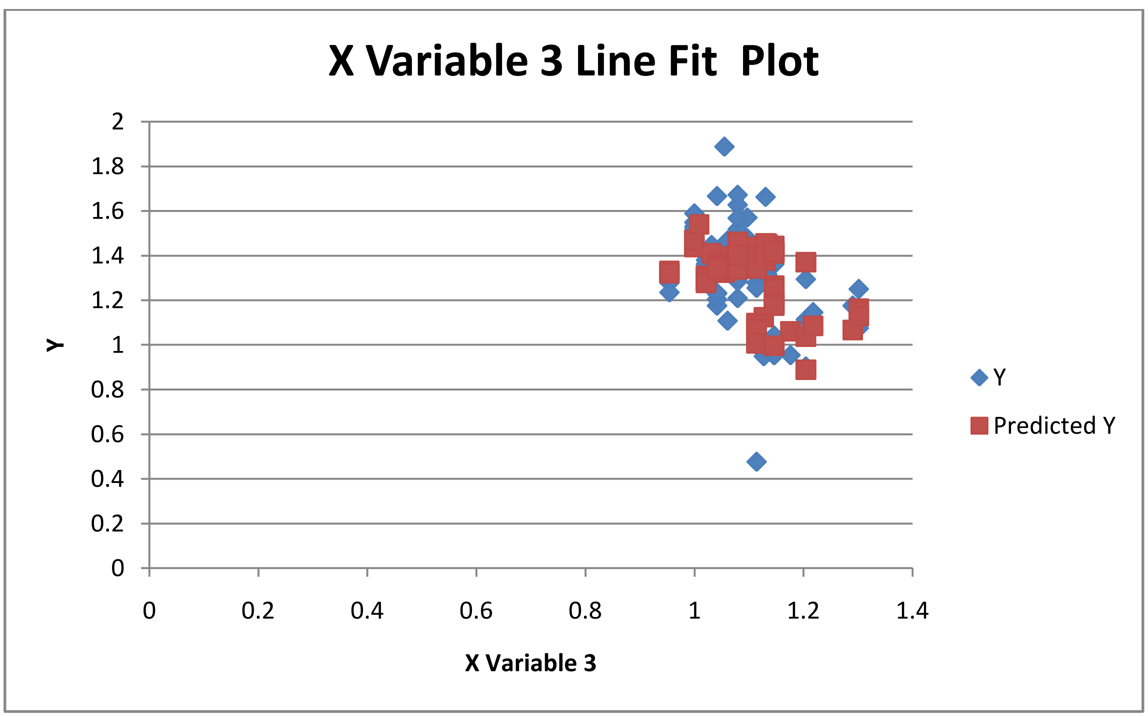

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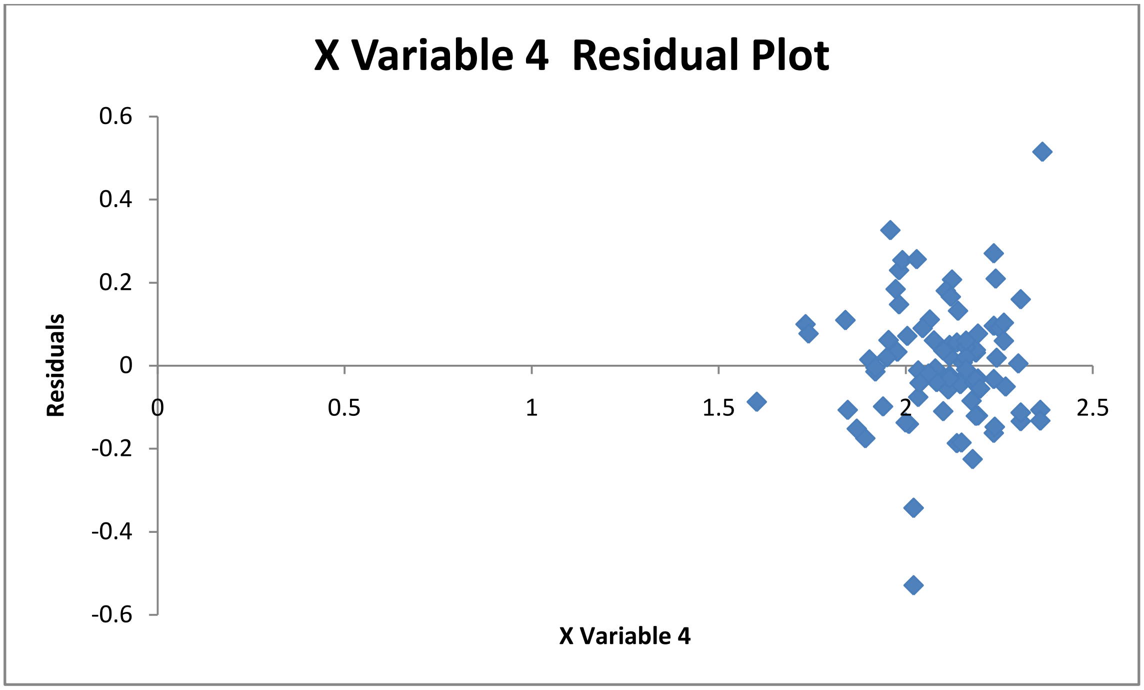

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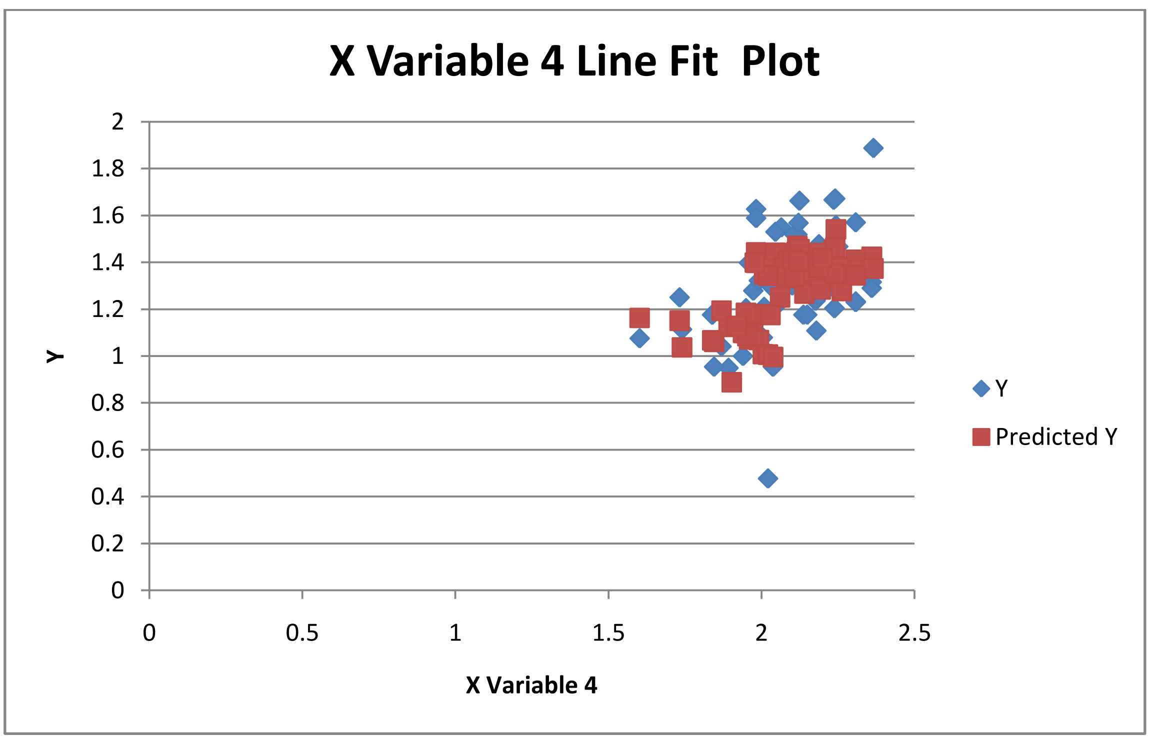

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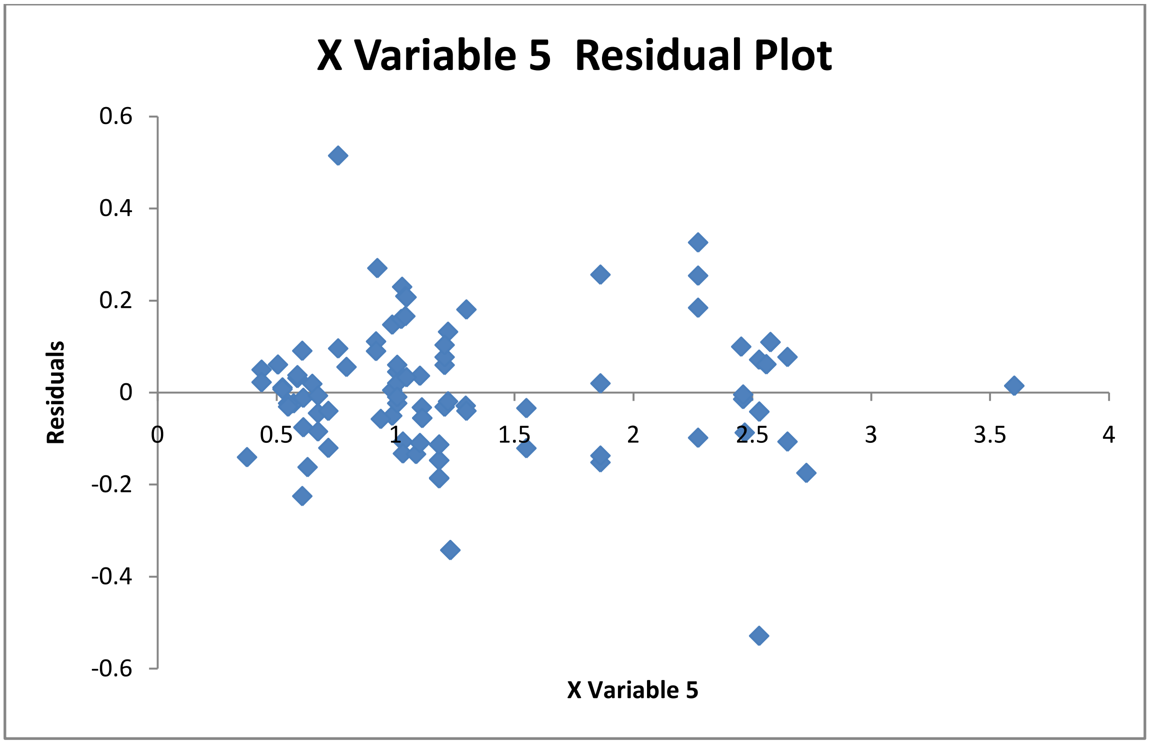

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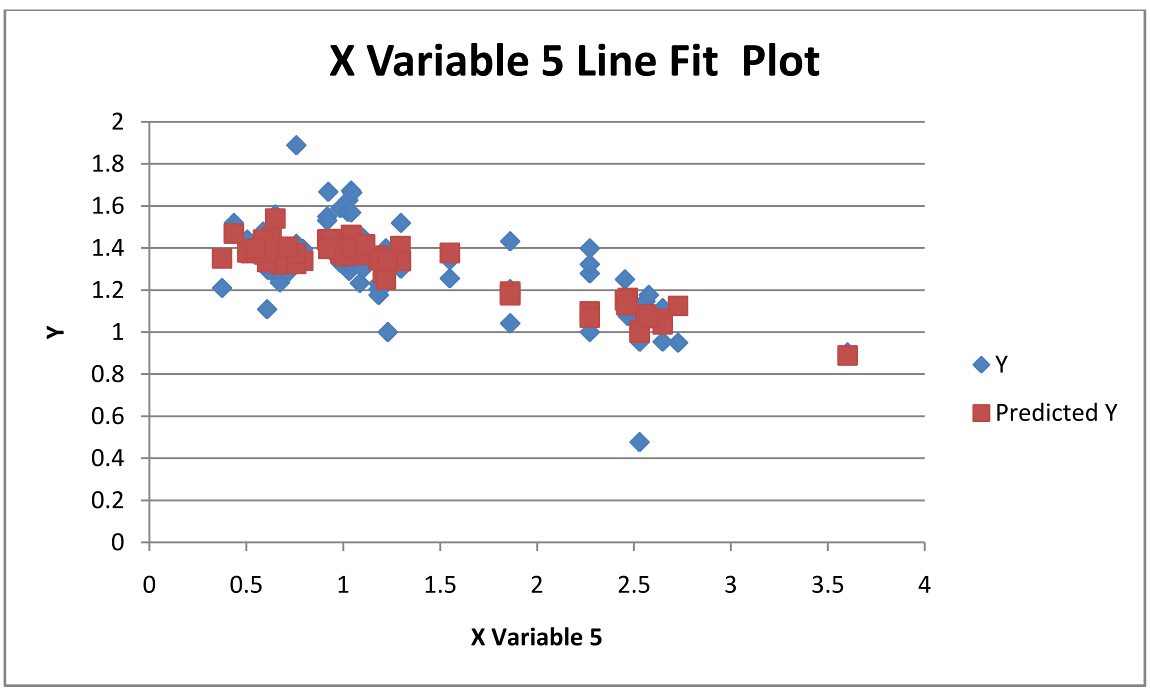

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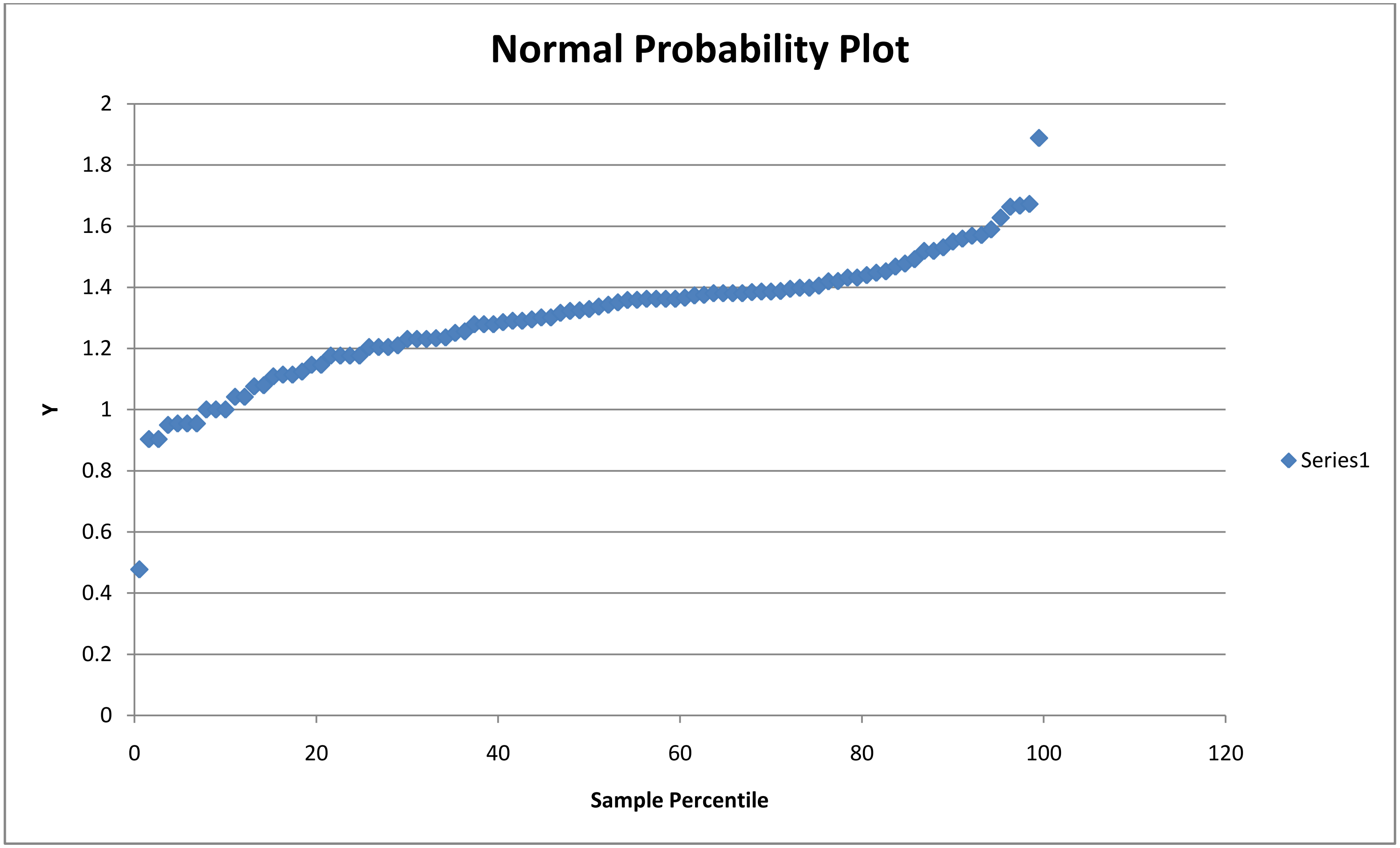

Normal Probability Plot for Regression Analysis of Final Model