

**PREDICTION OF WATERFLOODING PERFORMANCE IN NON-COMMUNICATING  
LAYERED RESERVOIR**

By

**MUHAMMAD IRSHAD BIN BANURI**

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the requirements for the  
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**Universiti Teknologi PETRONAS  
Bandar Seri Iskandar  
31750 Tronoh  
Perak Darul Ridzuan**

# CERTIFICATION OF APPROVAL

## Prediction of Waterflooding Performance in Non-Communicating Layered Reservoir

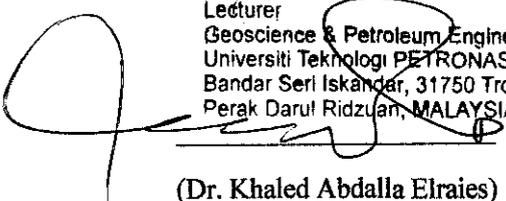
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Muhammad Irshad Bin Banuri

A project dissertation submitted to the  
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(Petroleum Engineering)

Approved by,

Dr. Khaled Abdalla Elraies  
Lecturer  
Geoscience & Petroleum Engineering Department  
Universiti Teknologi PETRONAS  
Bandar Seri Iskandar, 31750 Tronoh,  
Perak Darul Ridzuan, MALAYSIA

  
(Dr. Khaled Abdalla Elraies)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

August 2011

## CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgement, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.



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MUHAMMAD IRSHAD BIN BANURI

## ABSTRACT

The lack of sufficient natural drive in most reservoirs has led to the practice of supplementing the natural reservoir energy by introducing some form of artificial drive, the most basic method being the injection of gas or water. One of the objectives of waterflooding is to displace oil from reservoir.

The purpose of the project is to study the performance of the waterflooding on non-communicating layered reservoir. Analytical works based on Buckley-Leverett Method has been used and an enhance method for predicting waterflooding performance has been implemented. With different cases on mobility ratio, waterflooding performance such as oil and water production is varied as the viscosity of the displacing fluid helps in recovering the oil.

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# CHAPTER 1

## INTRODUCTION

### 1.1 Background

The discovery of a crude oil by Edwin L. Drake at Titusville, PA, on Aug. 27, 1859 marked the beginning of the petroleum era. As early as 1880, Carll raised the possibility that oil recovery might be increased by the injection of water into the reservoir to displace oil to producing wells. The terms primary oil recovery, secondary oil recovery, and tertiary (enhanced) oil recovery are traditionally used to describe hydrocarbons recovered according to the method of production or the time at which they are obtained. Primary oil recovery describes the production of hydrocarbons under the natural driving mechanisms present in the reservoir without supplementary help from injected fluids such as gas or water. In most cases, the natural driving mechanism is a relatively inefficient process and results in a low overall oil recovery (Willhite, 1986).

The lack of sufficient natural drive in most reservoirs has led to the practice of supplementing the natural reservoir energy by introducing some form of artificial drive, *the most basic method being the injection of gas or water. Secondary oil recovery refers to the additional recovery those results from the conventional methods of water injection and immiscible gas injection. Usually, the selected secondary recovery process follows the primary recovery but it can also be conducted concurrently with the primary recovery (Tarek, 2001).*

Waterflooding also called secondary recovery because the process yielded a second batch of oil after a field was depleted by primary production (Doug, 2003). *C. K. Chang, 1985 stated in his paper that waterflooding technique usually is used for two purposes. One is to maintain reservoir pressure by injecting water into aquifer zone. Waterflooding too is used to displace oil from reservoir by injecting water into oil zone.*

## 1.2 Historical Development of Waterflooding

The practice of waterflooding apparently began accidentally. Many wells were abandoned in the Bradford field following the flush production period of the 1880's. Some were abandoned by pulling casing without plugging, while in other wells the casing was left in the wells, where it corroded. In both cases, fresh water from shallower horizons apparently entered the producing interval. Water injection began, perhaps as early as 1890, when operators realized that water entering the productive formation was stimulating production.

By 1907, the practice of water injection had an appreciable impact on oil production from the Bradford field. The first flooding pattern, termed a circle flood, consisted of *injecting water into a well until surrounding producing wells watered out*. The watered-out production wells were converted to injection to create an expanding circular waterfront. Many operators were against the injection of water into sand. A Pennsylvania law requiring plugging abandoned wells and dry holes to prevent water from entering oil and gas sands was interpreted as prohibiting waterflooding, so waterflooding was done secretly. In 1921, the Pennsylvania legislature legalized the injection of water into the Bradford sands.

The practice of water injection expanded rapidly after 1921. The circle-flood method was replaced by a line flood, in which two rows of producing wells were staggered on both sides of an equally spaced row or line of water intake wells. By 1928, the line flood was replaced by a new method termed the five-spot because of the resemblance of the pattern to the five spots on dice. Waterflooding was quite successful in the Bradford field (Willhite, 1986).

### 1.3 General Consideration of Waterflooding

For many years analytical models have been used to estimate performance of waterflood projects. The Buckley-Leverett frontal advance theory and Dykstra-Parsons method for stratified reservoirs have been used for this purpose, but not in combination for stratified reservoirs with different  $kh$  and oil-water relative permeability. The Dykstra-Parsons method has a major drawback in that it assumes the displacement of oil by water is piston-like (Gasimov, 2005).

The goal of the research is to modify the Dykstra-Parsons method for 1-D oil displacement by water in such a manner that it would be possible to incorporate the Buckley-Leverett frontal advance theory. This would require modeling fractional flow behind the waterflood front instead of assuming piston-like displacement. By incorporating Buckley-Leverett displacement, a more accurate analytical model of oil displacement by water is expected. Permeability-thickness and oil-water relative permeability will be different for each layer, with no crossflow between the layers. The analytical model results (injection rate, water and oil production rate) will be compared against simulation results to ensure the validity of the analytical model.

The new analytical model has these assumptions:

- I. Pressure drop for all layers is the same.
- II. Total water injection rate is constant.
- III. Oil-water relative permeabilities may vary for each layer.
- IV. Water injection rate in each layer may vary.

Elraies and Yunan used similar method to Prats et al. A model similar to that of Prats et al (1954) was used in this study. The reservoir was considered to be composed of three layers that communicate only at the wellbores. Each layer is individually homogeneous, but may be different from every other layer. The following properties were allowed to vary between layers: absolute permeability, porosity and thickness. The following assumptions were made:

- I. Constant width and length for all layers,
- II. Negligible capillary and gravity forces,
- III. Constant pressure drop for all layers at a given time,
- IV. Constant oil-water relative permeability for all layers,
- V. Constant total injection rate for the reservoir (for ease of comparison with other method),
- VI. Water enters each layer in direct proportional to its capacity,  $kh$ ,
- VII. Uniform initial water saturation,
- VIII. There is no cross-flow between layers.

## 1.4 Description of the Problem

From the starting of oil production, an average of 30-35% can be produced through primary recovery methods. Primary recovery such as aquifer-driven and gas-drive could not displace most of the oil due to the pressure depletion or the heterogeneity of the formation.

Due to these problems, secondary recovery is needed to increase the oil production. One of the secondary recovery methods is waterflooding. Waterflooding can increase the oil production to about 30-50% and with it capital cost lower than EOR techniques. There is no doubt that it has become the preferred process applied after primary recovery.

## 1.5 Objectives of the Study

1. To investigate the performance of waterflooding in multi-layered reservoir.
2. To determine the breakthrough time for each layer in a multi-layered reservoir.
3. To study the effect of mobility ratio in the waterflooding performance.

## 1.6 Scope of the Study

1. Understanding of Waterflooding
2. Set the conditions and assumptions based on the reservoir
3. Buckley-Leverett Displacement Theory:
  - a. Derivation of the Fractional Flow Equation
  - b. Graphical Analysis of the Fractional Flow Curve
  - c. Mobility Ratio
  - d. Breakthrough Time Determination
4. Derivation of Buckley and Leverett Frontal Advance Equation
5. Predicting the Performance of Five Spot Waterflooding:
  - a. Application of the Recovery Curve Calculation
  - b. Application of the Total Injectivity Curve Calculation
  - c. Reduce Time Curve Calculation
  - d. Converting Reduce Time Data to Production versus Injection

## CHAPTER 2

### FUNDAMENTAL THEORY AND LITERATURE REVIEW

#### 2.1 Mechanism of Fluid Displacement

The production of oil is accomplished as a result of its displacement from the reservoir by either gas or water, and the amount of oil recovery is limited by the extent to which the displacing gas or water accumulates.

Crude oil has no inherent ability to expel itself from the pores of the reservoir rocks in which it is found. It must be forcibly injected or displaced by the accumulation of *the other fluids*. The displacing fluids normally available are gas and water, either or both of which may exist originally associated with the oil in a potentially usable form or may be supplied to the reservoir from external sources.

From the point of entry of the water, no substantial change in the water saturation results as the water first advances. Then a very sudden rise in the water saturation takes place as the transition zone reaches and passes the plane. This period of rapid increase of water saturation may be considered the initial phase of the displacement. During this phase, the displacement is quite effective as most of the water reaching the plane remains in the sand, ejecting oil.

After the initial process, this period increase in water saturation is much more gradual. This final period of gradual water accumulation may be termed the subordinate phase of the displacement. During this period, water flows more readily than does the oil, so that relatively large volumes of water flowing through the sand effect the removal of only small and continuously decreasing volumes of oil.

#### **Oil Displacement**

Under ideal conditions, water would displace oil from pores in a piston-like manner or at least in a manner representing a leaky piston. However, because of various wetting conditions, relative permeability of water and oil are important in determining where flow of each fluid occurs, and the manner in which oil is

displaced by water. In addition, the higher viscosity of crude oil in comparison to water will contribute to non-ideal displacement behavior (Lyons, 1996).

### Mechanism of Displacement

If the rate of production is such that the water table rises slowly enough to permit the maintenance of capillary equilibrium, the water saturation in the coarse sand will gradually increase simultaneously with the rise in the water table. As the water saturation in the adjacent coarse sand increases, the tight lens will imbibe water and expel oil, both by absorbing water at the bottom and expelling oil at the top and by counterflow of water and oil over the entire surface of the lens, tending always to maintain a higher water saturation than that reached by the surrounding coarse sand. Thus the tight sand will at all times be more completely flushed than the coarse sand and will become depleted while oil is still flowing in the surrounding sand.

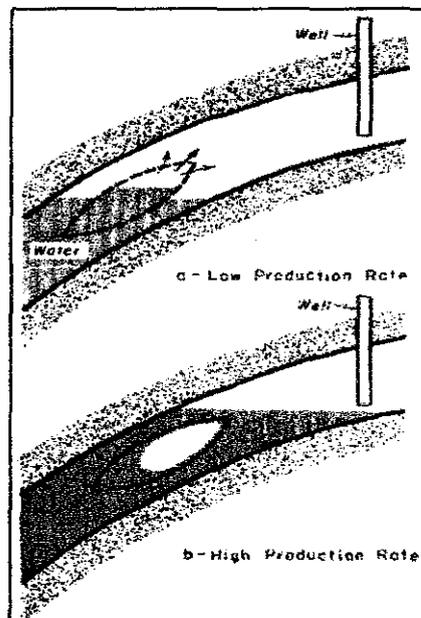


Figure 1: Effect of Production rate on Flooding of Oil by Water from a Low Permeability Lens

For this particular situation, it is evident that the slower the rate of water advance, the higher the recovery. The magnitude of advancement of the water depends upon the degree and nature of the irregularities of the sand and upon the viscosity of the oil (Buckley and Leverett, 1941).

## **Factors Affecting Oil Displacement**

There are conditions affecting the relative magnitude of initial phases of displacement which are:

- **Viscosity**

Since the rate of advance of a plane of given water saturation is directly proportional to  $\frac{df_w}{dS_w}$  (slope between water fraction and water saturation) and since  $f_w$  (water fraction) is related to the ratio of viscosity of oil to that of water as well as to the relative permeability of the sand to oil and to water, the course of the curves of water saturation vs. distance is influenced by the oil viscosity. The more viscous the oil, the less readily it flows under a given pressure gradient. Increased oil viscosity therefore results in the attainment of lower water saturation during initial phase of the displacement as well as a more gradual approach to the residual oil saturation during the subordinate phase of the displacement.

- **Effect of Initial Fluid Saturation**

If before invasion by the displacing fluid in the sands exceeds that which would be obtained during the initial phase of the displacement, this phase will be absent and only the subordinate phase will occur. Such a condition would be encountered in a water-drive operation where the original or connate-water content of the sand is excessive and in practice is most likely to be met in tight sands, with viscous oils, or in thin oil sands immediately overlying water. It is not possible to produce oil free from water in the part of the sand where this condition prevails.

- **Capillary and Gravitational Effects**

Capillary forces tend to oppose the formation of saturation discontinuities in homogenous sand, while gravitational forces tend to promote complete vertical segregation of oil, gas and water. Thus in any reservoir in which water is advancing upward or gas downward to displace oil, the capillary and gravitational effects oppose each other and tend to somewhat to cancel. At high rates of displacement the frictional forces may exceed both, with the result that their effects are obscured and

the flow is regulated primarily by the relative permeabilities and viscosities as was indicated in equation 3. At extremely low displacement rates, however the frictional forces may be negligible and the balance between the capillary and gravitational forces control the saturation distribution.

When water advances into the reservoir as a result of oil production, the level of zero capillary pressure rises, creating a tendency for the water saturation throughout the reservoir to increase in order to attain a new balance between capillary pressure and gravity (Buckley and Leverett, 1941).

## 2.2 Displacement Theory

### 2.2.1 The Buckley and Leverett Displacement Theory

In 1941, Leverett in his pioneering paper presented the concept of fractional flow. Beginning with the Darcy's Law for water and oil in 1-D flow, he formulated the following fractional flow equation:

$$f_w = \frac{1 + \frac{k k_{ro}}{q_t \mu_o} \left( \frac{\partial P_c}{\partial x} - g \Delta \rho \sin \alpha \right)}{1 + \frac{\mu_w k_{ro}}{\mu_o k_{rw}}}$$

Equation 1: Fractional Flow Equation

where  $f_w$  is the fractional flow of water,  $q_t$  is the total flow rate of oil and water,  $k_{ro}$  and  $k_{rw}$  are relative permeabilities of oil and water respectively,  $\mu_o$  and  $\mu_w$  are viscosities of oil and water respectively,  $\frac{dP_c}{dx}$  is the capillary pressure gradient,  $\Delta \rho$  is the density difference ( $\rho_o - \rho_w$ ),  $\alpha$  is the reservoir dip angle, and  $g$  is the gravitational constant.

For the case where the reservoir is horizontal ( $\alpha = 0$ ), Eq. 1.1 reduces to:

$$f_w = \frac{1}{1 + \frac{\mu_w k_{ro}}{\mu_o k_{rw}}}$$

Equation 2: Simplified Fractional Flow Equation

In 1946, Buckley and Leverett presented the frontal advance equation. Applying mass balance to a small element within the continuous porous medium, they expressed the difference at which the displacing fluid enters this element and the rate at which it leaves it in terms of the accumulation of the displacing fluid. This led to a description of the saturation profile of the displacing fluid as a function of time and distance from the injection point. The most remarkable outcome of their

displacement theory was the presence of a shock front. The frontal advance equation obtained was:

$$\boxed{\left(\frac{\partial x}{\partial t}\right)_{S_w} = \frac{q_t}{A\phi} \left(\frac{\partial f_w}{\partial S_w}\right)_t}$$

Equation 3: Frontal Advance Equation

Where  $q_t$  is a total volumetric liquid rate, equal to  $q_o+q_w$ , A is the cross-sectional area of flow,  $f$  is porosity,  $S_w$  is water saturation (Gasimov, 2005).

For many years analytical models have been used to estimate performance of waterflood projects. The Buckley-Leverett frontal advance theory and Dykstra-Parsons method for stratified reservoirs have been used for this purpose, but not in combination for stratified reservoirs with different kh and oil-water relative permeability. The Dykstra-Parsons method has a major drawback in that it assumes the displacement of oil by water is piston-like.

## 2.3 Theoretical Foundation

### 2.3.1 Fluid Saturation Distribution

Engineer's first assumptions without any theoretical knowledge of the initial saturation distribution are that initial saturations are uniform throughout the water zone, oil zone or gas cap of a reservoir. As production takes place in a reservoir, the gas cap expands or the water encroaches and the saturation in the invaded zone of the reservoir becomes uniform. Such assumptions often referred to as piston-like displacement. Just as uniform saturation distribution seldom exists in a reservoir, piston-like displacement seldom takes place in the reservoir.

To illustrate what happens when one fluid displaces another in the reservoir, consider a water-drive reservoir such as shown in Fig. 2. in this figure, water is encroaching up dip at a relatively slow rate as the oil is produced near the top of the structure. Now, consider the saturation in the horizontal slices of the reservoir as the displacement proceeds. We do this by identifying each horizontal slice as being some distance,  $X$ , from the initial minimum depth of the 100% water saturation with the distance  $X$ , measured along the bed dip of the formation. When the initial saturation in these horizontal slices is plotted versus the  $X$  position of each slice, we obtain the relationship as shown in Fig 2

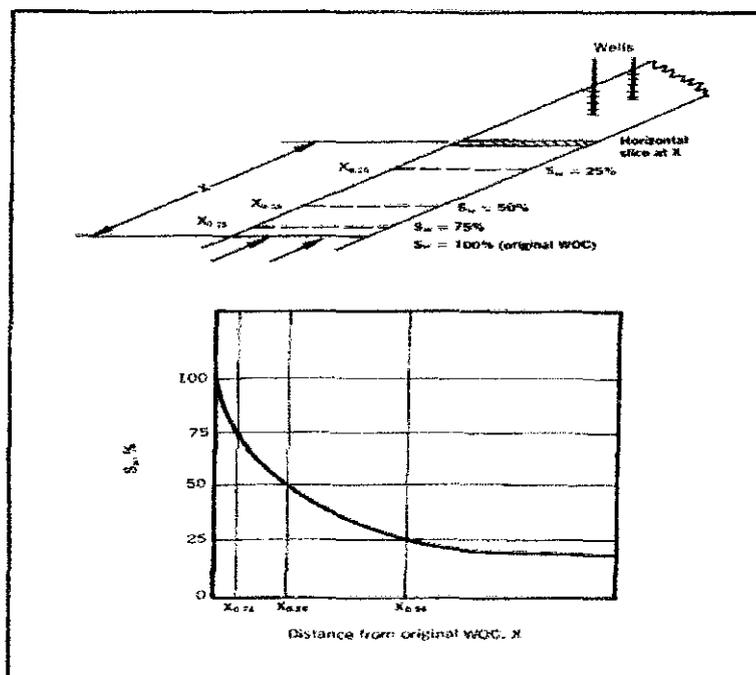


Figure 2: Saturation Profile in Water-Drive Reservoir

Consider the nature of the saturation distribution after 1 year, when a considerable amount of oil has been produced from this reservoir and the same amount of water has encroached into the initial oil-bearing portion of the reservoir. The resulting saturation distribution is shown in Fig. 3 along with the initial saturation distribution and the saturation distribution at later time of 2 and 3 years.

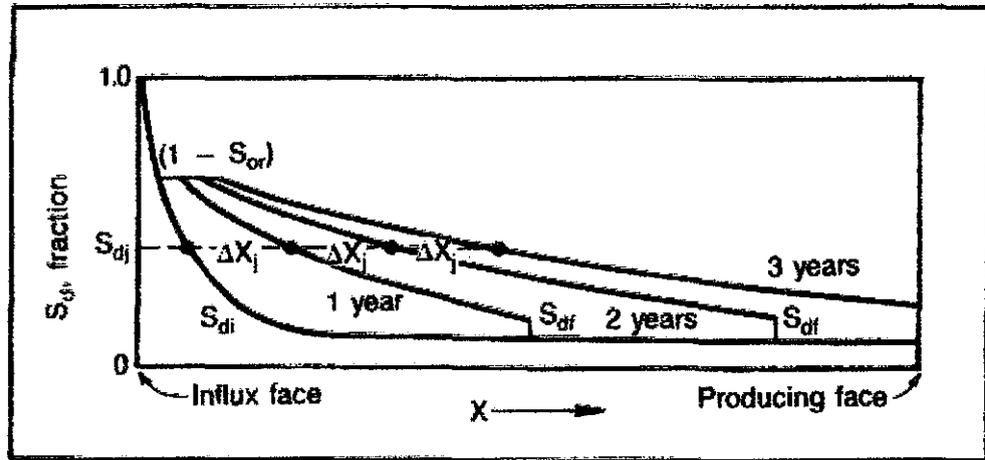


Figure 3: Fluid Displacement Characteristics with Initial Saturation Distribution

Notice that the only saturation discontinuities that exist are at the front or furthestmost advance of the displacing fluid. Note that the displacing-phase saturation is increasing at all times at most points in the reservoir behind the advancing front. In other words the advancing front does not displace all of the mobile oil as it moves through the reservoir. Instead, it acts more like a very inefficient piston. The front of the displacing fluid corresponds to the first stroke of the inefficient piston, which displaces some fraction of mobile oil. As water continues to flow through the same pore volume, it acts like a successive piston strokes with some percentage of the mobile oil that is left are displaced. Finally, after many pore volumes have flowed through the same pore space, many piston strokes have taken place, and all of the mobile oil has been displaced. The zone behind the displacing front is sometimes referred to as the drag zone, which seems to be fairly descriptive of what takes place physically in this part of the displacement.

There are two general characteristics of fluid displacements in porous media that are clearly indicated in Fig. 3. First, there is a saturation below which the saturation of the displaced (oil) phase cannot be driven regardless of the amount of the displacing fluid that passes through the porous media. The second general characteristic of immiscible displacement is that saturation travels through the reservoir at some fixed velocity as long as the displacement rate is constant.

## CHAPTER 3

### RESEARCH METHODOLOGY

#### 3.1 Introduction

##### 3.1.1 Understanding of Waterflooding

Waterflooding is the core topics for my project and understanding of the concept of waterflooding will gave me wide perspective of my project. This will also help me in completing the project.

The practice of waterflooding was applied to oilfields in the early 1900s and was soon termed secondary recovery as it followed the primary production phase and resulted in a second production surge from the same reservoir. When the technology suggested that the addition of chemicals or heat to the injected fluid could result in yet another surge the term tertiary recovery was coined. Eventually, engineers began to realize the value of maintaining reservoir pressure and waterflooding and gas injection began to be included in the development plans for new fields. In some cases, injection was initiated soon after the first barrel of oil was produced. Consequently, the term secondary recovery began to have less meaning.

Waterflooding is a method of secondary recovery in which water is injected into the reservoir formation to displace residual oil. The water from injection wells physically sweeps the displaced oil to adjacent production wells. Potential problems associated with waterflood techniques include inefficient recovery due to variable permeability, or similar conditions affecting fluid transport within the reservoir, and early water breakthrough that may cause production and surface processing problems.

### 3.1.2 Set the conditions and assumptions based on the reservoir

Based on my readings and understanding of other people's work on Buckley-Leverett's Theory, my assumptions on the reservoir properties and conditions are as below:

- I. constant width and length for all layers
- II. negligible capillary and gravity forces
- III. constant pressure drop for all layers at a given time
- IV. constant oil-water relative permeability for all layers
- V. constant total injection rate for the reservoir
- VI. water enters each layer in direct proportional to its capacity,  $kh$
- VII. uniform initial water saturation
- VIII. there is no cross-flow between layers

## 3.2 Buckley-Leverett Displacement Theory

### 3.2.1 Derivation of the Fractional Flow Equation

Consider displacement of oil by water in a system of dip angle  $\alpha$

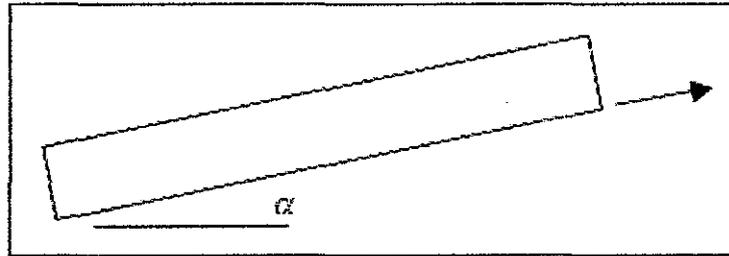


Figure 4: Example of a Displacement System with an Angle

Start with Darcy's equations

$$q_o = -\frac{kk_{ro}A}{\mu_o} \left( \frac{\partial P_o}{\partial x} + \rho_o g \sin \alpha \right)$$

$$q_w = -\frac{kk_{rw}A}{\mu_w} \left( \frac{\partial P_w}{\partial x} + \rho_w g \sin \alpha \right)$$

And replace the water pressure by  $P_w = P_o - P_{cow}$ , so that

$$q_w = -\frac{kk_{rw}A}{\mu_w} \left( \frac{\partial (P_o - P_{cow})}{\partial x} + \rho_w g \sin \alpha \right)$$

After rearranging, the equations may be written as:

$$-q_o \frac{\mu_o}{kk_{ro}A} = \frac{\partial P_o}{\partial x} + \rho_o g \sin \alpha$$

$$-q_w \frac{\mu_w}{kk_{rw}A} = \frac{\partial P_o}{\partial x} - \frac{\partial P_{cow}}{\partial x} + \rho_w g \sin \alpha$$

Subtracting the first equation from the second one, we get

$$\frac{1}{kA} \left( q_v \frac{\mu_v}{k_{rv}} - q_o \frac{\mu_o}{k_{ro}} \right) = -\frac{\partial P_{cap}}{\partial x} + \Delta \rho g \sin \alpha$$

Substituting for

$$q = q_w - q_o$$

$$f_w = \frac{q_w}{q}$$

and solving for the fraction of water flowing, we obtain the following expression for the fraction of water flowing:

$$f_w = \frac{1 + \frac{k k_{ro} A}{q \mu_o} \left( \frac{\partial P_{cap}}{\partial x} - \Delta \rho g \sin \alpha \right)}{1 + \frac{k_o \mu_v}{\mu_o k_{rv}}}$$

For the simplest case of horizontal flow, with negligible capillary pressure, the expression reduces to:

$$f_w = \frac{1}{1 + \frac{k_{ro} \mu_w}{k_{rw} \mu_o}}$$

Equation 4: Fractional Flow Equation

### 3.2.2 Graphical Analysis of the Fractional Flow Curve

From the relative permeability curve, Fractional Flow Curve can be constructed using the data of relative permeability.

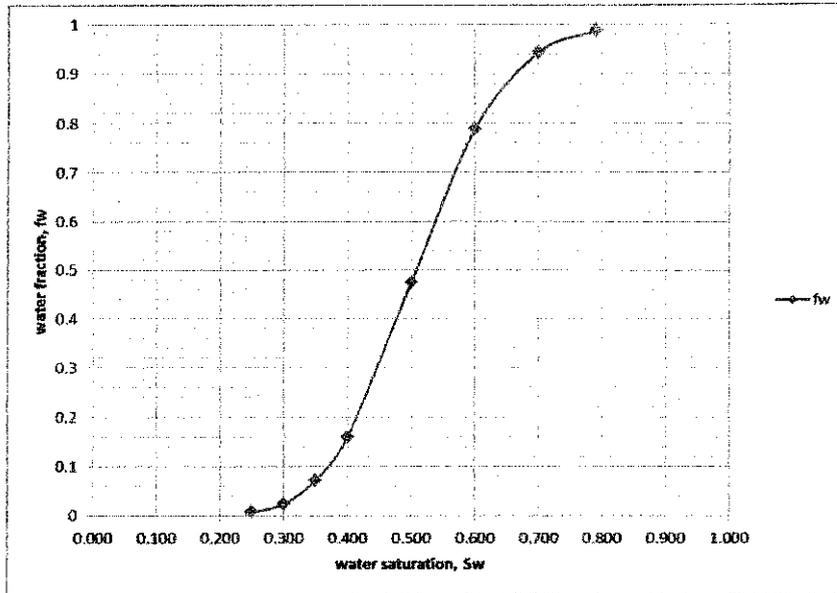


Figure 5: Fractional Flow Curve

Using the fractional flow curve, we could find the water saturation at the front of the waterflood and also the water saturation behind the front. Water saturation at the front can be determined by sketching a tangent line from  $S_{wc}$  to the fractional flow curve, the saturation value at the tangent point is equivalent to the water saturation at the front,  $S_{wf}$ .

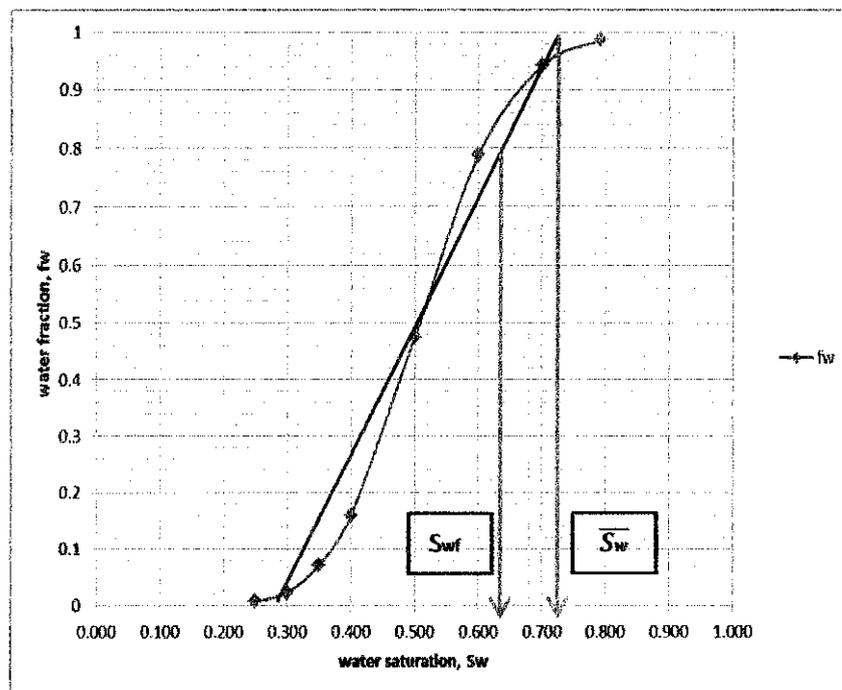


Figure 6: Fractional Flow with Tangent Line

For the average  $S_w$  behind front at breakthrough, the  $S_w$  value is intercept between the tangent line and the water fraction value of 1.

### 3.3 Derivation of Buckley and Leverett Frontal Advance Equation

Since

$$S_w(x,t)$$

We can write the following expression for saturation change

$$dS_w = \frac{\partial S_w}{\partial x} dx + \frac{\partial S_w}{\partial t} dt$$

In the Buckley-Leverett solution, we follow a fluid front of constant saturation during the displacement process, thus:

$$0 = \frac{\partial S_w}{\partial x} dx + \frac{\partial S_w}{\partial t} dt$$

Substituting into the Buckley-Leverett equation, we get

$$\frac{dx}{dt} = \frac{q}{A\phi} \frac{df_w}{dS_w}$$

Integration in time

$$\int \frac{dx}{dt} dt = \int \frac{q}{A\phi} \frac{df_w}{dS_w} dt$$

Yields an expression for the position of the fluid front:

$$x_f = \frac{qt}{A\phi} \left( \frac{df_w}{dS_w} \right)_f$$

Equation 7: Frontal Advance Equation

This called the frontal advance equation.

### 3.4 Predicting the Performance of Five-spot Waterflooding

#### 3.4.1 Application of the Recovery Curve Calculation

Much data are available that are usable for determining recovery curves for individual strata. Figs. 8 and 9 Dyes, Caudle and Erickson published similar data for other patterns. Since the problem lends itself to an interesting and not too difficult analysis either by use of flow models or computer models, there is much other data that can be used for the same purpose.

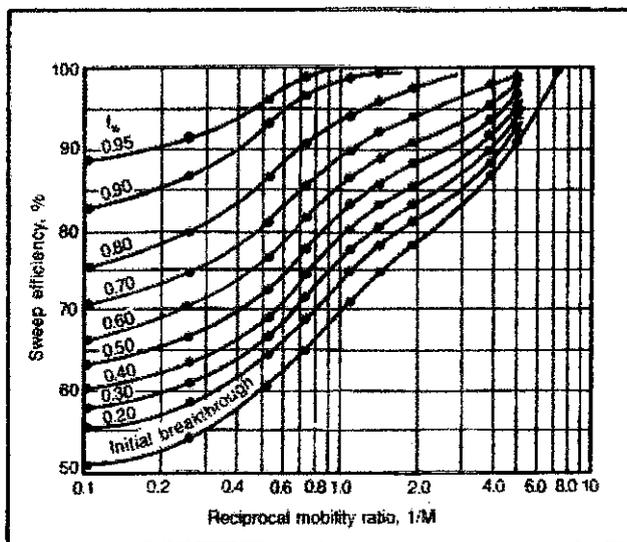


Figure 8: Effect of Mobility Ratio on Sweep Efficiencies for the five-spot pattern

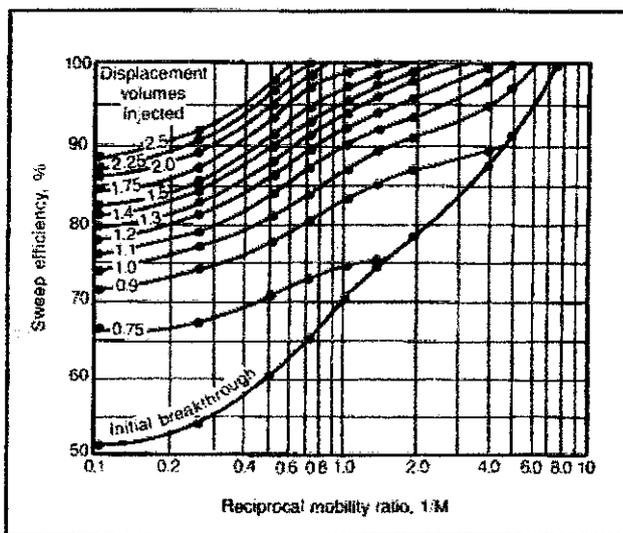


Figure 9: Effect of Mobility Ratio on the Displaceable Volumes Injected for Five-Spot Pattern

Most of the data assume piston-like movement or a minimal drag zone, a zone containing mobile oil behind the flood front, characteristic of very permeable sand. However, the error introduced using the piston-like concept seems minimal compared to the effect of sweep efficiency.

Consequently, we can assume that mobile free gas in the strata is produced before displaced oil is produced. Thus, for particular strata we can assume that before gas fill-up the injected water volume is equal to the reservoir volume of free gas produced. After gas fill-up, the oil produced is equal to the injected water in the reservoir, less the reservoir volume of mobile free gas in the strata at the start of the flood. When the swept area is 100%, all of the mobile oil and gas have been displaced from the reservoir. Thus, when the swept area is less than 100%, we can state the oil and gas production as:

$$B_o N_{pt} / V_p = E_H (S_{os} - S_{or} + S_{gi} - S_{ge}) - (S_{gi} - S_{ge})$$

If the initial mobile gas volume is subtracted from the oil and gas production and the resulting expression is solved for the reservoir barrels of oil production stated as a fraction of pore volume, we obtain:

$$B_o N_{pt} / V_p = E_H (S_{os} - S_{or} + S_{gi} - S_{ge}) - (S_{gi} - S_{ge})$$

To obtain the corresponding cumulative water injected from the displacement volumes injected, it is only necessary to recognize that one displacement volume is equal to the mobile oil saturation at the start of the flood, plus the mobile gas saturation at the start multiplied by the pore volume. Then, the cumulative water injection as a fraction of the pore volume is:

$$\left( \frac{W_i}{V_p} \right) = (DVI) (S_{os} - S_{or} + S_{gi} - S_{ge})$$

Using Fig. 9, it is then possible to read corresponding values of the percentage of the five spot swept and the displacement volumes injected for a particular reciprocal of the mobility ratio. These corresponding values can then be converted to reservoir barrels of oil production, as a fraction of pore volume, and water injection, as a fraction of the pore volume using both equation above.

### 3.4.2 Application of the Total Injectivity Curve Calculation

To approximate the Injectivity curve, we must first recognize that most pressure drops in a pattern occur at the wells, whether they are injection or production wells. The pressure distribution in a five spot pattern quadrant emphasizes this point.

Physically, the cross-sectional area between the injection and production wells is very large compared with the small cross section at the wells. Therefore, the wells provide most of the resistance to flow. This is true in five spot patterns as well as in virtually any waterflood pattern. Thus, we model the flood pattern by assuming it represents two radial systems back to back, as shown in Fig 10. to illustrate the use of simple geometry to approximate the behavior of a more complex geometry, the model of Fig 10 gives a flow equation almost the same as the exact analytical equation. We further expand this idea by assuming that saturations on the injection side are the same as saturations at the injection well and that saturations on the production side are the same as saturations at the production well.

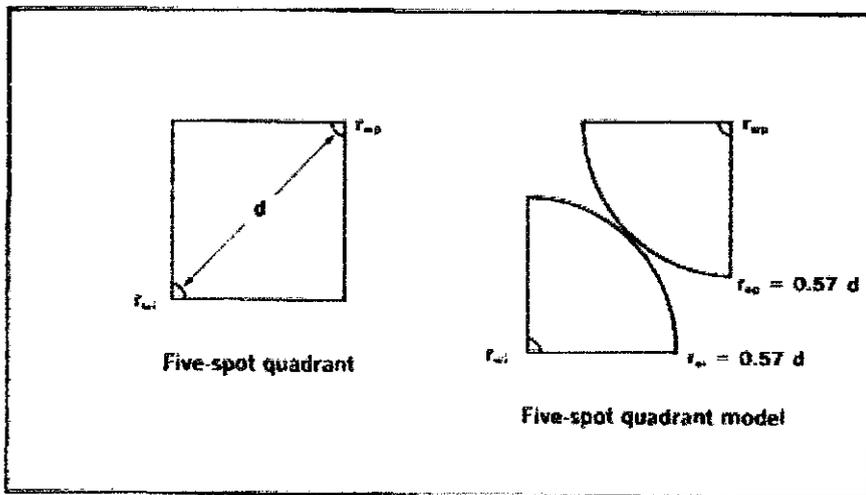


Figure 10: Modeling a Five-Spot

Using the model of Fig. 10, we can say that the total pressure drop between the injection and production wells is the sum of the pressure drop in the injection and production sides of the pattern:

$$p_{wi} - p_{wp} = \Delta p_i + \Delta p_p$$

We can determine an expression for the two pressure drops from the steady-state radial flow equation as a function of the injection rate,  $i$ :

$$p_{wi} - p_{wp} = 0.141i \ln(r_{oi}/r_{wi})(\mu_i/k_i)/h \\ + 0.141i \ln(r_{ep}/r_{wp})(\mu_p/k_p)/h$$

Equation 8: Pressure Drop on the Producing Side

When we solve Eq. 8 for the injection rate and assume the radius ratio on the injection and production sides is equal, we obtain:

$$i = \frac{(p_{wi} - p_{wp})h/0.141 \ln(r_o/r_w)}{(\mu_i/k_i) + (\mu_p/k_p)}$$

Equation 9: Injection Rate with the Value of Pressure Drop

Note that the numerator of Eq. 9 is the same for all the zones at any particular time. Thus, the ratio of the injection rates into two zones at a particular time is the ratio of the denominator of Eq. 9. By writing the numerator of Eq. 9 as a constant, we obtain:

$$i = \frac{\text{constant}}{[(\mu_i/k_i) + (\mu_p/k_p)]}$$

Equation 10: Injection Rate with a Constant

To obtain the initial injection rate into each zone,  $i_s$ , we consider only water to be flowing at the injection well and free gas to be flowing at the production well. This represents a typical situation when the reservoir has been substantially depleted by solution-gas drive prior to the start of the flood. However, if the gas saturation is very low at the start of the flood, the initial injection rate should be based on the assumptions that oil is being produced.

If we assume that production is substantially free gas, note that the initial injection based on Eq. 10 is:

$$i_s = \frac{\text{constant}}{[(\mu_w/k_w) + (\mu_g/k_g)]}$$

Equation 11: Injection Rate based on Water and Gas

When Eq. 11 is applied to a substantially depleted solution gas drive reservoir, note that the reciprocal of gas mobility is negligible compared to the reciprocal of water mobility in the water bank. Thus, we can consider the reciprocal gas mobility to be zero, and Eq 11 becomes:

$$i_s = \frac{\text{constant}}{(\mu_w k_w)}$$

Equation 12: Injection Rate without Negligible Gas Mobility

Based on the assumptions that Injectivity does not change until a substantial change in saturations occurs at the injection or production well, and that all of the free gas is produced from a particular strata before any oil rate increase occurs, the injection rate remains constant until gas fill-up has occurred in the strata. Thus, the injectivity ratio,  $i_s/i$  is 1.0 until  $W_i/V_p$  equals  $S_{gi}$ . At that time the oil bank reaches the producing well and oil starts flowing. Then, during the time between gas fill-up and water breakthrough into the producing well, the injectivity ratio according to Eq. 11 is:

$$i \text{ for fillup to BT} = \text{constant}/[(\mu_w/k_w) + (\mu_o/k_o)]$$

Equation 13: Injection Rate for fill-up to Breakthrough

In Eq. 13 water mobility should still be evaluated at the saturations existing in the water bank, and oil mobility should be evaluated at the saturations existing in the oil bank. A ratio of Eqs. 12 and 13 provides the injectivity ratio for the period from the gas fill-up for first water breakthrough:

$$(i_s/i) \text{ for fillup to BT} = 1 + (\mu_o/k_o)(k_w/\mu_w)$$

Equation 14: Injection Ratio for fill-up to Breakthrough

Recognizing the last term as the mobility ratio,  $M$ , we obtain:

$$(i_s/i) \text{ for fillup to BT} = 1 + M$$

Equation 15: Simplified Injection Ratio for Fill-up to Breakthrough

During the period when water and oil are both being produced from particular strata, the evaluation of the injectivity ratio is more difficult. It is necessary to modify the five spot models, Fig. 10, so both oil and water are flowing in parallel stream lines in the producing side of the model as shown in Fig 11. Using this model, the pressure drop on the production side of the model must be based on the oil or water flow rate, but the flow rate base is not the same as the flow or injection side. Thus, Eq. 11

cannot be used to determine the injectivity ratio when both oil and water are being produced from the strata.

It is convenient to state the pressure drop on the producing side of the five spot as a function of the water flow rate,  $q_w$ , and  $\theta_w$ , the fraction of the well radius flowing water.

$$p_{wi} - p_{wp} = \Delta p_p + \Delta p_i$$

Equation 16: Pressure Drop on the Producing Side

$$p_{wi} - p_{wp} = 0.141i (n(r_e/r_w)(\mu_w/k_w)/h) \\ + 0.141i (n(r_e/r_w)(\mu_w/k_w)/h)\theta_w$$

Equation 17: Pressure Drop with Water Mobility included

If the mobility ratio were 1.0,  $\theta_w$  would be equal to the water cut. However, the mobility ratio is seldom 1.0, so we must adjust the actual water cut on the basis of the mobility ratio to obtain the water cut that would exist if the mobility ratio were 1.0. This adjusted  $f_w$  then equals  $\theta_w$ .

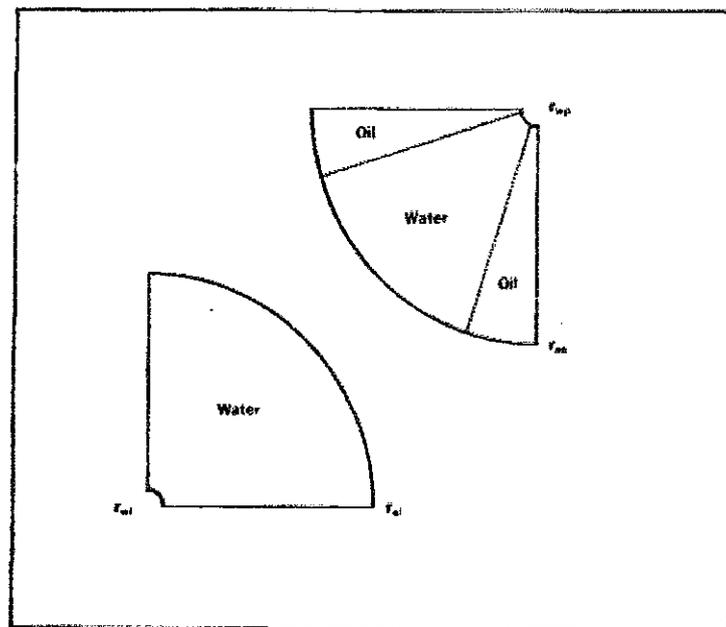


Figure 11: Five-Spot Quadrant Model After Water Breakthrough

On equation form we can write  $\theta_w$  as:

$$\theta_w = f_w / [f_w + (1 - f_w)M]$$

Equation 18: Theta of Water

Substituting for  $\theta_w$  in Eq. 17 according to Eq. 18, substituting  $f_w$  for  $q_w$ , and solving the equation for the injection rate, we obtain:

$$i \text{ after fillup} = \frac{(p_{wi} - p_{wp})h/0.141 \ln(r_e/r_w)}{(\mu_w/k_w)[1 + f_w + M(1 - f_w)]}$$

Equation 19: Injection Rate after Fill-up

Note that the numerator is the same as the numerator of Eq. 9, which is the constant of Eq. 19. Thus, the injection ratio is obtained by dividing Eq. 19 by Eq. 18.

$$(i_g/i) \text{ after fillup} = (1 + M) - (M - 1)f_w$$

Equation 20: Injection Ratio after Fill-up

Eq. 20 actually applies to any time after the gas fill-up, as indicated, which includes the period of time during which the water cut is zero. We discussed the period from fill-up to breakthrough and showed that the injectivity ratio as in Eq. 20 gives the same expression when  $f_w$  is zero. Thus, Eq. 20 and the realization that the injectivity ratio is 1.0 prior to the time of gas fill-up provide the entire history of the injectivity ratio for particular strata.

When using Figs. 8 and 9 to evaluate the injectivity ratio history for strata, it is necessary to determine the displacement volumes injected (DVI) at particular water cut to calculate the injectivity ratio at particular stage of the injection. Since the water cut and DVI injected do not appear on the same graph, it is necessary to relate them on basis of the swept area, which appears on both graphs.

### 3.4.3 Reduce Time Curve Calculation

Since our basic data include recovery versus cumulative injection for each zone, we can determine oil production for each zone at this particular time represented by this particular  $t_r$  value. Since we know the pore volume for each zone, we can convert recovery and cumulative injection data to barrels. The cumulative injection and injection rate then can be used to calculate injection time in days.

$$t_r = (\phi/k)_n \Sigma \left( \frac{i_s}{i} \right)_n \Delta(W_i/V_p)_n$$

Equation 21: Reduced Time Equation

#### Injectivity Ratio:

For fill-up to breakthrough

$$\frac{i_s}{i} = 1 + M$$

Equation 22: Simplified Injection Ratio for Fill-up to Breakthrough

After fill-up

$$\frac{i_s}{i} = (1 + M) - (M - 1)f_w$$

Equation 23: Injection Ratio after Fill-up

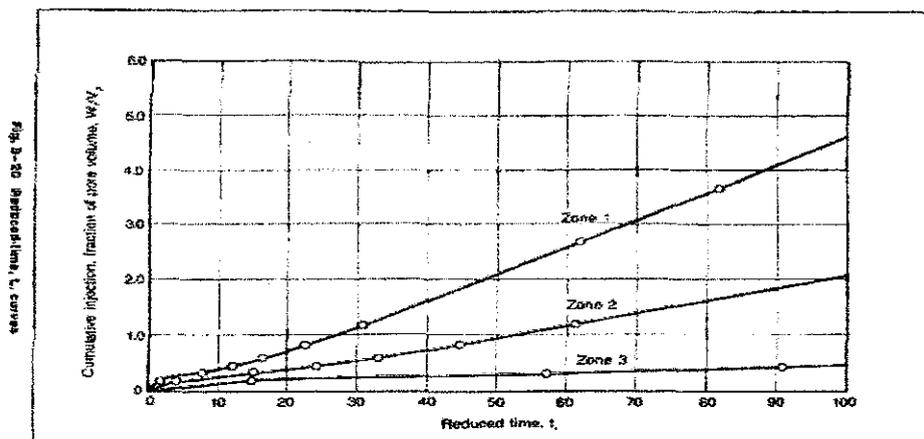


Figure 12: Example of Reduced Time Curve

In constructing  $tr$  curves is to first determine the ratio of the porosity and the absolute permeability for a particular zone. Any consistent units can be used since we are only interested in the relationship of the  $(W_i/V_p)$  values for different strata at a particular  $tr$ . Then to calculate the  $tr$  values assume some value for  $(W_i/V_p)$ . When this area is multiplied by the  $(\phi/k)$  ratio for those strata the reduced time  $tr$ , corresponding to that particular assumed  $(W_i/V_p)$  is determined and one point on the reduced time curve for that strata has been defined. The procedure is repeated until all the curves has been evaluated for all of the strata for the range of water injection desired.

### 3.4.4 Converting Reduce Time Data to Production versus Injection

Once reduced-time curves have been generated they can be interpreted in term of cumulative total injection and production of both oil and water. The reduced time curves show the relationship between cumulative injections into each zone at any particular reduced time. Consequently, even though we do not know directly the calendar time represented by any particular time, we can conclude that at that unknown real time the cumulative total injection will be some particular value.

This becomes meaningful when we recognize that a particular cumulative injection into a particular zone represents a particular cumulative production from that zone. Thus, for some unknown time we can determine the cumulative total injection and corresponding total oil and water recovery when such numbers are calculated for enough reduced times we have data representing cumulative oil and water production versus cumulative injection. By assuming some value for the injection rate, these data can in turn be used to calculate the cumulative oil and water production versus time.

$$\frac{W_i}{V_p} = (DVI)(1 - S_{wc} + S_{or})$$

Equation 24: Cumulative Water Injected as Fraction of Pore Volume

$$B_o N_{pf} / V_p = E_h(1 - S_{wc} + S_{or}) - S_{gi}$$

Equation 25: Oil Production as Fraction of Pore Volume

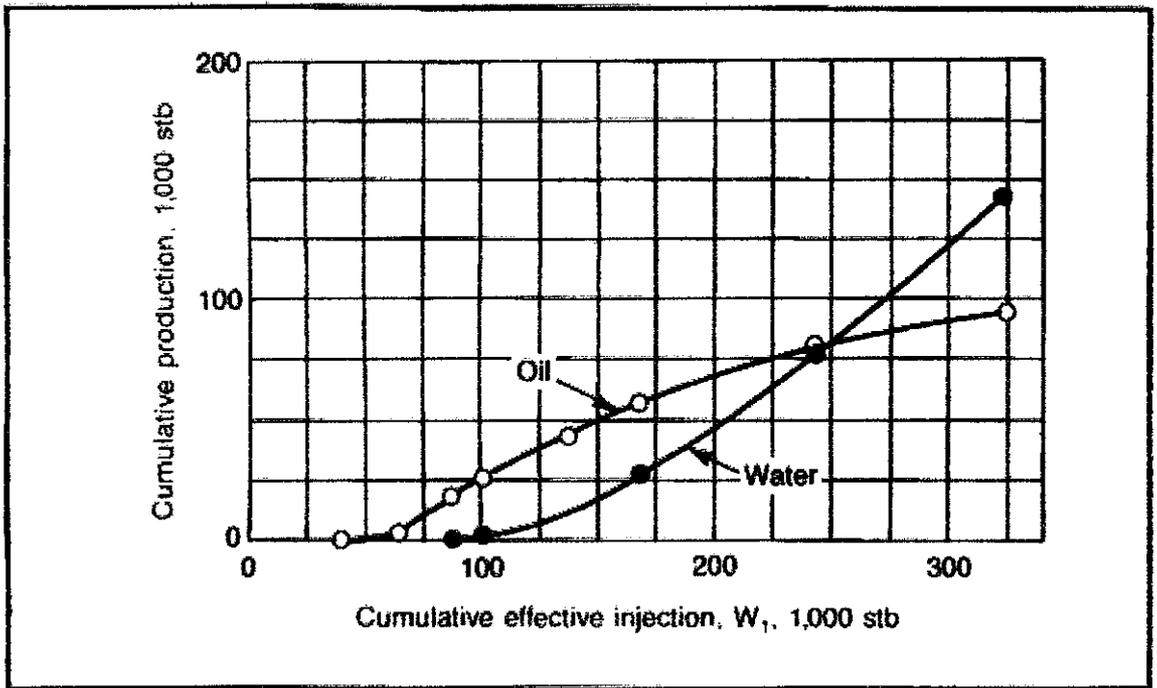
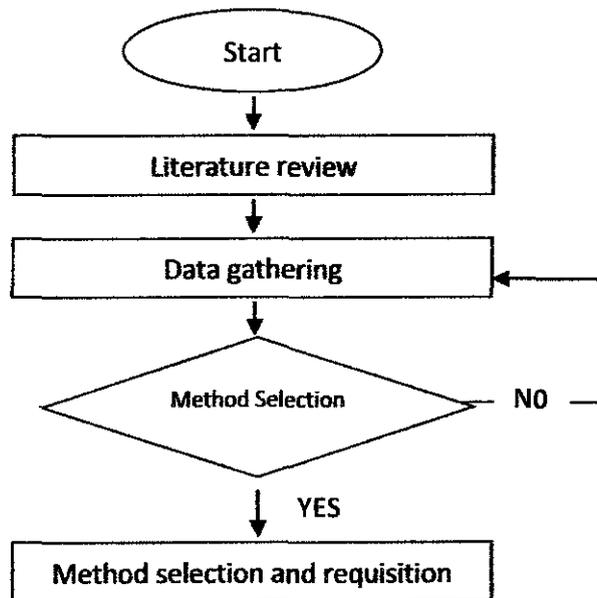


Figure 13: Example of Oil and Water Production Rates from Reduced Time Curves

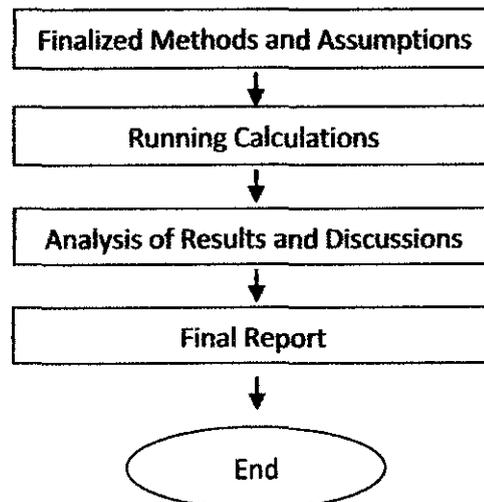
### 3.5 Key Milestone

In completing the project, student plays an important role as an investigator/researcher; doing all the literature study and look for his/her own approach to work on the topic. Thus, assistance and supervision from the assigned supervisor is essential to ensure the student is on the right path and follow the schedule. This could be done through a good communication medium such as weekly meeting, progress report and consultations. Progress report shall be submitted according to the schedule so that any corrective measure can be taken and indirectly both student and supervisor will have good and up-to-date information.

#### Final Year Project I



#### Final Year Project II



### 3.6 Gantt chart

No.	Activities /Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Project Work	█	█	█	█	█	█	█							
2	Progress Report Submission								█						
3	Project Work									█	█				
4	Pre-EDX											█			
5	Draft Report Submission												█		
6	Dissertation Submission													█	
7	Technical Paper Submission													█	
8	Oral Presentation														█
9	Project Dissertation Submission														█

## CHAPTER 4

### RESULTS AND DISCUSSIONS OF PART I: DISPLACEMENT ANALYSIS

#### 4.1 Introduction

##### Average Reservoir Fluid and Fluid Properties (Well 1)

<b>Distance between the injection and the producing wells, ft</b>	1414
<b>Average reservoir thickness, ft</b>	86
<b>Average reservoir porosity, fraction</b>	0.21
<b>Initial water saturation , fraction</b>	0.153
<b>Residual oil saturation, fraction</b>	0.21
<b>Total reservoir production, rb/day</b>	1977
<b>Average porosity, %</b>	21.7%
<b>Average water saturation, %</b>	15.3%
<b>Water viscosity, cp</b>	0.27
<b>Oil viscosity, cp</b>	0.423

Table 1: Average Reservoir Fluid and Fluid Properties Data

##### Reservoir Properties for Well 1

<b>Layers</b>	<b>Average Porosity</b>	<b>Average Permeability</b>	<b>Thickness</b>	<b>Pore Volume</b>
<b>Layer 1</b>	28%	430md	21ft	319MSTB
<b>Layer 2</b>	21%	224md	61ft	705MSTB
<b>Layer 3</b>	14.5%	110md	4ft	31MSTB

Table 2: Data of Reservoir Properties for Well 1

## Data for Different Case

Case	1	2	3
<b>Kro:</b>	0.98	0.98	0.98
<b>Krw:</b>	0.63	0.63	0.63
<b><math>\mu_o</math>:</b>	0.423	0.423	0.423
<b><math>\mu_w</math>:</b>	0.544	0.276	0.138
<b>M</b>	<b>0.50</b>	<b>1.00</b>	<b>2.00</b>

Table 3: Data for Different Mobility Ratio Cases

## 4.2 Oil-Water Relative Permeabilities

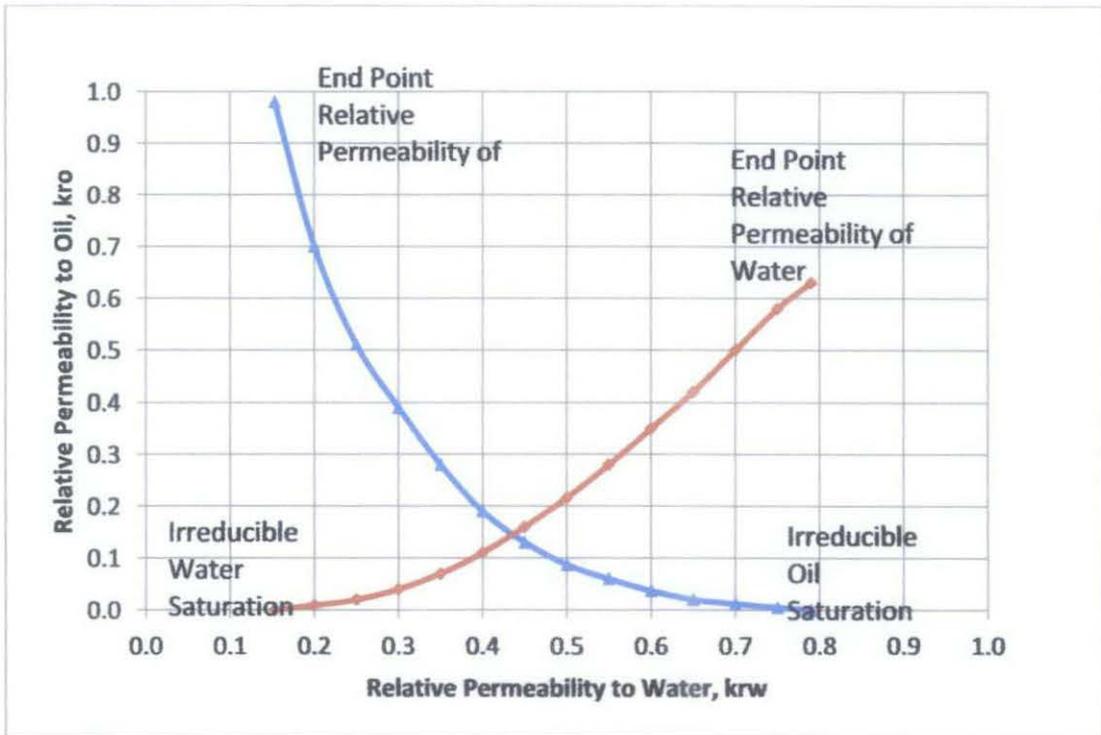


Figure 14: Relative Permeability Curves

When a wetting and a nonwetting phase flow together in a reservoir rock, each phase follows separate and distinct paths. The distribution of the two phases follows separate and distinct paths. The distribution of the two phases according to their wetting characteristics results in characteristic wetting and nonwetting phase relative permeabilities. Since the wetting phase occupies the smaller pore openings at small saturations, and these pore openings do not contribute materially to flow, it follows that the presence of a small wetting phase saturation will affect the nonwetting phase permeability only to a limited extent. Since the nonwetting phase occupies the central or larger pore openings which contribute materially to fluid flow through the reservoir, however, small nonwetting phase saturation will drastically reduce the wetting phase permeability.

Based on the relative permeability curves, we could see that the reservoir is slightly oil-wet. This is because the intersection of the curve is less than 0.5 of  $k_{rw}$ .

### 4.3 Fractional Flow Curve

#### Case 1 (M: 0.5)

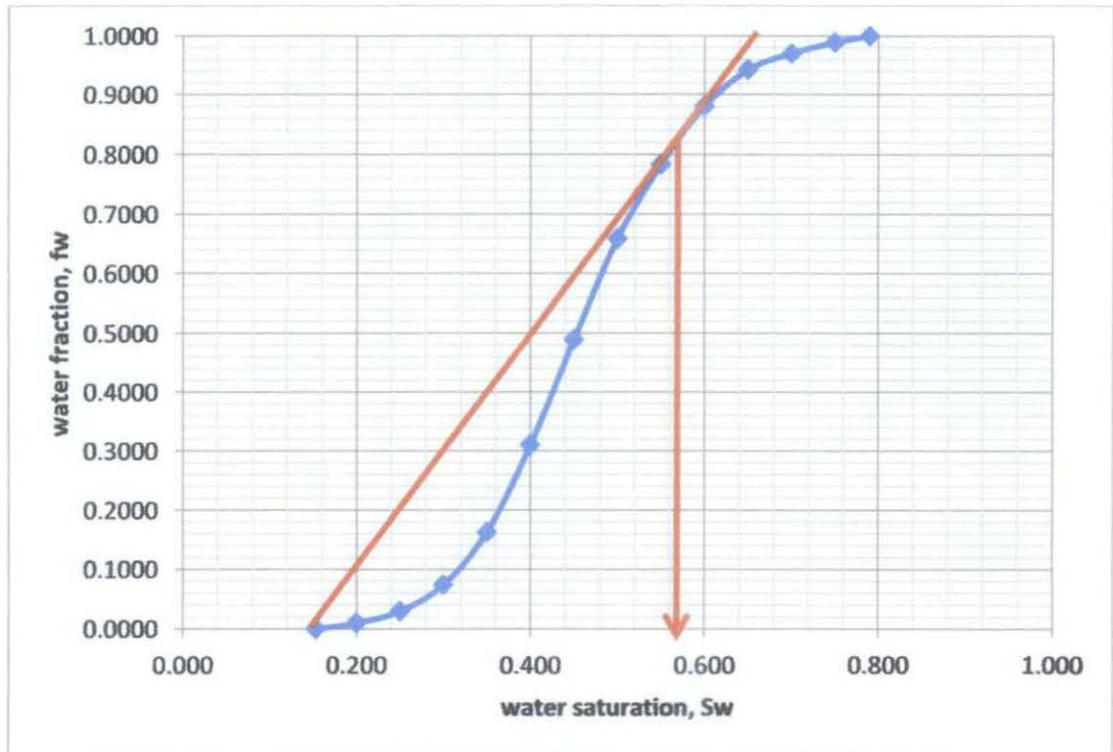


Figure 15: Fractional Flow Curves for Case 1 (M: 0.5)

Based on the intersection between the fractional flow curves and the tangent line, we could predict that the water saturation at the breakthrough is **0.57**.

Case 1 (with mobility ratio of 0.5) has a more efficient displacement process based on the fractional flow curve. The curve is shifted a bit lower which produce a lower value of water fraction. Decrease in value of water fraction increase the oil fraction and therefore increase the mobility of oil which results in better fluid displacement process.

## Case 2 (M: 1.0)

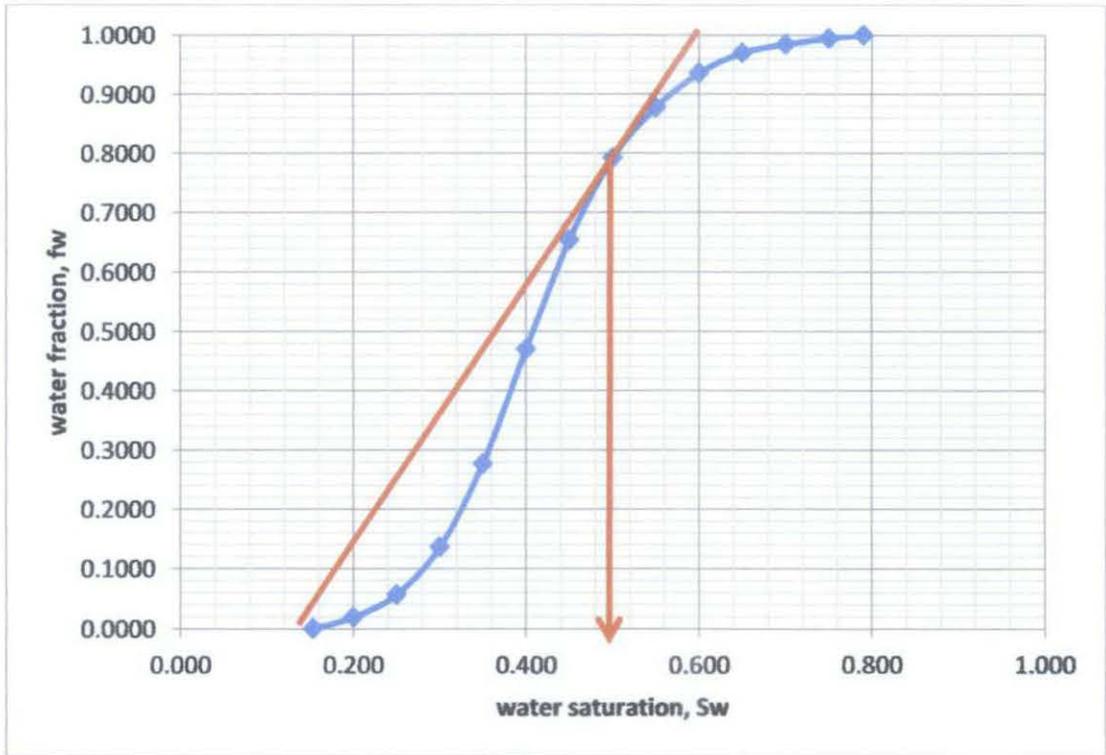


Figure 16: Fractional Flow Curves for Case 2 (M: 1.0)

Based on the intersection between the fractional flow curves and the tangent line, we could predict that the water saturation at the breakthrough is **0.50**.

Case 2 (with mobility ratio of 1.0) has a less efficient displacement process compared with case 2 based on the fractional flow curve. The curve is shifted a bit higher from case 1 which produces a higher value of water fraction. Increase in value of water fraction decrease the oil fraction and therefore decrease the mobility of oil which results in less efficient fluid displacement process.

### Case 3 (M: 2.0)

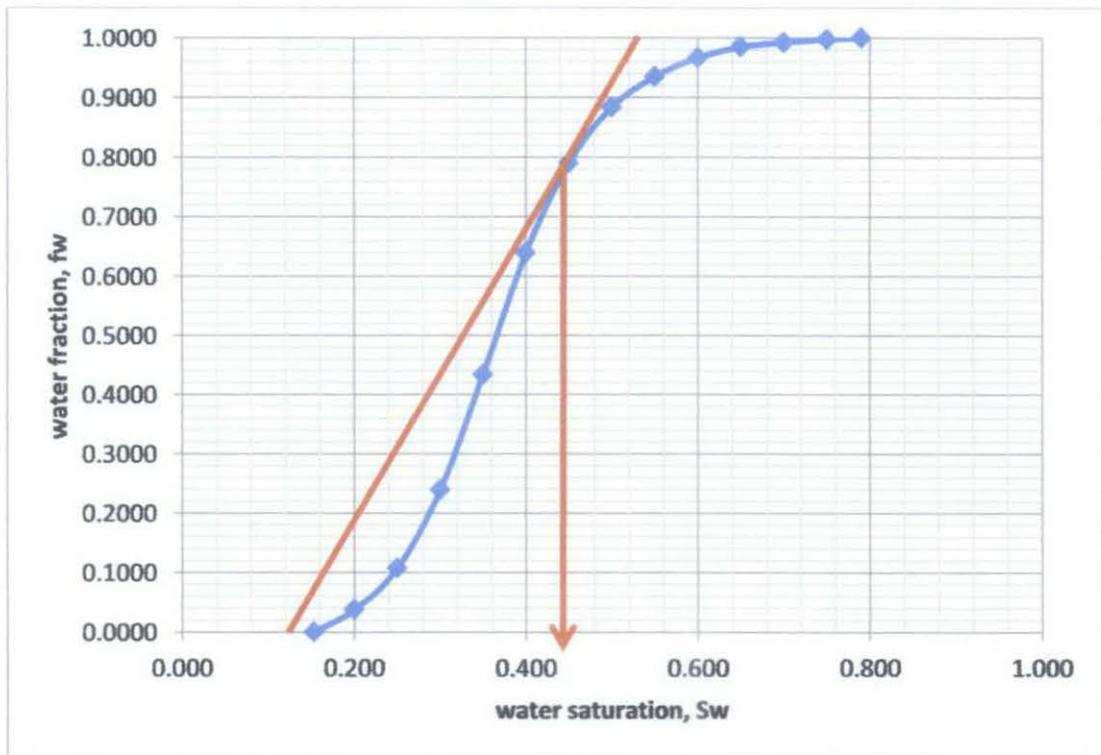


Figure 17: Fractional Flow Curve for Case 3 (M: 2.0)

Based on the intersection between the fractional flow curves and the tangent line, we could predict that the water saturation at the breakthrough is **0.45**.

Case 3 (with mobility ratio of 2.0) has a least efficient displacement process based on the fractional flow curve. The curve is shifted higher which produces a higher value of water fraction. Increase in value of water fraction decrease the oil fraction and therefore decrease the mobility of oil which results in least efficient fluid displacement process.

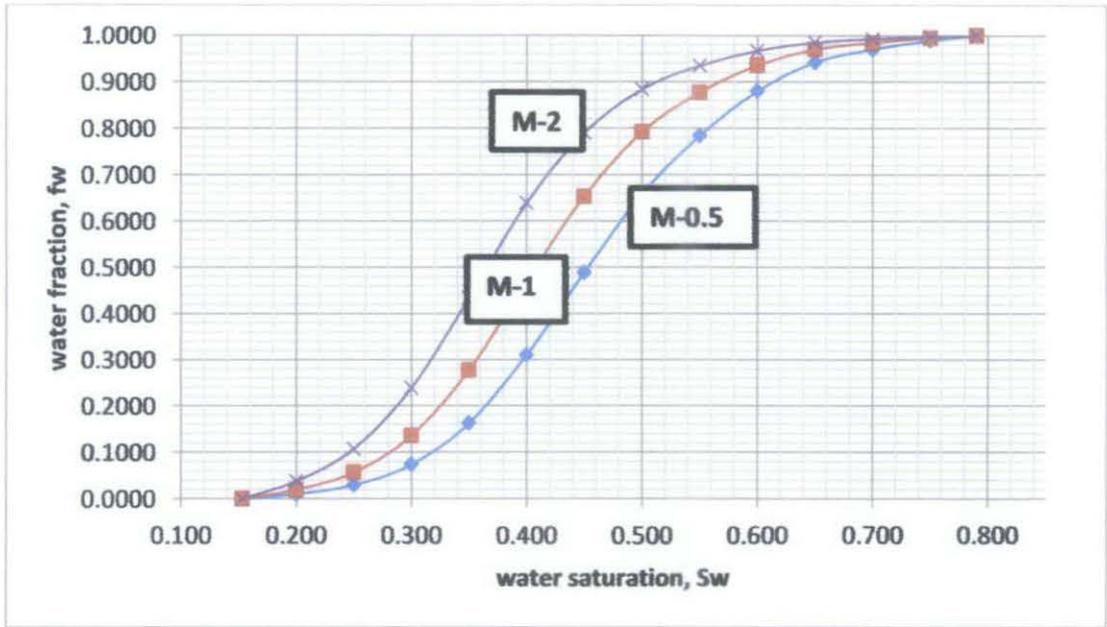


Figure 18: Comparison between Fractional Flow Curves of 3 cases

These are the comparison between three Fractional Flow Curve with different Mobility Ratio values. From the graph, we could see that viscosity of water (which influences the value of mobility ratio) affect the shape of fractional flow curve. Lower water viscosity produced higher water fraction and shift the curve positively upward. If fractional flow curve shift upward, it means less efficient displacement process. This is because, increase in water fraction cause decrease in oil fraction and oil mobility. Decrease in oil mobility means that the oil has less ability to move, which affect the waterflooding process as the water injected having a hard time to displace the oil with low mobility. Therefore, produces least efficient displacement process and results in lower recovery of oil.

In these situations, we could see that the viscosity of the displacing fluid play a big role in the displacement process. For the case 1 (mobility ratio of 0.5) with viscosity of 0.554cp, the displacement process is great as the viscosity of displacing fluid is higher compared to the viscosity of oil of 0.423cp. These conditions caused the water to flow behind the oil as it cannot bypass it due to the viscosity difference. If the viscosity of the displacing fluid is lower than the displaced fluid, such as in case 3, the displacement process is poor as the low viscosity water will flow bypass through the oil, minimizing the volume of oil that it can displaced.

#### 4.4 Determination of the Breakthrough Time

To find the breakthrough time of the waterflooding, Frontal Advance Equation is used.

$$x = \frac{5.615qt}{\phi A} \left( \frac{df_w}{dS_w} \right)$$

Equation 26: Frontal Advance Equation

$x$  is a distance between the injector and the producer, which in this case is 1414ft.  $q$  is an injection rate. Assumed that the displacement is piston-like displacement, injection rate is equal to the production rate which in this case is 1977bbl/day.  $t$  is the time taken for the injected water to flow along the  $x$  distance.  $\phi$  is the porosity value of the reservoir layer and  $A$  is the displacement area which is 1000ft of the reservoir width times 86 ft of the reservoir thickness.  $\left( \frac{df_w}{dS_w} \right)$  is the slope of the fractional flow curve. The value is difference in case of different viscosity of displacing fluid values. This slope plays a big role in determining the breakthrough time.

## 4.5 The Effect of the Mobility Ratio in Water Saturation Distribution during the Displacement

### Case 1 (M: 0.5)

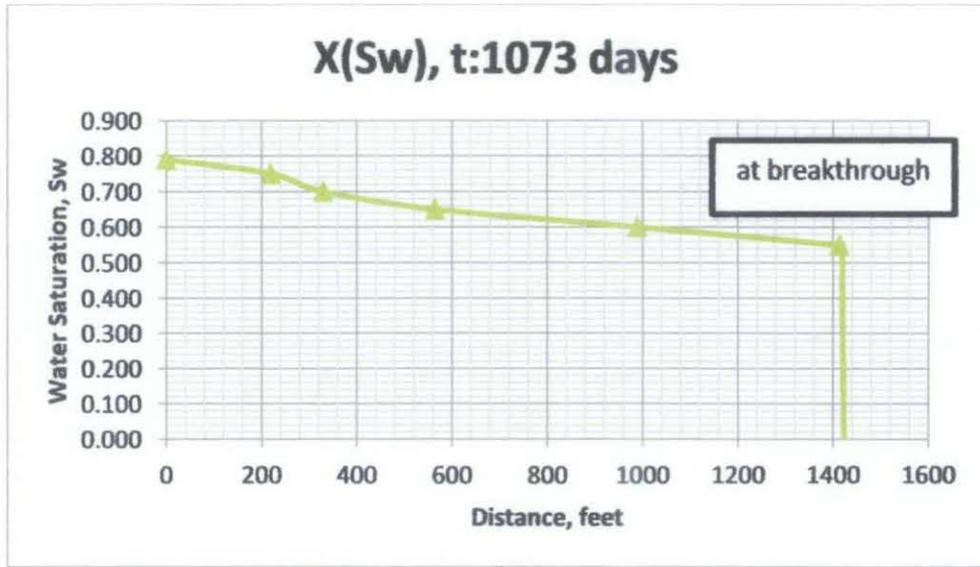


Figure 19: Fluid Saturation Distribution for Case 1 (M: 0.5)

### Case 2 (M:1.0)

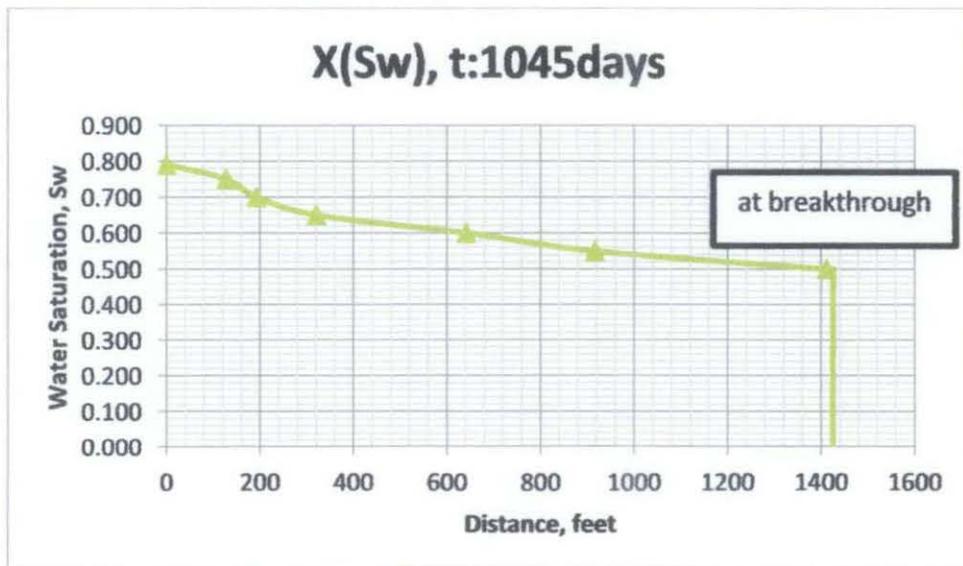


Figure 20: Fluid Saturation Distribution for Case 2 (M: 1.0)

**Case 3 (M:2.0)**

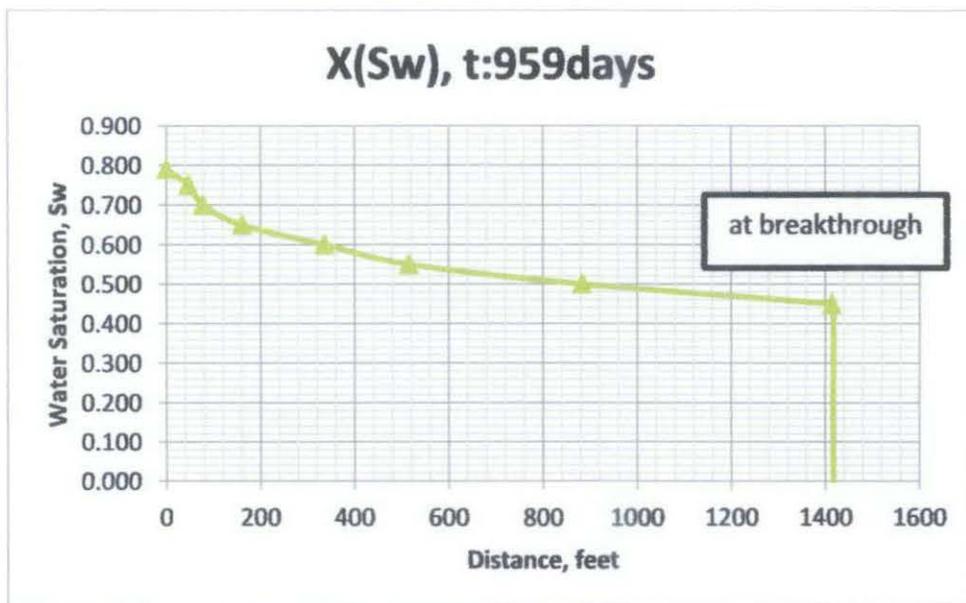


Figure 21: Fluid Saturation Distribution for Case 3 (M: 2.0)

By calculating the values using the Frontal Advance Equation, we could calculate the breakthrough time. The results are shown below:

Case	Mobility Ratio	Breakthrough Time, days
1	0.5	1073
2	1.0	1045
3	2.0	959

Table 4: Comparison of Breakthrough Time for 3 different Mobility Ratio

From the results, it is clearly indicated that case 1 takes the longest time to reach production well compared to case 3 which takes the shortest time to reach the production side. This can be explained by comparing the viscosity values of the displacing fluid possessed by each case. For case 1, the viscosity is 0.554cp which is higher than the viscosity of oil which is 0.423. Higher value of viscosity in displacing fluid cause a great displacing process as the water cannot bypass the oil.

To reach the production well, displacing fluid has to push the oil together without a chance to flow through it due to the viscosity difference. That is why it takes longest time to reach the end.

Different situation in case 3 as it has viscosity of water of 0.136, much lower compared to the oil viscosity of 0.423. In this case, the water, which was injected, *will try to move forward to the production well. As the viscosity of the displacing fluid is lower compared to the displaced fluid, its can bypass through the oil and reach the production well faster compared to the other 2 cases.*

## CHAPTER 5

### RESULTS AND DISCUSSIONS OF PART II: FIVE-SPOT WATERFLOODING PERFORMANCE

#### 5.1 General Description

The five-spot waterflooding performance prediction method that was used assumes knowledge of Injectivity of each permeability zone as a function of cumulative injection into that zone and knowledge of recovery of each zone as a function of cumulative injection into that zone.

This method which is also called as bookkeeping method is necessary because of the variation in the Injectivity, resistance to flow, or conductance of particular permeability strata during its flood life. The strata may start its flood life with essentially only gas flowing which would mean that the fluid flowing would have high mobility. Later on all of the gas may have been displaced from the strata and much less mobile oil and water will be flowing in the strata. Various strata do not take water at the same rate due to permeability differences. Consequently, no two strata of different permeabilities will be at the same state of depletion at the same time. What this means then is that the relation between the injectivity and resistance to flow in the various strata is continually changing as the flood develops.

## 5.2 Cumulative Oil and Water Production as Function of Cumulative Water Injection

### 5.2.1 Recovery and Injectivity Curves

#### 5.2.1.1 Recovery Curves for Layered Reservoir

Before the bookkeeping procedure can be initiated, recovery curves for each case. The recovery data, as presented below consist of three cases which are case 1, 2 and 3 with different mobility ratio.

#### Case 1 (M:0.5)

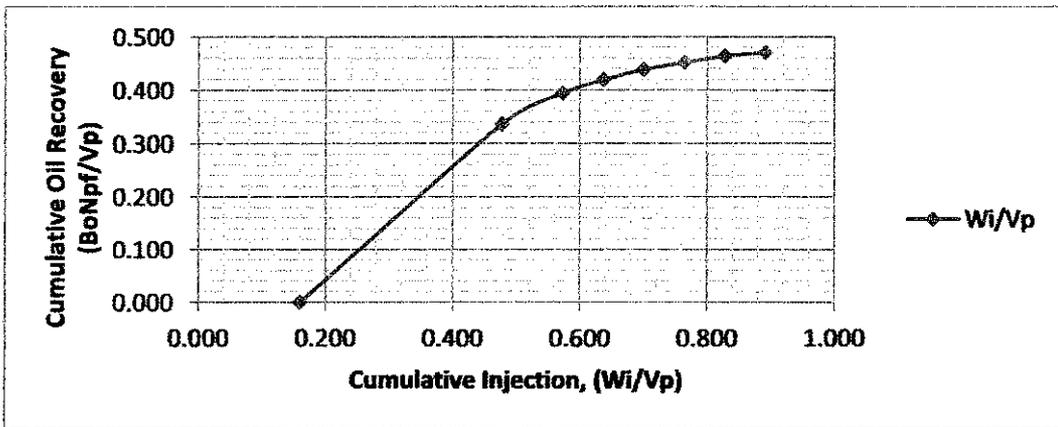


Figure 22: Recovery Curves for Case 1 (M: 0.5)

#### Case 2 (M:1.0)

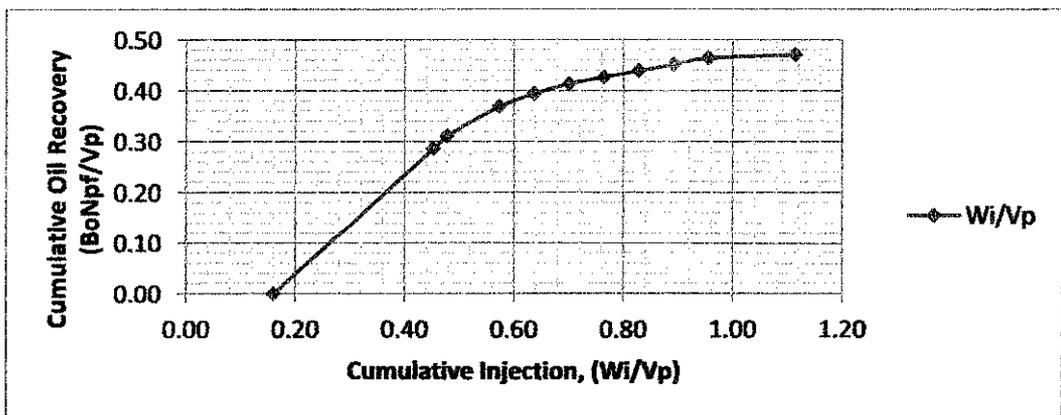


Figure 23: Recovery Curves for Case 2 (M: 1.0)

### Case 3 (M:2.0)

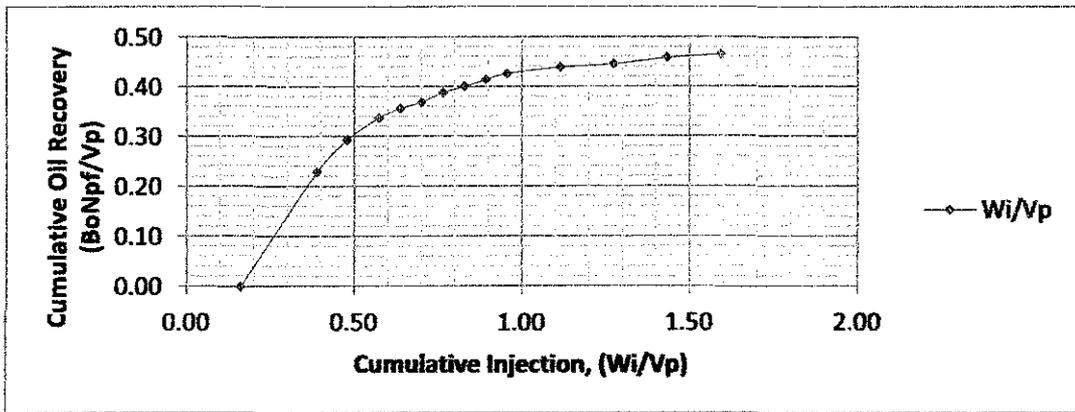


Figure 24: Recovery Curves for Case 3 (M: 2.0)

On the recovery curve, the cumulative oil production from the strata stated as a function of pore volume and cumulative injection again stated as a function of pore volume. The oil formation volume factor is included in the recovery term so that it will represent a fractional recovery in the reservoir with recovery in barrels at stock-tank conditions being Npf.

5.2.1.2 Total Injectivity Curve for Layered Reservoir

In order to construct a plot  $(W_i/V_p)$  versus a function of time,  $t_r$  for each of the cases, Injectivity data, permeability distribution and other reservoir parameters are needed.

**Case 1 (M:0.5)**

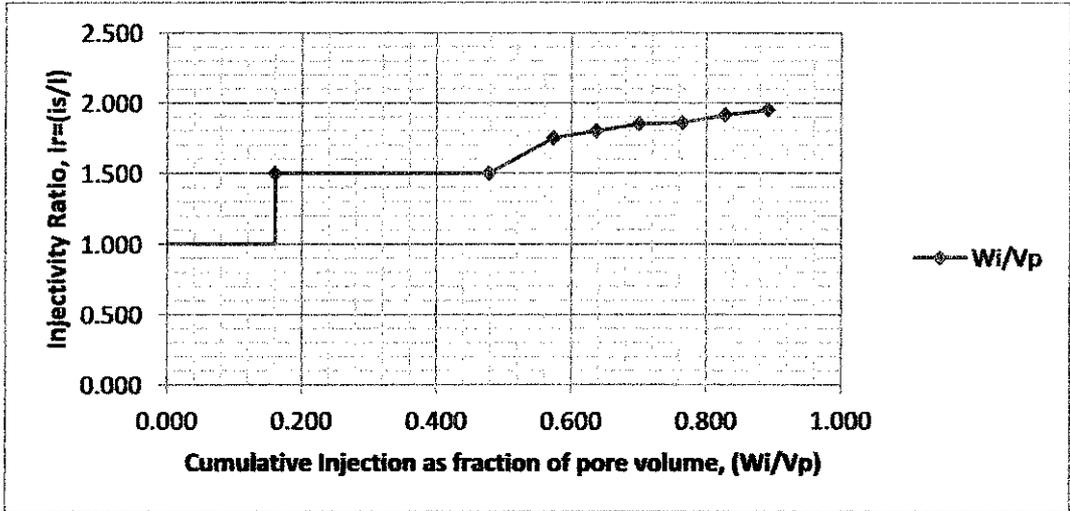


Figure 25: Injectivity Curve for Case 1 (M: 0.5)

**Case 2 (M:1.0)**

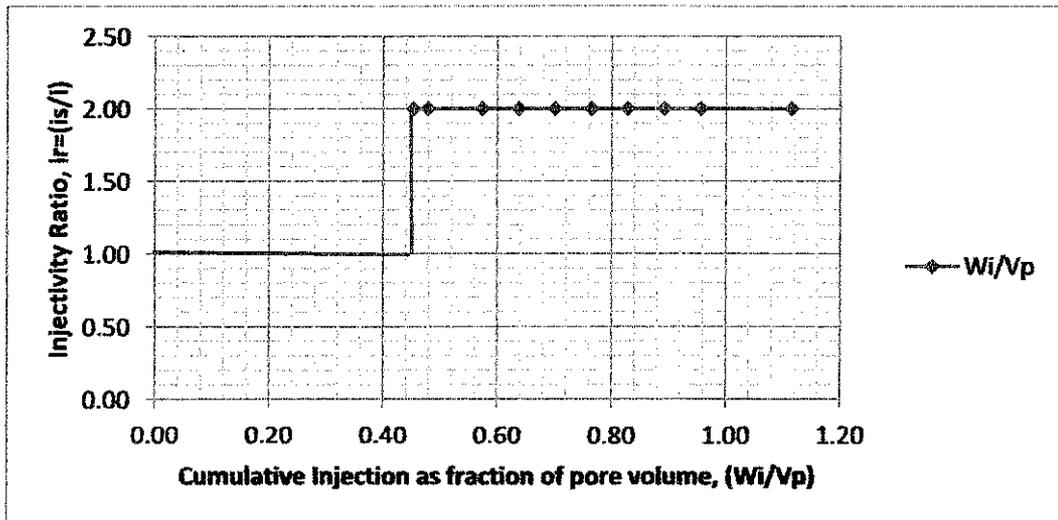


Figure 26: Injectivity Curve for Case 2 (M: 1.0)

### Case 3 (M:2.0)

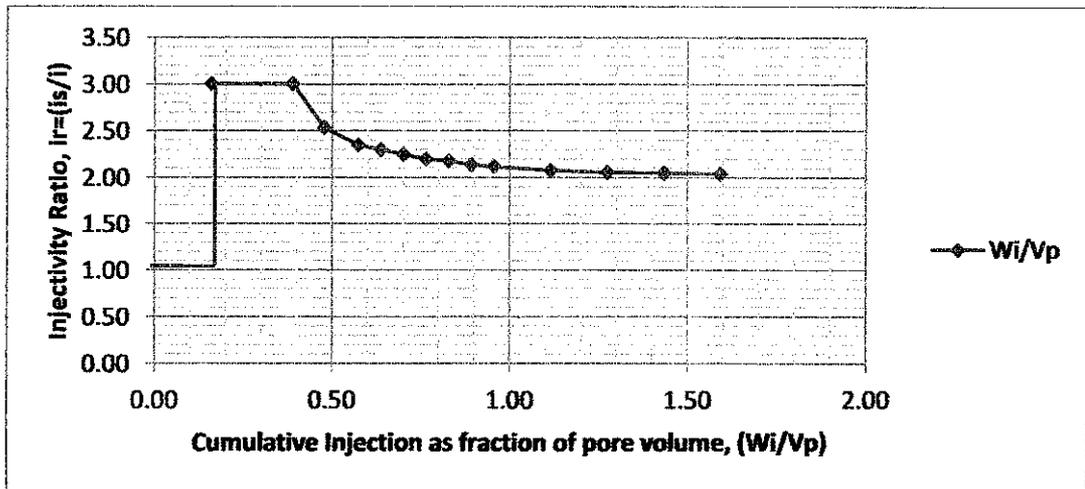


Figure 27: Injectivity Curve for Case 3 (M: 2.0)

Injectivity will not change until a substantial change in saturations occur at the injection or producing wells, and that all of the free gas will be produced from a particular strata before any oil-rate increase occurs, the injection rate will remain constant until gas fillup has occurred in the strata. Thus, the Injectivity ratio  $i_s/i$  would be 1.0 until  $W_i/V_p$  equals  $S_{gi}$ . At that time the oil bank reached the producing well and oil will start flowing. Then during the time between gas fillup and water breakthrough into the producing well, the injectivity are calculated by evaluating the saturations existing in the oil bank.

5.2.1.3 Reduce Time Curves

Case 1 (M:0.5)

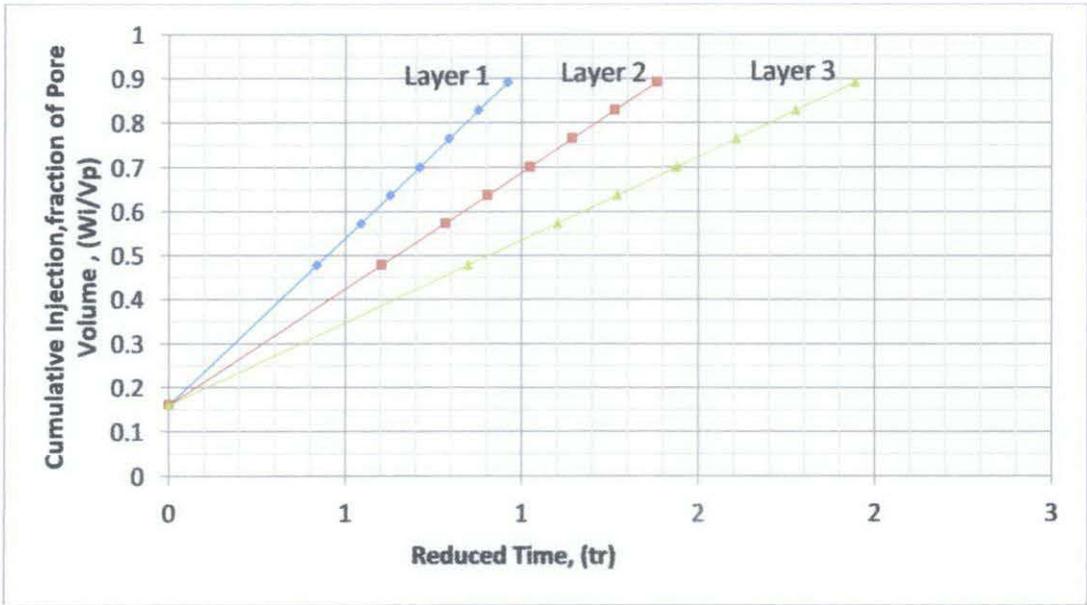


Figure 28: Reduce Time Curve for Case 1 (M: 0.5)

Case 2 (M:1.0)

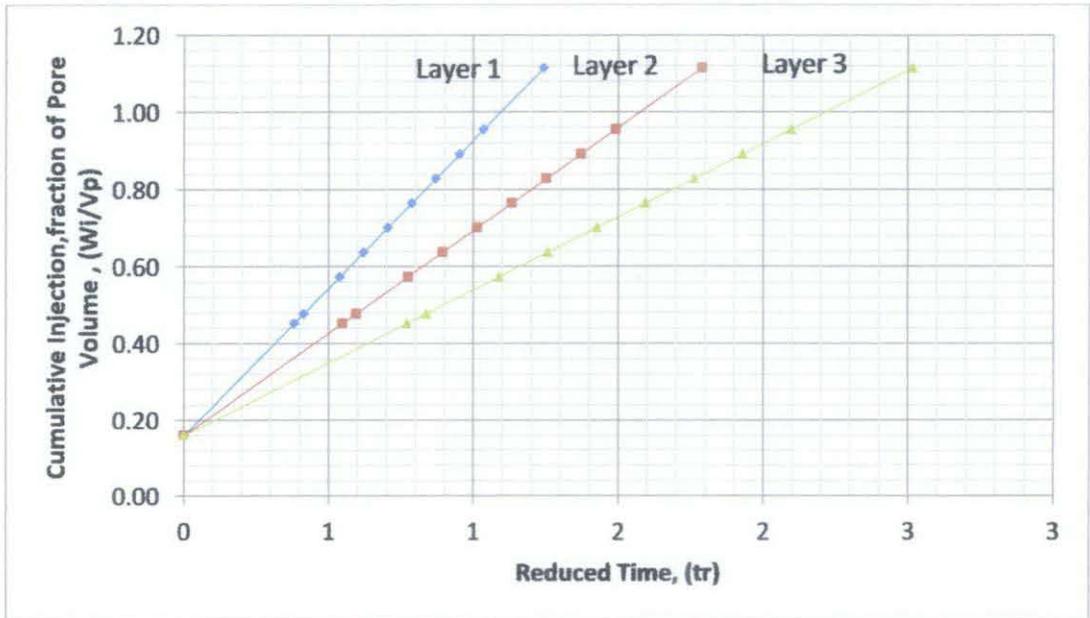


Figure 29: Reduce Time Curve for Case 2 (M: 1.0)

### Case 3 (M:2.0)

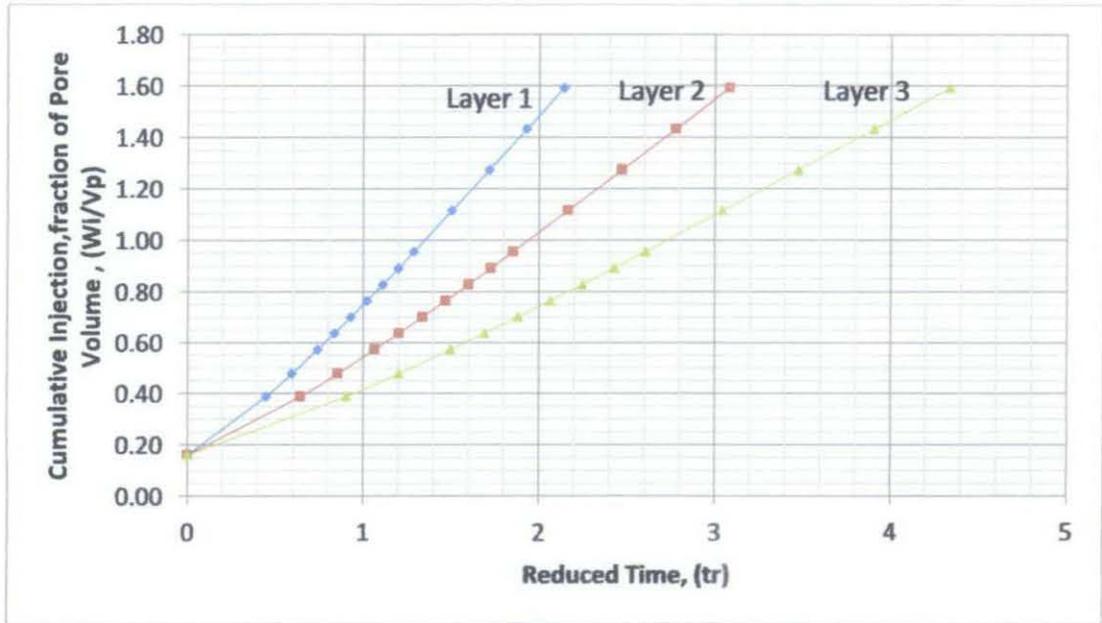


Figure 30: Reduce Time Curve for Case 3 (M: 2.0)

These 3 graphs show a connection between reduced time,  $t_r$  and cumulative injection in fraction of pore volume for 3 different reservoir strata. Layer 1 has the highest injection volume in all three cases as it has the highest value of permeability with 430md while Layer 3 has the lowest cumulative injection due to its low permeability of 110md. Meanwhile, Layer 2 has an average cumulative injection because of its permeability value is between Layer 1 and Layer 3 which is 110md.

### 5.2.2 Cumulative Oil and Water Production for each Layer versus Cumulative Water Injection

As the recovery curves and reduced time curves are now available, both data can be converted to Production versus Injection curves. Using the reduced time as a reference, a curve of Cumulative Oil Recovery and Cumulative Water Injection can be plotted.

#### Case 1 (M: 0.5)

Layer 1

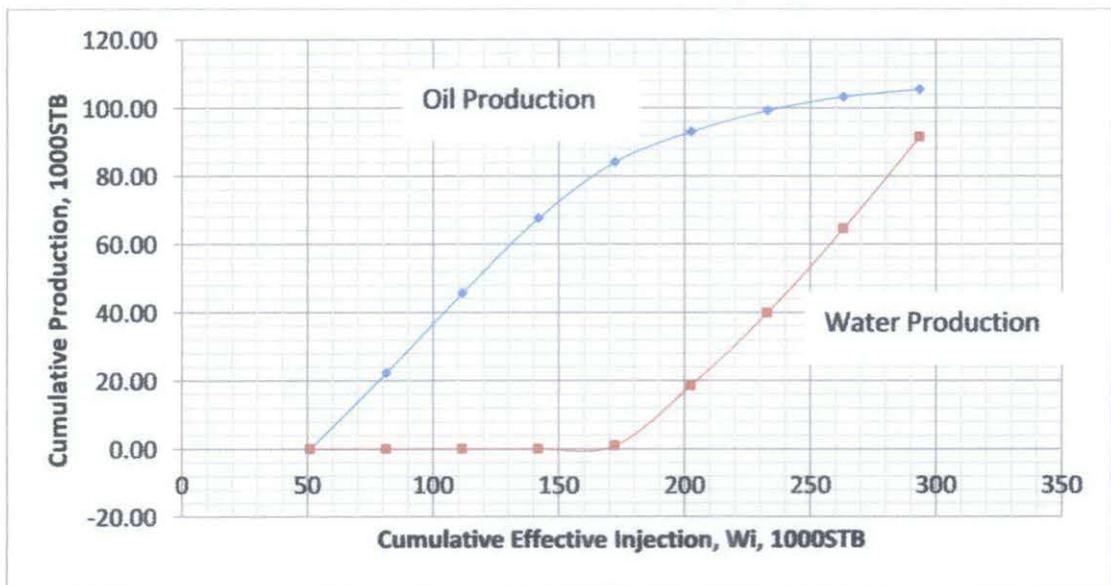


Figure 31: Production vs Injection Curves for Case 1 Layer 1

Layer 1 with permeability value of 430md produced greatest results in oil and water production. This is because the value of permeability for Layer 1 is the highest compared to others two. Therefore, the oil and water production for the layer is the highest as the fluid can easily flow from through the layer due to its high permeability.

## Layer 2

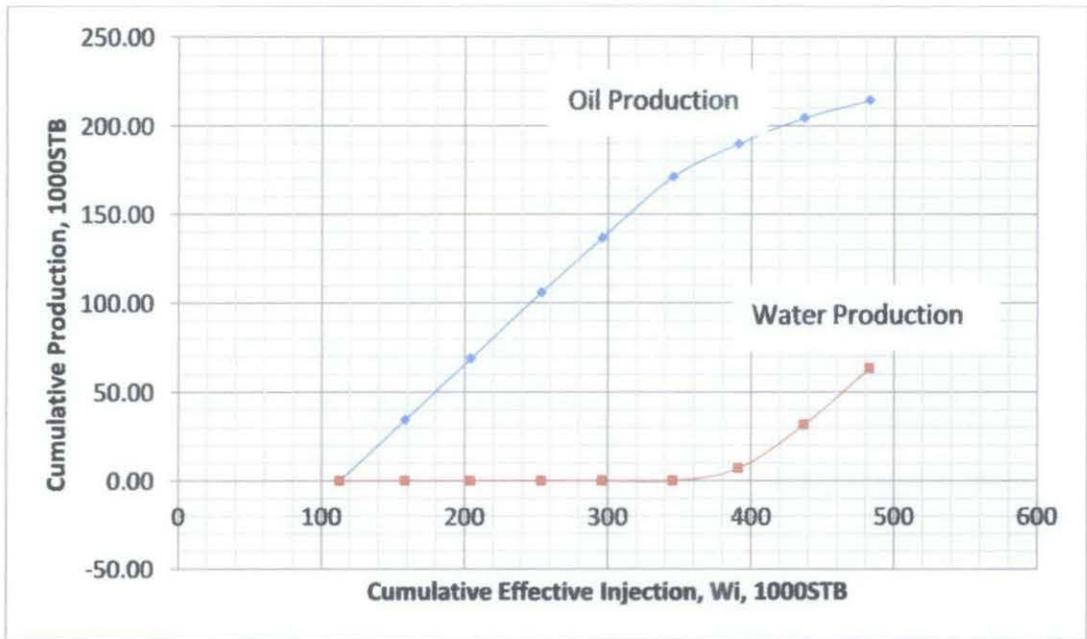


Figure 32: Production vs Injection Curves for Case 1 Layer 2

Layer 2 with permeability value of 224md produced much lower results in oil and water production. This is because the value of permeability for Layer 2 is the lower than Layer 1. Therefore, the oil and water production for the layer is the average as the fluid can flow from through the layer due to its average permeability.

### Layer 3

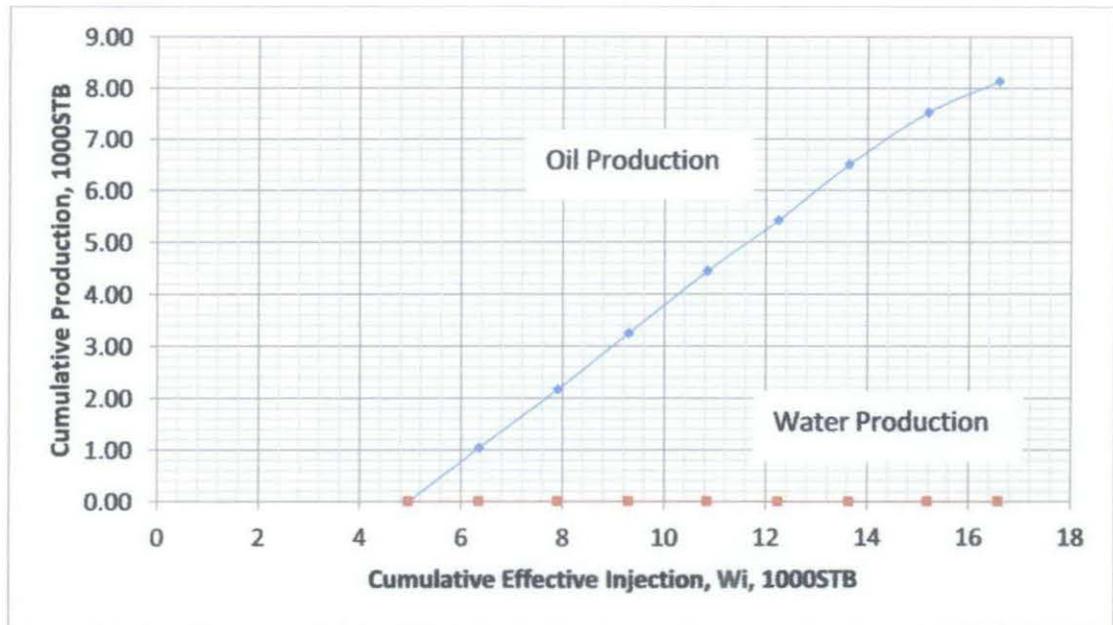


Figure 33: Production vs Injection Curves for Case 1 Layer 3

Layer 3 with permeability value of 110md produced lowest results in oil and water production. This is because the value of permeability for Layer 3 is the lowest compared to others two. Therefore, the oil and water production for the layer is the least as the fluid can barely flow from through the layer due to its low permeability.

## Case 2 (M: 1.0)

### Layer 1

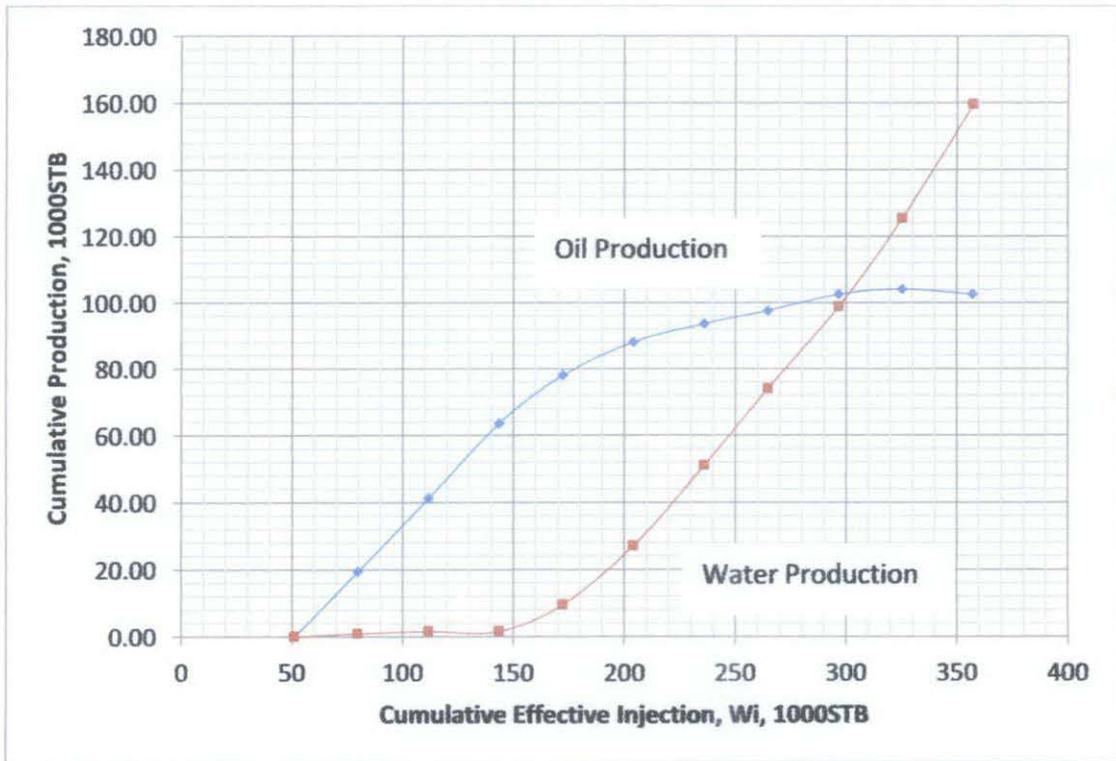


Figure 34: Production vs Injection Curves for Case 2 Layer 1

Layer 1 with permeability value of 430md produced greatest results in oil and water production. This is because the value of permeability for Layer 1 is the highest compared to others two. Therefore, the oil and water production for the layer is the highest as the fluid can easily flow from through the layer due to its high permeability.

## Layer 2

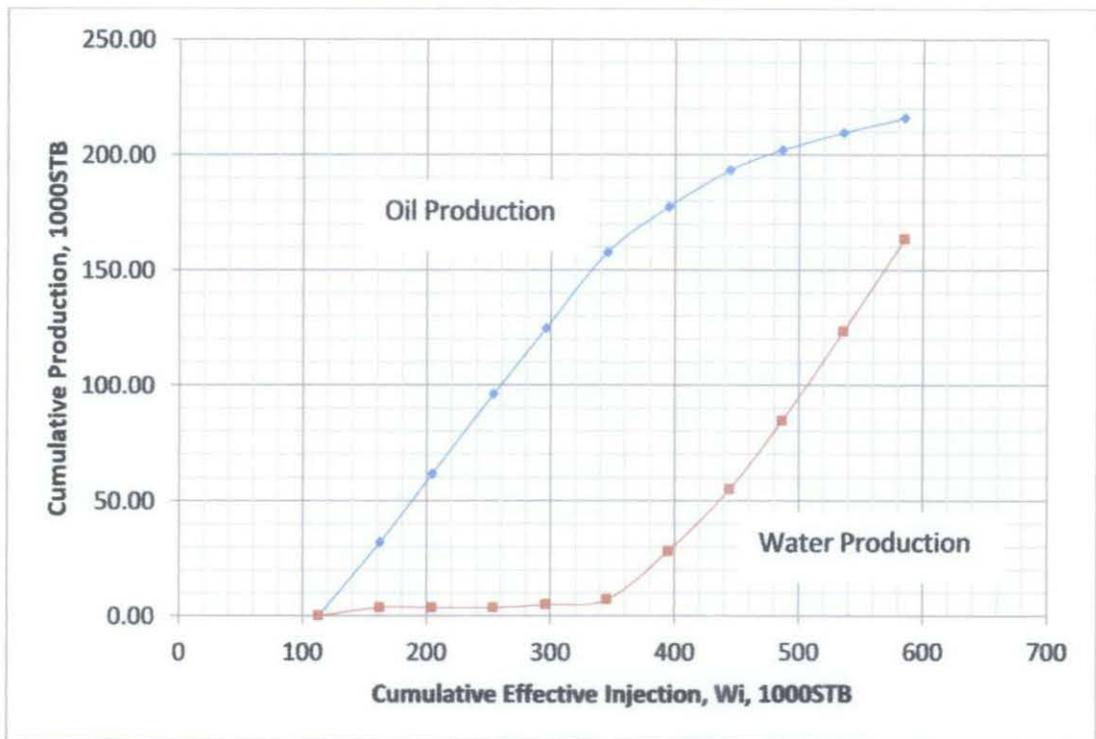


Figure 35: Production vs Injection Curves for Case 2 Layer 2

Layer 2 with permeability value of 224md produced much lower results in oil and water production. This is because the value of permeability for Layer 2 is the lower than Layer 1. Therefore, the oil and water production for the layer is the average as the fluid can flow from through the layer due to its average permeability.

### Layer 3

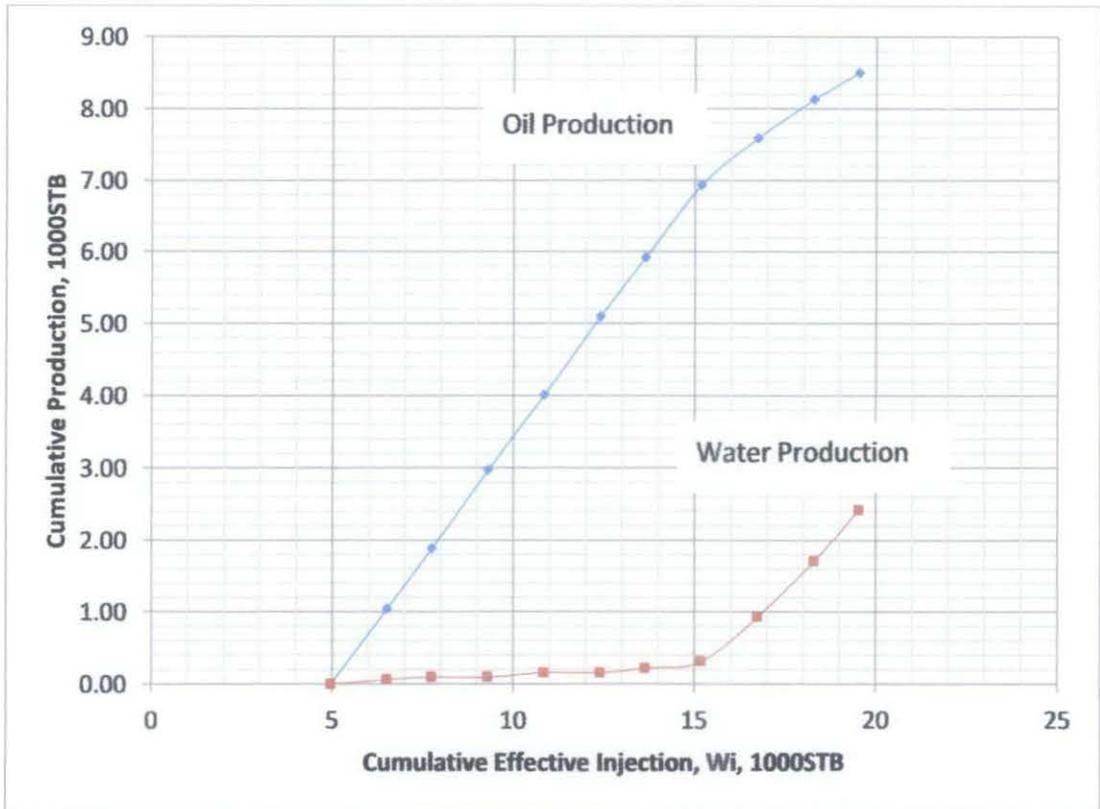


Figure 36: Production vs Injection Curves for Case 2 Layer 3

Layer 3 with permeability value of 110md produced lowest results in oil and water production. This is because the value of permeability for Layer 3 is the lowest compared to others two. Therefore, the oil and water production for the layer is the least as the fluid can barely flow from through the layer due to its low permeability.

### Case 3 (M: 2.0)

#### Layer 1

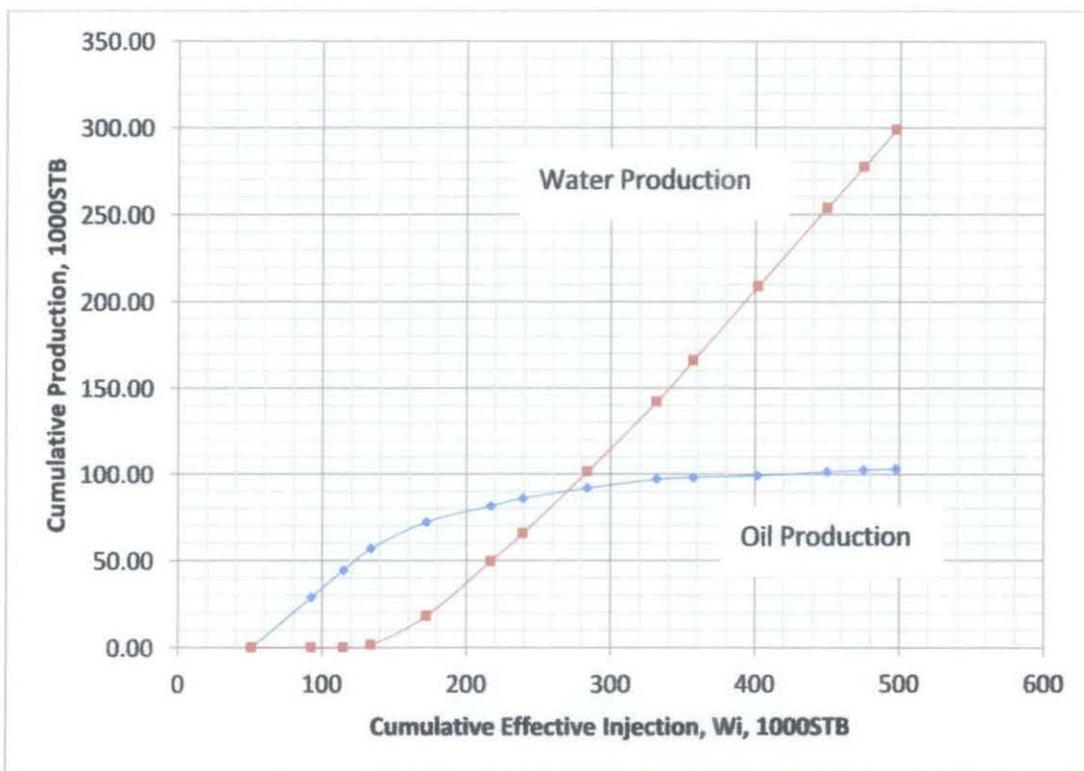


Figure 37: Production vs Injection Curves for Case 3 Layer 1

Layer 1 with permeability value of 430md produced greatest results in oil and water production. This is because the value of permeability for Layer 1 is the highest compared to others two. Therefore, the oil and water production for the layer is the highest as the fluid can easily flow from through the layer due to its high permeability.

## Layer 2

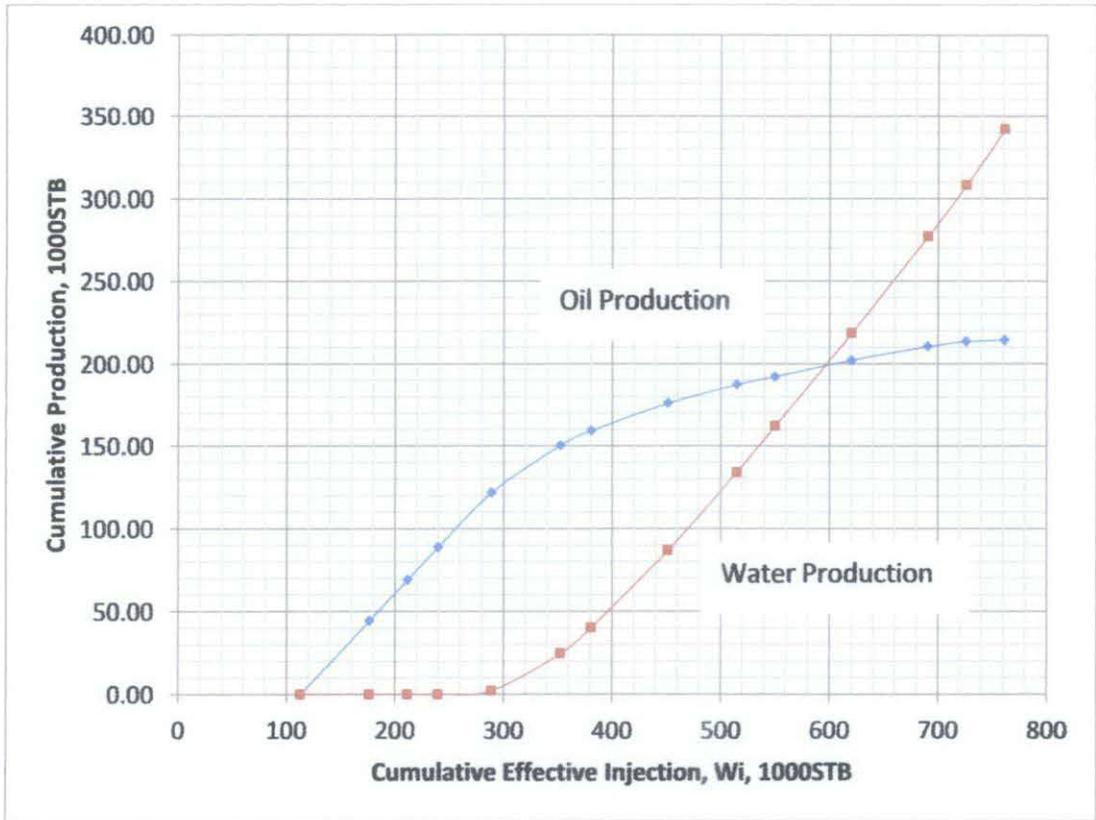


Figure 38: Production vs Injection Curves for Case 3 Layer 2

Layer 2 with permeability value of 224md produced much lower results in oil and water production. This is because the value of permeability for Layer 2 is the lower than Layer 1. Therefore, the oil and water production for the layer is the average as the fluid can flow from through the layer due to its average permeability.

### Layer 3

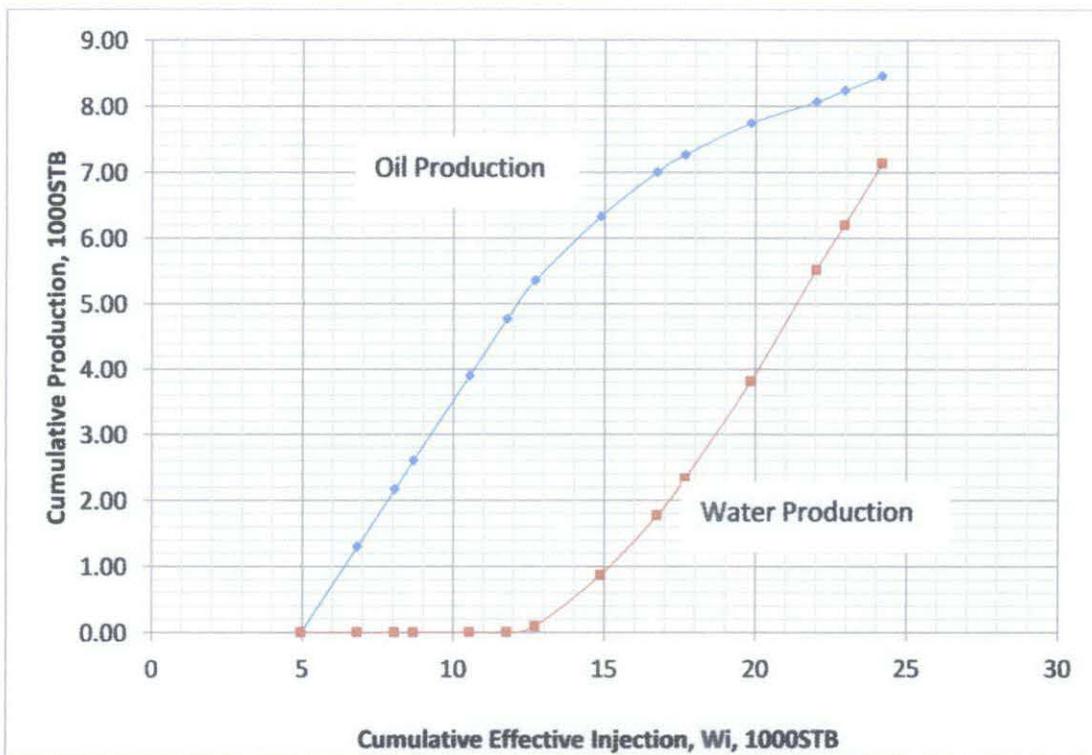


Figure 39: Production vs Injection Curves for Case 3 Layer 3

Layer 3 with permeability value of 110md produced lowest results in oil and water production. This is because the value of permeability for Layer 3 is the lowest compared to others two. Therefore, the oil and water production for the layer is the least as the fluid can barely flow from through the layer due to its low permeability.

For the Oil and Water Production on different layers, the same situation can be seen from the previous section. But this time, different layers have different values of permeability and porosity, which produced results as above. Let's compare the trend on 1<sup>st</sup> layer which have the highest values of permeability and porosity with the 3<sup>rd</sup> layers which have the lowest values of permeability and porosity. For the first layer, we could see that the water productions are higher due to the higher value of porosity and permeability compared to the 3<sup>rd</sup> layer which has the lowest water production due to its low porosity and permeability. These situations are happening regardless of mobility ratio value as porosity and permeability play bigger parts in terms of fluid movement in reservoir.

### 5.2.3 Total Cumulative Oil and Water Production versus Cumulative Water Injection

#### Case 1 (M: 0.5)

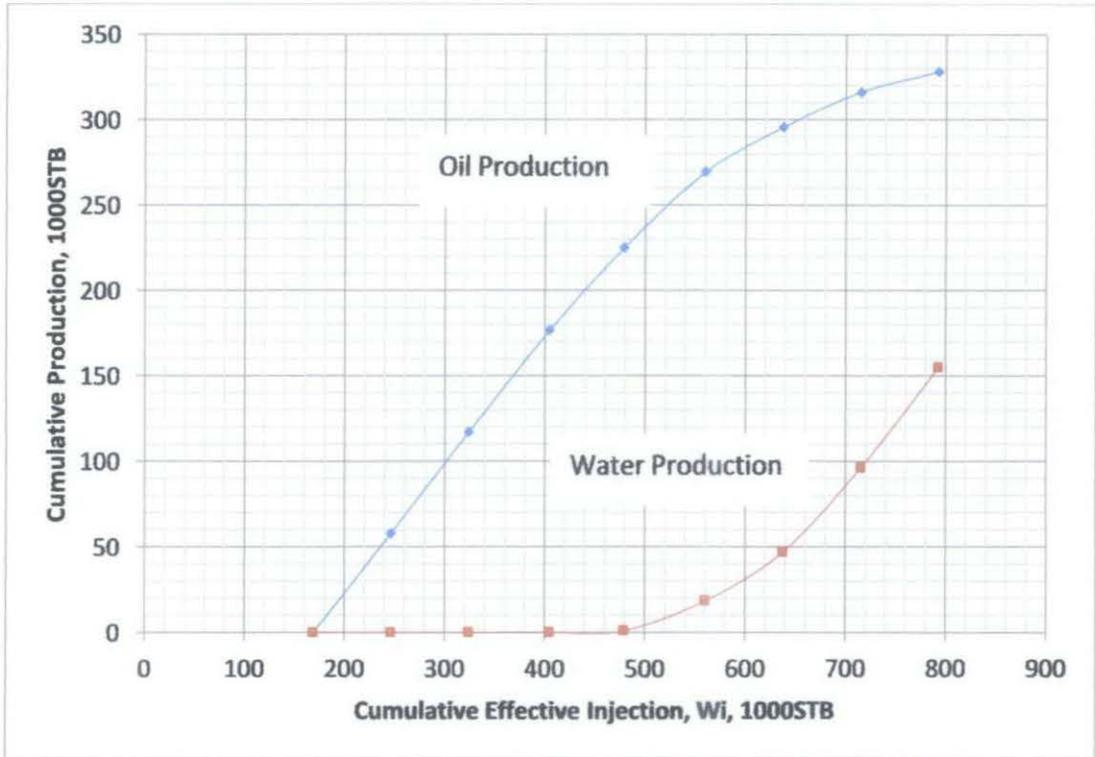


Figure 40: Production vs Injection Curves for Total Layer of Case 1

Case 2 (M: 1.0)

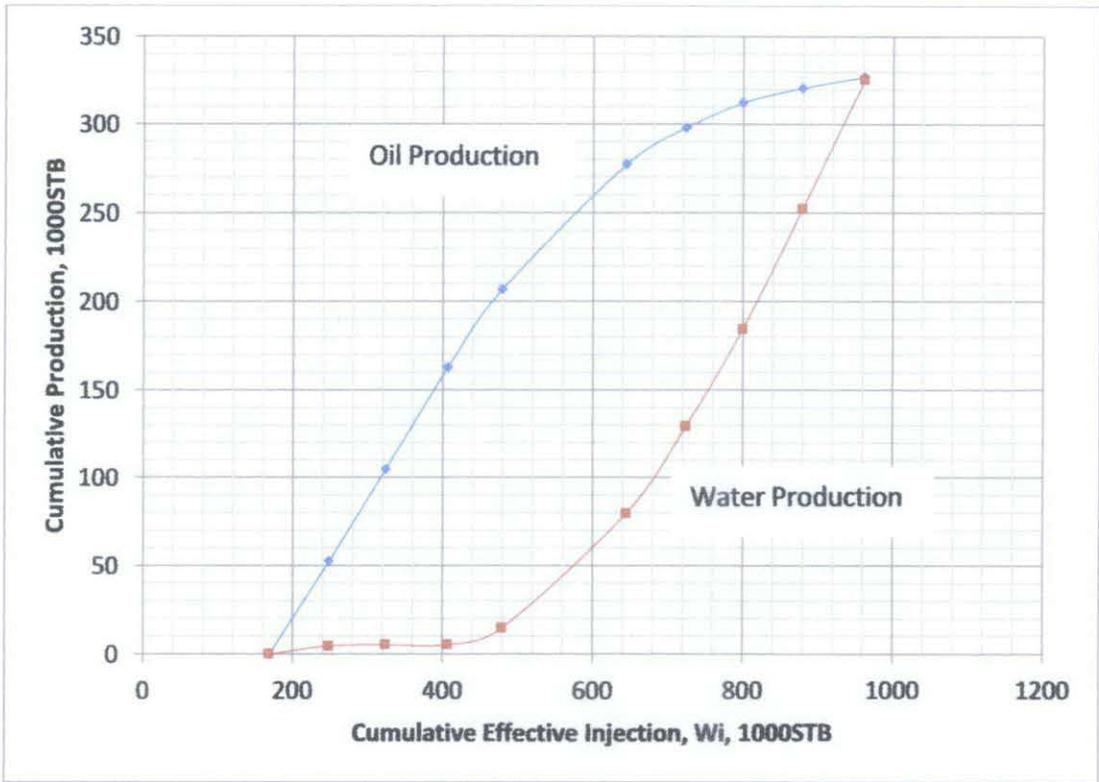


Figure 41: Production vs Injection Curves for Total Layer of Case 2

### Case 3 (M: 2.0)

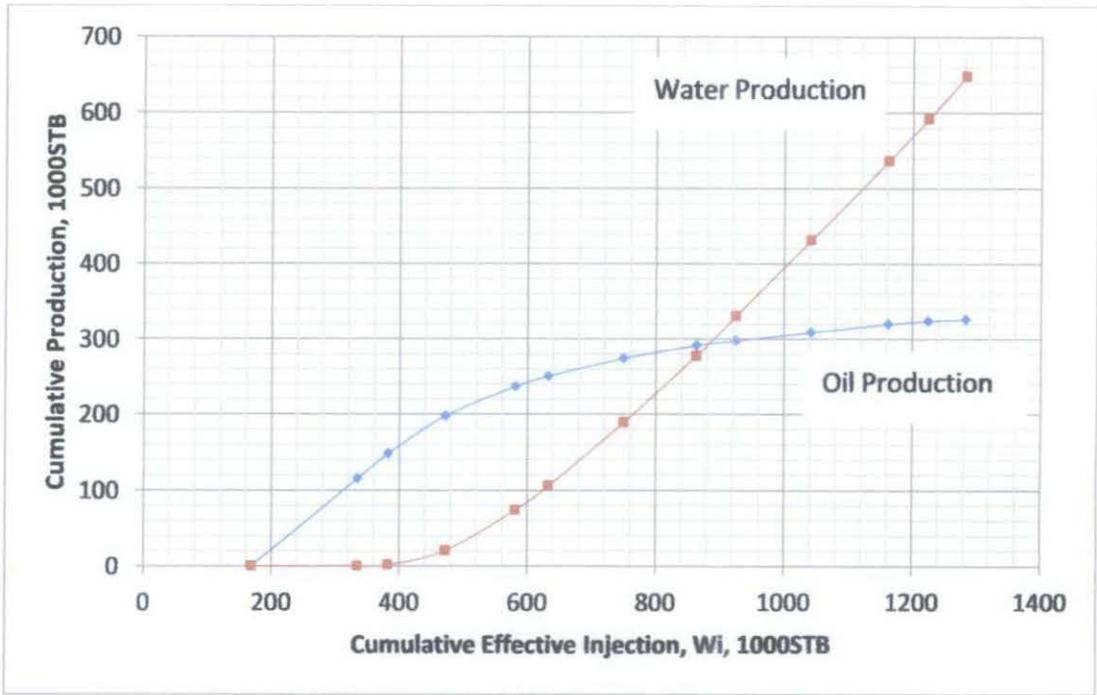


Figure 42: Production vs Injection Curves for Total Layer of Case 3

On this section, the graph of Cumulative Oil Production vs Cumulative Water Production in terms of Cumulative Effective Injection shows a trend of oil and water production. At the start of water injection, there is a process called fill-up. This is when the water displaced the free gas in the reservoir. For all cases, the value of gas saturation is 0.16. After the water has displaced all the gas, it will continue to displace the oil. The increase of water production on the graph from value 0 shows the breakthrough of water at the production well. This means that the water that we inject has reach production well.

In terms of mobility ratio, the graphs show different trend parallel to different values of mobility ratio. For the 1<sup>st</sup> case, high viscosity of water, the displacing fluid have a great displacement process of displacing oil. Therefore, the production of water did not exceed the production of oil. Different situation happens for 3<sup>rd</sup> case. For these cases which have lower water viscosity compared to oil viscosity, the water move faster than oil, bypass through it. Therefore, the water production exceeds the production of oil.

## CHAPTER 6

### CONCLUSIONS AND RECOMMENDATION

#### 6.1 Conclusion

In a nutshell, the Buckley-Leverett and Bookkeeping method used in predicting waterflooding performance in a multi-layered reservoir yields a good result as it jive with the effect of other parameters. In predicting breakthrough time with different water viscosity in different layers produce an outcome which is higher water viscosity takes longer time to reach breakthrough while the lower water viscosity is the fastest. Mobility ratio value affects the oil and water production. Lower mobility ratio produces a high oil production with low water production while higher mobility ratio produces higher water production compared to oil production. Based on the performance of the methods chosen to run this study, it can be concluded that it has successfully done great.

### Case 3

Sw	Fw
0.153	-
0.200	0.0033
0.250	0.0099
0.300	0.0254
0.350	0.0596
0.400	0.1281
0.450	0.2379
0.500	0.3864
0.550	0.5421
0.600	0.7059
0.650	0.8420
0.700	0.9136
0.750	0.9671
0.790	1.0000

#### 4.5 The Effect of the Mobility Ratio in Water Saturation Distribution during the Displacement

##### Case 1

<b>X(Sw), t:300days</b>	<b>X(Sw), t:600days</b>	<b>X(Sw), t:1073 days</b>
0.000	0.000	0.000
46.100	92.200	164.884
110.640	221.280	395.722
234.741	469.482	839.589
399.595	799.190	1429.215
590.080	1180.160	2110.515
614.605	1229.210	2198.233
614.605	1229.210	2198.233
395.169	790.338	1413.386
276.600	553.200	989.304
158.031	316.062	565.222
92.200	184.400	329.768
61.405	122.810	219.625
0.000	0.000	0.000

##### Case 2

<b>X(Sw), t:300days</b>	<b>X(Sw), t:600days</b>	<b>X(Sw), t:1210days</b>
0.000	0.000	0.000
40.937	81.874	165.110
73.760	147.520	297.496
184.400	368.800	743.740
316.062	632.123	1274.770
479.440	958.880	1933.724
626.960	1253.920	2528.716
583.995	1167.990	2355.425
491.795	983.590	1983.555
391.850	783.700	1580.448
234.741	469.482	946.781
129.080	258.160	520.618
81.874	163.747	330.221
0.000	0.000	0.000

**Case 3**

<b>X(Sw), t:300days</b>	<b>X(Sw), t:600days</b>	<b>X(Sw), t:1586days</b>
0.000	0.000	0.000
23.050	46.100	121.857
46.100	92.200	243.715
92.200	184.400	487.430
184.400	368.800	974.859
307.395	614.790	1625.090
450.674	901.347	2382.555
553.200	1106.400	2924.577
579.569	1159.138	3063.982
553.200	1106.400	2924.577
345.750	691.500	1827.861
239.720	479.440	1267.317
184.400	368.800	974.859
0.000	0.000	0.000

## 5.2 Cumulative Oil and Water Production as Function of Cumulative Water Injection

### 5.2.1 Recovery and Injectivity Curves

#### Case 1

DVI	Eh	Fw	ir: is/i	BoNpf/Vp	Wi/Vp
-	-	-	1.500	0.000	0.160
0.750	0.780	0.000	1.500	0.337	0.478
0.900	0.870	0.500	1.750	0.394	0.573
1.000	0.910	0.600	1.800	0.420	0.637
1.100	0.940	0.700	1.850	0.439	0.701
1.200	0.960	0.720	1.860	0.452	0.764
1.300	0.980	0.830	1.915	0.464	0.828
1.400	0.990	0.900	1.950	0.471	0.892

Wi/Vp	$\Delta$ Wi/Vp	ir = is/i	ir $\Delta$ (Wi/Vp)	cir $\Delta$ (Wi/Vp)	(0.28/0.43) $\epsilon$	(0.21/0.224) $\epsilon$	(0.145/0.11)
0.16	0.000	2.024	0.000	0.000	0.00000	0.00000	0.000
0.478	0.318	2.024	0.643	0.643	0.41874	0.60287	0.847
0.573	0.096	2.018	0.193	0.836	0.54430	0.78364	1.101
0.637	0.064	2.011	0.128	0.964	0.62770	0.90373	1.270
0.701	0.064	2.009	0.128	1.092	0.71103	1.02369	1.439
0.764	0.064	2.007	0.128	1.220	0.79427	1.14353	1.607
0.828	0.064	2.005	0.128	1.347	0.87742	1.26326	1.776
0.892	0.064	2.004	0.128	1.475	0.96056	1.38295	1.944

**Case 2**

<b>DVI</b>	<b>Eh</b>	<b>Fw</b>	<b>ir: is/i</b>	<b>BoNpf/Vp</b>	<b>Wi/Vp</b>
-	-	-	2.00	0.00	0.16
0.71	0.70	0.00	2.00	0.29	0.45
0.75	0.74	0.25	2.00	0.31	0.48
0.90	0.83	0.55	2.00	0.37	0.57
1.00	0.87	0.62	2.00	0.39	0.64
1.10	0.90	0.71	2.00	0.41	0.70
1.20	0.92	0.77	2.00	0.43	0.76
1.30	0.94	0.81	2.00	0.44	0.83
1.40	0.96	0.83	2.00	0.45	0.89
1.50	0.98	0.87	2.00	0.46	0.96
1.75	0.99	0.90	2.00	0.47	1.11

<b>Wi/Vp</b>	<b>ΔWi/Vp</b>	<b>ir = is/i</b>	<b>irΔ(Wi/Vp)</b>	<b>εirΔ(Wi/Vp)</b>	<b>(0.28/0.43)ε</b>	<b>(0.21/0.224)ε</b>	<b>(0.145/0.11)ε</b>
0.16	0.000	2.00	0.000	0.000	0.00000	0.00000	0.00000
0.45	0.292	2.00	0.585	0.585	0.38063	0.54801	0.77053
0.48	0.025	2.00	0.051	0.636	0.41381	0.59578	0.83770
0.57	0.096	2.00	0.191	0.827	0.53825	0.77494	1.08961
0.64	0.064	2.00	0.127	0.954	0.62121	0.89438	1.25755
0.70	0.064	2.00	0.127	1.081	0.70417	1.01381	1.42548
0.76	0.064	2.00	0.127	1.209	0.78713	1.13325	1.59342
0.83	0.064	2.00	0.127	1.336	0.87008	1.25269	1.76135
0.89	0.064	2.00	0.127	1.464	0.95304	1.37213	1.92929
0.96	0.064	2.00	0.127	1.591	1.03600	1.49156	2.09723
1.11	0.159	2.00	0.319	1.910	1.24340	1.79016	2.51707

**Case 3**

DVI	Eh	Fw	ir: is/i	BoNpf/Vp	Wi/Vp
-	-	-	3.00	0.00	0.16
0.61	0.61	0.00	3.00	0.23	0.39
0.75	0.71	0.47	2.53	0.29	0.48
0.90	0.78	0.65	2.35	0.34	0.57
1.00	0.81	0.70	2.30	0.36	0.64
1.10	0.83	0.75	2.25	0.37	0.70
1.20	0.86	0.80	2.20	0.39	0.76
1.30	0.88	0.82	2.18	0.40	0.83
1.40	0.90	0.86	2.14	0.41	0.89
1.50	0.92	0.88	2.12	0.43	0.96
1.75	0.94	0.92	2.08	0.44	1.11
2.00	0.95	0.94	2.06	0.45	1.27
2.25	0.97	0.95	2.05	0.46	1.43
2.50	0.98	0.96	2.04	0.46	1.59

Wi/Vp	$\Delta$ Wi/Vp	ir = is/i	ir $\Delta$ (Wi/Vp)	cir $\Delta$ (Wi/Vp)	(0.28/0.430)€	(0.21/0.224)€	(0.145/0.110)
0.16	0.000	3.00	0.000	0.000	0.00000	0.00000	0.0000
0.39	0.229	3.00	0.686	0.686	0.44697	0.64352	0.9048
0.48	0.089	2.53	0.226	0.912	0.59399	0.85518	1.2024
0.57	0.096	2.35	0.225	1.137	0.74027	1.06579	1.4985
0.64	0.064	2.30	0.147	1.283	0.83571	1.20319	1.6917
0.70	0.064	2.25	0.143	1.427	0.92907	1.33761	1.8807
0.76	0.064	2.20	0.140	1.567	1.02035	1.46903	2.0655
0.83	0.064	2.18	0.139	1.706	1.11079	1.59925	2.2486
0.89	0.064	2.14	0.136	1.842	1.19958	1.72707	2.4283
0.96	0.064	2.12	0.135	1.977	1.28753	1.85370	2.6064
1.11	0.159	2.08	0.331	2.309	1.50325	2.16427	3.0431
1.27	0.159	2.06	0.328	2.637	1.71688	2.47185	3.4755
1.43	0.159	2.05	0.326	2.963	1.92948	2.77793	3.9059
1.59	0.159	2.04	0.325	3.288	2.14104	3.08252	4.3342

5.2.2 Cumulative Oil and Water Production for each Layer versus Cumulative Water Injection

**Case 1**

Layer 1

<b>tr</b>	<b>Wi/Vp</b>	<b>BoNpf/Vp</b>	<b>Wi(MSTB)</b>	<b>BoNpf(MSTB)</b>	<b>Wp(MSTB)</b>	<b>Npf(MSTB)</b>
0.00	0.16	0	51.04	0	0.000	0.00
0.13	0.255	0.100	81.345	31.900	0.000	22.31
0.25	0.350	0.205	111.650	65.395	0.000	45.73
0.38	0.445	0.303	141.955	96.657	0.000	67.59
0.50	0.540	0.377	172.260	120.263	0.957	84.10
0.63	0.635	0.417	202.565	133.023	18.502	93.02
0.75	0.730	0.445	232.870	141.955	39.875	99.27
0.88	0.825	0.463	263.175	147.697	64.438	103.28
1.00	0.920	0.473	293.480	150.887	91.553	105.52

Layer 2

<b>tr</b>	<b>Wi/Vp</b>	<b>BoNpf/Vp</b>	<b>Wi(MSTB)</b>	<b>BoNpf(MSTB)</b>	<b>Wp(MSTB)</b>	<b>Npf(MSTB)</b>
0.00	0.16	0	112.8	0	0.000	0.00
0.13	0.225	0.070	158.625	49.350	0.000	34.51
0.25	0.290	0.140	204.450	98.700	0.000	69.02
0.38	0.360	0.215	253.800	151.575	0.000	106.00
0.50	0.420	0.277	296.100	195.285	0.000	136.56
0.63	0.490	0.347	345.450	244.635	0.000	171.07
0.75	0.555	0.385	391.275	271.425	7.050	189.81
0.88	0.620	0.415	437.100	292.575	31.725	204.60
1.00	0.685	0.435	482.925	306.675	63.450	214.46

Layer 3

<b>tr</b>	<b>Wi/Vp</b>	<b>BoNpf/Vp</b>	<b>Wi(MSTB)</b>	<b>BoNpf(MSTB)</b>	<b>Wp(MSTB)</b>	<b>Npf(MSTB)</b>
0.00	0.16	0	4.96	0	0.000	0.00
0.13	0.205	0.048	6.355	1.488	0.000	1.04
0.25	0.255	0.100	7.905	3.100	0.000	2.17
0.38	0.300	0.150	9.300	4.650	0.000	3.25
0.50	0.350	0.205	10.850	6.355	0.000	4.44
0.63	0.395	0.250	12.245	7.750	0.000	5.42
0.75	0.440	0.300	13.640	9.300	0.000	6.50
0.88	0.490	0.347	15.190	10.757	0.000	7.52
1.00	0.535	0.375	16.585	11.625	0.000	8.13

**Case 2***Layer 1*

<b>tr</b>	<b>Wi/Vp</b>	<b>BoNpf/Vp</b>	<b>Wi(MSTB)</b>	<b>BoNpf(MSTB)</b>	<b>Wp(MSTB)</b>	<b>Npf(MSTB)</b>
0.00	0.16	0	51.04	0	0.000	0.00
0.13	0.25	0.09	79.75	27.75	0.96	19.41
0.25	0.35	0.19	111.65	59.02	1.59	41.27
0.38	0.45	0.29	143.55	90.92	1.60	63.58
0.50	0.54	0.35	172.26	111.65	9.57	78.08
0.63	0.64	0.40	204.16	126.01	27.12	88.12
0.75	0.74	0.42	236.06	133.98	51.04	93.69
0.88	0.83	0.44	264.77	139.72	74.01	97.71
1.00	0.93	0.46	296.67	146.74	98.89	102.62
1.13	1.02	0.47	325.38	148.97	125.37	104.18
1.25	1.12	0.46	357.28	146.74	159.50	102.62

*Layer 2*

<b>tr</b>	<b>Wi/Vp</b>	<b>BoNpf/Vp</b>	<b>Wi(MSTB)</b>	<b>BoNpf(MSTB)</b>	<b>Wp(MSTB)</b>	<b>Npf(MSTB)</b>
0.00	0.16	0	112.8	0	0.000	0.00
0.13	0.230	0.065	162.150	45.825	3.525	32.05
0.25	0.290	0.125	204.450	88.125	3.525	61.63
0.38	0.360	0.195	253.800	137.475	3.525	96.14
0.50	0.420	0.253	296.100	178.365	4.935	124.73
0.63	0.490	0.320	345.450	225.600	7.050	157.76
0.75	0.560	0.360	394.800	253.800	28.200	177.48
0.88	0.630	0.392	444.150	276.360	54.990	193.26
1.00	0.690	0.410	486.450	289.050	84.600	202.13
1.13	0.760	0.425	535.8	299.625	123.375	209.53
1.25	0.830	0.438	585.15	308.79	163.56	215.94

Layer 3

<b>tr</b>	<b>Wi/Vp</b>	<b>BoNpf/Vp</b>	<b>Wi(MSTB)</b>	<b>BoNpf(MSTB)</b>	<b>Wp(MSTB)</b>	<b>Npf(MSTB)</b>
0.00	0.16	0	4.96	0	0.000	0.00
0.13	0.210	0.048	6.510	1.488	0.062	1.04
0.25	0.250	0.087	7.750	2.697	0.093	1.89
0.38	0.300	0.137	9.300	4.247	0.093	2.97
0.50	0.350	0.185	10.850	5.735	0.155	4.01
0.63	0.400	0.235	12.400	7.285	0.155	5.09
0.75	0.440	0.273	13.640	8.463	0.217	5.92
0.88	0.490	0.320	15.190	9.920	0.310	6.94
1.00	0.540	0.350	16.740	10.850	0.930	7.59
1.13	0.590	0.375	18.29	11.625	1.705	8.13
1.25	0.630	0.392	19.53	12.152	2.418	8.50

**Case 3**

## Layer 1

<b>tr</b>	<b>Wi/Vp</b>	<b>BoNpf/Vp</b>	<b>Wi(MSTB)</b>	<b>BoNpf(MSTB)</b>	<b>Wp(MSTB)</b>	<b>Npf(MSTB)</b>
0.00	0.16	0	51.04	0	0.000	0.00
0.25	0.290	0.130	92.510	41.470	(0.000)	29.00
0.4	0.360	0.200	114.840	63.800	(0.000)	44.62
0.5	0.420	0.255	133.980	81.345	1.595	56.88
0.7	0.540	0.323	172.260	103.037	18.183	72.05
0.9	0.680	0.365	216.920	116.435	49.445	81.42
1	0.750	0.385	239.250	122.815	65.395	85.88
1.2	0.890	0.412	283.910	131.428	101.442	91.91
1.4	1.04	0.435	331.760	138.765	141.955	97.04
1.5	1.12	0.44	357.280	140.360	165.880	98.15
1.7	1.26	0.445	401.940	141.955	208.945	99.27
1.9	1.41	0.455	449.790	145.145	253.605	101.50
2	1.49	0.46	475.310	146.740	277.530	102.62
2.1	1.56	0.463	497.640	147.697	298.903	103.28

## Layer 2

<b>tr</b>	<b>Wi/Vp</b>	<b>BoNpf/Vp</b>	<b>Wi(MSTB)</b>	<b>BoNpf(MSTB)</b>	<b>Wp(MSTB)</b>	<b>Npf(MSTB)</b>
0.00	0.16	0	112.8	0	0.000	0.00
0.25	0.250	0.090	176.250	63.450	0.000	44.37
0.4	0.300	0.140	211.500	98.700	(0.000)	69.02
0.5	0.340	0.180	239.700	126.900	0.000	88.74
0.7	0.410	0.247	289.050	174.135	2.115	121.77
0.9	0.500	0.305	352.500	215.025	24.675	150.37
1	0.540	0.323	380.700	227.715	40.185	159.24
1.2	0.640	0.357	451.200	251.685	86.715	176.00
1.4	0.73	0.38	514.650	267.900	133.950	187.34
1.5	0.78	0.39	549.900	274.950	162.150	192.27
1.7	0.88	0.41	620.400	289.050	218.550	202.13
1.9	0.98	0.427	690.900	301.035	277.065	210.51
2	1.03	0.433	726.150	305.265	308.085	213.47
2.1	1.08	0.435	761.400	306.675	341.925	214.46

Layer 3

<b>tr</b>	<b>Wi/Vp</b>	<b>BoNpf/Vp</b>	<b>Wi(MSTB)</b>	<b>BoNpf(MSTB)</b>	<b>Wp(MSTB)</b>	<b>Npf(MSTB)</b>
0.00	0.16	0	4.96	0	0.000	0.00
0.250	0.220	0.060	6.820	1.860	0.000	1.301
0.400	0.260	0.100	8.060	3.100	0.000	2.168
0.500	0.280	0.120	8.680	3.720	0.000	2.601
0.700	0.340	0.180	10.540	5.580	0.000	3.902
0.900	0.380	0.220	11.780	6.820	0.000	4.769
1.000	0.410	0.247	12.710	7.657	0.093	5.355
1.200	0.480	0.292	14.880	9.052	0.868	6.330
1.400	0.540	0.323	16.740	10.013	1.767	7.002
1.500	0.570	0.335	17.670	10.385	2.325	7.262
1.700	0.640	0.357	19.840	11.067	3.813	7.739
1.900	0.710	0.372	22.010	11.532	5.518	8.064
2.000	0.740	0.380	22.940	11.780	6.200	8.238
2.100	0.780	0.390	24.180	12.090	7.130	8.455

5.2.2 Total Cumulative Oil and Water Production versus Cumulative Water Injection

Case 1

tr	zones	Wi/Vp	BoNpf/Vp	Vp (MSTB)	Wi(MSTB)	BoNpf(MSTB)	Wp(MSTB)
0	1	0.16	0	319	51.04	0	0.000
	2	0.16	0	705	112.8	0	0.000
	3	0.16	0	31	4.96	0	0.000
<b>TOTAL</b>					<b>168.8</b>	<b>0</b>	<b>0</b>
0.125	1	0.255	0.100	319.000	81.345	31.900	(1.595)
	2	0.225	0.070	705.000	158.625	49.350	(3.525)
	3	0.205	0.048	31.000	6.355	1.488	(0.093)
<b>TOTAL</b>					<b>246.325</b>	<b>82.738</b>	<b>0.000</b>
0.25	1	0.350	0.205	319.000	111.650	65.395	(4.785)
	2	0.290	0.140	705.000	204.450	98.700	(7.050)
	3	0.255	0.100	31.000	7.905	3.100	(0.155)
<b>TOTAL</b>					<b>324.005</b>	<b>167.195</b>	<b>0.000</b>
0.375	1	0.445	0.303	319.000	141.955	96.657	(5.742)
	2	0.360	0.215	705.000	253.800	151.575	(10.575)
	3	0.300	0.150	31.000	9.300	4.650	(0.310)
<b>TOTAL</b>					<b>405.055</b>	<b>252.882</b>	<b>0.000</b>
0.5	1	0.540	0.377	319.000	172.260	120.263	0.957
	2	0.420	0.277	705.000	296.100	195.285	(11.985)
	3	0.350	0.205	31.000	10.850	6.355	(0.465)
<b>TOTAL</b>					<b>479.210</b>	<b>321.903</b>	<b>0.957</b>
0.625	1	0.635	0.417	319.000	202.565	133.023	18.502
	2	0.490	0.347	705.000	345.450	244.635	(11.985)
	3	0.395	0.250	31.000	12.245	7.750	(0.465)
<b>TOTAL</b>					<b>560.260</b>	<b>385.408</b>	<b>18.502</b>
0.75	1	0.730	0.445	319.000	232.870	141.955	39.875
	2	0.555	0.385	705.000	391.275	271.425	7.050
	3	0.440	0.300	31.000	13.640	9.300	(0.620)
<b>TOTAL</b>					<b>637.785</b>	<b>422.680</b>	<b>46.925</b>
0.875	1	0.825	0.463	319.000	263.175	147.697	64.438
	2	0.620	0.415	705.000	437.100	292.575	31.725
	3	0.490	0.382	31.000	15.190	11.842	(1.612)
					<b>715.465</b>	<b>452.114</b>	<b>96.163</b>
1.000000	1	0.920	0.473	319.000	293.480	150.887	91.553
	2	0.685	0.435	705.000	482.925	306.675	63.450
	3	0.535	0.375	31.000	16.585	11.625	0.000
					<b>792.990</b>	<b>469.187</b>	<b>155.003</b>

Case 2

tr	zones	Wi/Vp	BoNpf/Vp	Vp (MSTB)	Wi(MSTB)	BoNpf(MSTB)	Wp(MSTB)
0	1	0.16	0	319.000	51.04	0	0.000
	2	0.16	0	705.000	112.8	0	0.000
	3	0.16	0	31.000	4.96	0	0.000
<b>TOTAL</b>					<b>168.8</b>	<b>0</b>	<b>0</b>
0.125	1	0.250	0.087	319.000	79.750	27.753	0.957
	2	0.230	0.065	705.000	162.150	45.825	3.525
	3	0.210	0.048	31.000	6.510	1.488	0.062
<b>TOTAL</b>					<b>248.410</b>	<b>75.066</b>	<b>4.544</b>
0.25	1	0.350	0.185	319.000	111.650	59.015	1.595
	2	0.290	0.125	705.000	204.450	88.125	3.525
	3	0.250	0.087	31.000	7.750	2.697	0.093
<b>TOTAL</b>					<b>323.850</b>	<b>149.837</b>	<b>5.213</b>
0.375	1	0.450	0.285	319.000	143.550	90.915	1.595
	2	0.360	0.195	705.000	253.800	137.475	3.525
	3	0.300	0.137	31.000	9.300	4.247	0.093
<b>TOTAL</b>					<b>406.650</b>	<b>232.637</b>	<b>5.213</b>
0.5	1	0.540	0.350	319.000	172.260	111.650	9.570
	2	0.420	0.253	705.000	296.100	178.365	4.935
	3	0.350	0.185	31.000	10.850	5.735	0.155
<b>TOTAL</b>					<b>479.210</b>	<b>295.750</b>	<b>14.660</b>
0.625	1	0.640	0.395	319.000	204.160	126.005	27.115
	2	0.490	0.320	705.000	345.450	225.600	7.050
	3	0.400	0.235	31.000	12.400	7.285	0.155
<b>TOTAL</b>					<b>562.010</b>	<b>358.890</b>	<b>34.320</b>
0.75	1	0.740	0.420	319.000	236.060	133.980	51.040
	2	0.560	0.360	705.000	394.800	253.800	28.200
	3	0.440	0.273	31.000	13.640	8.463	0.217
<b>TOTAL</b>					<b>644.500</b>	<b>396.243</b>	<b>79.457</b>
0.875	1	0.830	0.438	319.000	264.770	139.722	74.008
	2	0.630	0.392	705.000	444.150	276.360	54.990
	3	0.490	0.320	31.000	15.190	9.920	0.310
<b>TOTAL</b>					<b>724.110</b>	<b>426.002</b>	<b>129.308</b>
1.000000	1	0.930	0.460	319.000	296.670	146.740	98.890
	2	0.690	0.410	705.000	486.450	289.050	84.600
	3	0.540	0.350	31.000	16.740	10.850	0.930
<b>TOTAL</b>					<b>799.860</b>	<b>446.640</b>	<b>184.420</b>
1.125	1	1.020	0.467	319.000	325.38	148.973	125.367
	2	0.760	0.425	705.000	535.8	299.625	123.375

	3	0.590	0.320	31.000	18.29	9.92	3.41
<b>TOTAL</b>					<b>879.47</b>	<b>458.518</b>	<b>252.152</b>
1.25	1	1.120	0.460	319.000	357.28	146.74	159.5
	2	0.830	0.438	705.000	585.15	308.79	163.56
	3	0.630	0.392	31.000	19.53	12.152	2.418
<b>TOTAL</b>					<b>961.96</b>	<b>467.682</b>	<b>325.478</b>

Case 3

tr	zones	Wi/Vp	BoNpf/Vp	Vp (MSTB)	Wi(MSTB)	BoNpf(MSTB)	Wp(MSTB)
0.25	1	0.290	0.130	319.000	92.510	41.470	(0.000)
	2	0.250	0.090	705.000	176.250	63.450	0.000
	3	0.220	0.060	31.000	6.820	1.860	0.000
<b>TOTAL</b>					<b>275.580</b>	<b>106.780</b>	<b>(0.000)</b>
0.4	1	0.360	0.200	319.000	114.840	63.800	(0.000)
	2	0.300	0.140	705.000	211.500	98.700	(0.000)
	3	0.260	0.100	31.000	8.060	3.100	0.000
<b>TOTAL</b>					<b>334.400</b>	<b>165.600</b>	<b>(0.000)</b>
0.5	1	0.420	0.255	319.000	133.980	81.345	1.595
	2	0.340	0.180	705.000	239.700	126.900	0.000
	3	0.280	0.120	31.000	8.680	3.720	0.000
<b>TOTAL</b>					<b>382.360</b>	<b>211.965</b>	<b>1.595</b>
0.7	1	0.540	0.323	319.000	172.260	103.037	18.183
	2	0.410	0.247	705.000	289.050	174.135	2.115
	3	0.340	0.180	31.000	10.540	5.580	0.000
<b>TOTAL</b>					<b>471.850</b>	<b>282.752</b>	<b>20.298</b>
0.9	1	0.680	0.365	319.000	216.920	116.435	49.445
	2	0.500	0.305	705.000	352.500	215.025	24.675
	3	0.380	0.220	31.000	11.780	6.820	0.000
<b>TOTAL</b>					<b>581.200</b>	<b>338.280</b>	<b>74.120</b>
1	1	0.750	0.385	319.000	239.250	122.815	65.395
	2	0.540	0.323	705.000	380.700	227.715	40.185
	3	0.410	0.247	31.000	12.710	7.657	0.093
<b>TOTAL</b>					<b>632.660</b>	<b>358.187</b>	<b>105.673</b>
1.2	1	0.890	0.412	319.000	283.910	131.428	101.442
	2	0.640	0.357	705.000	451.200	251.685	86.715
	3	0.480	0.292	31.000	14.880	9.052	0.868
<b>TOTAL</b>					<b>749.990</b>	<b>392.165</b>	<b>189.025</b>
1.4	1	1.04	0.435	319.000	331.760	138.765	141.955
	2	0.73	0.38	705.000	514.650	267.900	133.950
	3	0.54	0.323	31.000	16.740	10.013	1.767
<b>TOTAL</b>					<b>863.150</b>	<b>416.678</b>	<b>277.672</b>
1.5	1	1.12	0.44	319.000	357.280	140.360	165.880
	2	0.78	0.39	705.000	549.900	274.950	162.150
	3	0.57	0.335	31.000	17.670	10.385	2.325
<b>TOTAL</b>					<b>924.850</b>	<b>425.695</b>	<b>330.355</b>
1.7	1	1.26	0.445	319.000	401.940	141.955	208.945
	2	0.88	0.41	705.000	620.400	289.050	218.550

	3	0.64	0.357	31.000	19.840	11.067	3.813
<b>TOTAL</b>					<b>1042.180</b>	<b>442.072</b>	<b>431.308</b>
1.9	1	1.41	0.455	319.000	449.790	145.145	253.605
	2	0.98	0.427	705.000	690.900	301.035	277.065
	3	0.71	0.372	31.000	22.010	11.532	5.518
<b>TOTAL</b>					<b>1162.700</b>	<b>457.712</b>	<b>536.188</b>
2	1	1.49	0.46	319.000	475.310	146.740	277.530
	2	1.03	0.433	705.000	726.150	305.265	308.085
	3	0.74	0.38	31.000	22.940	11.780	6.200
<b>TOTAL</b>					<b>1224.400</b>	<b>463.785</b>	<b>591.815</b>
2.1	1	1.56	0.463	319.000	497.640	147.697	298.903
	2	1.08	0.435	705.000	761.400	306.675	341.925
	3	0.78	0.39	31.000	24.180	12.090	7.130
<b>TOTAL</b>					<b>1283.220</b>	<b>466.462</b>	<b>647.958</b>