UNIVERSITI TEKNOLOGI PETRONAS

FINAL EXAMINATION
MAY 2024 SEMESTER

COURSE	**:**	**TEB1093/TFB2083 - STATISTICS AND EMPIRICAL METHOD**
DATE	**:**	**30 JULY 2024 (TUESDAY)**
TIME	**:**	**9:00 AM - 12:00 NOON (3 HOURS)**

<u>INSTRUCTIONS TO CANDIDATES</u>

1. Answer **ALL** questions in the Answer Booklet.
2. Begin **EACH** answer on a new page in the Answer Booklet.
3. Indicate clearly answers that are cancelled, if any.
4. Where applicable, show clearly steps taken in arriving at the solutions and indicate **ALL** assumptions, if any.
5. **DO NOT** open this Question Booklet until instructed.

Note :

i. There are **EIGHTEEN (18)** pages in this Question Booklet including the cover page

ii. **DOUBLE-SIDED** Question Booklet.

1. a. Let revision of a software module-X is a normally distributed with a mean of 10 months and a standard deviation of 2 months. Find the probability that revision of software module-X lies between 11 months and 13.6 months.

[6 marks]

b. Quality for a computer memory chip is determine by it test scores. Assume that the test scores for all the available memory chips in the market are normally distributed with a mean of 77 and standard deviation of 8. If you invent a new computer memory chip and the test score must be at least 80% better than other available memory chip in the market before proceeds to the production line. Find the minimum test score you should achieve.

[4 marks]

c. A study was conducted to find the amount of time spent on doing study revision at library. Simple random samples of 50 boys and 75 girls were gathered. For boys, the average time spent was 20 hours, with a standard deviation of 3 hours. For girls, it was 17 hours, with a standard deviation of 2 hours. Assume that the two populations are independent and normally distributed. Find the 95% confidence interval for the revision time spending difference between boy and girl. Based on the confidence interval, conclude the difference of the revision time spending between boy and girl.

[10 marks]

2 a. A software quality assurance manager claimed that the mean user acceptance rate for one of their software-X was 80%. A random sample of $n = 100$ responses were collected and produced a sample mean of $\bar{x} = 79.7\%$ with standard deviation of $s = 8\%$. Do the data present sufficient evidence to refute the manager's claim? Let $\alpha = 0.01$.

i. State the hypotheses to be tested.

[2 marks]

ii. Use the critical value approach to conduct a statistical test of the null hypothesis and state your conclusion.

[4 marks]

iii. Use the p-value approach with $\alpha = 0.05$ to compare the conclusions from **part (a)(ii)**.

[4 marks]

b. An experiment was planned to compare the mean time (hours) required to complete a given programming challenge for a student with 4 hours of programming tutorial versus those who were not taking any programming tutorial. Suppose 35 students were randomly selected from each treatment category and that the mean completion times and standard deviations for two groups were as follows:

TABLE Q2: Mean Completion Times and Standard Deviations

	No Tutorial	4 Hours Programming Tutorial
Sample Mean	6.9	5.8
Sample Standard Deviation	2.9	1.2

i. Suppose your research objective is to show that programming tutorial reduces the mean time required to complete a programming exercise. Give the null and alternative hypotheses for the test.

[4 marks]

ii. Conduct the statistical test of the null hypothesis in **part (b)(i)** and state your conclusion. Test using $\alpha = 0.05$.

[6 marks]

3. Ten programmers have independently developed two different programs. They have measured the effort (in hours) that was required to develop the programs and the result is displayed as below:

TABLE Q3: Average Detected Bugs

Programmer	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Program 1	105	137	124	111	151	150	168	159	104	102
Program 2	86.1	115	175	94.9	174	120	153	178	70.3	110

a. Suppose the researcher wanted to detect a difference of effort required to develop the two programs. State the null and alternative hypotheses for the test.

[4 marks]

b. Give the critical value for the hypothesis testing at $\alpha = 0.05$

[2 marks]

c. Based on the value of the test statistics and critical value in **part (b)**, state your conclusions on whether you can reject the null hypothesis or not.

[7 marks]

d. State your conclusions based on the *p*-value and confidence interval of the test statistics in the results above.

[7 marks]

4. An experimenter is worried that the variability of responses using two different experimental procedures may not be the same. Before doing his research, he has conducted a pre-study with random samples of 16 and 20 responses and found sample variance $s_1^2 = 55.7$ and $s_2^2 = 31.4$, respectively.

a. Set up the null and alternative hypotheses.

[4 marks]

b. Is this one-tailed or two-tailed test? What is the critical value to reject the null hypothesis at α = 0.05.

[4 marks]

c. Do the sample variances present sufficient evidence to indicate that the population variances are unequal? Conduct the hypothesis test using α = 0.05.

[6 marks]

d. Approximate the p-value for the test. Does this confirm your conclusion in **part (c)**?

[6 marks]

5. Data below show the size of program wrote by a programmer in a specific period. The size of program is measured by number of modules, x and time to finish his program is measured by number of days, y.

TABLE Q5: Programmer's Productivity

Number of modules, x	10	20	30	40	49
Time in day, y	23	35	42	47	51

a. Assume that the number of modules, x and number of days, y are linearly related. Find the least-squares line relating y to x.

[10 marks]

b. Construct the ANOVA table for the linear regression

[5 marks]

c. Is module useful in predicting the number of days taken to finish the program? Use the appropriate statistical test and measures to explain the usefulness of the regression model for predicting time taken to finish the program.

[5 marks]

- END OF PAPER -

Formula Sheet

Arithmetic Mean	$\bar{x} = \frac{\sum x_i}{n}$
Population Variance	$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$
Sample Variance	$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$
Population Standard Deviation	$\sigma = \sqrt{\sigma^2}$
Sample Standard Deviation	$s = \sqrt{s^2}$
z-score	$z = \frac{x-\bar{x}}{s}$ $z = \frac{x-\mu}{\sigma}$
Permutations	$p_r^n = \frac{n!}{(n-r)!}$
Combination	$c_r^n = \frac{n!}{r!\,(n-r)!}$
Discrete random variable with probability distribution *p(x)*	$\mu = \sum xp(x)$ $\sigma^2 = \sum (x-\mu)^2 p(x)$ $\sigma = \sqrt{\sigma^2}$
Binomial experiment with *n* trials and probability *p* of success on a given trial	$\mu = np$ $\sigma^2 = npq$ $\sigma = \sqrt{npq}$
Standard Normal Sampling Distribution	$z = \frac{x-\mu}{\sigma}$ $z = \frac{x-np}{\sqrt{npq}}$ $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ $z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}$

Large Sample Point Estimators, Margin of Error, Confidence Interval

Parameter	$(1-\alpha)100\%$ Confidence Interval
μ	$\bar{x} \pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$
p	$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$
$\mu_1 - \mu_2$	$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
$p_1 - p_2$	$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$

Parameter	Point Estimator	Margin of Error
μ	$\bar{x}$	$\pm 1.96\left(\frac{s}{\sqrt{n}}\right)$
p	$\hat{p} = \frac{x}{n}$	$\pm 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$
$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$\pm 1.96\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
$p_1 - p_2$	$(\hat{p}_1 - \hat{p}_2) = \left(\frac{x_1}{n_1} - \frac{x_2}{n_2}\right)$	$\pm 1.96\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$

Large Sample Test Statistics

Parameter	Test Statistic
μ	$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
p	$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$
$\mu_1 - \mu_2$	$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
$p_1 - p_2$	$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

Small Sample Test Statistics

Parameter	Test Statistic	Degrees of Freedom
μ	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$n - 1$
$\mu_1 - \mu_2$ (equal variances)	$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$	$n_1 + n_2 - 2$
$\mu_1 - \mu_2$ (unequal variances)	$t \approx \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	Satterthwaite's approximation
$\mu_1 - \mu_2$ (paired samples)	$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$	$n - 1$
σ^2	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$n - 1$
σ_1^2/σ_2^2	$F = s_1^2/s_2^2$	$n_1 - 1$ and $n_2 - 1$

Analysis of Variance, Completely Randomized Design

$$\text{Total SS} = \Sigma x_{ij}^2 - \text{CM}$$
$$= (\text{Sum of squares of all } x\text{-values}) - \text{CM}$$

with

$$\text{CM} = \frac{(\Sigma x_{ij})^2}{n} = \frac{G^2}{n}$$

$$\text{SST} = \Sigma\frac{T_i^2}{n_i} - \text{CM}$$

$$\text{SSE} = \text{Total SS} - \text{SST}$$

and

G = Grand total of all n observations
T_i = Total of all observations in sample i
n_i = Number of observations in sample i
$n = n_1 + n_2 + \cdots + n_k$

Source	df	SS	MS	F
Treatments	k-1	SST	SST/(k-1)	MST/MSE
Error	n - k	SSE	SSE/(n-k)	
Total	n -1	Total SS		

Analysis of Variance, Randomized Block Design

$$\text{CM} = \frac{G^2}{n} \quad \text{where } G = \sum x_{ij}$$

$$\text{Total SS} = \sum x_{ij}^2 - \text{CM}$$

$$\text{SST} = \frac{\sum T_i^{\,2}}{b} - \text{CM} \quad \text{where } T_i = \text{ total for treatment } i$$

$$\text{SSB} = \frac{\sum B_j^{\,2}}{k} - \text{CM} \quad \text{where } B_j = \text{ total for block } j$$

$$\text{SSE} = \text{Total SS-SST-SSB}$$

Source	df	SS	MS	F
Treatments	k-1	SST	SST/(k-1)	MST/MSE
Blocks	b-1	SSB	SSB/(b-1)	MSB/MSE
Error	(b-1)(k-1)	SSE	SSE/(b-1)(k-1)	
Total	n-1	Total SS		

Correlation Coefficient, Regression Line, Least Squares Estimators

$$r = \frac{s_{xy}}{s_x s_y} \qquad s_{xy} = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{n-1}$$

$$S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} \qquad S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n} \qquad S_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

$$\hat{y} = a + bx \qquad b = \frac{S_{xy}}{S_{xx}} \qquad a = \bar{y} - b\bar{x}$$

$$Total\ SS = SSR + SSE \qquad Total\ SS = S_{yy} \qquad SSR = \frac{(S_{xy})^2}{S_{xx}}$$

$$F = \frac{MSR}{MSE}_{\ df=1,n-2} \qquad OR \qquad t = \frac{b-\beta_0}{\sqrt{MSE/S_{xx}}}_{\ df=n-2}$$

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

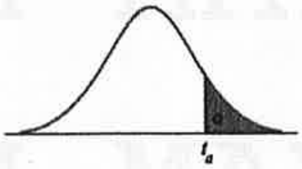
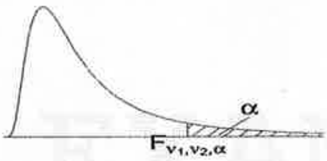


TABLE 4
Critical Values of t

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
∞	1.282	1.645	1.960	2.326	2.576	∞

Source: From "Table of Percentage Points of the t-Distribution," *Biometrika* 32 (1941):300. Reproduced by permission of the *Biometrika* Trustees.

PERCENTAGE POINTS OF THE F-DISTRIBUTION



The value given is $F_{\nu_1,\nu_2,\alpha}$ where $P(F_{\nu_1,\nu_2} > F_{\nu_1,\nu_2,\alpha}) = \alpha$ for the F-distribution with degrees of freedom ν_1 (numerator) and ν_2 (denominator).

Upper 5% points

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

ν_2 \ ν_1	12	15	20	24	30	40	60	120	∞
1	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.31
2	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1,76
24	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1,73
25	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Upper 2.5% points

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05

Upper 2.5% points continued

ν_2 \ ν_1	12	15	20	24	30	40	60	120	∞
1	976.71	984.87	993.10	997.25	1001.41	1005.60	1009.80	1014.02	1018.26
2	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	4.67	4.57	4.47	4.41	4.36	4.31	4.25	4.20	4.14
8	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
16	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
∞	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

Upper 1% points

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32

Upper 1% points continued

ν_2 \ ν_1	12	15	20	24	30	40	60	120	∞
1	6106.32	6157.28	6208.73	6234.63	6260.65	6286.78	6313.03	6339.39	6365.86
2	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

Upper 0.5% points

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10
1	16210.72	19999.50	21614.74	22499.58	23055.80	23437.11	23714.57	23925.41	24091.00	24224.49
2	198.50	199.00	199.17	199.25	199.30	199.33	199.36	199.37	199.39	199.40
3	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88	43.69
4	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14	20.97
5	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77	13.62
6	18.63	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.39	10.25
7	16.24	12.40	10.88	10.05	9.52	9.16	8.89	8.68	8.51	8.38
8	14.69	11.04	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21
9	13.61	10.11	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42
10	12.83	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85
11	12.23	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42
12	11.75	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09
13	11.37	8.19	6.93	6.23	5.79	5.48	5.25	5.08	4.94	4.82
14	11.06	7.92	6.68	6.00	5.56	5.26	5.03	4.86	4.72	4.60
15	10.80	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42
16	10.58	7.51	6.30	5.64	5.21	4.91	4.69	4.52	4.38	4.27
17	10.38	7.35	6.16	5.50	5.07	4.78	4.56	4.39	4.25	4.14
18	10.22	7.21	6.03	5.37	4.96	4.66	4.44	4.28	4.14	4.03
19	10.07	7.09	5.92	5.27	4.85	4.56	4.34	4.18	4.04	3.93
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85
21	9.83	6.89	5.73	5.09	4.68	4.39	4.18	4.01	3.88	3.77
22	9.73	6.81	5.65	5.02	4.61	4.32	4.11	3.94	3.81	3.70
23	9.63	6.73	5.58	4.95	4.54	4.26	4.05	3.88	3.75	3.64
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59
25	9.48	6.60	5.46	4.84	4.43	4.15	3.94	3.78	3.64	3.54
26	9.41	6.54	5.41	4.79	4.38	4.10	3.89	3.73	3.60	3.49
27	9.34	6.49	5.36	4.74	4.34	4.06	3.85	3.69	3.56	3.45
28	9.28	6.44	5.32	4.70	4.30	4.02	3.81	3.65	3.52	3.41
29	9.23	6.40	5.28	4.66	4.26	3.98	3.77	3.61	3.48	3.38
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12
60	8.49	5.79	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52

Upper 0.5% points continued

ν_2 \ ν_1	12	15	20	24	30	40	60	120	∞
1	24426.37	24630.21	24835.97	24939.57	25043.63	25148.15	25253.14	25358.57	25464.46
2	199.42	199.43	199.45	199.46	199.47	199.47	199.48	199.49	199.50
3	43.39	43.08	42.78	42.62	42.47	42.31	42.15	41.99	41.83
4	20.70	20.44	20.17	20.03	19.89	19.75	19.61	19.47	19.32
5	13.38	13.15	12.90	12.78	12.66	12.53	12.40	12.27	12.14
6	10.03	9.81	9.59	9.47	9.36	9.24	9.12	9.00	8.88
7	8.18	7.97	7.75	7.64	7.53	7.42	7.31	7.19	7.08
8	7.01	6.81	6.61	6.50	6.40	6.29	6.18	6.06	5.95
9	6.23	6.03	5.83	5.73	5.62	5.52	5.41	5.30	5.19
10	5.66	5.47	5.27	5.17	5.07	4.97	4.86	4.75	4.64
11	5.24	5.05	4.86	4.76	4.65	4.55	4.45	4.34	4.23
12	4.91	4.72	4.53	4.43	4.33	4.23	4.12	4.01	3.90
13	4.64	4.46	4.27	4.17	4.07	3.97	3.87	3.76	3.65
14	4.43	4.25	4.06	3.96	3.86	3.76	3.66	3.55	3.44
15	4.25	4.07	3.88	3.79	3.69	3.58	3.48	3.37	3.26
16	4.10	3.92	3.73	3.64	3.54	3.44	3.33	3.22	3.11
17	3.97	3.79	3.61	3.51	3.41	3.31	3.21	3.10	2.98
18	3.86	3.68	3.50	3.40	3.30	3.20	3.10	2.99	2.87
19	3.76	3.59	3.40	3.31	3.21	3.11	3.00	2.89	2.78
20	3.68	3.50	3.32	3.22	3.12	3.02	2.92	2.81	2.69
21	3.60	3.43	3.24	3.15	3.05	2.95	2.84	2.73	2.61
22	3.54	3.36	3.18	3.08	2.98	2.88	2.77	2.66	2.55
23	3.47	3.30	3.12	3.02	2.92	2.82	2.71	2.60	2.48
24	3.42	3.25	3.06	2.97	2.87	2.77	2.66	2.55	2.43
25	3.37	3.20	3.01	2.92	2.82	2.72	2.61	2.50	2.38
26	3.33	3.15	2.97	2.87	2.77	2.67	2.56	2.45	2.33
27	3.28	3.11	2.93	2.83	2.73	2.63	2.52	2.41	2.29
28	3.25	3.07	2.89	2.79	2.69	2.59	2.48	2.37	2.25
29	3.21	3.04	2.86	2.76	2.66	2.56	2.45	2.33	2.21
30	3.18	3.01	2.82	2.73	2.63	2.52	2.42	2.30	2.18
40	2.95	2.78	2.60	2.50	2.40	2.30	2.18	2.06	1.93
60	2.74	2.57	2.39	2.29	2.19	2.08	1.96	1.83	1.69
120	2.54	2.37	2.19	2.09	1.98	1.87	1.75	1.61	1.43
∞	2.36	2.19	2.00	1.90	1.79	1.67	1.53	1.36	1.00