# DESIGN AND ANALYSIS OF CONTROLLER FOR A COUPLED TANK SYSTEM VIA STATE SPACE APPROACH

By

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DISSERTATION

Submitted to the Electrical & Electronics Engineering Programme

in Partial Fulfillment of the Requirements

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(Electrical & Electronics Engineering)

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# CERTIFICATION OF APPROVAL

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Approved:

______________________

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Project Supervisor

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

December 2010

# CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

Yoganathan Gonasagran

# ABSTRACT

This paper describes the designing, simulation and analysis of controller for a coupled tank system via state-space approach. Many industries uses the conventional control system approach, as opposed to the modern control approach commonly used in aerospace industries. Conventional control posses several drawbacks, for example PID controllers are not adaptive and not robust. Thus qualities such as robustness, optimality and adaptivity could have been overlooked. This project is looking at modern control approach for plant control which is expected to be better in terms of the system's controllability and stability. The entire project involves understanding process control and state space, as well as understanding the concept of system identification and mastering the function of MATLAB and Simulink for controller and observer design and simulation. The system is modeled mathematically using physics law such as the Bernoulli's law. Then the model is simulated in MATLAB using script file and in Simulink with Control System tool box. Further studies were done to improvise the controller performance. The preliminary results shows that the model developed is controllable and observable.

# ACKNOWLEDGEMENTS

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# LIST OF ABBREVIATIONS

| | |
|---|---|
| **DCS** | Distributed Control System |
| **GUI** | Graphical User Interface |
| **SCADA** | Supervisory Control and Data Acquisition |
| **PID** | Proportional–Integral–Derivative |
| **PLC** | Programmable Logic Controller |
| **ODE** | Ordinary Differential Equation |
| **CO2** | Carbon Dioxide |
| **AISB** | Augmented Innovation Sdn. Bhd. |
| **LTI** | Linear Time Invariant |
| **UTP** | Universiti Teknologi Petronas |
| **UTM** | Universiti Teknologi Malaysia |

# CHAPTER 1

# INTRODUCTION

This chapter introduces and explains the project topic, "Design and Analysis of Controller for a Coupled Tank System via State-Space Approach". A background study on this topic is highlighted followed by problem statement, objectives, and finally the scope of the study.

## 1.1 Background of Study

Nowadays, the process industries such as petro-chemical industries, paper making and water treatment industries require liquids to be pumped, stored in tanks, and then flowed to another tank. The control of liquid in tanks and flow between tanks is a basic problem in the process industries. The above mentioned industries are the vital industries where liquid level is essential. Many times the liquids will be processed by chemical or mixing treatment in the tanks, but always the level fluid in the tanks must be controlled and regulated. Level control in tanks is the heart of all chemical engineering systems. All these can be done with the help of the controller.

This project aims to apply the concepts in modern control to a coupled-tank system control. This will involves the understanding of the system dynamics, the equations associated to it, analysis of its controllability and observability, which leads to the controller and observer design. Namely, the coupled tank system has wide applications in many commercial and industrial sectors. The coupled tank system has two vertical tanks joined with an orifice and has inlet liquid pumps and discharge valves. Presently, different control strategies such as PID type controllers, fuzzy logic based methods, and genetic algorithms based performance improvement methods have all been investigated for liquid level control of coupled tank systems.

The first and foremost important step before formulating a controller, a mathematical relationship or the governing dynamics between the input and the output of the system should be known. The underlying principle and knowledge of the system should be investigated to comprehend the occurrence of nonlinearity in the system dynamics.

## 1.2 Problem Statement

In process control, precise liquid level control of storage tanks and reaction vessels is essential in many industrial operations and mainly in chemical engineering systems where the liquids are pumped to the tanks, stored and flowed through the coupled tanks. The liquids will be mostly processed by chemical or mixing treatment in the tanks, and the level fluid in the tanks must be controlled and regulated continuously in order to achieve the desired output and fulfilling the control objectives such as product quality. Level control in tanks is the heart of all chemical engineering systems. The control of liquid in tanks and flow between tanks is a basic problem in the chemical industries. The conventional control approaches are widely used for controlling these processes. Conventional control posses several drawbacks, for example PID controllers are not adaptive and not robust. This project is looking at modern control approach for plant control which is expected to be better in terms of the system's controllability and stability.

## 1.3 Objectives

The main objective of this project is to design and analyze a controller for coupled tank system via state space approach. In doing so, the following objectives below are set:

- To model the control system of a coupled tank system in second-order state space representation.
- To design a controller and observer strategy that is robust, optimal and adaptive.
- To theoretically apply the concepts of modern control engineering in the coupled tank system.

## 1.4 Scope of Study

The scope of this project is on the coupled tank system solely. Therefore, this project requires detailed study of the coupled-tank principle. Besides the following should be also achieved:

- Modelling a coupled tank system using MATLAB and Simulink.
- Designing and implementing observer and controller strategies for plant processes.

# CHAPTER 2

# LITERATURE REVIEW

The overall theory and critical points of the project are reviewed in this part.

## 2.1 Modern control theory

In contrast to the frequency domain analysis of the classical control theory, modern control theory utilizes the time-domain state space representation, a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors and the differential and algebraic equations are written in matrix form (the latter only being possible when the dynamical sstem is linear).

### 2.1.1 State Space

The state space representation (also known as the "time-domain approach") provides a convenient and compact way to model and analyze systems with multiple inputs and outputs. With inputs and outputs, we would otherwise have to write down Laplace transforms to encode all the information about a system. Unlike the frequency domain approach, the use of the state space representation is not limited to systems with linear components and zero initial conditions. "State space" refers to the space whose axes are the state variables. The state of the system can be represented as a vector within that space.

State space modeling, is just another way of presenting differential equations describing a dynamic system. It uses a set of $1^{st}$ order differential equations. This modeling method appear novel to practitioners who are accustomed to thinking in terms of frequency response function or transfer function but it is not a new way of looking at dynamic systems. Physicists and control engineers have been using this modeling technique for years.

State space modeling enables noise and vibration engineers to have access to and put to use a wealth of knowledge and analysis technique from the linear system discipline, including designing estimators and controller for single-input-single-output system.

### 2.1.2 State Space Advantage

There are many advantages to modeling systems in state space. The most important advantage is the matrix formulation. Computers can easily manipulate matrices. Having the A, B, C, and D matrices, one can calculate stability, controllability, observability and many other useful attributes of a system. The second most important aspect of state space modeling is that it allows us to model the internal dynamics of the system, as well as the overall input/output relationship as in transfer function. And, as stated earlier, state space modeling makes the vast, existing, linear system knowledge such as estimation and optimal control theory available to the user.

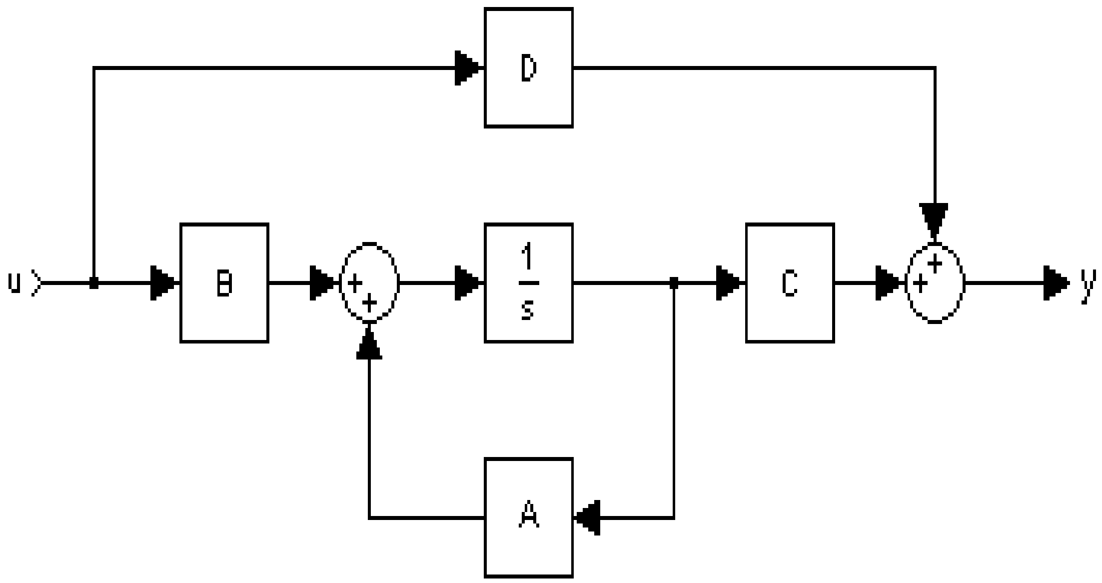


**Figure 2** Block diagram representation of the state space equations

## 2.2 Process Control

Process control is a statistics and engineering discipline that deals with architectures, mechanisms, and algorithms for controlling the output of a specific process..

For example, heating up the temperature in a room is a process that has the specific, desired outcome to reach and maintain a defined temperature (e.g. 20°C), kept constant over time. Here, the temperature is the controlled variable. At the same time, it is the input variable since it is measured by a thermometer and used to decide whether to heat or not to heat. The desired temperature (20°C) is the set point. The state of the heater (e.g. the setting of the valve allowing hot water to flow through it) is called the manipulated variable since it is subject to control actions.

A commonly used control device called a programmable logic controller, or a PLC is used to read a set of digital and analog inputs, apply a set of logic statements, and generate a set of analog and digital outputs. Using the example in the previous paragraph, the room temperature would be an input to the PLC. The logical statements would compare the set point to the input temperature and determine whether more or less heating was necessary to keep the temperature constant. A PLC output would then either open or close the hot water valve, an incremental amount, depending on whether more or less hot water was needed. Larger more complex systems can be controlled by a Distributed Control System (DCS) or SCADA system.

In practice, process control systems can be characterized as one or more of the following forms:

- Discrete – Found in many manufacturing, motion and packaging applications. Robotic assembly, such as that found in automotive production, can be characterized as discrete process control. Most discrete manufacturing involves the production of discrete pieces of product, such as metal stamping.

- Batch – Some applications require that specific quantities of raw materials be combined in specific ways for particular durations to produce an intermediate or end result. One example is the production of adhesives and glues, which normally require the mixing of raw materials in a heated vessel for a period of time to form a quantity of end product. Other important examples are the production of food, beverages and medicine. Batch processes are generally used to produce a relatively low to intermediate quantity of product per year (a few pounds to millions of pounds).

- Continuous – Often, a physical system is represented through variables that are smooth and uninterrupted in time. The control of the water temperature in a heating jacket, for example, is an example of continuous process control. Some important continuous processes are the production of fuels, chemicals and plastics. Continuous processes in manufacturing are used to produce very large quantities of product per year (millions to billions of pounds).

Applications having elements of discrete, batch and continuous process control are often called hybrid applications.

## 2.3 Terminology and Definitions

In controlling a process there exist two types of classes of variables.

1. **Input Variable** – This variable shows the effect of the surroundings on the process. It normally refers to those factors that influence the process. An example of this would be the flow rate of the steam through a heat exchanger that would change the amount of energy put into the process. There are effects of the surrounding that are controllable and some that are not. These are broken down into two types of inputs.

a. Manipulated inputs*:* variable in the surroundings can be control by an operator or the control system in place.

b. Disturbances*:* inputs that cannot be controlled by an operator or control system. There exist both measurable and immeasurable disturbances.

2. **Output variable -** Also known as the control variable. These are the variables that are process outputs that affect the surroundings. An example of this would be the amount of CO2 gas that comes out of a combustion reaction. These variables may or may not be measured.

As considering a controls problem, two major control structures can be looked on.

1. Single input-Single Output (SISO) - for one control(output) variable there exist one manipulate (input) variable that is used to affect the process

2. Multiple input-multiple output (MIMO) - There are several control (output) variable that are affected by several manipulated (input) variables used in a given process.

## 2.4 MATLAB

MATLAB (short for MATrix LABoratory) is a special-purpose computer program optimized to perform engineering and scientific calculation. It started life as a program designed to perform matrix mathematics, but over the years it has grown into a flexible computing system capable of solving essentially any technical problem.

The MATLAB program implements the MATLAB programming language, and provides an extensive library of predefined functions that make technical programming task easier and more efficient.

MATLAB is huge program, with an incredibly rich variety of functions. Even the basic version of MATLAB without any toolkits is much richer than other technical programming languages. There are more than 1000 functions in the basic MATLAB product alone, and the toolkits extend this capability with many more functions in various specialties.

## 2.5 SIMULINK

SIMULINK is a software package for modeling, simulating, and analyzing dynamic systems. It supports linear and nonlinear systems, modeled in continuous time, sampled time, or a hybrid of the two. Systems can also be multirate, i.e., have different parts that are sampled or updated at different rates. For modeling, SIMULINK provides a graphical user interface (GUI) for building models as block diagram, using click-and-drag mouse operation. With this interface, user can draw the models just as user would with pencil and paper (or as most textbooks depict them). This is cry from previous simulation packages that require user to formulate differential equations and difference equations in a language or program.

SIMULINK includes a comprehensive block library of sinks, sources linear and nonlinear components and connectors. User can also customize and create the own blocks. For information on creating the own blocks, the separate writing S-Functions guide is handy. Models are hierarchical, so user can build models using the top-down and bottom-up approaches. User can view the system at a high level, and then double click blocks to go down through the levels to see increasing levels of models detail. This approach provides insight into how a model is organized and how its part interacts. After user define a model, user can simulate it, using a choice of integration methods, either from the SIMULINK menus or by entering commands in the MATLAB Command Window.

The menus are particularly convenient for interactive works, while the command-;line approach is very useful for running a batch of simulation(for example, if user are doing Monte Carlo simulations or want to sweep a parameter across a range of values). Using scopes and other display blocks, user can see the simulation results while the simulation is running. In addition, user can change the parameters and immediately see what happens, for “what if” exploration.

## 2.6 Controllability and Observability

Controllability and observability represent two major concepts of modern control system theory. These originally theoretical concepts, introduced by R. Kalma in 1960, are particularly important for practical implementation. It can be defined as follows. Controllability mean in order to be able to do whatever we want with the given dynamic system under control input, the system must be controllable and observability mean in order to see what is going on inside the system under observation, the system must be observable. Even though the concepts of controllability and observability are almost abstractly defined, now intuitively understand their meanings. The remaining problem is to produce some mathematical check up tests and to define controllability and observability more rigorously. The observability of linear discrete systems is very naturally introduced using only elementary linear algebra.

Controllability is an important property of a system to be controlled. A linear controllable system may be defined as a system which can be steered to any state from the zero initial state. For example, if the linear system is a circuit consisting of capacitors, inductors and resistors controlled by an external voltage, then controllability means that by varying in time the external voltage, can achieve at some point in time any combination of voltages on capacitors and currents through inductors. The controllability property plays an important role in many control problems, such as stabilization of an unstable system by feedback or optimal control.

Since the output is a linear combination of the input and states, one or more poles can be canceled by the zeros induced by this linear combination. When that happens, the cancelled modes are said to be unobservable. Of course, since started with a transfer function, any pole-zero cancellations should be dealt with at that point, so that the state space realization will always be controllable and observable. If a mode is uncontrollable, the input cannot affect it; if it is unobservable, it has no effect on the output. Therefore, there is usually no reason to include unobservable or uncontrollable modes in a state-space model.

The fundamental controllability problem is associated with the question whether an input can be found such that the system states can be steered from an initial value $x_0$ to any final value $x_1$ in a given time interval. In general, the answer to this question depends on the time interval. This induces different notions of controllability all of which become equivalent for linear time invariant system. In the following discussion, only this simpler case is considered.

Kalman's canonical decomposition provides the basic theory and computational algorithm to remove unnecessary states from a realization, while preserving the input-output map. It reliance on the controllability and observability matrices makes the approach somewhat susceptible to numerical problems, as these matrices are often poorly conditioned. A more serious drawback, however, is that this reduction is based on structural properties of the system (linear independence) but without explicitly considering the quantitative aspects of the problem. In practical applications, especially when numerical computations are involved, one is rarely faced with perfectly dependent or perfectly orthogonal vectors. Moreover, a commonly encountered problem is that of a model reduction where modes that have independent but small contributions should be eliminated. With such an objective in mind the previous algorithm is inadequate.

## 2.7 Mathematical Formulation

### 2.7.1 Single Tank System

Control theory has typically been based on dynamic models that are represented as Ordinary Differential Equations (ODEs) and/or transfer functions. This single tank system concerns the relationship between the flow of liquid into the tank, $q_{in}(t)$ and the level of liquid, $h$ . From this model a feedback control system can be designed to maintain the level of liquid in the tank.

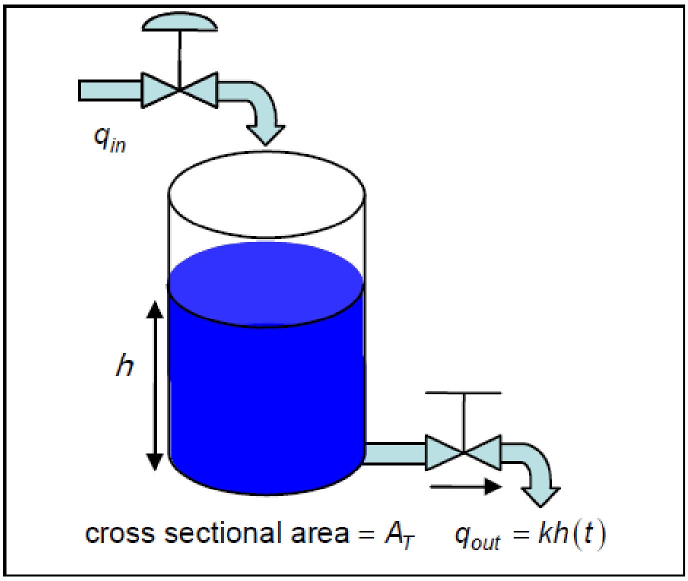


**Figure 2** Single Tank System

The flow of liquid out of the tank is proportional to level of liquid *h* if the flow is laminar:

$$q_{out}(t) = kh(t)$$

where *k* is a constant parameter.

A mass balance across the tank gives:

$$q_{in}(t) - q_{out}(t) = dV(t)/dt$$

i.e. the rate of change of liquid volume, *V*, is equal to the volumetric flow rate of liquid in minus the flow rate of liquid out.

The mass balance can be re-arranged to give:

$$q_{in}(t) - q_{out}(t) = \frac{dA_T h(t)}{dt}$$

and since $A_T$ is constant:

$$q_{in}(t) - q_{out}(t) = \frac{A_T dh(t)}{dt} .$$

Replacing $q_{out}$ by $kh(t)$ gives:

$$q_{in}(t) - kh(t) = \frac{A_T dh(t)}{dt}$$

and more re-arrangement gives:

$$\frac{dh(t)}{dt} + \frac{k}{A_T} h(t) = \frac{1}{A_T} q_{in}(t) \qquad (1)$$

Equation (1) describes a linear, first order relationship between the flow into the tank and the liquid level. Equation (1) is re-considered and a new variable, $x_1$ is now defined, such that $x_1(t) = h(t)$ .

Equation (1) can be re-written as:

$$\frac{dx_1(t)}{dt} = -\frac{k}{A_T} x_1(t) + \frac{1}{A_T} q_{in}(t)$$

which in a more general form can be expressed a dynamic *state equation :*

$$\frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

where

$$\mathbf{x}(t) = x_1(t), \ \mathbf{u}(t) = q_{in}(t), \ \mathbf{A} = -\frac{k}{A_T} \text{ and } \mathbf{B} = \frac{1}{A_T}$$

The earlier equation is the general, linear state equation, where the vector **x** represents the *state variables* of the process. It anticipates more complex systems by use of vectors and matrices. In the tank example, however, there is a single state variable *h(t)* and a single input variable $q_{in}$ *(t).* So the **A** and **B** matrices are scalars.

State variables are the smallest subset of system variables that describe the entire dynamic characteristics of the system. They can be thought of as internal elements of the system that are related to, or in some case are actually equal to the output variables. Although it can sometimes be beneficial, it is not necessary for the state variables to be measured or even have physical meaning.

In the even more general non-linear case the state equation is defined as:

$$\frac{d\mathbf{x}(t)}{dt} = f(\mathbf{x}, \mathbf{u}, t)$$

where *f* is a non-linear function.

### 2.7.2 Coupled Tank System

The coupled two-tank system shown in **Figure 3** is an example of a system with more than one state.

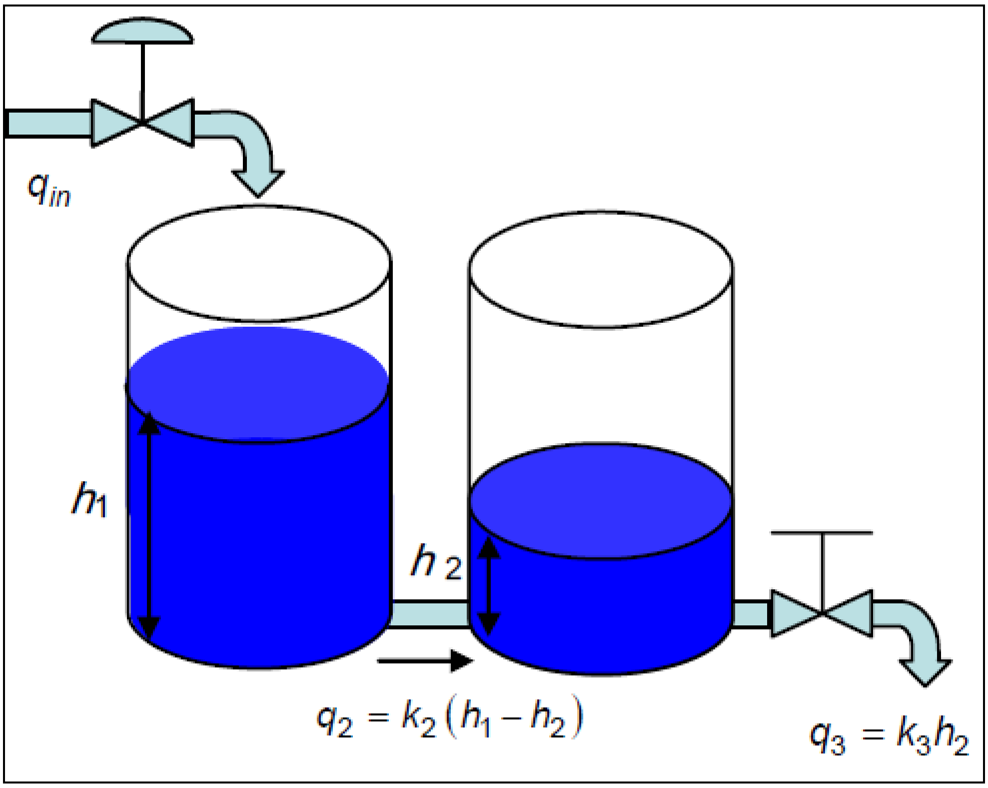


**Figure 3** Coupled Tank System

The coupled tank system has two physical states, $h_1$ and $h_2$ and, the tanks each have its own cross sectional area, the state equation is:

$$\begin{bmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-k_2}{A_1} & \frac{k_2}{A_1} \\ \frac{k_2}{A_2} & \frac{-(k_2+k_3)}{A_2} \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} q_{in}$$

Expanding out the matrices and rearranging shows that the state equation is nothing more than a volumetric balance for each tank.

$$\frac{dh_1}{dt} = \frac{-k_2(h_1 - h_2)}{A_1} + q_{in}$$

$$\frac{dh_2}{dt} = \frac{k_2(h_1 - h_2)}{A_2} - \frac{k_3 h_2}{A_2}$$

Thus, there is no new physics in a state equation. The reason why control engineers use it is that is a convenient way of studying the mathematical properties of the physical equations. Using state-space leads to some very powerful controller designs that would be too cumbersome to consider without the compact matrix formulation.

Referring back to the general state equation:

$$\frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

In the two-tank example, the state vector **x(t** ) is $\begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix}$, the input u(t) is $q_{in}$ and the constant-coefficient **A** and **B** matrices are:

$$A = \begin{bmatrix} \frac{-k_2}{A_1} & \frac{k_2}{A_1} \\ \frac{k_2}{A_2} & \frac{-(k_2 + k_3)}{A_2} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix}$$

The state equation describes how the states vary with time. In the case of the coupled tank, the output variables were the liquid levels, which were also defined as the state variables. However, in many examples, the state variables will be different from the output variables. It is therefore necessary to have another equation which describes the relationship between the process outputs and state variables. For general systems, this equation is defined as following:

$$\mathbf{y}(t) = g(\mathbf{x},\mathbf{u},t)$$

where $g$ is a non-linear function. For a linear, time-invariant system this equation always takes the form:

$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t)$$

where $\mathbf{y}(t)$ is a vector of output variables. In the coupled tank system:

$$C = [0\ 1] \qquad D = [0]$$

The above equation for $\mathbf{y}(t)$ is referred to as the *observer equation*. The combination of the state equation and observer equation is called a *state-space model*. The state space model is as following:

$$\begin{bmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-k_2}{A_1} & \frac{k_2}{A_1} \\ \frac{k_2}{A_2} & \frac{-(k_2+k_3)}{A_2} \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} q_{in}$$

$$\mathbf{y}(t) = [0\ 1] \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + [0]$$

# CHAPTER 3

# METHODOLOGY

This chapter discusses the project procedure identification as well as the tools utilized throughout the course of completing the project.

## 3.1 Procedure Identification

The flowchart and Gantt Chart of the entire project is included in **Appendices A** and **B** respectively. The project is generally divided into four main phase:

- **Phase 1**: Literature Review
  Thorough study on the coupled tank system, the mathematical modeling of the system, model in state space and as well the coding required for simulating in MATLAB. Other required studies which are related to the project were done too.

- **Phase 2**: Performance Analysis
  The model of the coupled tank system is modeled using the MATLAB with the help of Simulink. The performance of the system was analysed.

- **Phase 3**: Design of Controller and Observer
  The controller and observers poles and gains are determined based on the model parameters.

- **Phase 4**: Simulate controller and observers in Simulink
  The designed controller and observers are then simulated on different system types on Simulink to observe their effects. Results found to be documented and compiled in the report.

## 3.2 Tools and Equipment

The tools and equipment required for the project are as such:

- MATLAB
- Simulink
- Required tools to build a real model

# CHAPTER 4

# RESULTS AND DISCUSSION

In this chapter, the overall results or outcome of every stage and phase of the project are discussed.

## 4.1 State Space Equation

From the system analyzed the following state space was model:

$$\begin{bmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-k_2}{A_1} & \frac{k_2}{A_1} \\ \frac{k_2}{A_2} & \frac{-(k_2+k_3)}{A_2} \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} q_{in}$$

$$\mathbf{y}(t) = [0\ 1] \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + [0]$$

With comparison with the *AISB* Coupled-Tank control apparatus model CT-100, the cross sectional area and the proportionality constant are as following:

**Table 1** Parameters of Coupled-Tank system

| Name | Expression | Value | |
|---|---|---|---|
| Cross Sectional Area of the coupled tank reservoir | $A_1$ and $A_2$ | 32 $cm^2$ | |
| Proportionality constant that depends on discharge coefficient, orifice cross sectional area and gravitional constant | $k_i$ subscript *i* denotes which tank it refers | $k_1$ | $k_2$ |
| | | 14.3 $cm^{3/2}$/sec | 14.3 $cm^{3/2}$/sec |

Thus, the state space function are derived as such

$$\begin{bmatrix} \dot{h_1} \\ \dot{h_2} \end{bmatrix} = \begin{bmatrix} -0.4469 & 0.4469 \\ 0.4469 & -0.8938 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 0.03125 \\ 0 \end{bmatrix} q_{in}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + [0]$$

## 4.2 Coupled Tank System in SIMULINK

The coupled tank system was modeled using SIMULINK as shown in **Figure 4**. The output is connected to a scope to be observed. A step input is applied and the corresponding input and output is observed simultaneously at the scope.

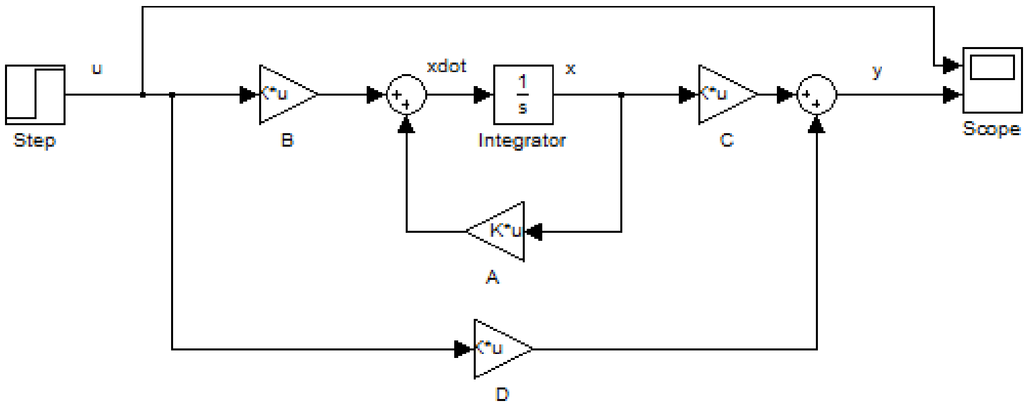


**Figure 4** Block diagram of the system

The output which is the level in the second tank $h_2$, has the *S*-shaped response that is characteristic of a second order system. System does not have any overshoot or offset but it does not reach the desired output.

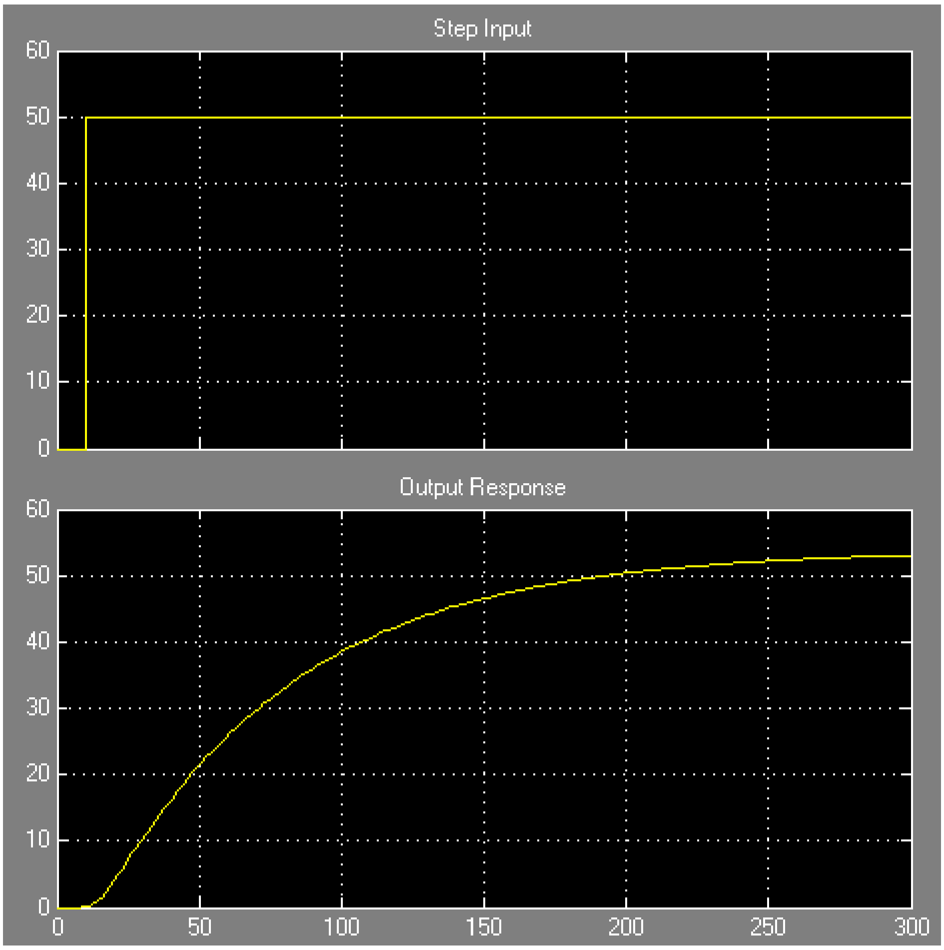


**Figure 5** Output of Coupled tank system

## 4.3 Controllability and Observability

Controllability and observability play important role in the design of the system in state space. The conditions of controllability and observability may govern the existence of a complete solution to the control system design problem.

The solution to this problem may not exist if the system considered is not controllable. Thus before proceeding to the modeling and simulation, the controllability and observability of this system are determined as shown in **Appendices C** and **D** respectively. The thorough calculation and computation using MATLAB is provided as in **Appendice E**.

The system is found to be controllable and observable as proven using calculation and as well computing in MATLAB. This is because in both calculations the rank is not equivalent to zero. Since the system is controllable and observable, thus the state feedback control can be designed.

## 4.4 Controller and Observers Design

In order to design controller and observers using pole placement method, the full-state feedback controller closed-loop poles are chosen to be further from 0 the system poles are(note: using continuous-time model). On the other hand, to design full-state and reduced-order observers with faster response, the observer closed loop poles for both types are chosen to be further from zero than the controller's ones are.

From the transfer function computation as in **Appendice F** the following closed-loop poles are assumed and chosen for the controller and observer:

**Table 2** Assumed and chosen poles

| Type | Poles |
|---|---|
| Controller | -0.171, -1.17 |
| Full-state observer | -1.2, -0.2 |
| Reduced-order observer | -1.2 |

From the data in **Table 2**, the controller and observer gains are obtained as shown in **Table 3**. The steps taken to obtain the gains are shown in **Appendices G** and **H.**

**Table 3** Controller and observer gains

| Type | Poles |
|---|---|
| Controller, K | [0.0096 0.0059] |
| Full-state observer, L | [0.0308 ; 0.0593] |
| Reduced order observer, L1 | 1.685 |

## 4.5 Simulation of Controller and Observer

To simulate the controller and full-state observer in Simulink, the observer shown in **Figure 6** is constructed in a subsystem.

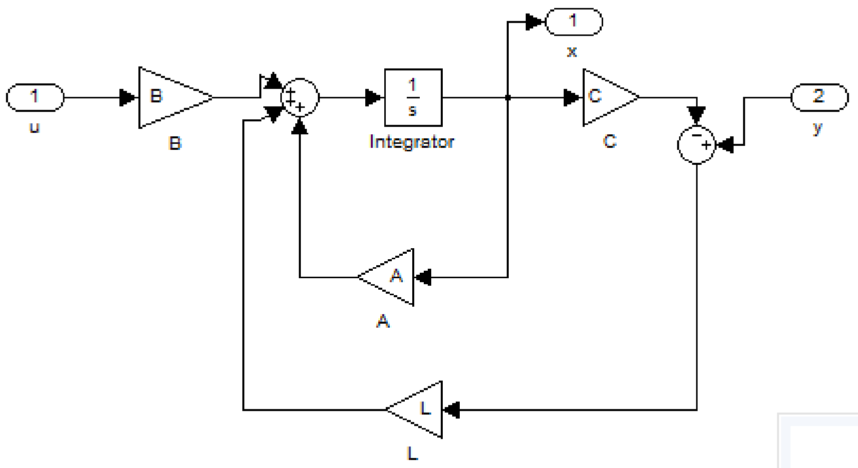


**Figure 6** Full state observer (constructed in subsystem)

To simulate the controller and reduced-order observer in Simulink, the observer shown in **Figure 7** is constructed in a subsystem.

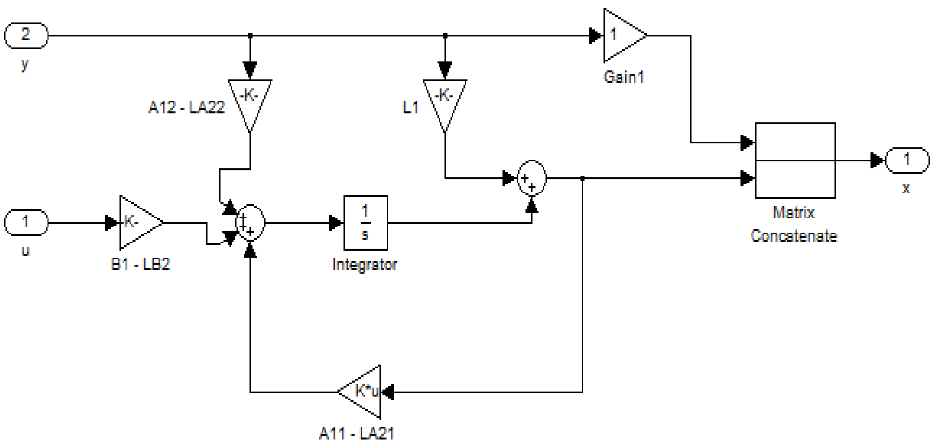


**Figure 7** Reduced state observer (constructed in subsystem)

These subsystems are used in all the following block diagrams shown in subsections to follow. The values of the model parameters and gains used in all Simulink block diagram are taken from **Section 4.3**.

### 4.6 Coupled Tank with Controller

The state feedback gain was determined as in **Table 5**, to shape the transient response, by solving the regulator problem. The Simulink as in **Figure 8** was developed with the feedback controller and the response was observed with a unit step input.

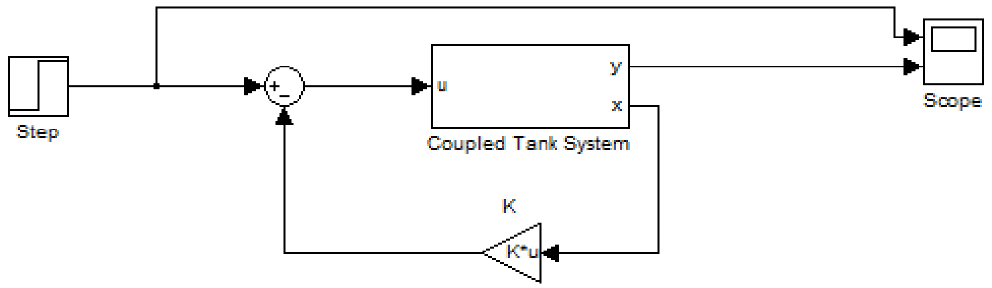


**Figure 8** Block diagram of Coupled Tank with Controller

The next step is to design the forward gain, N as shown in calculation in **Appendice I**. The Simulink as in **Figure 9** was developed with the feedback controller, K and forward gain, N and the response was observed with a unit step input.

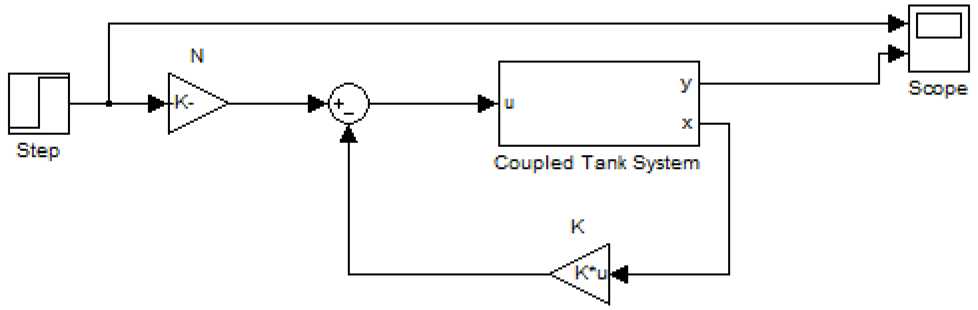


**Figure 9** Block diagram of Coupled Tank with Controller and Forward Gain

The corresponding output of the coupled tank system was observed as in **Figure 10** and **11** respectively.

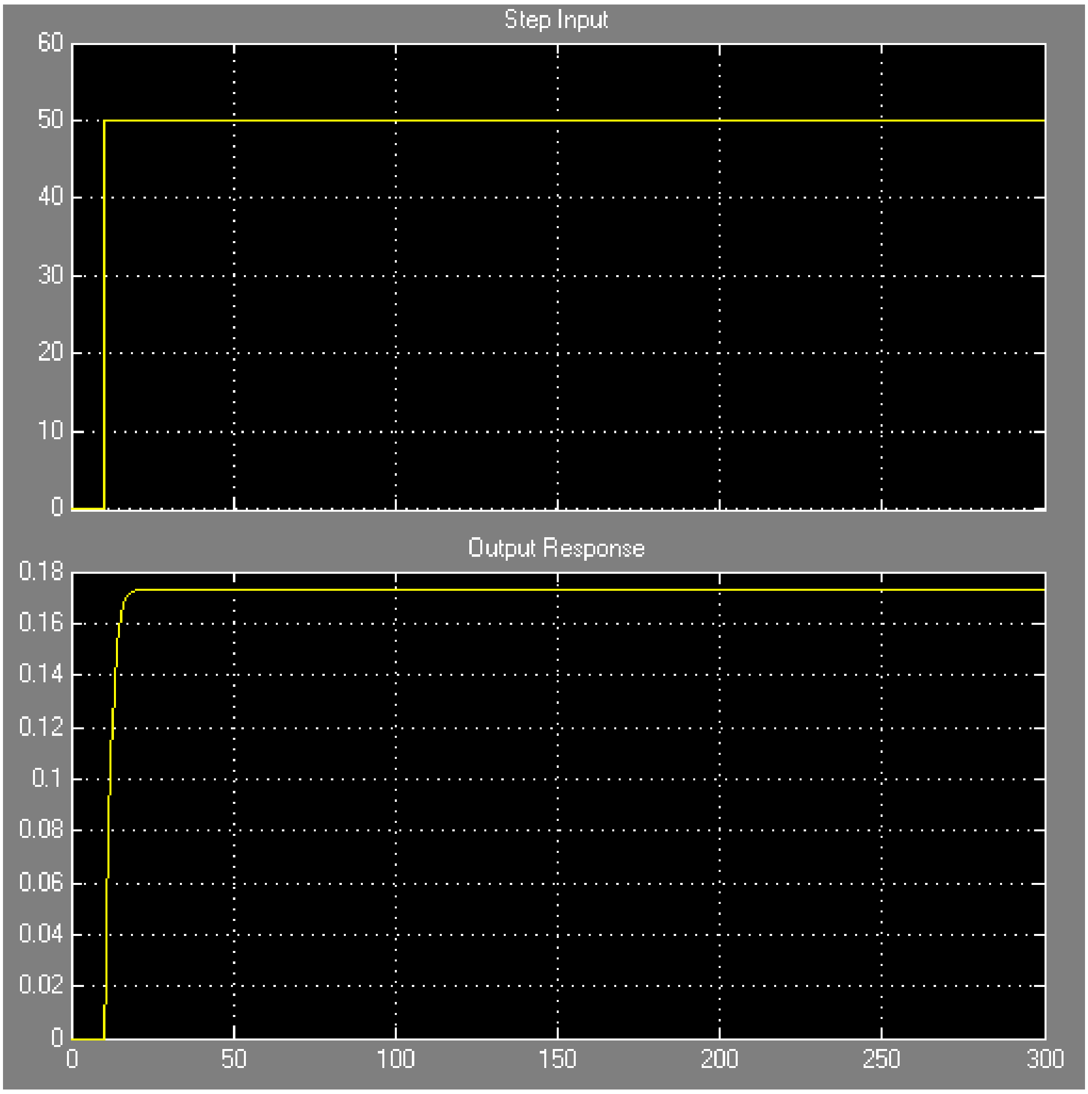


**Figure 10** The output response of the system with the feedback gain, *K*

The desired poles chosen are -0.171 and -1.17. These poles are chosen because they are located far from the *jw* axis and on the left hand plane. From **Figure 10**, it is shown the system settles at approximately 0.17 which is far away from the value applied to the step input. The rise time was 7s when observed in the scope.

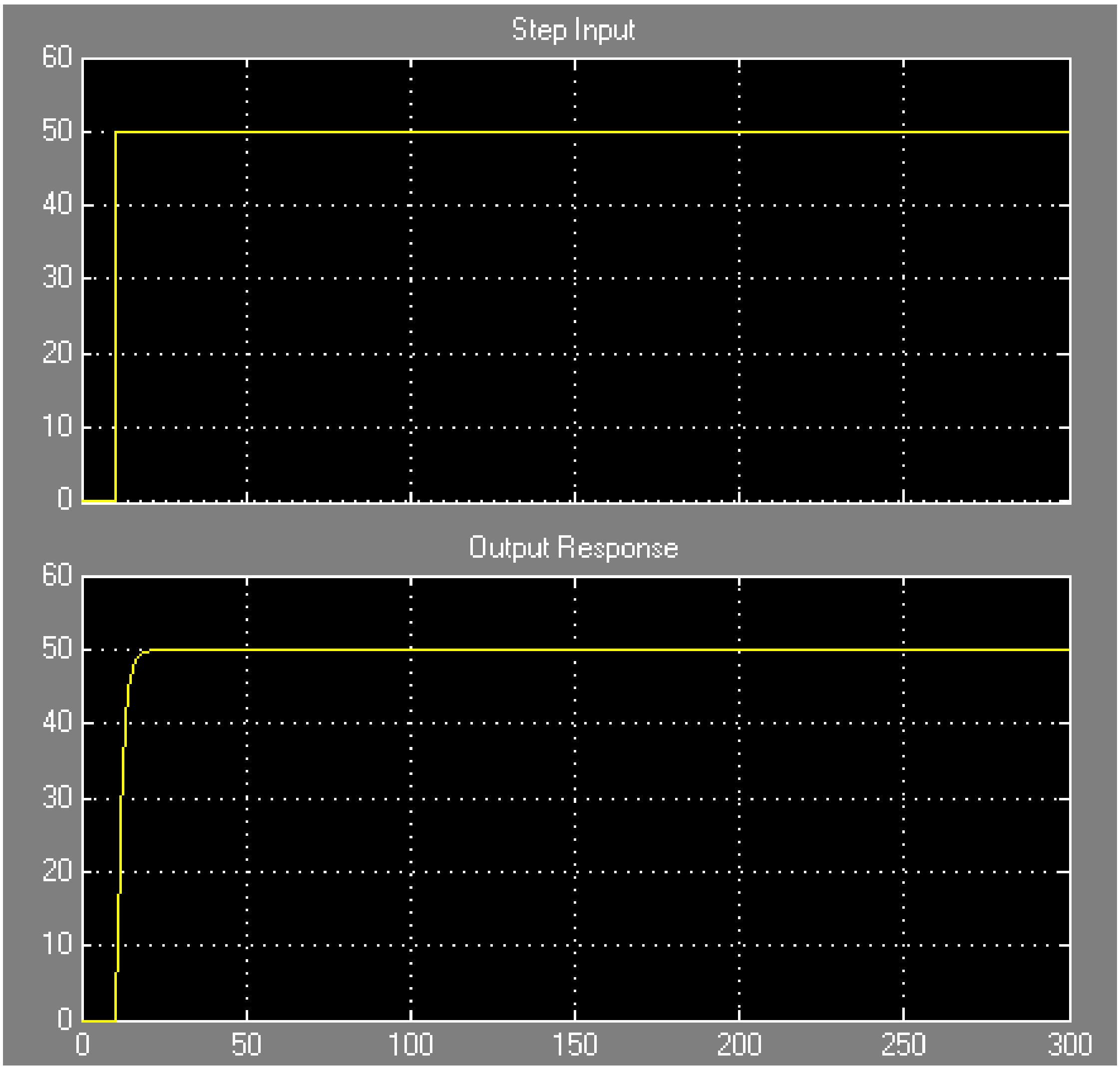


**Figure 11** The output response of the system with the feedback gain, *K* and forward gain, *N*

However, when the feed forward gain N was added, the graph in **Figure 11** was obtained. The system settles at 50, which is the same value applied to the step input and the rise time was only 4s. In both cases, it was observed that there was no overshoot and offset, system being stable.

### 4.7 Coupled Tank with Full State Observer

The earlier response was good, but was found assuming full-state feedback, and forward gain which most likely will not be a valid assumption. To compensate for this, a full-state observer is designed to estimate those states that are not measured. The full state observer was developed in Simulink as in **Figure 12.** A step input was applied and the respond was observed in the scope. The observer gain was obtained with the computation using MATLAB as in **Appendice G**.

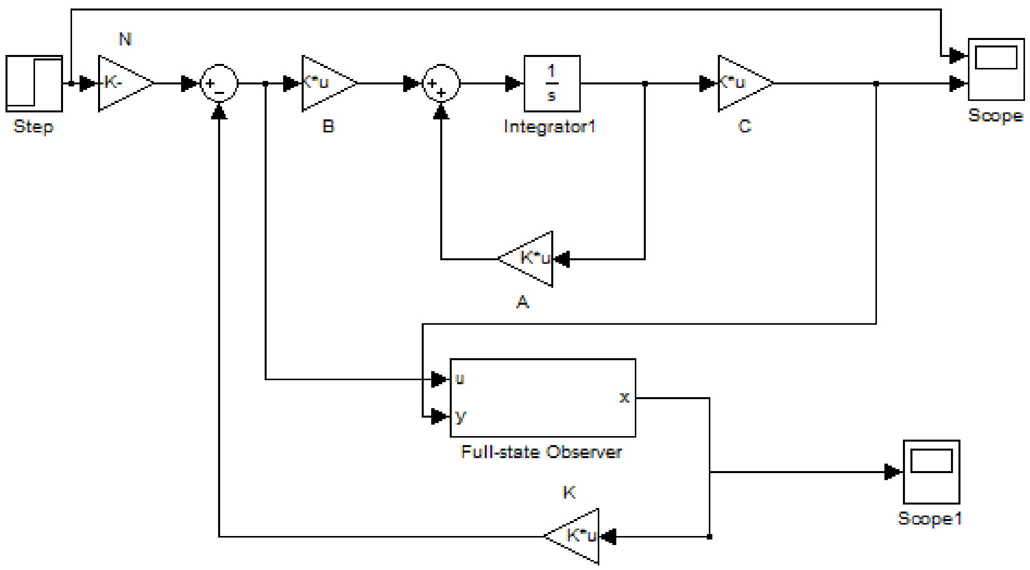


**Figure 12** Block diagram of Coupled Tank with Full state Observer

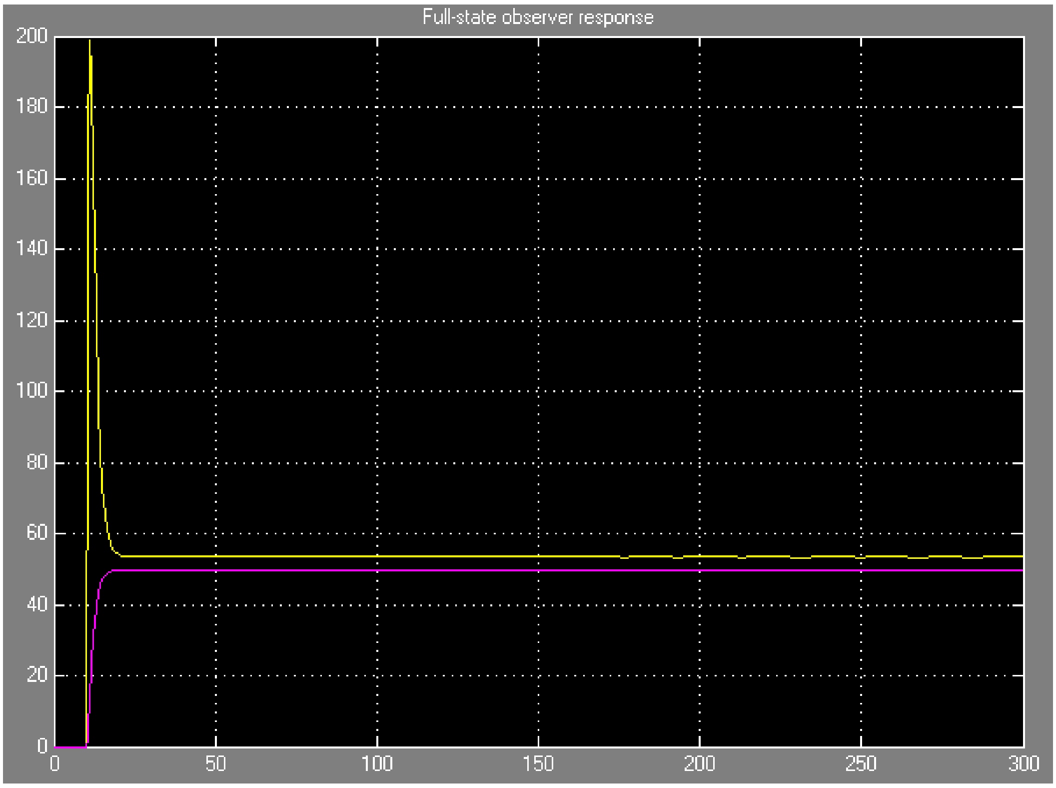


**Figure 13** Full state observer Output Response

The observer action was observed from the scope as in **Figure 13**, estimating both the state variables and matching the output Y with slight difference. This is probably due to the disturbance or noise in the system. The bandwidth of the system of the system increases as the selected desired poles are further onto the left hand side of the plane. This causes it to be more sensitive to noise and disturbance thus likely to alter the output response of the system.

### 4.8 Coupled Tank with Reduced Order Observer

The observer later was only studied using reduced order observer which observes the unmeasured state, $h_1$. The reduced order observer was developed in Simulink as in **Figure 14.** A step input was applied and the respond was observed in the scope. The reduced order observer gain was obtained with the computation as in **Appendice H**.

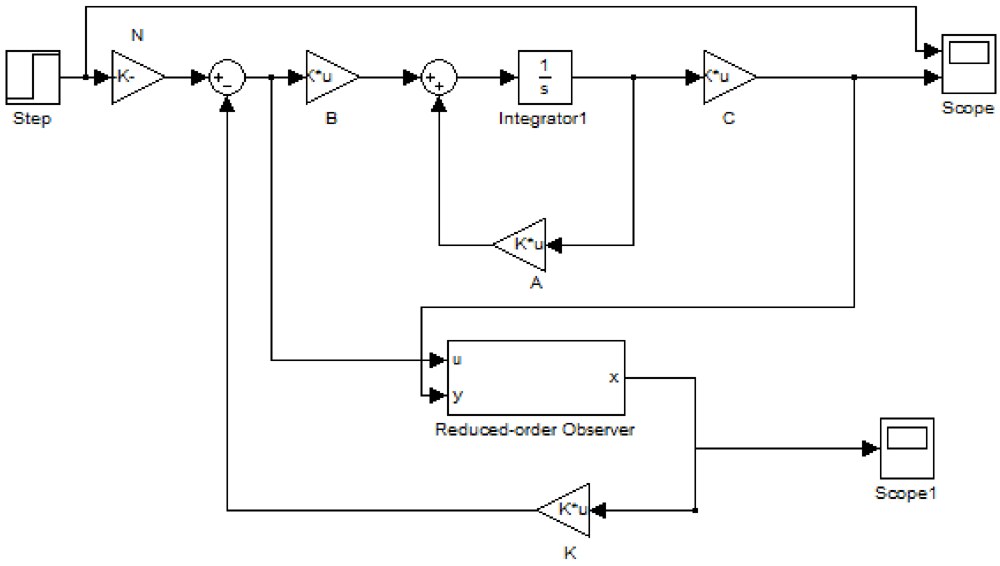


**Figure 14** Block diagram of Coupled Tank with Reduced Order Observer

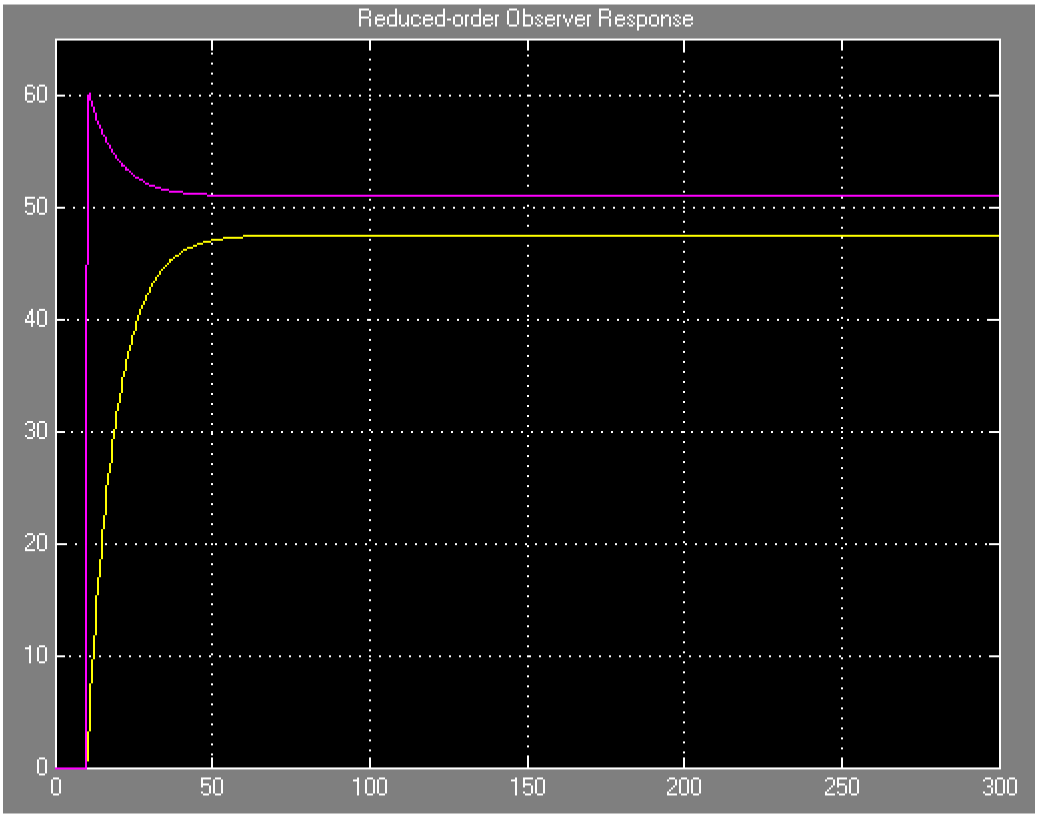


**Figure 15** Reduced Order Observer Output Response

The reduced order observer can estimate the unmeasurable states, and a direct feedback path can be used to obtain the measured state values. Here it is observed the system measured state values are 48 which is slightly lower than the input. This is probably the cause of the noise or disturbance in the system.

# CHAPTER 5

# CONCLUSION AND RECOMMENDATIONS

This chapter concludes the entire project and proposes several recommendations, which could improve the outcome of the project.

## 5.1 Recommendation

Despite the overall objective of the project was achieved, there are several recommendations, which could be considered in order to improve the project outcomes much further as such:

- **Implement controller and observer on a real coupled tank:** The results obtained were promising from the simulation done with Simulink and MATLAB, but implementing the controller and observer on an actual coupled tank will prove its effectiveness as an alternative control strategy.

- **Introduction of disturbance to test the controller and observer effect:** A disturbance should be introduced to check on the erroneous of the system output and how the controller and observer correct the error.

- **A thorough study on disturbance and noise in the system:** The bandwidth of the system increases as the desired poles are further onto the left hand side of the plane. This causes the system to be more sensitive to noise and disturbance that alter the output of the system. This problem can be encountered by having a thorough study on it.

- **Design a SIMO system:** Both the levels of the tank can be studied if the SIMO system is design and studied in this project. A detailed simulation can be carried out and both the levels of the tank can be regulated as compared here whereby only the output tank is studied.

## 5.2 Conclusion

A coupled tank system was successfully modeled on Simulink from thorough study on the mathematical modeling. Controller and observer were also then effectively designed using the pole placement method, producing promising results that indicate the practicality of modern control in plant process control system. The success of this project signifies that an alternative to the current implementation of plant process control system can be made possible with the design of a new controller and observer strategy that are robust, optimal and adaptive via modern control approach.

This paper also presents the objective, scope and methodology in modeling and analysis of a coupled tank system. The simulations have been run to analyse the characteristic of the system. The simulation results showed the characteristic of the coupled tank with and without the introduction of controller and observer. These simulations have improved my skill on using the MATLAB software.

Besides that, this project has achieved its objective that is to design the controller and observer by using the state space approach and it has been a successful. This has been proven in the simulation results obtained using MATLAB and Simulink.

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[6] G.F Franklin, J.D Powell and A. Emami-Naeini, *Feedback Control of Dynamic System*, 5$^{th}$ ed. Upper Saddle River, NJ: Pearson Prentice Hall: 2006

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**Online Resources**

[8] Wikipedia “Process Control” http://en.wikipedia.org/wiki/Process_control

[9] Wikipedia “Modern Control Theory” http://en.wikipedia.org/wiki/Control_theory#Modern_control_theory

[10] Controls Wikipedia “Process Control Definition” http://controls.engin.umich.edu/wiki/index.php

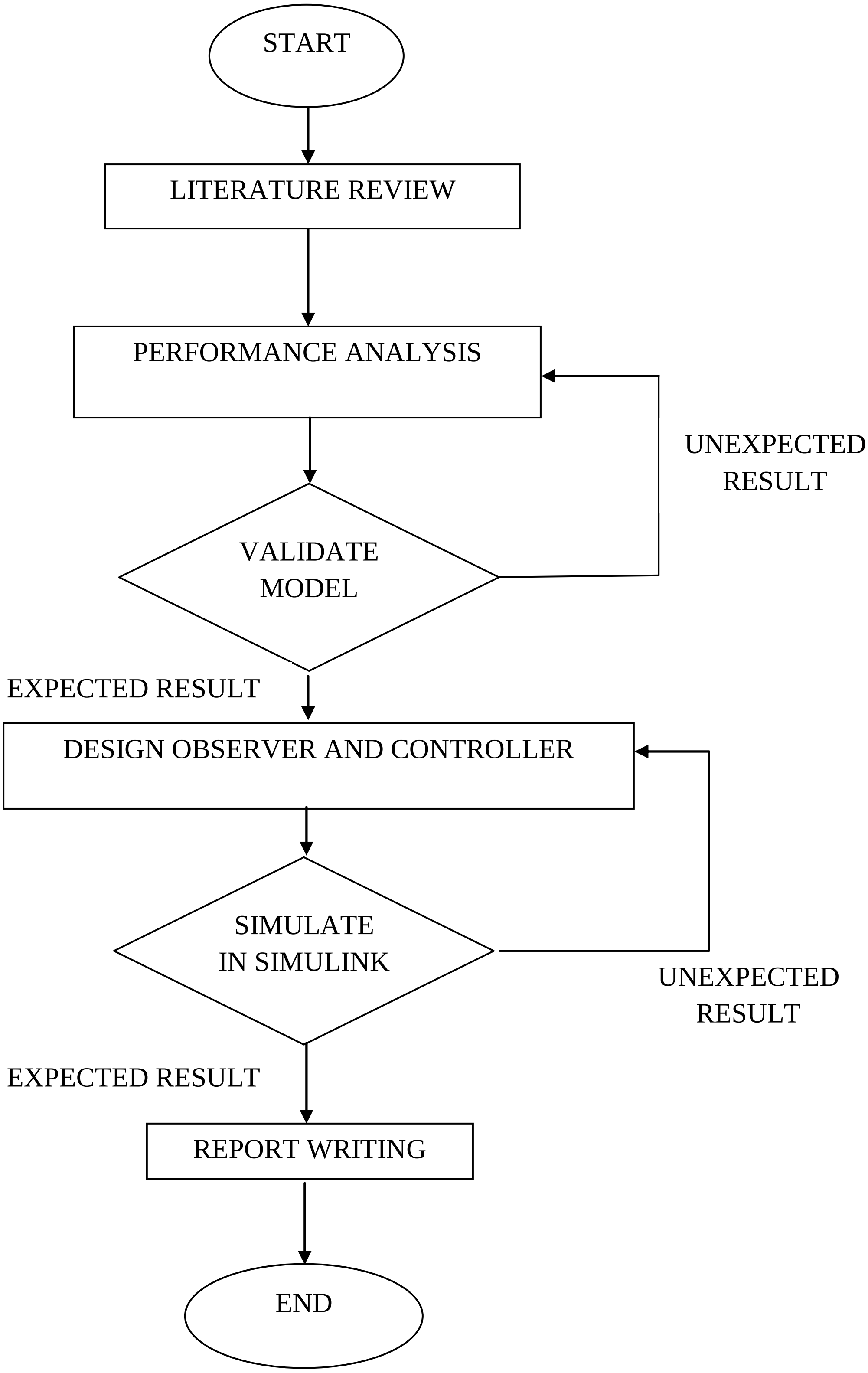
[11] Control System Principles “Coupled Tank System” http://www.control-systems-principles.co.uk/whitepapers/coupled-tanks-systems.pdf

[12] State Space Article “Modeling Coupled Tank System” http://personalpages.ps.ic.ac.uk/~nina/Research/PDFPapers/StateSpaceArticleJune2009.pdf

# APPENDICES

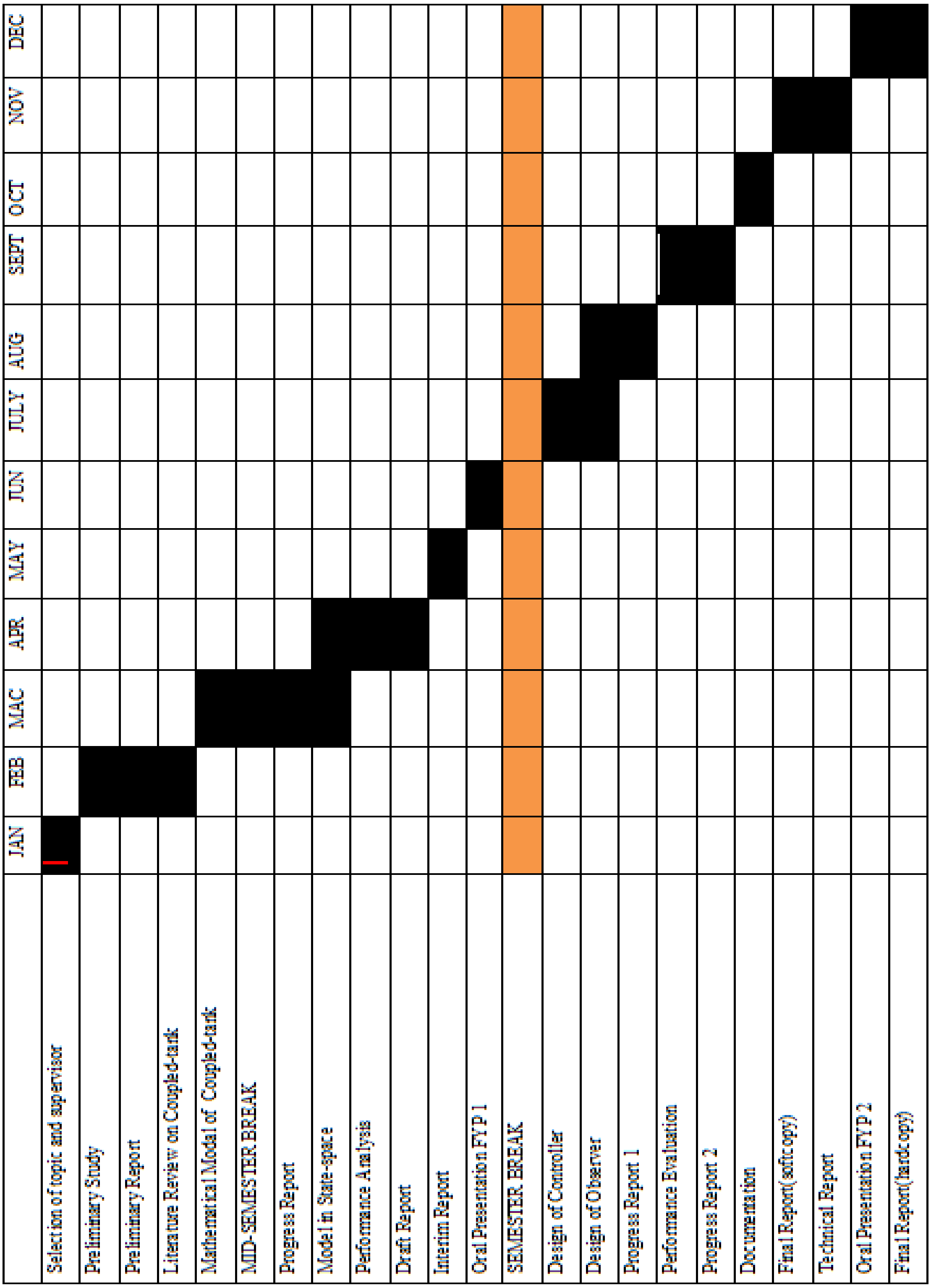
# APPENDIX A

# PROJECT FLOWCHART



# APPENDIX B

# PROJECT GANTT CHART



# APPENDIX C

# DETERMINING CONTROLLABILITY

Controllability of the system was checked using the following equation:

**Controllability**

Using criterion 2:

$$\boldsymbol{M_C} = [\boldsymbol{B} \quad \boldsymbol{AB}]$$

$$[\boldsymbol{B} \quad \boldsymbol{AB}] = \begin{bmatrix} \frac{1}{A_1} & \frac{-k_2}{{A_1}^2} \\ 0 & \frac{k_2}{A_2 A_1} \end{bmatrix} = \begin{bmatrix} 0.03125 & -0.001396 \\ 0 & 0.001396 \end{bmatrix}$$

$$\det\begin{bmatrix} \frac{1}{A_1} & \frac{-k_2}{{A_1}^2} \\ 0 & \frac{k_2}{A_2 A_1} \end{bmatrix} = \det\begin{bmatrix} 0.03125 & -0.001396 \\ 0 & 0.001396 \end{bmatrix}$$

$$= 0.00004 \neq 0$$

Thus, the system is controllable.

# APPENDIX D

# DETERMINING OBSERVABILITY

Observability of the system was checked using the following equation:

**Observability**

$$[\boldsymbol{C^T} \quad \boldsymbol{A^T C^T}] = \begin{bmatrix} 0 & \frac{-k_2}{A_1} \\ 1 & \frac{k_2}{A_2} \end{bmatrix} = \begin{bmatrix} 0 & -0.4469 \\ 1 & 0.4469 \end{bmatrix}$$

$$\det\begin{bmatrix} 0 & \frac{-k_2}{A_1} \\ 1 & \frac{k_2}{A_2} \end{bmatrix} = \det\begin{bmatrix} 0 & -0.4469 \\ 1 & 0.4469 \end{bmatrix}$$

$$= -0.4469 \neq 0$$

Thus, the system is observable.

# APPENDIX E

## DETERMINING CONTROLLABILITY & OBSERVABILITY USING MATLAB

```
>> A=[-0.4469  0.4469 ; 0.4469 -0.8938];
>> B=[0.03125;0];
>> C=[0 1];
>> D=[0];
>> %Checking Controlability
>> p=ctrb(A,B);
>> rankp=rank(p)

rankp =

     2

>> %Checking Observability
>> q=obsv(A,C);
>> rankq=rank(q)

rankq =

     2
```

## APPENDIX F

## OBTAINING TRANSFER FUNCTION USING MATLAB

```
>> A=[-0.4469  0.4469 ; 0.4469 -0.8938];
>> B=[0.03125;0];
>> C=[0 1];
>> D=[0];
>> [num,den]=ss2tf(A,B,C,D,1)

num =

         0         0    0.0140


den =

    1.0000    1.3407    0.1997
```

# APPENDIX G

# COMPUTING GAINS FOR CONTROLLER & FS OBSERVER

```
>> A=[-0.4469  0.4469 ; 0.4469 -0.8938];
>> B=[0.03125;0];
>> C=[0 1];
>> D=[0];
>> %Compute Controllor Gain, K
>> jk=[-0.171 -1.17];
>> k=acker(A,B,jk)

k =

    0.0096    0.0059

>> %Computing full-state Observer Gain, L
>> jl=[-1.2 -0.2];
>> %Assuming both poles are on the left hand plane on nearer and the other far
>> ll=acker(A',C',jl)'

ll =

    0.0308
    0.0593
```

# APPENDIX H

## COMPUTING GAINS FOR REDUCED ORDER OBSERVER

Since $x_2$ is measured, we want the observer to observe the unmeasured state, $x_1$

$\dot{x_2}$ = $A_{21}\ x_1 + A_{22}\ x_2$ + $B_2 u$

Define $w$ = $\dot{x_2}$ - $A_{22}\ x_2$ - $B_2 u$ = $\hat{y}$ - $A_{22}\ x_2$ - $B_2 u$

Basically $w$ = $A_{21}\ x_1$

$\dot{x_1}$ = $A_{11}\ x_1 + A_{12}\ x_2$ + $B_1 u$

Replace the unmeasured state by estimate, and add a correction term multiplied by observer gain, L

The correction term is $w$ - $\hat{w}$

$\hat{\dot{x}}$ = $A_{11}\ \widehat{x_1} +\ A_{12}\ x_2$ + $B_1 u + L\ (\dot{y} - A_{22}\ x_2 - B_2 u$ - $A_{21}\ \widehat{x_1}$ )

Re-arrange;

($\hat{\dot{x}}$ - L$\dot{y}$) = $A_{11}\ \widehat{x_1} +\ A_{12}\ x_2$ + $B_1 u + L\ (-A_{22}\ x_2 - B_2 u$ -$A_{21}\ \widehat{x_1}$)

($\hat{\dot{x}}$ - L$\dot{y}$) = -0.4469 $\widehat{x_1}$ + 0.4469 $x_2$ + 0.03125 $u + L$ (0.8938 $x_2 -$ 0.4469$\widehat{x_1}$ )

= (-0.4469 – $L$ 0.4469) $\widehat{x_1}$ + (0.4469 + $L$ 0.8938 ) $x_2$ + 0.3125 $u$

-0.4469 – $L$ 0.4469 = - 1.2

$L$ = **1.685**

$\hat{\dot{x}}$ – 1.685$\dot{y}$ = **-1.2**$\widehat{x_1}$ + **1.953** $x_2$ + **0.3125** $u$

# APPENDIX I

# COMPUTING FORWARD GAIN, N

N=$N_U$ + K $N_X$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} N_X \\ N_U \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$\begin{bmatrix} N_X \\ N_U \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} N_X \\ N_U \end{bmatrix} = \begin{bmatrix} -0.4469 & 0.4469 & 0.03125 \\ 0.4469 & -0.8938 & 0 \\ 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} N_X \\ N_U \end{bmatrix} = \begin{bmatrix} 0 & 2.2376 & 2 \\ 0 & 0 & 1 \\ 32 & 32 & 14.3008 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$N_X = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\quad$ $N_U$=14.3008

N=$N_U$ + K $N_X$ $\quad$ ; K=$[0.0096 \quad 0.0059]$

N=14.3259 $\approx$ 14