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UNIVERSITI TEKNOLOGI PETRONAS

Optimization of Reservoirs Operation using the Genetic Algorithm and Seasonally Varied Models: Case of Perak Cascading, Malaysia

By
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OPTIMIZATION OF RESERVOIRS OPERATION USING THE GENETIC ALGORITHM AND SEASONALLY VARIED MODELS: CASE OF PERAK CASCADING, MALAYSIA

By

TILAHUN DERIB ASFAW

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DECLARATION OF THESIS

Title of thesis	Optimization of Reservoirs Operation using the Genetic Algorithm and Seasonally Varied Models: Case of Perak Cascading, Malaysia

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hereby declare that the thesis is based on my original work except for quotations and citations, which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UTP or other institutions.

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I dedicate this work to my late father Derib Asfaw Weldeamanuel and my mother Ejigayehu Shiferaw Agize

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ABSTRACT

The Perak cascading scheme located in the state of Perak, Malaysia, consists of four reservoirs, namely, Temenggor, Bersia, Kenering and Chenderoh. The reservoirs are used for hydroelectric power generation and flood control. The hydroelectric power potential of the cascading scheme is 578 MW, while the annual long-term historical average (HA) hydroelectric power generation was around 228 MW. It was about 39.46% of the potential capacity. Accordingly, the study aimed to improve the hydroelectric power generation of the scheme.

The genetic algorithm and the seasonally varied models have been developed to maximize the annual hydroelectric power generation of the Perak cascading reservoir. The fitness function of the genetic algorithm model (GAM) was to minimize the difference between the potential capacity and actual generation of the scheme. GAM was established with a total of 208 and 104 equality and inequality constraints, respectively. The optimal release decisions were found after checking the optimality of the population size (PS), the crossover probability (CRP) and the generation number (GN). Whereas, a seasonally varied model (SVM) has been developed after the analysis of the long-term HA operation data. Consequently, from the annual variation of the headrace level of Temenggor, the most upstream reservoir in the cascading scheme, four seasons are identified. The seasons are the refill, upper level, deplete and lower level. The two seasons that require a ranking order to maximize the energy-storage and to minimize the spill of water in the scheme are the refill and deplete. Hence, the refill rank order performed according to the decrease order of the change of power production in the change of storage volume, while the order of depletion has been conducted with the increase order of the storage effectiveness ratio.

Additional 12.17 MW and 4.56 MW of hydroelectric power per day were achieved using the operations of the GAM and SVM, respectively. The additional power using the operation of the GAM improved the capacity factor of the HA hydroelectric power generation to 41.56% and that of the SVM to 40.25%. In addition, the annual economic benefit is estimated at about RM 22 million and RM 8 million with the operation of the GAM and SVM, respectively. It showed that the result found from the GAM was better than the corresponding values of the SVM and HA. GAM also provided high-energy storage with respect to time to the cascading scheme. Meanwhile, the optimal value for GAM was achieved with a PS of 150, CRP of 0.75 and GN of 60 using four iterations. The analysis also indicated that the GN has also a significant impact on the computation of fitness value like PS and CRP. The test of computational runtime of the GAM indicated that the increase of the PS provided an increase of the runtime, while the increase in CRP resulted a decrease in the runtime. The refill and deplete operation of the Perak cascading scheme should be performing with the increasing order of storage and generation capacity of the reservoirs, respectively. In general, the additional hydroelectric power found using the operation of the GAM and SVM would have a significant contribution to the growing energy needs of the country.

ABSTRAK

Skim takungan bertingkat yang terletak di negeri Perak, Malaysia, terdiri daripada empat takungan iaitu, Temenggor, Bersia, Kenering dan Chenderoh. Takungan-takungan ini digunakan untuk penjanaan kuasa hidroelektrik dan kawalan banjir. Potensi kuasa hidroelektrik skim takungan bertingkat ini adalah 578 MW, manakala penjanaan kuasa hidroelektrik purata tahunan jangka panjang (HA) adalah 228 MW. Namun ia hanyalah adalah kira-kira 39.46% daripada kapasiti potensi. Sehubungan itu, kajian ini bertujuan untuk meningkatkan penjanaan kuasa hidroelektrik skim berkenaan.

Algoritma genetik dan model bermusim pelbagai telah dibangunkan untuk memaksimumkan penjanaan kuasa tahunan hidroelektrik takungan bertingkat di Perak. Fungsi keserasian model algoritma genetik (GAM) telah digunakan untuk meminimumkan perbezaan antara kapasiti potensi dan penjanaan sebenar skim ini. GAM telah dibangunkan dengan jumlah 208 dan 104 kesaksamaan dan kekangan ketidaksamaan, masing-masing dengan pembolehubah keputusan hanya bersandarkan kepada kadar pembebasan turbin. Nilai keserasian telah digunakan untuk menilai kesan optima saiz penduduk (PS), kebarangkalian silangatas (CRP) dan bilangan generasi (GN). Manakala, model bermusim pelbagai (SVM) telah dibangunkan selepas analisa jangka panjang data operasi HA. Sehubungan itu, empat musim dikenal pasti dari perubahan tahunan aras takungan Temenggor. Musim-musim berkenaan adalah isi semula, aras atas, peringkat penyusutan dan aras yang lebih rendah, yang berlaku berurutan dan berulang kali sepanjang tahun. Dua musim yang memerlukan kedudukan untuk memaksimumkan tenaga simpanan dalam skim ini adalah pengisian semula dan penyusutan. Oleh itu, urutan isi semula dilakukan mengikut urutan penurunan perubahan penjanaan kuasa untuk perubahan isipadu penyimpanan, manakala urutan penyusutan telah dijalankan dengan aturan peningkatan nisbah simpanan yang berkesan.

Tambahan 12.17 MW dan 4.56 MW kuasa hidroelektrik sehari didapati dengan perlaksanaan GAM dan SVM. Kuasa tambahan ini telah meningkatkan kadar kapasiti penjanaan kuasa hidroelektrik HA kepada 41.56% dan 40.25%. Di samping itu, menafaat ekonomi tahunan dianggarkan kira-kira RM 22 juta dan RM 8 juta dengan pengoperasian GAM dan SVM, masing-masing. Hasil yang didapati dari GAM adalah lebih baik daripada SVM dan HA. Kajian ini juga mendapati GAM telah menghasilkan penyimpanan tenaga tinggi berdasar masa kepada skim takungan bertingkat di atas. Parameter optima GAM telah dicapai dengan PS 150, CRP 0.75 dan GN 60 menggunakan empat lelaran. Analisa juga menunjukkan bahawa jumlah lelaran (berjalan) menurun dengan peningkatan PS. Ujian masalaksana pengiraan GAM menunjukkan impak daripada GN adalah terlalu kecil, tetapi peningkatan HP dan CRP mempunyai kesan langsung dan tidak langsung pada pengiraan masalaksana masing-masing. Refill dan mengurangkan operasi skim melata Perak perlu melaksanakan dengan perintah yang semakin meningkat penyimpanan dan kapasiti penjanaan takungan, masing-masing. Secara umum, kuasa hidroelektrik tambahan yang didapati menggunakan operasi GAM dan SVM akan mempunyai sumbangan penting kepada keperluan tenaga yang semakin meningkat di negara ini.

TABLE OF CONTENT

LIST OF FIGURES

LIST OF TABLES

LIST OF NOMENCLATURE

General Abbreviations

AI	Artificial Intelligence
ARF	Area Reduction Factor
ASL	Above Sea Level
CF	Capacity Factor
CGA	Chaotic Genetic Algorithm
CI	Cumulative Inflow
CP	Critical Period
CRP	Crossover Probability
DID	Department of Irrigation And Drainage
DP	Dynamic Programming
DS	Deplete Season
DSL	Dead Storage Level
ES	Energy Storage
FER	Firm Energy Requirement
FSL	Full Supply Level
GA	Genetic Algorithm
GAM	Genetic Algorithm Model
GN	Generation Number
HA	Historical Average
HEC	Hydrologic Engineering Center
HWL	Headrace Water Level
IP	Integer Programming
IQR	Inter-Quartile Range
LLOS	Lower Level Operating Season
LP	Linear Programming
MMGA	Multi-Objective Genetic Algorithm
MPR	Mutation Probability
MWL	Maximum Water Level

NDL	Normal Drawdown Level
NLP	Non-Linear Programming
NR	Number of Run
OPE	Overall Plant Efficiency
PM	Peninsular Malaysia
PS	Population Size
PSO	Particle Swarm Optimization
RM	Ringgit Malaysia
RS	Refill Season
SA	Simulated Annealing
SDDP	Stochastic Dual Dynamic Programming
SDP	Stochastic Dynamic Programming
SER	Storage Effectiveness Ratio
Sg	Sungai (River)
SVM	Seasonally Varied Model
TGR	Three Gorges Reservoir
TNB	Tenaga Nasional Berhad
TS	Tabu Search
TWL	Tailrace Water Level
ULOS	Upper Level Operating Season
UN	United Nation
WT	Wavelet Transform

General Notations

A	Water Surface Area
E	Evaporation
F	Rainfall
f	Fraction
GW	Gigawatt
GWh	Gigawatt-hour
h	Head
h_{eff}	Net head

h_L	Headloss
hr	Hour
I	Total Inflow
km	Kilometer
K_p	Pan Coefficient
kW	Kilowatt
kWh	Kilowatt-hour
l	Distance between Rainfall Stations
L	Length of Season
m	Meter
MW	Megawatt
MWh	Megawatt-hour
n	Data Size
No	Number
O	Outflow
P	Hydroelectric Power
q	Number of the Tie Group
Q	Natural Inflow
R	Turbine Release
r	Rank of the Data
R^2	Regression Coefficient
S	Reservoir Storage Volume
Ss	Statistics of the Data Trend
t	Time
t_p	Number of Data Point in the Tie Group
V	Increase Power Production with Increase in Storage
v^2	Variance
w	Seasonal Constant
X	Data Value
y	Number of Rainfall Stations
Y	Location of the Quartile Data
Z	Normalized Test Statistics
μ	Mean

γ	Unit Weight of Water
η	Overall Plant Efficiency
σ	Standard Deviation

CHAPTER 1

INTRODUCTION

1.1. Water Resource Development Challenges

Water is a crucial issue in every nation because of its occurrence, quantity and quality. All living things consume water, and it requires a continuous and sufficient supply. However, the supply issues are challenging due to the spatial and time variation of water. The challenges of a water resource development can be grouped into three categories. The first challenge relates to the collection of water. Some areas are rich in water, while scarce in the other areas. An abundant quantity of water can be obtained in rainy periods, and insufficient amount of water during the off-periods (low rainfall periods). Thus, the spatial variation and the timely occurrence of rainfall make it difficult to collect an ample quantity of water. The impact of the time variation of water can reduce by storing water during the rainy (high-flow) period. The purpose of storing water is to supplement the deficit that can occur during the off-periods. The second category of the challenge in water resource development relies on the operation, and management of the collected (stored) water. This challenge arises due to the dynamic nature of water demands and the sustainability of the supply. The third category of the challenge relates to the disposal of surplus (excess) water [1].

Dam is a hydraulic structure; it is constructed across a river/stream to store water. The water store behind the dam is known as a reservoir. The operation and management of a reservoir may become an issue after the storage is accomplished. The development and management of water in the reservoir end with the optimization of the operation system [2]. Optimization of a reservoir operation varies with the reservoir's purposes and objectives. For instance, in a multi-purpose reservoir operation, the objectives can be contradicting to each other, and it becomes challenging to manage. This research focused on the analysis and the operation of a cascading reservoir system. The cascading scheme is used for hydroelectric power generation and flood control.

1.2. Water and Energy

According to the UN statement, water and energy are among the eleven global challenges in the future due to the tremendous increasing of the world population [1]. Thus, the rate of energy production should be increased in parallel with the population growth to satisfy the additional demands. There are two ways to increase the production of the electricity: by the construction new schemes, and by enhancing of the efficiency of the present system. It can apply both choices simultaneously or either of the method to increase the production of electricity. The choice of the methods can vary with the problem that will be faced. In this study, the latter approach has been chosen because the future management of water will be shifted from constructing new systems to improving the existing ones [3]. In addition, the global water challenge in the twenty-first century will be focused on the management of the source than the development [1]. As mentioned in Section 1.1, the operation and management of a reservoir are the second type of challenge in the water resource developments. Hence, the main target of this research was to improve the hydroelectric power generation from a cascading scheme through the development of an optimal reservoir operation model.

Hydroelectric power is a renewable energy source because the water is continuously being replenished by the precipitation. The source is advantageous because:

- the cost of a power station is generally much lower than the same capacity of a fossil-fuel power station,
- it is sustainable (naturally replenished source),
- it is generally environmentally friendly and no emission of pollutants into the air or water,
- it can be switched on and off rapidly and
- it is easy to control the generation quantity [4].

In hydroelectric power generation, energy and power are important terminologies. Energy refers to the capacity of water to do the work. Energy is measured in kilowatt-hours (kWh), megawatt-hours (MWh), etc. Whereas, power is the rate of transferring energy per unit time, and it can be measured in kilowatt (kW), megawatt (MW), etc. Capacity factor (CF) is the ratio of the actual hydroelectric power generation to the maximum potential that can be produced. CF is used to evaluate the performance of the hydroelectric power generation plant. The value of a CF depends on the availability of the fuel (it is water in the case of a hydroelectric power), the transmission system efficiency, the electricity demands and the length of the plant maintenance.

1.3. General Description of the Study Area

Malaysia is located in the Southeast of Asia. It has two land masses, namely, the west (Peninsular) and east parts, which are separated by the South China Sea. The 2010 census data showed that the total population of the country has reached 28.3 million. The average growth rate of the population from 1991 to 2010 was 2.6% [5]. The total area of the country is about 329,750 km^2. The altitude in Peninsular Malaysia (PM) varies within the range of zero to 2187 m above sea level (ASL). However, two-thirds of the Peninsular land lay above an altitude of 200 m with steep and densely forested mountains that rise from the flat coastal area [6].

The country has a huge rainfall potential; it receives about 990 billion cubic meters annually. An average annual rainfall in PM is around 2420 mm. Out of this annual quantity of rainfall, 36.3% is lost through evapotranspiration, 57.2% is a surface run-off and 6.5% of it recharges the groundwater. In the country, the average annual temperature varies between 25.5 to 32 degrees Celsius [7] and that of the relative humidity in the range of 75-95%. The four principal rivers in the PM are Sg. Perak (390 km), Sg. Muar (190 km), Sg. Pahang (500 km) and Sg. Kelantan (250 km) [6]. In the year 2008, the country had seventy-three dam reservoirs that are used for water supply, irrigation, hydroelectric power generation, flood mitigation and other purposes [8]. Among these, forty-seven were single purpose reservoirs, and the other sixteen were multi-purpose reservoirs. Only eleven dam reservoirs were used for hydroelectric power generation.

Electricity generation in Malaysia is from two main sources: renewable and non-renewable [9]. Currently, around 87% of the country's source of energy are from the non-renewable sources, and it is highly depleting. To combat the problem of the depletion of the non-renewable sources of electricity, the country has planned to increase the generation mixes of the renewable resources. Accordingly, hydroelectric power is one of the best alternative options [10], because of its sustainability, minor impact on the environment, and low operating cost [11]. Thus, development of a hydroelectric power in Malaysia would be viable due to the plenty of rainfall [12]; in addition, the country has a huge hydroelectric power potential, about 29 GW [11].

In 2008, the installed hydroelectric power generation capacity throughout the country from the seven schemes was about 2.091 GW; it was about 7% of the total potential of the country. The schemes are developed in the different river basins of the country. The Perak scheme, locates in the state of Perak of the PM, is among one of the seven schemes. It is the most utilized basin in terms of hydroelectric power development [13]. The capacity of the Perak scheme is about 649.1 MW that constitutes about 31% of the developed hydroelectric potential of the country. The scheme comprises six stations, namely, Temenggor, Bersia, Kenering, Chenderoh, upper Piah, and lower Piah. The first four stations (Temenggor, Bersia, Kenering and Chenderoh) are found in cascade as shown in Figure 1.1 (a). The cascading hydroelectric power potential is about 578 MW that constitutes around 89% of the total scheme's potential [14]. The Perak cascading reservoirs are also used for flood mitigation.

Figure 1.1 (b) shows the elevation of the reservoirs above sea level (ASL), the storage capacity, the water surface area at full supply level (FSL), and the distance between the dams. The figure also indicates that the largest storage capacity reservoir, Temenggor, is located at the most upstream side. Whereas, the smallest power generation capacity plant, Chenderoh, is located at the most downstream side. In addition, Table 1.1 illustrates the basic information of the Perak cascading reservoirs and the plants' specifications. The table shows that the largest storage capacity in the cascading system, Temenggor, is about eighty-six times greater than the smallest reservoir, Bersia. The installed capacity of the largest power plant, Temenggor, is nearly nine times greater than the smallest, Chenderoh.

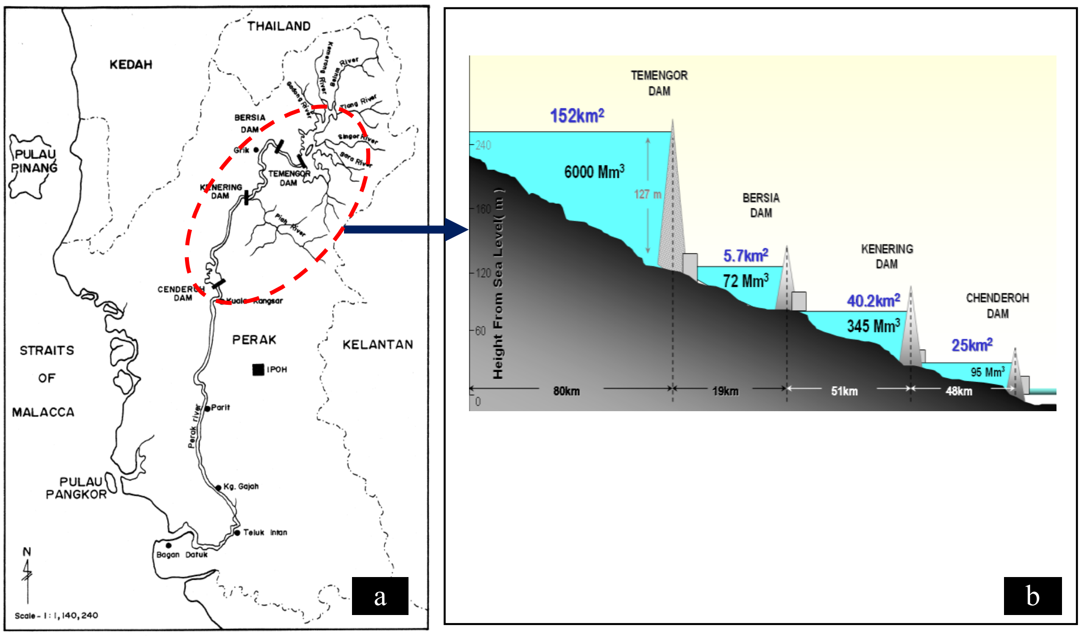


Figure 1.1 Perak River cascading reservoirs (a) location [15], (b) longitudinal view

Table 1.1 Descriptions of the Perak cascading hydroelectric power reservoirs

Description	Unit	Temenggor	Bersia	Kenering	Chenderoh
Year of construction	Year	1977	1983	1983	1926
Installed power capacity	MW	348	72	120	38
Number of turbines	No	4	3	3	4
Full supply level (FSL)	m (ASL)	248.41	141.43	111.31	60.42
Normal drawdown level (NDL)	m (ASL)	239.30	139.90	108.50	59.13
Tailrace water level (TWL)	m (ASL)	142.07	115.52	75.87	41.41
Warning level	m (ASL)	246.00	140.00	111.00	60.00
Storage capacity at FSL	10^6 m^3	6000	70	345	95.4
Water surface area at FSL	km^2	152	5.7	40.2	25
Range of operating level	m	9.11	1.53	2.81	1.29
Rated head	m	101.00	26.50	34.70	18.29
Operation hours per day	hr	24	24	24	24

1.4. Model Development

Various optimization and simulation models are applied to optimize the operation of the multi-reservoir system. In this study, two models have been developed to maximize the hydroelectric power generation of the Perak cascading scheme. The first model is developed using an optimization technique. It is known as the genetic algorithm model (GAM). The second model is developed considering a simulation technique, it is known as a seasonally varied model (SVM).

Genetic algorithm (GA) optimization technique is robust to find the global optimal value and it creates high-quality solutions for problems that are little known [16]. In addition, GA can easily apply in the complex systems [17]. The genetic algorithm model (GAM) applies after the determination of the optimal parameters of the population size, crossover probability, and generation number.

The SVM developed after the study of the headrace level variation of the most upstream side of the reservoir, Temenggor and the computations of the refill and depletion ranking orders of the Perak cascading reservoirs. The decisions on the rate of release and hydroelectric power generation using the operations of the GAM and SVM are compared to the corresponding long-term historical average (HA) values. The comparisons of the result are accomplished by taking an equal cumulative annual volume of the turbine releases. The models decision on the rate of the turbine releases and the energy-storage in the cascading scheme with respect to time are also used to evaluate and prove its advancement in the operation of the cascading reservoirs.

1.5. Rationale of the Study

In the twenty-first century, the global energy consumption rate is growing faster than the supply [18]. Currently, over 80% of the world electricity are from thermal resources such as coal, gas, oil, etc. [18]. However, the non-renewable resources are not sustainable. On the other hand, the energy from a nuclear plant is viable, but it has significant environmental concerns. The other alternative source of energy is the renewable technology (wind, solar, geothermal, hydro, biomass, etc.). Among these renewable sources, hydro is the most viable, clean and sustainable [18].

In Malaysia, the long-term energy security (the sustainability of production) is a crucial problem [14]. Emission is also another challenge of the country due to the fuel that is used to generate electricity. The Tenth Malaysia Plan (2010-2015) outlines that the country has planned to reduce high emissions. The plan was implemented through the diversification of the fuel composition that is used for power generation. Mainly, this can be achieved by reducing the share of the thermal plants [14] and replacing it with the renewable sources. The adverse effects of the renewable sources are negligible as compared to the thermal plants.

Studies indicated that the electricity needs of the country were growing at higher rates due to the change of living standards of the people and technological advancements. For example, the year 2030 energy consumption of the country is projected at nearly three times that of the 2002 level [19]. The electricity consumption of the country was increased by 200% from 1990 to 2008; the growth was equivalent to 6-8% per annum [14]. The prediction also showed that the energy demand of the country is expected to grow by 5-7.9% in the next 20 years [9]. In addition, prediction showed that the energy demand of Malaysia in the year 2020 is expected to be increased by threefold [20]. However, the global total electricity generation from 1990-2020 could grow between 2.3 and 3.6% per annum [21]. Therefore, studies revealed that the growth of electricity demands in Malaysia is greater than the corresponding average development of the global energy production.

As shown in Table 1.2, the average generation mixes of the hydroelectric power in Malaysia from 1980 to 2010 was increased; however, the share was still below the global average. For example, in the year 2008, the share of hydroelectric power to the overall energy production in Malaysia and in the world were about 3.1 and 6.4%, respectively [14]. The global average hydroelectric contribution in the mid 1990s was about 19% [21]. Even though the country has started generating hydroelectricity since 1900 [12], the share of the hydroelectric power is still below the corresponding global average.

Therefore, the country has proposed a new generation mixes strategy because the current electricity generation proportion cannot provide a sustainable development [9]. According to the new generation mixes strategy, the year 2000 energy mix of

74.9% of gas, 9.7% of coal, 10.4% of hydro and 5% of petroleum will be converted to 40% of gas, 29% of coal, 30% of hydro and 1% of petroleum in the year 2020. It shows that the strategy mainly focused on the energy generation from hydro and coal. This can have two advantages. First, the new generation mix minimizes the cost of power production since the expense of fuel for hydroelectric is small. Second, the strategy can reduce the emissions problem since power generation from hydro is environmentally friendly.

Table 1.2 Hydroelectric generation mixes status of Malaysia

Year	Generation mix (%)	Total hydro generation (GWh)	Researcher(s)
1990	4.5		Ong et al. [14]
1995	13.1		Mohamed and Teong [11]
2000	10.0	6,928	Mekhilef et al. [22]
2005	5.5	5,186	Mekhilef et al. [22]
2010	5.6	7,722	Mekhilef et al. [22]

Hence, the share of hydro can increase in two ways. Primarily, with the development of new schemes because the country utilized around 2 GW out of the 29 GW of the hydroelectric power potential [14]. This choice requires high investment cost. The second alternative is improving the efficiency of the current schemes. The second alternative is economically advantageous (less or negligible cost), not time-consuming, and does not have an additional environmental impact. This research considered the second alternative to study the Perak cascading hydroelectric power schemes. In terms of hydroelectric power development, the Perak cascading is the largest scheme found in the country.

The hydroelectric power potential of the Perak cascading scheme is 578 MW, and the average annual power generated from the scheme was about 228 MW. This shows that the average capacity factor of the scheme was below 40%. For example, the average capacity factor of the world's hydroelectric power plants in the year 2009 was around 44% [23]. This indicated that the hydroelectric power generation from the Perak cascading was below the global average. Hence, the efficiency of the cascading scheme should increase. The most important aspect to enhance the scheme hydroelectric power generation is the development of an optimal reservoir operation rule. The approach to find an optimal reservoir operation rule relies on the:

- increase of the unit water efficiency in the cascading scheme through the development of an optimal release schedule,
- development of a refill and deplete ranking order of the reservoirs that increases the energy-storage of the scheme and also maximizes the volume of water passing through the turbine and/or
- sensitivity analysis of the hydroelectric power generation from of each reservoir, and then the operation design should consider the impact of the reservoir variables.

Therefore, the basic research questions include:

- how much water should be released from each reservoir at a specific time to maximize the hydroelectric power generation from the cascading reservoirs that also avoids the risk of flooding?
- in what order should the reservoirs' refill and deplete to maximize the energy-storage and the unit water efficiency among the cascading scheme and to increase the quantity of water passing through the turbine?
- what are the optimal GA model parameters to maximize the hydroelectric power generation from a cascading reservoir system? and,
- in the Perak cascading scheme, which plant is the most sensitive to the change of hydroelectric power generation variables and to what extent does the variable affect the rate of production?

1.6. Problem Statement

The role of reservoir in water resource development is very important [24] and yet very complex [25], [26]. The complexity arises due to the trade-off in the wide range of conflicting objectives, the stochastic nature of the hydrological events and the dynamic nature of the demands [27]. The most complex problem in water management and hydropower engineering is the operation of cascading reservoirs [28]. This is due to several variables involving in the operation of a cascading system.

The refill and deplete ranking orders in a cascading reservoir is another important concern for efficient utilization inflow and then to maximize the hydroelectric power generation. The ranking order can boost the unit water efficiency and the energy-storage among the reservoirs. Inappropriate refill and deplete order lead to unnecessary spillage. The spillage has two drawbacks. First, it reduces the capability of the scheme to generate power. Second, it reduces the unit water efficiency in the cascading scheme and the cost of fuel (water) increases.

Various models are formulated and applied to optimize a reservoir operation, but optimization models that considered all the variables have not yet been developed [29]. The traditional optimization techniques such as the linear programming (LP) and the dynamic programming (DP) in a reservoir operation have drawbacks. The application of LP in a reservoir operation is limited due to the non-linear nature of the problems involved in the system [30]. Moreover, the application of DP in multi-reservoir system optimization is also limited due to the computational inefficiency. For example, Karamouz et al. [31] tested the capability of non-linear programming (NLP) and DP to develop the monthly operation planning of a multi-reservoir system. The conclusion showed that DP had limited capability to solve the reservoir operation problems because of the “curse of dimensionality”. However, the genetic algorithm (GA) received much attention because of its ability to solve complex problems accurately, reliably and quickly [32]. It also gained importance in reservoir operation due to its random search capability close to the global optimal value [33]. Besides, GA does not require linearization like the LP and does not suffer on the “curse of dimensionality” like the DP, while initial parameter settings are very important [34].

Various studies outlined the application the GA model for the operation of a reservoir. However, the developed models did not clearly show all the optimal initial parameters setting such as the population size (PS), crossover probability (CRP) and generation number (GN). These GA parameters have a great impact on the finding of the optimal fitness value. For example, the study of the operation of the Upper Wardha reservoir in India did not show the impact of the GN on the fitness value [35]. Furthermore, on the operation of the multi-reservoir of the Greater Karoon system in Iran, Dariane and Momtahen [36] did not state the qualitative influence of GN.

In this study, the impact of the basic GA parameters, namely, PS, CRP and GN are analyzed thoroughly to find the optimal fitness value. In addition, the study showed the relationship between the GA parameters according to the computational runtime and the minimum numbers of the run requirements for the operation of a cascading of four-reservoir system. The minimum numbers of run requirement expressed through the introduction of the "*borderline of runs.*" The concept of the "*borderline of runs*" was not presented in similar studies before.

1.7. Objective of the Research

The overall objective of this research is to model a cascading reservoir operation in order to have a better and more efficient system that to meet the goal of the stored water. In connection to this, the research taken the Perak cascading scheme as a case study, and the specific objectives were:

- to develop a real-time cascading reservoir operation model using the genetic algorithm (GA) optimization and the seasonally varied model (SVM) aimed to maximize the annual average hydroelectric power generation of the scheme.
- to determine the optimal values of the basic GA parameters in the operation of cascading reservoirs.
- to develop refill and deplete ranking orders of cascading reservoirs to capture the inflow and to optimize the energy-storage within the scheme and
- to conduct the sensitivity analysis of the hydroelectric power generation in the cascading scheme.

1.8. Thesis Structure

As shown in Figure 1.2, this thesis comprises of five chapters, including the introduction as chapter one (this chapter). The contents of each chapter are presented in their respective sequential order.

The most important related studies and findings are examined under the literature review part, which is chapter two. In addition, chapter two explains the state-of-the-art reservoir operation, the basic variables that influence the operation of the reservoir, various optimization models that are applied to operate a reservoir, and the key terms of the hydroelectric power generation. The literature review also pointed out the gap in the research using the genetic algorithm optimization technique in the operation of a cascading reservoir. Chapter three explains the data, methods and structures of the analysis. The techniques used to analyze the data; the procedures that followed to develop and to validate the genetic algorithm and seasonally varied models have been explained thoroughly in sequential order. The results and discussion are presented under chapter four. The chapter mainly articulates the most important findings of this study. The results are discussed in comparison with the other similar research findings. Finally, chapter five summarizes the whole study and outlines the most important research outputs. In addition, chapter five mentions the future work that relates to the optimization of a cascading reservoir operation using the genetic algorithm approach.

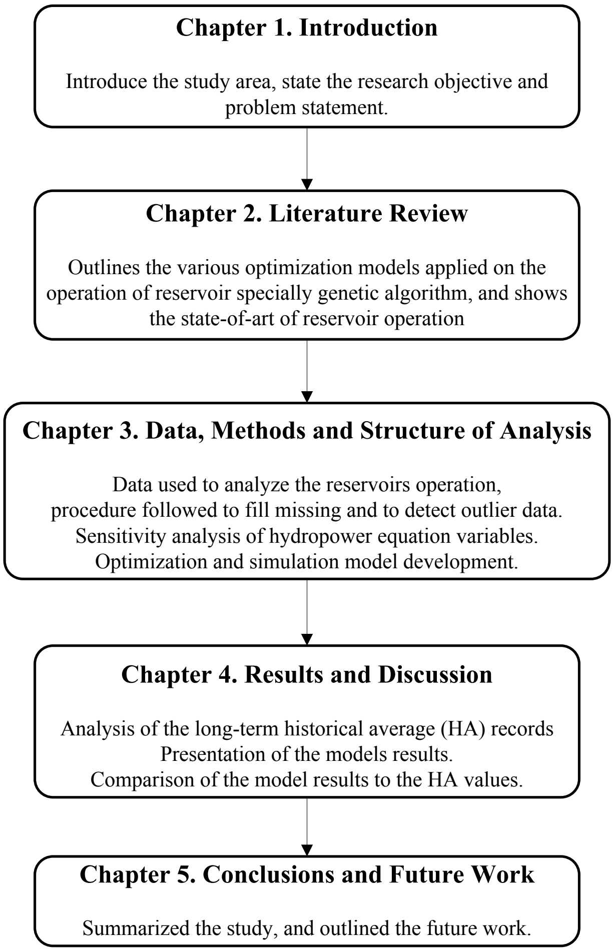


Figure 1.2 Framework of chapters organization

CHAPTER 2

LITERATURE REVIEW

2.1. Introduction

Optimization and simulation models that were used to optimize the operation of the reservoir are reviewed under this chapter. The majority of the models were developed using a conventional (traditional) and/or modern optimization techniques. The targets of the models are to enhance the efficiency of the water stored in the reservoirs through the development of an optimal reservoir operation rule. The robustness and the limitations of the models are presented in various sections of the chapter. It was found that some models are advanced to capture the global optimal value. As a result, the study selected the genetic algorithm (GA) model to optimize the operation of a cascading system because GA creates a high-quality solution, superior capacity to handle a complex problem and robust in the sampling space. However, the result of GA depends on the initial parameters setting.

2.2. Optimization Techniques

Optimization is a process of finding the optimal value of a certain system. The history of optimization revealed that the principle was started during the World War II to optimize the trajectory of missiles. Subsequently, mathematical programming has been developed. Later in 1970s, the meta-heuristic approach (the modern optimization techniques) was started. The meta-heuristic approach is developed with the simulation the behavior of a living organism, and then it applies on the actual condition [37].

Optimization has two parts, the objective and the constraint. The objective function is the target to meet (either to minimize or maximize), while the constraints show the domains that the system can work. In water resource optimization, the objective function relates to the benefit gained and/or the technical parts. The technical parts vary with the purposes of the system. For example, in a hydroelectric power reservoir operation, the technical part can be to maximize the energy production and/or to reduce spillage. The constraints also relate to the physical, economical, social and/or technical aspects of the hydroelectric power generation.

Optimization techniques can be classified in several ways based on different criterion. As shown in Figure 2.1, the optimization techniques are grouped into two: the mathematical programming and the modern (non-traditional) optimization techniques according to the methods that used to develop it. The modern optimization techniques are developed by simulating the behavior of biological, molecular and swarm insects [38].

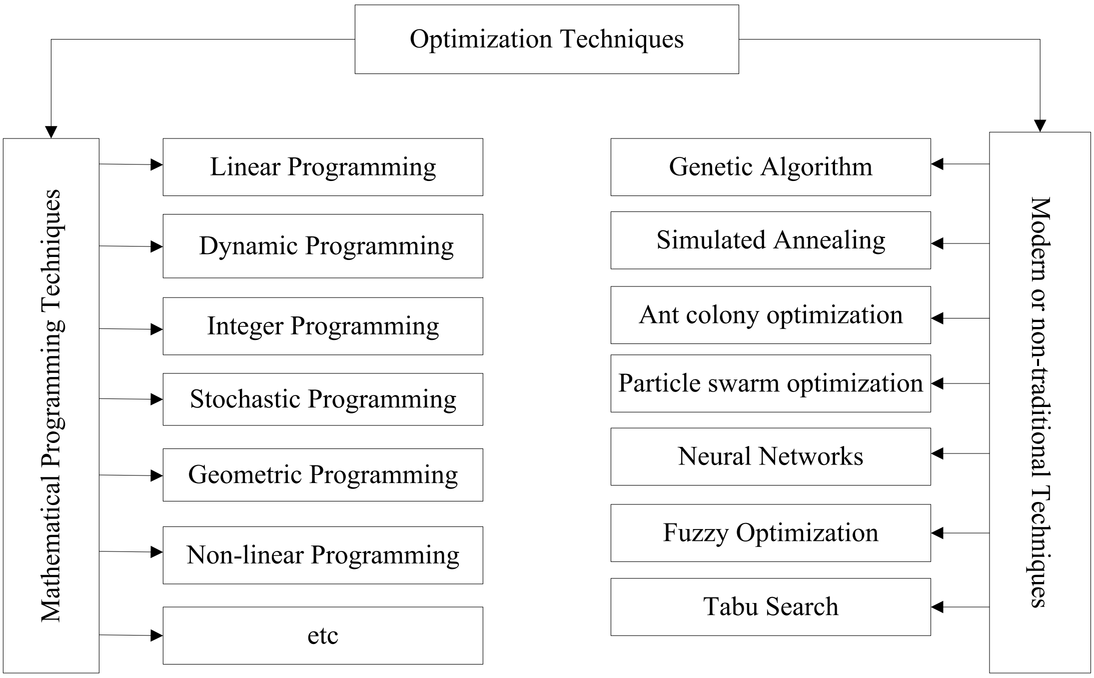


Figure 2.1 Classification of optimization techniques [38]

In the process of finding the optimal value using the optimization techniques, two types of solution encountered, the local and the global values. The local value represents the optimal point of a system within the specified range. Whereas, the global value represents the optimal point within the entire feasible range. Figure 2.2 shows the variation of a certain function, *f(x)* between the arbitrary points of X_1 and X_2. In the relationship, A_2 represents the global maximum and B_2 the global minimum values, while A_1 and A_3 are the local maximum and B_1 is the local minimum point.

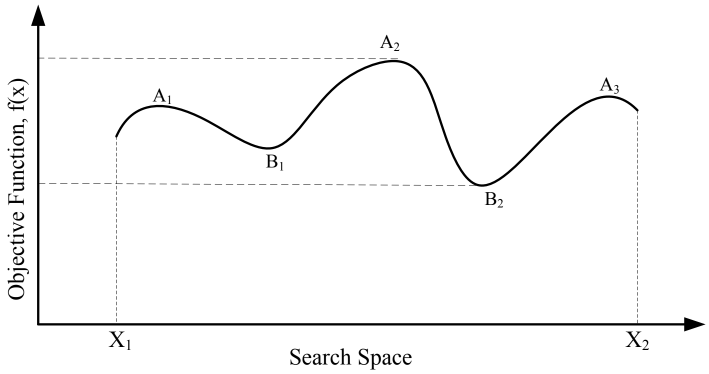


Figure 2.2 Relative and Global Optimal Values

The interest of optimization is to find either the global minimum or maximum point. Hence, one of the challenges in optimization problem is differentiating the relative (local) and global optimal points. Some optimization techniques are not robust enough to find the global optimal point. Therefore, the model applied to search for the optimal value should be robust (good performance), efficient (not take too much computation time), and should be accurate (minimum error). An optimization technique such as the genetic algorithm (GA) is quite robust to search for the global optimum point. GA can handle any type of objective function [39] and uses the heuristic search approach.

There are three types of search mechanisms in optimization: *analytical*, *blind* and *heuristic*. An analytical search guides with a mathematical function. It has a guarantee to find the optimal solution, if it exists. A blind search, sometimes known as an unguided search, has no guarantee to find the optimal solution since it is usually biased. However, a heuristic search is widely used in practice. It is a guided search, in most cases the solution found from the heuristic search methods are satisfactory [40].

2.3. Heuristic Search Approach

A heuristic is a branch of logic. The word originates from the Greek; the meaning is *to discover* or *to find out*. Heuristics are principles or methods that lead to achieve the goal with a proper decision among several alternatives. The concept of heuristic was begun in the 1950s with the notion of artificial intelligence (AI) [41]. Heuristic methods improved the problem solving efficiency with the principle of efficient administration and resource allocation. The process of heuristics are governed by three factors: availability, accessibility and applicability. The availability shows the knowledge structures that are stored in the memory; the accessibility is the capability to retrieve, and the applicability refers to how the stored knowledge is relevant to the current task.

Heuristic programming is a computer program that employs a problem solving procedure [41]. The heuristic algorithm is an optimization technique; it is classified under the family of local searches [39]. In the local search method, the movement from one state to another is based on well-define rules [42].

2.4. Genetic Algorithm, GA

Originally, genetic algorithm (GA), developed by John Holland, from the University of Michigan, in the 1970s is based on the principle of "*Natural Selection*" and "*Genetic Inheritance*" using the Darwin Theory of Evolution of "*survival of the fittest*" [43]. GA is a probabilistic method; it uses an adaptive heuristic search technique [44]. The method is widely used in business, science and engineering disciplines [32], [45].

2.4.1. An Overview of the GA Optimization Technique

The search space (domain) in optimization is normally large. Among the large search spaces, only one value provides the global optimal result. The objective of optimization is to obtain an optimal value to satisfy the goal (it can be a minimization or maximization of a certain system function). Hence, one of the basic differences in optimization techniques is the mechanism used to search the minimum/maximum value. The technique like GA is robust to find the optimal value. The principle of the GA optimization technique is based on the reproduction of the living organisms. Those active organisms can survive and continue their generations until it is dominated by the others. Similarly, a value that provides the best result in the search space is selected and continues until it is replaced by another superior candidate.

The fitness function, population, generation, chromosome, offspring etc. are the most important terminologies in the GA optimization. Figure 2.3 shows the relationships of GA terminologies.

- **Fitness function** – has similar meaning to the objective function for other optimization techniques. Usually, the fitness function cannot have a negative value [38].
- **Generation** – a complete cycle (iteration step), it comprises the fitness determination and selection.
- **Chromosome** – is a set of genes.
- **Gene** – variable (subunit) of a chromosome
- **Population** – is a set of chromosomes.
- **Population size (PS)** – is the number of chromosomes in each generation. The size of the population influences on the performance of the GA.

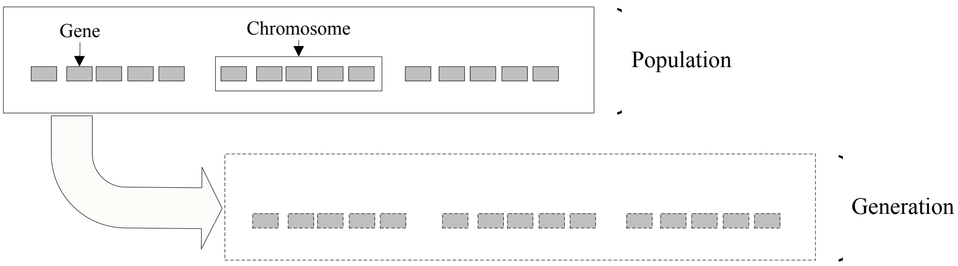


Figure 2.3 Important GA terminologies

The most important procedures in the GA optimization technique include the selection of the various alternative run options, the determination of the optimal number of generations, and the set-up of the model stopping criteria. The representation of the system, the solution process and the way to interpret the results are also the basics in the GA model development.

2.4.1.1. System Representation

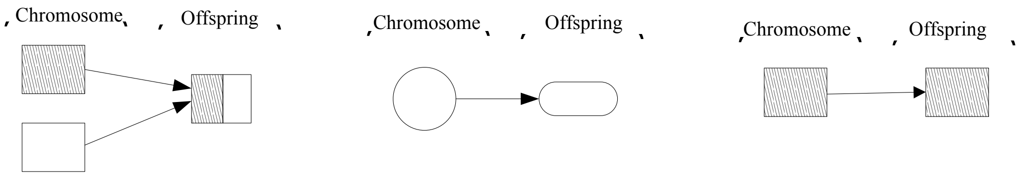
Defining the fitness function, representing the function with the genetic algorithm (GA) terms, and using appropriate operators is the most important aspects in the development of a GA model. The GA optimization solver associated with the

MATLAB package allows a minimization of the fitness function. Hence, the maximization function should change to the equivalent minimization function. The GA operators include selection, crossover and mutation.

- **Selection** – shows the mechanism that was applied to choose individuals from the current population for reproduction. Selection is one of the most important processes in GA [46]. The common techniques used for the selection procedure are the roulette wheel, tournament, and uniform. In the roulette wheel selection, parents are selected according to the fitness value. The better the fitness value a parent has, the more chance to be selected. In tournament selection, all have an equal probability to select. Reproduction using the tournament selection is accomplished with a randomly chosen set of chromosomes (parents) followed by picking out of the best chromosome (parent) [47].
- **Crossover** – is the process to create offspring according to the user-specified probability value. As shown in Figure 2.4 (a), the crossover operation accomplishes through the combination of two parents. The crossover operation has been conducted after the selection process [38]. The offspring has a better fitness value than the parents.
- **Mutation** – is also another method used to produce an offspring. As shown in Figure 2.4 (b), the mutation operation produces an offspring with the random change of the parents' behavior. The performance of the offspring is near to that of the parents.
- **Elite** – in this case, the parents continue without any modification as shown in Figure 2.4 (c) because the fitness value found from the parents is better than the offspring. Hence, elite parents automatically survive and continue to the next generation.

As indicated in Figure 2.4, the offspring that were produced in the process of the crossover, mutation and elite are according to the behavior of the parents. The ratio that the mating accomplishes with the crossover rule is called the crossover probability (CRP), and for that of the mutation is called mutation probability (MPR). The common trial and error initial value of the CRP is one [34]. However, a CRP of one does not provide an optimal value. In the study of Mathur and Nikam [35], the optimal value of CRP was found to be at 0.75. Similarly, the study of

Jothiprakash and Shanthi [48] showed that the optimal fitness value was found at a CRP of 0.76. Usually, the value of CRP varies in the range of 0.5 to 1.0 [17]. Hence, the optimal CRP is determined after the test results of the fitness function.



(a). Crossover (b). Mutation (c). Elite

Figure 2.4 GA reproduction operators

2.4.1.2. Evaluation Procedure

The solution procedure of the GA differs from the traditional optimization methods in four ways [33]. The first difference relies on the coding of the variables. GA uses the coding of the decision variable set, not the decision variable itself. The second is on the searching space; GA searches from the population of the decision variables in the set, not the single decision variable set. The third relates to the objective function. GA uses the objective function itself, not the derivative information. The forth disparity is about the rule of searching. The GA algorithm uses probabilistic, not deterministic search rules.

Basically, the evaluation process of GA follows four sequential steps [49]. In the first step, it randomly generates an initial population. The second is producing the next generation using the crossover and mutation operators. Evaluation of the fitness of the new generation is the third step. At the fourth step, the old generation is replaced with the new. Until the stopping criteria is reached, the third and fourth steps are repeated.

2.4.1.3. Stopping Criteria

GA uses five different stopping criteria with the exclusion of the manual setup. The first stopping criterion is the specified generation number. The second is the specific time. The third and fourth are the fitness and the stall time limits. The fitness limit is a desired fitness value specified by the user. The algorithm stops if the fitness reaches the specified value. The stall time limit is the specified time in seconds that the fitness function has no improvement. The fifth criterion is the stall generations. If the fitness function has no improvement within the specified number of generations, then it will be stopped. It is possible to apply more than one stopping criteria at the same time. However, the iteration is stopped if any one of the stopping criteria reaches first.

Various studies have applied different stopping criteria. For instance, in the operation of the multi-reservoir using the hybrid GA and LP, Ries et al. [50] used the fitness value limit and generation number as a stopping criterion. Whereas, Hashemi et al. [51] only used the generation number as a stopping criterion in the operation of the Jiroft dam reservoir in Iran. The conclusions of Hashemi et al. [51] indicated that if the choice of a stopping criterion is a generation number, the result would be advanced.

In the GA optimization procedure, except for the stopping criteria, all other steps are well-defined [52]. The selection of the appropriate stopping criteria is challenging because the global optimum point is unknown in advance. In addition, it is difficult to predict how the computed value far from the global value [53]. In such case, repeatedly iterate the model can be minimized the difference between the computed value to the global optimal point. This can be accomplished by taking the generation number as a stopping criterion.

2.4.2. Advantages of GA

Many researchers have shown the advantages of GA over other similar optimization techniques. For example, Kitagawa et al. [37] compared the general features of the meta-heuristic techniques, namely, the genetic algorithm (GA), simulated annealing (SA), Tabu Search (TS), and Particle Swarm Optimization (PSO). These are the modern optimization techniques as shown in Figure 2.1. As far as the comparative computational runtime and the result guarantee, GA provides a better result as compared to the other modern optimization techniques. In addition, Jones [54] compared the GA and PSO in the optimization of parameters during model identification. The comparison showed that the computational efficiency of the GA is better than the PSO. Zhang et al. [55] also evaluated GA and PSO in the calibration of SWAT model, it was found that the performance of the GA was advanced.

The study of Chaves and Kojiri [56] showed that GA is the most popular optimization method due to its superior capacity to handle complex problems among the various artificial intelligence (AI) approaches. Complex problems are discontinuous, non-differentiable, stochastic and have a nonlinear objective function [57]. GA is best suited for such problems that could not be solved with other standard optimization methods because it allows any type of objective function [39]. In addition, GA uses a probabilistic search technique [58] and solves both constrained and unconstrained optimization problems.

Bajpai and Kumar [59] also presented the approach of GA to solve optimization problems. The research reveals that GA creates high-quality solutions for problems that are little known, and it can mimic the natural conditions. GA does not require a derivative, robust in the sampling space [16], generates better data fitting solutions, it provides close to the global optimum value [60]. These are the advantages of GA over the other analytical methods. Due to the advantages of the GA optimization technique over the other similar approaches, various researchers have developed a GA model to optimize the reservoir operation problems. Reservoir operation problems are stochastic in nature; therefore, an advanced and robust optimization technique is necessary to handle such problems. In this regard, GA is the preferable technique to solve such problems.

2.5. Role of a Reservoir in Water Resource Development

Over 45,000 large dams were built in the world up to the year 2000 [61]. However, nearly 80% of the world's large dams are found in five countries: China, USA, India, Spain and Japan. China alone built half of the world's number (approximately 22,000 large dams), while the number of large dams found in the USA and India are around 6390 and 4000, respectively. Similarly, the number of large dams built in Spain and

Japan varies between 1000 to 1200 [62]. The data indicate that the developers, and the most populated countries are creating large storage of water through the construction of large dams. The water behind the dam is known as a reservoir.

A reservoir is an enclose area which can refill and empty over a period of time [63]. Refilling is normally accomplishes during high-flow periods. The water stores in the reservoir can be utilized to generate a hydroelectric power, irrigation, water supply etc. Reservoir also serves for flood mitigation, fishing, navigation and amenity uses. These are the demands of stored water. A reservoir can serve for more than one demand, which is call it a multi-purpose reservoir.

The stored water supplements the difference between the required and the actual water demand in the period when the inflow and outflow are out of balance. In most situations, the stored volume is less than the deficit quantity; hence, it is difficult to satisfy 100% of the water deficit. Due to this difficulty, the concern of a reservoir operation is an issue now a days. Its target is to optimize the deficit in order to reduce the risk associated with the water shortage. Simply a reservoir operation concern is an optimization of the stored water to enhance the system efficiency.

The role of the reservoir operation and management of water resources is very important [24], but it is complex [25], [26]. The complexity arises due to the trade-off in the wide range of conflicting objectives, the stochastic nature of the hydrological events and the dynamic behavior of the demands [27]. In the field of water resource engineering, the operations and management of a reservoir are complex activities; however, the most complex problem in water management and hydropower engineering is the operation of cascading reservoirs [28]. The main reason is due to the variables that are involved in the operation of the cascading reservoirs. Interpreting such variables, using mathematical terms are also challenging. Even though, data management with the recent computer technology could make the complexity become manageable [64], optimization models that considered all the variables in the reservoir system have not yet been developed [29]. There is no specific technology to handle the optimization of the reservoir [26]. Therefore, the development of an optimal reservoir operation is still an ongoing research.

2.6. Fundamentals in a Reservoir

Dam is a hydraulic structure, it is constructed across the river to impound water. A reservoir is created due to the impounding. A storage volume in the reservoir varies from time to time due to the variation of the rate of the inflow and outflow. The storage quantity can be managed with the rate of release. For example, if the storage volume is less than the recommended level of a reservoir, the release should be curtailed. The measure of reducing the rate of the release has an adverse impact on the water demands, since it requires a continuous and adequate supply. Hence, a compromise between the demand and the storage is mandatory, because it is difficult to always satisfy the water requirements. Hence, the notion of a reservoir operation rule is to balance the demand and the supply of water with a minimum discrepancy level. This shows the reservoir operation rule is to maximize the utilization of the stored water within the specified time horizon through an optimal release strategy. The release decision primarily requires the data of the available water in the reservoir, the current demand requirements, and the prevailing and the predicting inflow scenarios [65].

The reservoir operation rule that meets the demanding requirements with a minimum deficit with respect to time is considered the best. For instance, a reservoir used for hydroelectric power generation can have two main targets. The first aim is to augment the water during the high-flow season (period) and to supplement the supply deficit during the low-flow season. The second aim is to increase the energy head. The energy head varies according to the headrace level of the reservoir. However, the target of a flood-control reservoir is to reduce the peak flow rate. The strategy to reduce the peak flow rate depends on the characteristics of the reservoir and the threshold release value for flooding. In all cases, the release of the reservoir governs the satisfactory level.

Reservoirs can be classified according to the number, purpose, arrangements or configurations in the river basin. If the river basin has only one reservoir, it is known as single-reservoir, otherwise it is multi-reservoirs. In terms of purpose, a reservoir that serves for only one purpose is categorized as single-purpose reservoir; whereas, a multi-purpose reservoir meets more than one objective. Both single and multi-reservoirs can serve as single or multi-purposes. The configuration of multi-reservoirs can be in cascade/series, parallel or combination of both. Multi-reservoirs can have six types of layouts based on its purpose and configuration as shown in Figure 2.5. In the reservoir operation, the simplest is single purpose of single reservoir, while the most complex is multi-purposes of multi-reservoirs [66].

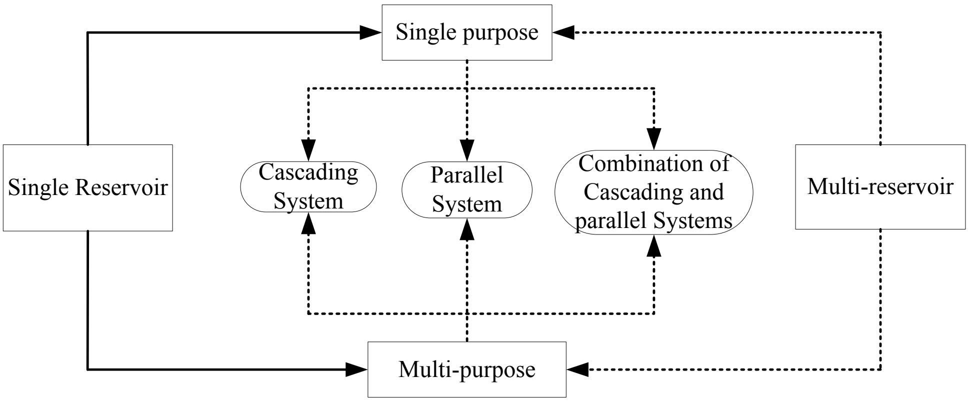


Figure 2.5 Possibilities of the reservoirs configuration in a river basin

2.6.1. Storage Zones in a Reservoir

The water stored in the reservoir is mainly divided into four zones; surcharge, flood control, conservation and dead storage as shown in Figure 2.6. Sometimes the storage between the maximum water level (MWL) and the full supply level (FSL) is treated as one zone. Of course, both zones use for the safe passage of floods, and it acts as a temporary storage [67]. In the operation of a reservoir, the dead storage is not important. Only the conservation storage is readily available for the normal operation of a reservoir.

In the operation of a reservoir, a part of the conservation storage maintains as a reserve, it implies that the normal minimum operating level (NMOL) is much above the level of the undersluices. The storage between NMOL and undersluices keep as a carryover, it is a buffer storage. The quantity of the buffer storage varies based on the operating constraints of the reservoir. The release from the buffer storage zone accomplishes in case of extreme conditions.

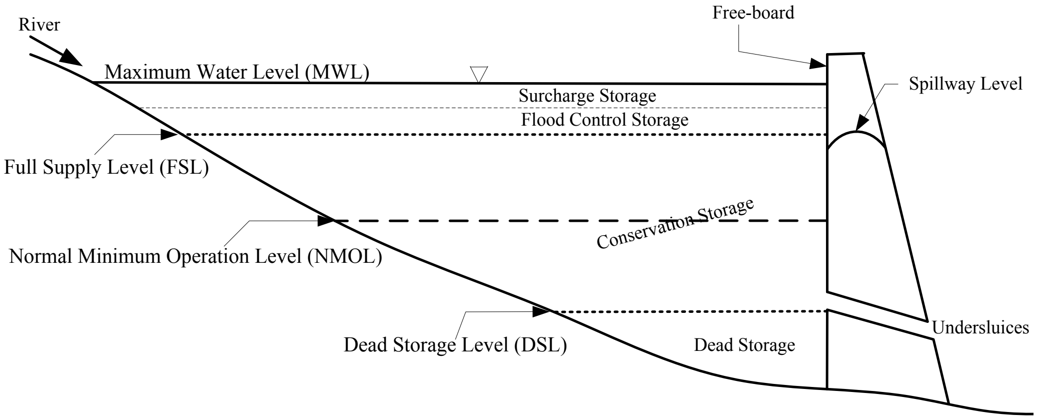


Figure 2.6 Zone of a reservoir storage

2.6.2. Reservoir System Variables

Stage, storage and water surface area are the most important system variables that interpret the physical characteristics of the reservoir. The common method to determine the reservoir system variables are with the help of a stage-storage-area curve. The curve develops during the design period, and it is important for the analysis and operation of a reservoir.

The development of the stage-storage-area usually begins from the topographic map of the reservoir area. Using the contour map of the reservoir, planimeter or grid technique is used to compute the water surface area at different elevations. The reading of the planimeter or the grid changes to the actual value according to the calibration scale. The storage volume between the successive contours is the product of the average area and the contour interval (the elevation difference). After successive determination of the surface area at various elevations, the stage-area and stage-storage relationships can be developed. Hence, the area-storage relationship constructs from the two curves as shown in Figure 2.7. The figure also shows the relationships of the three reservoir system variables.

Once the stage-storage-area relationship for a certain reservoir is developed, it requires modifying due to various reasons. For example, sedimentation affects the conservation (live) storage of a reservoir; therefore, it is important to modify the relationships. This is because the old relationship provides wrong data for the operation of the reservoir. However, the impact of the sedimentation on the operation of the reservoir is out of the scope of this study. While, the research is concerned about how to revise the reservoir system variable relationships in case that the old curve is inaccessible, missing, and/or damaged.

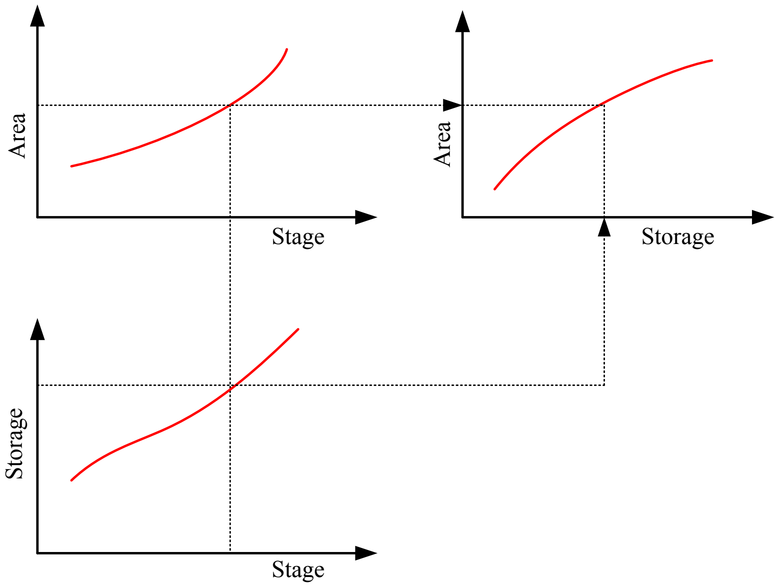


Figure 2.7 General stage-storage-area relationship curves

One of the important considerations in reinventing of the relationship of the reservoir system variables is the shape of the curves. The assumption of a linear relationship between the stage-storage has produced a serious error; hence, Bayon et al. [68] proposed a non-linear relationship (second-order polynomial function). In addition, Mohammadzadeh-Habili et al. [69] proved the non linearity of the stage-storage relationship and it has a concave shape. This is a fundamental hypothesis that was applied to develop the Perak cascading reservoir system variable relationships.

2.6.3. Inflow-outflow Process

Any form of water that enters the reservoir is considered as inflow; whereas, that leaves from the reservoir is taken as an outflow. The common inflows to the reservoir include the river flow, the rainfall falls directly on the surface of the reservoir, and the ground water recharges. On the other hand, evaporation, release, spillage and seepage are outflows from the reservoir. The computation of the ground water recharge and the seepage is challenging because of the difficulty to measure the values. Usually, analysis are made with the assumption that both quantities are annually balanced and the net effect on the change of storage volume equal to zero. The assumption might not represent the real condition, but the error found due to this assumption become insignificant.

The transition that water enters and leaves from the reservoir is partly a controlled process. Figure 2.8 shows the reservoir system that can be interpreted as a function of the change of the total inflow (*I*) and total outflow (*O*) at a certain time. The difference of the total inflow and outflow has a direct impact on the headrace level, storage volume and water surface area of a reservoir. The inflow hydrograph is important to predict the reservoirs' system variables and to decide on the rate of the release (the outflow hydrograph).

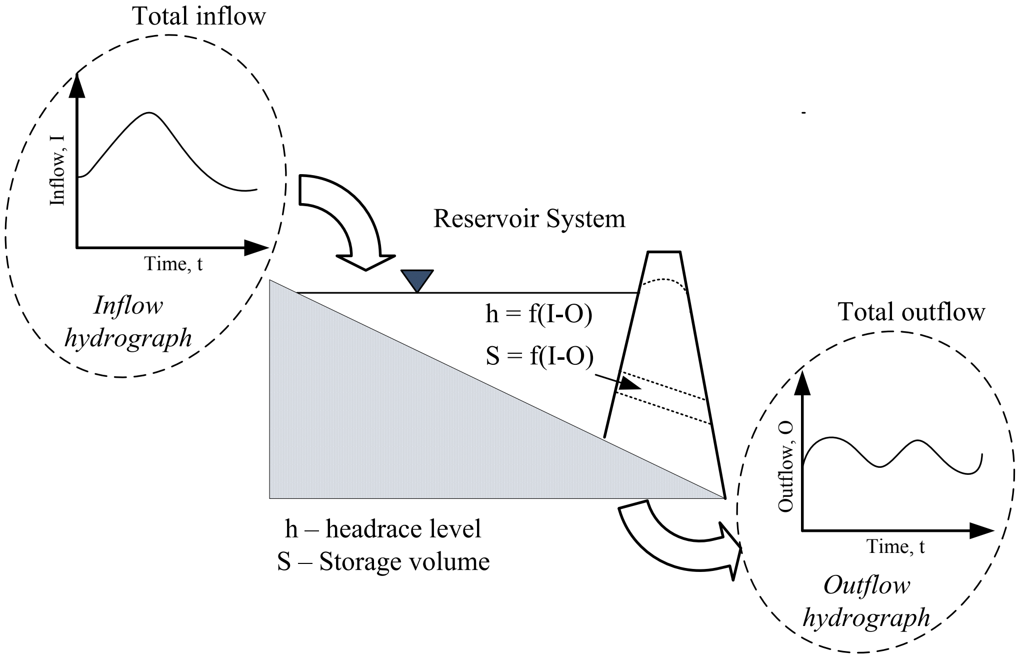


Figure 2.8 Conceptual representation of a reservoir system

The rate of the inflow and outflow in the cascading scheme can be mutually dependent or independent; it can be continuous or discrete. The basic equation of the reservoir system expresses as the total inflow minus the total outflow equal to the change in storage. The equation seems to be a very simple logic, but the real condition is a complex process. This is because of the dynamic nature of the reservoir system with respect to time. Hence, it is challenging to solve analytically.

In the operations of a reservoir, three possible conditions can exist; the first condition is the unsteady inflow and steady release. The second possible situation is the steady inflow and unsteady release, and the third case can be when both are in an unsteady condition. Practically, in the long-term reservoir operation analysis, the first and second conditions cannot exist; however, the third situation is common. The management of unsteady flow is a bit complicated, since it is dynamic in nature and difficult to predict the rate of inflow.

The reservoir system as shown in Figure 2.8 depends on the change of the rate of inflow and outflow. A positive difference between the total inflow and outflow indicates a refill operation, and the negative shows a depletion operation. If the inflow and outflow are balanced, the system becomes a steady state. During the refill period, the headrace of the reservoir increases. As a result, both the storage volume and water surface area are increasing. However, the rate of the increment or decrement of the storage volume and water surface area depends on the topography of the under-lying water. The largest water surface area reservoir has the smallest the rate of change of the headrace level due to the addition or withdrawal of water. For normal operations, the cumulative sum of the inflow volume in the hydrological year is always greater than or equal to the corresponding cumulative volume of the release. Hydrological year is the length of the analysis for a reservoir system, and it is almost equivalent to a critical period.

2.6.4. Critical Period, CP

There are two different outlooks in the definition of a critical period (CP). The first definition stated that a CP is the longest length that taking a full reservoir attains its minimum operating level under a normal operation condition without a spill. Whereas, the second definition of CP is showed that a full reservoir goes to minimum operating level and then back to the full level. In North America, the second definition is commonly used [70]. The main difference between the two definitions relies on the refill period. The first definition excludes the refill period to determine the CP. However, without the refill period, the full cycle of the reservoir is not completed. Hence, the second definition is quite logical, and this study used the second definition to determine the CP. The knowledge of CP is important to develop the operation rule-curve. However, the rule-curve developed with the concept of CP is very conservative in its decision [71].

2.6.5. Configurations of Multi-reservoirs

The configurations of the multi-reservoir show the relationship between the reservoirs that are located in the same river basin. The relationship of the reservoirs can either share the same inflow or discharges into the same river and/or both. Cascading reservoir systems share the same inflow and discharge into a common river, while the parallel reservoir systems discharge into the same river. Depending on the objectives of the reservoirs, the management and operation principles of each system require different approaches.

The operation of a parallel of hydroelectric power reservoirs are challenging in the period of high-flow season, because of the possibility of flooding. Hence, the objective of the operation should be included the flood mitigation measures too. The flood mitigation can be achieved with a careful design that considered the basic operational questions. These are when to release and in what proportion of release should be accomplished from each reservoirs within a certain specified time.

In the case of a cascading scheme that not consuming water (like hydropower generation), the release of the upstream reservoir is usually the main inflow of the next downstream reservoir. The operation of a cascading scheme depends on the number of reservoirs in the river system, the objectives, and the hydro-meteorological condition of the catchment area. In general, the operation of a cascading scheme is more complex than the single system [72]. However, the non-water consume scheme such as hydroelectric power generation, a cascading system utilizes the unit water as much as for its maximum capacity level. For instance, let consider a cascading of four hydroelectric power reservoirs, a turbine release from the most upstream reservoir that generates the power recaptures by the next downstream reservoir. The recaptured water again generates power the second time at the next downstream plant and then joins to the succeeding reservoir. This process continues until the water passes through the turbine of the last downstream reservoir. In this process, the unit water can generate power for four times. Therefore, the capacity of the unit of water to produce power is higher than a single system.

The main questions on the operation of cascading reservoirs are when and how much to release from a specific reservoir in a specific period. The reaction varies with the location and the situation of the reservoir along the cascading scheme. However, in the non-water consume scheme, the most upstream side reservoir in the cascading scheme has a great impact on the operation of all the other downstream reservoirs, because the rate of inflow depends on the operation of the preceding reservoir.

The most important aspect in the operation of a cascading hydroelectric power generation scheme is the storage volume within the system. Theoretically, all reservoirs should have maximum storage to provide maximum power. This is not possible throughout the operation seasons. Hence, development of the refill and deplete ranking of the reservoirs is important to have maximum energy-storage among the reservoirs and efficiently utilizes the inflow by reducing the possibility of spillage.

According to Jain and Singh [67], cascading reservoir operations can be conducted in three approaches. The first is the equal function method. The concept of this method is maintaining all reservoirs in the same zone in the operation periods. The second is according to the priority concept. Reservoirs are arranged in priority order. The priority criterion can be given according to the objectives of the reservoirs. Refill starts from the highest priority reservoir, and release accomplishes from the reservoir that has the lowest priority. The third concept is based on the storage lag principle. According to the principle of lag, the release should start from the downstream reservoir. Then after, the downstream reservoir should have enough storage to augment the release of the preceding reservoir.

Among the three possible operation approaches of a cascading reservoir mentioned by reference [67], the second is the preferable alternative for the Perak cascading reservoir operation because the storage capacity and range of operation levels of the reservoirs are different as illustrated in Table 1.1. Therefore, the first approach cannot be applicable; moreover, in the actual operation, the third approach seems rational, but it may not be applicable to all cascading systems. Cascading reservoirs vary in capacity, purpose and the sensitivity of the governing variables. In general, the approach of the operation of cascading reservoirs cannot conclude before the analysis of the exact phenomenon of the system.

2.7. Reservoir Operation

Release to water demand, evaporation, spillage and seepage are the outflow of a reservoir. Among these, evaporation and seepage are uncontrolled flow, while the release and spillage are the controlled flow. Hence, a reservoir operation deals with the decision about when and how much should be stored and released. The rule-curve is an important reference to decide on the rate of release. The reservoir operation rule-curve is developed in the design period considering the hydro-meteorological condition and the demand of water. However, if a real-time decision is required, it relies on the subjective judgment of the operator. In any regards, the target of a reservoir operation is to optimize the utilization of the stored water. Hence, optimization models are developed to maximize the efficiency of the system. Since, the robustness of the optimization models are different, the selection of an appropriate model is mandatory for a specific system condition.

2.7.1. Rule-curve in Reservoir Operation

A rule-curve is a guide used to operate a reservoir [73]. The curve shows the desired storage volume or the stage level that is required to meet during the operation process, but it cannot describe the quantity of the release [74]. A rule-curve varies with the objectives of the reservoir. For example, a rule-curve for the purpose of irrigation is different from the hydroelectric power generation, even for the same reservoir.

Rule-curve is developed during the design stage [24] by referring to the long-term hydro-meteorological situation of the area. Figure 2.9 shows the general practice used to develop a rule-curve. Point *A* and *B* are the start and the end of the rule-curve. The period between point *A* and *B* is equivalent to the critical period (CP). Within the CP, all the possible events of the reservoir (such as a refill, deplete, etc.) happen sequentially.

During the refill period (from *A* to *C*) as shown in Figure 2.9 (a), the rate of the inflow is greater than the corresponding outflow value; moreover, in the depletion period (from *C* to *B*) the rate of the outflow is over the rate of the inflow. When the reservoir reaches at the maximum storage volume, point *C* in Figure 2.9 (b), the difference between the cumulative inflow and outflow attain the maximum level as shown in Figure 2.9 (c). At point *C,* the rate of the inflow equal to the rate of the outflow, a maximum difference of the cumulative (cum.) inflow and outflow attain, and the reservoir reaches at the maximum storage volume.

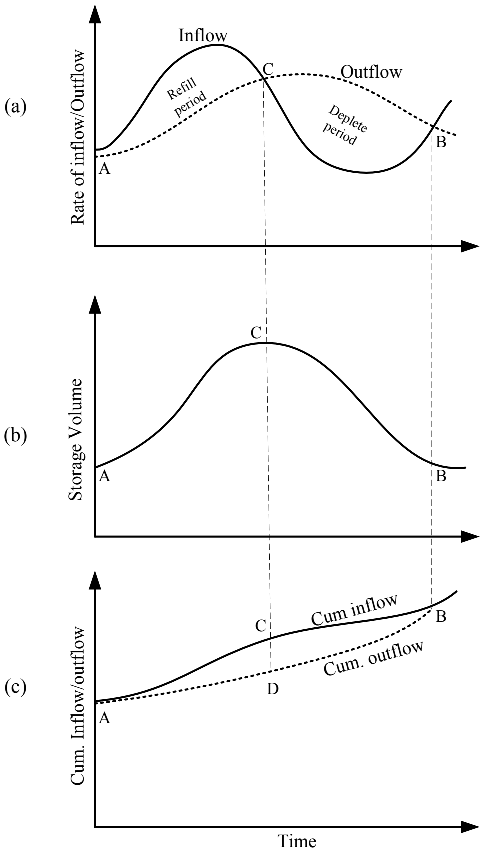


Figure 2.9 Development of rule-curve: Adapted from Jain and Singh, [67]

In the operation of a reservoir using the rule-curve, the actual level can have three possibilities. The first possible case is the existing reservoir level above the rule-curve level. In this case, the aim of operation is to satisfy all the demands. The second condition can be when the actual reservoir level is in the vicinity of the rule level. During such condition, the principle of the operation is maintaining the rule-curve, and the release is conducted in a restricted manner with consideration of the possible future storage. The third possibility is the actual reservoir level quite below the rule level. The focus of the third strategy intend on the refill operation. Refill is accomplished by reducing the rate of release until it reaches the rule level [67]. The first and the third situations show the extreme conditions of a reservoir operation. The first case happens due to an excessive inflow; whereas, the third case indicates an extreme deficit and it happens during a low-flow period. Thus, water should store during the first condition to supplement the shortage at the third condition.

Operation using a rule-curve is comparatively simple, but it is mostly used for single reservoirs and rarely for cascading schemes [75]. In this research, the reservoir operation rule is developed considering the seasonal variation of the hydro-meteorological variables. The release from each reservoir has been conducted in accordance with the principle of the rule-curve operation after the amendment of suitable modifications. The modifications rely on the test results of the refill and deplete ranking orders of the cascading reservoirs.

2.7.2. Real-time Operation of a Reservoir

In a real-time reservoir operation, the decision on the release is conducted using the short time information [76]. The length of the operation varies from an hour to a week. However, in the case of hydroelectric power generation, a continuous decision-making process to determine the water level and power generation using the release data is referred to as a real-time reservoir operation [77]. Generally, a real-time operation defined as a continuous decision-making process [78], [79] and it depends on the current prevailing situation of the demand and the hydro-meteorological condition of the catchment area.

The real-time reservoir operation is studied for more than half a century, but it has not yet been finalized. The technique relies on two components, outflow optimization and inflow forecasting [80]. This study focused on the outflow (release) optimization according to the definition of a real-time hydroelectric power reservoir operation made by reference [77].

2.7.3. Models in a Reservoir Operation

Models used in reservoir operation is either optimization or simulation [81]. The aim of optimization is to answer ‘what is best?’ Whereas, the simulation models are concerned about ‘what if?’ Simulation models are conceptually simple because it follows a rule-based algorithm [3]. The capability of the optimization models in solving a reservoir operation problem is not the same. Hence, the selection of an appropriate model is necessary to find the optimal result.

In a particular reservoir operation, the model performance can be compared. In general, some models advanced over the other. For instance, the research of Yi et al. [82] on the real-time operation of the three hydropower plants of the Lower Colorado River system in the USA showed that the operational efficiency and execution time of dynamic programming (DP) was superior than the mixed integer programming. For simplicity of analysis, optimization models are categorized into different clusters.

The study of Labadie [83] grouped the solution strategies of the reservoir operation models into four: heuristic programming, real-time control with forecasting, implicit and explicit stochastic optimization. Heuristic programming models work based on the rule-of-thumbs. The real-time control model uses a long-term data to analyze the short time operation. However, the implicit and explicit stochastic models refer for the optimization of the long-term operation with perfect knowledge of the future phenomenon.

The optimization models used in the operation of reservoirs can also classify into two groups: traditional and modern techniques. In the traditional methods, a reservoir operation problem is solved using analytical techniques; however, it has a drawback in the analysis of complex and non-linear problems. The modern optimization techniques, develop by mimicking the behavior of the biological organisms, have the ability to find the global optimum value and can apply any type of problem. Genetic algorithm (GA) optimization is one of the modern optimization techniques [84], it develops according to the Darwin’s theory of Evolution “*survival of the fittest*”.

2.7.3.1. Common Models Used for the Operation of a Reservoir

The models used in the operation of a reservoir are developed using a traditional, non-traditional (modern) optimization techniques or a combination of both. The common mathematical models that are used in the operation of a reservoir include: linear programming (LP) [85], [86]; stochastic dynamic programming [87], [88]; rule-based storage accounting [89], fuzzy logic [77], [90], [91], [92]; stochastic fuzzy neural network [56], ant colony optimization [93], [94]; genetic algorithm [95], [96], [97], [98]; a combination of the genetic algorithm and linear programming [50] etc. While, the commonly used computer packages in the operation of a reservoir include: HEC-ResSim [99] and MIKE 11 [100]. The results found from each of the models are subject to uncertainty. In many reservoir operation problems, various uncertainties are inherent [79]. The sources of uncertainties are characterized into two: natural, which is related to the stream flow, and the accuracy of the forecasting models [101].

2.7.3.2. Selection of a Reservoir Operation Model

The selection of the reservoir optimization technique depends on the availability of the data, the specified objective and the constraints of the operation [102], [103]. Traditional optimization techniques such as linear programming (LP) and dynamic programming (DP) in a reservoir operation have drawbacks. For instance, the application of LP in a reservoir operation is limited due to the non-linear nature of the problems involved in the system [30]. In addition, the application of DP in multi-reservoir system optimization is limited due to the computational inefficiency.

Karamouz et al. [31] tested the capability of non-linear programming (NLP) and DP for the development of the monthly operation planning of a multi-reservoir system. The conclusion indicated that DP had limited capability to solve the reservoir operational problems due to the “curse of dimensionality”. However, the genetic algorithm (GA) has received much attention because of its ability to solve complex problems. GA did not require linearization like LP and did not suffer on the “curse of dimensionality” like DP, however it required an initial parameter settings [34]. The method is robust to find the global optimal point [104]. GA is a popular and powerful approach for the analysis of a reservoir operational because of its advancement in finding an optimal result [95]. It is also flexible and versatile in solving optimization problems as compared to the other conventional/traditional optimization techniques [105]. The advantages of the GA is not only in its computational efficiency, but also in its robustness in solving non-linear and non-convex problems.

2.8. GA in Water Resource Optimization

The genetic algorithm (GA) applied in various sectors of water resources such as water supply and wastewater treatment applications, water distribution design and operations, hydrology and fluvial systems, urban drainage and sewer systems and groundwater system design [106]. Accordingly, Dandy et al. [107] used a GA to improve the pipe network optimization, and Afshar and Jabbari [108] also applied a GA to optimize the layout and size in the piping network; furthermore, Vuuren [109] developed a GA model to optimize the pipe diameter. Madsen and Perry [110] used a GA to run groundwater model problems using MODFLOW. This shows that the application of GA is versatile in the field of water resource engineering. The method also gained great attention because of its comparative advantage over other similar techniques. Various researchers also used GA for the optimization of reservoir operation problems.

2.8.1. GA in Reservoir Operation

Reservoir operation is one of the challenging problems for water planners and managers [33]. A number of studies used GA model to find the optimal operation of reservoirs. The model was mainly utilized to develop the operation rule-curve and to determine the optimal release from a reservoir. The results found from the GA model are robust as compared to the traditional optimization techniques. In addition, the algorithm is quite advanced to solve problems that is little known [59].

2.8.1.1. GA Applied to Develop a Reservoir Operation Rule-curve

Kuo et al. [111] developed GA to optimize the rule-curve of a multi-reservoir system in the Chou-Shui River Basin, Taiwan. The system comprises two reservoirs that are used for power generation, irrigation, public and industrial water supply. Analysis were conducted by considering the hydroelectric power and water supply as the principal purposes. Six distinct scenarios with combinations of two principal purposes under varied circumstances and weighted factors were analyzed using GA. The combinations of the main purposes of the different weighting factors provided various shapes of rule-curves, which led to obtain the Pareto optimal solution. Furthermore, Hormwichian et al. [73] used a GA to develop the rule-curve of a reservoir. The developed model was applied in the operation of the Lampao reservoir, Thailand. The result indicated that the pattern of the rule-curve was similar to the existing one that was developed using HEC-3. The advantage of the rule-curve developed using GA provided least shortage than the situation of the HEC-3.

Chen et al. [112] used the macro-evolutionary multi-objective genetic algorithm (MMGA) to develop the rule-curve for a multi-reservoir system in Taiwan. The method was applied on the Fei-Tsui reservoir that was used for hydroelectric power generation and water supply. Forty-one years of historical inflow data were employed to develop the curve. The reservoir is operated with the guide of three curves, namely, the upper, lower and critical limit of the operational levels. Each curve are expressed by two decision variables showing the timely range of storage level zones (high and low). The first objective was to minimize the 10-day shortage of water supply, and the second was to maximize the length of power generation per day. The constraints are related to the water balance of the reservoir system and the recommended operation level of the reservoir. The hedging rule also applied in the process of analyzing the objectives. The result indicated that the model could generate a smooth and well-spread Pareto frontier showing the trade-off water shortage and hydropower generation. In addition, the computation time was proportional to the square of population size. Finally, the research concluded that if the cost of the computation is a vital issue, the approach of MMGA was promising.

2.8.1.2. GA Applied to Optimize a Reservoir Operation

Reddy and Kumar [113] developed GA model to optimize the Bhadra multi-objective reservoir operation in India. The reservoir is used for irrigation and hydroelectric power generation. The objectives were to minimize the deficit of the irrigation, and to maximize the annual hydroelectric power production. The constraints were related to the storage continuity, the active storage limits, the maximum power production, the channel capacity, irrigation demands and the minimum release to satisfy the downstream water quality requirements. Analysis were carried out on monthly basis considering the three different inflow scenarios, namely, the dry, the normal and the wet seasons. A population size of 200 and a maximum generation number of 1000 were used to run the model. The result found from the model was advanced and suitable for decision makers. Meanwhile, on the same reservoir, Kumar and Reddy [66] tested the efficiency and the reliability of the swarm intelligence approach, and used GA for the comparison purpose. The result indicates that for the lower number of function evaluations, the GA model provides a better optimal solution.

Jothiprakash and Shanthi [33] used GA for the operation of the Pechiparai reservoir in India. The fitness function was to minimize the annual sum squared deviation between the desired irrigation release and the storage volume of the reservoir. The research concluded that GA could perform better, if it applies in a real-world operation of a reservoir. Dariane and Momtahen [36] used the direct search genetic algorithm (DSGA) model to optimize a multi-reservoir system operation. The method applied on the piecewise of a cascading of three, seven and sixteen reservoirs. It showed that GA was better than the traditional optimization model (Dynamic programming) in terms of the objective function value and computational runtime. Furthermore, with the objective to maximize the power generation from the three reservoirs that found in the Colorado River Storage Project, Hincal et al. [114] investigated the efficiency and effectiveness of GA. The result of GA was compared to the real operation values. As far as water management concern, GA was efficiently managed the system operation.

2.8.2. GA Compared to Other Optimization Models

Different studies were conducted in order to test the robustness of the GA decision on the operation of a reservoir. Most of the comparisons were made on the operations of the multi-purpose single-reservoir and multi-purpose of the multi-reservoir. The comparisons focused on the results of optimization, the policy that were derived to operate the reservoir, the variables used, and the computation time. The conclusions are outlined that the results found from GA were superior than the corresponding models that were used for comparison.

Ahmed and Sarma [96] compared the policy derived using GA and the stochastic dynamic programming (SDP) for the operation of multi-purpose reservoirs located in the Pagladia river, India. The reservoir is used for hydroelectric power generation and irrigation. The result showed that the operation policy derived using GA was more efficient than the SDP. Likewise, Jothiprakash and Shanthi [48] evaluated GA and SDP for the operation of a multi-purpose single reservoir, namely, the Perunchani reservoir in India. It found that the optimal value obtained from GA was better than the SDP.

In the operation of the Chiller reservoir in India, Azamathulla et al. [76] developed a real-time operation of an irrigation reservoir using the GA and linear programming (LP). The fitness function was to minimize the yield deficit through the maximization of the rate of the actual evapotranspiration. The result indicated that the yield found from the GA operation was better than that of the LP. This shows that the irrigation scheduling developed using the GA was more preferable than the corresponding of the LP. The research concluded that GA could apply to complex problems with little difficulty. Consequently, with the fitness function to minimize the squared deviation of the monthly irrigation demand in the mass balance equation of a cascading reservoir located in the Aras River Basin in Iran, Pilpayeh et al. [57] developed a GA model. The result found from GA compared to the standard operation policy using a simulation model. The comparison showed that GA had a higher benefit in terms of the production.

Cheng et al. [105] evaluated DP and GA in the operation of the Chaishitan hydroelectric power reservoir, China. The average annual energy generated from the plant was 183 GWh. Using 38-years' inflow data, both methods were applied to analyze the monthly reservoir operation. The annual average hydroelectric power generation using DP and GA were 189.3 and 192.8 GWh with computation runtimes of 175 and 14 seconds, respectively. The result shows the computation runtime of DP was obout ten times of the GA. Besides, and the hydroelectric power generation using GA was better than the DP. The majority of the studies revealed that the results found from GA were superior to the corresponding similar optimization models. There were also studies that embedded GA to other similar models to enhance the searching ability. The embedded models quite improved the capability of the model.

2.8.3. GA Model Embedded to Other Optimization Techniques

Chang and Yang [115] embedded GA and HEC-5 to develop the operation rule-curve of the water resource system in southern Taiwan. The embedded model significantly improved the capacity of the existing system. Reis et al. [50] proposed a new combined model using linear programming (LP) and GA to determine the decision variables of a multi-reservoir system. The model demonstrated on the hypothetical hydrothermal system of a four-reservoir. The result found from the combined approach of LP-GA compared to the stochastic dual dynamic programming (SDDP) method. The result of the new proposed GA-LP model was very close to that of SDDP. Moreover, Ebrahimi et al. [116] used GA and Wavelet Transform (WT) methods to operate the multi-purpose Vanyar dam reservoir in Iran. WT facilitated the convergence of the GA model. The combined of the WT and GA model improved the result.

Valeriano et al. [117] used a heuristic algorithm integrated with the physically based distributed hydrological model to reduce the downstream flood risk due to the releases of the reservoirs found in the upper Tone River in Japan. The objective was to minimize the difference between the threshold and simulated discharges. The proposed combined model reduced the basin flood risk effectively. Therefore, the study of the combination of GA to other optimization techniques in the operation of a reservoir indicated that the new combined models are quite advanced.

2.9. Reservoirs Operation for Hydroelectric Power Generation

The operation of the hydroelectric power generation reservoir is different from the others such as irrigation, water supply and flood control. Because, as shown in Figure 2.10, hydroelectric power generation mainly depends on the rate of the turbine release, R and the head, h. Whereas, in the cases of irrigation and water supply, only the rate of the release directly affects the target value, but the volume of the water stored in the reservoir influences on the future release decision.

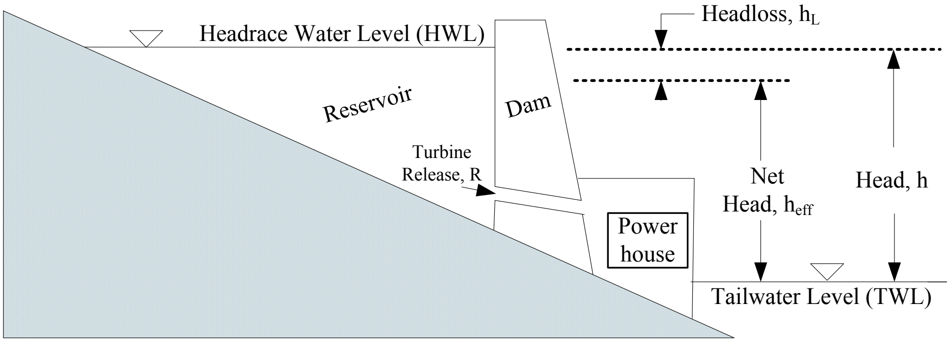


Figure 2.10 Components of a hydroelectric power reservoir

In hydroelectric power generation, the actual generation is less than the installed capacity. Thus, the capacity factor is used to express the relationship between the actual and the installed capacity of a plant. The higher capacity factor shows a better production is being achieved from that plant. Hence, how to operate the reservoir for the stated capacity to produce the largest amount of energy that can be delivered steadily is the concern of hydroelectric power reservoir operation [118].

As shown in Figure 2.10, if the Tailwater level (TWL) is constant, the head, h, varies with the variation of the headrace water level (HWL) only. Hence, the net head is a function of HWL. However, if any backwater effect exists, the TWL has a significant impact on the effective head of water.

The headrace water level (HWL) and the rate of turbine release, R are the two most important governing variables in hydroelectric power generation together with the overall plant efficiency. The general equation of hydroelectric power generation is expressed as:

$$P = \eta\gamma R h_{eff} \quad (2.1)$$

where P is the hydroelectric power, η is the plant efficiency, γ is the unit weight of water, R is the rate of the turbine release, and h_{eff} is the net head of water. However, net head of water is the difference of the head, h and headloss, h_L. It is determined from:

$$h_{eff} = h - h_L \quad (2.2)$$

while the head, h is the computed using the relationship of:

$$h = HWL - TWL \quad (2.3)$$

Thus, the target of a hydroelectric power reservoir operation is to find the best combination of the HWL and the turbine release quantity, R. The higher values of HWL and R favor the hydroelectric power generation. However, during a low flow period, both the values of HWR and R may not simultaneously found be at the maximum level. Hence, for optimal operation, the best combinations of the two values should be determined. The optimization of a hydroelectric power generation deals with the finding of the optimal combination of the turbine release and HWL within the critical period (CP). Hence, the challenges in the finding of the optimal value rely on not only the uncertainties of the various variables involving in the system, but also the potential of the optimization models that were used to find the optimal value of the turbine release with respect to time.

2.10. Cascading Reservoir Operation

Wardlaw and Sharif [17] formulated a GA model for the operation of a four-reservoir problem taking the rate of release as a decision variable. A four-reservoir is a combination of a series and parallel reservoir system as shown in Figure 2.11. The reservoirs are used for irrigation and hydroelectric power generation. According to Wardlaw and Sharif [17] a four-reservoir problem was formulated and solved for the first time by Larsen in 1968. The objective of the problem was to maximize the benefit from the system using the operation period of 12 two-hour periods. The variables, S and R represent the reservoir storage and release volume (a control variable), respectively. Hence, taking the random variable of the inflow volume, I the evolution of the four-reservoir from one stage to the next is expressed as [119]:

$$S_{i+1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} S_t + \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & -1 \end{bmatrix} R_t + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} I_t \tag{2.4}$$

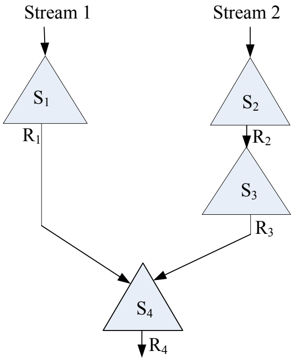


Figure 2.11 Configuration of a four-reservoir problem [17]

To develop the state transformation equation of the Perak cascading scheme, a similar concept of a four-reservoir problem was applied. Actually, the configuration of the Perak cascading scheme is different from a four-reservoir system, because all the four reservoirs in the Perak River are found in cascade. In addition, the reservoirs are used for hydroelectric power generation and flood mitigation. However, the the same procedure was used to develop the state transformation equation.

The conceptual operation rule of a cascading reservoir used for hydroelectric power generation during the refill period is to maximize the energy-storage, while the rule at the deplete period maximizes the power production with the available energy-storage [120]. Therefore, refill and deplete require ranking in order to fulfil the conceptual rule of operating a cascading reservoir.

Liu et al. [121] developed a dynamic programming neural-network simplex model to derive the refill operation of the Three Gorges Reservoir (TGR), China. The result showed that the model improved not only the probability of the refill of the TGR, but also the mean hydroelectric power generation from the reservoirs. Likewise, Liu et al. [75] used China’s Qing River cascading hydroelectric power reservoirs as a case study to develop the rule-curve with the concept of an aggregated reservoir. The large set of Pareto solutions identified through the development of a multi-objective genetic algorithm. The objective function was to maximize the annual hydroelectric power generation, and the constraint equations related to the water balance equation, storage capacity, release limit, power output limit and the state of the reservoir. The result revealed that the approach improved the conventional operation rule by 2.61%.

On a cascading of three reservoirs that was used for hydroelectric power generation and flood control, Guo et al. [122] proposed a model that has three components: combined guide curves, storage distribution and optimization. The storage effectiveness index was used to determine the distributions of the storage volume, while the particle swarm technique was applied to optimize the system. The result showed that the developed model produced additional 2.77% electricity, and flooding was reduced by 38.96% annually.

The developed reservoir operation models using GA improved the system efficiency. However, the reservoir operation studies did not clearly show the determination of the optimal value of all the GA model parameters such as population size, crossover probability, generation number, number of runs, and the impact of parameters on the computational runtime, simultaneously. Because the global optimal point is computed using the optimal combination of the GA parameters. In addition, the computation runtime is important to compare the most appropriate model among several alternatives. To make it more general, the computation runtime is better expressed in a relative manner, because computers have different specifications. In addition, in this study the impact of operational seasons were analyzed with the newly developed model known as the seasonally varied model. In the critical period, four seasons are identified. The name of the seasons is termed as refill, upper level, depletion, and lower level that occur in sequentially.

2.11. Summary of the Literature Review

Optimization is the process of finding the optimal value of a certain system. It can be classified as traditional (conventional) and non-traditional (modern) types. The traditional method employs mathematical analysis; whereas, the modern method is developed by mimicking the behavior of biological, molecular, and swarms of insects

[38]. Modern optimization uses a heuristic search approach. It is a guided search approach. In most cases the solution found from modern optimization techniques are satisfactory [40]. Genetic algorithm (GA) is one of the modern optimization techniques that are used to optimize the various water resource sectors, including a reservoir operation.

The most complex problem in water management and hydropower engineering is the operation of cascading reservoirs due to many variables involved in the system. Optimization models that consider all the variables have not been developed yet. Hence, it is still an ongoing research. However, the techniques like GA have had a great attention due to its ability to compute the reservoir operation problems near to the global optimum value. Literature indicates that the GA model for the development of the rule-curve, and in the optimization of a reservoir operation is more advanced than the traditional optimization techniques like linear programming, dynamic programming, etc. Besides, a combination of the GA model to other similar optimization and simulation techniques can boost its searching ability.

The best result from a GA model can be found after the determination of the optimal values of the model parameters such as the population size (PS), the crossover probability (CRP), the generation number (GN), and the numbers of runs (NR). The computation runtime is also another criterion to select and to compare among the various alternative run options of the GA. The operation of the reservoir using GA employed all the optimal parameters. This research analyzed the basic GA parameters and the optimal values of each used to develop the rule-curve and to maximize the hydroelectric power generation from a cascading scheme. Hence, this research differed from the previous reservoir operation models because of:

i. the optimality of the basic GA parameters especially the impacts of generation number has been determined,
ii. a minimum numbers of run were introduced to guide the iteration of the GA model and
iii. a relative computational runtime was used to explain the sensitivity of the GA model parameters.

CHAPTER 3

METHODOLOGY

3.1. Introduction

A genetic algorithm and seasonally varied models are developed to optimize the operation of the Perak cascading reservoir of Malaysia. The Perak cascading reservoirs are used for hydroelectric power generation and flood mitigation. In terms of the analysis, hydroelectric power generation is taken as a fundamental purpose and the flood mitigation as the operational constraint. The objective of both models is to maximize the annual average hydroelectric power generation from the scheme. Data are collected from Tenaga Nasional Berhad (TNB) and the Department of Irrigation and Drainage (DID) of Malaysia. Analysis are made after filling the missing and screening the outliers data. The areal rainfall and open water evaporation are determined according to the site constraints. Simpson one-third numerical integration technique is applied to develop the stage-storage relationship of the cascading reservoirs. In addition, the state transformation equation is expressed in a matrix form. The seasonally varied model is developed after the determination of the refill and deplete ranking order of the reservoirs with the introduction of a seasonal constant. Whereas, the genetic algorithm model is developed with the optimal values of population size, crossover probability and generation number. The fitness function was to minimize the difference between the potential capacity to the actual power generation of the scheme. The total number of equality and inequality constraint equations were 208 and 104, respectively.

3.2. Data and Analysis Framework

The most important data for this research were collected from the Tenaga Nasional Berhad (TNB) and the Department of Irrigation and Drainage (DID) of Malaysia. The length of the data were varied from 4-20 years (1991-2010). For simplicity, the data was classified into hydrological, meteorological, operational and permanent (constant) based on its variation with respect to time and with consideration of its impact on the operation of the reservoir.

- **Hydrological data** – consists of the daily data of the reservoirs water level (stage). The level is recorded every day at 8:00 am using an automatic recording unit located at the Bersia Office, headquarter of the Perak cascading scheme. For the computation the change of a reservoir storage, daily stage data are usually sufficient [123]. A staff gage is also used as an alternative device to measure the reservoir level. Figure 3.1 shows a staff gage located at the Bersia reservoir. In the figure, the grove shows the maximum level of the reservoir.

Figure 3.1 Stage measurement using a staff gage at Bersia reservoir, Malaysia

- **Meteorological data** - include the rainfall and evaporation records. The daily rainfall data was found from the respective site of the reservoirs and the daily evaporation data collected from the stations that are located nearby the study area having similar meteorological characteristics.
- **Operational data** – it comprises the daily turbine release volume and the hydroelectric power generation. These data are mutually interrelated, because hydroelectric power can generate with the availability of release. In the operation of a reservoir, only the rate of turbine release is a decision variable.

- **Permanent or constant data** – it includes the minimum and the maximum reservoir operating levels, the height of the dam, the minimum and the maximum turbine release rate, the maximum generation capacity of each plant, the rated head, and the threshold rate of release for flood control. The nature of such data are constant throughout the operation period, but very important for the analysis and operation of the reservoir.

Figure 3.2 shows the general framework that are used to develop the genetic algorithm model (GAM) and the seasonally varied model (SVM). Primarily, the missing and the outlier records has been checked and corrected. Inverse distance weighting method are used to fill in the missing rainfall data, while a regression equation is developed to fill in the missing turbine release and hydroelectric power generation data. Secondly, the methods such as Z-score, box-plot and eye-ball are applied to detect the outlier data.

As shown in the framework of analysis, the state transformation equation are developed after the analysis of inflow to each reservoir, areal rainfall and open water evaporation. In parallel to this, the stage-storage-area relationship of each reservoir are developed. It is important to analyze the basic hydroelectric power parameters relationship which indicated under Section 2.9. In addition, the non-parametric Mann-Kendall's method was used to test the trend of the hydroelectric power. The test result would show the performance of the scheme and it provided a reason why a new operational model is necessary to develop.

The development procedures of the genetic algorithm and seasonally varied models are shown at the bottom left and right sides of Figure 3.2, respectively. The genetic algorithm model (GAM) is developed after defining the fitness function and the constraints. The seasonally varied model (SVM) is also developed after the determination of the refill and the depletion ranking order of the cascading reservoirs. The purpose of both models is to maximize the total annual hydroelectric power generated from the entire scheme. The most important findings of the models on the energy-storage among the reservoirs, the variation of the release with respect to time, and the average hydroelectric power generation were compared to the long-term historic average (HA) values. The performance of the operation using GAM and SVM are compared to the corresponding HA by taking a similar inflow pattern and equal total annual quantity of releases. Hence, the ability of the models to find the optimal release rate of the cascading reservoirs with respect to time improved the annual hydroelectric power generation of the scheme.

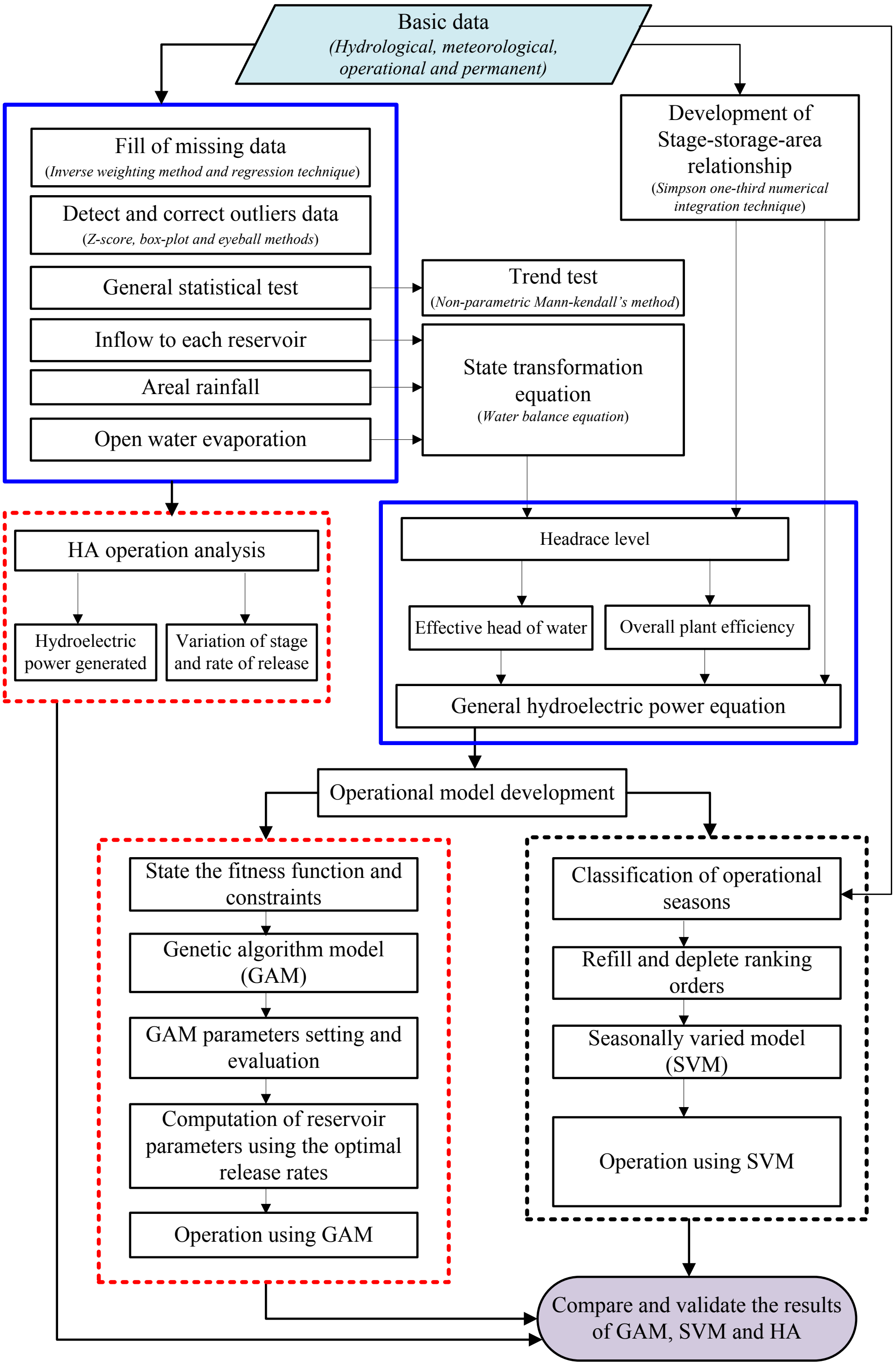


Figure 3.2 General flowchart showing the methodology used and the structures of analysis

3.3. Raw Data Analysis

The raw data are checked against missing and outliers. Missing data is found in rainfall, reservoir level, hydroelectric power generated, and daily turbine released volume records. The missing rainfall data is filled in using the inverse distance weighting method. Regression and correlation methods are applied to fill in the missing data of turbine release and power generation. In addition, outlier data is detected and corrected using the techniques of z-score, box-plot, and eye-ball methods. A statistical analysis is also conducted to show the relationships and distributions of the data. In addition, the trend of the hydroelectric power generation data are tested using the non-parametric Mann-Kendall's method.

3.3.1. Filling in Missing Values

The treatment of missing data can either be replaced by a new estimate or remove the value from the entire data set. The choice of the measure of treatment depends on the type of data and its impact on the overall operation of the reservoir. The preferable measure for missing rainfall, turbine release volume and hydroelectric power generation data can be replaced it with the new estimate.

3.3.1.1. Missing Rainfall Values

The inverse distance weighting method is used to fill in the missing rainfall values. The method is advantageous over the station average and the normal ratio techniques. The reason is the inverse distance method provides different weighting factors based on the distance between the station that the missing data was found to the other stations of interest. The value of the weighting factor is inversely proportional to the distance between the stations. The nearest station has the highest weighting factor. For instance, the missing rainfall at the Kenering is filled in using:

$$F_{Kt} = \frac{\sum_{i=1}^{y} l_i^{-x} F_{it}}{\sum_{i=1}^{y} l_i^{-x}} \tag{3.1}$$

where F_{Kt} is the missing rainfall value of the Kenering station; F_{it} is the rainfall value at the station i; l_i is the corresponding distance from Kenering to the station; x and y are a proportionality factor and the number of stations other than the missing station. The value of the proportionality factor is varied between one and six, but the most commonly value is two [124]. Hence, a proportionality factor of two is adopted to fill the missing data of a rainfall. The missing rainfall of the other stations are also filled in using a similar approach.

3.3.1.2. *Missing Turbine Release and Power Generation Values*

The turbine release and power generation are mutually related. If either of the data is available, the corresponding missing value is computed using a correlation equation of:

$$P_{it} = \chi_i R_{it} + \psi_i \tag{3.2}$$

where P_{it} is the power generated, R_{it} is the rate of the release for the reservoir i during the week t; whereas, χ and ψ are the regression constants. In most cases, either the release rate or the power generation data values were available, but on certain occasions, both data were missing. In that situation, the rate of the turbine release was determined first with the help of the reservoir water balance equation, and then the regression equation applied to estimate the value of the missing hydroelectric power generation.

3.3.2. Screening of Outliers' Data

Outliers are data that have inconsistent and abnormal distance from the entire values. The values are unusually large or small compared to the rest of the data recorded. Such data occur because of a failure to observe the actual measurement, recording error, or sometimes the measurements are correct, but it represents rare events. If the extreme values provide wrong information, the problem of the outlier would be similar to missing data. Outliers also affect the skew and the normality of the data; it distorts the regression results by pulling towards them. In general, the existence of outliers in a data set affects the decision-making process, which is related to the design, operation and the management of water resources [125]. Therefore, outliers records are initially screened and then replaced it with the corresponding new estimate.

Detection of outlier data is subjective in nature since the decision has no clear evidence. In general, representing an outlier in mathematical terms is not easy [126], because all approaches provide a probable result. In this study, z-score, box-plot and eye-ball (simple regression) methods have been used to detect the existence of outliers. The suspected outlier data using the z-score is identified with:

$$Z = \frac{X - \mu}{\sigma} \tag{3.3}$$

where X is the data value, μ is the mean and σ is the standard deviation. A data outside ±2 and ±3 of z-scores is suspected as outlier. If the z-score was beyond ±3, it is taken as an outlier data [127].

In the box-plot method, initially the data are arranged in ascending order. Five data points, namely, the lower and upper quartile values, the median, the minimum and the maximum data values are necessary to define the box and whiskers, as shown in Figure 3.3. The lower and upper quartiles are determined using the relationship of:

$$Y_f = r_{(n+1)f} \tag{3.4}$$

where Y is the location of the quartile data value, r is the rank in the ascending order, n is the data size (number of data) and f is a fraction (0.25, 0.50, and 0.75). $Y_{0.25}$ indicates the lower 25% data (lower or first quartile); $Y_{0.50}$ is the median; and $Y_{0.75}$ is the upper 75% data (upper or third quartile). The value of $(n+1)f$ should be an integer number. If it is a fractional number, the corresponding value of Y should be the average of the preceding and the succeeding integer number of $(n+1)f$.

The difference between the upper and lower quartile values is the Inter-quartile range (IQR). The length of the box in the box-plot equals to the IQR, while whiskers are lines drawn from the upper and lower quartiles up to the maximum and the minimum data values, respectively. The length of the whisker varies according to the minimum, and the maximum data value. Whiskers that are longer than 1.5 times the box-plot is suspected as outliers [128], and if the whisker length is beyond three times the box-plot, the value is highly suspected as an outlier. In this research, both z-score and box-plot methods were applied to detect outliers in the rainfall data.

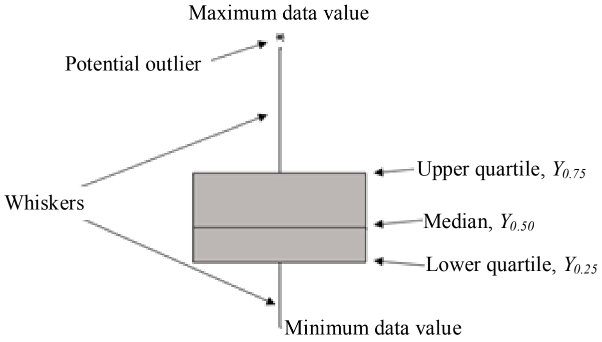


Figure 3.3 Outlier data detection using the box-plot method

Another technique used to screen the suspected outlier data was the eye-ball (simple regression) method. Figure 3.4 shows an exemplary relationship between the daily turbine release and the corresponding energy generated at the Kenering plant. The value of X_1 and X_2 in Figure 3.4 (a) are isolated from the rest of the data set. The values are suspected as outliers. After the removal of these suspected outliers, the regression coefficient (R^2) improved from *0.9784* to *0.9908* as shown in Figure 3.4 (b). Finally, the values of X_1 and X_2 were estimated using the newly developed regression equation and then replaced by the new estimate.

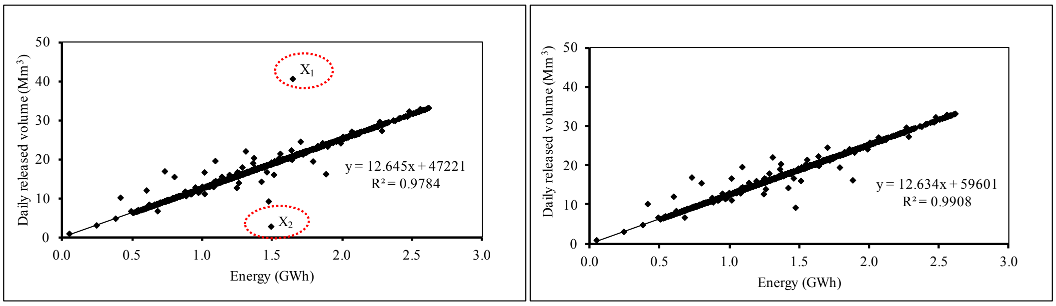


Figure 3.4 Eye-ball method to identify outliers data at Kenering

3.3.3. Statistical Analysis

General statistical analyses is conducted to evaluate the relationships of the data and their trends. The study of the statistical analysis of the hydrological data is important because the occurrences of the past provide some clues for the future conditions. The knowledge of the trend of data is also useful in the decision-making process of a reservoir operation.

3.3.3.1. General Statistical Test

In the general statistical test, the central tendency, dispersion and shape of the data set were evaluated. The aim of the central tendency is to identify a value that best represents the entire data. The common parameters that express the central tendency of the data are the mean, the median and the mode. The mean shows the average of the total value, and it is determined by:

$$\mu = \frac{\sum X}{n-1} \tag{3.6}$$

where μ is the mean, X is the data value, and n is the number of the data. The median and the mode provide less information for the analysis of the reservoir operation. However, dispersion describes how much the score deviates from the mean. It provides an overview of the reservoir operation. The measures of the spread of the data include the range, the variance and the standard deviation. The variance, v^2 (the second central moment about the mean) and the standard deviation, σ were determined using the relationships of:

$$v^2 = \frac{\sum (X-\mu)^2}{n-1} \tag{3.5}$$

$$\sigma = \sqrt{v^2} \tag{3.7}$$

In addition, the third and fourth central moments of the data express the shape of the distribution. Skewness, the third central moment of the data, provides information about the symmetry of the data and it is determined using the relationship of:

$$Skewness = \frac{\sum_{i=1}^{n} (X_i - \mu)^3}{(n-1)\sigma^3} \tag{3.8}$$

while kurtosis, the fourth central moment of the data, shows the peak or the flatness of the data in relative to the normal distribution. The value of kurtosis is computed from:

$$Kurtosis = \frac{\sum_{i=1}^{n}(X_i - \mu)^4}{(n-1)\sigma^4} - 3 \tag{3.9}$$

3.3.3.2. Trend Test

The trend analysis is an important tool to assess the hydrological process [129]. It is useful for effective water resource planning, design and management tasks. In this study, the non-parametric Mann-Kendall's method is used to obtain the trend of the data. The method has been commonly used to access the significance of trends in the time-series data [130]. The non-parametric Mann-Kendall's trend test method has a better performance than the t-test for skewed data [131]. In addition, the method is less sensitive to the outlier data [132].

The trend of the hydroelectric power generation of the Perak cascading reservoir is tested using the non-parametric Mann-Kendall's method. The data of hydroelectric power generation are arranged sequentially into *45* groups, and each represented a summation of 30 days of power generation. The statistic, *Ss* and the sign were computed using the relations of:

$$Ss = \sum_{k=1}^{n-1}\sum_{j=k+1}^{n} sign(X_j - X_k) \tag{3.10}$$

$$sign(X_j - X_k) = \begin{bmatrix} 1 & if & (X_j - X_k > 0) \\ 0 & if & (X_j - X_k = 0) \\ -1 & if & (X_j - X_k < 0) \end{bmatrix} \tag{3.11}$$

where, X_j and X_k are the sequential data values, and n is the length of the data set (group). The computation of the trend depends on the number of data groups. If the data groups are less than ten, the trend of the data can decide referring the results of Equations 3.10 and 3.11 only. Positive value of *Ss* indicates an upward (increasing), and the negative value shows the downward (decreasing) trend. However, when the number of data groups is greater than or equal to ten, the decision of the trend relies on the test result of *VAR (Ss)* which is given as:

$$VAR(Ss) = \frac{1}{18}\left[n(n-1)(2n+5) - \sum_{p=1}^{q} t_p(t_p-1)(2t_p+5) \right] \tag{3.12}$$

where *VAR (Ss)* is the variance of *Ss*, *n* is the number of data points (groups), *q* is the number of tied groups, and t_p is the number of data points in the p^{th} group. Since the data groups in the current study were greater than ten (it was forty-five), the decision of the trend is executed after the computation of the variance. The computation of the variance, *VAR (Ss)* considers the influence of tied groups, if any. The normalized test statistic, *Z*, was computed using the value of *Ss* and *VAR(Ss)* using:

$$Z = \begin{cases} \dfrac{Ss-1}{[VAR(Ss)]^{1/2}} & if \quad Ss > 0 \\ 0 & if \quad Ss = 0 \\ \dfrac{Ss+1}{[VAR(Ss)]^{1/2}} & if \quad Ss < 0 \end{cases} \tag{3.13}$$

and the value of *Z* was checked against the standard normal distribution to determine the critical region. The probability associated with the normalized distribution is computed using the normal probability density function. The function is expressed as:

$$f(Z) = \frac{1}{2\pi} e^{z^2/2} \tag{3.14}$$

A 5% level of significance was taken to reach a conclusion. The test result would have three possibilities:

- **Case one** - the computed probability could be greater than the level of significance, and the value of *Z* would be negative. In this case, it is an increasing trend.
- **Case two** - *Z* would be positive and the computed probability could be greater than the level of significance. For this condition, it is a decreasing trend.
- **Case three** – the computed probability could be less than the level of significance for any condition of *Z*. This is the situation of no trend [133].

3.4. Basic Data for Reservoir Operation

Point rainfall and pan evaporation data are not directly used to analyze a reservoir operation because it cannot represent the actual value. It required modifications to represent the actual condition. Therefore, the point rainfall data is changed to the equivalent areal (average) value; likewise, the pan evaporation values changed into the equivalent open water evaporation. In addition, in the cascading system, the rate of the inflow of the downstream reservoir depends on the release at the preceding reservoir. Hence, the most upstream side reservoir in the cascading scheme can determine the entire system of the inflow-release patterns of the downstream reservoirs.

3.4.1. Areal rainfall

In the analysis of a reservoir operation, the impact of a rainfall can be categorized into two groups: rainfall directly fall over the reservoir and rainfall falls outside the reservoir area. The rainfall that falls outside the reservoir area, but inside the catchment area has an impact on the rate of the river flow. Whereas, the effect of the rainfall over the entire water surface area has been analyzed after changing the point value into the equivalent areal (average) value. Areal rainfall represents the average depth of a single storm event over the entire area. The average (areal) rainfall is computed from the corresponding point value using the relationship of:

$$F_{Ait} = ARF_i \times F_{Pit} \tag{3.15}$$

where F_{Ait} is the average (areal) rainfall, ARF_i is the area reduction factor over the entire reservoir water surface area, and F_{Pit} is the point rainfall for reservoir i in time t. A proportionality constant between the point and the average (areal) value is known as the Areal Reduction Factor (ARF). Figure 3.5 shows the value of the ARF for Peninsular Malaysia (PM). The relationship was developed to design an urban stormwater drainage system. Since urban areas and reservoirs have similarities in terms of the area coverage, this study adopted the relationship to determine the areal rainfall over the reservoir water surface area.

The value of the ARF depends on the catchment area and the duration of the rainfall. For a catchment area less than or equal to 10 km^2, the value of the reduction factor is one [134]. In this study, the meaning of a chatchment area is equivalent to the water surface area of the reservoir.

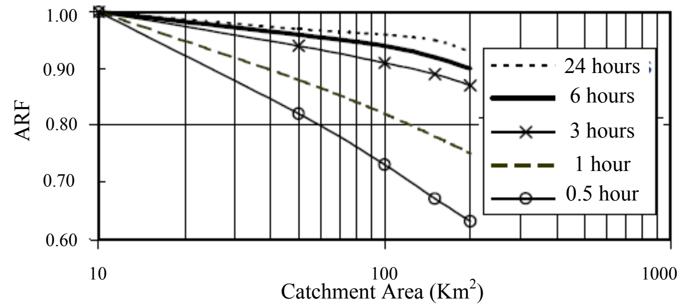


Figure 3.5 Area reduction factor (ARF) for Peninsular Malaysia [134]

3.4.2. Open Water Evaporation

The annual inflow to the reservoir can be utilized in three ways: evaporation, spill and release [135]. The rate of the evaporation can be estimated either using measuring equipment or with the help of prediction models. Different models are available to predict the rate of evaporation. The selection of the model mainly depends on the availability and suitability of the data. Measurement of the evaporation using a pan is common in regions that have less climate variability [136].

The data of pan evaporation is collected from six stations that are located close to the Perak cascading scheme. Normally, the pan evaporation data is greater than the actual rate of evaporation, hence an adjustment is required. The adjustment is conducted after the introduction of a pan coefficient. According to the world meteorological organization suggestion, the pan coefficient (the adjustment value) varies between 0.35 to 0.95 [137], but the actual value of the pan coefficient depends on the elevation, the temperature and the wind speed of the site. The rate of the evaporation also varies with the nature of the evaporating surfaces. Accordingly, the pan coefficient for the Peninsular Malaysia (PM) in the case of open water evaporation is about 0.90 [138]. Therefore, the recorded pan data is changed into the equivalent open water evaporation using the relationship of:

$$E_{it} = K_P E_{Pit} \tag{3.16}$$

where E_{it} is the open water evaporation, E_{Pit} is the pan evaporation from the reservoir i during the time t; and K_P is the pan coefficient which is 0.90 for the PM.

As shown in Figure 3.6, two distinct relationships are portrayed between the elevation and the open water evaporation in PM. The first relationship represents the north western coastal range extending from north to central Perak towards Baling, and the second represents the central mountain chain extending from just north of Malacca to the Thailand border. This study used the first curve that represents the north western coastal range of the country to evaluate the change of open water evaporation of the study sites.

The lowest and highest elevation points in the study sites are 58 and 248 m above sea level (ASL) at Chenderoh and Temenggor, respectively. The differences in the total annual open water evaporation within the range of the elevations are not significant as shown in Figure 3.6. In addition, Raman and Hussein [12] computed the annual average evaporation rate in the Perak area using the daily data of 1997 to 2007. The result showed that the total average annual evaporation was about 1534 mm; hence, it was close to the data shown in Figure 3.6. Therefore, the same rates of open water evaporation were adopted for all the reservoirs.

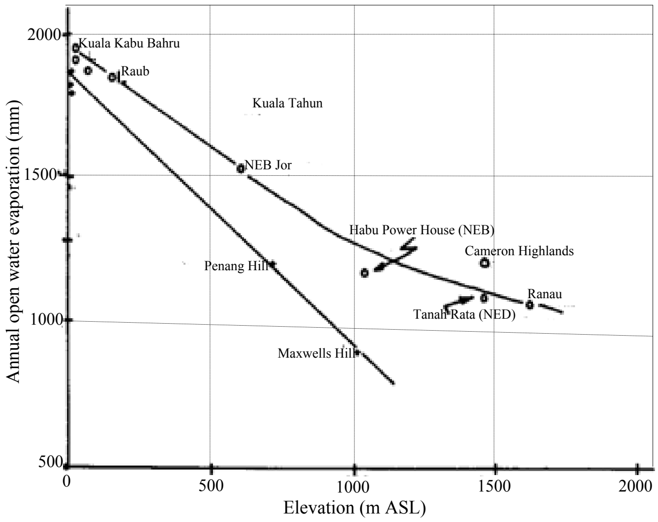


Figure 3.6 Relationship between the open water evaporation-elevation for Peninsular Malaysia [138]

3.4.3. Inflow to the Reservoir

Depending on the relative position of the reservoir in the cascading system, the inflow has two characteristics. Inflow to the most upstream reservoirs relies on the catchment characteristics and the hydro-meteorological conditions, while for downstream reservoirs; the major inflow comes from the release of the preceding reservoir. The total inflow to any downstream reservoir is the sum of the natural inflow and the release of the preceding reservoir. Natural inflow is from the catchment area between the reservoirs.

Figure 3.7 shows the exceedance probability of the long-term historical inflow, and Figure 3.8 the weekly average inflow variation to the most upstream and the largest storage capacity reservoir in the Perak cascading scheme, Temenggor. The figures show that 100% of the weekly average inflow to Temenggor was above100 m^3/s and the annual average inflow was about 141 m^3/s. In addition, it shows that from October to January, the rate of the inflow is above the annual average value.

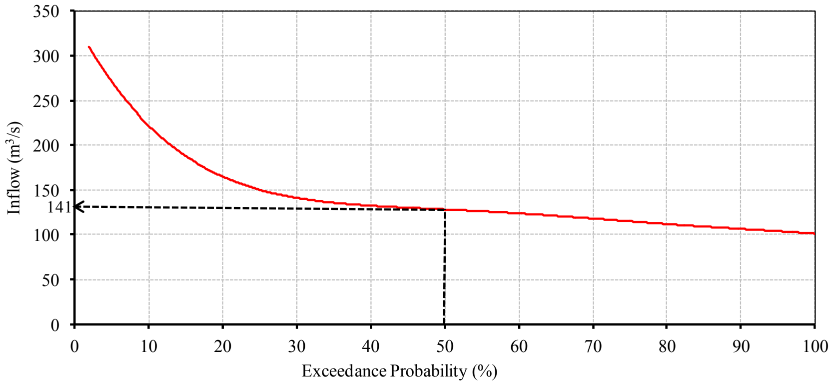


Figure 3.7 The exceedance probability of the weekly inflow to the Temenggor Reservoir

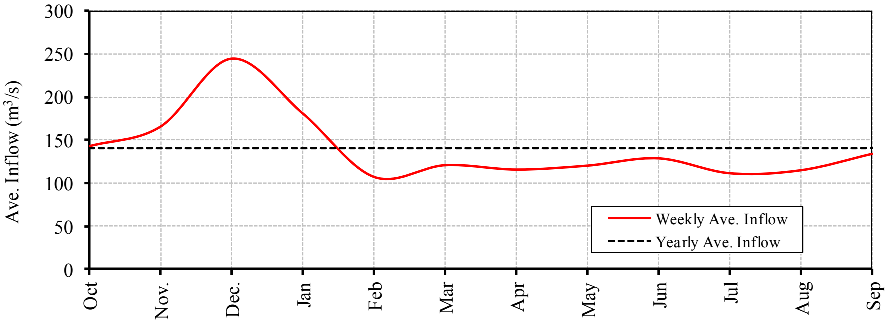


Figure 3.8 Average weekly inflow rate to the most upstream side reservoir, Temenggor

The rate of inflow to any downstream reservoirs is determined using the relations of:

$$S_{it} = S_{i(t-1)} + k_1(I-R)_{it} + k_2(F-E)_{it} A_{it} \tag{3.17a}$$

$$I_{it} = R_{it} + \left(\frac{S_{it} - S_{i(t-1)} - k_2 (F - E)_{it} A_{it}}{k_1} \right) \tag{3.17b}$$

where S_{it} is the storage volume, I_{it} is the rate of the total inflow, R_{it} is the rate of the turbine release, F_{it} is the average rainfall, E_{it} is the rate of the open water evaporation, and A_{it} is the reservoir water surface area for reservoir i during the time t; whereas, k_1 and k_2 are constants related to unit convrsions.

Neglecting spillage, the difference between the total inflow and the release of the preceding reservoir is equal to the natural inflow. The rate of the natural inflow was computed using the relationship of:

$$Q_{it} = I_{it} - R_{(i-1)t} \tag{3.18}$$

where I_{it} and Q_{it} are the total and the natural inflow to the reservoir i during the week t, respectively, and $R_{(i-1)t}$ is the release from the preceding reservoir during the time t. For all cases, the release of the preceding reservoir joins to the succeeding reservoir within a few hours. As illustrated in Table 3.1, a maximum lag time of seven hours was observed between Kenering and Chenderoh.

Table 3.1 Average flow travel time between the reservoirs

From	To	Distance between the dams (km)	Average travel time (hr)
Temenggor	Bersia	19	3
Bersia	Kenering	51	5
Kenering	Chenderoh	48	7

3.4.4. Determination of the Critical Period

A critical period (CP) is the maximum time that takes a full or an absolute full reservoir depletes and then regain its initial level. The study of CP conducted on the variation of the Temenggor reservoir because it is located in the most upstream position in the cascading scheme, and the inflow to the reservoir is based on the hydro-meteorological condition of the catchment area only. In addition, Temenggor reservoir has the largest storage and generation capacity in the scheme.

Figure 3.9 shows the ten consecutive years (2001-2010) headrace variations of the Temenggor reservoir. The relative maximum level was attained roughly at the beginning of February of each year. Therefore, the critical period (CP) is from the beginning of February of the year to the end of January of the following year. The length of CP is almost equal to the calendar year. The knowledge of the CP is important to study the cyclical effects of the various events on the reservoirs such as a refill, deplete, etc.

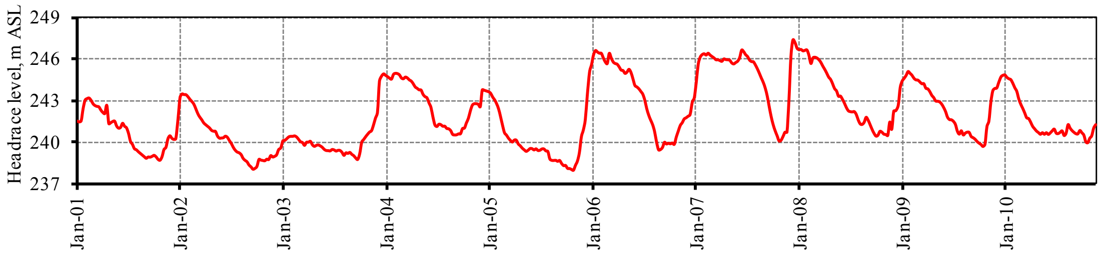


Figure 3.9 Headrace level variation of the Temenggor reservoir (2001-2010)

3.5. Basic Relationships in the Operation and Analysis of a Reservoir

Stage, storage and water surface area are the most important variables for a reservoir operation and analysis. All the variables are related to each other, but the relationship varies with the topography of the reservoir and the height of the dam. Computations of the storage volume and water surface area are challenging, while the measurement of the headrace level is relatively simple. Hence, the common approach used to compute the storage volume, and the water surface area of a reservoir is by relating the values to the headrace level. The stage-storage-area curve for a certain reservoir is developed during the design phase, but it would be subjected to change due to various causes such as sedimentation. This study proposed a method to develop the reservoir variable's relationship based on the operational data. The proposed approach will be tested on the Perak cascading reservoirs.

3.5.1. Stage-Area Relationship

The variation of a reservoir area varies with the stage level, the shape and the slope of the land under water. Computations of the topographic features using mathematical equations are complex, because the natural surface has no definite shape and slope. However, the measurement of the stage is easy and it does not require much effort. The stage of a natural reservoir and the corresponding water surface area can be related as:

$$\frac{\partial A}{\partial h} = \beta h + \lambda \tag{3.19}$$

where A is the water surface area, h is the stage, β is a coefficient and λ is a constant. The coefficient and the constant in Equation 3.19 convey the topographic features of the reservoir area. The values are normally computed from some pre-known data of A and h. This kind of calibration can be achieved with a minimum of three pre-known values, but the accuracy increases if the pre-known values are increased. In this study, the two boundary values (the base level of the dam and the maximum supply level), and arbitrary third and fourth values within the allowable operating levels were considered for the calibration procedure.

3.5.2. Stage-Storage Relationship

A linear relationship between the stage and storage could produce inaccuracies [3], [68]. Mohammadzadeh-Habili et al. [69] showed the non-linearity of the stage-storage relationship. Hence, the relationship of the stage-storage is expressed in the form of a quadratic equation [68]. In this study, the stage-storage relationship was developed using the information from the stage-area curve. The water surface area of each reservoir is expressed with respect to the stage using Equation 3.19, then the Simpson one-third numerical integration technique was used to compute the net storage volume between any two the stage levels. The Simpson one-third numerical integration technique is expressed as a function of the stage (headrace level), h as:

$$\int_{h_0}^{h_n} f(h)dh \approx \frac{(h_n - h_0)}{3}\left[f(h_0) + 4f(h_1) + 2f(h_2) + 4f(h_3) + \ldots + f(h_n)\right] \tag{3.20}$$

The analysis started from the full supply level (FSL), which is a known initial storage volume. The volume at any stage below the FSL is the difference between the initial storage and the net volume between them. Accordingly, the successive net storage volumes at various levels of the reservoir were computed.

As shown in Figure 3.10, the analysis is conducted by dividing the stage into the n sub-stage levels (h_0, h_1, h_2 ... h_n). As a rule, n should be an odd number. In this study, the value of n varied between 7 and 11. The value of n depends on the change of the volume with the change of the stage level. The value of n is proportional to change of the volume with the stage. A larger n value provides a better estimation. In addition, the accuracy of the estimation of the net storage volume is high if the differences between the stages h_0 to h_n, h_n to h_{2n}, h_{2n} to h_{3n}, etc. are small. A variable stage difference between h_0, h_n, h_{2n} ... h_{jn} are used to compute the net volume.

The live storage volume is used as a checkpoint to validate the newly developed relationship. Theoretically, the sum of the small sub-stage storage volumes between the FSL, h_0 and the intake level, h_{jn} is equal to the actual live storage volume of a reservoir. Finally, the stage-storage relationship of each reservoir in the Perak cascading scheme is expressed as a form of quadratic equation degree two.

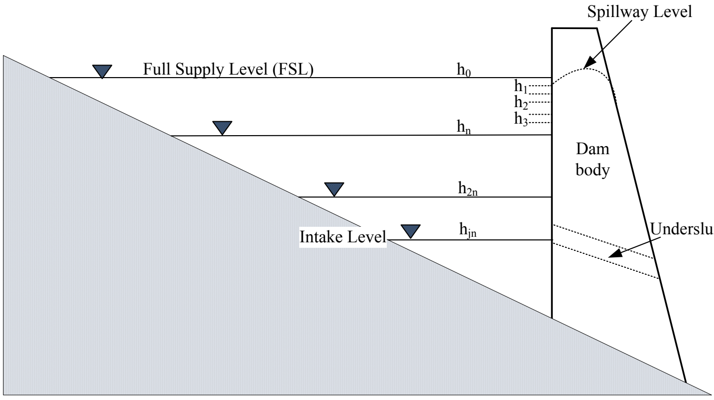


Figure 3.10 A typical dam cross-section

3.5.3. State Transformation Equation

The state transformation equation is developed by considering all the reservoirs initially at the warning level. In addition, the predicted values of the inflow, average rainfall and open water evaporation are applied to develop the equation. Since the aim of the operation is to avoid spillage, its value in the state transformation equation is considered as zero. The variation of the storage volume was computed on a weekly basis, and the computation period varied between the first week of February of the year to the last week of January of the following year. The selection of the period was in accordance with the occurrence of the warning level in the Temenggor reservoir as shown in Figure 3.9 and the consideration of the critical period. The Temenggor reservoir reaches at warning level around the first week of February every year.

The water balance of the reservoir can be expressed using Equation 3.17 (a), and the weekly successive storage volume is computed from:

$$S_{i1} = S_{i0} + k_1\left(I_{i1} - R_{i1}\right) + k_2\left(F_{Ai1} - E_{i1}\right)A_{i1} \tag{3.21a}$$

$$S_{i2} = S_{i1} + k_1\left(I_{i2} - R_{i2}\right) + k_2\left(F_{Ai2} - E_{i2}\right)A_{i2} \tag{3.21b}$$

The equation can be continued up to the week 52. If Equation 3.21 (a) is substituted into Equation 3.21 (b), it gives:

$$S_{i2} = S_{i0} + k_1\left(I_{i1} + I_{i2} - R_{i1} - R_{i2}\right) + k_2\left[\left(F_{Ai1} - E_{i1}\right)A_{i1} + \left(F_{Ai2} - E_{i2}\right)A_{i2}\right] \tag{3.21c}$$

The general form of the water balance equation at any week *t* can be formulated as:

$$S_{it} = S_{i0} + k_1\sum_{t=1}^{n}\left(I_{it} - R_{it}\right) + k_2\sum_{t=1}^{n}\left(F_{Ait} - E_{it}\right)A_{it} \tag{3.21d}$$

where S_{i0}, S_{i1}, S_{i2}, ..., and S_{in} are the storage volumes at the initial stage, after week *1*, *2*, ..., and *n* for reservoir *i,* respectively, I_{it} is the total inflow, F_{Ait} is the areal rainfall over the reservoir, E_{it} is the open water evaporation, and A_{it} is the average water surface area for reservoir *i* during the week *t*; whereas, k_1 and k_2 are unit conversion constants.

Equation 3.21 (d) is known as the state transformation equation. Considering the configuration of the Perak cascading reservoir, the state transformation equation can also be expressed in a matrix form as:

$$S_{t+1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \left[S_t + Q_t + \left(F - E\right)_t A_t\right] + \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \times R_t \tag{3.22}$$

where Q is the natural inflow to the reservoir: other variables are similar to Equation 3.21 (d).

Among the various variables involved in the state transformation equation, the only decision variable was the rate of release. The variation of the storage volume in the reservoir can be managed with the rate of release. Hence, the target of a reservoir operation model was to optimize the weekly rate of release in order to maximize the hydroelectric power generation of the Perak cascading scheme. In addition, the rate of release determines the headrace level variation and the overall plant efficiency.

3.5.4. Head of Water

As shown in Figure 2.10, the difference between the headrace and the tailrace water levels is referred to as the head of water. At a known storage volume, the headrace level can compute using the stage-storage relationship. In the Perak cascading reservoirs, the stage-storage relationship is expressed as:

$$h_{it} = c_i S_{it} + d_i \tag{3.23a}$$

where h_{it} is the headrace level, S_{it} is the storage volume, c_i and d_i are the coefficient and the constant of the stage-storage relationship for reservoir i, respectively. The storage volume, S_{it} is computed from Equation 3.21 (d) or 3.22, and subsequently, the weekly effective head of water is determined using:

$$h_{it(net)} = h_{it} - h_{i(tail)} \tag{3.23b}$$

where $h_{it(net)}$ is the effective head of water, and $h_{i(tail)}$ is the tailwater levels above the mean sea level for the reservoir i during the week t, respectively. In Equation 3.23 (b), the tailwater level was taken as a constant, because the impact of the tailwater variation on the system performance as compared with the headrace level is insignificant [139]. Moreover, the influence can be minimal if there is no backwater effect.

3.5.5. Overall Plant Efficiency

The overall plant efficiency (OPE) of a hydroelectric power generation plant is the combined effect of the hydraulic, turbine and generator efficiencies. The relationships between the long-term historical OPE to the corresponding headrace level and the rates of turbine release of the Perak cascading reservoirs is checked. As shown in Figure 3.11, there was no distinct relationship between the OPE and the rate of releases.

However, as shown in Figure 3.12 the OPE has a definite relationship to the headrace level. The value of OPE decreases with the increase of the headrace level (stage). In the analysis of a hydroelectric system, the study of Goor et al [88] expressed efficiency as a function of the average head. Similarly, this study presented the OPE as a form of the linear equation and it is expressed as:

$$\eta_{it} = a_i h_{it} + b_i \tag{3.24}$$

where η_{it} is the overall plant efficiency, h_{it} is the headrace level, while a_i and b_i are the regression constants for the reservoir i during the week t.

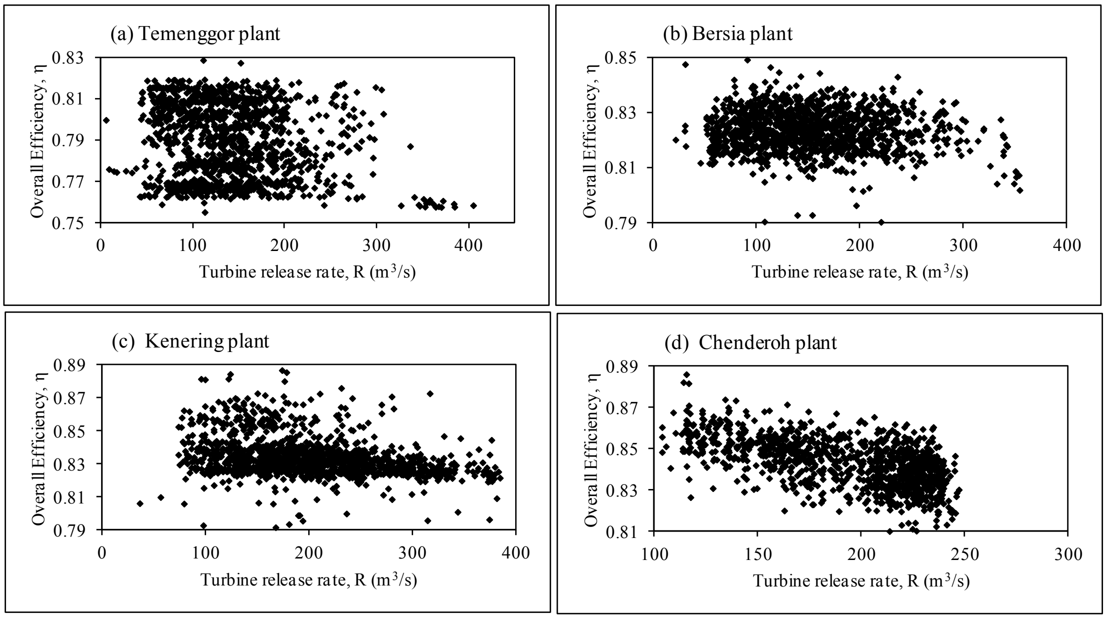


Figure 3.11 Relationship between turbine release rate and overall efficiency

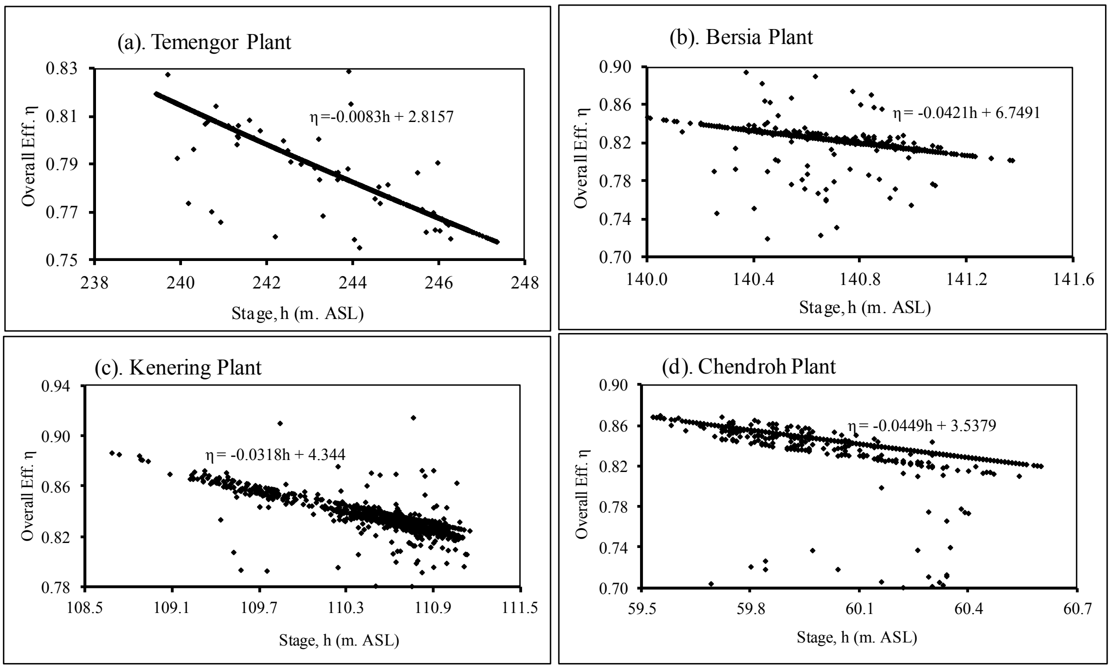


Figure 3.12 Relationship between headrace level and overall plant efficiency

3.6. Development of a Reservoir Operation Models

The rate of release determines the storage volume in the reservoir and the headrace level. However, in terms of the reservoir's objective achievement, the storage volume may not be equally important to the rate of the release. For example, the quantity of water in the reservoir does not have an abrupt impact for irrigation. The concern of irrigation is the rate of release that satisfies the crop water requirements, while for hydroelectric power generation; both parameters (release and storage) have a direct impact on the quantity of the power production. In the case of cascading reservoirs, the release can be also accomplished with the consideration of the downstream reservoir capacity. The main reason is the release of the preceding reservoir should effectively be recaptured by the succeeding reservoir. It indicates the operation of a reservoir is complex, and sometimes the purposes are contradicted to each other.

In this study, two models were developed to optimize the operation of the Perak cascading reservoirs. The first model was developed considering the long-term operation, and it is known as a seasonally varied model (SVM). The second model is known as a genetic algorithm model (GAM). The seasonally varied model (SVM) has been developed after the analysis of the long-term historical average (HA) operation and hydrological records. The headrace level variation of Temenggor reservoir taken for analysis. Temenggor is located in the most upstream side in the cascading scheme; moreover, the reservoir is the largest storage and generation capacity in the system. Accordingly, the critical period is sub-divided into four seasons, namely, deplete, lower level, refill and upper level. The refill and deplete seasons require a ranking order. The objectives of the refill and deplete ranking are to maximize the energy-storage among the cascading reservoirs within the hydrological year (critical period). The order also reduces the volume of spillage, and maximizes the total volume of water passing through the turbine.

The genetic algorithm model (GAM) was developed to maximize the hydroelectric power generation through the optimization of the rate of turbine release from each reservoir in the hydrological year. GAM evaluated the various alternative options of the turbine release rates of each reservoir. The optimal values were determined with the consideration of the total hydroelectric power generation and the energy-storage in the cascading scheme. The two main variables in the hydroelectric power equation are the effective head and the rate of the turbine release. Hence, the decision of the GAM optimization technique on the weekly rate of releases indirectly considered the headrace level variation, which has a direct relation to the storage volume of the reservoir.

The generic algorithm (GA) optimization method is widely used in water resource system optimization [105]; in addition, for a reservoir operation, GA has better performance than the dynamic programming [48]. GA can be applied for both convex and non-convex problems; it is another advantage of the technique. For example, Hosseini et al. [140] used GA for convex problems. However, traditional optimization methods such as linear programming, dynamic programming, inter programming, etc. cannot solve non-convex problems [141].

3.6.1. Seasonally Varied Model (SVM)

A seasonally varied model (SVM) has been developed after the analysis of the long-term HA operational and hydrological data of the most upstream and the largest hydroelectric power generation capacity reservoir in the cascading scheme. The analysis showed that in the hydrological year, there are four distinct seasons, namely, deplete, lower level, refill and upper level and it occurs consecutively. Since the most upstream reservoir governs the entire cascading scheme, the seasons also applied to all other downstream reservoirs. The total length of the four seasons is equal to one year, while the length of each season varies.

The operation of SVM is accomplished using the weekly-predetermined headrace level. The target of the release was to maintain the predetermined headrace level. However, in some situations, like at the high or low-flow period, maintaining the target level was quite difficult. Hence, the operation was performed with the adjustment of the new headrace level. The new headrace level would have a minor variation to that of the predetermined level.

In addition, the release from the preceding reservoir was carried out considering the storage capacity of the following reservoir. The maximum release potential and the available free storage of the succeeding reservoir governed the optimal release of the preceding reservoir. Hence, the total hydroelectric power generation of the scheme can be maximized with the optimal release of each reservoir.

3.6.1.1. Classification of the Operational Seasons

The long-term historical time-series data of the largest and the most upstream side reservoir, Temenggor headrace level variation is used to classify the operational season/period. Because inflow to the Temenggor reservoir is not obstructed. As shown in Figure 3.9, the headrace level variation of the Temenggor is periodical; however, the annual values of the maximum and minimum level are different. That means, the reservoir was not completely refilled or depleted in some years. After the year 2006, the lower level was almost similar. Hence, with the observation of the headrace level variation of the Temenggor reservoir, four important seasons are identified. The seasons are namely, deplete, lower level, refill and upper level operation seasons.

- **Deplete Season (DS)** – this is a continuous season in which the reservoir depletes from the relatively maximum/peak to the minimum operating level. This happens when the weekly inflow volume is below the corresponding outflow volume. The long-term HA operation data showed that the DS is the longest season and it starts not before February of each year.
- **Lower Level Operating Season (LLOS)** – the season starts at the end of DS. In the LLOS, the reservoir level stays at the lower operating level. In this season, the rate of release is equal to the rate of inflow. Operation during the LLOS is required great attention because if all the reservoirs are found on the lower level at the same time, it might be liable to a power shortage.
- **Refill Season (RS)** – this is the total length of the period that a reservoir is continuously under the refilling operation. During RS, the rate of the inflow is higher than the rate of the outflow.
- **Upper Level Operating Season (ULOS)** – this season starts at the end of RS. During this season, the stage level stays at or above the warning level, and the weekly inflow is equal to or above the release volume. Hence, the target of the operation during this season is maintaining the reservoir level at the warning level. During the period, the rate of the release is equal to the rate of inflow. Release from the preceding reservoir can be accomplished considering the turbine specifications, available free storage capacity of the succeeding reservoir and the threshold value for flood control of the reservoirs. The start and end period of ULOS is variable; it depends on the priority order of the refill operation. When the reservoir level is below the warning level, the ULOS is over and then depletion starts.

The seasons are arranged in the order of *DS–LLOS– RS– ULOS*. The cyclic period of the operation starts in the first week of February of the year, and ends at the last week of January of the following year. The historical time-series data showed that the starting period of RS was in the beginning of November, while the DS in February. LLOS and ULOS started at the end of DS and RS, respectively. The length of LLOS and ULOS depended on the corresponding length of the DS and RS, respectively. The exact start and end week of each season requires a subjective judgment of the operator and the prevailing hydro-meteorological conditions of the catchment area.

The RS and DS require ranking since the seasons could have a high impact on the operation of the cascading scheme. In addition, the optimality of hydroelectric power generation and the management of inflow depend on the two seasons. In the cascading scheme, the length of RS was variable. In addition, the priority of the refill order is proportional to the increase of the power production with the storage. The length of the depletion period is also proportional to the storage effectiveness ratio (SER). A higher SER value, provides a longer DS.

3.6.1.2. *Refill Ranking Order*

Refill season (RS) is a period that the headrace level continuously increases from a relatively lower to maximum level within the critical period (CP). Since the reservoirs have a different storage capacity and inflow situations, the length of a refill period is variable. In addition, the refill ranking order is important to manage the total inflow to the cascading reservoirs during the season. The refill order enhances the annual energy-storage of the cascading hydroelectric reservoir and minimizes the volume of spillage. Therefore, the objective of the refill ranking order is to reduce the total annual volume of the spill and to optimize the quantity of water that pass through the turbines.

The ranking order performed with a consideration of the reservoir storage capacities and the sensitivity of the power production to the change of the storage volume, which affects the headrace level of the reservoir. As shown in Figure 3.13, the ratio of the change of the head, Δh to the change of storage volume, ΔS is higher for smaller reservoirs. A similar situation also happens to the change of power generation, ΔP to changes of storage volume, ΔS.

The refill ranking order of the four Perak cascading reservoirs was determined using the model developed by Lund [120]. The model is expressed as:

$$V_i = \alpha_i \eta_i \left(\sum_{i=1}^{j} I_i \right) \tag{3.25}$$

where V_i is the increase in power production with an increase in the storage, α_i is the change in the headrace level per change in storage, η_i is the power generation efficiency, and I_i is the total inflow to reservoir j, and the summation was for all the reservoirs upstream of the reservoir i. Analysis of V started from the most upstream side of the the reservoir.

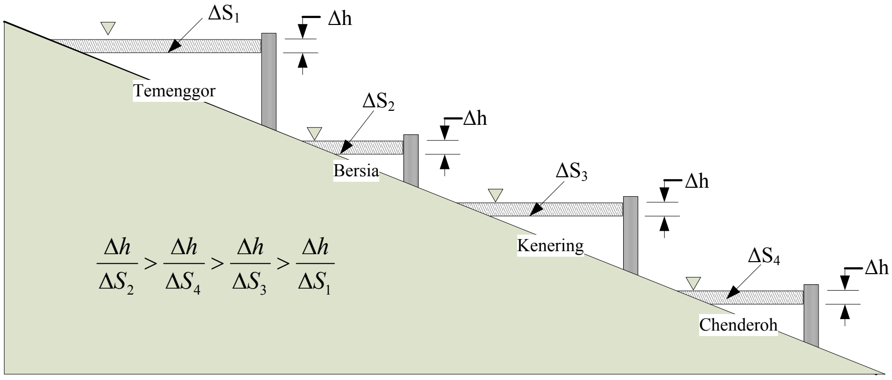


Figure 3.13 Changes in the storage volume with equal change of headrace level

The change of the hydroelectric power head per change of storage volume was computed by taking an equal rate of inflow to each reservoir. The headrace level is determined from Equation 3.23 (a). The rate of inflow to each reservoir and the overall plant efficiency (OPE) were determined from the respective methods shown in Sections 3.4.3 and 3.5.5, respectively. The cumulative total inflow volume to each reservoir up to the period of analysis was used to compute the increase of the power production to the change of storage. The refill operation started from the higher value of the power produced per storage volume, and the successive operations accomplished with the descending order of the value that was found from Equation 3.25.

3.6.1.3. Deplete Ranking Order

After RS, the reservoirs stay at a relatively maximum level for a certain period. The length of the period depends on the difference between the inflow and release rates. The depletion process starts immediately when the rate of the inflow is less than the rate of the outflow. Apart from the most upstream side reservoir, the rate of the inflow to the entire cascading depends on the rate of the release of the preceding reservoir.

The order of deplete ranking can be conducted by adjusting the rate of release of the reservoir. As in the refill case, the deplete ranking also accomplished to optimize the storage volume and to maximize the power generation of the scheme.

The deplete ranking order showed the priority of the reservoir headrace level that should be lowered first. The objective of the deplete ranking was to find the optimal headrace level and release rate in the cascading scheme during the low-flow period. In addition, during DS the reservoir should be effectively recaptured the inflow.

The depletion ranking order was also developed using the method presented by Lund [120]. Sequential procedures are followed to determine the value of the storage effectiveness ratio (SER), which governs the depletion ranking order. SER is determined using:

$$SER_i = \frac{P_{Li}}{ES} \tag{3.26}$$

where P_{Li} is the drawdown season power loss, and ES is the energy-storage. Both are equated using the relationship of;

$$P_{Li} = 168\left(CI_{(i-1)} + S_{pi}\right) \times h_i \times \left(S_i - \Delta S_i\right) \times \eta_i \tag{3.27}$$

$$ES = FER - 168\sum_{i=1}^{n} I_{(i-1)} \times h_i\left(S_i\right) \times \eta_i \tag{3.28}$$

The drawdown volume, ΔS_i is determined from:

$$\Delta S_i = \frac{ES}{168 \times h_i \times \eta_i} \tag{3.29}$$

where $CI_{(i-1)}$ is the cumulative natural inflow upstream of the reservoir for the remainder of the refill season, S_{pi} is the volume of upstream storage to be emptied during the remainder of the deplete season, h_i is the average head to the corresponding drawdown volume, S_i is the current reservoir storage volume, η_i is the overall plant efficiency, FER is the firm energy requirement, $I_{(i-1)}$ is the inflow from the upstream of the reservoir of interest, and $h_i(S_i)$ is the hydropower head as a function of the reservoir storage for reservoir *i*. The constant 168 represents the total hours of the week. Depletion is accomplished with the ascending order of *SER* values.

The value of SER for each reservoir determined using the long-term historical average (HA) hydroelectric power generation data of the cascading scheme is shown in Appendix A. The minimum inflow to each reservoir is used to compute the firm energy. The difference between the average cascading hydroelectric power generation and the cumulative of the firm energy provided the energy shortage of the scheme. The average volume of the release required to generate the energy shortage and the impact of the additional release of the storage volume of the specific reservoir was determined by using Equations 3.26 to 3.29.

3.6.1.4. Optimal Power Generation Model using the Ranking Order

The length of each season directly influences the total annual power generation from the cascading scheme. The most important period that highly influence the maximization of the total hydroelectric power generation from cascading plants are *RS* and *DS* because both seasons have a great impact on the net head of water. The net head of water is one of the major parameters in hydroelectric power generation as indicated in Equation 2.1.

The seasonal water balance of each reservoir is evaluated on a weekly basis. The change in the storage volume is computed using the relation:

$$S_{it} - S_{i(t-1)} = k_1\left(\frac{A_{it} + A_{i(t-1)}}{2}\right)\left(h_{it} - h_{i(t-1)}\right) = k_2\left(I_{it} - R_{it}\right) + k_3\left(F_{Ait} - E_{it}\right)\left(\frac{A_{it} + A_{i(t-1)}}{2}\right) \tag{3.30}$$

where S_{it} is the storage volume, A_{it} is the water surface areas, h_{it} is the stage, I_{it} is the rate of inflow, R_{it} is the rate of turbine release, F_{Ait} is the areal rainfall, and E_{it} is the rate of open water evaporation of the reservoir *i* during the period of *t*; k_1, k_2 and k_3 are the unit conversion constants.

The change of the stage with respect to time during a specific season was a predetermined value. However, the value depended on the length of the season (time) and the range of the operating head of the reservoir. The weekly small increment of the stage is computed with the introduction of a seasonal constant, *w*. The value of the season constant varies between -1 and 1. During RS, the value of *w* is positive, while it is negative during DS. The value is zero during ULOS and LLOS. The value of *w* is inversely proportional to the length of the season.

Initially, it is assumed that the total outflow of the reservoir is with in the permissible range of the turbine releases. Hence, the weekly change of the headrace level is computed using the relationship of:

$$\Delta h_{it} = h_{it} - h_{i(t-1)} = \begin{cases} w_{ij}\Delta h_{i\max} & \text{if } \; R_{i\min} \le I_{it} \le R_{i\max} \\ w_{ij}\Delta h_{i\max} + 2\Delta t\left(\dfrac{I_{it} - R_{it}}{A_{it} + A_{i(t-1)}}\right) & \text{if } \; I_{it} > R_{i\max} \; or \; I_{it} < R_{i\min} \end{cases} \tag{3.31}$$

where Δh_{it} is the weekly change of headrace level, $\Delta h_{i\,max}$ is the difference between the warning level and the minimum operating level of the reservoir, I_{it} is the total inflow, $R_{i\,min}$ and $R_{i\,max}$ are the minimum and the maximum rate of the turbine releases and O_{it} is the total outflow from the reservoir *i*, during the week *t* and season *j*. Thus, the actual rate of release is determined by:

$$R_{it} = \begin{cases} I_{it} - \left(\dfrac{A_{it} + A_{i(t-1)}}{2k_2}\right)\left[k_1\Delta h_{it} - k_3\left(F_{it} - E_{it}\right)\right] & \text{if } \; R_{i\min} \le O_{it} \le R_{i\max} \; or \; \left(R_{it} \equiv O_{it}\right) \\ R_{i\max} & \text{if } \; O_{it} \ge R_{i\max} \\ 0 & \text{if } \; O_{it} < R_{i\min} \end{cases} \tag{3.32}$$

If the initial assumption is not satisfied, the alternative condition of Equation 3.31 can be taken considering the rate of release that is computed with the initial assumption. Thus, the final release value is used to compute the hydroelectric power generation. As a result, the total seasonal hydroelectric power generation was the sum of all power generated from each plant in the cascading system and it was determined using:

$$P_{it} = \gamma\sum_{i=1}^{4}\sum_{t=1}^{L_j}\eta_i R_{it}\left[\left(h_{it} + \frac{\Delta h_{it}}{2}\right) - h_{it\,r}\right] \tag{3.33}$$

and the annual hydroelectric power generation, P from the cascading of four reservoirs in the 52 weeks using the seasonally varied model (SVM) was found from:

$$P = \sum_{i=1}^{4} \sum_{t=1}^{52} P_{it} \tag{3.34}$$

where P_{it} is the power generated from reservoir i during the period of t, $h_{it\ r}$ is the tailrace level, η_i is the overall plant efficiency and L_j is the total length of season j.

3.6.2. Optimization using the Genetic Algorithm Model (GAM)

An efficient optimization algorithm depends on its accuracy and ability for searching for the global optimum point [142]. The genetic algorithm (GA) is an efficient tool for large-scale nonlinear optimization problems [60], and it is powerful in searching for the optimal strategy for a reservoir operation [95]. Hence, the GA optimization technique was developed to analyze the Perak cascading hydroelectric power generation scheme.

After the selection of the optimal run option, the GAM algorithm is developed. The optimal run option is used to find and to evaluate the optimal GAM parameters. The population size (PS), the crossover probability (CRP) and the generation number (GN) are the basic GAM parameters. The optimal selection of the PS can be performed according to the result of the change of fitness value to the corresponding change of the PS. The optimal value of PS is selected, when the change of fitness with the change of the PS was negligible. Similarly, the optimal values of the CRP and GN was selected taking into account the change of the fitness value. Of the various alternative values of CRP and GN, the one that had a minimum fitness value was selected. Moreover, a relative computation runtime is also used to show the impact of the GAM parameters on the maximization of hydroelectric power generation.

3.6.2.1. Selection Procedure of the Basic GAM Parameters

In this study, the basic GAM parameters include the population size (PS), crossover probability (CRP) and generation number (GN). Tests were conducted to determine the optimal value of each parameter. The optimal value of each parameter is used to maximize the hydroelectric power generation of the cascading scheme.

i. **Population Size (PS):** deals with the determination of the optimal PS value. The main challenge in the analysis of PS is the initial estimated value maybe too-small or too-large. A too-large PS has a time penalty; whereas, a too-low PS has a quality penalty [143]. There is no clear criterion for the selection of the initial PS, but selection should be accomplished in order to meet the minimal time and quality penalties as shown in Figure 3.14. In this study, a random initial value of 20 is used to study the impact of PS on the corresponding fitness value. The test of PS was examined by randomly increasing the value up to 1000.

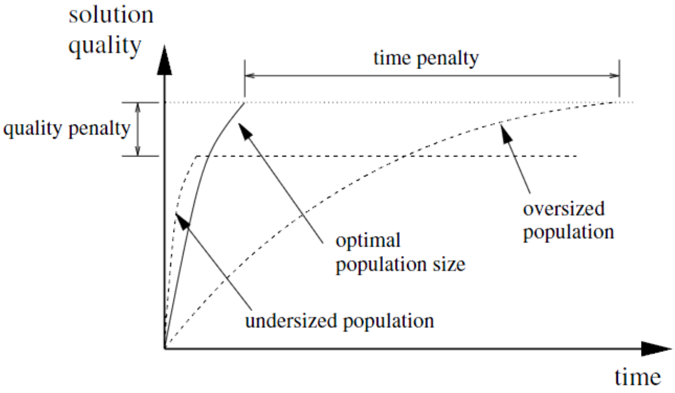


Figure 3.14 Population sizing character in GA [143]

ii. **Crossover and mutation:** crossover and mutation are processes that take place to produce an offspring. The majority of the offspring is produced by either crossover or the mutation operators. The crossover probability (CRP) is the ratio that parents follow the crossover rule to produce an offspring. A CRP of one indicates that all offspring has different parents. The value of CRP varies between 0.50 and 0.95 [17]. In this research, the impact of a CRP is studied between 0.60 to 0.95. However, in the study of Chang et al [95] the optimal value was achieved at a mutation probability (MUP) of 0.1; in addition, the study of Wu and Chau [144] applied a MUP of 0.1. It can be concluded that the optimal MUP value is close to 0.1. Therefore, in this study a MUP of 0.1 is adopted.

iii. **Generation number (GN):** at each step, the GA model randomly selects parents from the population and produces an offspring. The next generation is based on selection, crossover and mutation rules. The impact of GN evaluated in the range of 20 to 200 with consideration of the change of the fitness value. The GN influences the optimal search ability and at the same time, it acts as a stopping criterion.

Optimization using GA was accomplished using the processes of selection, mutation and crossover. Figure 3.15 shows the general flowchart of the GA model used. Optimization using GA starts with the creation of a random initial PS [97], and then repeated iteration (run) followed until the optimal condition achieved.

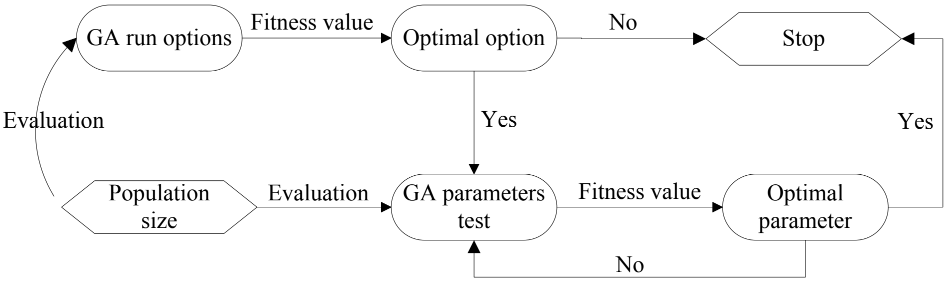


Figure 3.15 The GA flowchart

The general procedures of optimization using GA can be followed as:

- the fitness value computed at each end process
- more fitting points selected as parents
- offspring produced by
    - mutation – with random change of a single parent
    - crossover – with a combination of parents
    - elite – the best parent continues as it is
- the iteration should be repeated until the stopping criterion is reached

Using various combinations of fitness scaling, selection, and mutation options as shown in Table 3.2, eight model run options were analyzed because the other options provided infeasible solution. All the options initially used a CRP of 0.8, MUP of 0.1 and GN of 50 to find the optimal run option. Similarly, the study of Wardlaw and Sharif [17] on the computation of a four-reservoir problem was used a PS of 100, uniform crossover option and tournament selection to analyze the sensitivity of CRP.

The run option that provided a minimum fitness value per PS was selected as the optimal GA run option. The optimal PS, CRP and GN were also determined in a similar manner. The stopping criteria were when the fitness value had no more improved or when it arrived at the end of the GN, whichever is reached first. The best stopping criterion is the GN [51].

Table 3.2 The criterion used for the GAM run options

	Criteria	
	Selection	Crossover option
Option 1	Stochastic Uniform	Scattered
Option 2	Uniform	Scattered
Option 3	Roulette	Scattered
Option 4	Tournament	Scattered
Option 5	Stochastic Uniform	Heuristic
Option 6	Uniform	Heuristic
Option 7	Uniform	Intermediate
Option 8	Uniform	Single point

3.6.2.2. *The Genetic Algorithm Model Development*

As shown in Figure 3.16, the inflow and the outflow are classified into controlled and uncontrolled according to its nature and the way of management. For this study, the natural flow and rainfall over the reservoir surface are treated as uncontrolled inflow, while the release of the upstream reservoir that joins to the next downstream reservoir was a controlled inflow. Likewise, the turbine release and the spillage are classified under controlled and uncontrolled releases, respectively. The target of GAM was to optimize the rate of the controlled outflow taking into consideration the other three scenarios, namely, the controlled inflow, uncontrolled release and uncontrolled inflow.

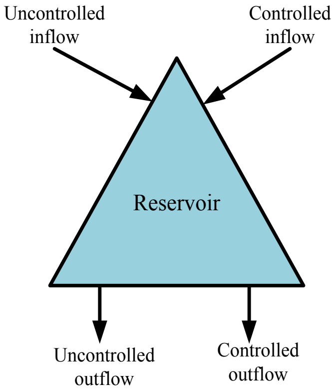


Figure 3.16 Group of reservoir variables

The average uncontrolled inflow to the most upstream side reservoirs in the cascading scheme was determined using Equation 3.17 (b), but there is no controlled inflow into the reservoir. Total inflow to Bersia, Kenering and Chenderoh are the sum of the natural flow determined using Equation 3.18 and the release of the immediate preceding reservoir. The relationship between the preceding release and the succeeding reservoir inflow are shown in Appendix B. The relationship is represented using a linear function.

The basic equation of hydroelectric power generation is expressed as:

$$P_{it} = \gamma \eta_i R_{it} h_{it(net)} \tag{3.35a}$$

where P_{it} is the hydroelectric power, R_{it} is the rate of release; and $h_{it(net)}$ is the effective head of water for reservoir i during the week t, respectively; γ is the unit weight of the water, η_i is the overall plant efficiency. Substituting the values of effective head of water and overall plant efficiency from Equation 3.23 (a) and 3.24, respectively into Equation 3.35 (a) gives,

$$P_{it} = \gamma\left[a_i\left(c_i S_{it} + d_i\right) + b_i\right] \times R_{it} \times \left[\left(c_i S_{it} + d_i\right) - h_{i(tail)}\right] \tag{3.35b}$$

The overall plant efficiency and the effective head of water are expressed as a function of the release rate. Equation 3.35 (b) is formulated after the substitution of Equations 3.23 (a), 3.23 (b), and 3.24 into Equation 3.34 (a). The final hydroelectric power equation has only one decision variable per week; that was the rate of release and it found after substituting S_{it} from Equation 3.21 (d) into Equation 3.35 (b). The target of GAM was to optimize the rate of the releases from the entire four reservoirs of the cascading scheme to maximize the hydroelectric power generation.

3.6.2.3. Fitness Function and Constraint Equations

The objective of GAM was to maximize the total annual hydroelectric power generation from the cascading of the four reservoirs. Hence, the fitness function was to minimize the difference between the potential and the actual generation of the cascading scheme. The objective function is expressed by:

$$\min(P_{\max} - k_o \sum_{i=1}^{4} \sum_{t=1}^{52} P_{it}) \tag{3.36}$$

where P_{max} is the total potential capacity of the cascading scheme (578 MW), P_{it} is the actual power generation, determined from Equation 3.35 (b) for reservoir i at the week t, and k_o is a constant. The algorithm of the fitness function was shown in Appendix C.

The constraints were related to the state transformation equation of each reservoir, the threshold value for flood release, the maximum and the minimum headrace level and the rate of release, the cumulative turbine releases and inflow volume at the end of the operating period. Mathematically, the constraints are expressed as:

$$k_{1i}\left(h_{i(t+1)}^2 - h_{it}^2\right) + k_{2i}\left(h_{i(t+1)} - h_{tj}\right) - k_{3i}\left(I_{it} - R_{it}\right) - F_{Ait} + E_{it} = 0 \tag{3.37}$$

$$R_{it\,\min} \le R_{it} \le R_{it\,\max} \tag{3.38}$$

$$h_{it\,\min} \le h_{it} \le h_{it\,\max} \tag{3.39}$$

$$\sum_{t=1}^{52} R_{it} \le \sum_{t=1}^{52} I_{it} \tag{3.40}$$

where R_{it} is the rate of turbine release, h_{ij} is the stage (headrace) level, F_{Ait} is the areal rainfall, and E_{it} is the rate of open water evaporation of the reservoir i for the week of t, k_{1i} and k_{2i} are constants that depend on the stage-storage relationship, and k_{3i} is the unit conversion factor.

The developed model has four decision variables per week and the total decision variables per year (for 52 weeks) would be 208. There were 208 equal and 104 non-equal constraint equations. The cumulative annual inflow to the reservoir should be greater than or equal to the cumulative annual releases as indicated in Equation 3.40. The minimum turbine release rates for each reservoir are selected after the analysis of the long-term historical operation data. Whereas, the maximum releases apart from the most downstream side reservoir (Temenggor) is decided according to the plants capacity. The maximum release of the most downstream side reservoir was decided with the consideration of the threshold value of the flood control. The threshold value for the flood control of the Perak River was 850 m^3/s [145]. This study adopts a factor of safety of two to determine the maximum release rate from the Chenderoh, the most downstream reservoir. Therefore, a maximum release of 425 m^3/s was taken according to the flood mitigation criteria.

3.6.2.4. Computational Runtime

The computational runtime varies based on the computer specifications. A computer that has a processor of Intel (R) i5 CPU M430 @ 2.27GHz with a RAM of 2 GB is used to analyze the computation runtime. The computational runtime with different combinations of population size (PS), crossover probability (CRP) and generation number (GN) were tested. The objective of the test was to conduct the sensitivity of the genetic algorithm (GA) parameters on the computation runtime. At each iteration, the length of the runtime to reach the stopping criteria was recorded. Repeated trials were conducted per each set of options, and the average time elapsed in seconds per run option was recorded. Out of all run options, the one that took the longest time was selected as a benchmark to evaluate the sensitivity of the parameters' on the computational runtime.

3.7. Sensitivity Analysis of a Cascading Hydroelectric Power Generation

The sensitivity of the hydroelectric power generation from each reservoir with the change of the reservoir variables are analyzed. The variables were the rate of turbine release, the headrace level and the overall plant efficiency. The test was conducted with the assumption of all reservoirs have the same rate of inflow patterns during the refill and deplete seasons. The effect of the change of the head of water on the corresponding rate of release was examined. The result of sensitivity analysis is provided the priority of the reservoir that require attension during the operation process. It is also directly related to the maximization of hydroelectric power generation of the Perak cascading scheme.

3.8. Summary of the Methodology

The data, methods and structures of the analysis comprise three parts. The first part shows the detail analysis of the raw data. The raw data are presented in accordance with the model requirements. The second part indicates the procedures that is used to develop the seasonally varied model (SVM) and the genetic algorithm model (GAM). The third part reveals the impact of the computational runtime on the fitness value and the sensitivity analysis of the hydroelectric power generation.

In the raw data analysis, the point rainfall changed into the equivalent areal (average) values by applying a reduction factor obtained from the urban drainage manual of Malaysia. Similarly, the pan evaporation values converted into the equivalent open water evaporation with the provision of a pan coefficient of 0.90, it is recommended for Peninsular Malaysia. The rate of inflow to each reservoir of the Perak cascading scheme is also computed. The computation is conducted with the consideration of the release of the preceding reservoir and the natural inflow from the catchment area that is located between the reservoirs. In addition, the stage-storage relationships of the reservoirs are revised using the Simpson one-third numerical integration technique. The storage-stage relationship is presented as a form of a quadratic equation of degree two.

After the study of the long-term historical average (HA) headrace level variation of the most upstream reservoir in the cascading scheme, Temenggor, four seasons are identified. These are the refill, upper level, deplete and lower level seasons. Maximization of hydroelectric power generation and energy-storage of the cascading scheme is accomplished after the development of the reservoirs' refill and deplete ranking order. Hence, the SVM was developed taking into account the refill and deplete ranking orders of the reservoirs. Alternatively, using the principle of the optimization technique, the GAM was developed to optimize the operation of the Perak cascading reservoirs. The fitness function of GAM was to minimize the difference between the potential and actual generation from the cascading scheme. The model has been developed using 208 equal and 104 non-equality constraints that had 208 decision variables. The optimal GAM parameters had been found after the determination of the run option. The GAM computational runtime was evaluated using a computer that has a processor of Intel (R) i5 CPU M430 @ 2.27GHz with a RAM of 2 GB. Hence, the impact of population size, crossover probability and generation number on the computation runtime were also evaluated. The variation of the fitness value was used to compare the impact of the computation runtime. The sensitivity analysis of the cascading hydroelectric power generation was conducted to identify which reservoir is more susceptible to the change of the rate of turbine release, headrace level variations and overall plant efficiency.

The capabilities of the models to maximize hydroelectric power have been tested with consideration of a similar pattern of inflow and an equal volume of annual release conditions. Finally, the results found from the models were compared to the long-term historical operation data values.

CHAPTER 4

RESULTS AND DISCUSSION

4.1. Introduction

This chapter, the results and the discussion, has three major parts. The first part shows the analysis of the long-term historical time-series data of the Perak cascading reservoirs. As shown in Figure 4.1, the rainfall increased towards the downstream direction. However, as indicated in Figure 4.2, the combined influence of evaporation and rainfall had a negative effect on the energy-storage of the Temenggor reservoir. The computation of the live storage volume using the newly revised stage-storage relationship showed that a maximum of 12% relative error found from the theoretically known value at the Temenggor reservoir. In addition, as shown in Figure 4.7, the discrepancies of the estimated live storage volume for all the downstream reservoirs were below a 10% of relative error.

In the second part, the results of the genetic algorithm model (GAM) and the seasonally varied model (SVM) are presented. It showed that the results of GAM mainly relied on the optimal values of the population size (PS), crossover probability (CRP) and generation number (GN). In addition, the impacts of the computational runtime and iteration number on the result of the GAM was also explained thoroughly. The annual average hydroelectric power generations using the operating of the GAM, SVM and HA were evaluated comparatively. The result showed that GAM and SVM improved the HA power generated value by 5.34% and 2.00%, respectively. The percentage of improvement of the GAM was equivalent to 12.17 MW per day and a relatively uniform rate of release and a high-energy storage than the SVM. The result that obtained using the operation of the GAM was quite good and it is convinced to apply the model for the real operation of the Perak cascading reservoirs.

The third important aspect of this study was on the economic analysis of the additional power generated from GAM and SVM. The economic benefit is analyzed taking the minimum tariff rate of the Tenaga Nasional Berhad (TNB), the electricity utility company in Malaysia. The additional power generated using the GAM and SVM could worth about RM 22 million and RM 8 million, respectively.

4.2. Long-term Historical Average (HA) Data Analysis

The long-term historical average (HA) data were essential to study the overall conditions of the reservoir. The most important historical data used for the analysis of reservoir operation performances were grouped into two: the hydro-meteorological and operational data. The hydro-meteorological data governs the reservoir operation, while the operation data show the historical decisions made due to the hydro-meteorological events. The reservoir operational data include the release rate, the output gained (hydroelectric power) from the reservoir and the overall efficiency of the system. In addition, the data trend analysis was made to evaluate the system performance and then to design the future reliable operation rule.

In the basic data analysis of the Perak cascading scheme, the rainfall around the reservoirs, the rate of evaporation, the rate of inflow to each reservoir, the headrace variation of the reservoirs within the critical period (CP), the rate of the turbine release and the hydroelectric power generation were shown in details. Among these, the two important variables that predominantly influence the hydroelectric power generation are the rate of the turbine release and the headrace level.

4.2.1. Rainfall Variability

The ten years (2001-2010) annual average intensity of rainfall at Temenggor, Bersia, Kenering and Chenderoh were 2.6, 5.4, 5.3 and 7.4 mm per day, respectively. The result showed that the intensity of the rainfall increases in the downstream direction. During the period, the standard deviations of the average weekly rainfall at Temenggor, Persia, Kenering and Chenderoh were 1.53, 2.53, 2.49 and 4.40 mm, respectively. The standard deviation also indicated that the variability of the rainfall towards the downstream direction had an increasing pattern. The data showed that the intensity and the variability of the rainfall were increasing towards the downstream direction. However, as shown in Figure 4.1, the modes of the rainfall patterns were different. A bi-modal rainfall pattern, the peak values during April and November are observed around Bersia and Kenering. Whereas, at Chenderoh, the rainfall had a tri-modal pattern, and an additional third peak value is observed in July. The general rainfall pattern indicates that a minimum value occurs during January and a maximum around November. Even though the entire four reservoirs had different patterns of rainfall, it can be concluded that from February to April, and again from August to November, the depth of rainfall in the cascading scheme was increasing.

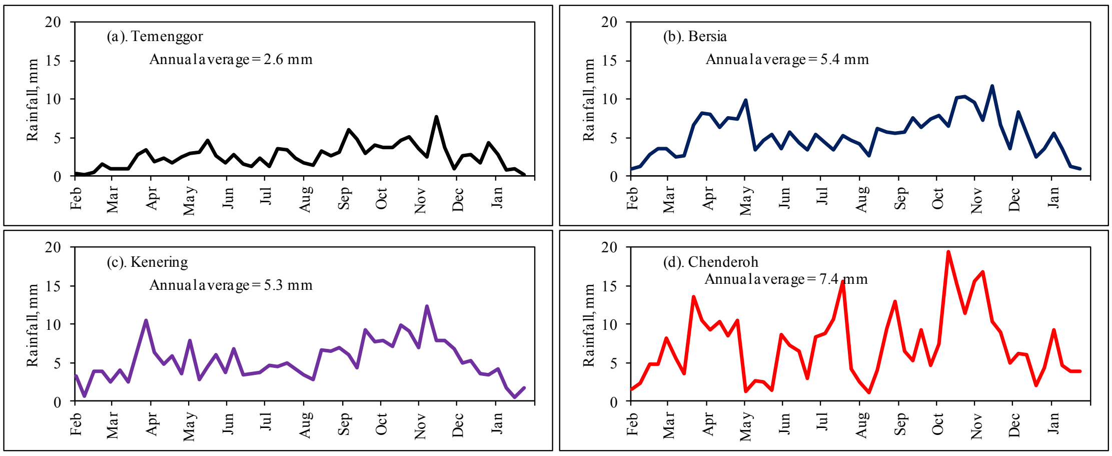


Figure 4.1 Weekly average rainfall variations (2001-2010)

The combined effect of the areal rainfall (F) and open water evaporation (E) are shown in Figure 4.2; both have different impacts on the energy-storage. A negative value of (F-E) shows the rate of open water evaporation exceeds the corresponding value of the areal (average) rainfall. Hence, the positive value of (F-E) indicated that an increase of the energy-storage in the reservoir. The analysis showed that the annual average difference between the areal rainfall and the open water evaporation in the entire three downstream reservoirs had a positive impact on the energy-storage. A positive impact of energy-storage provides an increase of the head of water in the reservoir that has a direct relationship with the hydroelectric power generation.

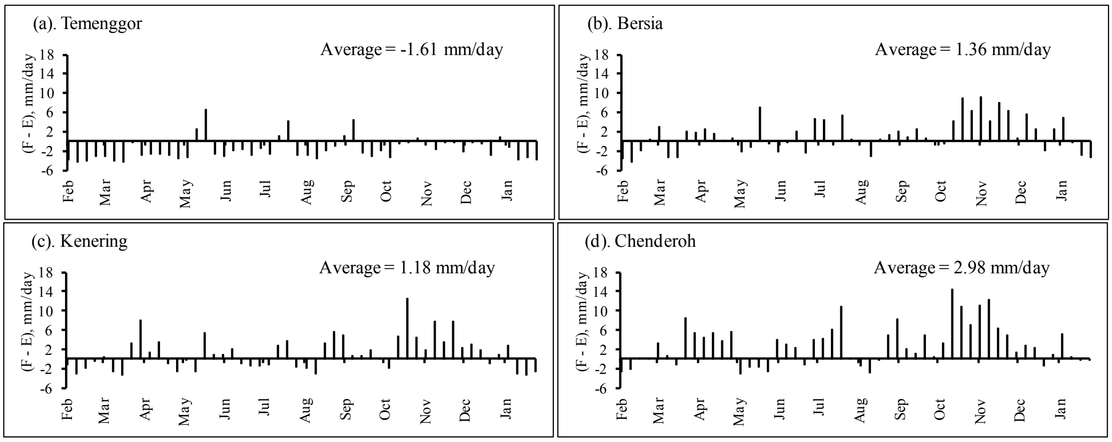


Figure 4.2 Difference between areal rainfall and open water evaporation rates (2001-2010)

4.2.2. Natural Inflow to the Reservoir

The total inflow to a reservoir in a cascading system is the sum of the release of the preceding reservoir, the natural inflow and the rainfall over it. The natural inflow to the reservoir refers to the flow from the catchment area that are found between the reservoirs. A natural inflow and the rainfall over the reservoir are uncontrolled flows, while the release of upstream reservoir is taken as a controlled inflow of the next downstream reservoir. The rate of controlled inflow is subjected to change due to the variation of operational decision. Therefore, the analysis of a controlled inflow should be separated from the uncontrolled flow. If the natural inflow is relatively smaller than the controlled inflow, then the total inflow rate to any of the downstream reservoirs in the cascading system is highly dependent on the release of the preceding reservoir.

The natural inflow to each reservoir from the respective catchment area is shown in Figure 4.3. The average annual natural inflow to Temenggor, Bersia, Kenering and Chenderoh reservoirs were 141, 17, 46 and 22 m^3/s, respectively. A relatively higher average annual natural inflow was observed at Temenggor because there was no abstraction of the flow to the reservoir. In addition, all the reservoirs had a variable peak natural inflow rate that was occurring at different periods. The peak natural inflow rate at Temenggor is found during December and January; whereas, at Kenering, the peak is found around November. At Bersia and Chenderoh, the relative peak values are observed around September and October. The refill operation depends on the occurance of the peak flow period of the most upstream reservoir, Temenggor and the rate of releases from the preceeding reservoir. Hence, the possible time to conduct the refill operation of the Perak cascading scheme is between September of the year and January of the next year.

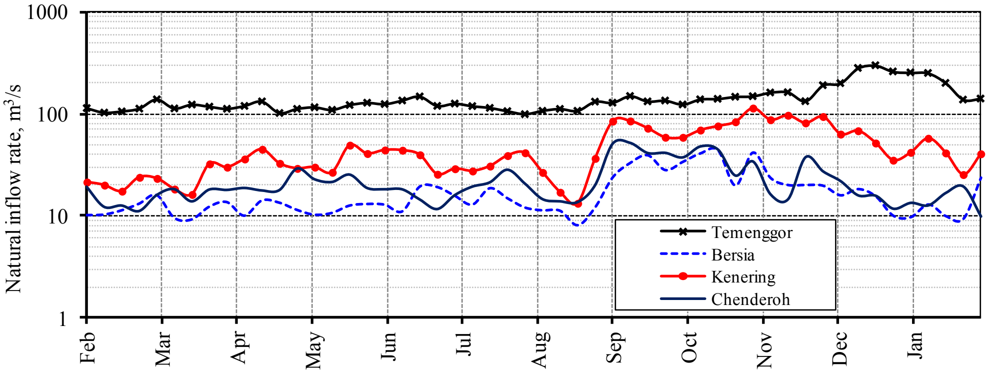


Figure 4.3 Average natural inflow rate to the reservoirs (2006-2010)

4.2.3. Variation of the Turbine Releases in the HA Operation

The minimum release rates from Temenggor, Bersia, Kenering and Chenderoh were 98, 111, 127 and 148 m^3/s, while the maximum rates were 174, 188, 271 and 265 m^3/s, respectively. This shows that both the minimum and maximum rate of the turbine release increase towards the downstream direction. As shown in Figure 4.4, the releases from Temenggor and Bersia followed a similar pattern. However, the release from the Bersia reservoir was higher than the Temenggor during the entire period. It indicated that the operation of Temenggor reservoir governs the release of Bersia.

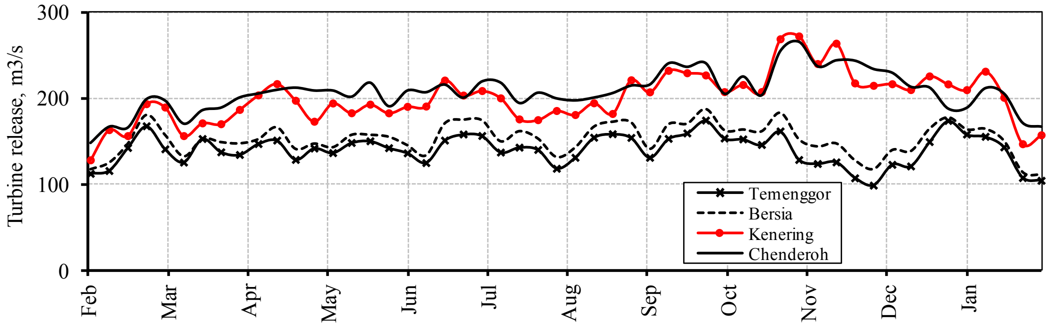


Figure 4.4 Weekly average turbine released rate of the HA operation (2006-2010)

4.2.4. Variation of the Headrace Level in the HA Operation

As shown in Table 1.1 under Section 1.1, the maximum allowable operating levels or FSL of Temenggor, Bersia, Kenering and Chenderoh are *248.41*, *141.43*, *111.31* and *60.42* m above sea level (ASL), respectively. The minimum and the extreme permissible operating levels of the three turbines out of the four at the Temenggor plant are *239.3* and *236.5* m ASL, respectively and the other turbine has a minimum operating level of 221.3 m ASL. Similarly, the minimum normal operating levels or NDL for all the turbines at Bersia and Kenering are 139.9 and 108.5 m ASL, respectively. The minimum operational level of the three turbines out of the four at the Chenderoh plant is 59.13 m ASL, and the other one has a minimum operating level of 57.43 m ASL.

Analysis of the headrace level was conducted between February of the year and January of the following year. The reason was in each year the largest storage capacity reservoir in the cascading scheme, Temenggor, reaches at a relative maximum level around the first week of February as shown in Figure 3.9.

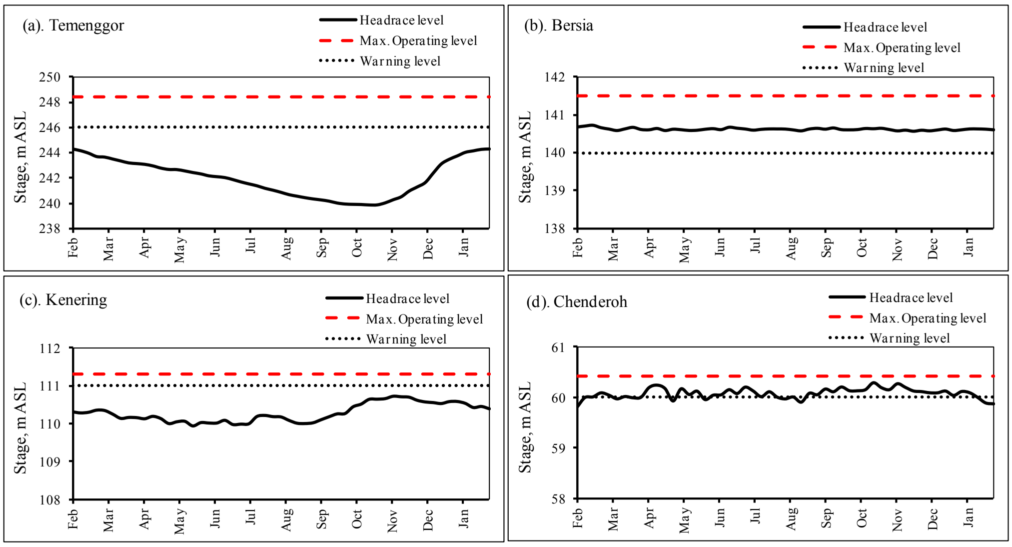


Figure 4.5 Weekly average headrace level variation of the HA operation (2006-2010)

The variation of the HA headrace level of the Temenggor reservoir, as shown in Figure 4.5 (a), continuously decreased from February to October. Whereas, the headrace level of the Kenering reservoir slightly decreased from February to August. As shown in Figures 4.5 (b) and (d), the change of headrace level at Bersia and Chenderoh were insignificant.

The long-term weekly HA of the headrace level variations of the Temenggor and Kenering reservoirs showed that the operations were below the warning level. About 100% of the Bersia and 74% of the Chenderoh reservoirs operation period indicated that the headrace levels were above the warning level. In addition, as illustrated in Table 4.1, except the Temenggor reservoir, the range of the headrace variation was below a meter. It showed that almost all three downstream reservoirs operated with nearly a constant headrace level. The analysis shows that the downstream three reservoirs of the Perak cascading were operated mainly with the consideration of the total rate of inflow.

Table 4.1 The weekly HA headrace variation of the Perak cascading reservoirs

Statistical Parameter	Unit	Temengor	Bersia	Kenering	Chendroh
Max. stage level	m ASL	244.30	140.82	110.74	60.30
Min. stage level	m ASL	239.82	140.58	109.94	59.80
Mean stage level	m ASL	241.98	140.69	110.29	60.08
Range	m	4.48	0.24	0.80	0.50
Standard deviation	m	1.48	0.03	0.24	0.10
Skewness		0.005	1.04	0.41	-0.34
Kurtosis		-1.36	1.80	-1.17	0.10

The HA headrace level data at Temenggor, Bersia and Kenering were positively skewed. Only the stage (headrace level) data at Chenderoh was negatively skewed. The negative skewed data show that for more than half of the operation period, the values were below the mean (average) level. However, the headrace level data of Temenggor and Kenering had a negative kurtosis. The negative and positive kurtoses are in accordance with the shape of the standard normal distribution curve. Data that had zero kurtosis follows the standard normal distribution. A negative kurtosis is flatter than the standard normal distribution, while a positive kurtosis is a comparatively higher peak than the normal distribution curve. The flatter shape indicates that the majority of the data were found far from the mean value; whereas, in the case of the higher the peak shape, the majority of the data values were found close to the mean value. Hence, the headrace level data of the Chenderoh and Bersia were situated near to the mean value, while the data of the headrace level variation of the Temenggor and Kenering reservoirs were far from the mean value.

From February to October, the average net outflow from the Temenggor reservoir was greater than the total net inflow; however, the net outflow and inflow for the other reservoirs, as indicated in Figures 4.2, 4.3 and 4.4 were almost equal. After the mid of October up to the first week of December, release from the two upstream reservoirs (Temenggor and Bersia) decreased as shown in Figure 4.4. Whereas, Figure 3.8 shows that the natural inflow to the Temenggor, during that period was above the annual average value. Similarly, Figure 4.5 (a) shows the Temenggor reservoir was in the refilling state. In the HA operation, the refill of the Temenggor reservoir was started with the reduction of the rate of the release. During this period, the releases from the downstream of the two reservoirs (Kenering and Chenderoh) were on average increased. Hence, the operation of the Temenggor reservoir influences the entire cascading scheme because the total inflows to all other reservoirs are directly or indirectly dependent on the release of the Temenggor.

4.2.5. The HA Hydroelectric Power Generation

The hydroelectric power generation potential of Temenggor, Bersia, Kenering and Chenderoh are 348, 72, 120 and 38 MW, respectively. It constitutes a total cascading hydroelectric potential of 578 MW. However, the actual annual average of the hydroelectric power generated from the cascading scheme was 228.07 MW. It was 39.46% of the potential. The overall weekly data of the minimum, average and maximum hydroelectric power generations of the cascading system were 170.57, 228.07 and 272.82 MW, respectively. The standard deviation (σ) of the weekly hydroelectric power generation from the Perak cascading scheme was 22.82 MW, and the coefficient of variation (COV) was 0.10. The value of COV indicated that the weekly average hydroelectric power generation was close to the annual average value, which is 228.07 MW.

The weekly variation of the hydroelectric power generated from each reservoir is shown in Figure 4.6. It shows that the hydroelectric power generation from the Bersia and Chenderoh were nearly equal; the average values were 31.39 MW and 30.21 MW, respectively. However, the hydroelectric power generation potential of the Bersia plant is 72 MW and that of Chenderoh is 38 MW. This indicated that Bersia was less efficient than the Chenderoh.

The total cascading hydroelectric power generation is sensitive to the operation of the Temenggor reservoir because the variation of the total hydroelectric power generated has a similar pattern to that of Temenggor. For instance, from the end of September to December, the power generated from Temenggor was decreasing. In the meantime, the total power generated from the cascading scheme during the same period had a decreasing pattern.

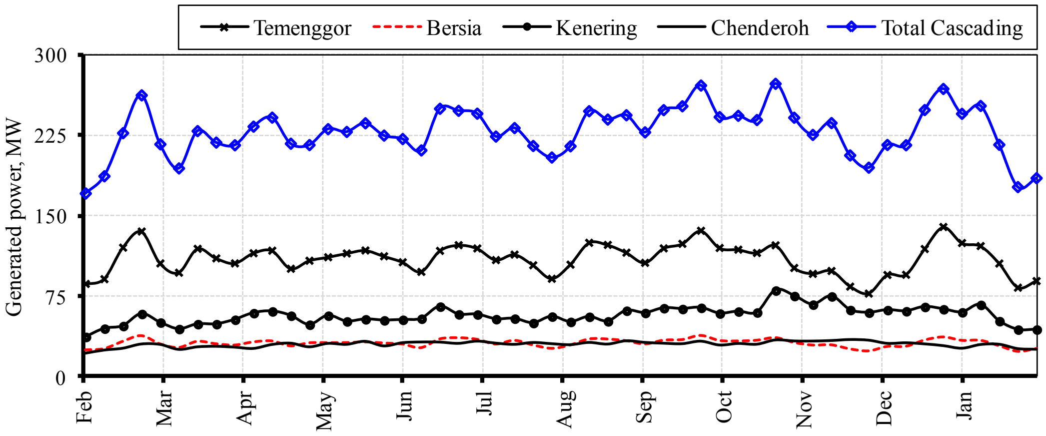


Figure 4.6 The weekly average HA hydroelectric power generation (2006-2010)

4.3. Trend of Hydroelectric Power Generation

The significance of trend in the Perak cascading hydroelectric power generation was tested using the non-parametric Mann-Kendall’s statistical method. In this method, the null hypothesis was the data had no trend. The test result showed that the values of the statistic, *Ss* and the normalized test statistic, Z were 26 and 0.24, respectively. As shown in Table 4.2, with a 95% confidence level, the null hypothesis was true. The probability associated with the normalized statistical, Z was 60%; it was less than the level of confidence. Therefore, it is clearly showed that hydroelectric power generation from the Perak cascading reservoirs were neither increasing nor decreasing with respect to time.

Table 4.2 The hydroelectric power generation trend test results

Statistical, S_s	$VAR(S_s)$	Z	$Prob(Z< -0.24 \text{ and } Z > 0.24)$	Conclusion
26	10,450	0.24	0.60	No trend

4.4. Comparative Error of the new Stage-Storage Curve

The research generated a new stage-storage relationship with the assumption of the stage-area relation being represented by a polynomial function of degree two. The function was calibrated with three known values of the water surface area at different levels of the reservoirs. Hence, the net storage volume between any two arbitrary reservoir levels was determined by integrating the stage-area equation (Simpson one-third numerical integration technique). Likewise, the live storage volume was determined by taking the maximum operating and the top of the dead storage levels. The accuracy of the determination of the live storage increases, if the level range is divided into large number of small increments as shown in Figure 3.10. Hence, the level found between the maximum operation and the intake is sub-divided into a number of segments with consideration of the range of the operating level. The live storage volume is the sum of successive segment storage volume from the maximum operating up to the top level of the dead storage volume. Finally, the calculated live storage was compared to the corresponding stated available data. It was found that the maximum relative live storage variation had about 12% relative error at the Temenggor, while a minimum of 4.2% relative variation was found at the Kenering as shown in Figure 4.7.

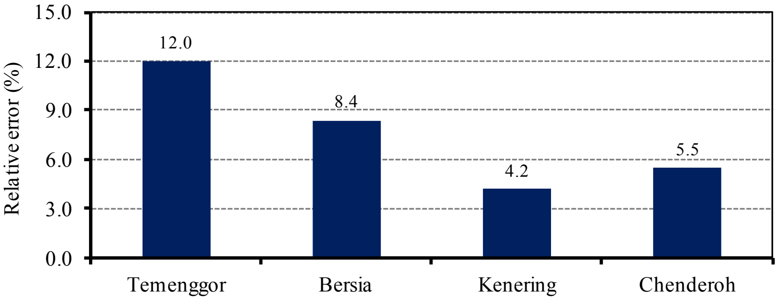


Figure 4.7 Relative errors of the new developed stage-storage relationship

The result showed that the predicted value using the new stage-storage relationship was close to the stated live storage volume data at the respecting reservoirs. Therefore, the assumption made to develop the relationship was viable because the minimum operating level of the reservoirs is much above the top dead storage level that hypothetically shown in Figure 2.6. Hence, the error made to predict a storage volume using the newly developed stage-storage relationship is minimal and its influence on hydroelectric power model has insignificant.

4.5. Effects of Variables change in Hydroelectric Power Generation

Addition or withdrawal of equal volumes of water among the reservoirs had different impacts on the hydroelectric power generation. The addition of water to the reservoir is due to the inflow, while withdrawal is mainly due to the release. A smaller reservoir has a higher change in the headrace level per change of storage volume. However, the combined effects of the headrace level and the rate of the turbine release on the hydroelectric power generation cannot be predetermined. Therefore, Figure 4.8 shows the sensitivity of headrace level, turbine release and overall plant efficiency of the Perak cascading reservoirs for hydroelectric power generation. Figures 4.8 (a) and (b) show the relative values of the storage and generation capacity of the reservoirs. It showed that the storage capacity of the Temenggor reservoir is about 100 times larger than that of Bersia. However, the relative generation capacity of the Temenggor is nearly five times greater than that of the Bersia and nine times greater than that of the Chenderoh.

The test result of the change in power generation, ΔP with the change in storage capacity, ΔS showed that the Bersia reservoir was the most sensitive. The change of storage volume can express with the headrace level that is one of the variables in hydroelectric power generation. As shown in Figure 4.8 (c), the volume of water that changes the headrace level of the Bersia reservoir by one unit is equivalent to *0.06*, *0.16*, and *0.40* units at the Temenggor, Kenering and Chenderoh, respectively. This indicates that an addition or withdrawal of water from the Bersia reservoir comparatively provide a higher change in the headrace level. The rate of change of the headrace level has a direct impact on the hydroelectric power generation.

Furthermore, the impact of the headrace level of the hydroelectric power generation was evaluated by taking the constant rate of the turbine release and overall plant efficiency. The result showed that the largest storage capacity of the reservoir has the smallest change in the power production per unit of change of the storage volume as shown in Figure 4.8 (c). On the other hand, as depicted in the same figure, the smaller the reservoir the more sensitive to the change in storage volume for the generation of a hydroelectric power from the cascading scheme. Hence, the most sensitive reservoir due to the change of the storage volume, should be given a priority during the refill and the least precedence during the deplete season.

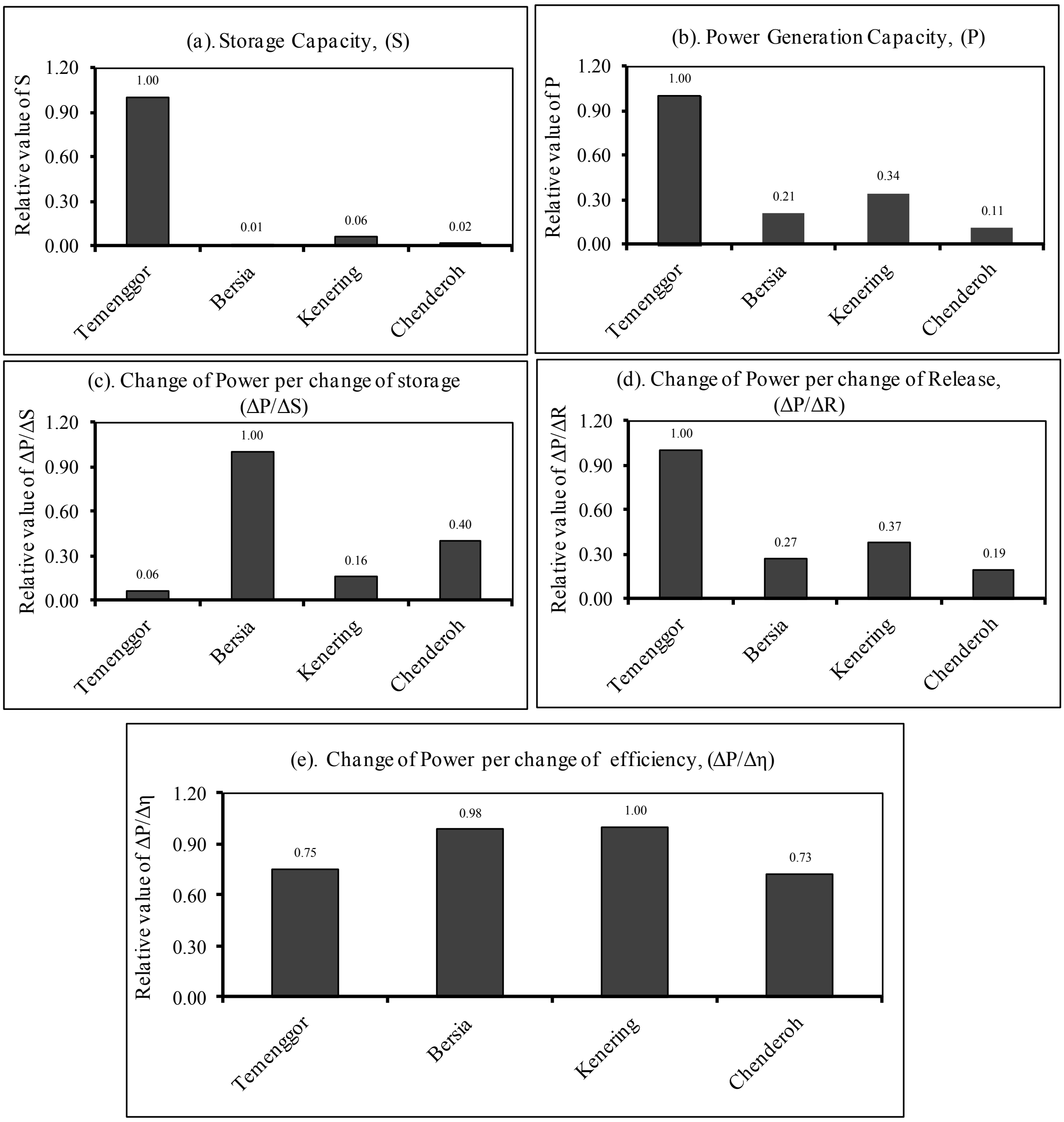


Figure 4.8 Sensitivity of the hydroelectric power generation on the variables change

The volume of the turbine release influences not only the hydroelectric power generation, but also the headrace level. As shown in Figure 4.8 (b) and (d), the highest generation capacity plant has the most sensitivity to the unit change of release. Kenering is relatively the most sensitive to change of the overall plant efficiency; whereas, Chenderoh is the least sensitive as shown in Figure 4.8 (e).

From the end of September to December, as shown in Figure 4.4, the rate of release from the Temenggor reservoir was decreasing, while the headrace level was increasing as indicated in Figure 4.5 (a). Based on the presentation of the two figures, it can be concluded that the power generated from the Temenggor plant is more sensitive to the rate of release than the headrace level; in addition, the refill operation is started with the reduction of the rate of the release. At the Temenggor plant, the effect of release rate is about 17 times to that of the storage volume to generate a hydroelectric power. Accordingly, the sensitivity analysis conducted by Yoo [146] showed that the effect of the release volume was 25 times greater than the storage volume in the hydroelectric power generation. The storage volume determines the headrace level of the reservoir; hence, the variation of the headrace level has a direct impact on the hydroelectric power generation.

4.6. Seasonally Varied Model (SVM) to Operate a Cascading Reservoir

The SVM operation rule is developed after the analysis of the refill and deplete ranking order of the cascading reservoirs. The order has an impact not only on the annual total energy-storage among the reservoirs, but also on the total power generation from the cascading system. The comparative results of SVM operation in relation to the HA are presented to show the advancement of the model.

4.6.1. Reservoirs' Ranking Orders to Maximize the Energy-Storage

As shown in Figure 3.8, from October to January the weekly average inflow to the Temenggor reservoir was above the mean annual rate. However, the inflow to the other three downstream reservoirs rely on the operation of the Temenggor because the major inflow is from the release of the preceding reservoir. Therefore, the period from

October to January was selected as a refilling season (RS). During the RS, the first important consideration was the selection of the appropriate time that the refill could begin. Hence, analysis was made between the first week of September and the last week of October, because the long-term historical average (HA) data indicated that inflow during this period was above the annual mean value. The optimal period to start a refilling operation and the corresponding impact on the total annual average power generation of the cascading scheme is shown in Figure 4.9. The appropriate time to begin a refill operation for all the reservoirs was around the first week of October. However, between the second weeks of September up to the third weeks of October, the generation was also above the annual average HA value, 228 MW.

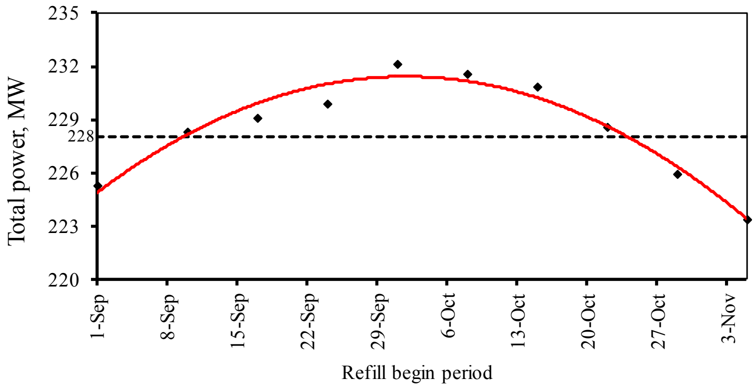


Figure 4.9 Period of the refill begins

After the determination of the refill start period, the next important consideration was the order of refilling. Similarly, in what order the depletion should be carried out to maximize the energy-storage among the cascading scheme. As illustrated in Table 4.3, the refill operation should be started from the smallest storage capacity reservoir, Bersia. The deplete should be started from the smallest generation capacity plant, Chenderoh.

Table 4.3 Test results of the refill and deplete ranking

	Temenggor	Bersia	Kenering	Chenderoh
V value	1.04	**21.66**	5.82	9.37
SER value	0.54	0.15	0.21	**0.11**

The priority order of the refilling operation was based on the descending order of the increase of power production to the increase of the storage (*V* value), and the deplete rank accomplished with the ascending order of the storage effectiveness ratio (*SER*) value. Accordingly, the refill operation are performed in the order of Bersia, Chenderoh, Kenering and Temenggor. Likewise, the deplete operation started from Chenderoh and then followed by Bersia, Kenering and Temenggor, respectively. The order of the operation showed that the largest capacity reservoir, Temenggor, ranked at last on both refill and deplete operations. It showed that more attention shold require for a small storage capacity reservoirs during the operation of a cascading of hydroelectric power scheme.

4.6.2. SVM Operation Rule-curve

As shown in Figure 4.10, based on the results of the refill and deplete ranking orders, the operation rule-curves were developed for each reservoir. For simplicity, the relative stage (headrace) was used to show the comparative level differences between the minimum and maximum operating levels. The relative stage varies from zero to one. Zero indicates the minimum operating and one for the warning levels of the respective reservoir.

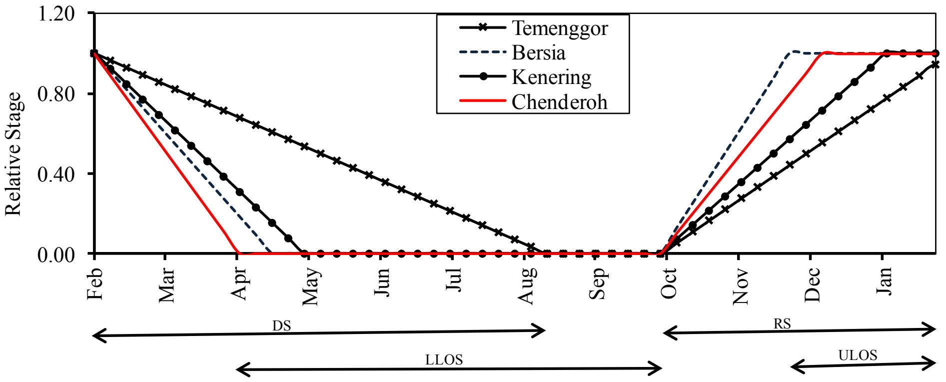


Figure 4.10 Reservoirs rule-curve using the concept of the refill and deplete ranking

At the Temenggor reservoir, both RS and DS take the longest time. However, about 40 to 50% of the total operation period of the downstream reservoirs were found at the LLOS; this period is from April to the end September, as shown in Figure 4.10. During the LLOS, more attention is required at the rate of the turbine release rather than maintaining the headrace level. However, the lower headrace level is provided the higher overall plant efficiency as shown in Figure 3.12. Hence, it has a minor advantage for the generation of hydroelectric power. The research showed that the hydroelectric power generation from the Perak cascading reservoir is more sensitive to the variation of the rate of the release than to the headrace level. The reason was the production of hydroelectricity from high head power stations are mainly depend on the rate of turbine release, not by the head [88].

Refill operation is accomplished, when the rate of the inflow exceeds the rate of the release. Similarly, if the release is higher than the rate of the inflow, then the depletion starts. The other two scenarios are ULOS and LLOS. The logical consecutive order of the seasons is RS, ULOS, DS, and LLOS. The operation rule-curve shown in Figure 4.10, shows maintaining the Temenggor reservoir at ULOS and LLOS do not lead to the optimal total annual average hydroelectric power generation from the cascading scheme. It also showed that between May and September, all the downstream reservoirs are at the LLOS. In addition, the SVM rule-curve indicates about 70-80% of the total operation period of the downstream of all the three reservoirs are found either in the ULOS or in LLOS.

4.6.3. Hydroelectric Power Generation using SVM

As illustrated in Table 4.4, SVM improved the HA annual average hydroelectric power generation of the cascading system by 2%; it is equivalent to 4.56 MW per day. A maximum improvement of 13.77%, about 4.16 MW per day was found from the Chenderoh plant, while a reduction of 1.21%, about 0.69 MW per day was obtained from the Kenering plant. The result showed that the capacity factor (CF) of the cascading hydroelectric power generation is improved from 39.46% to 40.25%.

The analysis indicated that the CF of the Temenggor reservoir was below the cascading average. However, the potential contribution of Temenggor in the cascading scheme is about 60%, but the long-term HA generated data showed that the actual contribution of Temenggor was only about 48%. This analysis shows that Temenggor was the least efficient in the scheme. As a paradox, as shown in Figure 4.6, the total power generation of the scheme is highly dependent on the Temenggor.

Table 4.4 Comparative power generation of HA and SVM

Description	Unit	Temenggor	Bersia	Kenering	Chenderoh	Total
HA	MW	109.57	31.39	56.90	30.21	228.07
SVM	MW	110.04	32.01	56.21	34.37	232.63
Change of power	%	0.43	1.98	-1.21	13.77	2.00
CF of HA	%	31.49	43.60	47.42	79.50	**39.46**
CF of SVM	%	31.62	44.46	46.84	90.45	**40.25**

Only the hydroelectric power generation from the Kenering plant was not improved due to the operation restraints of the Chenderoh reservoir. The maximum permissible release rate and the available free storage to augment the release of the Kenering were the operational constraints of the Chenderoh reservoir. Since the release of the Kenering should be recaptured by the Chenderoh reservoir, the constraint that related to the free storage has a great impact on the operation of the Kenering. If the available free storage capacity of the Chenderoh reservoir is small, the release of the Kenering are forced to be curtailed. As a result, the hydroelectric power generation from the Kenering decreases.

4.7. Optimal Reservoir Operation using GAM

The genetic algorithm model (GAM) analysis showed that the optimal value was reached at a population size (PS) of 150, a crossover probability (CRP) of 0.75 and a generation number (GN) of 60. The test of the iterations (runs) number also indicated that at the tenth iterations, the fitness values found from PS of 60, 100, 150 and 300 were equal. It showed that the impact of PS can be minimized with repeated iterations of the model. However, the minimum number of runs required to reach the optimal value are decreased with the increase of PS. The test of the model computation runtime on the result of the fitness value indicated that the impact of GN was negligible as compared to the PS and the CRP.

4.7.1. Optimality of the Basic GAM Parameters

Figure 4.11 shows the variation of the GA run options and the corresponding fitness value of the eight options of the model run. For the purpose of analysis, a population size (PS) of 150 and 300 were selected to evaluate the results found from each run option. PS of 150 and 300 showed the deflation of the fitness value. Up to PS of 150, the least result found from the run options 6 and 8, while the best was from option 2. Whereas, between the PS of 150 and 300, the fitness value found from run option 2 had a better performance, and the fitness value found from options 6 and 8 were highly improved as compared to the others. Likewise, after the PS of 300, the changes of the fitness value per change of PS for all options were negligible.

All the three run options (2, 6 and 8) used uniform selection. However, the run option 2 used a crossover option of scattered and the run options 6 and 8 applied heuristic and intermediate crossover options, respectively. The difference between the scattered and heuristic crossover options rely on the generation. In the scattered option, the next generation is produced by taking the genes from either of the parents. In the case of the heuristic crossover option, both parents are important to produce the next chromosome. However, the intermediate crossover option uses a random weighted average of the parents to produce an offspring.

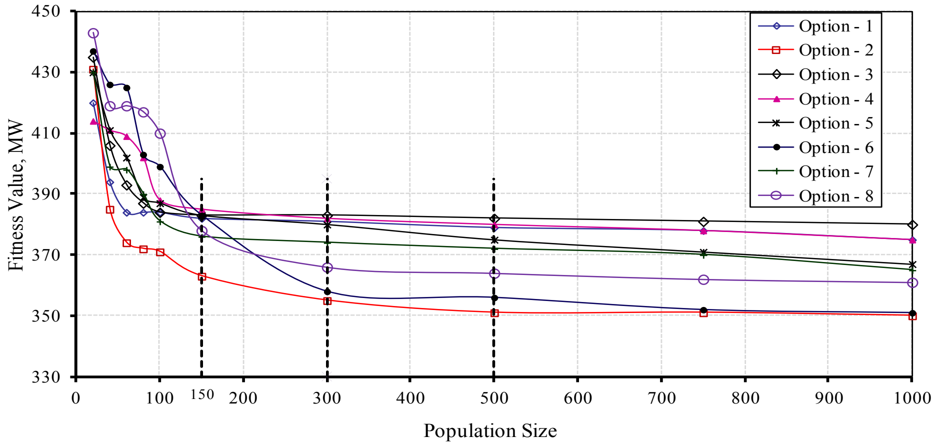


Figure 4.11 Option's fitness value at various PS

The GA run option 2 provided the smallest fitness value. Hence, it was selected as the optimal run option. The option is used a fitness scaling of rank, uniform selection, scattered crossover option, a crossover probability of 0.80 and mutation probability of

0.01. The result of the fitness value showed that uniform selection is better performed than the roulette and tournament selections. Furthermore, as shown in Figure 4.11, at the higher value of the PS, options 2 and 6 have the same fitness value. This indicates that with the increase of the PS value using a uniform selection, the difference of the the fitness value found from the scattered and heuristic crossover option become close each other.

4.7.1.1. Population Size, PS

As shown in Figure 4.11, the rate of improvement of the fitness value was different with the increase of the PS. This was analyzed using PS of 150, 300 and 500 as turning points. At the PS of 150, the best fitness value was found from option 2 and the worst from option 4. While after the PS of 300, the best value was found from option 2 and the least from option 3. The difference between the best and the least fitness value at the PS of 150, 300 and 500 were 22, 28, and 31 MW, respectively. The results indicated that the difference between the best and the least fitness value increased with the increase of PS. It also showed that with the increase of PS, the rate of improvement of the best fitness value was greater than the worst.

The change of the fitness value was also studied after classification of the PS into three groups as shown in Figure 4.11. The first group was below a PS of 150; the second was between PS of 150 and 300; and the third was for PS above 300. Up to the PS of 150, the improvement of the fitness value with the increase of PS was large for all options. Between the PS of 150 and 300, a major change of the fitness value observed only for options 2, 6 and 8. With the increase of PS from 150 to 300, the corresponding change of the fitness value was 8, 25 and 12 MW from options 2, 6 and 8, respectively. A larger change was found from option 6. However, after the PS of 300, the fitness value improved, but the rates of improvement for all the options were minimal. Even though the fitness value improved with the increments of PS, the optimal value was restricted in between the PS of 150 and 300 with consideration of the GAM computational runtime. The PS of 150 was the deflation point. Moreover, the computation runtime increased with the increase of the PS. Hence, a PS of 150 was selected as an optimal value. In the analysis of a four-reservoir problem, Wardlaw and Sharif [17] used a population size of 100. The optimal PS value in water resources is varied in the range of 64-300 [89].

4.7.1.2. Generation Number, GN

The generation number (GN) is one of the stopping criteria used in the GAM. The smaller the GN, the shorter the time required to stop. GN has an impact to determine the fitness value. In the analysis of a global optimal point, the smaller GN cannot meet the target; however, the larger GN has a problem of convergence.

Using the GAM run option 2 and PS of 150, the impact of the GN is presented and it is showed as in Figure 4.12. Up to the GN of 60, the fitness value improved; however, between the GN of 50 and 100, the value of the fitness had only a minor change. The fitness value increased after the GN of 100. Therefore, the optimality is attained at GN of 60. In this study, the influence of GN to determine the optimal fitness value of a cascading reservoir operation was demonstrated and it was one of the main contributions of the research.

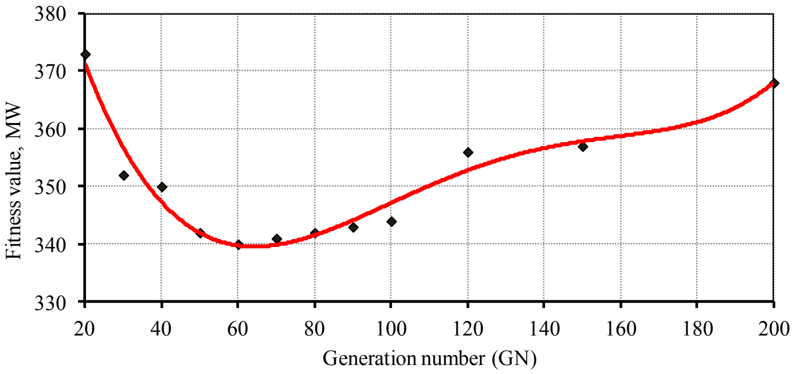


Figure 4.12 Impact of the GN on the fitness value

4.7.1.3. Crossover Probability, CRP

The crossover probability (CRP) influences the selection of the next generation. As indicated in Section 2.4.1.1, GA used three mechanisms to select the next generation: crossover, mutation and elite. The next generation using the crossover selection is accomplished with the modification of the parents. Hence, CRP shows the proportion (the ratio) that the next generations follow the crossover reproduction mechanism.

The impact of CRP was analyzed using the GAM run option 2, PS of 150 and GN of 60. The test result of the CRP is presented with respect to the fitness value as shown in Figure 4.13. The actual hydroelectric power generation improved, when the CRP was increased from 0.60 to 0.75. After a CRP of 0.75, the fitness value increased continuously, while the actual hydroelectric power generation decreased. Therefore, the optimum value of CRP was 0.75. Accordingly, the sensitivity analysis of CRP conducted by Jothiprakash and Shanthi [33] indicated that the system performance increased with the increase of the CRP up to a certain optimal point and then it decreased with further increments of the CRP.

The change of the fitness value due to the variation of CRP and GN was compared. The test of the sensitivity analysis of the GN as shown in Figure 4.12 indicated that with the PS of 150, CRP of 0.80 and GN of 20, the corresponding fitness value was about 373 MW. However, by only changing of the GN value to 60, the corresponding fitness value improved to 340 MW. It was a 33 MW improvement. Whereas, as shown in Figure 4.13, using the PS of 150, CRP of 0.60 and GN of 60, the fitness value found to be about 345 MW, while with the only change of the CRP to 0.75 the fitness value became 338 MW. The improvement was 7 MW. This showed that fitness value is more sensitive to the change of the GN than the CRP.

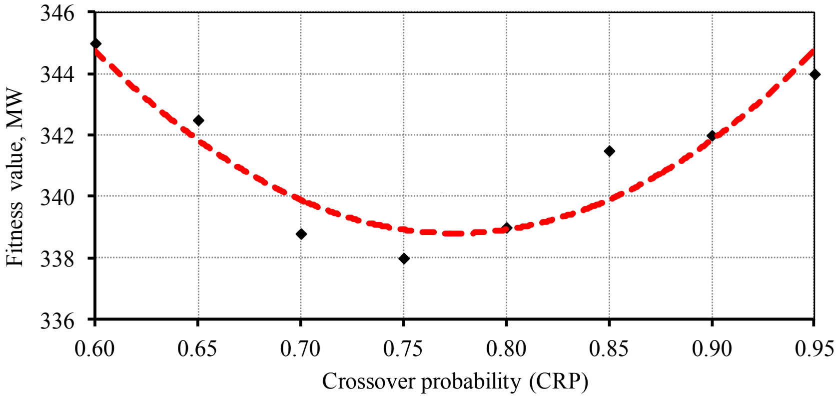


Figure 4.13 Selection of the optimal crossover probability (CRP)

Table 4.5 illustrates the study of optimal GA parameters conducted by various researches. It showed that the optimal value of CRP found from the various studies is almost similar (about 0.75). Hence, it can be concluded that in a reservoir operation analysis, the optimal value of CRP is around 0.75. However, the optimal value of PS varies with the reservoir condition.

The sensitivity test of the GA parameters for the operation of a single reservoir in China conducted by Yun et al. [98] showed that the fitness value decreased with the increase of PS. Likewise, the studies of Jothiprakash et al. [33] and Pilpayeh et al.[57] showed that the fitness value decreased with the increase of PS. This study also showed that the fitness value decreased with the increase of PS as shown Figure 4.11. The performance of the GAM can improve in two ways: by the changing model run option, and/or by repeatedly iterating the model.

Table 4.5 Optimal test results of the GA parameters of some selected studies

The study of	The optimal value of			Remark
	PS	CRP	GN	
Mathur and Nikam [35]	250	0.75		For multipurpose reservoir operation
Pilpayeh et al. [57]	700	0.75		For cascading of two reservoirs
Dariane and Momtahen [36]	200	0.80		For Multi-reservoir operation
This study	150	0.75	60	For cascading of four reservoirs

4.7.1.4. Optimal Number of Runs

Optimization using GAM requires a repeated number of runs since the optimal value could not prematurely converge [147]. There is no qualitative measure to check the probability of the success of GA; however, repeated runs provide a better alternative guarantee, since it improves the performance of the search for the global optimum value [147]. Figure 4.14 (a) shows the impact of PS and the corresponding fitness value improvements within the ten repeated runs. Tests of the repeated number of runs were conducted using the CRP of 0.75 and the GN of 60. As the number of run increases, the difference between the fitness values of the various population sizes decrease. For instance, after the first, the sixth and the tenth number of runs, the maximum differences of the best and the least fitness value found among the entire alternative runs were 36, 27, and 24 MW, respectively. It indicates that if the number of run increases, the fitness value converged. As shown in Figure 4.14 (a), the values of the fitness that was found from the PS of 100, 150 and 300 were equal at iteration number ten. Since the optimal value of PS was 150, ten iterations would be sufficient to show the impacts of the repeated runs because after ten iterations the variation of PS has no impact on the fitness value.

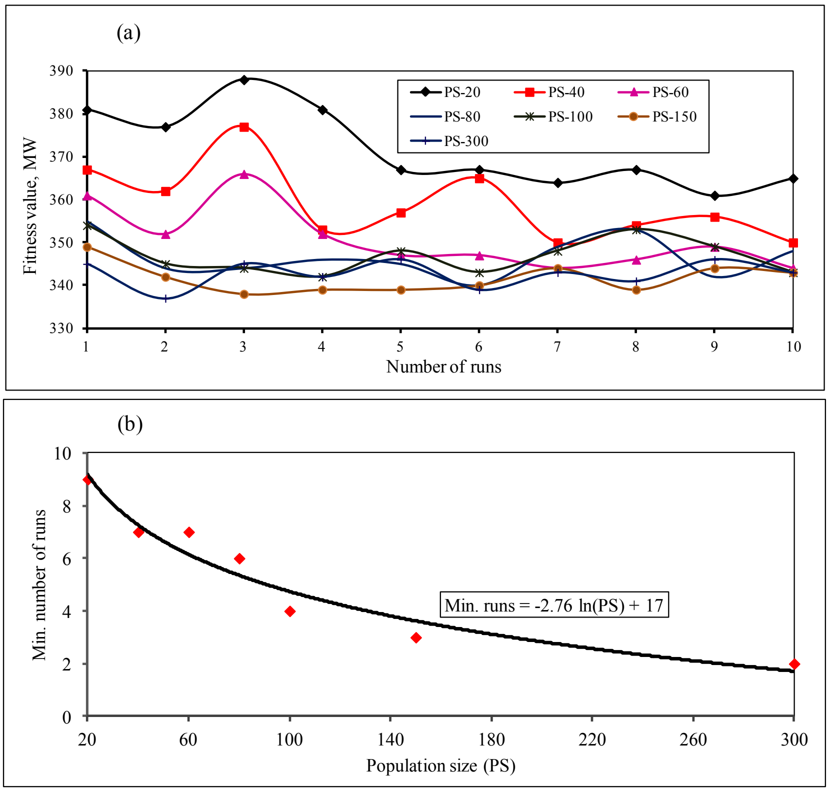


Figure 4.14 Minimum numbers of runs required per PS

The iteration number that provided a minimum fitness value within the ten repeated runs were selected to develop the relationship to PS. As shown in Figure 4.14 (b), the minimum number of runs required to obtain the optimal value decreased with the increase of PS. For example, a minimum of nine runs are required to reach the optimal value using a PS of 20. While, for a PS of 150, the optimal fitness value is reached at the third iteration. This study presented the relationship between PS and minimum number of runs required to obtain the optimal value with the introduction of the "*borderline of run*". The *borderline of run* is presented using a logarithmic equation as shown in Figure 4.14 (b), and this is also another contribution of the study. However, a similar study conducted by Dariane and Momtahen [36] considered an average of five to ten runs to test the model performance. The study also concluded that the number of runs to determine the best model performances do not have a fixed value.

4.7.2. GAM Relative Runtime

The computational runtime varies with the computer's specifications. A computer with a processor of Intel(R) i5 CPU M430 @ 2.27GHz and a RAM of 2 GB was used to analyze the computational runtime of the GAM. A relative runtime is used to present in order to make the results would be generalized. The relative runtime is a more explanatory and indicative parameter to compare the computational runtime. PS, CRP and GN are selected as variables to conduct the computational runtime. The maximum runtime was used as a benchmark, and the results presented in accordance to that. The result showed that the minimum time taken was 4125 seconds (about 69 minutes) for a PS of 100, CRP of 0.80 and GN of 65, while the maximum runtime was 21,125 seconds (about 5 hours and 52 minutes) using a PS of 300, CRP of 0.65 and generation number of 100 to reach the stopping criteria. The fitness values were 363 MW and 355 MW for the minimum and maximum computational runtime, respectively. The runtimes of all other GAM run options were in between 4,125 and 21,125 seconds.

The computational runtime was highly influenced by CRP and PS, while the GN had the least impact. As shown in Figures 4.15 (a) to (d), the relative computational runtime increases with the increase of PS and with the decrease of CRP. The double rise of the PS with the same the value of CRP and the GN resulted a twofold increase of the corresponding computational runtime. For instance, the relative computational runtime for a PS of 150 and CRP of 0.80 was about 0.30; however, with the same CRP value doubling only the PS (to 300), the relative computational runtime was about 0.60.

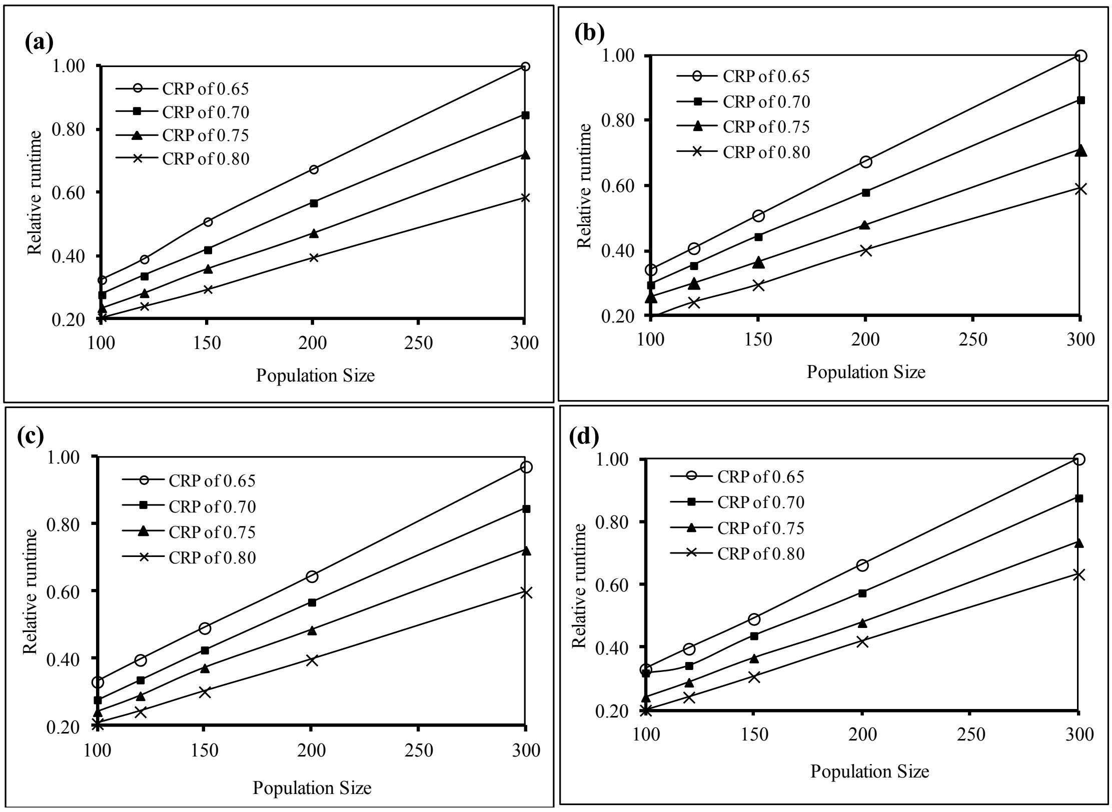


Figure 4.15 Relative runtime with the GN of (a) 50, (b) 65, (c) 75 and (d) 100

The computational runtime was also used as another criterion for selecting the optimal GAM parameters. However, the decision on the optimal CRP based on the computational runtime is subjective in nature. For example, as shown in Figure 4.13, the difference of the fitness value between the CRP of 0.75 and 0.80 is one megawatt, while Figure 4.15 shows the relative difference of the computation runtime between the CRP of 0.75 and 0.80 at a PS of 150 was about 0.06; it is equivalent to 24 minutes. Therefore, the decision on the optimal CRP with the consideration of computational runtime requires a penalty on the fitness value.

4.8. Results of the GAM and SVM compared to HA

The criterion of the headrace level variation, release decision and annual average hydroelectric power generation were used to compare the GAM and SVM results with the corresponding values of the HA. The annual cumulative volume of release from each reservoir for all the three operation cases was the same. The difference mainly relied on the release decision with respect to time. Consequently, the release decision influences the headrace level variation. The combination of the rate of release and headrace level (head of water) governs the quantity of the hydroelectric power generation.

4.8.1. Annual Variation of the Headrace Level

The headrace level variation of the reservoirs in the Perak cascading scheme was studied between the first week of February of the year to the last week of January of the following year. The variation of the headrace level of Temenggor was different from the cascading scheme, because of its location in the scheme, and the storage capacity. In the HA operation, all the three downstream reservoirs refilled and depleted a number of times during the study period. In addition, the range of the operational levels of the three downstream reservoirs in the cascading scheme was small as compared to the Temenggor, it is the most upstream reservoir. The comparison also showed that the GAM and HA provided a higher energy-storage with respect to time than the SVM did.

4.8.1.1. Temenggor Reservoir

The headrace level variations at the Temenggor reservoir using the GAM, SVM and HA operations' rule were presented. The variation of the headrace level within the critical period (from February to January) indicated that the refilling and the depletion operations using the GAM and SVM had similar patterns to the corresponding HA. As shown in Figure 4.16, the time to reach the minimum level using SVM and GAM was different; GAM was about two months ahead. In addition, the minimum level also varied; nearly two meters difference were observed between the GAM and SVM operations. However, the period that the reservoir level reached the minimum level using the GAM and HA operations were almost the same (at the end of October).

The length of the depletion period using the GAM and HA operations was longer than the corresponding of the SVM, while the refilling period of SVM was longer than the GAM and HA. Relatively, the SVM operation had a higher rate of depletion and lower rate of refilling than the corresponding rates of the GAM and HA. From

February to October, the headrace level of the Temenggor reservoir found from the GAM operation was above the corresponding SVM and HA, with the exclusion of the first week of May.

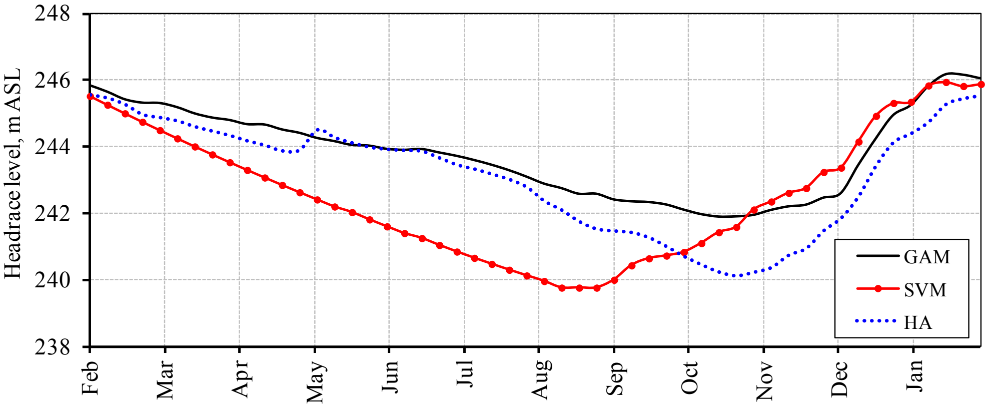


Figure 4.16 Headrace variations of the Temenggor reservoir

At the beginning of May, a slight rise was observed due to the impact of rainfall as shown in Figure 4.2 (a). Likewise, the SVM operation decision indicates that the headrace level from the end of October to the first week of January was above that of the GAM and HA levels. The initial and final reservoir levels for all three operations were the same. This indicates that the total volume of releases was equal. Therefore, with an equal total annual volume of release, the GAM operation provides a larger energy-storage than the corresponding SVM and HA for nine months. In hydroelectric power generation, a high-energy storage had two advantages; a high headrace level, which leads to greater power production and a greater quantity of water stored in the reservoir, which is important for further utilization.

4.8.1.2. Bersia Reservoir

The HA operation provided the minimum range of headrace level, which is 0.24 m as illustrated in Table 4.6. The range of the headrace level variation using the operations the GAM and HA were almost equivalent. In addition, the minimum headrace level found from the operation of SVM and HA was about the same. The highest maximum level was found using the operation of the HA, while the lowest maximum level with the operation of SVM. A higher maximum level provides a better energy-storage.

Table 4.6 Summary of the Bersia reservoir headrace variation

	Maximum level	Minimum level	Average level	Range
GAM	141.35	140.00	140.77	1.35
SVM	141.07	139.65	140.18	1.42
HA	140.82	140.58	140.69	0.24

The HA headrace variation as shown in Figure 4.17 indicates that the Bersia reservoir operated at nearly a constant level. However, the operation using GAM indicates that the reservoir refilled and depleted repeatedly in the operation period. While, the result of the SVM operation showed that the reservoir has a definite time to refill and to deplete. The deplete season (DS) was from February to April, and then follow the lower level operating season (LLOS), which is extended up to mid September. The refill season (RS) started at the end of September, and it reached the maximum operational level in the beginning of November. Then, after the upper level operating season (ULOS) starts, and it stays until the DS commences.

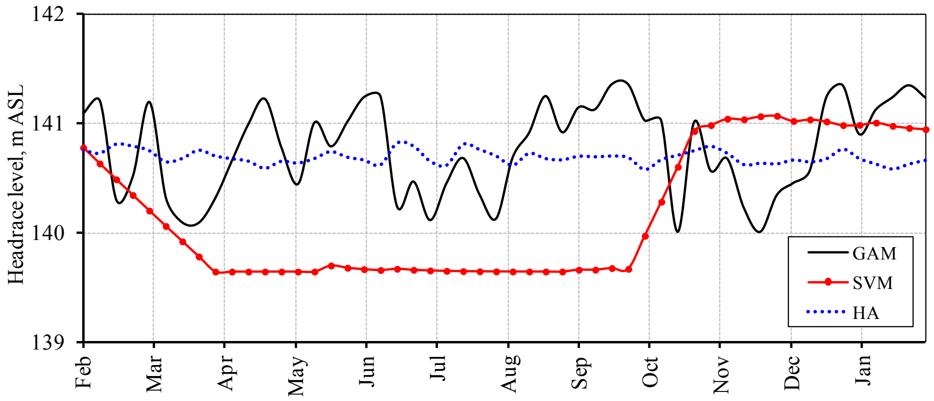


Figure 4. 17 Headrace variations in the Bersia reservoir

Between April and October, the energy-storage of the Bersia reservoir using the SVM operation was smaller than the corresponding GAM and HA. The sensitivity analysis, as shown in Figure 4.8, indicates that the hydroelectric power generation from the Bersia plant is more sensitive to the change of the storage capacity. Hence, operating the Bersia reservoir at a relatively constant headrace level, specially at higher-level is preferable.

4.8.1.3. Kenering Reservoir

The average operation levels of the Kenering reservoir were 110.57, 109.52 and 110.57m using GAM, SVM and HA, respectively. It showed that the difference between the average operation level using SVM and GAM was about a meter. While, as indicated in Figure 4.18, HA and GAM provided almost a similar decision.

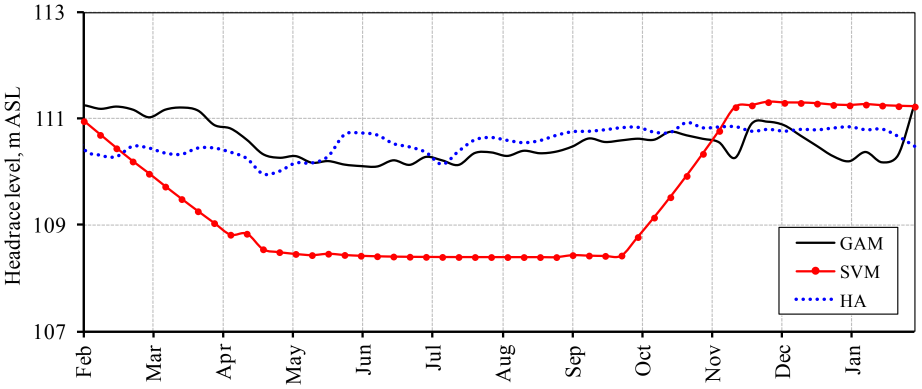


Figure 4.18 Headrace variations in the Kenering reservoir

The ranges of operation using the GAM, SVM and HA were 1.16, 2.92 and 0.96 m, respectively. This indicated that the HA operation had the least variability of the headrace level. In addition, the energy-storage from May to November was higher than the corresponding GAM and SVM operations.

4.8.1.4. Chenderoh Reservoir

Chenderoh reservoir is the most downstream side in the Perak cascading scheme. Hence, the headrace level at Chenderoh can be controled for two reasons: to meet the flood control requirement and to maintain the recommended headrace level for the generation of hydroelectric power. According to the design recommendation, the headrace level should vary within the range of 57.43 and 60.42 m ASL to satisfy the latter requirement. As shown in Figure 4.19, the operation using GAM, SVM and HA indicates that the variation of the headrace level of the Chenderoh reservoir was similar to that of the Bersia and Kenering. SVM has provided a lower energy-storage variation, while GAM and HA allowed relatively higher-energy storage throughout the analysis period.

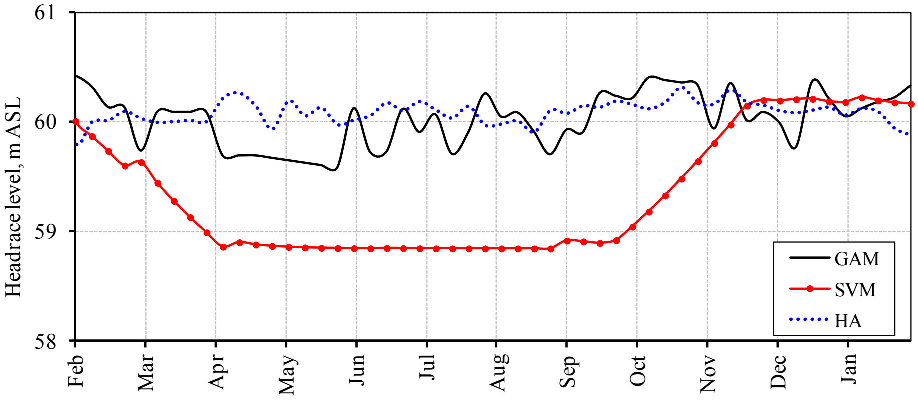
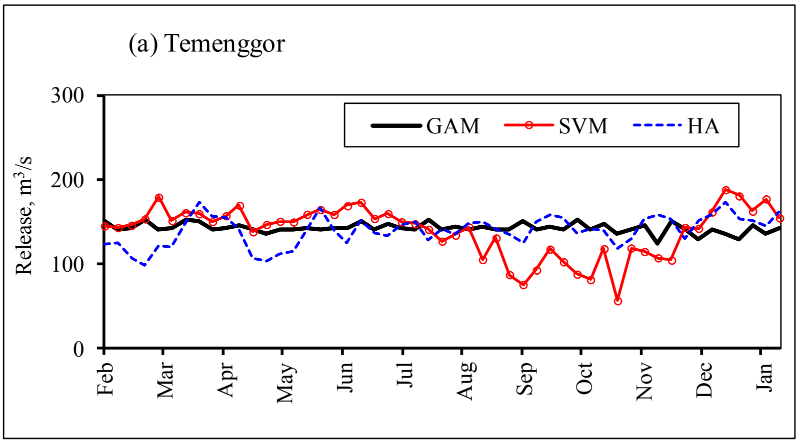


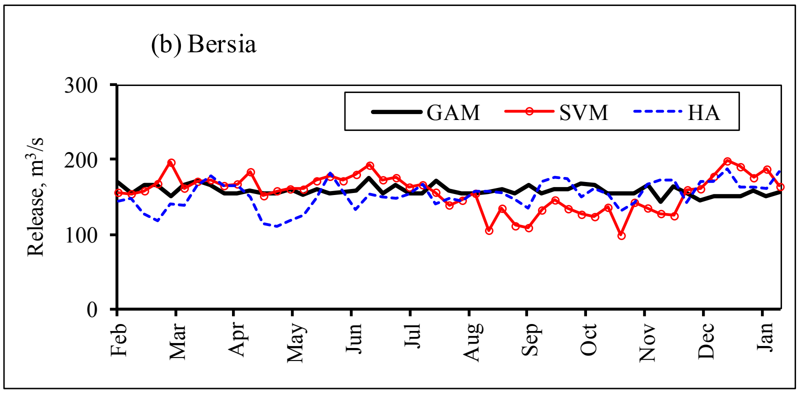
Figure 4.19 Headrace variations in the Chenderoh reservoir

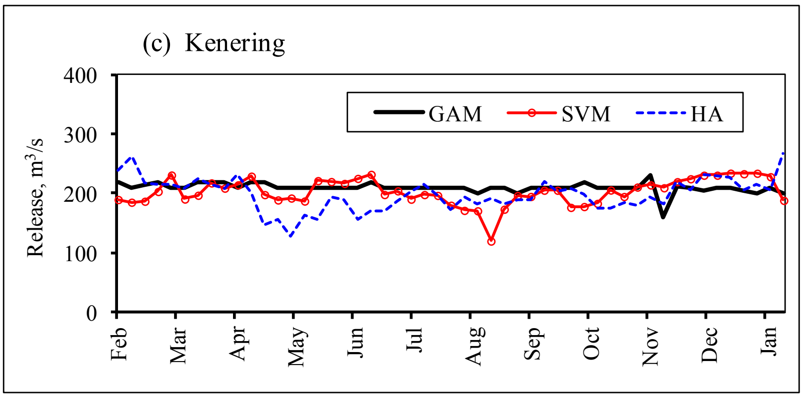
4.8.2. Release Decisions of the Models

Release from the Perak cascading reservoir was conducted mainly with consideration of two conditions. The first consideration relates to the turbine constraints because it operates in a definite range of minimum and maximum rate of releases. The second consideration was the storage capacity of the subsequent reservoir. The succeeding reservoir should have enough storage space to augment the release of the previous reservoir to avoid spillage. Hence, for efficient operation, the total inflow should pass through the turbines.

As shown in Figures 4.20 (a) and (b), the release of Temenggor and Bersia follow similar patterns. Similar patterns were also observed between Kenering and Chenderoh as shown in Figures 4.20 (c) and (d). This indicates that the release of Temenggor is based on the operational constraints of the Bersia reservoir, and a similar condition was also shown between the Kenering and Chenderoh reservoirs. In the cascading scheme, Bersia and Chenderoh have smaller storage capacities than their immediate preceding reservoir. The influence of the Bersia reservoir on the operation of the Kenering was minor because the storage capacity of the Kenering is greater than the Bersia.







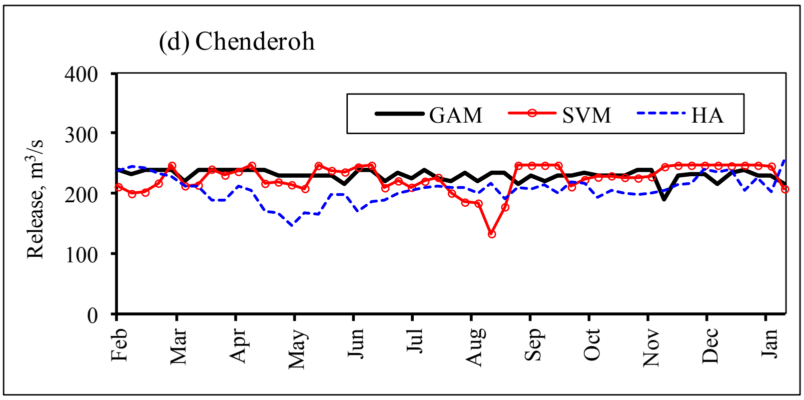


*Figure 4.**Error! Unknown switch argument.** Average weekly releases decisions from each reservoir*

As illustrated in Table 4.7, the operation of the GAM had a higher minimum and a lower maximum release as compared to the SVM and HA. This showed that the operation of GAM provided a uniform release. Like the variation of the headrace levels, the difference between the releases using the operation of the SVM had a larger range than the GAM and HA. However, the cumulative total annual volume of release of all operations was the same. In practice, a uniform release is comparatively quite easier to manage. Hence, the GAM operation provided a uniform release and it seems better to apply.

Table 4.7 Weekly average minimum (Min) and the maximum (Max) release decisions

Reservoir	Unit	Rate of release using					
		GAM		SVM		HA	
		Min	Max	Min	Max	Min	Max
Temenggor	m^3/s	125	153	57	188	98	174
Bersia	m^3/s	143	175	99	198	111	188
Kenering	m^3/s	160	230	120	234	127	271
Chenderoh	m^3/s	165	240	134	247	148	265

During the period of August to December, the SVM release decisions from Temenggor and Bersia reservoirs were less than the corresponding of the GAM and HA. As shown in Figure 4.16, in the period, the Temenggor reservoir was in the refilling state, also as shown in Figure 4.17, the Bersia reservoir was also somewhat in the refilling state. This indicates that the refill operation of the Temenggor reservoir started by reducing the rate of release.

The release decisions of the GAM and SVM were compared to the corresponding values of the HA. The comparisons were conducted by taking an equal total annual release volume of water in all operations. In addition, the rate of inflow was the same for all operations. The main difference in the result happened because of the decision ability of the models' on the rate of the release with respect to time. It can be concluded that for the same rate of inflow pattern, the higher release reduces the level of the headrace. However, both the rate of release and the headrace level can determine the hydroelectric power generation. Hence, the rate release is a key decision to maximize the hydroelectric power generation in the cascading scheme.

4.8.3. Hydroelectric Power Generation using the Models

The objective of the developed reservoirs' operation models is to maximize the annual hydroelectric power generation from the Perak cascading scheme. The maximization of the hydroelectric power generation requires a careful management of the inflow and an optimal decision-making on the rate of releases with respect to time. A better operation provides a high-energy storage and efficient power generation in the cascading scheme. However, during the low-flow season, it is impossible to have maximum generation and energy-storage at the same time. Hence, a compromise between the two quantities would lead to a fair decision. The optimal power generation can be achieved with a reasonable quantity of energy-storage.

The maximum annual average hydroelectric power generated from the Perak cascading scheme was found using the operation of the GAM. It was about 240.24 MW as shown in Table 4.8. The corresponding annual average hydroelectric power generation using the SVM and HA operations were 232.63 MW and 228.07 MW, respectively. The result showed that the GAM improved the HA annual average power generated by 5.34%, and the SVM by 2%. The daily additional hydroelectric power generation using the GAM and SVM were equivalent to 12.17 MW and 4.56 MW, respectively.

As shown in Table 4.8, apart from the Kenering reservoir using the operation of the SVM, the power generations improved at different proportions. A larger improvement of 17.54% found from the Chenderoh plant using the operation of the GAM. However, a reduction of 1.21% as compared with HA found at Kenering using the SVM operation. The possible reasons would be due to the operational constraints of the Chenderoh reservoir. As per the SVM operation, the available free storage capacity of the Chenderoh reservoir should augment the release of Kenering. Likewise, the operation of the China's Three Gorge reservoirs using GA improved the efficiency of the hydroelectric power generation by 17.4% [27]. Hence, it can be concluded that GA is robust and efficient technique in the operation of a hydroelectric power scheme.

Table 4.8 Comparative results of the power generated using GAM, SVM and HA

Criteria	Unit	Plant				Total
		Temenggor	Bersia	Kenering	Chenderoh	
Potential power = a	MW	348	72	120	38	578
HA generated = b	MW	109.57	31.39	56.90	30.21	228.07
SVM generated = c	MW	110.04	32.01	56.21	34.37	232.63
GAM generated = d	MW	112.27	32.17	60.29	35.51	**240.24**
c – b = e	MW	0.47	0.62	-0.69	4.16	4.56
d – b = f	MW	2.7	0.78	3.39	5.30	12.17
e/b	%	0.43	1.98	-1.21	13.77	2.00
f/b	%	2.46	2.48	5.96	17.54	**5.34**
b/a	%	31.49	43.60	47.42	79.50	39.46
c/a	%	31.62	44.46	46.84	90.45	40.25
d/a	%	32.26	44,68	50.24	93.45	**41.56**

As shown in Figure 4.21 from June to November, the total hydroelectric power generation using the SVM operation was smaller than the corresponding GAM and HA. During this time, the release from Temenggor and Bersia with the operation of SVM was relatively minimal. Consequently, the power generated from the Temenggor was low. On the other hand, the power generation using GAM operation was almost constant throughout the period. Because, the release conducted according to the GAM operation was relatively uniform throughout the period. This shows that the variation of the headrace level is not a key factor for the generation of electricity in the cascading system.

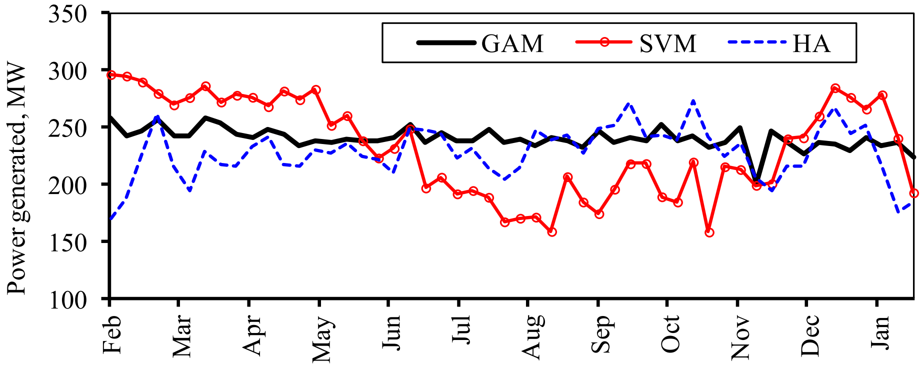
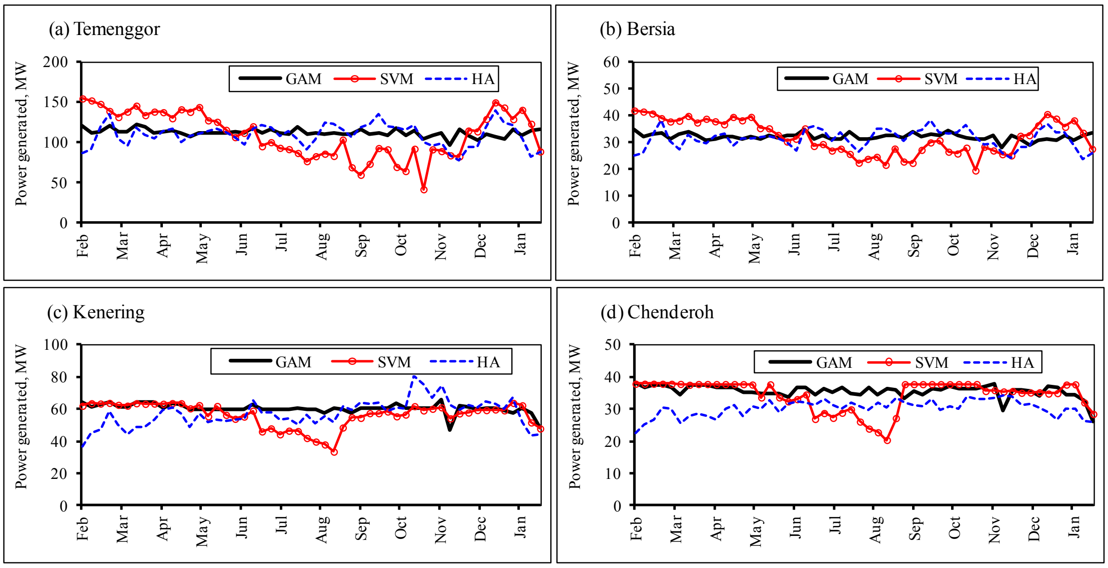


Figure 4.21 The variation of the hydroelectric power generation of the total scheme

The operation of the Perak cascading scheme relied on the operation of Temenggor reservoir. As shown in Figures 4.22 (a) and (b) the pattern of the hydroelectric power generation from Temenggor and Bersia were similar to that of the total cascading scheme, while the electricity generation from Kenering and Chenderoh also follow a similar pattern each other as shown in Figures 4.22 (c) and (d).


(a) Temenggor

(b) Bersia

(c) Kenering

(d) Chenderoh

Figure 4.22 The variation of the hydroelectric power generation of each plant

4.9. Relative Advantages and Disadvantages of GAM and SVM

The advantages and disadvantages of GAM and SVM were analyzed with consideration of the ability of the model to maximize the hydroelectric power generation. The three operations, namely, GAM, SVM and HA were evaluated with the same natural inflow patterns, with equal total annual volume of releases from the reservoirs and with the same initial reservoir level. The differences were found due to the ability of the models to optimize the rate of turbine release and energy-storage of the entire reservoir system with respect to time. The rate of turbine release determines the power generation and the energy-stored in the cascading scheme.

Table 4.9 shows the relative advantages and disadvantages of the GAM, SVM and HA. The relative comparisons were presented with consideration of the hydroelectric power generation models, the release patterns, the subjectivity and the complexity of the operation to the operator. Since the objective of the reservoir operation was to maximize the annual hydroelectric power generation, relatively the operation of the GAM met the objective, and it can be taken as the best model for the operation of the Perak cascading scheme.

Table 4.9 Advantages and disadvantages of GAM and SVM

GAM	Advantage	- The operation provides maximum hydroelectric power - Not require subjective judgment of the operator during the operation time.
	Disadvantage	- The result is sensitive to the variation of GA parameters, and it varies with the number of iterations (runs). - Comparatively complex to the operator and the model requires a continuous update
SVM	Advantage	- Comparatively simple for the operator - The operation is advanced than the HA
	Disadvantage	- The end of the refill and the begin of the deplete periods depend on the hydro-meteorological condition.

4.10. Economic Benefit

One of the main concerns of the hydroelectric scheme is the economic benefit of the system. The benefit of the hydroelectric power can be measured in monetary terms. The total income gained from the additional hydroelectric power of the Perak cascading scheme is estimated taking into account the current energy prices in Malaysia. According to the Tenaga Nasional Berhad (TNB), the electric utility company in Malaysia, consumers is classified into different tariff rates [148]. The tariff rate is based on the consumer type and the quantity of the energy consumed per the billing month. For example, the tariff rate for domestic/residential consumers who utilize up to 200 kilowatt hours (kWh) per month in Ringgit Malaysia (RM) is 0.218 per kWh.

Table 4.10 illustrates the economic advantage of the use of the GAM and SVM in the operation of the Perak cascading reservoirs. The benefit is predicted adopting a transmission efficiency of 95 % [20]. Moreover, the analysis conducted without the consideration of the cost incurred to generate the additional power. Since there are no additional infrastructures required to generate the additional power, the extra cost could be negligible. The benefit of the additional power clearly showed that utilizing the GAM and SVM operations is worth being applied.

Table 4.10 Gross income of the additional power generation

Model	Additional power in			Tariff rate (RM per kWh)	Gross additional income	
	MW	MW	kWh		RM per day	RM per year
a	b	c = 0.95 × b	d = c × 24,000	e	f = e × d	g = f × 365
SVM	4.56	4.33	103,920	0.218	22,655	8,269,075.00
GAM	12.17	11.56	277,440	0.218	60,482	22,075,930.00

The benefit of the additional hydroelectric power were predicted with consideration of the domestic electricity tariff rate. According to the TNB annual report of 2010, the domestic/residential consumers utilized about 20.4% of the total generated [149]. The prediction was very conservative because it considered the minimum tariff rate that the utility applying for the customers.

4.11. Summary of the Results and Discussion

Two models, namely, the genetic algorithm model (GAM) and the seasonally varied model (SVM) were developed to maximize the total annual hydroelectric power generation from the Perak cascading scheme, Malaysia. The results of the models were presented in the comparison with the long-term historical average (HA) operational data of the cascading scheme. It showed that the operation of the GAM improved the system performance by 5.34%; whereas, the SVM by 2.00%. The corresponding hydroelectric power was equal to 12.17 MW and 4.56 MW using GAM and SVM operation, respectively. In terms of enhancing the system's performance, the result found from the GAM was advanced. In addition, the headrace level and the release variation of the reservoirs showed that GAM provided a higher energy-storage and relatively uniform releases as compared to the SVM. The higher headrace level was advantageous to generate a better hydroelectric power. The operation of SVM also provided a better hydroelectric power generation than the HA. In addition, the implementation of SVM was quite easy, but it requires a strong subjective judgement of the operator, especially at the beginning of depletion season. The decision of the operator depends on the real-time and future hydro-meteorological condition of the catchment area.

The maximum hydroelectric power generation with the operation of GAM was achieved with the optimal value of the population size (PS) of 150, crossover probability (CRP) of 0.75 and generation number (GN) of 60. This research has shown the impact of the value of GN on the fitness function, which has not been demonstrated in the previous similar studies. The research also analyzed the sensitivity of the GAM parameters based on the computational runtime. It showed that the computational runtime was sensitive to the variation of the PS and CRP. In addition, the research outlined the relationship between the PS and iteration number with the introduction of "*border line of run*". The number of iterations decreased with the increase of PS, and the relation was expressed using a logarithmic function.

Another approach that used to optimize the annual energy-storage in the cascading reservoir is the refill and deplete ranking. The ranking order maximizes the volume of water passing through the turbine and reduces the quantity of spillage. The test result of the refill and deplete order of the Perak cascading reservoirs showed that the refill operation should start from the Bersia, while the deplete from the Chenderoh. Moreover, the appropriate time to start the refill operation is around the first week of October.

The advantages and the disadvantages of each method was analyzed to show the robustness of the model. One of the drawbacks of GAM was the current operation strictly relied on the future condition. Hence, the decision taken at every step considered not only the current prevailing condition, but also conditions that would happen within the year. The advantage of GAM over the other similar models was the ability to find the global optimal value. Meanwhile, the economic benefit of the additional hydroelectric power generation was estimated without the consideration of the costs incurred. The costs incurred to apply the model can be expected zero, since it would not be necessary to construct any additional infrastructures. The benefit earned from the additional hydroelectric power generation was more than RM 22 million and RM 8 million using GAM and SVM, respectively. The economic analysis showed that the benefit is worthily enough to implement it.

Another important result of the study was the development of the stage-storage relationships of the reservoir. Using the newly revised relationship, the live storage volume of the reservoirs were determined. The analysis showed that at the Temenggor reservoir, the relative variation of the theoretical live storage value and the determined new value using the revised relationship had a 12% difference; and the rest below a 10% relative variation. Therefore, the newly revised relationship can represent the system. It can be applied for the purpose of a reservoir operation, because for the small change of the storage volume variation (the error) in the determination of headrace level becomes insignificant.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

5.1. Conclusions

Nowadays, electricity is one of the basic requirements for human life. The demand of electricity is increasing due to the increasing population and changing the living standards. To stratify the new requirements of electric power due to the increase of demands, it is mandatory to generate additional power. The two possible measures that can be used to increase the power generation are developing new schemes and improving the capacity of the existing systems. The first measure, the development of new schemes, requires huge investment cost than the latter choice of improving the existing capacity. Even though the additional power generated from the latter alternative is small, it provides a fast response to the power shortage, and has a negligible extra adverse impact on the environment.

In this study, the Perak cascading scheme located in Malaysia was selected to maximize the hydroelectric power generation using the newly developed operational models. The scheme comprises four reservoirs, namely, Temenggor, Bersia, Kenering and Chenderoh that have different storage and generation capacity. The hydroelectric potential of the scheme is 578 MW. However, the long-term historical average (HA) data showed that the actual hydroelectric power generation was about 228 MW, which was below 40% of the potential. In this research, four specific objectives were established to analyze the Perak cascading reservoir operation scheme.

- The first specific objective was to maximize the hydroelectric power generation of the scheme through the development of the genetic algorithm and seasonally varied models. The overall result showed that the genetic algorithm model (GAM) and the seasonally varied model (SVM) improved the long-term historical average (HA) of the hydroelectric power generation of the Perak cascading scheme by 5.34% and 2%, respectively. A daily improvement of 12.17 MW was found using the GAM operation, and the corresponding value of SVM was equal to 4.56 MW. This additional hydroelectric power using the operation of the GAM and SVM found with an equal total cumulative annual release and similar patterns of the rate of inflow to that of the HA. The difference was due to the ability of models to optimize the release decision with respect to time. In terms of the power generation and energy-storage, the operation of the GAM is advanced. Whereas, the operation of the SVM was superior than the HA since it improved the generation of the HA. GAM was robust enough to find the optimal value close to the global point. GAM also improved the capacity factor of the Perak hydroelectric power generation. In general, the additional power has a positive impact on the growing electricity demand of the country.

- The second important objective of this research was to evaluate the optimal parameters of the GAM. The test results of the GAM parameters showed that the optimal value achieved at the population size (PS) of 150, the crossover probability (CRP) of 0.75 and the generation number (GN) of 60. Like the PS and CRP, the GN also has a great influence on the finding of an optimal value. The previous studies has not clearly determined the impact of the GN. In this study, the impact of GN on the fitness value was clearly outlined. In addition, the test on the minimum number of repeated runs showed that the increases of the PS resulted in the decrease in the minimum iteration number. A larger the PS requires a lower minimum iteration number. At the PS of 150, the minimum number of iterations was four. However, if the number of iterations (runs) were increased the difference between the fitness values found from the various PS decreased. After 10 runs, the difference between the fitness values found from the PS of 100, 150 and 300 were equal. When the number of run increases, the fitness value found from the largest and the smallest PS was roughly close to each other. The GAM computation runtime also evaluated comparatively, the result showed that the computation runtime increased with the increase of the PS and with the decrease of the CRP.

- In the third specific objective of the research, the long-term historical average (HA) operational and hydrological data of the Perak cascading reservoirs were analyzed, and four distinct seasons identified, namely, the refill, upper level, deplete and lower level. Refill and deplete were the two major seasons that influence the operation of the cascading reservoirs. Hence, in order to manage the inflow and to maximize the energy-storage of the scheme, the seasons require a ranking. The refill operation was accomplished with the increasing order of the storage capacity of the reservoirs, while the depletion ranking followed the increasing order of the plant's generating capacity. Therefore, the refill operation started from Bersia, and followed by Chenderoh, Kenering and Temenggor, respectively. However, the depletion is in order of Chenderoh, Bersia, Kenering and Temenggor. The result indicated that the refill order follows the increase in the reservoir storage capacity; whereas, the depletion is based on the decreasing order of the power generation capacity of the plants. The appropriate time to start the refill operation is around the first week of October of each year. The length of the refill operation varies with the reservoirs, but it is not extended beyond the end of January of each year.

- Under the fourth specific objective of this research, the sensitivity analysis of the hydroelectric power generation of the cascading reservoir was examined. Hydroelectric power generation from the Bersia reservoir is highly sensitive to the change of the storage volume, while the production of the energy from Temenggor is more sensitive to the change of the release as compared with the other plants in the cascading scheme. However, the hydroelectric power generation from the Kenering plant is more sensitive to the change of the overall plant efficiency. In general, the larger storage capacity reservoir is more sensitive to the rate of the release; whereas, the smaller storage capacity reservoir is more sensitive to the change of the headrace level with an equal addition and withdrawal of water. The overall plant efficiency varies linearly with the headrace level, while there is no definite relation to the rate of the turbine release. The overall plant efficiency decreases with the increase of the headrace level. Therefore, in the computation of the hydroelectric power generation using a constant efficiency could lead to inaccuracy.

A new approach using the Simpson one-third numerical integration technique is proposed to revise a stage-storage curve. The proposed approach was verified using four reservoirs. The new stage-storage curve has a maximum of 12% deviation from the known live storage volume at Temenggor, while less than 10% discrepancies found in the other three reservoirs. There is no margin of error in the computation of the storage volume using the relationship of the stage-storage. However, the developed relationship is quite capable to represent the reservoirs' system because the error might be very small for a small change in the storage volume. Therefore, the approach that used to develop the stage-storage relationship could be an alternative method to predict the reservoir variables such as stage, storage and water surface area.

5.2. Future work

This study presents the approach of optimizing a cascading reservoir using the genetic algorithm and the seasonally varied models. The models improved the capacity factor of the Perak cascading hydroelectric power generation to a maximum of 41.56%. The additional power also had a significant impact on the growing energy requirements of the country.

However, as compared to the global average hydroelectric power generation's capacity factor, the Perak cascading still require further improvement. For example, the world average hydroelectric power capacity factor in 2009 was about 44%. It showed that the research on the Perak cascading should continue to improve the hydroelectric power generation of the scheme in further. Hence, in the future work of the Perak cascading reservoir operation can be focused on:

- improving the searching capacity of the GAM. It can be achieved by embedding the GA to other optimization models such as linear programming technique, computer packages like MIKE 11, HEC-ResSim etc.

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APPENDIX A

Storage Effectiveness Ratio (SER) computation

General Steps used to compute SER

Step 1. Find the firm energy, E_f

Step 2. Estimate the shortfall of firm hydropower

$$P_f = E_f - 168\sum_{i=1}^{n} I_{Ui} h_i(S_i)\eta_i$$

Step 3. For each reservoir, estimate the drawdown requires

$$\Delta S_i = \frac{P_f}{168 \times h_i \times \eta_i}$$

Step 4. For each reservoir, estimate the energy loss

$$E_{Li} = 168(CI_{Ui} + V_{pi})h_i(S_i - \Delta S_i)\eta_i$$

Step 5. Calculate the storage effectiveness ratio (SER) for reservoir *i*

$$SER_i = \frac{E_{Li}}{P_f}$$

A. Temenggor Reservoir

Step 1

Temenggor

Minimum inflow rate = 103 m^3/s
Headrace level = 246 m ASL
Net head = 101 m (rated head)
Overall eff. = 0.79
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.79 * 9.81 * 103 * 101
F_P = 80,622 kW

Bersia

Minimum inflow rate = 126 m^3/s
Headrace level = 140 m ASL
Net head = 26.5 m (rated head)
Overall eff. = 0.82
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.82 * 9.81 * 126 * 26.5
F_P = 26,860 kW

Kenering

Minimum inflow rate = 163 m^3/s
Headrace level = 111 m ASL
Net head = 34.7 (rated head)
Overall eff. = 0.85
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.85 * 9.81 * 163 * 34.7
F_P = 47,163 kW

Chenderoh

Minimum inflow rate = 167 m^3/s
Headrace level = 60 m ASL
Net head = 18.29 m
Overall eff. = 0.85
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.85 * 9.81 * 167 * 18.29
F_P = 25,470 kW

Step 2 Average hydroelectric power generated = 228,000 kW

P_f = 228,000 – 80,622 – 26,860 – 47,163 – 25,470

P_f = 47,885 kW

Step 3 Average release requires to produce this generation

$$R = \frac{P}{\gamma \sum_{i=1}^{4} \eta_i h_i}$$

Where
R - average release required (m^3/s)
P - power (kW)
γ - unit weight of water (kN/m^3)
η - overall average efficiency for reservoir *i*
h - net stage for reservoir *i*

R = 33.31 m^3/s

The volume of water release per week equal to

V = 33.31 * 7 * 24 * 3,600

V = 20.1 Mm^3 Volume of water emptied

For Temenggor reservoir the change in head due to the release of the above volume (from stage-storage relationship) is

Stage, m	Storage, Mm^3
246.00	5732
245.55	5712

Change in head = 0.45 m

Step 4 New net head = 101 - 0.45 = 100.55 m

New release rate = 103 + 33.31 = 136.31 m^3/s
New power = 0.79 * 9.81 * 100.55 * 136.31
P = 106,220 kW
Loss of power, E_{Li} =10,6220 – 80,622 = 25,598 kW

Step 5 Storage Effectiveness Ratio (SER)
SER = 25,598 / 47,885 = **0.54**

B. Bersia Reservoir

Step 1

Temenggor
Minimum inflow rate = 103 m^3/s
Headrace level = 246 m ASL
Net head = 101 m (rated head)
Overall eff. = 0.79
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.79 * 9.81 * 103 * 101
F_P = 80,622kW

Bersia
Minimum inflow rate = 126 m^3/s
Headrace level = 140 m ASL
Net head = 26.5 m (rated head)
Overall eff. = 0.82
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.82 * 9.81 * 126 * 26.5
F_P = 26,860 kW

Kenering
Minimum inflow rate = 163 m^3/s
Headrace level = 111 m ASL
Net head = 34.7 (rated head)
Overall eff. = 0.85
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.85 * 9.81 * 163 * 34.7
F_P = 47,163 kW

Chenderoh
Minimum inflow rate = 167 m^3/s
Headrace level = 60 m ASL
Net head = 18.29 m
Overall eff. = 0.85
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.85 * 9.81 * 167 * 18.29
F_P = 25,470 kW

Step 2 Average hydroelectric power generated = 228,000 kW

$P_f = 228{,}000 - 80{,}622 - 26{,}860 - 47{,}163 - 25{,}470$

$P_f = 47{,}885$ kW

Step 3 Average release requires to produce this generation

$$R = \frac{P}{\gamma \sum_{i=1}^{4} \eta_i h_i}$$

Where
R - average release required (m^3/s)
P - power (kW)
γ - unit weight of water (kN/m^3)
η - overall average efficiency for reservoir *i*
h - net stage for reservoir *i*

$$R = \frac{47885}{9.81 \times (0.79 \times 101 + 0.82 \times 26.5 + 0.85 \times 34.7 + 0.85 \times 18.29)}$$

R = 33.31 m^3/s

The volume of water release per week is equal to

V = 33.31 * 7 * 24 * 3,600

V = 20.1 Mm^3 Volume of water emptied

For Bersia reservoir the change in head due to the release of the above volume (20.1 Mm^3) is zero, because inflow and outflow were balanced each other

Step 4 New net head = 26.5 m

New release rate = 126 + 33.31 = 159.31 m^3/s

New power = 0.82 * 9.81 * 26.5 * 159.31

P = 33,960 kW

Loss of power, E_{L2} =33,960 – 26,860 = 7,100 kW

Step 5 Storage Effectiveness Ratio (SER)

SER = 7100 / 47885 = **0.15**

C. Kenering Reservoir

Step 1

Temenggor

Minimum inflow rate = 103 m^3/s
Headrace level = 246 m ASL
Net head = 101 m (rated head)
Overall eff. = 0.79
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.79 * 9.81 * 103 * 101
F_P = 80,622 kW

Ber sia

Minimum inflow rate = 126 m^3/s

Headrace level = 140 m ASL
Net head = 26.5 m (rated head)
Overall eff. = 0.82
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.82 * 9.81 * 126 * 26.5
F_P = 26,860 kW

Kenering

Minimum inflow rate = 163 m^3/s
Headrace level = 111 m ASL
Net head = 34.7 (rated head)
Overall eff. = 0.85
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.85 * 9.81 * 163 * 34.7
F_P = 47,163 kW

Chenderoh

Minimum inflow rate = 167 m^3/s
Headrace level = 60 m ASL
Net head = 18.29 m
Overall eff. = 0.85
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.85 * 9.81 * 167 * 18.29
F_P = 25,470 kW

Step 2 Average hydroelectric power generated = 228, 000 kW

P_f = 228,000 – 80,622 – 26,860 – 47,163 – 25,470

P_f = 47,885 kW

Step 3 Average release requires to produce this generation

$$R = \frac{P}{\gamma \sum_{i=1}^{4} \eta_i h_i}$$

Where
R - average release required (m^3/s)
P - power (kW)
γ - unit weight of water (kN/m^3)
η - overall average efficiency for reservoir *i*
h - net stage for reservoir *i*

$$R = \frac{47885}{9.81 \times (0.79 \times 101 + 0.82 \times 26.5 + 0.85 \times 34.7 + 0.85 \times 18.29)}$$

R = 33.31 m^3/s

If the above flow rate added and release, the volume of water release per week would be

$V = 33.31 * 7 * 24 * 3{,}600$

$V = 20.1\ Mm^3$ Volume of water emptied

For Kenering Reservoir the change in head due to the release of the $20.1\ Mm^3$ is zero, because inflow and outflow were balanced each other

Step 4 New net head = 34.7 m

New release rate = 163 + 33.31 = 196.31 m^3/s

New power = 0.85 * 9.81 * 34.7 * 196.31

P = 56,801 kW

Loss of power, E_{L2} =56,801 – 47,163 = 9,638 kW

Step 5 Storage Effectiveness Ratio (SER)

SER = 9,638 / 47,885 = **0.21**

D. Chenderoh Reservoir

Step 1

Temenggor

Minimum inflow rate = 103 m^3/s
Headrace level = 246 m ASL
Net head = 101 m (rated head)
Overall eff. = 0.79
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.79 * 9.81 * 103 * 101
F_P = 80,622 kW

Bersia

Minimum inflow rate = 126 m^3/s
Headrace level = 140 m ASL
Net head = 26.5 m (rated head)
Overall eff. = 0.82
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.82 * 9.81 * 126 * 26.5
F_P = 26,860 kW

Kenering

Minimum inflow rate = 163 m^3/s
Headrace level = 111 m ASL
Net head = 34.7 (rated head)
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.85 * 9.81 * 163 * 34.7
F_P = 47,163 kW

Chenderoh

Minimum inflow rate = 167 m^3/s
Headrace level = 60 m ASL
Net head = 18.29 m
Overall eff. = 0.85
Unit weight of water = 9.81 kN/m^3
Firm Energy = 0.85 * 9.81 * 167 * 18.29
F_P = 25,470 kW

Step 2 Average hydroelectric power generated = 228,000 kW

P_f = 228,000 – 80,622 – 26,860 – 47,163 – 25,470

Pf = 47,885 kW

Step 3 Average flow required to produce this generation

$$R = \frac{P}{\gamma \sum_{i=1}^{4} \eta_i h_i}$$

Where
R - average release required (m^3/s)
P - power (kW)
γ - unit weight of water (kN/m^3)
η - overall average efficiency for reservoir *i*
h - net stage for reservoir i

$$R = \frac{47885}{9.81 \times (0.79 \times 101 + 0.82 \times 26.5 + 0.85 \times 34.7 + 0.85 \times 18.29)}$$

R = 33.31 m^3/s

If the above flow rate added and release, the volume of water release per week would be

V = 33.31 * 7 * 24 * 3,600
V = 20.1 Mm^3 Volume of water emptied

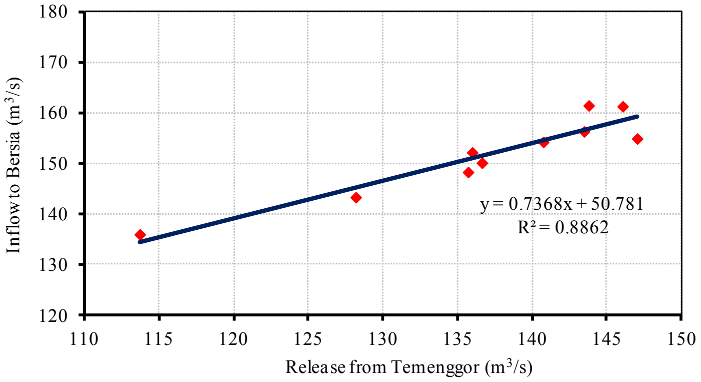
For Chenderoh Reservoir the change in head due to the release of the above volume is Zero, because inflow and outflow were balanced each other

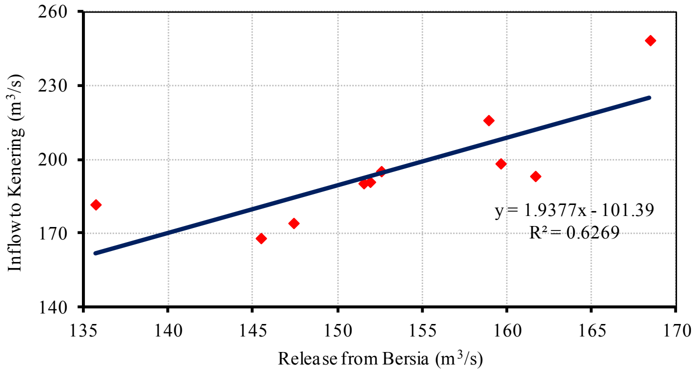
Step 4 New net head = 18.29 m
New release rate = 167 + 33.31 = 200.31 m^3/s
New power = 0.85 * 9.81 * 18.29 * 200.31
P = 30,550 kW
Loss of power, E_{L2} =30,550 – 25,470 = 5,080 kW

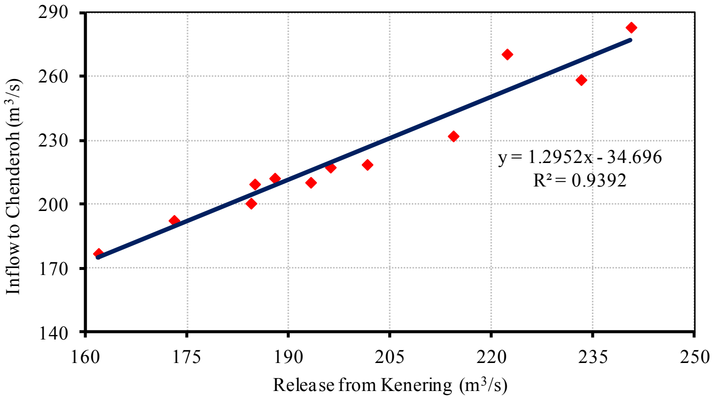
Step 5 Storage Effectiveness Ratio (SER)
SER = 5,080 / 47,885 = **0.11**

APPENDIX B

Relationship between release of preceding and total inflow to the reservoir in the Perak cascading scheme







APPENDIX C

Genetic Algorithm Model (GAM) Fitness Function

```
% This is the fitness function of the Perak cascading hydroelectric power reservoirs
% to maximize the annual average generation
% Tilahun Derib Asfaw ( PhD student Department of Civil Engineering)
% Assoc. Prof. Dr. Khamaruzaman Wan Yusof (Main Supervisor)
% Assoc. Prof. Ahmad Mustafa Hashim (Co-Supervisor)
% Universiti Teknologi PETRONAS (UTP), Malaysia
% =======September 7, 2011=======
%
function f = perak_cascadeobjav(x)
f = sqrt(578000^2-9.81^2*((0.0152*(x(1)*(101+0.0057*(113-x(1))^2)+...
    x(2)*(101+0.0057*(219-x(1)-x(2))^2)+x(3)*(101+0.0057*(321-x(1)-x(2)-...
    x(3))^2)+x(4)*(101+0.0057*(455-x(1)-x(2)-x(3)-x(4))^2)+x(5)*(101+...
    0.0057*(594-x(1)-x(2)-x(3)-x(4)-x(5))^2)+x(6)*(101+0.0057*(714-...
    x(1)-x(2)-x(3)-x(4)-x(5)-x(6))^2)+x(7)*(101+0.0057*(834-x(1)-x(2)-...
    x(3)-x(4)-x(5)-x(6)-x(7))^2)+x(8)*(101+0.0057*(962-x(1)-x(2)-x(3)-...
    x(4)-x(5)-x(6)-x(7)-x(8))^2)+x(9)*(101+0.0057*(1090-x(1)-x(2)-x(3)-...
    x(4)-x(5)-x(6)-x(7)-x(8)-...
    x(9))^2)+x(10)*(101+0.0057*(1210-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-...
    x(7)-x(8)-x(9)-x(10))^2)+x(11)*(101+0.0057*(1353-x(1)-x(2)-x(3)-...
    x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11))^2)+x(12)*(101+0.0057*...
    (1468-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-...
    x(12))^2)+x(13)*(101+0.0057*(1585-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-...
    x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13))^2)+x(14)*(101+0.0057*(1699-...
    x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-...
    x(13)-x(14))^2)+x(15)*(101+0.0057*(1823-x(1)-x(2)-x(3)-x(4)-x(5)-...
    x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15))^2)+...
    x(16)*(101+0.0057*(1945-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-...
    x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16))^2)+x(17)*...
```

```
(101+0.0057*(2082-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-...
x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17))^2)+x(18)*(101+...
0.0057*(2208-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-...
x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18))^2)+x(19)*(101+...
0.0057*(2345-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-...
x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19))^2)+x(20)*...
(101+0.0057*(2499-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-...
x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-...
x(20))^2)+x(21)*(101+0.0057*(2622-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-...
x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-...
x(18)-x(19)-x(20)-x(21))^2)+x(22)*(101+0.0057*(2751-x(1)-x(2)-...
x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-...
x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-x(22))^2)+x(23)*(101+...
0.0057*(2871-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-...
x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-...
x(22)-x(23))^2)+x(24)*(101+0.0057*(2987-x(1)-x(2)-x(3)-x(4)-x(5)-...
x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-...
x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24))^2)+x(25)*(101+0.0057*...
(3110-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-...
x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-...
x(22)-x(23)-x(24)-x(25))^2)+x(26)*(101+0.0057*(3217-x(1)-x(2)-x(3)-...
x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-...
x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-...
x(26))^2)+x(27)*(101+0.0057*(3324-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-...
x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-...
x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27))^2)+...
x(28)*(101+0.0057*(3441-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-...
x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-...
x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28))^2)+x(29)*...
(101+0.0057*(3555-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-...
x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-...
x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-x(29))^2)+...
x(30)*(101+0.0057*(3696-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-...
```

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x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-...
x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-x(29)-...
x(30))^2)+x(31)*(101+0.0057*(3807-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-...
x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-...
x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-...
x(29)-x(30)-x(31))^2)+x(32)*(101+0.0057*(3947-x(1)-x(2)-x(3)-x(4)-...
x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-...
x(17)-x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-...
x(28)-x(29)-x(30)-x(31)-x(32))^2)+x(33)*(101+0.0057*(4084-x(1)-...
x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-...
x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-...
x(25)-x(26)-x(27)-x(28)-x(29)-x(30)-x(31)-x(32)-x(33))^2)+x(34)*...
(101+0.0057*(4215-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-...
x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-...
x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-x(29)-x(30)-x(31)-...
x(32)-x(33)-x(34))^2)+x(35)*(101+0.0057*(4329-x(1)-x(2)-x(3)-x(4)-...
x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-...
x(17)-x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-...
x(28)-x(29)-x(30)-x(31)-x(32)-x(33)-x(34)-x(35))^2)+x(36)*(101+...
0.0057*(4457-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-...
x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-...
x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-x(29)-x(30)-x(31)-x(32)-...
x(33)-x(34)-x(35)-x(36))^2)+x(37)*(101+0.0057*(4585-x(1)-x(2)-...
x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-...
x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-...
x(26)-x(27)-x(28)-x(29)-x(30)-x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-...
x(37))^2)+x(38)*(101+0.0057*(4734-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-...
x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-...
x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-...
x(29)-x(30)-x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-x(38))^2)+...
x(39)*(101+0.0057*(4877-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-...
x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-...
x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-x(29)-x(30)-...
```

```
x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-x(38)-x(39))^2)+...
x(40)*(101+0.0057*(5043-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-...
x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-...
x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-x(29)-x(30)-...
x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-x(38)-x(39)-x(40))^2)+...
x(41)*(101+0.0057*(5207-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-...
x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-...
x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-x(29)-x(30)-...
x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-x(38)-x(39)-x(40)-...
x(41))^2)+x(42)*(101+0.0057*(5341-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-...
x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-...
x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-...
x(29)-x(30)-x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-x(38)-x(39)-...
x(40)-x(41)-x(42))^2)+x(43)*(101+0.0057*(5528-x(1)-x(2)-x(3)-x(4)-...
x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-...
x(17)-x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-...
x(28)-x(29)-x(30)-x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-x(38)-...
x(39)-x(40)-x(41)-x(42)-x(43))^2)+x(44)*(101+0.0057*(5692-x(1)-x(2)-...
x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-...
x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-...
x(26)-x(27)-x(28)-x(29)-x(30)-x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-...
x(37)-x(38)-x(39)-x(40)-x(41)-x(42)-x(43)-x(44))^2)+x(45)*(101+...
0.0057*(5971-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-...
x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-...
x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-x(29)-x(30)-x(31)-x(32)-...
x(33)-x(34)-x(35)-x(36)-x(37)-x(38)-x(39)-x(40)-x(41)-x(42)-x(43)-...
x(44)-x(45))^2)+x(46)*(101+0.0057*(6251-x(1)-x(2)-x(3)-x(4)-x(5)-...
x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-...
x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-...
x(29)-x(30)-x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-x(38)-x(39)-...
x(40)-x(41)-x(42)-x(43)-x(44)-x(45)-x(46))^2)+x(47)*(101+0.0057*...
(6511-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-...
x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-...
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x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-x(29)-x(30)-x(31)-...
x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-x(38)-x(39)-x(40)-x(41)-...
x(42)-x(43)-x(44)-x(45)-x(46)-x(47))^2)+x(48)*(101+0.0057*...
(6694-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-...
x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-x(22)-...
x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-x(29)-x(30)-x(31)-x(32)-x(33)-...
x(34)-x(35)-x(36)-x(37)-x(38)-x(39)-x(40)-x(41)-x(42)-x(43)-x(44)-...
x(45)-x(46)-x(47)-x(48))^2)+x(49)*(101+0.0057*(6949-x(1)-x(2)-x(3)-...
x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-...
x(16)-x(17)-x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-...
x(27)-x(28)-x(29)-x(30)-x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-...
x(38)-x(39)-x(40)-x(41)-x(42)-x(43)-x(44)-x(45)-x(46)-x(47)-x(48)-...
x(49))^2)+x(50)*(101+0.0057*(7148-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-...
x(7)-x(8)-x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-...
x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-...
x(29)-x(30)-x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-x(38)-x(39)-...
x(40)-x(41)-x(42)-x(43)-x(44)-x(45)-x(46)-x(47)-x(48)-x(49)-x(50))^2)+...
x(51)*(101+0.0057*(7288-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-x(9)-...
x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-x(18)-x(19)-x(20)-...
x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-x(29)-x(30)-x(31)-...
x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-x(38)-x(39)-x(40)-x(41)-x(42)-...
x(43)-x(44)-x(45)-x(46)-x(47)-x(48)-x(49)-x(50)-x(51))^2)+...
x(52)*(101+0.0057*(7414-x(1)-x(2)-x(3)-x(4)-x(5)-x(6)-x(7)-x(8)-...
x(9)-x(10)-x(11)-x(12)-x(13)-x(14)-x(15)-x(16)-x(17)-...
x(18)-x(19)-x(20)-x(21)-x(22)-x(23)-x(24)-x(25)-x(26)-x(27)-x(28)-...
x(29)-x(30)-x(31)-x(32)-x(33)-x(34)-x(35)-x(36)-x(37)-x(38)-x(39)-...
x(40)-x(41)-x(42)-x(43)-x(44)-x(45)-x(46)-x(47)-x(48)-x(49)-x(50)-...
x(51)-x(52))^2)))+(0.0158*(x(53)*(26.5+0.1119*(1.18*x(1)-x(53))^2)+...
x(54)*(26.5+0.1119*(1.18*(x(1)+x(2))+(x(53)+x(54))))^2+...
x(55)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3))-(x(53)+x(54)+x(55))))^2+...
x(56)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4))-(x(53)+x(54)+x(55)+...
x(56))))^2+x(57)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5))-...
(x(53)+x(54)+x(55)+x(56)+x(57))))^2+x(58)*(26.5+0.1119*(1.18*(x(1)+...
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x(2)+x(3)+x(4)+x(5)+x(6))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58))))^2+...
x(59)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7))-(x(53)+...
x(54)+x(55)+x(56)+x(57)+x(58)+x(59))))^2+x(60)*(26.5+0.1119*(1.18*...
(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8))-(x(53)+x(54)+x(55)+x(56)+...
x(57)+x(58)+x(59)+x(60))))^2+x(61)*(26.5+0.1119*(1.18*(x(1)+x(2)+...
x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9))-(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+x(59)+x(60)+x(61))))^2+x(62)*(26.5+0.1119*(1.18*(x(1)+x(2)+...
x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10))-(x(53)+x(54)+x(55)+x(56)+...
x(57)+x(58)+x(59)+x(60)+x(61)+x(62))))^2+x(63)*(26.5+0.1119*(1.18*...
(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11))-(x(53)+...
x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63))))^2+...
x(64)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+...
x(9)+x(10)+x(11)+x(12))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+...
x(60)+x(61)+x(62)+x(63)+x(64))))^2+x(65)*(26.5+0.1119*(1.18*(x(1)+...
x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13))-...
(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+...
x(64)+x(65))))^2+x(66)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+...
x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14))-(x(53)+x(54)+...
x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+...
x(66))))^2+x(67)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+...
x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15))-(x(53)+x(54)+...
x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+...
x(66)+x(67))))^2+x(68)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+...
x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16))-(x(53)+...
x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+...
x(65)+x(66)+x(67)+x(68))))^2+x(69)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+...
x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+...
x(16)+x(17))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+...
x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69))))^2+x(70)*(26.5+...
0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+...
x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18))-(x(53)+x(54)+...
x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+...
x(66)+x(67)+x(68)+x(69)+x(70))))^2+x(71)*(26.5+0.1119*(1.18*(x(1)+...
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x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+...
x(14)+x(15)+x(16)+x(17)+x(18)+x(19))-(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+...
x(69)+x(70)+x(71))))^2+x(72)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+...
x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+...
x(17)+x(18)+x(19)+x(20))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+...
x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+...
x(71)+x(72))))^2+x(73)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+...
x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+...
x(18)+x(19)+x(20)+x(21))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+...
x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+...
x(71)+x(72)+x(73))))^2+x(74)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+...
x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+...
x(17)+x(18)+x(19)+x(20)+x(21)+x(22))-(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+...
x(69)+x(70)+x(71)+x(72)+x(73)+x(74))))^2+x(75)*(26.5+0.1119*(1.18*...
(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+...
x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23))-...
(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+...
x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+...
x(75))))^2+x(76)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+...
x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+...
x(19)+x(20)+x(21)+x(22)+x(23)+x(24))-(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+...
x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76))))^2+x(77)*(26.5+...
0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+...
x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+...
x(22)+x(23)+x(24)+x(25))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+...
x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+...
x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77))))^2+x(78)*(26.5+0.1119*...
(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+...
x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+...
x(25)+x(26))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+...
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x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+...
x(73)+x(74)+x(75)+x(76)+x(77)+x(78))))^2+x(79)*(26.5+0.1119*(1.18*...
(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+...
x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+...
x(25)+x(26)+x(27))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+...
x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79))))^2+x(80)*(26.5+...
0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+...
x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+...
x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28))-(x(53)+x(54)+x(55)+x(56)+...
x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+...
x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+...
x(79)+x(80))))^2+x(81)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+...
x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+...
x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+...
x(29))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+...
x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+...
x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81))))^2+x(82)*(26.5+...
0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+...
x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+...
x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+x(30))-(x(53)+x(54)+...
x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+...
x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+...
x(77)+x(78)+x(79)+x(80)+x(81)+x(82))))^2+x(83)*(26.5+0.1119*(1.18*...
(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+...
x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+...
x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31))-(x(53)+x(54)+x(55)+x(56)+...
x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+...
x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+...
x(79)+x(80)+x(81)+x(82)+x(83))))^2+x(84)*(26.5+0.1119*(1.18*(x(1)+...
x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+...
x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+...
x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+x(32))-(x(53)+x(54)+x(55)+...
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x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+...
x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+...
x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84))))^2+x(85)*(26.5+0.1119*...
(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+...
x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+...
x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+x(32)+x(33))-(x(53)+x(54)+...
x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+...
x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+...
x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85))))^2+x(86)*...
(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+...
x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+...
x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+...
x(32)+x(33)+x(34))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+...
x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+...
x(83)+x(84)+x(85)+x(86))))^2+x(87)*(26.5+0.1119*(1.18*(x(1)+x(2)+...
x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+...
x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+x(25)+...
x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+x(32)+x(33)+x(34)+x(35))-(x(53)+...
x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+...
x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+...
x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+...
x(87))))^2+x(88)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+...
x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+...
x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+...
x(30)+x(31)+x(32)+x(33)+x(34)+x(35)+x(36))-(x(53)+x(54)+x(55)+x(56)+...
x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+...
x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+...
x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88))))^2+...
x(89)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+...
x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+...
x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+...
x(32)+x(33)+x(34)+x(35)+x(36)+x(37))-(x(53)+x(54)+x(55)+x(56)+x(57)+...
```

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x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+...
x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+...
x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89))))^2+...
x(90)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+...
x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+...
x(20)+x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+...
x(31)+x(32)+x(33)+x(34)+x(35)+x(36)+x(37)+x(38))-(x(53)+x(54)+x(55)+...
x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+...
x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+...
x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+...
x(89)+x(90))))^2+x(91)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+...
x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+...
x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+...
x(29)+x(30)+x(31)+x(32)+x(33)+x(34)+x(35)+x(36)+x(37)+x(38)+x(39))-...
(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+...
x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+...
x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+...
x(86)+x(87)+x(88)+x(89)+x(90)+x(91))))^2+x(92)*(26.5+0.1119*(1.18*...
(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+...
x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+...
x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+x(32)+x(33)+x(34)+...
x(35)+x(36)+x(37)+x(38)+x(39)+x(40))-(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+...
x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+...
x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+...
x(91)+x(92))))^2+x(93)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+...
x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+...
x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+...
x(29)+x(30)+x(31)+x(32)+x(33)+x(34)+x(35)+x(36)+x(37)+x(38)+x(39)+...
x(40)+x(41))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+...
x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+...
x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+...
x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93))))^2+...
```

```
x(94)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+...
x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+...
x(20)+x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+...
x(31)+x(32)+x(33)+x(34)+x(35)+x(36)+x(37)+x(38)+x(39)+x(40)+x(41)+...
x(42))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+...
x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+...
x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+...
x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+x(94))))^2+...
x(95)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+...
x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+...
x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+...
x(32)+x(33)+x(34)+x(35)+x(36)+x(37)+x(38)+x(39)+x(40)+x(41)+x(42)+...
x(43))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+...
x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+...
x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+...
x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+x(94)+x(95))))^2+...
x(96)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+...
x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+...
x(20)+x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+...
x(31)+x(32)+x(33)+x(34)+x(35)+x(36)+x(37)+x(38)+x(39)+x(40)+x(41)+...
x(42)+x(43)+x(44))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+...
x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+...
x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+...
x(94)+x(95)+x(96))))^2+x(97)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+...
x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+...
x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+...
x(28)+x(29)+x(30)+x(31)+x(32)+x(33)+x(34)+x(35)+x(36)+x(37)+x(38)+...
x(39)+x(40)+x(41)+x(42)+x(43)+x(44)+x(45))-(x(53)+x(54)+x(55)+x(56)+...
x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+...
x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+...
x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+...
x(90)+x(91)+x(92)+x(93)+x(94)+x(95)+x(96)+x(97))))^2+x(98)*(26.5+...
```

```
0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+...
x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+...
x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+x(32)+...
x(33)+x(34)+x(35)+x(36)+x(37)+x(38)+x(39)+x(40)+x(41)+x(42)+x(43)+...
x(44)+x(45)+x(46))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+...
x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+...
x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+...
x(94)+x(95)+x(96)+x(97)+x(98))))^2+x(99)*(26.5+0.1119*(1.18*(x(1)+...
x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+...
x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+...
x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+x(32)+x(33)+x(34)+x(35)+...
x(36)+x(37)+x(38)+x(39)+x(40)+x(41)+x(42)+x(43)+x(44)+x(45)+x(46)+...
x(47))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+...
x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+...
x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+...
x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+x(94)+x(95)+...
x(96)+x(97)+x(98)+x(99))))^2+x(100)*(26.5+0.1119*(1.18*(x(1)+x(2)+...
x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+...
x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+x(25)+...
x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+x(32)+x(33)+x(34)+x(35)+x(36)+...
x(37)+x(38)+x(39)+x(40)+x(41)+x(42)+x(43)+x(44)+x(45)+x(46)+x(47)+...
x(48))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+...
x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+...
x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+...
x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+x(94)+x(95)+...
x(96)+x(97)+x(98)+x(99)+x(100))))^2+x(101)*(26.5+0.1119*(1.18*(x(1)+...
x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+...
x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+...
x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+x(32)+x(33)+x(34)+x(35)+...
x(36)+x(37)+x(38)+x(39)+x(40)+x(41)+x(42)+x(43)+x(44)+x(45)+x(46)+...
x(47)+x(48)+x(49))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+...
x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
```

```
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+...
x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+...
x(94)+x(95)+x(96)+x(97)+x(98)+x(99)+x(100)+x(101))))^2+x(102)*(26.5+...
0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+...
x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+...
x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+...
x(32)+x(33)+x(34)+x(35)+x(36)+x(37)+x(38)+x(39)+x(40)+x(41)+x(42)+...
x(43)+x(44)+x(45)+x(46)+x(47)+x(48)+x(49)+x(50))-(x(53)+x(54)+x(55)+...
x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+...
x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+...
x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+...
x(89)+x(90)+x(91)+x(92)+x(93)+x(94)+x(95)+x(96)+x(97)+x(98)+x(99)+...
x(100)+x(101)+x(102))))^2+x(103)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+...
x(4)+x(5)+x(6)+x(7)+x(8)+x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+...
x(16)+x(17)+x(18)+x(19)+x(20)+x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+...
x(27)+x(28)+x(29)+x(30)+x(31)+x(32)+x(33)+x(34)+x(35)+x(36)+x(37)+...
x(38)+x(39)+x(40)+x(41)+x(42)+x(43)+x(44)+x(45)+x(46)+x(47)+x(48)+...
x(49)+x(50)+x(51))-(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+...
x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+...
x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+...
x(94)+x(95)+x(96)+x(97)+x(98)+x(99)+x(100)+x(101)+x(102)+x(103))))^2+...
x(104)*(26.5+0.1119*(1.18*(x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)+x(8)+...
x(9)+x(10)+x(11)+x(12)+x(13)+x(14)+x(15)+x(16)+x(17)+x(18)+x(19)+x(20)+...
x(21)+x(22)+x(23)+x(24)+x(25)+x(26)+x(27)+x(28)+x(29)+x(30)+x(31)+...
x(32)+x(33)+x(34)+x(35)+x(36)+x(37)+x(38)+x(39)+x(40)+x(41)+x(42)+...
x(43)+x(44)+x(45)+x(46)+x(47)+x(48)+x(49)+x(50)+x(51)+x(52))-(x(53)+...
x(54)+x(55)+x(56)+x(57)+x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+...
x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+...
x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+...
x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+x(94)+x(95)+x(96)+x(97)+...
x(98)+x(99)+x(100)+x(101)+x(102)+x(103)+x(104))))^2))+(0.0163*...
(x(105)*(34.7+0.0192*(1.35*x(53)-x(105)))^2+x(106)*(34.7+0.0192*...
```

```
(1.35*(x(53)+x(54))-(x(105)+x(106))))^2+x(107)*(34.7+0.0192*(1.35*...
(x(53)+x(54)+x(55))-(x(105)+x(106)+x(107))))^2+x(108)*(34.7+0.0192*...
(1.35*(x(53)+x(54)+x(55)+x(56))-(x(105)+x(106)+x(107)+x(108))))^2+...
x(109)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57))-(x(105)+...
x(106)+x(107)+x(108)+x(109))))^2+x(110)*(34.7+0.0192*(1.35*(x(53)+...
x(54)+x(55)+x(56)+x(57)+x(58))-(x(105)+x(106)+x(107)+x(108)+x(109)+...
x(110))))^2+x(111)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+(59))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111))))^2+...
x(112)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+...
x(60))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112))))^2+...
x(113)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+...
x(60)+x(61))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+...
x(113))))^2+x(114)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+(59)+x(60)+x(61)+x(62))-(x(105)+x(106)+x(107)+x(108)+x(109)+...
x(110)+x(111)+x(112)+x(113)+x(114))))^2+x(115)*(34.7+0.0192*(1.35*...
(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63))-...
(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+...
x(114)+x(115))))^2+x(116)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+...
x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64))-(x(105)+x(106)+...
x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+…
x(116))))^2+    x(117)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+…
x(58)+(59)+ x(60)+x(61)+x(62)+x(63)+x(64)+x(65))-(x(105)+x(106)+…
x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+…
x(116)+x(117))))^2+x(118)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+…
x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66))-(x(105)+…
x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+…
x(116)+x(117)+x(118))))^2+...
x(119)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+...
x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67))-(x(105)+x(106)+...
x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+...
x(117)+x(118)+x(119))))^2+x(120)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+...
x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+...
x(68))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+...
```

```
x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120))))^2+x(121)*(34.7+...
0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+...
x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69))-(x(105)+x(106)+...
x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+...
x(117)+x(118)+x(119)+x(120)+x(121))))^2+x(122)*(34.7+0.0192*(1.35*…
(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+…
x(65)+x(66)+x(67)+x(68)+x(69)+x(70))-(x(105)+x(106)+x(107)+x(108)+x(109)+...
x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+...
x(120)+x(121)+x(122))))^2+x(123)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+...
x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+...
x(68)+x(69)+x(70)+x(71))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123))))^2+x(124)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+...
x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+...
x(68)+x(69)+x(70)+x(71)+x(72))-(x(105)+x(106)+x(107)+x(108)+x(109)+...
x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+...
x(120)+x(121)+x(122)+x(123)+x(124))))^2+x(125)*(34.7+0.0192*(1.35*...
(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+...
x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73))-(x(105)+...
x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+...
x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+…
x(125))))^2+x(126)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+…
x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+…
x(70)+x(71)+...
x(72)+x(73)+x(74))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+...
x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+...
x(122)+x(123)+x(124)+x(125)+x(126))))^2+x(127)*(34.7+0.0192*(1.35*...
(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+...
x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+...
x(75))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+...
x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+...
x(122)+x(123)+x(124)+x(125)+x(126)+x(127))))^2+x(128)*(34.7+0.0192*...
(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+...
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x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+...
x(74)+x(75)+x(76))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+...
x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128))))^2+x(129)*…
(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+...
(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77))-(x(105)+x(106)+x(107)+x(108)+...
x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+...
x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+...
x(129))))^2+x(130)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+...
x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78))-(x(105)+x(106)+...
x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+...
x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+...
x(127)+x(128)+x(129)+x(130))))^2+x(131)*(34.7+0.0192*(1.35*(x(53)+x(54)+...
x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+...
(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+...
x(79))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+...
x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+...
x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131))))^2+x(132)*…
(34.7+ 0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+…
x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+…
x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80))-(x(105)+x(106)+x(107)+x(108)+…
x(109)+...
x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+...
x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+...
x(130)+x(131)+x(132))))^2+x(133)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+...
x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+...
x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+...
x(80)+x(81))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+...
x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+...
x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+...
x(133))))^2+x(134)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+...
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x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+...
x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+...
x(82))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+...
x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+...
x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+...
x(134))))^2+x(135)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+...
x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+...
x(82)+x(83))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+...
x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+...
x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+...
x(133)+x(134)+x(135))))^2+x(136)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+...
x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+...
x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+...
x(80)+x(81)+x(82)+x(83)+x(84))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+...
x(131)+x(132)+x(133)+x(134)+x(135)+x(136))))^2+x(137)*(34.7+0.0192*(1.35*...
(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+...
x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+...
x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85))-(x(105)+x(106)+...
x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+...
x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+...
x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+...
x(137))))^2+x(138)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+...
x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+...
x(82)+x(83)+x(84)+x(85)+x(86))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+...
x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138))))^2+...
x(139)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+...
x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
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x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+...
x(84)+x(85)+x(86)+x(87))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+…
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+…
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+…
x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139))))^2+...
x(140)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+...
x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+...
x(83)+x(84)+x(85)+x(86)+x(87)+x(88))-(x(105)+x(106)+x(107)+x(108)+...
x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+...
x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+...
x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+...
x(139)+x(140))))^2+x(141)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+...
x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+...
x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+...
x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89))-(x(105)+x(106)+...
x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+...
x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+...
x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+...
x(137)+x(138)+x(139)+x(140)+x(141))))^2+x(142)*(34.7+0.0192*(1.35*...
(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+...
x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+...
x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+...
x(88)+x(89)+x(90))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+...
x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+...
x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+...
x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+...
x(140)+x(141)+x(142))))^2+x(143)*(34.7+0.0192*(1.35*(x(53)+x(54)+...
x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+...
x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+...
x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+...
x(88)+x(89)+x(90)+x(91))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
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x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+...
x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+...
x(139)+x(140)+x(141)+x(142)+x(143))))^2+x(144)*(34.7+0.0192*(1.35*...
(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+...
x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+...
x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+...
x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92))-(x(105)+x(106)+x(107)+...
x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+...
x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+...
x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+...
x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+x(143)+x(144))))^2+...
x(145)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+...
x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+...
x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93))-(x(105)+…
x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+…
x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+…
x(126)+x(127)+   x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+…
x(136)+x(137)+x(138)+   x(139)+x(140)+x(141)+x(142)+x(143)+x(144)+…
x(145))))^2+x(146)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+…
x(58)+x(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+…
x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+...
x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+…
x(89)+x(90)+x(91)+x(92)+x(93)+x(94))-(x(105)+x(106)+x(107)+x(108)+…
x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+…
x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+…
x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+…
x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+x(143)+x(144)+x(145)+...
x(146))))^2+x(147)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+...
x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+...
x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+...
x(94)+x(95))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+...
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x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+...
x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+…
x(133)+...
x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+x(143)+...
x(144)+x(145)+x(146)+x(147))))^2+x(148)*(34.7+0.0192*(1.35*(x(53)+x(54)+...
x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+...
x(67)+x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+...
x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+...
x(91)+x(92)+x(93)+x(94)+x(95)+x(96))-(x(105)+x(106)+x(107)+x(108)+x(109)+...
x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+...
x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+...
x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+...
x(140)+x(141)+x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+x(148))))^2+...
x(149)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+...
x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+...
x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+x(94)+x(95)+...
x(96)+x(97))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+...
x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+...
x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+...
x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+...
x(143)+x(144)+x(145)+x(146)+x(147)+x(148)+x(149))))^2+x(150)*(34.7+...
0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+x(60)+x(61)+...
x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+...
x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+...
x(94)+x(95)+x(96)+x(97)+x(98))-(x(105)+x(106)+x(107)+x(108)+x(109)+...
x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+...
x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+...
x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+...
x(140)+x(141)+x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+x(148)+x(149)+...
x(150))))^2+x(151)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+...
```

```
x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+...
x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+...
x(92)+x(93)+x(94)+x(95)+x(96)+x(97)+x(98)+x(99))-(x(105)+x(106)+...
x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+...
x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+...
x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+...
x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+...
x(143)+x(144)+x(145)+x(146)+x(147)+x(148)+x(149)+x(150)+x(151))))^2+...
x(152)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+...
x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+x(83)+...
x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+x(94)+x(95)+...
x(96)+x(97)+x(98)+x(99)+x(100))-(x(105)+x(106)+x(107)+x(108)+x(109)+...
x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+...
x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+...
x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+...
x(140)+x(141)+x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+x(148)+x(149)+...
x(150)+x(151)+x(152))))^2+x(153)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+...
x(56)+x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+...
x(68)+x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+...
x(79)+x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+...
x(90)+x(91)+x(92)+x(93)+x(94)+x(95)+x(96)+x(97)+x(98)+x(99)+x(100)+...
x(101))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+...
x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+...
x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+...
x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+...
x(143)+x(144)+x(145)+x(146)+x(147)+x(148)+x(149)+x(150)+x(151)+x(152)+...
x(153))))^2+x(154)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+...
x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+...
x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+...
x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+...
x(92)+x(93)+x(94)+x(95)+x(96)+x(97)+x(98)+x(99)+x(100)+x(101)+x(102))-...
(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+...
```

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x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+...
x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+...
x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+x(143)+x(144)+...
x(145)+x(146)+x(147)+x(148)+x(149)+x(150)+x(151)+x(152)+x(153)+…
x(154))))^2+...
x(155)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+x(57)+x(58)+(59)+...
x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+x(69)+x(70)+x(71)+...
x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+x(80)+x(81)+x(82)+...
x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+x(91)+x(92)+x(93)+...
x(94)+x(95)+x(96)+x(97)+x(98)+x(99)+x(100)+x(101)+x(102)+x(103))-...
(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+...
x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+...
x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+...
x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+x(143)+...
x(144)+x(145)+x(146)+x(147)+x(148)+x(149)+x(150)+x(151)+x(152)+x(153)+...
x(154)+x(155))))^2+x(156)*(34.7+0.0192*(1.35*(x(53)+x(54)+x(55)+x(56)+...
x(57)+x(58)+(59)+x(60)+x(61)+x(62)+x(63)+x(64)+x(65)+x(66)+x(67)+x(68)+...
x(69)+x(70)+x(71)+x(72)+x(73)+x(74)+x(75)+x(76)+x(77)+x(78)+x(79)+...
x(80)+x(81)+x(82)+x(83)+x(84)+x(85)+x(86)+x(87)+x(88)+x(89)+x(90)+...
x(91)+x(92)+x(93)+x(94)+x(95)+x(96)+x(97)+x(98)+x(99)+x(100)+x(101)+...
x(102)+x(103)+x(104))-(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+...
x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+...
x(141)+x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+x(148)+x(149)+x(150)+...
x(151)+x(152)+x(153)+x(154)+x(155)+x(156))))^2))+(0.0163*(x(157)*...
(18.29+0.0376*(1.13*x(105)-x(157)))^2+x(158)*(18.29+0.0376*(1.13*(x(105)+...
x(106))-(x(157)+x(158))))^2+x(159)*(18.29+0.0376*(1.13*(x(105)+x(106)+...
x(107))-(x(157)+x(158)+x(159))))^2+x(160)*(18.29+0.0376*(1.13*(x(105)+...
x(106)+x(107)+x(108))-(x(157)+x(158)+x(159)+x(160))))^2+x(161)*(18.29+...
0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109))-(x(157)+x(158)+x(159)+...
x(160)+x(161))))^2+x(162)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+...
x(109)+x(110))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162))))^2+x(163)*...
```

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(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111))-...
(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163))))^2+x(164)*(18.29+...
0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112))-...
(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+x(164))))^2+...
x(165)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+...
x(164)+x(165))))^2+x(166)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+...
x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114))-(x(157)+x(158)+x(159)+...
x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166))))^2+x(167)*(18.29+...
0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+...
x(113)+x(114)+x(115))-(x(157)+x(158)+x(159)+x(160)+x(161)+...
x(162)+x(163)+x(164)+x(165)+x(166)+x(167))))^2+x(168)*(18.29+0.0376*...
(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+...
x(114)+x(115)+x(116))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+...
x(164)+x(165)+x(166)+x(167)+x(168))))^2+x(169)*(18.29+0.0376*(1.13*...
(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+...
 x(115)+x(116)+x(117))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+...
 x(164)+x(165)+x(166)+x(167)+x(168)+x(169))))^2+x(170)*(18.29+0.0376*(1.13*...
 (x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+...
 x(115)+x(116)+x(117)+x(118))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+...
x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170))))^2+x(171)*…
(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+…
x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119))-(x(157)+x(158)+…
x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+…
x(169)+x(170)+x(171))))^2+x(172)*(18.29+0.0376*(1.13*(x(105)+x(106)+…
x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+…
x(116)+x(117)+x(118)+x(119)+x(120))-(x(157)+…
x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+...
x(167)+x(168)+x(169)+x(170)+x(171)+x(172))))^2+x(173)*(18.29+0.0376*(1.13*...
(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+...
x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121))-(x(157)+x(158)+x(159)+...
x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+...
x(170)+x(171)+x(172)+x(173))))^2+x(174)*(18.29+0.0376*(1.13*(x(105)+...
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x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+...
x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122))-(x(157)+x(158)+x(159)+...
x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+...
x(170)+x(171)+x(172)+x(173)+x(174))))^2+x(175)*(18.29+0.0376*(1.13*(x(105)+...
x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+…
x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123))-(x(157)+…
x(158)+x(159)+...
x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+...
x(170)+x(171)+x(172)+x(173)+x(174)+x(175))))^2+x(176)*(18.29+0.0376*...
(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+...
x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+…
x(123)+x(124))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+…
x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+...
x(175)+x(176))))^2+x(177)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+...
x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+...
x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125))-(x(157)+x(158)+x(159)+...
x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+...
x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+x(177))))^2+...
x(178)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123)+x(124)+x(125)+x(126))-(x(157)+x(158)+x(159)+...
x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+...
x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+…
x(178))))^2+...
x(179)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+…
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127))-(x(157)+x(158)+…
x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+…
x(169)+x(170)+x(171)+...
x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179))))^2+...
x(180)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128))-(x(157)+x(158)+...
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x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+...
x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+...
x(179)+x(180))))^2+x(181)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+...
x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+...
x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+...
x(128)+x(129))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+...
x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+...
x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181))))^2+...
x(182)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130))-...
(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+...
x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+...
x(177)+x(178)+x(179)+x(180)+x(181)+x(182))))^2+x(183)*(18.29+0.0376*...
(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+...
x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+...
x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131))-...
(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+...
x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+...
x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+x(183))))^2+x(184)*(18.29+...
0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+...
x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+...
x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+...
x(132))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+...
x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+...
x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+x(183)+x(184))))^2+...
x(185)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+...
x(131)+x(132)+x(133))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+...
x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+...
x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+...
x(183)+x(184)+x(185))))^2+x(186)*(18.29+0.0376*(1.13*(x(105)+x(106)+...
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x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+...
x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+...
x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134))-(x(157)+...
x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+...
x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+...
x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+x(183)+x(184)+x(185)+...
x(186))))^2+x(187)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+...
x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+...
x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+...
x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135))-...
(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+...
x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+...
x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+x(183)+x(184)+...
x(185)+x(186)+x(187))))^2+x(188)*(18.29+0.0376*(1.13*(x(105)+x(106)+...
x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+...
x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+...
x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+...
x(134)+x(135)+x(136))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+...
x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+...
x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+...
x(181)+x(182)+x(183)+x(184)+x(185)+x(186)+x(187)+x(188))))^2+...
x(189)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+...
x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+...
x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+...
x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+...
x(137))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+...
x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+...
x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+...
x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189))))^2+...
x(190)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+...
x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+...
x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+...
x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+...
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x(137)+x(138))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+...
x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+...
x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+x(183)+...
x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190))))^2+x(191)*(18.29+...
0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+...
x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+...
x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+...
x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139))-(x(157)+x(158)+...
x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+...
x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+...
x(179)+x(180)+x(181)+x(182)+x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+...
x(189)+x(190)+x(191))))^2+x(192)*(18.29+0.0376*(1.13*(x(105)+x(106)+...
x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+...
x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+...
x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+...
x(137)+x(138)+x(139)+x(140))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+...
x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+...
x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+...
x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190)+x(191)+...
x(192))))^2+x(193)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+...
x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+...
x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+...
x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+...
x(139)+x(140)+x(141))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+...
x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+...
x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+...
x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190)+x(191)+x(192)+...
x(193))))^2+x(194)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+...
x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+...
x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+...
x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+...
x(139)+x(140)+x(141)+x(142))-(x(157)+x(158)+x(159)+x(160)+x(161)+...
x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+...
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x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+...
x(180)+x(181)+x(182)+x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+...
x(189)+x(190)+x(191)+x(192)+x(193)+x(194))))^2+x(195)*(18.29+0.0376*...
(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+...
x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+...
x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+...
x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+...
x(143))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+...
x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+...
x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+x(183)+x(184)+x(185)+...
x(186)+x(187)+x(188)+x(189)+x(190)+x(191)+x(192)+x(193)+x(194)+…
x(195))))^2+...
x(196)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+...
x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+...
x(141)+x(142)+x(143)+x(144))-(x(157)+x(158)+x(159)+x(160)+x(161)+...
x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+...
x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+...
x(182)+x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190)+x(191)+...
x(192)+x(193)+x(194)+x(195)+x(196))))^2+x(197)*(18.29+0.0376*(1.13*...
(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+...
x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+...
x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+...
x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+...
x(143)+x(144)+x(145))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+...
x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+...
x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+...
x(181)+x(182)+x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+...
x(190)+x(191)+x(192)+x(193)+x(194)+x(195)+x(196)+x(197))))^2+...
x(198)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+...
x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+...
x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+...
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x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+...
x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+x(143)+x(144)+x(145)+...
x(146))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+...
x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+...
x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+...
x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190)+x(191)+...
x(192)+x(193)+x(194)+x(195)+x(196)+x(197)+x(198))))^2+...
x(199)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+...
x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+...
x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+...
x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+...
x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+x(143)+x(144)+x(145)+...
x(146)+x(147))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+...
x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+...
x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+...
x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190)+x(191)+x(192)+...
x(193)+x(194)+x(195)+x(196)+x(197)+x(198)+x(199))))^2+x(200)*(18.29+...
0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+...
x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+...
x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+...
x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+...
x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+x(148))-(x(157)+x(158)+...
x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+...
x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+...
x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+x(183)+x(184)+x(185)+...
x(186)+x(187)+x(188)+x(189)+x(190)+x(191)+x(192)+x(193)+x(194)+...
x(195)+x(196)+x(197)+x(198)+x(199)+x(200))))^2+x(201)*(18.29+...
0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+...
x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+...
x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+...
x(139)+x(140)+x(141)+x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+...
x(148)+x(149))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+...
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x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+...
x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+...
x(182)+x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190)+x(191)+...
x(192)+x(193)+x(194)+x(195)+x(196)+x(197)+x(198)+x(199)+x(200)+…
x(201))))^2+...
x(202)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+…
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+…
x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+…
x(141)+x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+x(148)+x(149)+…
x(150))-(x(157)+x(158)+x(159)+x(160)+…
x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+…
x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+…
x(181)+x(182)+x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190)+...
x(191)+x(192)+x(193)+x(194)+x(195)+x(196)+x(197)+x(198)+x(199)+x(200)+...
x(201)+x(202))))^2+x(203)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+...
x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+...
x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+...
x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+...
x(138)+x(139)+x(140)+x(141)+x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+...
x(148)+x(149)+x(150)+x(151))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+...
x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+...
x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+...
x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190)+x(191)+x(192)+...
x(193)+x(194)+x(195)+x(196)+x(197)+x(198)+x(199)+x(200)+x(201)+x(202)+...
x(203))))^2+x(204)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+...
x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+...
x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+...
x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+...
x(139)+x(140)+x(141)+x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+x(148)+...
x(149)+x(150)+x(151)+x(152))-(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+...
x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+...
x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+...
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x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190)+x(191)+x(192)+...
x(193)+x(194)+x(195)+x(196)+x(197)+x(198)+x(199)+x(200)+x(201)+x(202)+...
x(203)+x(204))))^2+x(205)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+...
x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+...
x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+...
x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+...
x(138)+x(139)+x(140)+x(141)+x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+...
x(148)+x(149)+x(150)+x(151)+x(152)+x(153))-(x(157)+x(158)+x(159)+x(160)+...
x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+...
x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+...
x(181)+x(182)+x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190)+...
x(191)+x(192)+x(193)+x(194)+x(195)+x(196)+x(197)+x(198)+x(199)+x(200)+...
x(201)+x(202)+x(203)+x(204)+x(205))))^2+x(206)*(18.29+0.0376*(1.13*...
(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+...
x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+...
x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+...
x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+x(141)+x(142)+x(143)+x(144)+...
x(145)+x(146)+x(147)+x(148)+x(149)+x(150)+x(151)+x(152)+x(153)+x(154))-...
(x(157)+x(158)+x(159)+x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+...
x(166)+x(167)+x(168)+x(169)+x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+...
x(176)+x(177)+x(178)+x(179)+x(180)+x(181)+x(182)+x(183)+x(184)+x(185)+...
x(186)+x(187)+x(188)+x(189)+x(190)+x(191)+x(192)+x(193)+x(194)+x(195)+...
x(196)+x(197)+x(198)+x(199)+x(200)+x(201)+x(202)+x(203)+x(204)+x(205)+...
x(206))))^2+x(207)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+...
x(109)+x(110)+x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+...
x(119)+x(120)+x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+...
x(129)+x(130)+x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+...
x(139)+x(140)+x(141)+x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+x(148)+...
x(149)+x(150)+x(151)+x(152)+x(153)+x(154)+x(155))-(x(157)+x(158)+x(159)+...
x(160)+x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+...
x(170)+x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+...
x(180)+x(181)+x(182)+x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+...
x(190)+x(191)+x(192)+x(193)+x(194)+x(195)+x(196)+x(197)+x(198)+x(199)+...
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x(200)+x(201)+x(202)+x(203)+x(204)+x(205)+x(206)+x(207))))^2+...
x(208)*(18.29+0.0376*(1.13*(x(105)+x(106)+x(107)+x(108)+x(109)+x(110)+...
x(111)+x(112)+x(113)+x(114)+x(115)+x(116)+x(117)+x(118)+x(119)+x(120)+...
x(121)+x(122)+x(123)+x(124)+x(125)+x(126)+x(127)+x(128)+x(129)+x(130)+...
x(131)+x(132)+x(133)+x(134)+x(135)+x(136)+x(137)+x(138)+x(139)+x(140)+...
x(141)+x(142)+x(143)+x(144)+x(145)+x(146)+x(147)+x(148)+x(149)+x(150)+...
x(151)+x(152)+x(153)+x(154)+x(155)+x(156))-(x(157)+x(158)+x(159)+x(160)+...
x(161)+x(162)+x(163)+x(164)+x(165)+x(166)+x(167)+x(168)+x(169)+x(170)+...
x(171)+x(172)+x(173)+x(174)+x(175)+x(176)+x(177)+x(178)+x(179)+x(180)+...
x(181)+x(182)+x(183)+x(184)+x(185)+x(186)+x(187)+x(188)+x(189)+x(190)+...
x(191)+x(192)+x(193)+x(194)+x(195)+x(196)+x(197)+x(198)+x(199)+x(200)+...
x(201)+x(202)+x(203)+x(204)+x(205)+x(206)+x(207)+x(208))))^2))));
```