## Appendix Two

## Proof for equation of Measuring UPD

## 1. Proof for equation of Measuring UPD for BT

Assume that $h$ is the number of hops (they are represented with the black circles). The following cases are considered:
a. If $c=1 \rightarrow h=1$

b. If $c=2 \rightarrow h=3$

This is because if $\mathrm{c}=2$, this implies existing of one zone server according to our assumption that two or more cell servers need a zone server for resolving their conflicts.

Thus, $c=2 \leftrightarrow \mathrm{z}=1$

c. If $z=2 \rightarrow h=5$

Also, if $\mathrm{z}=2$, this implies existing of one master server for resolving their conflicts.

Thus, $\mathrm{z}=2 \leftrightarrow m=1$

d. For $m \geq 2$, we use mathematical induction as follows:

If $m=2 \rightarrow h=6$
This is because updates should be propagated in P2P manner in case of existing more than one master server since there is no higher level than the master level in our strategy. Update conflicts are resolved by delegating the responsibility of resolving to the next peer.


Accordingly, the If $\mathrm{m}=k \rightarrow h=k+4$
Thus, if $\mathrm{m}=k+1 \rightarrow h=(k+1)+4$

## 2. Proof for equation of measuring UPD for P2P-CONCENTRATE

By using mathematical induction and assuming $h$ is the number of hops, we consider the following cases:
a. $M H_{i}$ and $M H_{j}$ in the same cell

If $n=2 \rightarrow h=0$ (There are no hops between $M H_{i}$ and $M H_{j}$ )
$M H_{j}$

$M H_{i}$
If $n=3 \rightarrow h=1$


Accordingly, If $n=\mathrm{k} \rightarrow h=k-2$


Since the equation holds for $n=k$, this implies that:

$$
\text { If } n=k+1 \rightarrow h=(k-2)+1=(k+1)-2
$$

b. $M H_{i}$ and $M H_{j}$ in different cells in the same zone

$$
\text { If } c=2 \rightarrow h=2
$$



If $\mathrm{c}=k \rightarrow h=k$


Thus, if $c=k+1 \rightarrow h=k+1$
c. $M H_{i}$ and $M H_{j}$ in different zones in the same master area


If $\mathrm{z}=k \rightarrow h=k+2$


Thus, if $z=k+1 \rightarrow \mathrm{~h}=(\mathrm{k}+1)+2$
d. $M H_{i}$ and $M H_{j}$ in different master areas

The proof is performed in same manner as above.

