MODAL ANALYSIS OF A TENNIS RACQUET

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the

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in partial fulfilment of the requirement for the

BACHELOR OF ENGINEERING (Hons)

(MECHANICAL ENGINEERING)

Approved by,

____

(Ir Dr Mohd Amin bin Abd Majid)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

September 2012

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

MOHD AMIN BIN HARON

ABSTRACT

This project is carried out to investigate the behavior of tennis racquet with varying parameters and to obtain the modal characteristics of a tennis racquet. Modal analysis is the study of the dynamic properties of structures under vibrational excitation. In this project, the structure put to the test is a tennis racquet. Tennis racquet is subjected to a force from the velocity of the tennis ball when it is hit. This paper presents structural and modal analysis for a tennis racquet under certain assumptions which had been set. The scope of the project has been narrowed down to develop modal parameters of a tennis racquet with respect to natural frequency and mode shape. This project also examined the behavior of the model subjected to different forces at different locations with varying materials. In this project, ANSYS software was used for the analysis on the tennis racquet. The methods used in this project were structural and modal analysis, both using ANSYS for modeling and simulation. The findings are reported and based on the data; the natural frequency of the model is obtained and compared to the theoretical values. Recommendation is pointed out as the outcome of the project which can be further continued by selecting maximum values of the racquet standard as well as conducting analysis on more different materials.

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CHAPTER 1

PROJECT BACKGROUND

1.1 Background of Study

Tennis is one of the most widely-played sports which are enjoyed by players of all ages around the world. In a tennis match, each player uses a racquet to hit the ball into opponent's side of the court. A tennis racquet might look like simple but there is actually a deep study to produce a good tennis racquet. The design and material of the racquet are important in order to determine its quality.

Up until the 1970's, virtually everyone engaged in racket sports was using wooden rackets with leather gripped handles and natural gut strings [1]. The introduction of aluminum and steel frames paved the way for increasingly lightweight and highly durable materials. Today most racket frames are made from light-weight graphite or graphite composites that incorporate materials such as titanium, kevlar or fiberglass, giving added levels of frame flexibility, while remaining cost effective.

As the sporting goods industry has grown way more competitive nowadays, it has become increasingly important for manufactures to come up with a better product to offer. That is because a tennis racquet involves with varies impact or vibration when hitting the ball which might affect the performance of the player [2]. With a good racquet, the impact could be reduced significantly which is crucial for all tennis players especially the top ones competing in major tournaments. Therefore, this project of modal analysis of a tennis racquet is conducted to obtain the dynamic characteristics for a tennis racquet.

1.2 Problem Statement

The tennis racquet has been designed to ensure the structure can withstand heavy impact force by a tennis ball during serve as well as normal play. Some distortion may happen on the structure due to the impact [3]. The racquet is also designed to ensure less vibration will be transmitted to the players hand by properly sizing the sweet spot which is the area when hit by the ball will transmit less vibration [4]. Sweet spot on the tennis racquet is a spot where minimum vibration is transmitted to the player's hand. It can be located anywhere on the longitudinal axis between the tip and throat, depending on the incident speed of the ball [5].

Unfortunately the data is not easily accessible to the general public for further research. Thus, this project is undertaken to develop modal parameters of a tennis racquet and examine its dynamic characteristics using finite element analysis modeling method. This project's significance is that the analysis will provide the dynamic characteristics of the racquet which can be used to make advancement in the model for a better tennis racquet. This advancement of the model also includes the recommendation for the best material and damping required to lessen the vibration.

1.3 Objective of Project

The objective of this project is to study the dynamic characteristics of a tennis racquet with modal analysis using different types of materials, forces and at various locations.

1.4 Scope of Study

The scope of study for this project includes:

- Develop modal parameters of a tennis racquet using finite element analysis (FEA) modeling method.
- Examine the dynamic characteristics of the tennis racquet subjected to force at various locations.
- Examine different materials for the analysis with varying forces.

1.5 Relevancy of Project

This project of modal analysis is closely related to vibration, which is one of the core subjects in Mechanical Engineering programme. Thus, the knowledge in vibration learnt before this is used back and can be related to this Final Year project. Besides that, the modeling of the tennis racquet is done in ANSYS software. This software is widely used nowadays and it will help to improve skills in handling this software as it might be useful when working in the future. It will not be a difficult task as the basic knowledge on ANSYS has been learnt in Computer Aided Engineering Design course earlier.

1.6 Feasibility of Project

Due to limitation of time, it is not feasible to perform this project experimentally. However, with the help of compatible software such as ANSYS, it is feasible to perform a numerical simulation and modeling on the racquet tennis as well as all of the required analysis. Besides that, the scope of the project has also been narrowed down in order to make sure that the project will be completed within the required time frame. Overall, the project is justified to be completed within the time frame.

CHAPTER 2

LITERATURE REVIEW

2.1 Vibration Theory

Generally, vibration is a mechanical oscillation about a reference point. Vibration can be defined as an oscillation wherein the quantity is a parameter defining the motion of a mechanical system [6]. In a basic vibration theory, every system has a stable position in which all forces are equivalent, and when this equilibrium is disturbed, the system will try to regain its stable position [7]. In order to remain stable, any structure exhibits vibration at different magnitude when excited, the degree of vibration varies from point to point, due to the variation of dynamic responses of the structure and the load applied.

The types of loading can be classified into four categories based on the nature of the load and time [8]. Those four loadings are shown below:

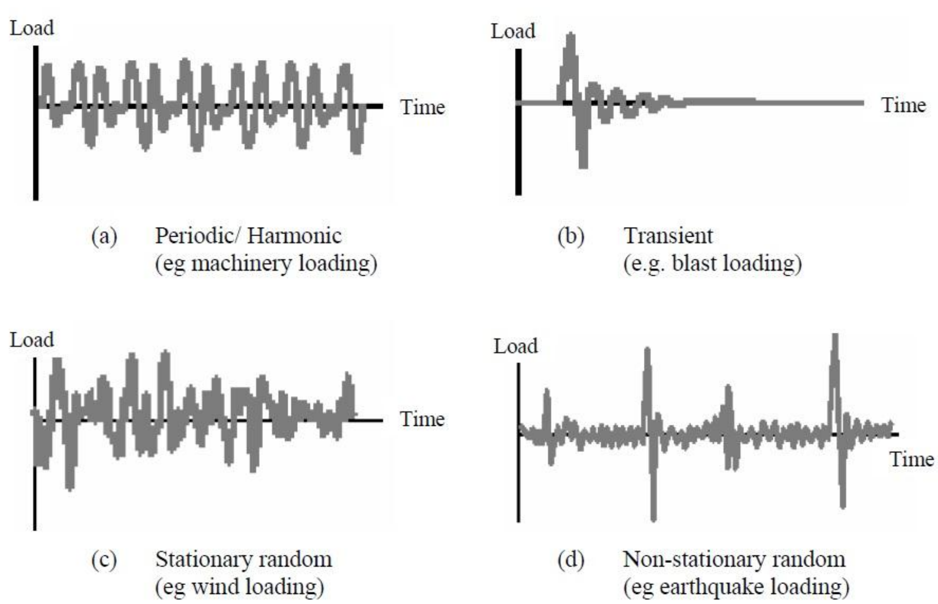

Figure 2.1: Dynamic loading based on time-domain representation [6]

Maguire J.R. et al. [6] state that the periodic/ harmonic type has constant amplitudes and repeats itself regularly numerous times, or to an infinite extend if damping is absent. It is also known as sinusoidal. Machinery loading is one of the examples to construct this type of loading. Transient loading varies with time and does not repeat itself continuously.

This type of loading happens suddenly and often with a high amplitude. Stationary random is a type of loading that does not have precise magnitude; its statistical properties vary only very slowly. One example of this type is wind loading. Non-stationary random on the other hand is much similar as stationary random, except that the statistical properties vary more rapidly.

2.2 Modal Analysis Concept

Modal analysis studies the dynamic properties or structural characteristics of a mechanical structure under dynamic excitation which are resonant frequency, mode shapes and damping [11]. The resonant frequencies of a structure need to be identified and quantified to better understand any structural problem.

Natural frequency is the frequency at which a system naturally vibrates once it has been set into motion. In other words, natural frequency is the number of times a system will oscillate (move back and forth) between its original position and its displaced position, if there is no outside interference [12]. For example, consider a simple beam fixed at one end and having a mass attached to its free end, as shown in Figure 2.2. If the beam tip is pulled downward, then released, the beam will oscillate at its natural frequency.

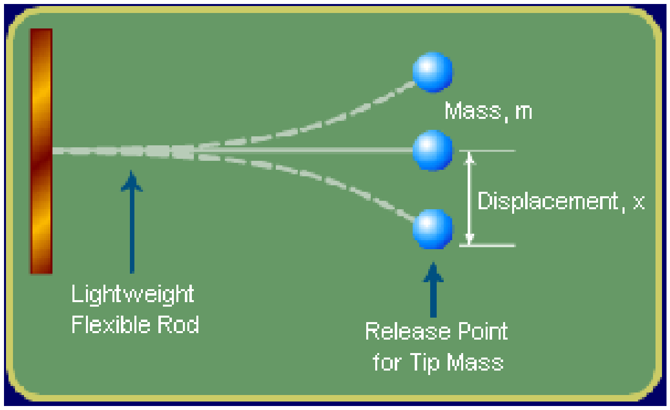

Figure 2.2: Example of natural frequency phenomenon [18]

If the mass weighs much more than the beam to which it is attached, the natural frequency can be calculated using the formula:

$$f = \frac{1}{2\Pi}\sqrt{\frac{k}{m}} \qquad \text{(Eq. 2.1)}$$

where f is the natural frequency, k is the beam stiffness and m is the mass of the weight attached at the end of the flexible rod.

The frequencies of the modes and the mode shapes can also be derived from Euler-Bernoulli Beam Theory. The formula is shown below;

$$\omega_n = \alpha_n{}^2\sqrt{\frac{EI}{ml^3}} \qquad \text{(Eq. 2.2)}$$

Where ω_n = natural frequency
E = Young's Modulus
I = second moment of area
M = mass
l = length of the beam

Modal analysis has become a widespread means of finding the modes of vibration of a structure [13]. In every development of a new or improved mechanical product, structural dynamics testing on prototype is used to assess its real dynamic behavior.

The modal parameters or dynamic characteristics occur in all structures due to the fact that all structures have a mass, including a tennis racquet. Observing the dynamic responses of a structure under its natural frequencies, one could determine the point of weakness on the structure [5]. That is because damaged structure inclines to have shown excessive deformation under the same magnitude of excitation. The dynamic characteristics of any structure can also portray whether the resonant vibration is occurring or not.

Even though mode shape variations demonstrate some of the effects of the vibration dampers, a better way involves the transmission of vibrations through the racquet itself. The transmission of vibrations from an impact on the sweet spot to several points on the handle of the racquet can be calculated to investigate the vibration transferred to the player. The vibration transmissibility, T_{pq} is calculated by the following equation:

$$T_{pq} = \frac{H_{pq}}{H_{qq}} \qquad \text{(Eq. 2.3)}$$

where H_{pq} is the Frequency Response Function (FRF) for striking the racquet at the handle and receiving at the sweet spot and H_{qq} is the FRF for the drive point measurement at the sweet spot.

2.3 Finite Element Method (FEM)

Finite element is a numerical technique used to obtain approximate solutions of boundary value problems in engineering. Boundary value problem is a mathematical problem in which one or more dependent variables must satisfy a differential equation everywhere within a known domain of independent variables and satisfy specific conditions on the boundary of the domain [14].

Finite element method is to determine solutions only at some finite locations instead of providing the infinite number of solutions as in the exact solutions. This is done by first discretizing the geometry of the model into a number of finite elements. In FEM, the force acting between the nodes cannot be determine since only points on the nodes or grid are able to be measured.

A model will have a lot of nodes after being discretized as shown in Figure 2.3. These finite elements are connected at grid points or nodes at which the unknowns are to be determined.

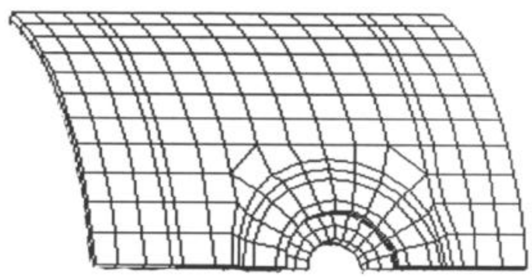

Figure 2.3: A finite element model [18]

The key idea of the finite element method is to transform the differential equations into a set of algebraic equations for each element. The finite element equations from all elements are then combined together to form a large set of simultaneous equations. The boundary conditions of the problem are applied prior to solving for the unknowns at all nodes.

As noted by researcher [11], finite element analysis of real structures, the actual structure is broken down into many small pieces of various types, shapes and sizes. They solve the field with discrete model. The field variables may include temperature, vibration and also displacement.

The finite element method is widely used nowadays for analysis and design of new products as the method is suitable for problems with complex geometry. The method has been applied to analyze problems in different fields such as in solid, structures and fluid flows. Today, the finite element method has played an important role in the sporting goods industry including the analysis of tennis racquet.

There are few proposed models in which the tennis racquet is going to be designed. The first one is assuming the racquet handle is a rigid cantilever beam and the force is exerted to the beam in downwards direction. This is shown in Figure 2.3 where the force comes from the velocity of the tennis ball. An object is taken as a rigid body when its size is considerable in the context and rotation of the object is also to be taken in analysis. In this case, the tennis racquet will be taken as a rigid body.

A rigid body, as the name tells, does not deform, that is, the positions of all the points on the rigid body remain fixed relative to each other even when a force is applied [17]. There is no physical body which is rigid. Rigid body is only an approximation, applicable when the deformations are very small relative to the size of the body or are not important for analysis.

Rigid body assumption simplifies the analysis as the material properties of the object will not have to be considered for the analysis. For example, to find the load on the supports of a structure it can be considered as a rigid body. Though this structure and its supports deform but to find the load at supports the deformation need not be considered. The consideration of an object as a rigid body or a particle does not change the object or its behavior, what changes is the scope and the method of its engineering mechanics analysis.

The second one is the racket handle is assumed to be a flexible rod in which it can deform when load or force is applied to it. This is different than the previous proposed model because the flexible cantilever beam can bend according to how much force is exerted on it. This concept is illustrated in Figure 2.2 with the natural frequency can be calculated using Eq. 2.1.

2.4 Tennis Racquet Analysis

The vibration that occurs when a player hits the tennis ball to the opposite direction is transmitted to the player's hand. There has been debate for many years as to whether the hand plays a significant or a negligible role in determining the dynamics of the impact of a racquet and ball. The collision of a tennis ball with a racquet can be modeled according to researches [14] [15] which studied the effects of grip firmness on the coefficient of restitution (COR). In these studies, a ball was projected onto the strings of a racquet and the ball rebound speed was measured under various grip conditions and for impacts at several different locations on the strings.

Early racquets were made of wood, which was not as good as modern material nowadays, since wood has inconsistencies which results in different feels when striking the ball. Later designs used metals, experimenting with metals such as aluminum, magnesium and titanium. The advancement in technology sees that recent tennis racquets are made by materials such as boron, ceramics, graphite and composites due to their lightweight properties but strong to withstand the impact of the tennis ball. Each material had its own desirable qualities but ceramics and graphite were the best picks for being very stiff as well as being very good with vibration reduction.

Despite the available testing technology, setting up a materials testing regime to faithfully replicate the dynamic stresses and distortions experienced by racket frames during a game is a real challenge [1]. The compression of the frame, the flexibility of the head, the torsional twist of the racket in the hand upon impact - are all dynamic conditions that affect performance. Thus, it is important for the players to choose the best tennis racquet for them.

The tennis racquet from different manufacturer has a different structure as well as its material but the overall look of a modern tennis racquet is almost similar. A tennis racquet consists of three main parts which are handle, throat and head. Modern rackets are made in a variety of shapes and lengths, and testing grips can be adapted to accommodate that. A common tennis racquet nowadays is illustrated in the Figure 2.4 below:

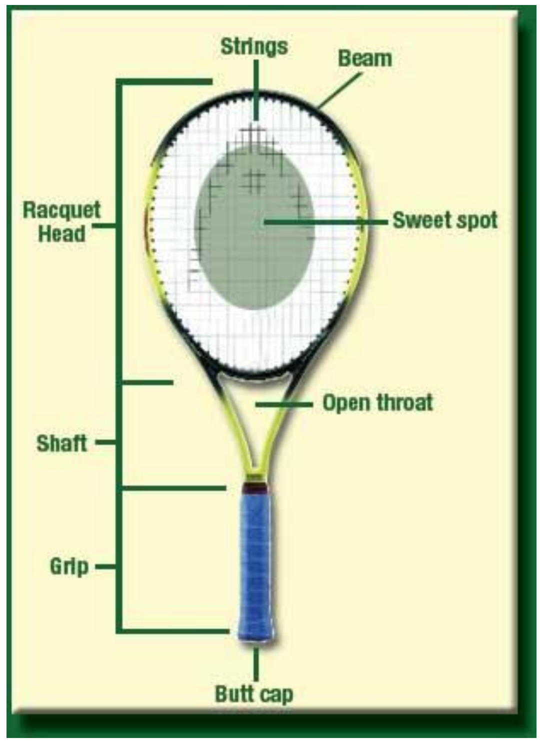

Figure 2.4: Common parts of a tennis racquet [9]

Based on the figure of the racquet, a tennis racquet can be categorized into three main parts which are head, shaft and grip. The head of the racquet is where the ball must be hit to properly served and returned: it is where the strings are contained. Specifically within the head area is an area within the strings known as the "sweet spot" - the area of the strung surface that creates the most amount of power with the least amount of effort [16]. The sweet spot is where the player aims to always hit the ball, unless deliberately trying to mishit the ball after charging the net in order to barely touch the ball.

The shape and size of the tennis racquet head can vary and be classified in the following three categories [19]:

- Over-sized racquet

  A type of racquet mostly used by the beginners who appreciate the greater string area of over-sized racquets, even jumbo racquets are available to cater with the needs. These racquets can present a string area between 100 (over-sized) and a huge 140 square inches. The string area is larger; the same goes to the sweet spot which makes it suitable to be used as the training racquet.

- Mid-sized racquet

  Mid-sized racquet heads range between 85 to 100 square inches of string area. Most professional tennis players use head sizes that are standard or mid-sized in design, averaging a range more closely within 85 to 95 square inches. The benefit of a smaller racquet head size is greater maneuverability and speed in swing. Therefore, they are considered a great compromise providing the dual benefits of a larger sweet spot, while still allowing competitive speed and handling of the racquet.

- Standard-sized racquet

  Ranging in size from 80 to 85 square inches, standard-sized racquet head designs are now considered "old school" or obsolete, given the downside created by a smaller sweet spot and more limited flexibility in shot strategy.

The beam of the racquet on the other hand is the area on either side of the head. It does not increase the overall length or width of the racquet, but is considered its thickness. Comparing racquets can be done by placing the racquets flat on a table, and it can be seen that their designs may differ in that some have wider beams than others. Wider beams can add power to the shots, however many say that a wider beam affects how the strings are contained and, therefore, how they perform. This creates a greater flex, or trampoline effect that can affect control and direction of shots.

The open throat design has become a design standard in most of today's racquets, eliminating single main shaft directly attached to the racquet head as can be observed in badminton racquet. The open throat design was created to better stabilize the racquet head, better compensating for off-center shots and, along with the larger head areas, has worked to enlarge the effective sweet spot.

Meanwhile, the shaft of the racquet is the point at which the two sides curve down from the throat, where they come closest to extend down directly to connect to the racquet handle. The throat, as it goes down, becomes the shaft, and then joins to the racquet handle.

Grip is located at the end of the shaft, and it joins with the racquet grip. Different grip sizes are made available for both hand size and best comfort preference. Grips range in diameter between 4 and 4-5/8 inches. Choosing the size of grip is important to make sure it has the right feel and will not affect the performance of the player. All of those parts mentioned play a part in the performance function but there is the last part of the tennis racquet that does not involved – the butt cap. It simply provides closure to the racquet handle and creates a convenient place for placement of manufacturer logos.

Tennis racquets nowadays are made from various types of materials. Among the well-known materials are graphite, boron, Kevlar, composites, aluminium, and titanium. Vast majority of racquets manufactured today use graphite in one form or another as the base ingredient [19]. Graphite is the technological generation's equivalent of the trusty laminated wooden racquet that was so popular until about the 1970s. Graphite is remarkably strong for its relatively light weight. It provides terrific power, as well as good control and feel for the ball.

Both Boron and Kevlar fibers both resemble graphite, but boron and Kevlar are even lighter and stiffer than graphite. Kevlar is best known for its use to make bulletproof vests. Unless mixed with other materials, however, Kevlar's stiffness can transmit a lot of shock and vibration to the arm and shoulder, especially if the player hits the ball off the sweet spot.

Aluminum on the other hand is still used in less expensive racquets. Aluminum offers decent power and a surprising amount of feel. Feel is the sensation players get for how they are striking the ball and where it is going. Some racquet materials are more sensitive than others to things like impact and vibration, so they transmit in a different way.

More recently, a new technology has emerged in the manufacturing of tennis racquets - titanium. Made from a very strong, extremely light material, titanium has been a hit with professionals and serious recreational tennis players. Titanium is similar to aluminum. Either aluminum or titanium is an acceptable choice for beginners.

In January 1997, the International Tennis Federation introduced manufacturing guidelines covering design dimensions. It is the standard of any tennis racquet to be used for any tennis tournament. While a modern 135 square inch ($340cm^2$) head on a 29 inch (73.5cm) long racket remains legal, it is still twice the head size of the older wooden rackets. This has allowed manufacturers to open up the world of tennis to a wider market.

Another model of the effect of the arm on racquet dynamics is shown in Figure 2.5 below:

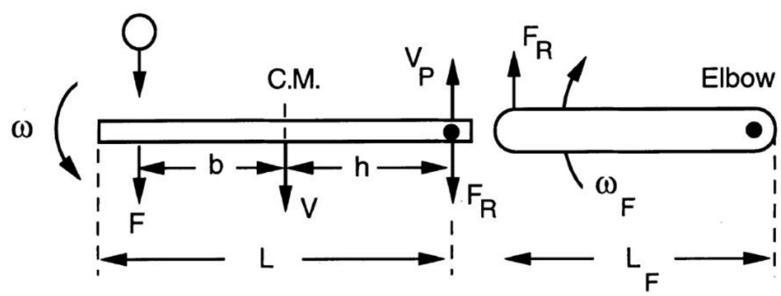

Figure 2.5: Diagram of racquet pivoted at wrist and forearm pivoted at the elbow [16]

Based on the model by Cross R. [16], the racquet is assumed as a beam of mass M and length L connected by a pivot joint to the forearm. It can be assumed that the other end of the forearm is pivoted about the elbow, but it is assumed for simplicity that the elbow does not translate during the impact. The velocity of the tennis ball is the force applied on the structure to investigate the dynamic characteristics of the tennis racquet.

2.5 Summary

All of the concept of vibration, modal analysis and finite element method will be adopted in this project. All of them are useful to conduct the modal analysis of a tennis racquet and those concepts will be further expanded in this project.

CHAPTER 3

METHODOLOGY

3.1 Research Methodology

The methodology starts with problem identification, research for literature review on vibration, modal analysis, finite element analysis and tennis racquet analysis, followed by modeling/simulation, varying the parameters, comparison of results, post-processing results, conclusion and finally recommendation. Figure 3.1 shows the methodology of the project:

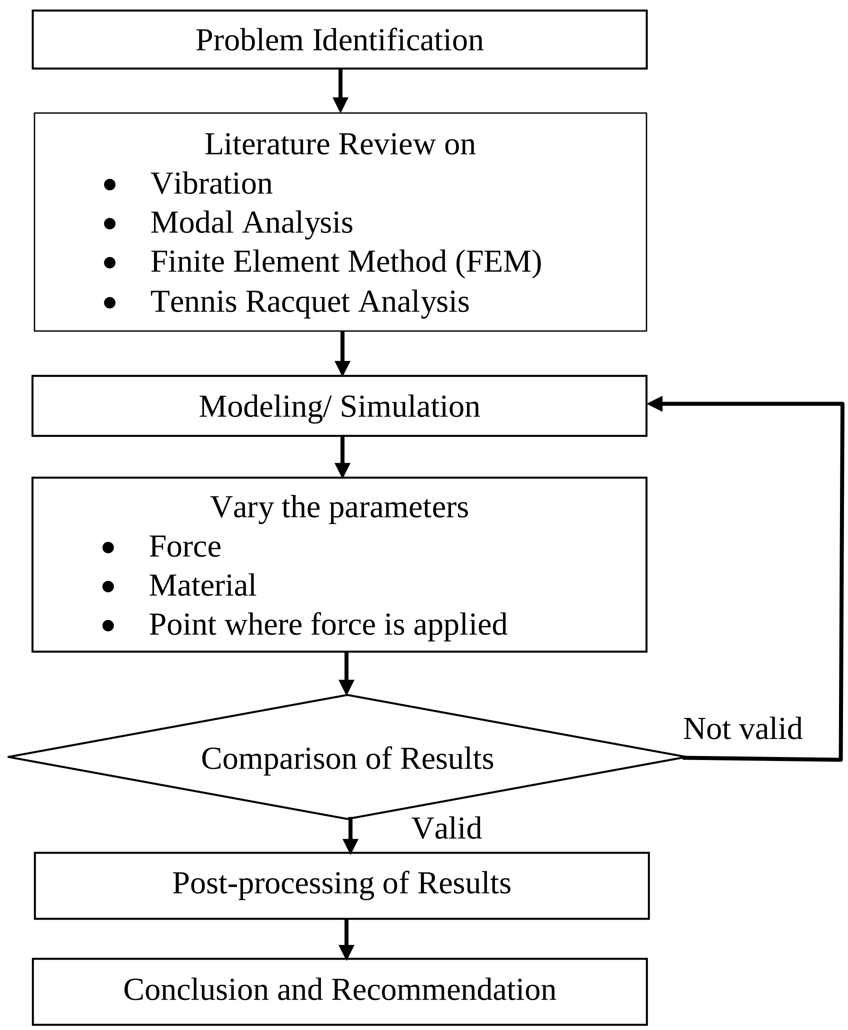

Figure 3.1: Research Methodology

3.2 Project Activities

Table 3.1: Project Activities

Methodology	**Activities**
Problem Identification	• Confirmation of project title with supervisor • Problem statement identification • Scope of study identification
Literature Review • Vibration • Modal Analysis • Finite Element Method (FEM) • Tennis Racquet Analysis	• Understanding the theory of vibration • Understanding the principle of modal analysis and the method of analyzing it • Understanding the function of FEM and how it affects the model • Understanding the tennis racquet itself from the parts to the manufacturing level
Modeling/ Simulation	• Make the racquet model based on assumptions made and simulate the analysis in ANSYS
Vary the parameters • Force • Material • Point where force is applied	• Change the parameters in the analysis including force, material and nodal point to investigate the effects
Comparison of Results	• Dynamic characteristic of the tennis racquet is compared with theoretical value to verify whether it is valid or not
Post-processing of Results	• All results from the varying parameter is plotted and discussed
Conclusion and Recommendation	• Natural frequency of the racquet is obtained and the best material for the model is recommended

Table 3.1 shows the proposed project activities that will be conducted during the progress of the project.

3.3 Tools/ Method

This project used the software ANSYS for modeling and simulation of the tennis racquet analysis. It was the one of the tools used to perform all of the analysis required in this project. ANSYS is basically engineering simulation software for computer-aided engineering (CAE). ANSYS offers a comprehensive software suite that spans the entire range of physics, providing access to virtually any field of engineering simulation that a design process requires. It is universal as organizations around the world trust ANSYS to deliver the best value for their engineering simulation software investment.

Structural mechanics solutions from ANSYS provide the ability to simulate every structural aspect of a product. That includes linear static analysis that simply provides stresses or deformations, modal analysis that determines vibration characteristics, through to advanced transient nonlinear phenomena involving dynamic effects and complex behaviors. In this project, structural mechanics are important because the model of tennis racquet will be analyzed to get the deformations as well as the modal analysis that is needed for the vibration characteristics.

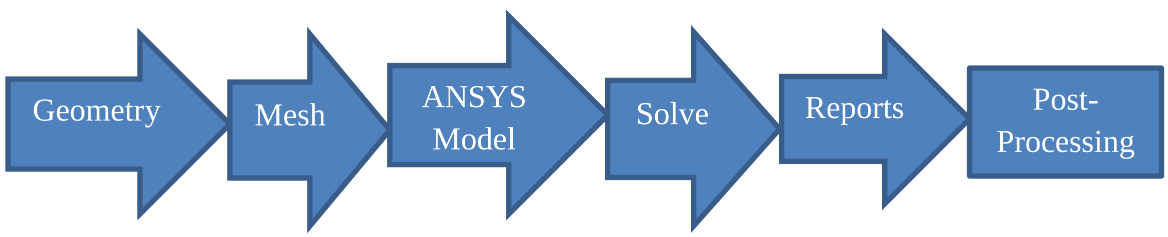

Figure 3.2: Process Flow for Analysis Modeling and Simulation

Figure 3.2 shows the process flow that need to be conducted for the analysis of modeling and simulation.

3.3.1 Geometry

In order to define the geometry of the racquet, the standard size of a tennis racquet set by International Tennis Federation (ITF) is used. ITF has created rules governing tennis racquets for legal play in order to prevent cheating and players from using new technologies that may provide an unfair advantage. The geometry dimensions are shown in Table 3.2 below:

Table 3.2: Dimensions for the Tennis Racquet [19]

Parameter	Racquet standard range by ITF	Selected dimension
Head width	Not more than 317 mm	268 mm
Shaft (include grip) length	-	327 mm
Shaft width	-	40 mm
Thickness	-	22 mm
Overall length	Not more than 737 mm	700 mm

Figure 3.3 below illustrates the basic dimensions for the tennis racquet geometry.

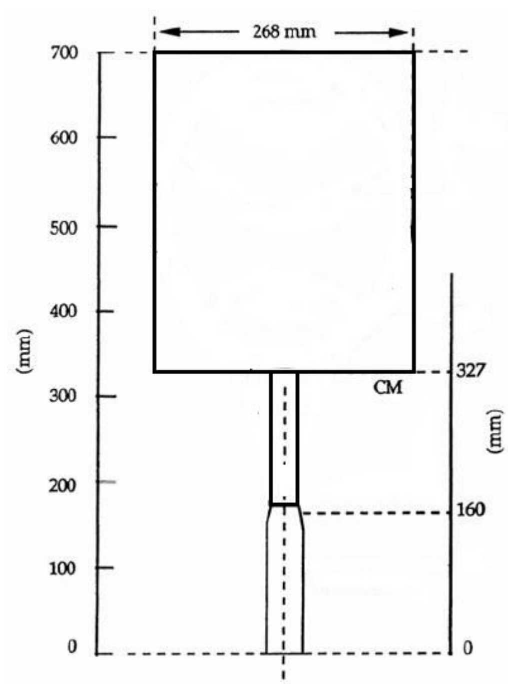

Figure 3.3: Dimensions of the Model of Tennis Racquet

For the model, a rectangle head is used instead of an oval shape to simplify the geometry for the analysis purpose. Figure 3.4 shows the model of the racquet used for the whole analysis.

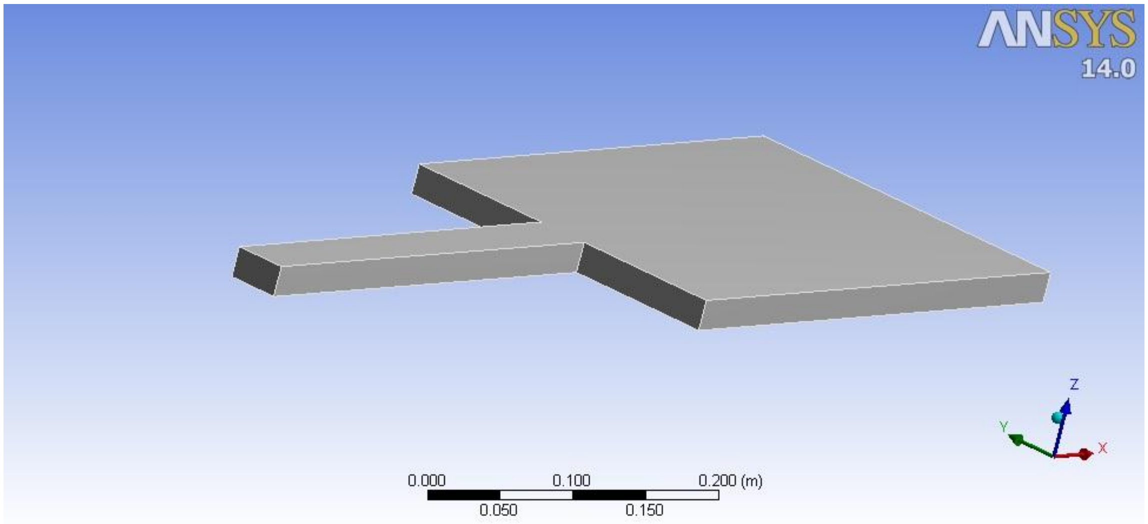

Figure 3.4: Geometry of the ANSYS Model for the Tennis Racquet

For the modeling, structural analysis and modal analysis are conducted to investigate the dynamic characteristic of the tennis racquet model tested. Structural analysis and modal analysis flow chart are shown in Figures below:

3.3.2 Mesh

In mesh process, the geometry is given a volume to enable the analysis to be solved by the solver. This meshing is related to the finite element method explained earlier. For this project, default parameters based on ANSYS as the solver preference had set for the meshing process. In order to make sure the geometry will be meshed correctly, the minimum edge length is set to 22 mm. The meshed model is shown in Figure 3.5:

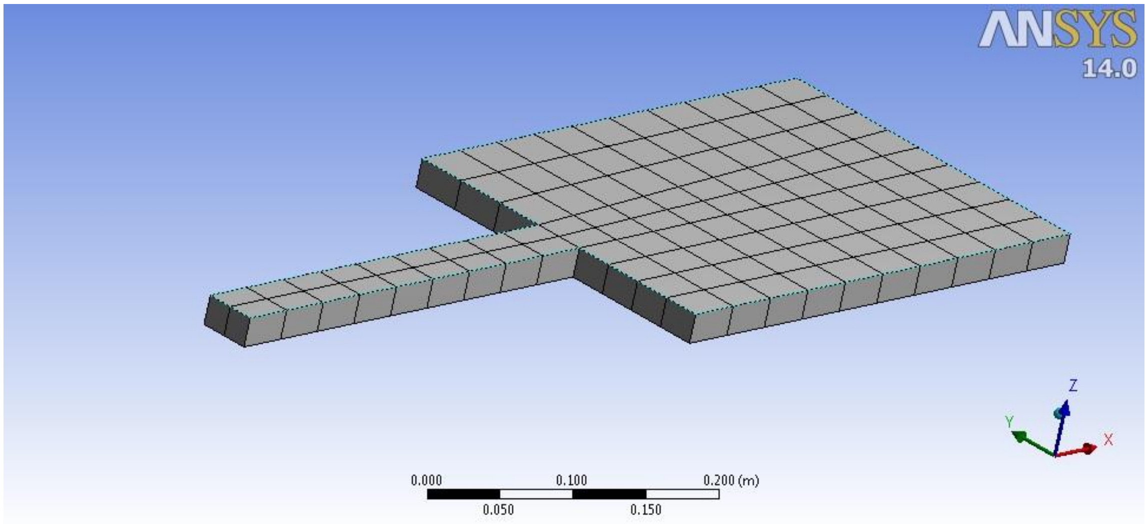

Figure 3.5: Meshed Model of the Tennis Racquet

3.3.3 ANSYS Model

Material variations are practiced in this analysis in order to investigate the effect of the model and to look for the best material for the racquet. There are four materials used which are titanium, carbon fiber composites, graphite and Kevlar. Properties of those materials including Young's Modulus, density and Poisson ratio are tabulated in Table 3.3 below:

Table 3.3: Properties of Materials Used [19]

Properties \ Materials	Graphite	Kevlar	Titanium	Carbon fiber composites
Young's Modulus	11 GPa	186 GPa	110 GPa	220 GPa
Density	1800 kg/m^3	1470 kg/m^3	4420 kg/m^3	1780 kg/m^3
Poisson ratio	0.31	0.36	0.35	0.74

Those are the properties of the materials used in the analysis. The forces used are different as well with 300N, 400N and 500N are applied to the racquet model.

3.3.4 Solve

For the linear static structural analysis, the displacements {x} are solved for in the matrix equation below:

$$[K]\{x\}=\{F\} \qquad \text{(Eq. 2.2)}$$

Assumptions:

- [K] is constant
  - Linear elastic material behavior is assumed
  - Small deflection theory is used
  - Some nonlinear boundary conditions may be included
- {F} is statically applied
  - No time-varying forces are considered
  - No inertial effects (mass, damping) are included

3.3.5 Report and Post-Processing

In the report and post-processing, the time history of the results can be saved. Besides that, all the deformation can be simulated and viewed. All the data for maximum displacement are recorded to be plotted and discussed. Contours are usually shown on the deformed geometry. Figure 3.6 shows the contour of the model in post-processing of the results:

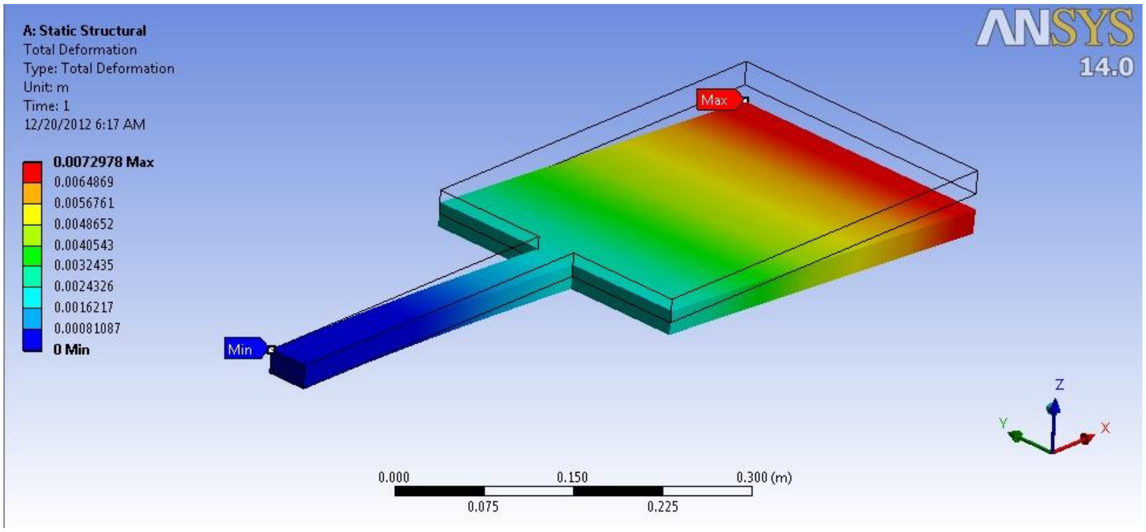

Figure 3.6: Contour of the Model in Post-Processing

The figure above shows that the model deformed from the original shape and the blue contour represent the minimum deformation while the red contour is the maximum deformation that occurs to the model. We can observe the colour of the contour changing from dark blue to dark red depending on the deformation experienced by each part of the model. The post-processing also involves the collection of all the data for the analysis to be plotted.

3.4 Key Milestone

Table 3.4: Key Milestone

Week	Objectives
	FYP I
5	Completion of preliminary research work
6	Submission of extended proposal
9	Completion of proposal defense
13	Submission of Interim draft report
14	Submission of Interim report
	FYP II
8	Submission of progress report
11	Pre-SEDEX
12	Submission of draft report
13	Submission of technical paper and dissertation
14	Oral presentation
15	Submission of project dissertation

Table 3.4 shows the key milestones for the project which is the objective that must be achieved within the specific week.

3.5 Gantt Chart

Table 3.5: FYP 1 Project Gantt Chart

Topic \ Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Title Selection														
Literature Review														
Submission of Draft Proposal														
Submission of Extended Proposal														
Identify Design Criteria														
Proposal Defense														
Modeling Execution														
Submission of Draft Report														
Submission of Interim Final Report														

Table 3.6: FYP II Project Gantt Chart

Topic \ Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Modeling, Simulation & Experiment														
Progress Report Submission														
Comparative Study														
Pre SEDEX														
Submission of Draft Report														
Submission of Technical Paper														
Oral Presentation														
Submission of Final Report														

Table 3.5 and 3.6 show the Gantt Chart for the project implementation for both FYP I and II. Based on the Gantt Chart, the project is feasible to be completed within the given amount of time.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Introduction

In this analysis, a beam is used since it is the assumption made considering the tennis ball momentum on a tennis racquet is similar to the force applied on the beam. For the analysis in ANSYS, the material properties of graphite are used because common tennis racquet nowadays is made of that material. It is based on the famous current players' tennis racquet such as Roger Federer, Novak Djokovic and Rafael Nadal in which all of them use tennis racquets that basically made of graphite. Although some of them have other materials to strengthen the frame in the manufacturing, but the basic material involve is graphite. That is the reason the first analysis is conducted by using graphite.

4.2 Variation of Forces

The force applied to the tennis racquet is assumed to be downwards with a varying magnitude of 300, 400 and 500N and the beam is flexible graphite. Figure 4.1 below show the free body diagram of the force on the tennis racquet.

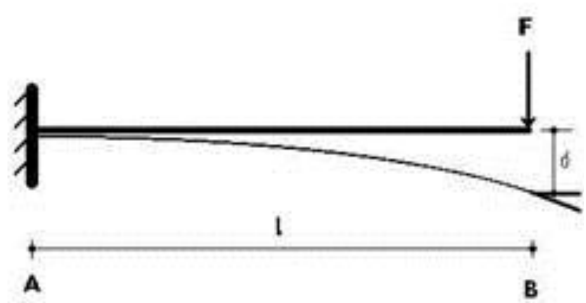

Figure 4.1: The free body diagram of the force applied on tennis racquet

For the analysis, the end A is fixed and force is applied to the end B of the racquet. The distance, *l* from the fixed end to the point where the force is exerted is varies to observe the deformation, δ of the racquet. Figure 4.2 shows the geometry of the tennis racquet used for the analysis.

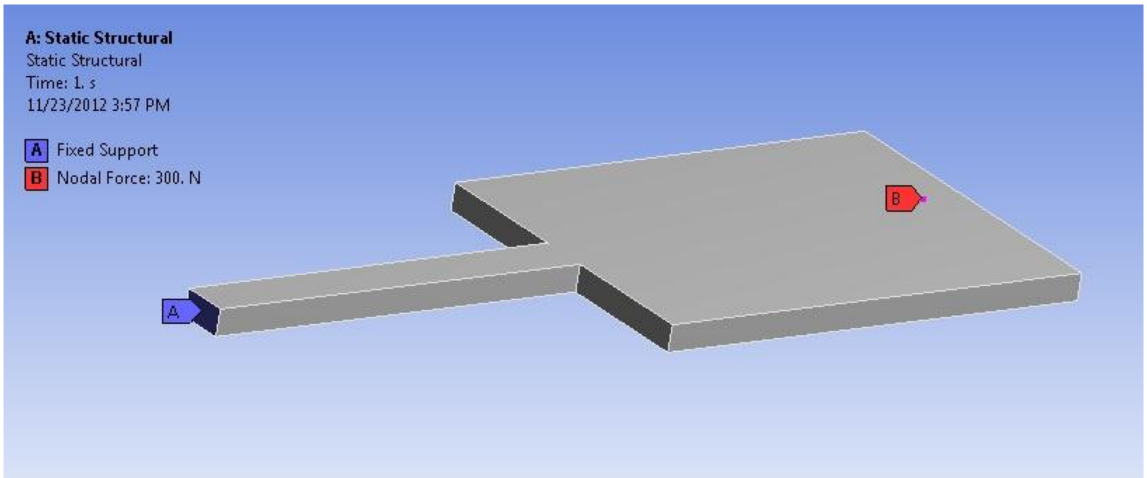

Figure 4.2: The model of tennis racquet used in ANSYS

It is observed that the end A is assumed to be a fixed end while the entire model is a flexible beam. A force is applied to the racquet to investigate the effect to the model. Table 4.1 illustrates the variations of the distance with the deformation occurred for the material graphite that has been tested.

Table 4.1: Variations of distance with displacement of the racquet for different forces

Distance, l (mm)	Max displacement, m (deformation)		
	300N	400N	500N
550.8	0.022556	0.030074	0.037592
513.5	0.020359	0.027145	0.033932
476.2	0.018174	0.024232	0.030290
438.9	0.016003	0.021338	0.026672
401.6	0.013850	0.018467	0.023083

Based on the table, a plot is made to visualize the relationship of horizontal distance of the force from the fixed end with the deformation of the tennis racquet. Figure 4.3 below demonstrates the correlation of those two at different forces using graphite.

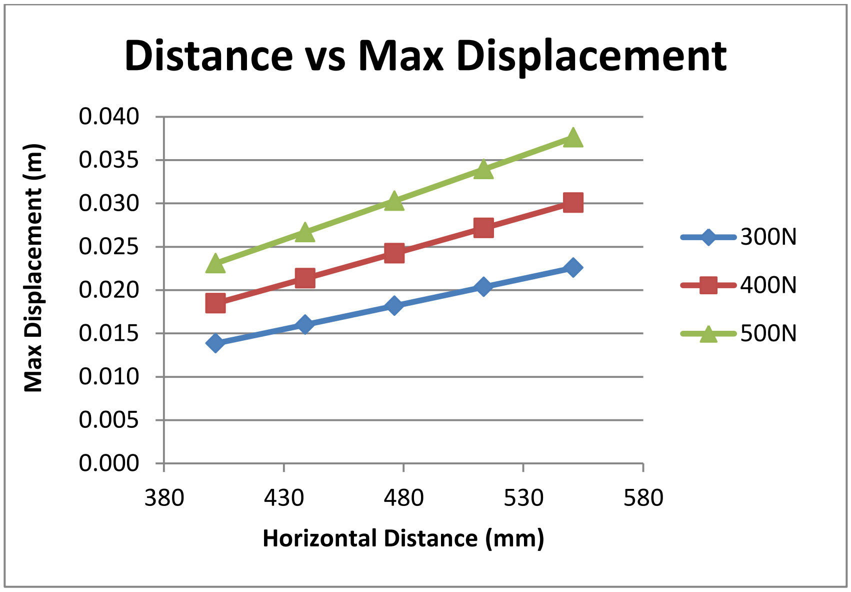

Figure 4.3: Graph of horizontal distance of the force vs max displacement for graphite

Looking at the graph, it is observed that the further the distance of the force applied from the fixed end, the maximum displacement increases. This is true because the force away from the fixed support tends to deform the beam more. For all the three forces applied, the highest force which is 500N gives the greatest impact to the displacement of the tennis racquet model while 300N force gives the smallest displacement. This step is the first in investigating the behavior of the flexible beam before moving to the modal analysis.

The graph also deduces that in order to reduce the impact of the high velocity ball to the tennis racquet, the ball must be hit at the sweet spot for the best effect in terms of less vibration received by the hand of the player as well as more power being exerted to the ball for the return. Hitting the ball around the sweet spot can lessen the wasted energy from swinging of the racquet and provide more control of the ball as well.

4.3 Variation of Materials

The analysis is continued by changing the materials for the tennis racquet. Apart from graphite, three more materials namely titanium, Kevlar and carbon fiber are also analyzed to observe the effect to the tennis racquet. These materials are the constants for the analysis and the force of 300N, 400N and 500N are exerted to the tennis racquet model separately. Figures below show the relations of the distance of the applied force with the deformation occurred using all of the materials for each of the force that have been tested.

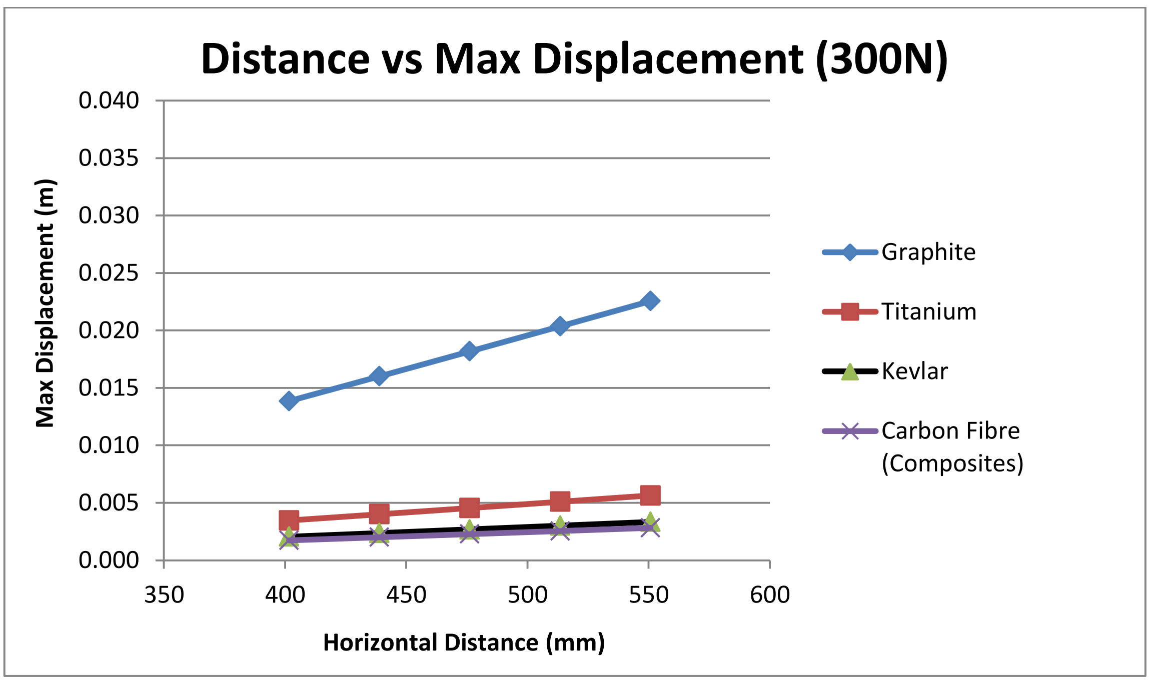

Figure 4.4: Graph of horizontal distance of the force vs max displacement for 300N with different materials

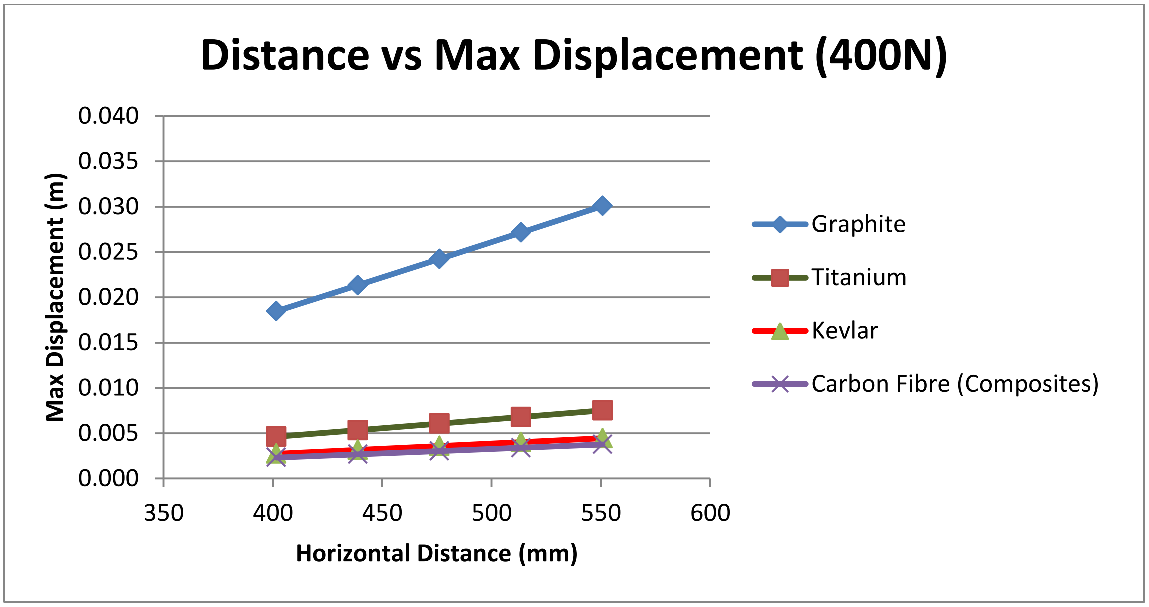

Figure 4.5: Graph of horizontal distance of the force vs max displacement for 400N with different materials

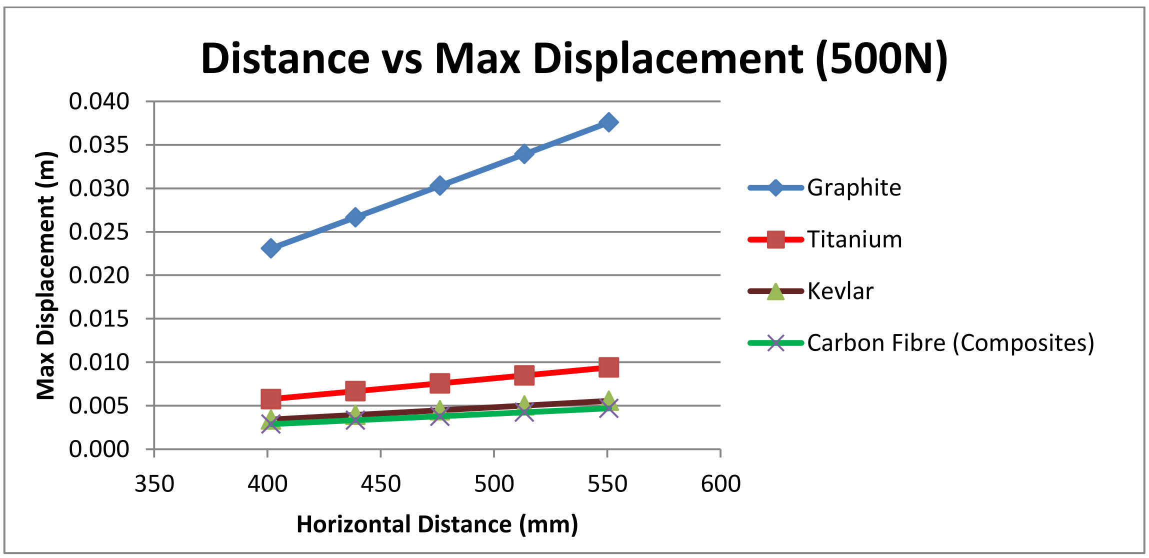

Figure 4.6: Graph of horizontal distance of the force vs max displacement for 500N with different materials

Looking at those three graphs, we can observe that the trend or pattern for each material is almost the same for all of the forces analyzed. For all the forces, graphite has the most displacement when the force is applied, followed by titanium, Kevlar and carbon fiber (composites).

In terms of stiffness, we can conclude that carbon fiber (composites) is the stiffest material for the tennis racquet followed by Kevlar, titanium and last but not least graphite. For this analysis, carbon fiber which is a composite is the best material for the tennis racquet.

4.4 Variation of Location of the Forces

For all of the analysis done above, the force is applied to the center of the tennis racquet. The next analysis is done by applying the force not at the center but shifting to the side of the tennis racquet head. The best material which is carbon fiber composites is used for this analysis with 300N force. Figures below illustrate the difference of the point of the applied forces on the model.

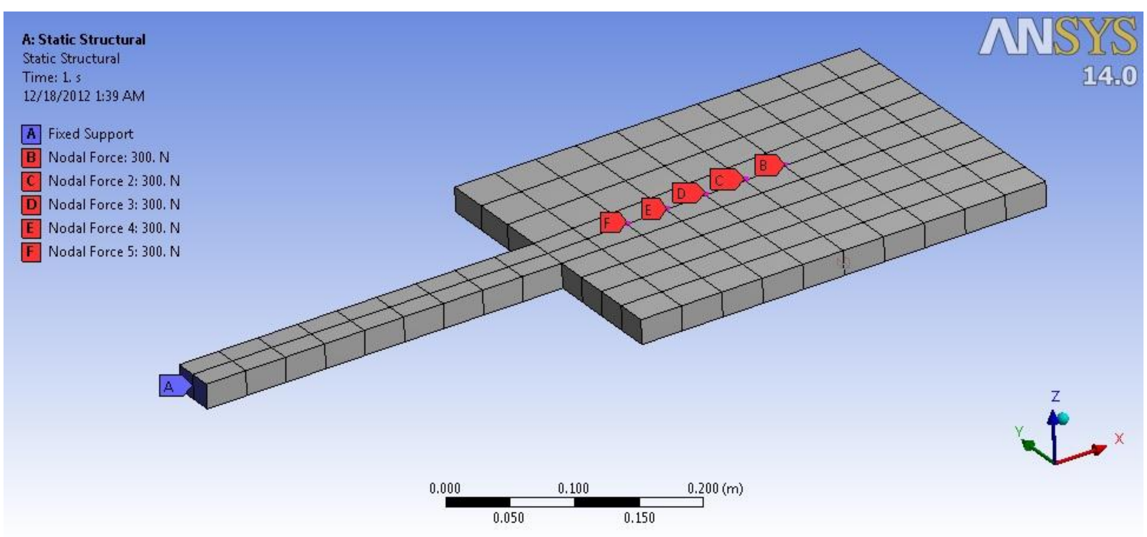

Figure 4.7: All forces are applied to the center of the model

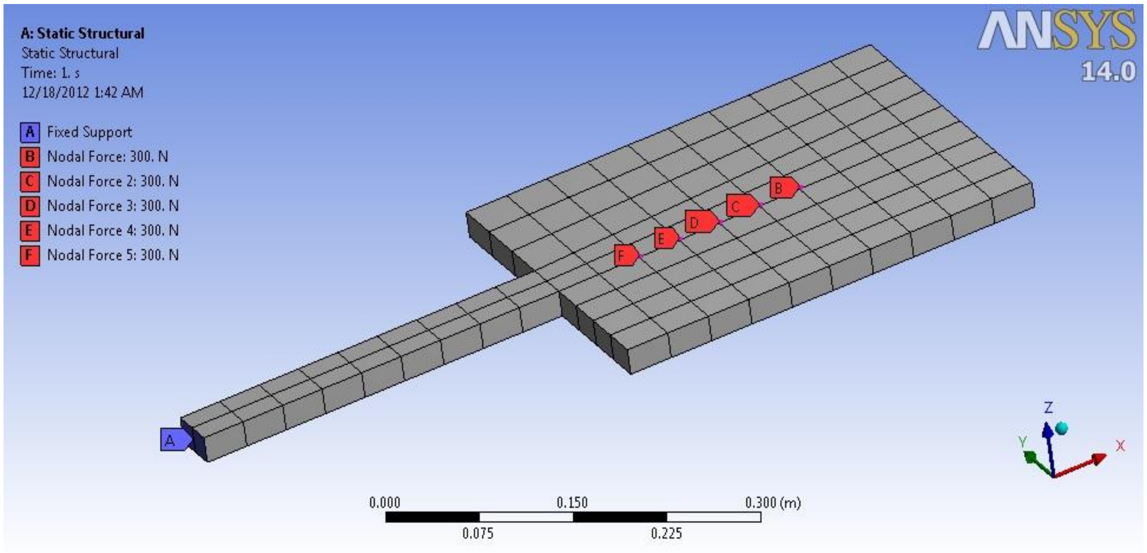

Figure 4.8: All forces are shifted to the side of the model

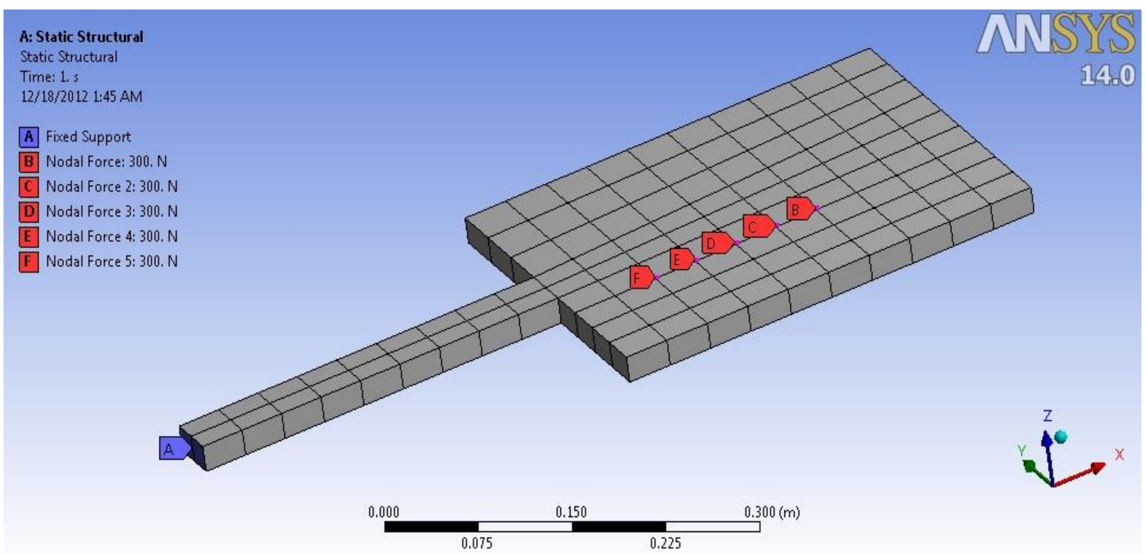

Figure 4.9: All forces are further shifted to the side of the model

Observing all of the Figures above, it can be noticed that the forces are indeed applied at different nodes on the mesh. While all of the forces are exerted at the center in Figure 4.7, they are shifted sideways on the adjacent mesh node which is 48.5mm away from the center axis/line which is portrayed in Figure 4.8. In Figure 4.9, the forces are further shifted to a line 77mm away from the center line.

The effect when the force is applied to different points can be translated in real life by a player hits a tennis ball at different point of the racquet, not only along the center line. Logically, even a professional player could not hit tennis ball 100% at the center of the racquet as there are range of area on the racquet the player could hit to return the ball.

For the analysis of the different nodal points where the force is applied, carbon fiber is used since it is the best material from the previous analysis. The result for the different points of the applied force can be shown in Table 4.2 below:

Table 4.2: Variations of point of force applied on the model with the displacement for different point on the racquet model

Distance, l (mm)	Max displacement, m (deformation)		
	Center	48.5mm from center	77mm from center
550.8	0.0028194	0.0028351	0.0028896
513.5	0.0025449	0.002564	0.0026176
476.2	0.0022717	0.0022943	0.002347
438.9	0.0020004	0.0020263	0.0020781
401.6	0.0017313	0.0017603	0.0018111

Based on the table, a plot is made to clearly note the effect of varying the point of force on the tennis racquet. Figure 4.10 below shows the resulting plot:

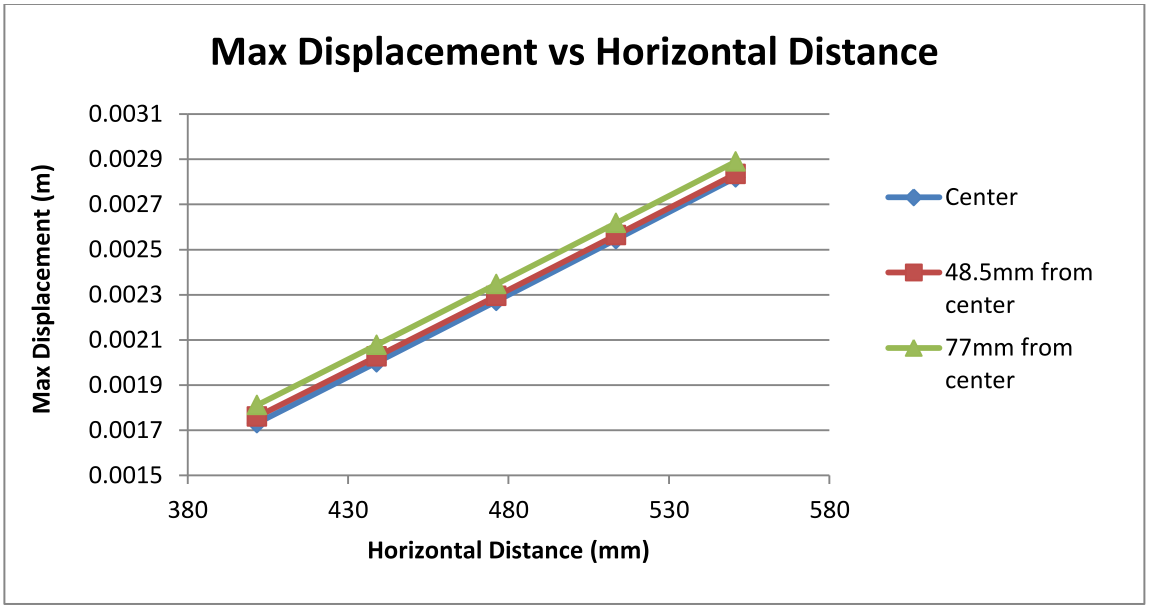

Figure 4.10: Plot of varying point of force applied on the model

According to the plot, it can be seen that the displacement when the force applied is furthest from the center is the highest which means it deforms more than the rest. It follows the pattern as 77mm away from the center line has highest displacement followed by 48.5mm from the center line and force applied at the center is the last. This proves that hitting the ball at the center or perfect sweet spot gives more power for the return ball and transmit less vibration to the arms of the player. Although the ball is hit not perfectly at the center, it can be returned still, as long it stays in the range around the center. This is called as sweet spot, which gives more power and control of the ball if it is being hit inside that area.

4.5 Modal Analysis

The modal analysis simulation is done by using ANSYS as well. For the modal analysis, the same boundary conditions are applied with the fixed support at one end and the nodal force applied to the racquet. The natural frequencies of the modes in the analysis are obtained as shown in Figure 4.11.

	Mode	Frequency [Hz]
1	1.	14.594
2	2.	25.685
3	3.	69.353
4	4.	149.22
5	5.	262.
6	6.	676.65

Figure 4.11: Natural frequencies of the modes

For each of the modes analyzed, the result can be simulated in ANSYS with the movement of the racquet tennis model can be observed. The image of each mode is also captured and is shown in Figures below:

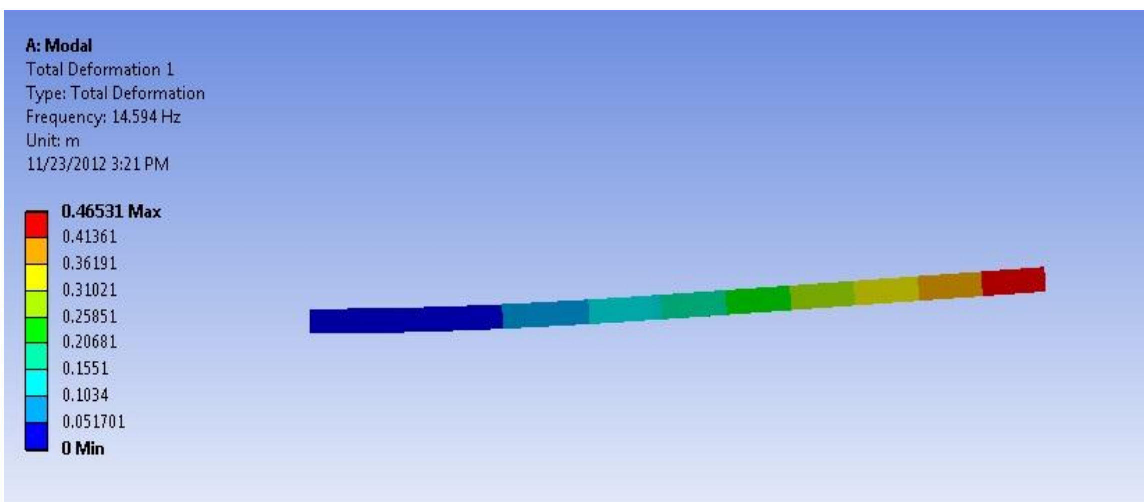

Figure 4.12: Mode 1

Figure 4.12 shows mode 1 for the modal analysis of the tennis racquet. Notice that the shape of the deformation is bending motion.

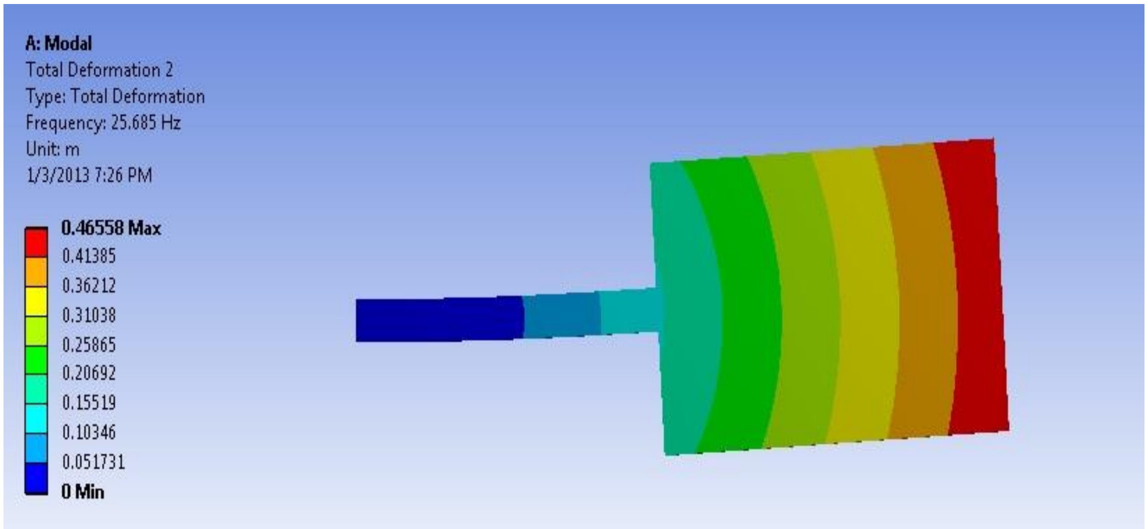

Figure 4.13: Mode 2

Figure 4.13 shows mode 2 for the modal analysis of the tennis racquet. It vibrates at full of the sine curve of dynamic loading curve.

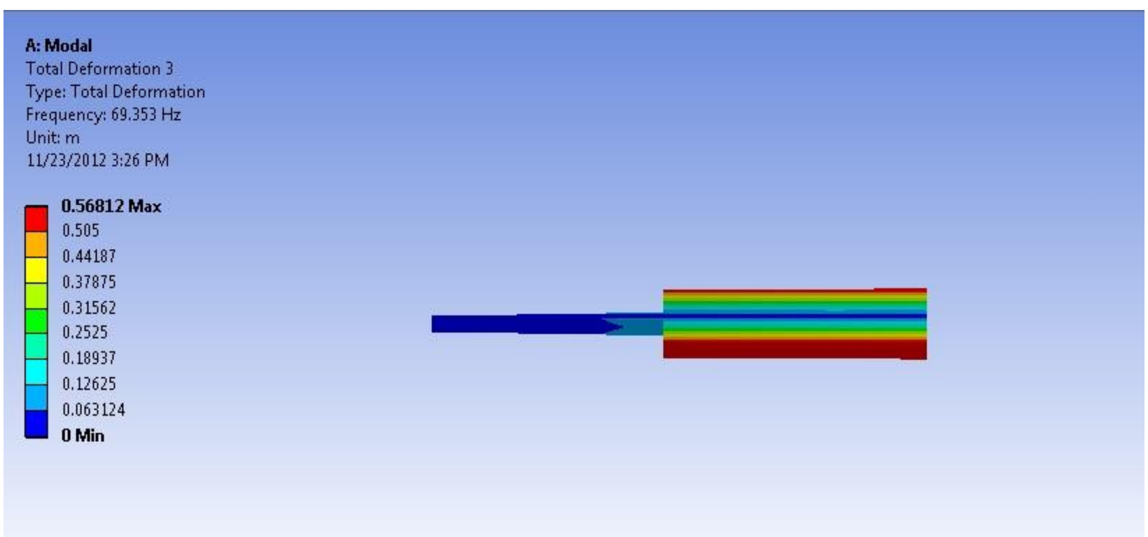

Figure 4.14: Mode 3

Figure 4.14 shows mode 3 for the modal analysis of the tennis racquet. The shape of the deformation is a twisting motion.

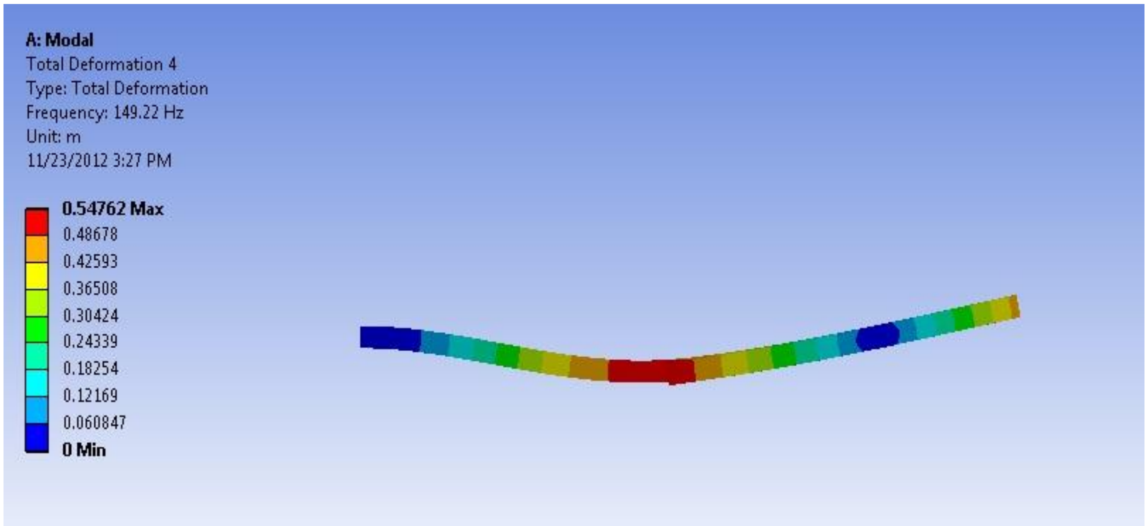

Figure 4.15: Mode 4

Figure 4.15 shows mode 4 for the modal analysis of the tennis racquet. The shape of the deformation is a second bending motion.

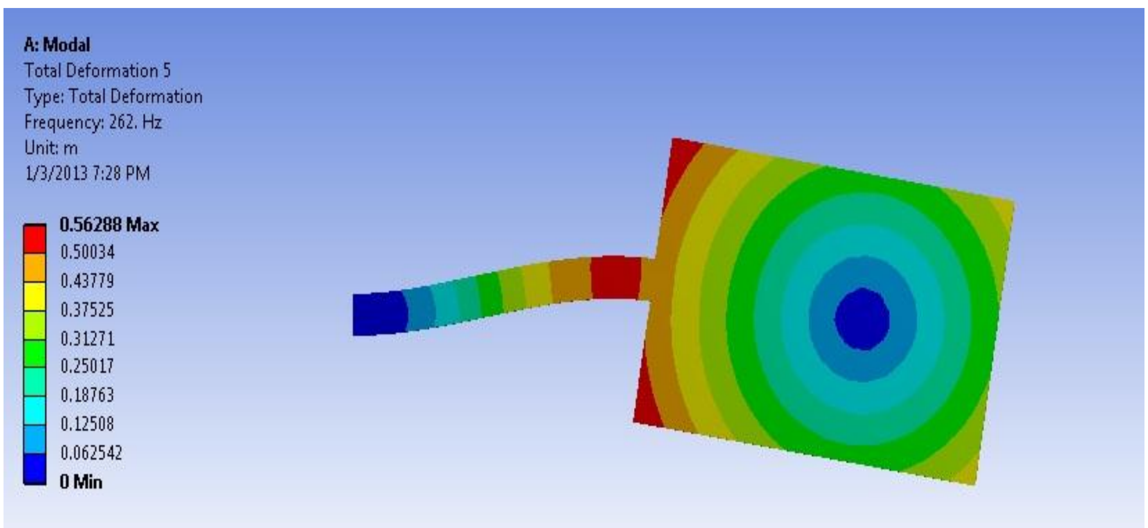

Figure 4.16: Mode 5

Figure 4.16 shows mode 5 for the modal analysis of the tennis racquet. The shape of the deformation is a third bending motion.

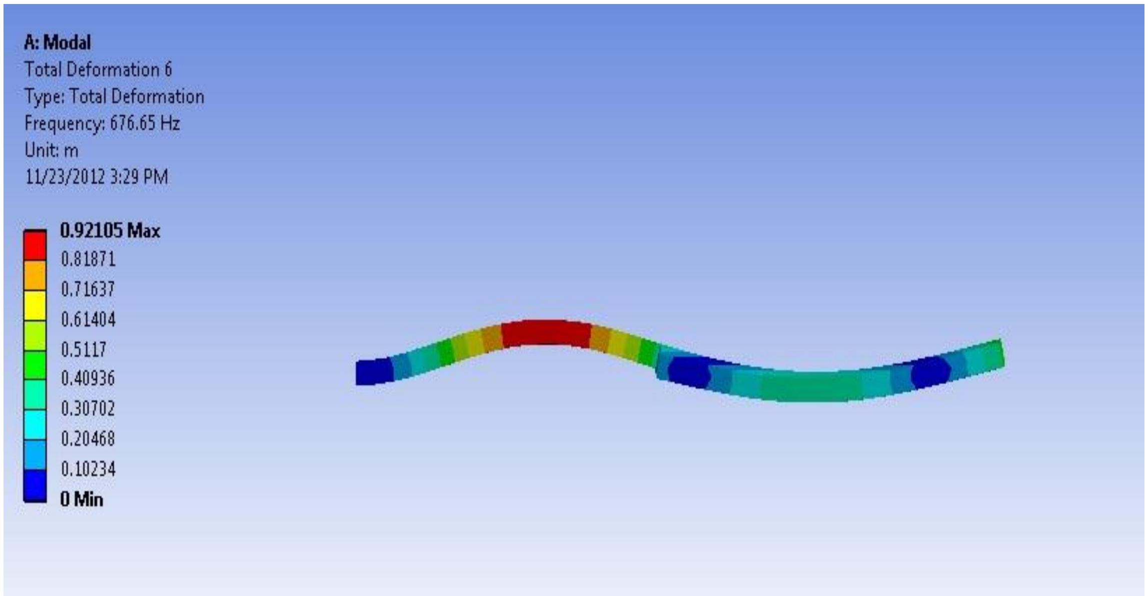

Figure 4.17: Mode 6

Figure 4.17 shows mode 6 for the modal analysis of the tennis racquet. The shape of the deformation is a fourth bending motion.

The modal analysis gives the modes of the structure which is being tested. In the analysis, the force to the beam is applied in the y-direction. Therefore, we need to make sure that the effective mass in the y-direction is higher than 90% of the total mass as most codes use this as a requirement for the analysis. It is noticed that there are six modes in the figure above which have the overall 100% participating mass in the y-direction. The overall table for the whole modes is shown in Figure 4.18:

***** PARTICIPATION FACTOR CALCULATION ***** Y DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	37.5025	0.26665E-01	4.4403	1.000000	19.7158	0.704926	0.608542
2	198.512	0.50375E-02	-0.34960E-10	0.000000	0.122220E-20	0.704926	0.377240E-22
3	233.263	0.42870E-02	-2.4756	0.557533	6.12854	0.924047	0.189161
4	405.357	0.24670E-02	0.45514E-10	0.000000	0.207155E-20	0.924047	0.639396E-22
5	628.438	0.15912E-02	0.21037E-10	0.000000	0.442565E-21	0.924047	0.136600E-22
6	650.735	0.15367E-02	1.4575	0.328246	2.12429	1.00000	0.655676E-01
sum					27.9687		0.863270

Figure 4.18: Participation of all modes in y-direction

It is observed that two of the modes (mode 1 and 3) are contributing with approximately 90% of the effective mass and consequently can be expected that the response will be dominated by these modes. For the subsequent analysis, we will only use the first three modes as input as these modes participates with 92% of the effective mass in y-direction.

The frequencies of the modes and the mode shapes are derived from Euler-Bernoulli Beam Theory in the pre-analysis. It is done manually using the formula to be compared with the result from ANSYS. The formula is shown below;

$$\omega_n = \alpha_n^{\,2} \sqrt{\frac{EI}{ml^3}} \qquad \text{(Eq. 2.2)}$$

From the equation, the natural frequency for each mode is calculated;

ω_1 = 15.22 Hz

ω_2 = 28.67 Hz

ω_3 = 188.46 Hz

These values are compared to the natural frequency after ANSYS analysis. For verification, we will focus on the first three modes since there are the ones that affect the result mostly. ANSYS uses a different type of beam element to compute the modes and frequencies, and therefore provides more accurate results for relatively short, stubby beams.

From the pre-analysis, based on Euler-Bernoulli beam theory, we have calculated the frequencies of 36.2, 226.7 and 634.9 Hz for the three modes. The ANSYS simulation yielded results of 37.7, 203.1 and 233.8 Hz. Those results are illustrated in the Table 4.3 below:

Table 4.3: Difference of natural frequencies

Mode no.	Euler-Bernoulli Beam Theory	ANSYS result	Percentage difference
1	15.22	14.594	4.1
2	28.67	25.685	10.4
3	188.46	69.353	63.2

These results give percent differences of 4.1%, 10.4% and 63.2%. The results are acceptable for the first two modes, but are way off for the third mode. This is explained by the inaccuracy of Euler-Bernoulli beam theory for high order modes in short, stubby beams.

CHAPTER 5

CONCLUSION

The project of modal analysis of a tennis racquet enables the learning on the dynamic characteristics of a tennis racquet which includes the natural frequency, mode shape and damping. In this project, the deformation of the tennis racquet model is analyzed with respect to varying forces, materials and locations of the force applied on the model. Based on the result, the model deformed the most at the highest force exerted, with location of the force furthest away from the center line, using graphite as the material. It is deduced that the best spot to hit the ball is at the center of the racquet as it will give more power for the ball return with the least vibrations transmitted to the arms of the player. This center spot of the racquet is also known as the sweet spot. Meanwhile, the best material for the racquet is carbon fiber composites based on the different forces at different locations on the racquet. The natural frequency is also observed and the difference with theoretical values is considered acceptable. Besides, mode shape of the model is obtained as well which gives information on the damping that can be used to reduce the vibration received by the racquet.

RECOMMENDATION

For future work, it is recommended that some other parameters are analyzed such as a different dimension of the tennis racquet model. Other than that, more other materials can be used to continue this project apart from all four materials tested. This might result in finding a better parameter, size or material for the manufacturing of the tennis racquet. Other than that, it is also recommended to analyze on variations of ball speed and string tension since those criteria affect the performance of a tennis player.

CHAPTER 6

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APPENDIX A

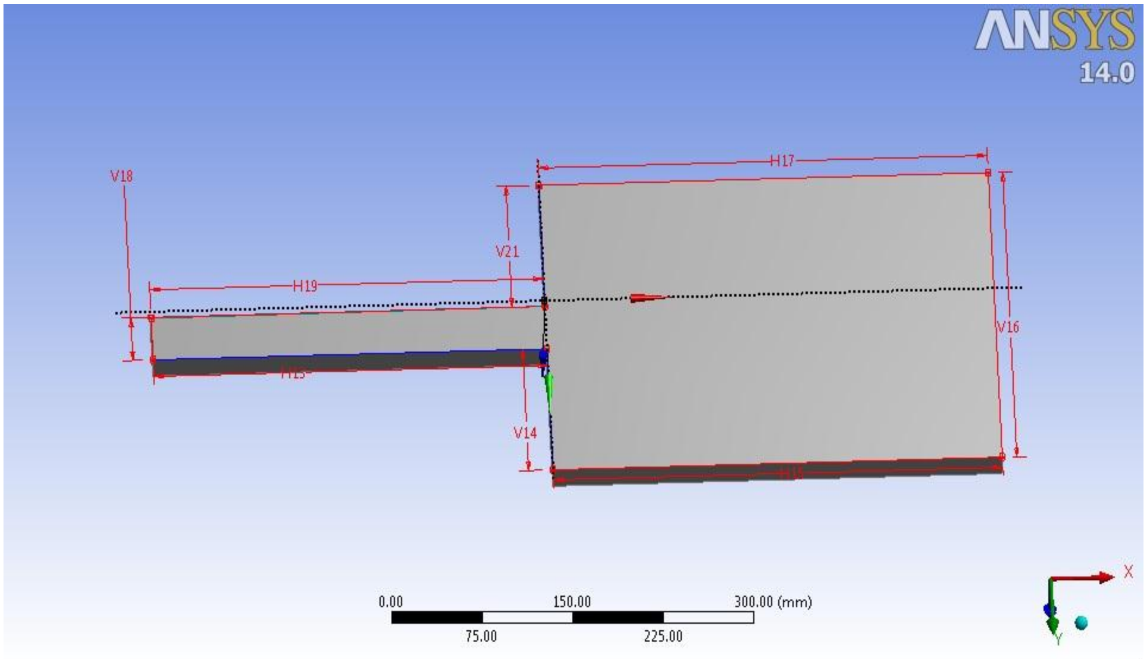

Figure A-1

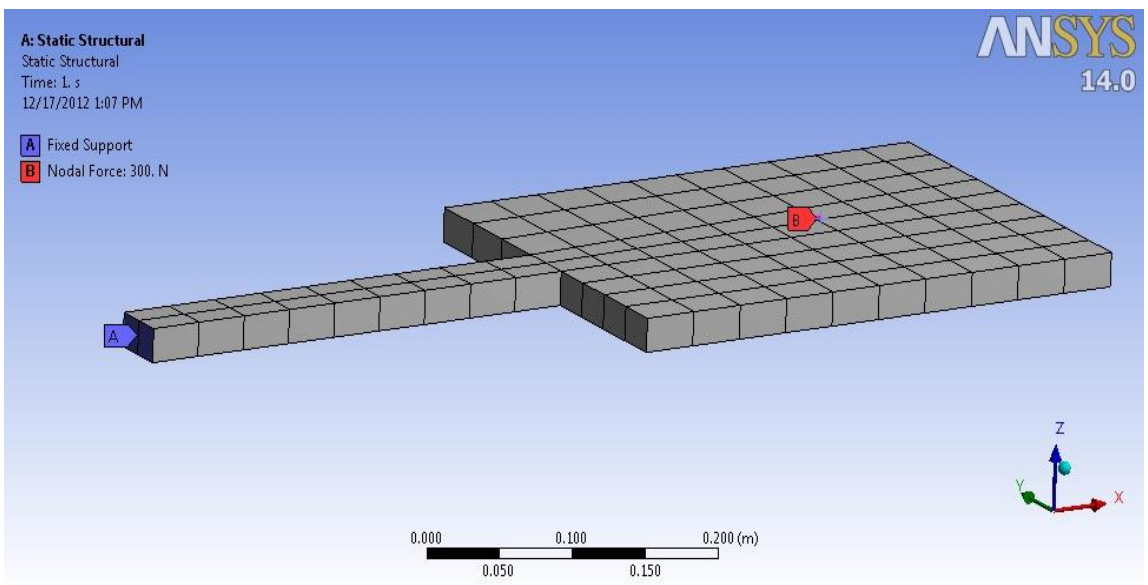

Figure A-2

APPENDIX B

***** PARTICIPATION FACTOR CALCULATION ***** X DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	14.5941	0.68521E-01	0.12016E-11	0.000000	0.144382E-23	0.293415E-19	0.131510E-24
2	25.6851	0.38933E-01	0.19897E-02	0.295799	0.395909E-05	0.804572E-01	0.360612E-06
3	69.3528	0.14419E-01	0.31160E-12	0.000000	0.970930E-25	0.804572E-01	0.884368E-26
4	149.222	0.67014E-02	0.36895E-12	0.000000	0.136127E-24	0.804572E-01	0.123990E-25
5	261.998	0.38168E-02	0.67267E-02	1.000000	0.452484E-04	1.00000	0.412143E-05
6	676.651	0.14779E-02	-0.53498E-11	0.000000	0.286207E-22	1.00000	0.260691E-23
sum					0.492074E-04		0.448204E-05

***** PARTICIPATION FACTOR CALCULATION ***** Y DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	14.5941	0.68521E-01	-0.71146E-09	0.000000	0.506169E-18	0.481299E-19	0.461042E-19
2	25.6851	0.38933E-01	2.9933	1.000000	8.95989	0.851966	0.816108
3	69.3528	0.14419E-01	0.46767E-09	0.000000	0.218716E-18	0.851966	0.199216E-19
4	149.222	0.67014E-02	-0.72809E-11	0.000000	0.530109E-22	0.851966	0.482847E-23
5	261.998	0.38168E-02	1.2477	0.416840	1.55683	1.00000	0.141803
6	676.651	0.14779E-02	0.50434E-12	0.000000	0.254363E-24	1.00000	0.231685E-25
sum					10.5167		0.957911

***** PARTICIPATION FACTOR CALCULATION ***** Z DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	14.5941	0.68521E-01	3.0447	1.000000	9.27001	0.874975	0.844355
2	25.6851	0.38933E-01	0.72050E-09	0.000000	0.519115E-18	0.874975	0.472834E-19
3	69.3528	0.14419E-01	-0.75658E-03	0.000248	0.572412E-06	0.874975	0.521379E-07
4	149.222	0.67014E-02	1.0882	0.357427	1.18428	0.986757	0.107870
5	261.998	0.38168E-02	0.14423E-10	0.000000	0.208037E-21	0.986757	0.189489E-22
6	676.651	0.14779E-02	0.37458	0.123027	0.140307	1.00000	0.127798E-01
sum					10.5946		0.965005

Figure B-1

***** PARTICIPATION FACTOR CALCULATION *****ROTX DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	14.5941	0.68521E-01	0.77143E-01	0.318456	0.595105E-02	0.891228E-01
2	25.6851	0.38933E-01	-0.32926E-01	0.135924	0.108415E-02	0.105359
3	69.3528	0.14419E-01	0.24224	1.000000	0.586805E-01	0.984157
4	149.222	0.67014E-02	0.27874E-01	0.115066	0.776941E-03	0.995792
5	261.998	0.38168E-02	-0.13725E-01	0.056659	0.188377E-03	0.998613
6	676.651	0.14779E-02	0.96222E-02	0.039722	0.925861E-04	1.00000
sum					0.667736E-01	

***** PARTICIPATION FACTOR CALCULATION *****ROTY DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	14.5941	0.68521E-01	-0.64584	1.000000	0.417112	0.898856
2	25.6851	0.38933E-01	0.21887E-04	0.000034	0.479043E-09	0.898856
3	69.3528	0.14419E-01	-0.25937E-03	0.000402	0.672725E-07	0.898856
4	149.222	0.67014E-02	0.19764	0.306022	0.390622E-01	0.983033
5	261.998	0.38168E-02	0.73994E-04	0.000115	0.547505E-08	0.983033
6	676.651	0.14779E-02	0.88733E-01	0.137391	0.787349E-02	1.00000
sum					0.464047	

***** PARTICIPATION FACTOR CALCULATION *****ROTZ DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION
1	14.5941	0.68521E-01	-0.16102E-09	0.000000	0.259261E-19	0.501513E-19
2	25.6851	0.38933E-01	0.67900	1.000000	0.461035	0.891823
3	69.3528	0.14419E-01	0.10609E-09	0.000000	0.112548E-19	0.891823
4	149.222	0.67014E-02	0.38701E-11	0.000000	0.149774E-22	0.891823
5	261.998	0.38168E-02	-0.23648	0.348279	0.559229E-01	1.00000
6	676.651	0.14779E-02	0.69890E-12	0.000000	0.488460E-24	1.00000
sum					0.516958	

Figure B-2