UNIVERSITI TEKNOLOGI PETRONAS

FINAL EXAMINATION
SEPTEMBER 2016 SEMESTER

COURSE : TDB1013 – DISCRETE MATHEMATICS

DATE : 29 DECEMBER 2016 (THURSDAY)

TIME : 9.00 AM – 12.00 NOON (3 HOURS)

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions from the Question Booklet.
2. Begin **EACH** answer on a new page in the Answer booklet given.
3. Indicate clearly answers that are cancelled, if any.
4. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions, if any.
5. Do not open this Question Booklet until instructed.

Note : This Question Booklet contains **NINE (9)** pages including the cover page and **APPENDIX I**.

1. a. Let ***p:*** Iqbal is nimble,
q: Muhammad is fast,
r: Ameira jumped over the fence.

Translate the statement "*Iqbal is nimble and Muhammad is fast, or Ameira did not jump over the fence*", into logical expressions using predicates and logical connectives.

[2 marks]

b. Using the three propositions stated in **part(a)**, write an English sentence that corresponds to each of the following logical expressions:

i. $(p \wedge q) \leftrightarrow \sim r.$

[2 marks]

ii. $(r \vee q) \leftrightarrow p.$

[2 marks]

c. If the logical expression $p \rightarrow q$ is true, determine the corresponding truth value for the expression $(p \vee q) \rightarrow \sim q$. Justify your answer.

[4 marks]

d. Given two propositions $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \vee \neg q)$, determine whether they are logically equivalent.

[5 marks]

e. Prove the first absorption law of sets using a membership table by showing that if A and B are sets, then $A \cup (A \cap B) = A$.

[5 marks]

2. a. The *big-O* notation is used to estimate the growth of functions and the number of operations required as the input size grows. Show that each of these functions is $O(x^2)$.

i. $f(x) = x^2 + 1000$.

[3 marks]

ii. $f(x) = x \log x$.

[3 marks]

b. Let $f(x) = (7x + 2)^2$.

i. Determine the *big-O* function.

[3 marks]

ii. Determine the *big-omega* function.

[3 marks]

iii. Determine the *big-theta* function.

[3 marks]

iv. Using graphical representation, clearly illustrate the relationship between $f(x)$, *big-O, big-omega* and *big-theta* functions.

[5 marks]

3. a. Define inductive hypothesis in mathematical induction.

[2 marks]

b. The train stations provided by Keretapi Tanah Melayu Berhad (KTMB) from Ipoh to Kuala Lumpur include Kampar, Tapah, Sungkai, Slim River, Rawang, Sg Buluh and KL Sentral. Suppose that the KTMB train stops at Kampar, then it stops at the next station of Tapah. Show using mathematical induction that the train stops at all stations.

[4 marks]

c. Mathematical induction is used to prove statements that assert $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function. Let $P(n)$ be the statement:

$$\frac{1}{(1)(4)}+\frac{1}{(4)(7)}+\cdots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$$

Prove that the statement $P(n)$ is true, whenever n is a positive integer.

[8 marks]

d. Use mathematical induction to prove that 43 divides $(6^{n+1}+7^{2n-1})$ for every positive integer n.

[6 marks]

4. a. The High Performance Computing Cloud Center (HPC^3) has a vacancy of 10 interns under a research attachment program. However 25 students have applied. Determine the number of ways to select a team of 10 interns given that all students fulfill the minimum CGPA criteria.

[3 marks]

b. Company X conducted an inquiry into their phone subsidiary section with respect to their latest brand Z flagship series. Findings concluded that 30% of the phone buyers initially had a charging problem. Furthermore, when investigations were done on the lithium-ion batteries, 92% of the buyers who initially got charging problems later their phone exploded. 40% of the buyers initially had no charging problems but their phone also exploded. Find the probability that;

i. the phone will explode given that the buyer initially had a charging problem.

[2 marks]

ii. the phone will not explode given that the buyer initially had a charging problem.

[2 marks]

iii. the phone will explode given that the buyer had no charging problem.

[2 marks]

iv. the phone will not explode given that the buyer had no charging problem.

[2 marks]

c. Suppose that B and D are events in a sample space, given $P(B) = \frac{1}{3}$, $P(D) = \frac{1}{2}$, and $P(B \mid D) = \frac{2}{5}$. Find the probability of $P(D \mid B)$.

[3 marks]

d. Given two matrices, A and B.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

i. Find the join of A and B.

[3 marks]

ii. Find the Boolean product of A and B.

[3 marks]

5. a. Daily airline routes for ASEAN countries are shown in **TABLE Q5**.

TABLE Q5

Departing Airport	Arrival Airport	No of Flights
Kuala Lumpur	Hanoi	4
Hanoi	Kuala Lumpur	2
Hanoi	Jakarta	3
Jakarta	Hanoi	2
Hanoi	Singapore	1
Singapore	Hanoi	2
Hanoi	Bangkok	3
Bangkok	Hanoi	2
Bangkok	Jakarta	1

Draw graphs to represent daily airline routes for ASEAN countries where there is an edge,

i. between vertices representing cities that have a flight between them (in either direction).

[3 marks]

ii. between vertices representing cities for each flight that operates between them (in either direction).

[3 marks]

iii. between vertices representing cities for each flight that operates between them (in either direction), plus a loop for a special sightseeing trip that takes off and lands in Jakarta.

[2 marks]

iv. from a vertex representing a city where a flight starts to the vertex representing the city where it ends.

[3 marks]

b. For the directed multigraph in **FIGURE Q5**, determine the number of vertices, edges and find the in-degree and out-degree of each vertex.

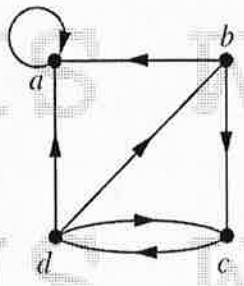

FIGURE Q5

[5 marks]

c. Draw an undirected graph represented by the given adjacency matrix.

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

[4 marks]

- END OF PAPER -

APPENDIX I

Formulae	
Permutation	$P(n,r) = n(n-1)(n-2)\ldots.(n-r+1)$
Combination	$C(n,r) = \frac{n!}{r!\,(n-r)!} = C(n,n-r)$
Bayes' Theorem	$P(A\,\|B) = \frac{P\big((B\|A\big).\ P(A)}{P\big((B\|A\big).\ P(A)P\big((B\|\bar{A}\big).\ P(\bar{A})}$