

UNIVERSITI
TEKNOLOGI
PETRONAS

FINAL EXAMINATION MAY 2023 SEMESTER

COURSE : VEB4022 - DESIGN OF REINFORCED CONCRETE
STRUCTURES 2
DATE : 2 AUGUST 2023 (WEDNESDAY)
TIME : 9.00 AM - 12.00 NOON (3 HOURS)

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions in the Answer Booklet.
2. Begin **EACH** answer on a new page in the Answer Booklet.
3. Indicate clearly answers that are cancelled, if any.
4. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions, if any.
5. **DO NOT** open this Question Booklet until instructed.

Note :

- i. There are **TWENTY-THREE (23)** pages in this Question Booklet including the cover page and appendix.
- ii. **DOUBLE-SIDED** Question Booklet.

1. **FIGURE Q1** shows a reinforced concrete slender column cross section with dimensions of 300 mm x 200 mm. The column is subjected to an ultimate axial load of 1250 kN and moment of 87 kN.m about minor axis ($y-y$). Design the longitudinal reinforcements and shear links for this column. The characteristic strength of concrete is 30 MPa, the characteristic strength of steel is 500 MPa and axis distance is 50 mm. Assume the following information:

Slenderness ratio, $l_y = 39$

Effective length; $L_{oy} = 4.1\text{m}$

[35 marks]

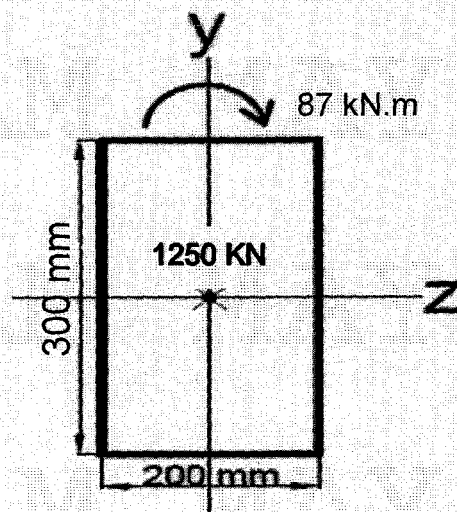


FIGURE Q1

2. **FIGURE Q2** shows the longitudinal section of a reinforced concrete stair connecting to the ground floor and the first floor of a shop house. The 1.5 m width stairs have 170 mm risers and 300 mm going along the flight. The stair is monolithically casted with supporting beams and connecting slabs at the first-floor level. However, at the ground level it is only casted monolithically with a supporting beam. Other dimensions of the staircase are given in the figure.

With a characteristic live load of 5.0 kN/m² and a characteristic dead load of 1.4 kN/m² (excluding of self-weight), design the reinforcement required in the stair using Grade C40 concrete, Grade 500 steel reinforcement, and a concrete cover of 25 mm. Assume a suitable value for the waist of the stair. All checking of deflection, shear and cracking should be done.

[35 marks]

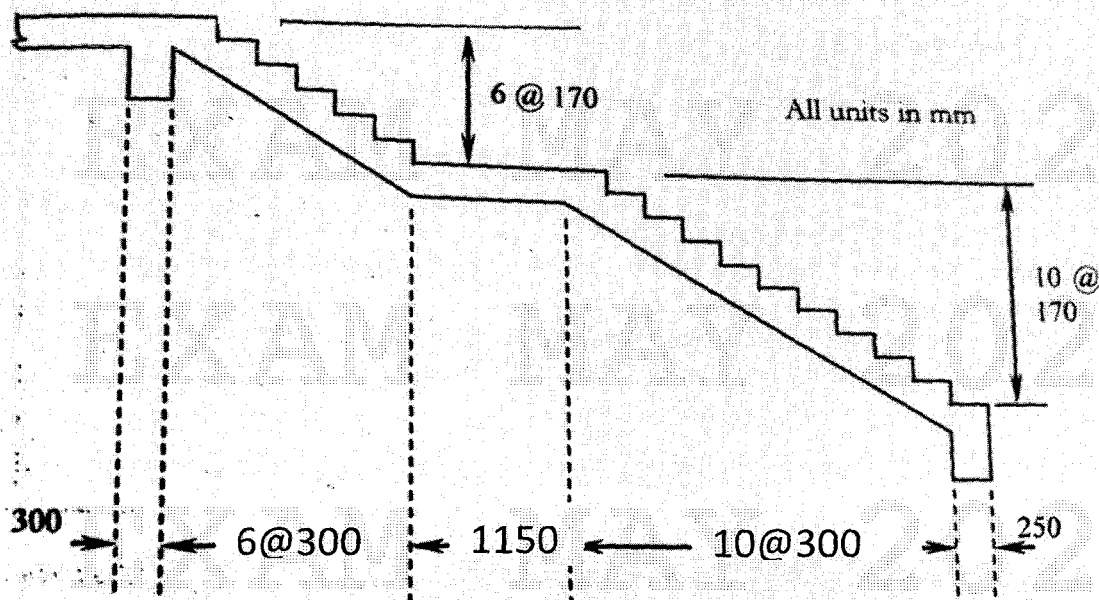


FIGURE Q2

3. FIGURE Q3 shows a simply supported rectangular beam with span of 10 m carrying a uniformly distributed vertical load of 15 kN/m. Calculate the minimum prestressing force P required for this beam for two cases as the following:

a) P applied along the line of centre of gravity. [15 marks]

b) P applied at a distance (e) = 175 mm. [15 marks]

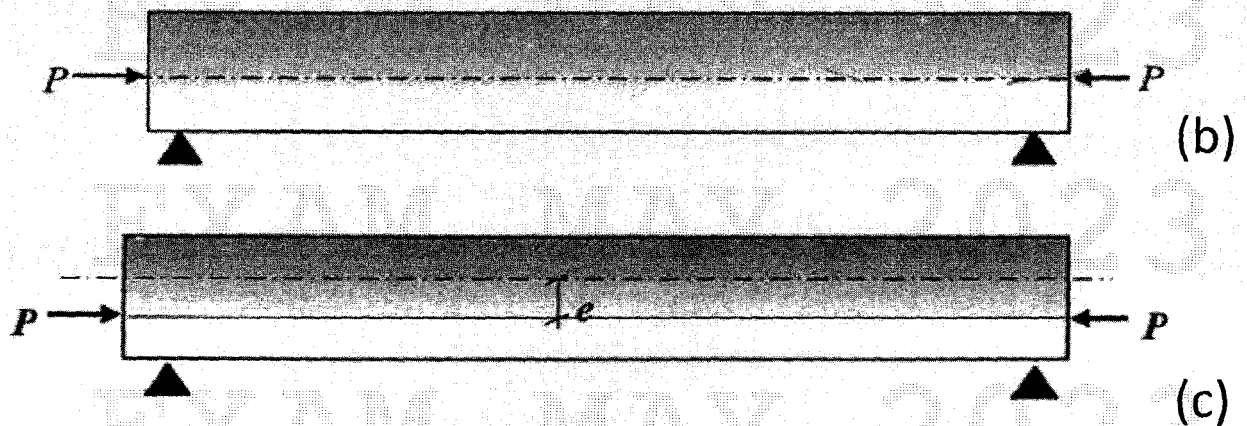
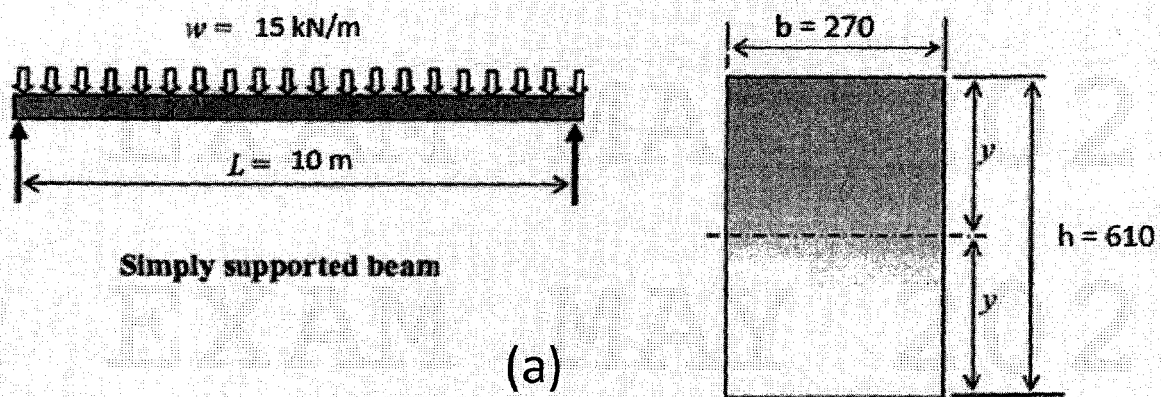


FIGURE Q3: (a) Beam dimensions; (b) P applied along the line of centre of gravity; (c) P applied at a distance of $e = 175$ mm.

-END OF PAPER-

APPENDIX

Stair design

$$y = h \cdot [(G^2 + R^2)^{1/2} / G]$$

$$r = y + (R/2)$$

General

$$K = M / bd^2 f_{ck}$$

$$z = d [0.5 + \sqrt{0.25 - K/1.134}] = 0.93 d \leq 0.95d$$

$$A_s = M / 0.87 f_{yk} z$$

$$A_{s,min} = 0.26 (f_{ctm} / f_{yk}) bd$$

$$A_{s,max} = 0.04 A_c$$

Secondary reinforcement

$$A_s = 20\% \text{ of the main reinforcement}$$

Design shear resistance,

$$V_{Rd,c} = [0.12 k (100 \rho_1 f_{ck})^{1/3}] bd$$

$$\rho_1 = A_{st} / bd \leq 0.02$$

$$V_{min} = [0.035 k^{3/2} f_{ck}^{1/2}] bd$$

DEFLECTION

$$\rho = A_{s,req} / bd$$

$$\rho_o = (f_{ck})^{1/2} \times 10^{-3}$$

$$\frac{l}{d} = K \left[11 + 1.5 \sqrt{f_{ck}} \frac{\rho_o}{\rho} + 3.2 \sqrt{f_{ck}} \left(\frac{\rho_o}{\rho} - 1 \right)^{3/2} \right] \quad \text{if } \rho \leq \rho_o$$

$$\frac{l}{d} = K \left[11 + 1.5 \sqrt{f_{ck}} \frac{\rho_o}{\rho - \rho'} + \frac{1}{12} \sqrt{f_{ck}} \sqrt{\frac{\rho'}{\rho_o}} \right] \quad \text{if } \rho > \rho_o$$

Table 7.4N: Basic ratios of span/effective depth for reinforced concrete members without axial compression

Structural System	K	Concrete highly stressed $\rho = 1,5\%$	Concrete lightly stressed $\rho = 0,5\%$
Simply supported beam, one- or two-way spanning simply supported slab	1,0	14	20
End span of continuous beam or one-way continuous slab or two-way spanning slab continuous over one long side	1,3	18	26
Interior span of beam or one-way or two-way spanning slab	1,5	20	30
Slab supported on columns without beams (flat slab) (based on longer span)	1,2	17	24
Cantilever	0,4	6	8

Note 1: The values given have been chosen to be generally conservative and calculation may frequently show that thinner members are possible.

Note 2: For 2-way spanning slabs, the check should be carried out on the basis of the shorter span. For flat slabs the longer span should be taken.

Note 3: The limits given for flat slabs correspond to a less severe limitation than a mid-span deflection of span/250 relative to the columns. Experience has shown this to be satisfactory.

CRACKING

Main bar :

$$S_{\max, \text{slabs}} = 3h \leq 400 \text{ mm}$$

Secondary bar :

$$S_{\max, \text{slabs}} = 3.5h \leq 450 \text{ mm}$$

Assume steel stress under quasi permanent loading ,

$$= 0.6 (f_{yk}/1.15) (A_{s, \text{req}}/A_{s, \text{prov}})$$

Based on C30/37 concrete and 25mm cover

Steel stress (N/mm ²)	Maximum bar size (mm)
160	32
200	25
240	16
280	12
320	10
360	8
400	6
450	5

EG2 Class / Value

Main Bars in Tension

$A_{s,min}$ 9.2.1.1 (1):
 $0.26 \frac{f_{ctm}}{f_{yk}} bd \geq 0.0013 bd$

$A_{s,max}$ 0.04 bd

Secondary Transverse Bars

$A_{s,min}$ 9.3.1.1 (2):
 0.2 A_s for single way slabs

$A_{s,max}$ 9.2.1.1 (3): 0.04 bd

Spacing of Bars

S_{min} 8.2 (2): $d_g + 5 \text{ mm}$ or ϕ or 20mm
 9.3.1.1 (3): main $3h \leq 400 \text{ mm}$

S_{max} secondary: $3.5h \leq 450 \text{ mm}$

places of maximum moment:

main: $2h \leq 250 \text{ mm}$

secondary: $3h \leq 400 \text{ mm}$

Columns**Main Bars in Compression**

$A_{s,min}$ 9.5.2 (2): $0.10N_{Ed}/0.87f_{yk} \leq 0.002bh$

$A_{s,max}$ 9.5.2 (3): 0.04 bh (out of laps zones)
 0.08 bh (in laps zones)

Links

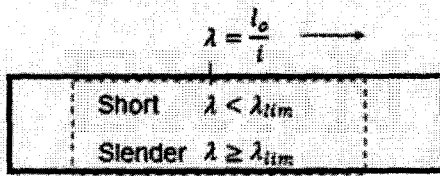
Min size 9.5.3 (1) 0.25 ϕ or 6 mm

$S_{cl,tmax}$ The least of the following:
 a. 20 x min diameter of bars
 b. Lesser dimension of the column (b,h)
 c. 400 mm

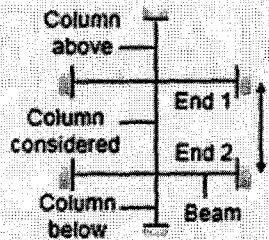
Slenderness ratio

The slenderness ratio of a column bent about an axis:

Slenderness check



Next slide

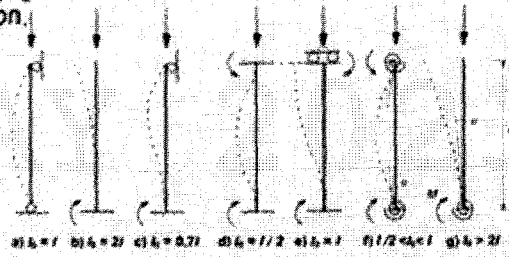


Assumption

- column above or below gives no rotational restraint of the joint
- the stiffness beam = $2EI$ to allow for cracking effects in the beam

Effective length, l_o

(a) General definition, CI 5.8.3.2 EC2,



(b) Equation EC2

Braced

$$l_o = 0.5l \sqrt{\left(1 + \frac{k_1}{0.45 + k_1}\right) \left(1 + \frac{k_2}{0.45 + k_2}\right)}$$

Unbraced

$$l_o = l \cdot \max \left\{ \sqrt{\left(1 + 10 \frac{k_1 \cdot k_2}{k_1 + k_2}\right)}; \left(1 + \frac{k_1}{1 + k_1}\right) \cdot \left(1 + \frac{k_2}{1 + k_2}\right) \right\}$$

$$k = \frac{k_{col}}{\sum k_{beam}} = \frac{\left(\frac{EI}{l}\right)_{col}}{2 \sum \left(\frac{EI}{l}\right)_{beam}} \geq 0.1$$

(c) BS8110 Tables 3.19, 3.20

Radius of gyration, $i = \sqrt{\frac{I}{A}}$

relative flexibilities

- $k_1 = \text{end 1}$,
- $k_2 = \text{end 2}$

$$\lambda_{lim} = \frac{20 \cdot A \cdot B \cdot C}{\sqrt{n}}$$

A = $1 / (1 + 0.2\varphi_{ef})$ (if φ_{ef} is not known, A = 0.7 may be used)

B = $\sqrt{1 + 2\omega}$ (if ω is not known, B = 1.1 may be used)

C = $1.7 - r_m$ (if r_m is not known, C = 0.7 may be used)

φ_{ef} effective creep ratio; see 5.8.4;

$\omega = A_s f_{yd} / (A_c f_{cd})$, mechanical reinforcement ratio;

A_s is the total area of longitudinal reinforcement

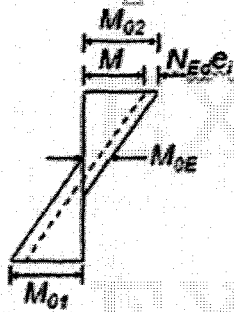
$n = N_{Ed} / (A_c f_{cd})$, relative normal force

$r_m = M_{01} / M_{02}$, moment ratio

M_{01}, M_{02} are the first order end moments, $|M_{02}| \geq |M_{01}|$

Determine M_{Ed}
SHORT COLUMN

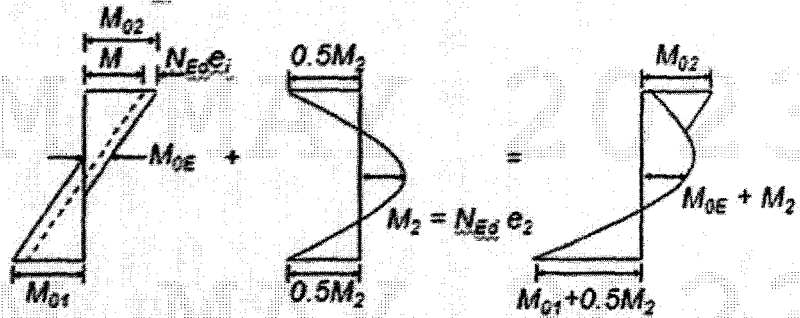
$$M_{Ed} = \max\{M_{02}, M_{min}\}$$



SLENDER COLUMN

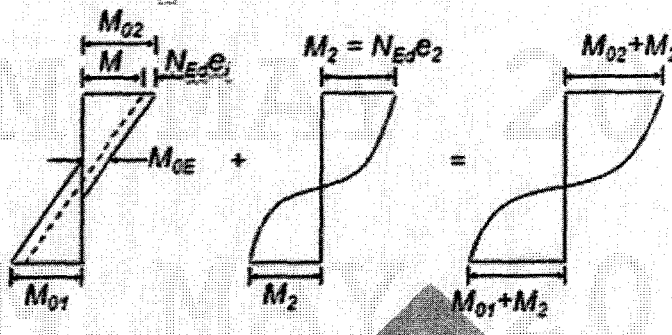
(a) Braced

$$M_{Ed} = \max\{M_{02}, M_{0E} + M_2, M_{01} + 0.5M_2, M_{min}\}$$



(b) Unbraced

$$M_{Ed} = \max\{M_{02} + M_2, M_{min}\}$$



$$(a) M_{02} = M + N_{Ed} \cdot e_i \quad \text{effective length} \quad (e) M_{01} = M_{bot} + N_{Ed} \cdot e_i$$

Equivalent first order
end moment, M_{0E}

$$(b) M_{0E} = 0.6M_{02} + 0.4M_{01} \geq 0.4M_{02} \quad \text{CI 5.8.8.2(1)}$$

$$(c) M_2 = N_{Ed} \cdot e_2 \quad \text{CI 5.8.8.2(3)}$$

$$\text{nominal second order moment, } M_2 \quad e_2 = \left(\frac{1}{r}\right) \left(\frac{l_o^2}{c}\right) \quad \text{CI 5.8.8.2(4)} \quad \varepsilon_{yd} = \frac{f_{yd}}{E_s} - f_{yd} = 0.87f_{yk}$$

$c = 10 (\approx \pi^2)$
Factor

$$\text{CI 5.8.8.3(1) curvature} \quad \frac{1}{r} = K_r K_\varphi \left(\frac{1}{r_o}\right) - \frac{1}{r_o} = \frac{\varepsilon_{yd}}{0.45d}$$

$$\text{correction factor (axial load)} \quad K_r = \frac{n_u - n}{n_u - n_{bal}} \leq 1 \quad n = \frac{N_{Ed}}{A_c f_{cd}} \quad \text{relative axial force, } n \quad \text{effective creep ratio} \quad \varphi_{ef} = \varphi_{(\infty, t_0)} \cdot \frac{M_{0Eq}}{M_{0Ed}} \quad \text{CI 5.8.4}$$

$$\text{CI 5.8.8.3(3)} \quad K_\varphi = 1 + \beta \varphi_{ef} \geq 1 \quad \text{factor (creep)} \quad \text{CI 5.8.8.3(4)}$$

$$\beta = 0.35 + \frac{f_{ck}}{200} - \frac{\lambda}{150} \quad \text{slenderness ratio}$$

$$\varpi = \frac{A_s f_{yd}}{A_c f_{cd}}$$

$$(d) M_{min} = N_{Ed} \cdot e_o \quad \text{CI 6.1(4)} \quad \left. \begin{array}{l} e_o = \frac{h}{30} \geq 20 \end{array} \right\} \text{For construction tolerance}$$

$$N_{ult} = 0.567 f_{ct} A_o + 0.87 f_{yk} A_s$$

EC 2 does not directly give a method for designing biaxially bent columns other than working from first principle. Clause 5.8.9 of EC 2 stated that separate design in each principal direction, disregarding biaxial bending, may be made as a first step. No further check is necessary if,

- (a) $\lambda_y/\lambda_z \leq 2$ and $\lambda_x/\lambda_y \leq 2$, and
 (b) $(e_y/h)(e_z/b) \leq 0.2$ or $(e_z/b)(e_y/h) \leq 0.2$

where

b, h are the width and depth of a section

λ_y, λ_z are the slenderness ratio with respect to y- and z- axis respectively

$e_y = M_{Edz} / N_{Ed}$; eccentricity along y-axis

$e_z = M_{Edy} / N_{Ed}$; eccentricity along z-axis

M_{Edy} is the design moment about y-axis. Including second order moment

M_{Edz} is the design moment about z-axis. Including second order moment

N_{Ed} is the design value of axial load in the respective load combination

- (a) if $M_{Edz} / h^* \geq M_{Edy} / b^*$ then $M_{Edz}^* = M_{Edz} + \beta(h^*/b^*) M_{Edy}$
 (b) if $M_{Edz} / h^* < M_{Edy} / b^*$ then $M_{Edy}^* = M_{Edy} + \beta(b^*/h^*) M_{Edz}$

h^* and b^* are the effective depth as indicated in Figure 10.9, and the coefficient β can be obtained from the equation

$$\beta = 1 - N_{Ed}/bhf_{ck} \quad (0.3 < \beta < 1.0)$$

$$\left(\frac{M_{Edz}}{M_{Rdz}}\right)^\alpha + \left(\frac{M_{Edy}}{M_{Rdy}}\right)^\alpha \leq 1.0$$

where

M_{Rdy} is the moment resistance in y-axis.

M_{Rdz} is the moment resistance in z-axis.

α is the exponent;

for circular and elliptical cross section: $\alpha = 2$

for rectangular cross sections :

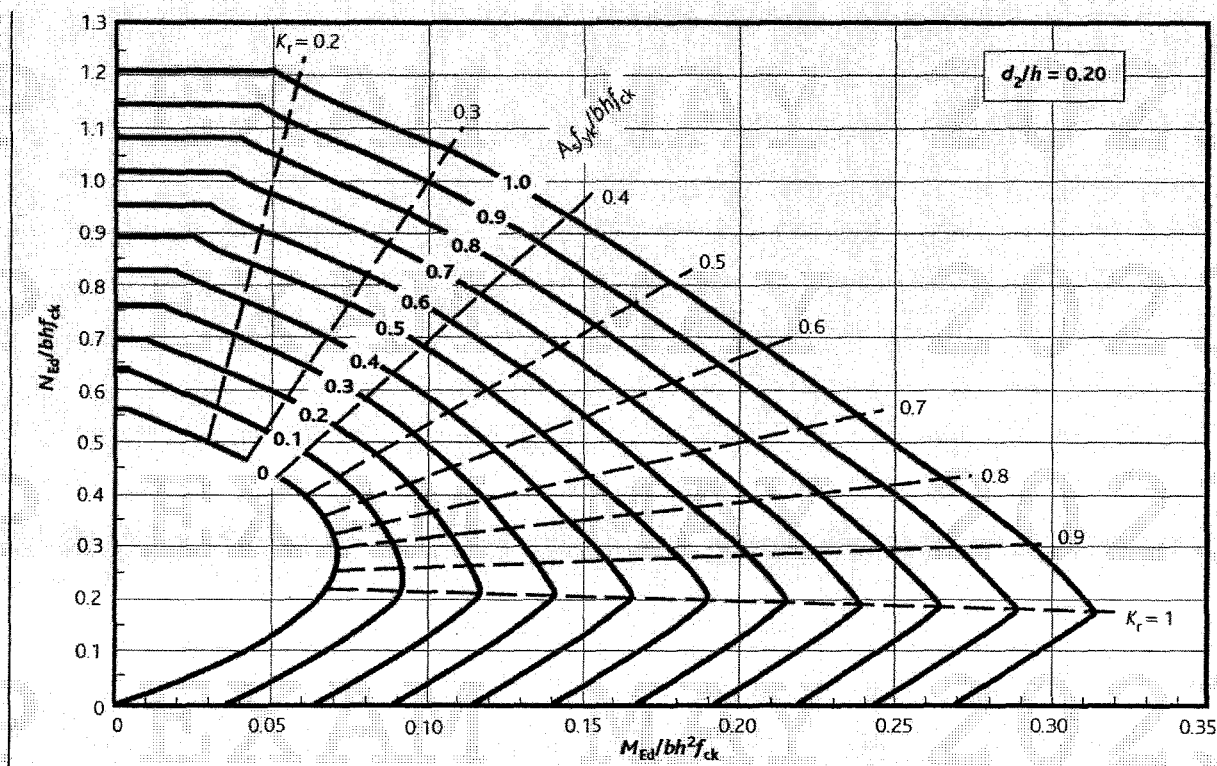
N_{Ed}/N_{Rd}	0.1	0.7	1.0
α	1.0	1.5	2.0

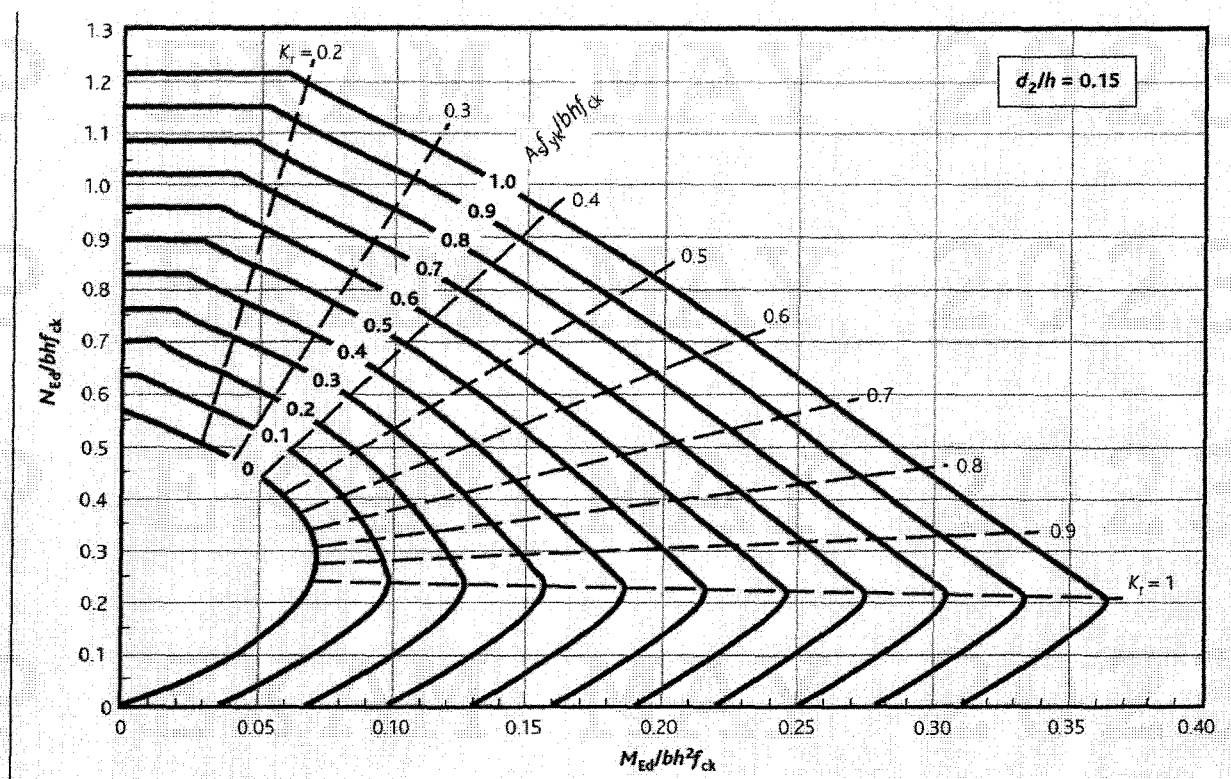
with linear interpolation for intermediate values

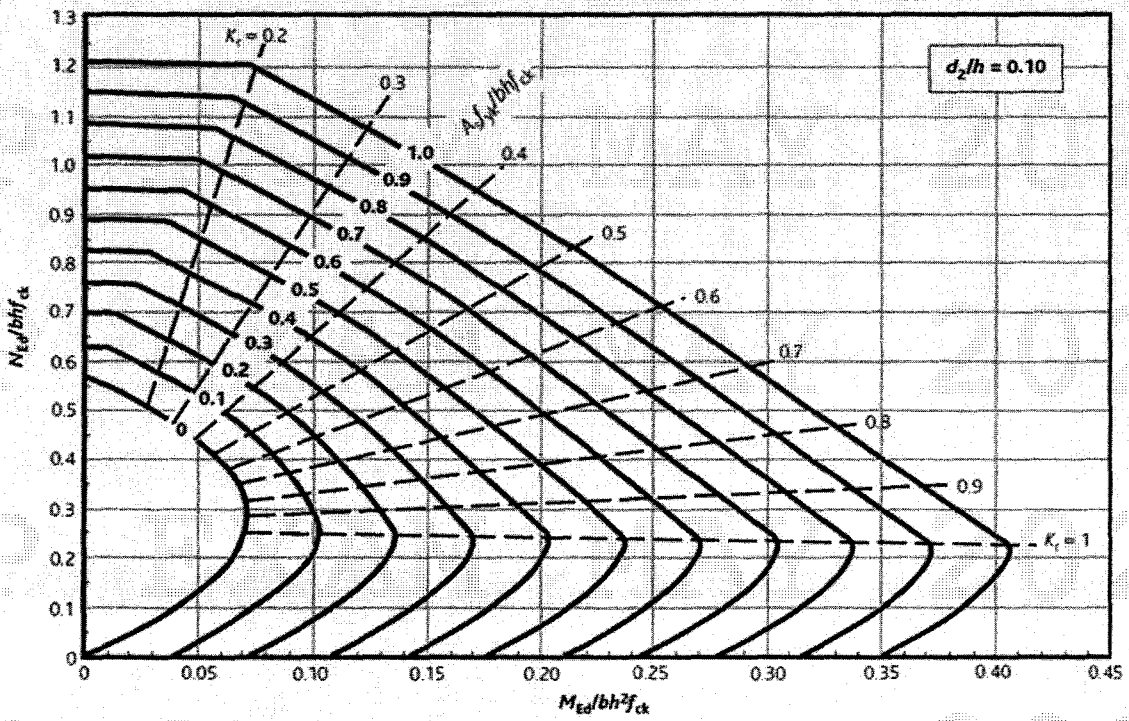
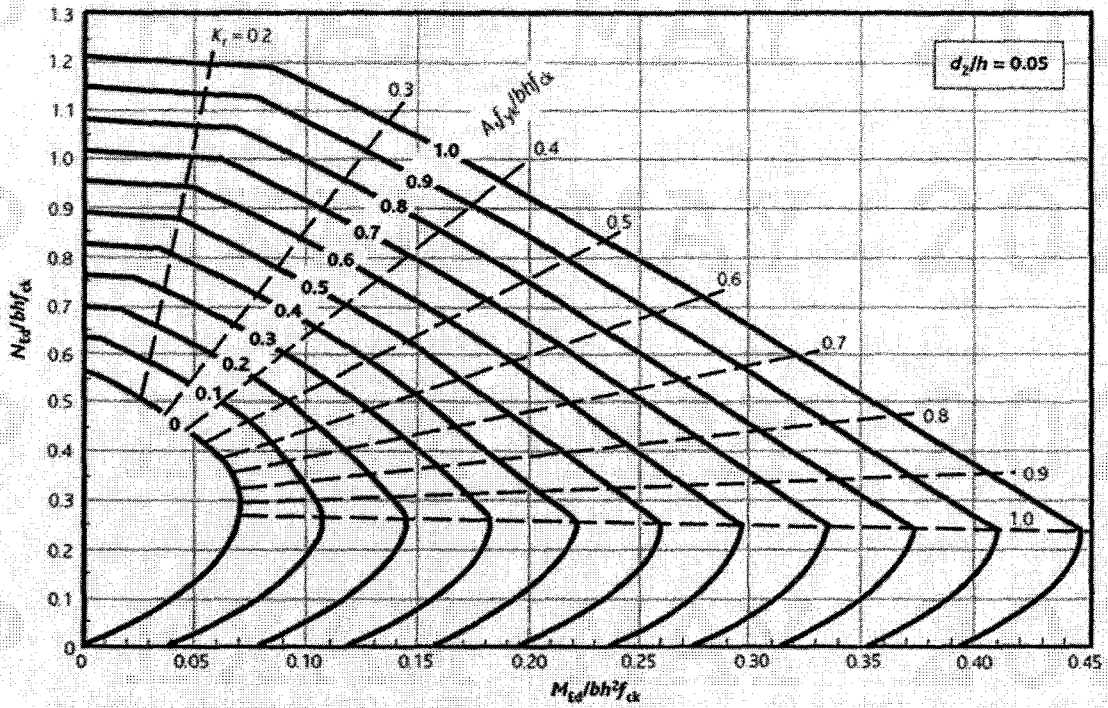
$N_{Rd} = A_c f_{cd} + A_s f_{yd}$, design axial resistance of section

A_c is the gross area of the concrete section

A_s is the area of longitudinal reinforcement







$$V_{Rd,max} = 0.5 \gamma_1 f_{cd} u d = 0.5 \gamma_1 \left(\frac{f_{ck}}{1.5} \right) u d$$

where:

u is the perimeter of the column

d is the effective depth of the footing

γ_1 is the strength reduction factor = $0.6 (1 - f_{ck}/250)$

$$V_{Rd,c} = v_{Rd,c} b d$$

$$v_{Rd,c} = C_{Rd,c} k (100 \rho_1 f_{ck})^{0.3333} \geq v_{min} = 0.035 k^{3/2} \sqrt{f_{ck}}$$

$$C_{Rd,c} = 0.18/\gamma_c = 0.12$$

$$k = 1 + \sqrt{200/d} \leq 2.0$$

$$\rho_1 = \frac{A_{sl}}{b_w d} \leq 0.02$$

Punching shear stress $v_{Ed} = \frac{V_{Ed}}{\text{perimeter} \times d}$

And

$$v_{Rd,c} = 0.12 K (100\rho_1 f_{ck})^{1/3} \geq v_{Rd,cmin} = 0.035 K^{3/2} f_{ck}^{1/2}$$

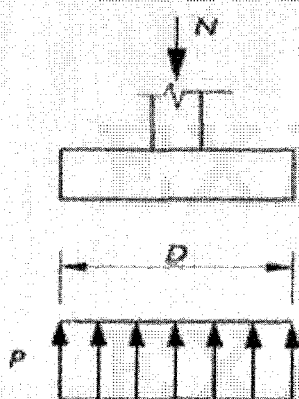
where

$$K = \left(1 + \sqrt{\frac{200}{d}} \right) \leq 2.0 \text{ with } d \text{ in mm}$$

$$\rho_1 = \frac{A_{s1}}{b_w d} \leq 0.02$$

Breadth of footing = B

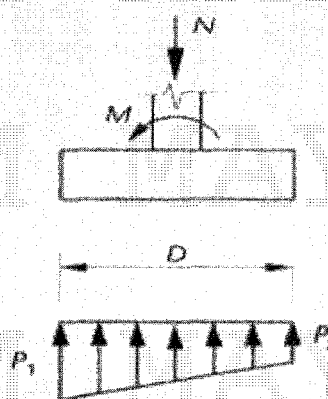
Eccentricity (e) = M/N



$$e = 0$$

$$p = \frac{N}{BD}$$

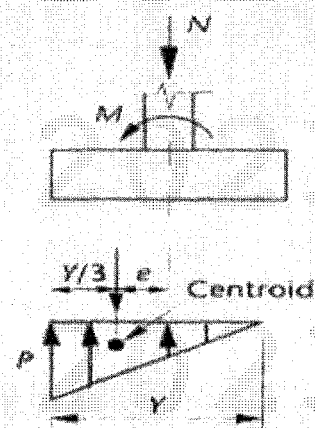
(a)



$$e \leq D/6$$

$$p = \frac{N}{BD} \pm \frac{6M}{BD^2}$$

(b)



$$e > D/6$$

$$p = \frac{2N}{BY}$$

where:

$$Y = 3 \left(\frac{D}{2} - e \right)$$

(c)

If $h_p \leq 550 \text{ mm}$: $h = 2h_p + 100$

If $h_p > 550 \text{ mm}$: $h = \frac{1}{3}(8h_p - 600)$

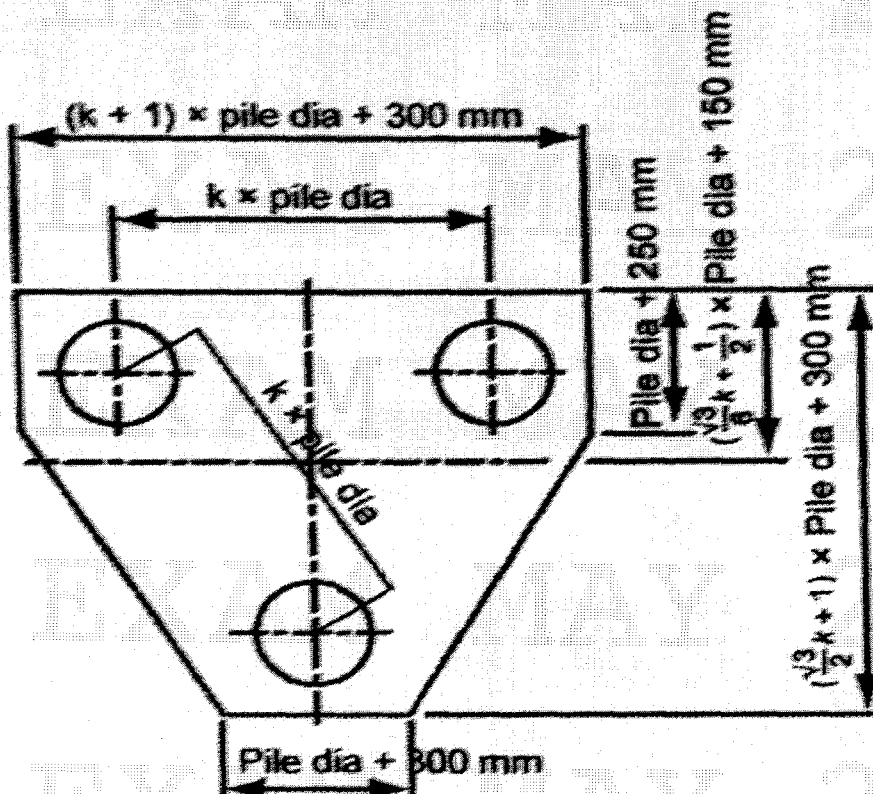
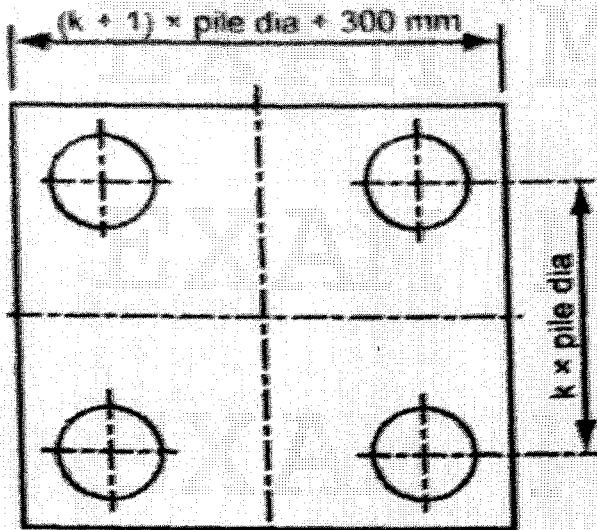

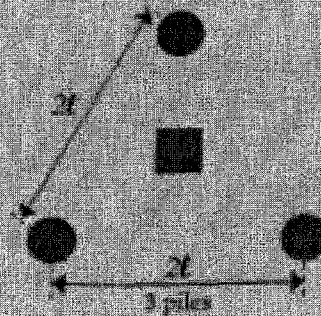
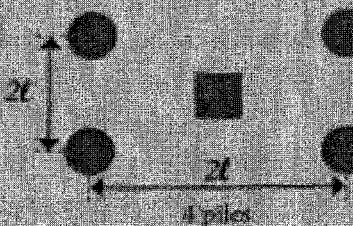
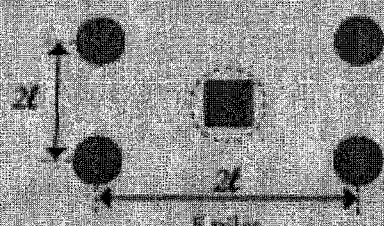
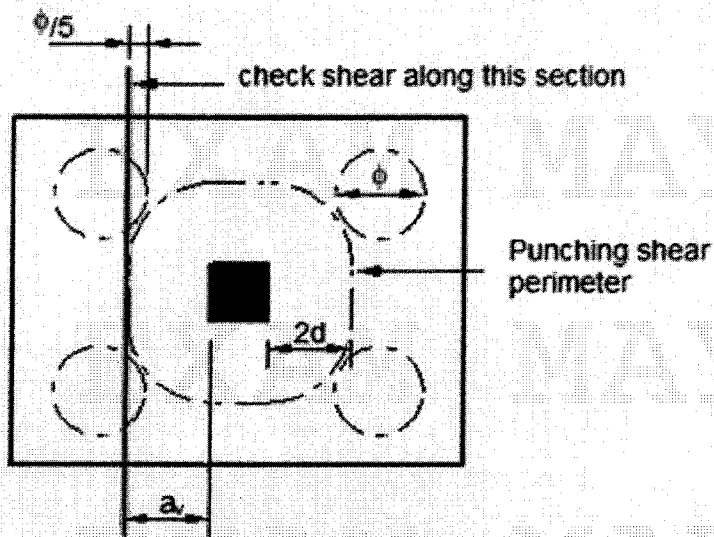


Table 8.1 Tensile force across pile cap

Pile Group	Tensile Force
	$T = \frac{N}{2l}$
	$T = \frac{2N}{3l}$
	$T = \frac{N}{l}$
	$T = \frac{0.8Nl}{4l}$



Shear enhancement

$$a_v / 2d \times V_{Ed}$$

CRACKING

Main bar :

$$s_{\max, \text{slabs}} = 3h \leq 400 \text{ mm}$$

Secondary bar :

$$s_{\max, \text{slabs}} = 3.5h \leq 450 \text{ mm}$$

Assume steel stress under quasi permanent loading ,

$$= 0.55 (\sigma_{yk} / 1.15) (A_{s, \text{req}} / A_{s, \text{prov}})$$

Class designation	Description of the environment	Informative examples where exposure classes may occur
1 No risk of corrosion or attack		
X0	For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry	Concrete inside buildings with very low air humidity
2 Corrosion induced by carbonation		
XC1	Dry or permanently wet	Concrete inside buildings with low air humidity Concrete permanently submerged in water
XC2	Wet, rarely dry	Concrete surfaces subject to long-term water contact Many foundations
XC3	Moderate humidity	Concrete inside buildings with moderate or high air humidity External concrete sheltered from rain
XC4	Cyclic wet and dry	Concrete surfaces subject to water contact, not within exposure class XC2
3 Corrosion induced by chlorides		
XD1	Moderate humidity	Concrete surfaces exposed to airborne chlorides
XD2	Wet, rarely dry	Swimming pools Concrete components exposed to industrial waters containing chlorides
XD3	Cyclic wet and dry	Parts of bridges exposed to spray containing chlorides Pavements Car park slabs

4 Corrosion induced by chlorides from sea water		
XS1	Exposed to airborne salt but not in direct contact with sea water	Structures near to or on the coast
XS2	Permanently submerged	Parts of marine structures
XS3	Tidal, splash and spray zones	Parts of marine structures
5. Freeze/Thaw Attack		
XF1	Moderate water saturation, without de-icing agent	Vertical concrete surfaces exposed to rain and freezing
XF2	Moderate water saturation, with de-icing agent	Vertical concrete surfaces of road structures exposed to freezing and airborne de-icing agents
XF3	High water saturation, without de-icing agents	Horizontal concrete surfaces exposed to rain and freezing
XF4	High water saturation with de-icing agents or sea water	Road and bridge decks exposed to de-icing agents Concrete surfaces exposed to direct spray containing de-icing agents and freezing Splash zone of marine structures exposed to freezing
6. Chemical attack		
XA1	Slightly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA2	Moderately aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA3	Highly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water

Table for determining final Structural Class – EC 02-1-1: Table 4.3N

Structural Class	Exposure Class according to Table 4.1						
	X0	XC1	XC2 / XC3	XC4	XD1	XD2 / XS1	XD3 / XS2 / XS3
Design Working Life of 100 years	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2
Strength Class ^{1) 2)}	≥ C30/37 reduce class by 1	≥ C30/37 reduce class by 1	≥ C35/45 reduce class by 1	≥ C40/50 reduce class by 1	≥ C40/50 reduce class by 1	≥ C40/50 reduce class by 1	≥ C45/55 reduce class by 1
Member with slab geometry (position of reinforcement not affected by construction process)	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1
Special Quality Control of the concrete production ensured	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1

Table 4.4N: Values of minimum cover, $c_{min,dur}$, requirements with regard to durability for reinforcement steel in accordance with EN 10080.

Structural Class	Environmental Requirement for $c_{min,dur}$ (mm)						
	X0	XC1	XC2 / XC3	XC4	XD1 / XS1	XD2 / XS2	XD3 / XS3
S1	10	10	10	15	20	25	30
S2	10	10	15	20	25	30	35
S3	10	10	20	25	30	35	40
S4	10	15	25	30	35	40	45
S5	15	20	30	35	40	45	50
S6	20	25	35	40	45	50	55

Table 5.1: Minimum cover, $c_{min,b}$, requirements with regard to bond

Arrangement of bars	Minimum cover $c_{min,b}$ *
Separated	Diameter of bar
Bundle	Equivalent diameter $\phi_n = \phi \sqrt{n_b} \leq 55 \text{ mm}$ Where n_b is the number of bars in the bundle, which is limited to $n_b \leq 4$ for vertical bars in compression $n_b \leq 3$ for all other cases
* If the nominal maximum aggregate size is > 32 mm, $c_{min,b}$ should be increased by 5 mm	

(Source: Table 4.2 EN 1992-1-1)

Table 6.12 Final creep coefficient of normal weight concrete ($\phi(\infty, t_0)$)

Age at loading (days)	Notional size ($2A_c/u$) mm							
	100	200	300	500	100	200	300	500
	Dry atmosphere (inside: 50% RH)				Humid atmosphere (outside: 80% RH)			
1	5.5	5.0	4.7	4.3	3.8	3.5	3.4	3.3
3	4.6	4.0	3.8	3.6	3.1	2.9	2.8	2.8
7	3.8	3.5	3.2	2.9	2.6	2.4	2.3	2.2
28	3.0	2.8	2.6	2.3	2.1	2.0	1.9	1.9
100+	2.7	2.5	2.3	2.1	1.9	1.8	1.7	1.6

Note: A_c = cross-sectional area of concrete, u = perimeter of that area exposed to drying.

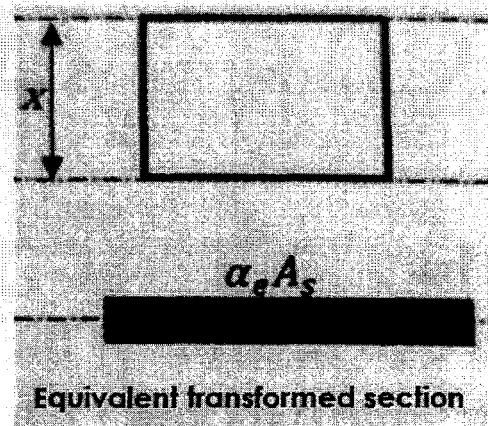
$$f_{ctm} = 0.3 f_{ck}^{2/3} \text{ (MPa)}$$

$$E_{cm} = 22 \left[\frac{(f_{ck} + 8)}{10} \right]^{0.3} \text{ kN/mm}^2$$

$$E_{c,eff} = \frac{E_{cm}}{1 + \phi(\infty, t_0)}$$

$$\alpha_e = \frac{E_s}{E_{c,eff}}$$

$$w_k = s_{r,max} (\epsilon_{sm} - \epsilon_{cm})$$



$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_f \frac{f_{ct,eff}}{\rho_{p,eff}} (1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

$$\alpha_e = \frac{E_s}{E_{cm}}$$

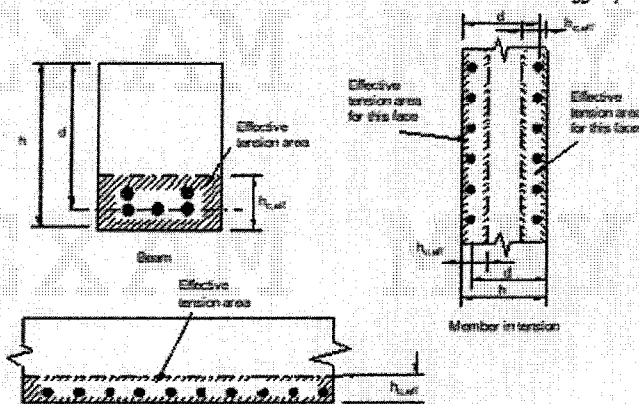
where σ_s is the stress in the tension steel calculated using the cracked concrete section. k_1 is a factor that accounts for the duration of loading (0.6 for short-term load, 0.4 for long-term load).

$$s_{r,max} = 3.4c + 0.425k_1k_2\phi / \rho_{p,eff}$$

k_1 is a coefficient accounting for the bond properties of the reinforcement (0.8 for high bond, 1.6 for plain bars)

k_2 is a coefficient accounting for the nature of the strain distribution which for cracking due to flexure can be taken as 0.5.

$\rho_{p,eff}$ is the effective reinforcement ratio, $A_s / A_{c,eff}$, where A_s is the area of reinforcement within an effective tension area of concrete $A_{c,eff}$.



$$h_{c,eff} = \text{lesser of } 2.5(h - d), (h - x)/3 \text{ or } h/2$$

Type of stress:

Axial Stress = P/A

Bending stress = M/z

