UNIVERSITI TEKNOLOGI PETRONAS

FINAL EXAMINATION
MAY 2024 SEMESTER

COURSE	:	**TEB1053/TFB1113 - DISCRETE MATHEMATICS**
DATE	:	**31 JULY 2024 (WEDNESDAY)**
TIME	:	**2:30 PM - 5:30 PM (3 HOURS)**

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions in the Answer Booklet.
2. Begin **EACH** answer on a new page in the Answer Booklet.
3. Indicate clearly answers that are cancelled, if any.
4. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions, if any.
5. **DO NOT** open this Question Booklet until instructed.

Note :

i. There are **SEVEN (7)** pages in this Question Booklet including the cover page and the Appendix.

ii. **DOUBLE-SIDED** Question Booklet.

1. a. Write these propositions in logical connectives.

   i. You get an A on the final, you do every exercise in the textbook, but you don't get an A in this class.

   [2 marks]

   ii. Getting an A on the final and doing every exercise in the textbook is sufficient for getting an A in this class.

   [2 marks]

   iii. You will get an A in this class if and only if you either do every exercise in the textbook or you get an A on the final.

   [2 marks]

   b. Construct a truth table for each of the following compound propositions and determine if the propositions are tautologies.

   i. $(p \rightarrow (q \wedge r) \leftrightarrow (p \rightarrow q) \wedge (p \rightarrow r)$

   [8 marks]

   ii. $(p \leftrightarrow q) \leftrightarrow (p \wedge q) \vee (\sim p \wedge \sim q)$

   [6 marks]

2. a. 34 farmers answered a questionnaire in which 18 said that they produce apples, 20 said they produce pears and 2 said that they produce neither. Determine the number that

i. produce both apples and pears.

[4 marks]

ii. produce only apples.

[2 marks]

iii. produce either only apples or only pears.

[2 marks]

b. Students need to answer 8 out of 10 questions in the Discrete Mathematics examination. Determine the number of ways a student can choose

i. the 8 questions.

[2 marks]

ii. 8 questions if the first three questions are mandatory.

[2 marks]

iii. 8 questions if at least 4 of the first 5 questions must be answered.

[2 marks]

c. Write the first 6 terms in the sequence defined by the recurrence equation and initial condition.

$$f_n = f_{n-1} - f_{n-2}, \qquad f_0 = 0,\ f_1 = 1$$

[6 marks]

3. a. Let $A = \{1, 2, 3, 4\}$. For each of the following three relations on A, prove or disprove that it is an equivalence relation and, if it is one, write down its equivalence classes.

i. $R_1 = \{(1, 1), (2, 2), (3, 4), (3, 3), (4, 4)\}$

[4 marks]

ii. $R_2 = \{(1,1),(2,2),(3,4),(4,4),(1,2),(2,1),(3,3),(4,3),(1,3),(1,4),(3,1),(4,1)\}$

[6 marks]

iii. $R_3 = \{(1,1),(2,2),(3,4),(3,3),(4,4),(1,2),(2,1),(4,2),(2,3)\}$

[4 marks]

b. Consider the set A= {2,7,14,28,56,84} and the relation $a \leq b$ if and only if a divides b. Give the Hasse diagram for (A, $\leq$).

[6 marks]

4. a. Graph A is represented by the following adjacency matrix.

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

i. Draw the graph, A.

[4 marks]

ii. Determine whether A is a tree. Justify your answer.

[2 marks]

iii. Determine whether A is Eulerian graph. Justify your answer.

[4 marks]

b. Use the Euclidean algorithm to compute the greatest common divisor of

i. (42, 101).

[3 marks]

ii. (3, 26).

[3 marks]

c. There is a total of 11 people which consist of five distinct women and six distinct men that are eligible for the election.

Calculate the number of ways to select

i. two women.

[2 marks]

ii. three men.

[2 marks]

5. a. Let $f(x) = (x^4 + 9x^3 + 4x + 7)$.

Determine the *big-* O function when $O(x^4)$.

[6 marks]

b. Let P(n) be the statement that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ for all positive integers n.

i. Calculate P(1).

[1 mark]

ii. Compute the basis step of the proof.

[2 marks]

iii. Determine the inductive hypothesis.

[4 marks]

iv. Compute the inductive steps.

[7 marks]

- END OF PAPER –

Appendix

1. $$p(E) = \frac{|E|}{|S|}$$

2. $$p(E \cap F) = p(E)\,p(F)$$

3. $$p(E|F) = \frac{p(E \cap F)}{p(F)}$$

4 $$p(F|E) = \frac{p(E|F)\,p(F)}{p(E|F)\,p(F) + p(E|\overline{F})\,p(\overline{F})}$$