DISSERTATION

MCB 4044 FYP 2



AXIAL VIBRATION ANALYSIS OF DRILLSTRING

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MECHANICAL ENGINEERING

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Dissertation submitted in partial fulfillment of

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CERTIFICATION OF APPROVAL

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By

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A project dissertation submitted to the Chemical Engineering Programme Universiti Teknologi PETRONAS in partial fulfilment of the requirement for the BACHELOR OF ENGINEERING (Hons) (MECHANIICAL)

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UNIVERSITI TEKNOLOGI PETRONAS TRONOH, PERAK JANUARY 2015

CERTIFICATION OF ORIGINALITY

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

KEK JIA HOW

ABSTRACT

The problem of drill string vibration occurs in drilling operations and it will affect the usage life of drill pipe and drill bits. Three types of vibration, which are axial, lateral and torsional vibrations exist in the drill string. In order to being able to guide a drillstring in any direction, came the significant task of modelling the whole process mathematically and physically. This paper will study on the axial vibration in drill string by using finite element modelling analysis to observe the dynamic behavior of drill string under several conditions.

The structure of drill string mainly consists of drill pipe, which is 80% of the total drill string length, drill collars which are used to provide additional weight and stabilize the drill string, and drill bit which are used for drilling operation. Due to vibrations, are costly. Sometimes these parts can break unpredictably, or the bits may be dull while drilling. When this situation occurs, the whole assembly of drillstring has to be pulled out of the well, to replace the damaged parts in order to continue the drilling process. Axial vibration will increase the risk of early fatigue of components such as bit life reduction, pipe fatigue and failure in drill string.

In this project, finite element analysis method is used to find the modal and harmonic frequencies of drill string. Using real field data, a drill string model is designed using ANSYS simulation software. The case studies on drill string model include surface drilling, intermediate drilling, and slim hole drilling. Parametric studies of this project consists of three parts, which are the effect of drill pipe length on frequency of drill string, the effect of rotary speed on frequency of drill string, and the effect of Weight On Bit (WOB) on harmonic response of drill string. The simulation is expected to generate natural frequency of drill string under different depth and harmonic frequency of the drill string when subjected to axial force. Drill string dynamic behavior under influence of length, speed and force will be observed.

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Chapter 1 INTRODUCTION

1.1 Background

Oil and gas wells are predominantly drilled using rotary drilling. A rock-cutting tool driven from the surface, known as the drill bit, is used to create a borehole using rotary drilling system. The transfer of torque between torque generating unit and the drill bit is done by using a series of connected, hollow, thin-walled steel drill pipes, which is called drillstring. Drillstring is an important tool in the rotary drilling operation process and procedure. The drillstring extends through the water and carries drilling fluids from the drilling rig to the borehole. Each drill pipe is typically 27 to 32 feet long, and there is a threaded connection, known as tool joints at the end of the drill pipes for connecting two drill pipes. The typical tool joint is as shown in Figure 1-1.



Figure 1-1. Tool Joints. Reprinted from Oil & Gas Drilling Engineering. Retrieved January 16, 2015, from <u>http://www.oilngasdrilling.com/tag/tool-joint</u>

A torque generating unit or rotary unit, which consists of an electric motor and a gearbox unit, is used to drive the drillstring in a rotary fashion from the top end. The drillstring is sent at a prescribed rate through a rotating mass near the ground level. Through this system, a borehole is constructed in order to reach the reservoir of oil and

gas under the earth's surface. The borehole is usually lined with steel and the excess in the diameter of this cavity over the diameter of the drill pipes is called the over gauge. This annular gap, which varies along the length of the borehole, is necessary for fluid conduction.

A drilling operation can be categorized into several types, namely surface drilling, intermediate drilling and slim hole drilling. Surface drill rigs are used for blast hole drilling in construction, quarrying and open-pit mining. It is used in early stage of drilling operation until the drillstring reaches a pre-set depth. Intermediate drilling involves a casing string that is set in place after the surface casing and before the production casing. Intermediate drilling is used to isolate one or more zones of the openhole to enable deepening of the well. A slim hole means a small diameter boreholes drilled at the end of conventional wells. Typical applications for slim hole drilling are exploration wells in remote areas and reentry operations such as deepening or sidetracking in existing wells.

In oil and gas industry, vibration often occurs in drill string during the process of extracting the resources from well. This is a serious issue as it may cause huge loss in term of energy and finance. The vibrations of the drillstring are classified depending on the direction they appear. Drill string vibration consists of three types, which are lateral, axial and torsional vibrations (Hakimi & Moradi, 2009). Any of these forms of vibrations is likely to have a negative impact on overall drilling efficiency.

Drill strings are important in drilling for oil and gas. However, instability condition often occurs in drill string which may lead to excessive vibrations (Sangeetha & Kumar, n.d.). Hence, it is important to understand the mechanisms of vibration occurs in a drill string to achieve better work rate and performance. In this paper, the object to be studied is drill string, and different parametric studies will be done on the drill string model to obtain the natural frequency and harmonic response of the model. The impact of these variables onto the system are analyzed. In the study, the factors that affect the dynamic behavior are drill pipe length, drilling rotational speed and axial forces acting on the drill string.

1.2 Problem Statement

As the drilling technology develops, the factors that cause drill string vibrations can be investigated through more advanced methods. By field observation based on downhole and surface vibration measurements, drill string has been identified to have severe vibrations and the risk increases as the drilling depth increases. (Khulief, Al-Sulaiman, & Bashmal, 2006). The Bottom Hole Assembly (BHA) consists of stabilizers, drill collars and the drilling bit, which make up the lower part of the drill string. It was found out that the main factors of the vibrations are due to the contact of the bit with the formation and contact of the drill string (drill pipe, drill collars and stabilizers) with the borehole.

Of all the vibrations modes, axial vibration takes place more easily than others. Some of the effect axial vibrations can cause are bounce-bit, damage the bit cutters and bearings, and result in broken drill string. Besides, axial vibration can lead to other unwanted vibration modes too. As the drilling process is undergoing, coupled states of vibration could happen in the drill string, for example, axial-lateral, torsional-lateral and axial-torsional vibrations (Ghasemloonia, Rideout, & Butt, 2012). One of these is torsional vibration modes as it can cause movement of bit and breakage of cutters (Sangeetha & Kumar, n.d.). Once accidents happen, a huge amount of manpower and costs will be needed to fix them. The axial forces acting on a drillstring through the bit to rock interface have been studied by many researchers, and its causes and effects are further discussed in the literature review.

1.3 Objectives and Scope of Study

The main aim of this study is to explore and investigate the effects axial forces on a drillstring and approaches or alternatives are brought forward to reduce this phenomena.

The objectives of the project are as following:

- To analyze the modal and harmonic frequency of vertical oil well drillstring for surface drilling, intermediate drilling and slim hole drilling
- To observe the effect of Weight On Bit (WOB), drill string length and rotational speed on natural frequency and harmonic response of drillstring

The study will involve in identifying several parametric variables that affect the frequency of the drillstring. The drill string natural frequency and harmonic response frequency will be observed. These frequencies are important in understanding and controlling the vibration behavior in oil well drill string. The parametric will be varied in order to obtain results at different conditions. Vibration caused by resonance can be avoided if the oil well drill string operates at a frequency range away from the natural frequency.

Chapter 2

LITERATURE REVIEW AND THEORY

This chapter will discuss further in the background information on what has been done in the area of drilling, to better tie this work with the discipline of drilling. References are also given on different drill string models and on experimental results.

2.1 Drillstring Components

In oil and gas industry today, oil drilling and exploration become more advanced and feasible. This enables the expansion in deepwater drilling in recent years despite limitations in the past. The fundamental components of a rotary drilling assembly are standardized. Typical drillstring components are shown in Figure 2-1. At the top of the drillstring compromises of Kelly, the swivel and the mud pumping equipment. The function of kelly is to apply an upward force on the assembly and allows for connection of other drill collars as the well is being drilled. The Bottom Hole Assembly (BHA), consists of heavy weight pipes, stabilizers and the bit. The whole BHA is always in compression. The heavy weight drill pipe is used to increase the Weight on Bit (WOB) and stiffness of the assembly.

A drill string composes of four major components, which are the drill pipes, transition pipes, bottom hole assembly (BHA) and a bit. The BHA is a crucial part of drill string and composed of drill pipes, heavy-weight pipes, stabilizers, and the bit that is always in compression (Kotsonis, 1994). The bottom part of the BHA is the drill bit. There are two types of bits that are widely used, which are the tricone and the Polycrystalline Diamond Compact (PDC) bits. The tricone bit, or also known as Roller Cone bit, has lower rate of penetration, but is better for hard formations. For PDC, although it has higher rate of penetration in soft ground, but it can become dull quickly when operates in hard formation.



Figure 2-1. Components of Drillstring. (Rabia, 2002).

Collar section is one of the main parts of the drill string and is located at the bottom section. The collar section is centralized inside the wellbore and installed with stabilizers at several points over the BHA. One of the main functions of collars is to minimize the bit stability problems from vibrations and wobbling (Khulief et al., 2006). With proper configuration, the collar can be made to rise, drop or remain on a steady course to ensure successful directional drilling (Kotsonis, 1994).

2.2 Types of Drilling

There are three stages of drilling in a drilling operation, namely surface drilling, intermediate drilling and slim hole drilling. A typical drilling operation with different stages of casing can be seen in Figure 2-2.



Figure 2-2 An overview of drilling casing

For surface drilling, a large-diameter drill bit and relatively low pressure drillstring is set in shallow yet competent formations to predetermined depth. Surface drilling can reach several thousand feet in length. The surface casing provides a mounting place for the blowout preventer and serves as the support for the production casing that will be placed in the end of the well bore. As the drillstring reached predetermined depth, it will be removed from the well bore and cement will be pumped down the surface casing.

In some wells, in order to separate problem zones such as areas of high pressure or lost circulation, intermediate drilling is conducted. Intermediate drilling is carried out on longer drilling intervals where necessary drilling mud weight to prevent blowouts. Intermediate casing string is used to isolate one or more zones of the openhole to enable deepening of the well. This type of drilling is optional yet important so that the hydrostatic pressure of the drilling fluid remains at a pressure level that is between formation pore pressure and fracture pressure.

In slim hole drilling, the drilling operation has reaches the total depth. Production casing is installed inside the last casing string and the tubing annulus is sealed at the bottom of the tubing by a packer. At this depth, the diameter of the drill pipe used is relatively smaller as compared to intermediate drilling and surface drilling. Slim hole drilling has been fueled by oil companies seeking to exploit a number of potential advantages, predominantly cost savings. It is observed that significant savings can be made from a reduction in consumables such as rock bits, muds, cement and fuel oil.

2.3 Vibration of Drillstring and Its Impacts

Natural frequency of a system or an object is important for several reasons. By knowing the natural frequency of an object, one can know how it will vibrate and what kinds of waves it will create. In real world, waves are limited by the size of the system. Boundaries create conditions that favor special frequencies or wavelengths. There are many factors that influence the natural frequency, such as the length, weight and stiffness of the object. The natural frequency of the system can be altered by changing any of the factors that affect the size, inertia, or forces in the system.

Each and every physical substance vibrates at a natural frequency or rate of vibration. As drillstring consists of many components, each part has its own natural frequencies where vibration starts to happen. As drillstring consists of connected pipes in great length, vibration can be out of control when its resonance of harmonics is achieved. As the vibration occurs, energy is absorbed and the amplitude of string vibration is boosted leading to premature failure of tools and decrease lifespan of bit.

Drilling is one of the most costly and risky activities for exploration and development of oil fields. There are many new techniques being introduced for more efficient and energy saving drilling. However, vibrations of drill string remains one of the main causes of the deterioration in drilling performance. Strong vibration of drill string can cause drilling tool fatigue, as the pressure on the surface of the drill string is concentrated and form cracks in these parts (Han & Yan, 2009). Once the cracks continue to extend and left untreated, the drill string may break and cause accidents to take place. Other than that, vibration of the drill bit can affect the dimensional accuracy, roundness and surface quality of the wells (Ahmadi & Altintas, 2013). This situation will bring huge loss to the oil companies as the treatment for these accidents requires massive amount of manpower and cost.

Drillstring vibrations can be broken down into 3 modes, namely axial vibration, torsional vibration and lateral vibration, as illustrated in Figure 2-3. The axial forces that are considered acting on the drill string are weight on bit (WOB), drill string self-weight

and mud hydrostatic load (Sampaio, Piovan, & Lozano, 2007). The most significant of these are vibrations which include:

- a. Axial bit bounce
- b. Lateral whirl
- c. Torsional stick-slip
- d. Buckling or bending while rotating



Figure 2-3. Types of vibrations. Reprinted from Vibration Analysis of Drilling Operation. Retrieved January 17, 2015, from <u>https://www.academia.edu/4253436/VIBRATION_ANALYSIS_OF_DRILLING_OPERATION</u>

During the drilling process, the vibration instability due to feedback between subsequent cuts is known as self-excited or regenerative chatter (Ahmadi & Altintas, 2013).These vibrations are observed to be more severe as the drill pipe length increases. Hence, the occurrence of axial vibrations at the bottom of drillstring, usually BHA, will increase the risk of early fatigue of drillstring and its components such as bit life reduction, pipe fatigue and failure in drill string (Han & Yan, 2009). Besides, the production rate will be decreased once the drill string components such as Rotary Steerable (RSS) or Logging While Drilling (LWD) tools are damaged.

2.4 Main Factors Contributing to Vibration

There are many factors that can lead to vibration in drillstring. As drillstring is made up of complex components and mechanisms, each component has its own contribution to the drillstring vibration. The main factors that contributes to the vibration within drillstring are identified as below:

- i. WOB, Weight On Bit
- ii. q, buoyant weight per unit length of the drillstring
- iii. EI, bending stiffness of the drillstring
- iv. u, vibrating mass per unit length
- v. TOB, torque on bit
- vi. MW, mud weight
- vii. L, length of BHA, drill pipe, drill collars and other drillstring assembly
- viii. r, radial clearance between drillstring and wellbore
- ix. Es, rock (formation) compressive strength
- x. ρ a, rock density
- xi. Hs, Schmidt index

2.5 Equation of Motion of Drillstring

2.5.1 Elemental Equation of Motion

The Langrangean approach can be used to derive the equation of motion for drillstring. The Langragean function (L = T - U), is substituted into the following equation:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = Q \tag{1}$$

where

 $q = {\Omega e^T}^T$: generalized coordinates

Q = vector of generalized forces

T = total kinetic energy

U = total strain energy

By differentiating Equation 1, the element equations of motion can be determined as follows:

$$C_I \dot{\omega} = \{Q\} \tag{2}$$

$$[M]\{\ddot{e}\} + \dot{\omega} [G]\{\dot{e}\} + [K]\{e\} = \{Q\}$$
(3)

where

Cı	$=\frac{1}{2}\int_0^I I_p \ dx$
[M]	= augmented mass matrix
[G]	= gyroscopic matrix
[K]	= augmented stiffness matrix

As the rotational velocity, $\dot{\omega}$, of the drillstring is constant during drilling, Equation 2 will not be considered. Equation 3 represents the dynamic equation of motion at element level. The discretized element equations is the assembled to obtain the equation of motion of the whole drillstring.

2.5.2 The Assembled Equations of Motion

The equation of motion of the whole drillstring can be rewritten by using standard finite element assembly procedure:

$$[M]\{\ddot{e}\} + \dot{\omega} [G]\{\dot{e}\} + [K]\{e\} = \{Q\}$$
(4)

The solution of the general equation of motion is obtained by representing it in the following state space form:

$$\begin{bmatrix} \begin{bmatrix} 0 \\ - \begin{bmatrix} M \end{bmatrix} \\ \begin{bmatrix} G \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \ddot{e} \end{Bmatrix} + \begin{bmatrix} \begin{bmatrix} M \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ K \end{bmatrix} \begin{Bmatrix} \dot{e} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{Q\} \end{Bmatrix}$$
(5)

Or in more compact form:

$$[A]\{\dot{y}\} + [B]\{y\} = \{\bar{Q}\}$$
(6)

where

$$[A] = \begin{bmatrix} [0] & -[M] \\ [M] & [G] \end{bmatrix}$$
$$[B] = \begin{bmatrix} [M] & [0] \\ [0] & [K] \end{bmatrix}$$
$$\{\dot{y}\} = \begin{cases} \{\ddot{e}\} \\ \{\dot{e}\} \end{cases}$$
$$\{y\} = \begin{cases} \{\dot{e}\} \\ \{e\} \end{cases}$$
$$\{\bar{Q}\} = \begin{cases} \{0\} \\ \{Q\} \end{cases}, \text{ generalized force vector}$$

Assume free vibrations and ignore damping in the drillstring,

$$M\{\ddot{e}\} + K\{e\} = 0$$
(7)

Assume harmonic motion, ie. $U = U \sin (\omega t)$

$$[K] - \omega^2[M]\{e\} = 0 \tag{8}$$

The roots of this equation are ω_i^2 , the eigenvalues, where i ranges from 1 to number of DOF. Corresponding vector are {e}, the eigenvectors.

The square roots of eigenvalues are ω_i , the structure's natural circular frequencies (radians/sec). natural frequencies, f_i are then calculated as $f_i = \omega_i / 2\pi$ (cycles/sec). It is the natural frequencies f_i that are input by the user and output by ANSYS.

The eigenvectors $\{e_i\}$ represent the mode shapes, the shapes assumed by the structure when vibrating at frequency f_i .

2.6 Drillstring Vibration Studies

The axial vibration of drill string has to be studied in details to discover the effects it brings to the drill string. Drill string vibrations become a case to be studied since early sixties by Baily and Finnie (1960), who obtain the natural frequencies of the string by graphical approach (Hakimi & Moradi, 2009). Nowadays following the fast pace of technology development, the complex behavior can be observed by using simulation software.

Several dynamic formulation have been reported for investigating specific aspects of drillstring vibrational behavior and few of them have tackled axial vibrations. Christoforou and Yigit used a simple dynamic model to simulate the effects of varying operating conditions on stick-slip and bit bounce interactions. The equation of motion of such a system were developed by using a simplified lumped parameter model with only one compliance. The model did not account for the effect of higher nodes, the flow inside and outside the drill pipe and collars, or complicated cutting and friction conditions at the bit/formation interface.

Aarrested, Tonnesen, and Kyllingstad (1986) and Cook, Nicholson, Sheppard, and Westlake (1989) reported some of the first experimental results on drill-string vibrations. Kyllingstad and Halsey (1989) focused on the stick-slip phenomenon associated with torsion vibrations, which were modeled by using a single degree-of-freedom (DOF) system. Berlioz and Ferraris (1996) conducted experiments to examine the coupling between lateral and axial vibrations.

Khulief et al (2006) formulated a finite element dynamic model of the drillstirng including the drill pipes and drill collars. The model accounted for torsional-bending inertia coupling and the axial-bending geometric for the gyroscopic effect and the effect of gravitational force field. Complex modal transformations were applied and reducedorder models were obtained. The finite element formulation was then integrated into a computational scheme for calculating the natural frequencies of drillstring. The computational scheme was extended further to integrate the equations of motion, either in the full-order or the reduced-order form, to obtain the dynamic response. In the study, the simulation tool used was MATLAB. They did not considered hydrodynamic damping, due to drilling fluid circulation in the drill pipe and the annular space in their model. Stick-slip interaction was giving a coupling between axial and torsional vibration but did not have discussion about the complex effect of bit rotary speed and threshold force on torque on bit.

Cobern, (2007) detailed that the drill strings have relatively low resonance frequencies due to their huge lengths. They discussed the current drilling operations where the drill is operated below the resonant frequency. Optimal drilling cannot be achieved while drilling at this rate and the vibrations could be reduced also when the drill is operated above the resonant frequency.

Bailey, (2008) developed a BHA dynamic modeling tool and have designed the model such that for a particular operating range the critical modes causing resonance are avoided. The drillstring vibration and resonant frequencies are still a major research topic in the discussion of vibration minimizing procedures.

Jansen (1991) modeled the bottom hole assembly as an unbalanced rotor supported by two bearings. By using this model, effects of nonlinearities due to friction and gap between the outer shell and whirling motions of the drill string were studied. The work of Melakhessou, Berlioz, and Ferraris (2003), in which a four degree-of-freedom model is presented to study the bending and torsion motions of the drill string as well as the interactions with the outer shell, builds on the work of Jansen (1991).

Navarro-Lopez and Cortes (2007) have discussed that axial motions can lead to failures in a drill system. They focus on the drill-bit dynamics and study possible failures of the drill bit due to torsion, axial, and lateral oscillations. Self-excited stick-slip oscillation and sticking phenomena of the drill bit have also been studied in prior efforts (e.g., Richard, Germay, and Detournay, 2004). The results of Richard et al. (2004) indicate that by changing the weight on the bit (WOB) and the rotation speed of the drill string, undesired nonlinear oscillatory phenomena can be avoided.

In this paper, a finite element model is presented using ANSYS software to describe the axial vibration of drillstring in vertical oil wells. Dynamic behavior of drillstring is exhibited by modal and harmonic analyses to achieve mode shape and natural frequencies and frequency response respectively. It is assumed that the presence of drilling mud and friction between drillstring and borehole wall are neglected.

Chapter 3

METHODOLOGY

This section will discuss about the methods taken to ensure the completion of project within time frame and expectation.

3.1 Project Methodology

The procedure below briefly summarizes the proposed strategies involved in solving the problems:

Step1 Identify the problem of the vibrations in drillstring and the consequences of these vibrations

- Step 2 Conduct literature review on previous researches on topics related to the vibration analysis of drillstring in order
- Step 3 Develop a finite element model of drillstring dynamic using ANSYS to describe the axial behavior of a generic drillstring
- Step 4 Conduct parametric studies on the drillstring model based on real field data
- Step 5 Simulate the model using simulation software to obtain the results
- Step 6 Observe the results and make conclusions

3.2 Tools required

The project will be using ANSYS simulation software to generate results for modal frequency and harmonic frequency of drillstring. The simulation method is chosen over experimental data collection method due to the weighed advantages of the simulation method in contrast to the limitations imposed by experimental method. It is more feasible and effective in term of cost and time by using simulation method since any required amount of mass and energy can be provided in modeling. Experimental research method will require different equipment and manpower that are difficult to be obtained.

The software that will be used in this project is as follows:

ANSYS

ANSYS structural analysis software functions at solving complex structural engineering problems by using FEA (finite element) tools to simulate a model. ANSYS is suitable as it provides modal analysis that determines the vibration characteristics. For the study, a drillstring modal will be formulated and finite element analysis will be carried out on the modal to obtain the complex behavior and dynamic effects of the drill string. The model will be meshed and solved for several parametric variables. Modal analysis and harmonic analysis system are used to simulate the frequency result.

3.3 Project Flow Chart

A process flow chart for tasks to be done is constructed. The process flow chart is used to provide a framework to help accomplish the project. The flow chart for Final Year Project is as shown in Figure 3-1 below:



Figure 3-1 Project Flow Chart

3.4 Project Key Milestone

The project key milestone for FYP I and FYP II are as shown in Table 3-1 and Table 3-2 below:

Table 3-1. FYP 1 Project Key Milestone

	Final Year Project I														
No.	Item/ Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Project title														
	selection														
2	Extended														
	proposal														
	submission														
3	Proposal														
	defence														
4	Draft report														
	submission														
5	Final report														
	submission														

	Final Year Project II															
No.	Item/ Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Project work continues															
2	Submission of Progress Report															
3	Pre-SEDEX Poster Presentation															
4	Submission of Technical Paper															
5	Submission of Draft Dissertation (softbound)															
6	Oral Presentation															
7	Submission of Project Dissertation (Hardbound)															

Table 3-2. FYP 2 Project Key Milestone

3.5 Project Timeline (Gantt Chart)

The Gantt Chart for FYP I and FYP II are as shown in Table 3-3 and Table 3-4 below:

Table 3-3. FYP 1 Project Timeline

		Fina	l Yea	r Proje	ect I										
No.	Item/ Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Project title selection														
2	Background study and literature review on the														
	vibration analysis of drillstring														
3	Identify general issues of vibrations in														
	drillstring and its effects														
4	Extended proposal														
5	Formulate drill string model using ANSYS														
6	Derive equation of motions and generate														
	calculations using MatLab														
7	Proposal defence														
8	Analyse the results generated from simulation														
	on axial vibration of drill string														
9	Draw conclusion on the effect of axial														
	vibration analysis and suggest approaches														
10	Draft report														
11	Final report														

100000-4.117271000000000000000000000000000000000	Table	3-4.	FYP	2	Project	Timeline
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	Final Year Project I															
No.	Item/ Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Project work continues															
2	Submission of Progress Report															
3	Project work continues															
	Preparation for Poster Presentation															
4	Pre-SEDEX Poster Presentation															
5	Project work continues															
	Preparation for Technical Paper															
6	Submission of Technical Paper															
7	Submission of Draft Dissertation															
	(Softbound)															
8	Oral Presentation															
9	Submission of Project Dissertation															
	(Hardbound)															

3.6 Model Design

A conventional vertical drillstring model is designed in ANSYS workbench for simulation of modal analysis and harmonic response analysis. The drillstring model is simplified into two major parts, which are the drill pipe and the drill collar. These two components are being focused on as 90% of the drillstring is made up of drill pipe and drill collar.

The drillstring model is then used to be analysed for frequency results by using modal analysis and harmonic response analysis. Modal analysis is used to determine the natural frequency of the drillstring model under pre-stressed and rotation condition, while harmonic analysis can generate results of harmonic response of drillstring model when subjected to axial force. Figure 3-2 below shows an overview of the ANSYS workbench.



Figure 3-2. Overview of ANSYS workbench for modal and harmonic response analysis

The drillstring model designed in ANSYS workbench is as illustrated in Figure 3-3 and Figure 3-4 below:



Figure 3-3. Drillstring design in ANSYS



Figure 3-4. Drill pipe and tool joints section

Chapter 4 RESULTS AND DISCUSSION

4.1 A Dynamic Model of a Conventional Drillstring

A model describing all drillstring dynamical phenomena would be too complex for analysis purposes. Hence, simplifications are mandatory to allow more simple analysis and system simulation. A conventional drillstring consists of the bottom hole assembly and drill pipes connected end to end to each other to form a long pipe.

In designing the drillstring model in ANSYS software, the following assumptions are made:

- i. The borehole and the drillstring are both vertical and straight
- ii. No lateral bit motion is present
- iii. The friction in the pipe connections and between the pipes and the borehole are neglected
- iv. The drilling mud is neglected
- v. The drill pipes are considered to have same intertia
- vi. The motor dynamics is not considered, the drive torque is supposed to be constant and positive

Researches have been done to obtain the design specifications for a drillstring model. References from API standard for data and specifications of drillstring are collected. The common sized of drill pipes used in the drilling operation are shown in Appendix A. A common drill pipe has a length of 27 - 30 feet, and the range of lengths of drill pipe can be found in Appendix B. The material used for manufacturing drill pipes is also categorized into several grades, each with specific yield strength and tensile strength, which can be referred from Appendix C.

4.2 Parametric Studies

Three parametric studies have been done to identify the effect of these parameters on the natural and harmonic frequency response of drillstring. The three parameters are drillpipe length, rotational speed and the Weight On Bit (WOB) of drillstring.

The parameters that will be used in this project are obtained from real field data, which is provided by the author's co-supervisor, Dr. Tamiru Alemu Lemma. This is to ensure the effective study of the vibration analysis on drillstring. The analysis is applied on three case studies, which are surface drilling, intermediate drilling and slim hole drilling. All of the case studies have different drillpipe and drill collar length, as well as the Weight On Bit (WOB) applied on drillstring. These information are as shown in Table 4-1 below:

		Units	Surface	Intermediate	Slim hole
			drilling	drilling	drilling
70	Density (ρ)	kg/m ³		7850	
aterial perties	Modulus of elasticity (E)	N/m ²		210 x 10 ⁹	
Mf	Shear modulus (G)	N/m ²			
	Length of drill pipe (L _p)	Meter (m)	232	1940	3154
rties	Length of BHA (L _{BHA})	Meter (m)	224	173	352
Prope	Drill pipe internal diameter (D _{pi})	Meter (m)	0.10861	0.10861	0.10861
etrical	Drill pipe outer diameter (D _{po})	Meter (m)	0.127	0.127	0.127
Geom	Drill collar internal diameter (D _{ci})	Meter (m)	0.07325	0.07325	0.0762
	Drill collar outer diameter (D _{co})	Meter (m)	0.127	0.127	0.16764
	Weight on Bit (WOB)	Newton (N)	20000	35000	70000

Table 4-1. Case Study Data

4.3 Modal Analysis Results

In this section, the modal characteristics of the drillstring system are examined. Modal analysis is conducted to specify the natural modes and frequency of the drillstring. the free vibrational equations of the system are solved in the state-space form to extract the complex eigenvalues and eigenvectors of the system. The free vibrational conditions can be achieved by setting the forcing vector to zero in Equation 5. In modal analysis, [A] and [B] will retrieve their symmetric properties since the nonlinear coupling will vanish.

If external excitations exert at such frequencies, the resonance happens and the amplitude of axial vibration increases exceedingly, causing excessive vibrations and promote to failure of drillstring. To prevent happening of this destructive phenomenon, the frequency of external loads must be far from natural frequencies.

In modal analysis using ANSYS, case study one, two and three are being varied with two parameters, which are drillpipe length and rotary speed. The drill collar length is constant at all time, and drillpipe of different depth is manipulated to obtain the natural frequency. As for the rotary speed, each of the case study is tested with four speed range, which are 40rpm, 80rpm, 160rpm and 200rpm. The results obtained are inserted into tables and plotted into graphs as below.

Parameter 1: Drillpipe Length

Surface Drilling

For surface drilling, the total length of drillpipe excluding the drill collar is 232m. The drillpipe length is set at $L_p = 12m$, 36m, 60m, 120m, 180m, and 232m for modal analysis. Each setting produce 5 lowest natural frequency mode of drillstring at that particular point of the total length. The simulation result for frequency of drillstring for surface drilling is as shown in Table 4-2 below:

Mode	Frequency												
	$L_p = 12m$	$L_p = 36m$	$L_p = 60m$	$L_p = 120m$	$L_p = 180m$	$L_p = 232m$							
1	0.00211	0.00174	0.00146	0.00100	0.00072	0.00057							
2	0.01326	0.01093	0.00916	0.00625	0.00453	0.00355							
3	0.03715	0.03061	0.02565	0.01749	0.01268	0.00995							
4	0.07280	0.05999	0.05028	0.03427	0.02485	0.01950							
5	0.12035	0.09920	0.08314	0.05667	0.04109	0.03225							

Table 4-2. Surface Drilling: Frequency results of drillstring at points of length



Figure 4-1 Surface Drilling: Graph of Frequency vs Drillpipe Length

The results obtained are plotted into graph of natural frequency versus drillpipe length as shown in Figure 4-1. It can be seen that for each mode, as the drillpipe length increases, the natural frequency of drillstring decreases. The decreased value also becomes more significant from mode 1 to mode 5. The lowest natural frequency for each mode is at the greatest length of the drillstring, which is 232m in this case.

Intermediate Drilling

For intermediate drilling, the total length increases significantly from surface drilling, which is 2113m including 173m drill collars. Hence, the drillpipe length is set at $L_p = 300m$, 500m, 800m, 1300m, 1600m, and 1940m for modal analysis. Table 4-3 shows the natural frequency mode of drillstring at several point of interest of intermediate drilling.

Mode		Frequency					
	$L_p = 300m$	$L_p = 500m$	$L_{p} = 800m$	$L_p = 1300m$	$L_p = 1600m$	$L_p = 1940m$	
1	0.00053	0.00026	0.00012	0.00005	0.00004	0.00003	
2	0.00330	0.00163	0.00078	0.00034	0.00024	0.00017	
3	0.00925	0.00457	0.00219	0.00095	0.00066	0.00046	
4	0.01813	0.00895	0.00428	0.00187	0.00129	0.00091	
5	0.02997	0.01481	0.00708	0.00309	0.00213	0.00150	

Table 4-3.Intermediate Drilling: Frequency results of drillstring at points of length



Figure 4-2 Intermediate Drilling:Graph of Frequency vs Drillpipe Length

Compared to surface drilling, the natural frequency of intermediate drilling case is lower overall as observed in Figure 4-2. As the depth increases, the gap of frequency between each mode decreases. The most significant point can be found at length 1940m, where all the natural frequencies are close and has to be avoided with external loads.

Slim Hole Drilling

Similar to previous two case studies, slim hole drilling is analyzed with six different points of drillpipe length. The total drillpipe length is 3154m, hence the length is set at $L_p = 400m$, 800m, 1500m, 2000m, 2500m and 3154 m for modal analysis. The results for natural frequency for five lowest mode are as shown in Table 4-4 below:

Tuble 4 4. Smith hole Drining. Trequency results of utilisting at points of leng	Table 4-4. Slim Hole	Drilling: Fre	equency resu	Its of drillstring	at points	s of length
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Mode	Frequency						
	$L_p = 400m$	$L_p = 800m$	$L_p = 1500m$	$L_p = 2000m$	$L_p = 2500m$	$L_p = 3154m$	
1	0.00053	0.00026	0.00012	0.00005	0.00004	0.00003	
2	0.00330	0.00163	0.00078	0.00034	0.00024	0.00017	
3	0.00925	0.00457	0.00219	0.00095	0.00066	0.00046	
4	0.01813	0.00895	0.00428	0.00187	0.00129	0.00091	
5	0.02997	0.01481	0.00708	0.00309	0.00213	0.00150	



Figure 4-3 Slim Hole Drilling: Graph of Frequency vs Drillpipe Length

Referring to Figure 4-3, in slim hole drilling, due to the extensive length of drillpipe, the frequency at the bottom of 3154m is very low. The first natural frequency at this point of length is 0.00001 Hz, which can be excited easily and lead to excessive vibration if forced frequency is not monitored carefully.

From the three case study with different drillpipe length as point of interest, the analysis results show that as the drillpipe length increases, the natural frequency of the drillstring decreases. Hence, it can be deduced that the natural frequency of drillstring becomes lower as the length of the drillstring increases.

Parameter 2: Rotary Speed

The optimum rotary speed of the drillstring is between 20rpm to 200rpm. In this parametric study, four different values of rotational speed have been applied to the drillstring, which are 40rpm, 80rpm, 160rpm and 200rpm to observe the results of frequency under rotation.

The drillstring model is simulated with different rotational speed and the first 5 modes of lowest frequencies are obtained and tabulated as below:

Surface Drilling

Table 4-5 below shows the five lowest mode of natural frequency for surface drilling when subjected to different rotary speed.

Modes	1	2	3	4	5
f@40rpm (Hz)	0.00174	0.01090	0.03060	0.06000	0.09920
f@80rpm (Hz)	0.00174	0.01090	0.03060	0.06000	0.09920
f@160rpm (Hz)	0.00174	0.01090	0.03060	0.06000	0.09910
f@200rpm (Hz)	0.00174	0.01090	0.03060	0.06000	0.09910

Table 4-5. Surface Drilling: Frequency vs Rotational Speed

Intermediate Drilling

Table 4-6 below shows the five lowest mode of natural frequency for intermediate drilling when subjected to different rotary speed.

Table 4-6. Intermediate Drilling: Frequency vs Rotational Speed

Modes	1	2	3	4	5
f@40rpm (Hz)	3.75E-05	2.35E-04	6.58E-04	1.29E-03	2.13E-03
f@80rpm (Hz)	3.75E-05	2.35E-04	6.58E-04	1.29E-03	2.13E-03

f@160rpm (Hz)	3.75E-05	2.35E-04	6.58E-04	1.29E-03	2.13E-03
f@200rpm (Hz)	3.75E-05	2.35E-04	6.58E-04	1.29E-03	2.13E-03

Slim Hole Drilling

Table 4-7 below shows the five lowest mode of natural frequency for slim hole drilling when subjected to different rotary speed.

Modes	1	2	3	4	5
f@40rpm (Hz)	1.45E-05	9.09E-05	2.54E-04	4.99E-04	8.24E-04
f@80rpm (Hz)	1.45E-05	9.08E-05	2.54E-04	4.99E-04	8.24E-04
f@160rpm (Hz)	1.45E-05	9.08E-05	2.54E-04	4.98E-04	8.24E-04
f@200rpm (Hz)	1.45E-05	9.08E-05	2.54E-04	4.98E-04	8.24E-04

Table 4-7. Slim Hole Drilling: Frequency vs Rotational Speed

From the results tabulated, it can be observed that the rotational speed does not affect the frequency of the drillstring. Unlike drillpipe length, the frequency does not change as the rotational speed increases for each mode. For example, the first mode frequency is same at all rotational speed. Same pattern can be observed for the rest of the modes. This is due to the axial frequency is not affected by the rotation of drillstring, as it is not perpendicular exerted on the vertical drillstring.

4.4 Harmonic Analysis Results

Harmonic analysis is performed to achieve the frequency response of the drillstring. The result of this analysis is depicted for the displacement versus frequency of the axial load tolerated at the first node in place of rotary table as the excitation.

For each case study, different axial force were set to be acting perpendicular to the bottom of drillstring, which was the drill bit. The graph shown consists of two results, which are the harmonic response of drillstring without and with the force acting on it.

Surface Drilling

For surface drilling, the axial force acting on the drillstring model was set to be 20 kN. The harmonic response result is as shown in Figure 4-4.



Figure 4-4. Surface Drilling: Graph of Harmonic Response

Intermediate Drilling

For intermediate drilling, the axial force was 35 kN. Figure 4-5 showed the result of harminc response of intermedite drilling simulation.



Figure 4-5. Intermediate Drilling: Graph of Harmonic Response

Slim Hole Drilling

For slim hole drilling, the axial force was set to be 70 kN. The graph of harmonic response can be barved in Figure 4-6.



Figure 4-6. Slim Hole Drilling: Graph of Harmonic Response

By observing the graph patterns of harmonic response results, the frequency response of drillstring is higher when axial force is exerted. As the drillstring model has a compression force acting on it at drill bit, the amplitude of drillstring becomes higher than without Weight On Bit (prestressed). However, for both cases, the frequencies are still relatively low due to the great length of drillstring. These frequencies have to be avoided in order to prevent excessive vibrations due to resonance.

Chapter 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

In the end of the project, a drill string model accounts for analysis of vibrations of drillstring has been studied through finite element modelling. It is feasible that axial vibration to be analysed by using ANSYS. A drillstring model is developed and analysed to generate results of the drillstring dynamic behavior under the influence of several factors, which are drill pipe length, rotational speed and Weight On Bit. The axial vibration of drillstring is a pervasive vibration for drilling and it takes place easily. Hence, the vibration within the drillstring must be prejudged before drilling to achieve optimum drilling operation.

5.2 Recommendation

Some suggestions have been provided in this section for further studies in the following areas:

- i. Model selection
- ii. Nonlinear phenomena
- iii. Control schemes

5.2.1 Distributed parameter models with experiments

Reduced-order models focusing on the vertical drillstring dynamics have been studied in this work. In order to include more features such as initial curvature, a distributed parameter model is needed. Non-dimensional analysis could also be considered to make the predictions broader applicability. Experiments focused on horizontal drilling, along with a corresponding distributed parameter model, can be a bridge between the current work and studies with full size drillstring system.

5.2.2 More than two degree of freedom model

The continuation of work with the two degree of freedom model through numerical simulations and experiments can form another path to study the system. The additional degree of freedom, tile angle, can allow the linking of the model to a continuous model for a better description of the whole drillstring system.

5.2.3 Nonlinear phenomena

If feasible, nonlinear analysis can be conducted along the lines. Attention also need to be paid to possible nonlinear coupling between axial vibrations and torsional vibrations and other modes of vibrations.

5.2.4 Control scheme

Different control schemes can be studied with the experimental apparatus to determine their effectiveness. These schemes can include those that have been previously studied in the active control area and previous efforts related to drill stings.

Chapter 6

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APPENDICES

APPENDIX A

Common sizes of drill pipes

<u>Size(OD)</u> (inches)	$\frac{Weight}{(lb/ft)}$	ID (inches)
23/8	6.65	1.815
27/8	10.40	2.151
31/2	9.50	2.992
31/2	13.30	2.764
5	15.50	4.602
5	16.25	4.408
5	19.50	4.276
5 ¹ / ₂	25.60	4.000
5 ¹ / ₂	21.90	4.776
$5^{1}/_{2}$	24.70	4.670

APPENDIX B

Ranges of drill pipes

API Range	Length (ft)
Range 1	18-22
Range 2	27-30
Range 3	38-45

APPENDIX C

Grades of drill pipe

API Grade	Minimum Yield Stress (psi)	Minimum Tensile Stress (psi)	<u>Yield Stress</u> ratio Tensile Stress
D	55,000	95,000	0.58
Е	75,000	100,000	0.75
Х	95,000	105,000	0.70
G	105,000	115,000	0.91
S	135,000	145,000	0.93

APPENDIX D

Axial vibration On Drillstring

Free longitudinal vibrations exist in the drill which are produced due to selfweight of the drill. These vibrations can be analyzed with the help of the following formula:

$$\omega_n = \sqrt{\frac{k_{retaining spring}}{m}} rad/sec$$
(9)

$$\omega_n = \frac{1}{2\pi} \sqrt{\frac{k_{retaining spring}}{m}} Hz \tag{10}$$

The axial or longitudinal vibrations of a drillstring can also be characterized by one-dimensional wave equation. By referring to Besaisow and Payne (1988), the axial wave equation is developed from elemental-force-balance considerations,

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{f_l} \frac{\partial^2 u}{\partial t^2} \tag{11}$$

Where the longitudinal wave speed, $f_1 = \sqrt{E/\rho} = 5136$ m/s for steel. u is the axial dynamic displacement, and z is the displacement along the drillstring.

Further, t is the time, E is the modulus of elasticity, and ρ is the density of steel. When the continuity conditions are applied, it can be seen that the axial wave amplitude is inversely proportional to the cross-sectional area. The axial natural frequency, f_n, can be calculated for simple drillstring by assuming a fixed-at-top, free-at-bottom boundary condition, specifically it is found that

$$f_n = \frac{4214}{L} (2n - 1) \tag{12}$$

where $n = 1, 2...\infty$.

Forced axial vibration exists when the drillstring touches the rock formation and starts drilling to produce the hole. The upward resistive force acts on the drill because of the tensile stress of the rock formation. The force exerted in the upward axial direction during the drilling can be calculated using the following formula:

$$F = \sigma_{rock\ formation} * \frac{\pi}{4} * d^2 \tag{13}$$

The amplitude of the forced axial vibration can be computed using the formula given below:

$$\frac{A}{B} = \frac{\sqrt{1 + (2\xi\frac{\omega}{\omega_n})^2}}{\sqrt{(1 - \frac{\omega}{\omega_n}^2)^2 + (2\xi\frac{\omega}{\omega_n})^2}}$$
(14)