

**RESPONSES OF CLASSIC SPAR PLATFORM SUBJECTED TO REGULAR WAVES**

By

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**FINAL PROJECT REPORT**

**Submitted to the Civil Engineering Programme  
In Partial Fulfilment of the Requirements  
For the Degree  
Bachelor of Engineering (Hons)  
(Civil Engineering)**

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# **CERTIFICATION OF APPROVAL**

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A project dissertation submitted to the  
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Approved:



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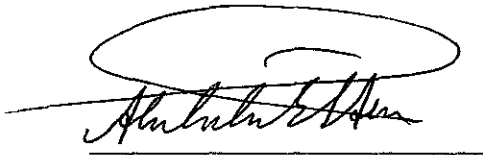
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UNIVERSITI TEKNOLOGI PETRONAS  
TRONOH, PERAK

June 2008

## **CERTIFICATION OF ORIGINALITY**

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

A handwritten signature in black ink, appearing to read 'Abubakr Elamin Abdalla El-khalifa', is written over a horizontal line. The signature is stylized and cursive.

Abubakr Elamin Abdalla El-khalifa

## **ABSTRACT**

Increasing demand for Oil and Gas and decreasing onshore reserves has lead to Deep water exploration therefore it has accelerated the need to explore structures suitable for these depths, which operate more economically in deep water. A Spar platform is one such compliant offshore floating structure used for deep water applications for the drilling, production, processing, storage, and offloading of ocean deposits. The Spar platform was modelled as a rigid body with six degrees-of freedom, connected to the sea floor by multi-component catenary mooring lines, which are attached to the Spar platform at the fairleads. The aim of this project was to conduct a simple dynamic rigid body analysis in time domain for a typical classic spar subjected to regular waves and then conduct a parametric analysis. The results obtained are within the permissible limits where the maximum Surge, Heave, & Pitch recorded were 1.2 m, 0.95 m, and -0.04 rad respectively for wave height of 3.6 m and wave period of 7 sec.

## ACKNOWLEDGEMENTS

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## LIST OF ABBREVIATIONS

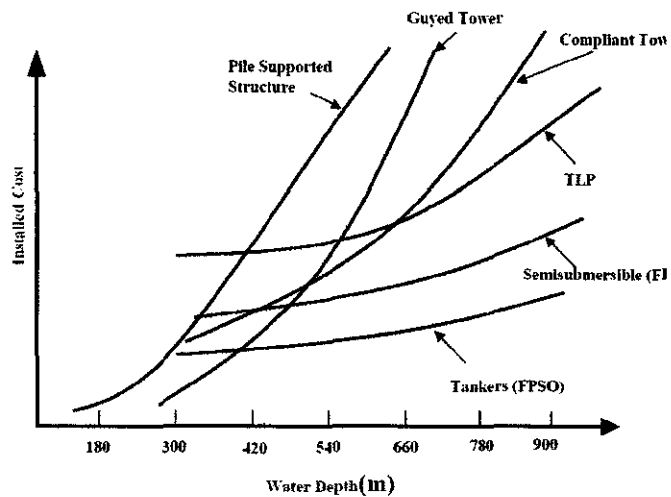
$c$	wave celerity
CB	Centre of Buoyancy
$C_D$	Drag coefficient
CG	Centre of Gravity
$C_M$	Inertia coefficient
$D$	Hull diameter
$d$	water depth
$f$	frequency
$F_x$	Surge forces
$F_y$	Heave forces
$H$	wave height
$K$	stiffness
$k$	Wave number
$L$	wave length
$RAO$	Response amplitude operator
$T$	wave period
$T_n$	Spar natural period
TLP	Tension Leg Platform
$u$	Horizontal water particle velocity
$\ddot{u}$	Horizontal water particle acceleration
$v$	vertical water particle velocity
$\ddot{v}$	Vertical water particle acceleration
$r_x$	Radius of gyration
$\eta$	Wave profile
$\eta_x$	Surge profile
$\eta_y$	Heave Profile
$\rho$	Water density
$\omega$	Natural frequency
$\zeta$	Damping ratio

# CHAPTER 1

## INTRODUCTION

### 1.1 Background of Study

As the search for oil and gas is pushed into deeper and deeper water, it is expected that pile-supported platforms will not be used and that is due to the high cost of fabrication and installation constraint. As an example a comparison of the relative cost trends for pile supported platforms, compliant towers, and tension leg platforms for the Gulf of Mexico, is shown in **Figure. 1.1**



**Figure 1.1** Platform cost comparison for Gulf of Mexico (Günther et al, 1988)

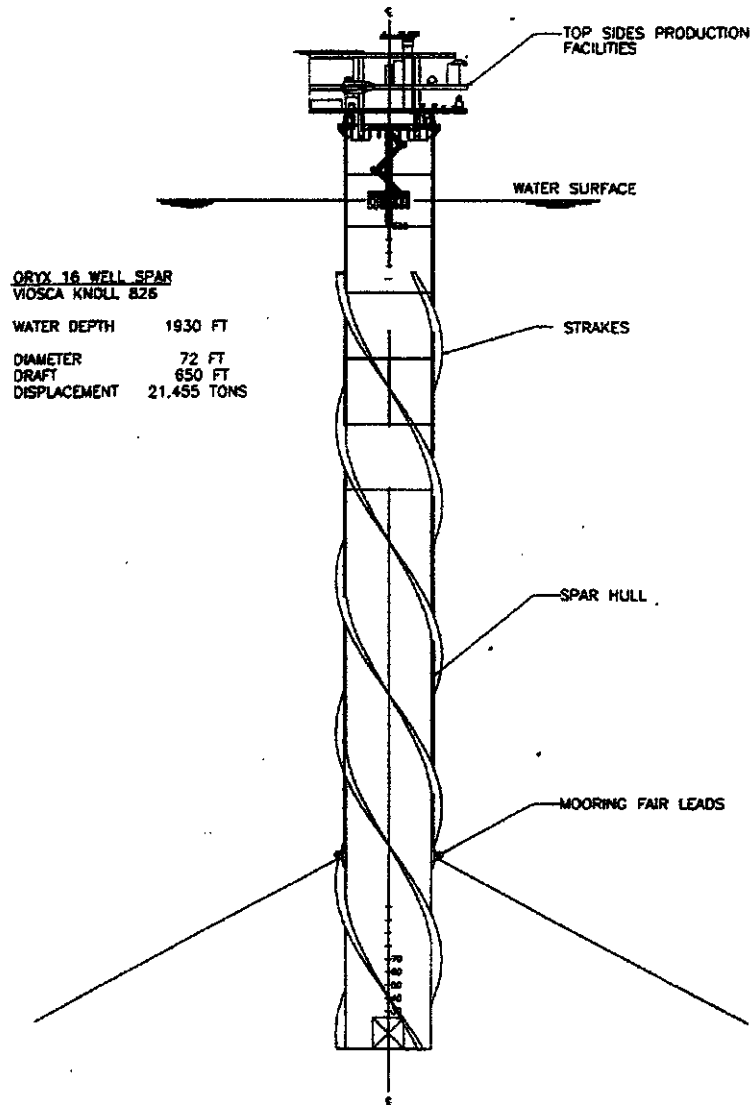
In addition to that the cost of exploration and production has increased significantly, and due to that many innovative floating offshore structures are being proposed for cost savings. And in order to reduce wave induced motion, the natural frequency of these newly proposed offshore structures, they are designed to be far away from the peak frequency of the force power spectra. Spar platforms are one of

the famous types of deep water structures. It is being considered the next generation of deep water offshore structures by many oil companies.

Spar platforms are essentially a deep draft cylinder with rigid risers protected inside the central part of the cylinder. The spar is gaining very small motions from having a small water plane area and large mass with the advantage that taut steel catenary moorings can be utilized for all water depths. They are moored to the seabed like the Tension Leg Platforms TLP, but whereas the TLP has vertical tension tethers the Spar has more conventional mooring lines. Spars have been designed in three configurations: the "conventional" one-piece cylindrical hull, the "truss spar" where the midsection is composed of truss elements connecting the upper buoyant hull (called a hard tank) with the bottom soft tank containing permanent ballast, and the "cell spar" which is built from multiple vertical cylinders. The Spar may be more economical to build for small and medium sized rigs than the TLP, and has more inherent stability than a TLP since it has a large counterweight at the bottom and does not depend on the mooring to hold it upright. The first production Spar was Kerr-McGee's Neptune (See Fig. 1.2) , which is a floating production facility anchored in 1,930 feet (588 m) in the Gulf of Mexico, and the world's deepest spar is PERDIDO truss spar; it's located at 7817 feet (2,383 m) of water depth in the Gulf of Mexico. The first cell spar is Kerr-McGee's Red Hawk. The present generation of Spar has the following features:

- They can be used as a mobile drilling rig.
- They have minimum hull-deck interface.
- They can be operated at depths of up to 3000 m from full drilling and production to production only.
- They are always stable because the centre of buoyancy (CB) is above the centre of gravity (CG).
- They can have large range of topside payloads.
- Oil can be stored at low marginal cost.
- Rigid steel production risers are supported in the centre well by separate buoyancy cans.
- They have favourable motions compared to other floating structures.
- They have sea keeping characteristics superior to all other mobile drilling units.

- They can have steel or concrete hull.
- The mooring system is easy to install, operate and relocate.
- The risers, which normally take breathing in the wave zone from high waves on semi-submersible, drilling units would be protected inside the Spar. Sea motion inside the Spar's centre well would be minimal.



**Figure 1.2** Neptune Spar Platform (TECHNIP OFFSHORE 2006)

## **1.2 Problem statement**

Design analysis of Spar platform is a difficult job, and that is mainly due to the variation and uncertainties associated with the specification of the environmental loads. In the Analysis and Design of offshore structure wave loading is being highly considered, in fact the wave loading of an offshore structure is usually the most important of all environmental loadings for which the structure must be designed. The forces on the structure are caused by the motion of the water due to the waves which are generated by the action of the wind on the surface of the sea. The aim of this project is to conduct a simple dynamic rigid body analysis in time domain for a typical classic spar platform subjected to regular waves

## **1.3 Objectives**

- To prepare a detailed literature survey about the spar technology.
- To determine the responses of a classic spar platform due to regular waves and determine spar motions by conducting a dynamic analysis in frequency domain.
- To study the effect of various parameters on the spar responses.

## **CHAPTER 2**

### **LITERATURE REVIEW & THEORY**

Many researches have been done before for such case “classic spar subjected to regular waves” such as Agrawal and Jain (2001) “*Dynamic behaviour of offshore spar platforms under regular sea wave*”, Agrawal (2001) mentioned that “The analysis, design and operation of Spar platform turn out to be a difficult job, primarily because of the uncertainties associated with the specification of the environmental Loads.”

Ran *et al*, (1996) investigated the response characteristics of a large slack-moored floating spar in regular waves, bichromatic waves, and unidirectional irregular waves with or without sheared currents by experiment and numerical method. They conducted the experiment with 1:55 scale model in the Offshore Technology Research Center (OTRC), the spar model was tested in a variety of environments including:

- 12 different regular waves
- 14 different combinations of bi-chromatic waves
- 4 different random wave sea states,
- 3 different currents,
- 3 different variable winds, including the gust spectra,
- Various combined random waves & currents, and
- Various combined random waves, currents & variable winds.

However for this particular project only regular waves were considered

BON-JUN KOO (2003) in other hand have done further analysis and evaluation of the effect of contact between risers and guide frames on offshore spar platform motions.



Larsen (2002) has conducted hydrodynamic analysis based on the use of MOSES (MultiOperational Structural Engineering Simulator) but for the latest tow they have considered random sea waves instead the simplified regular waves. However in this project the focus is on the effect of regular sea wave on a classic spar platform.

Among the important properties of ocean waters are the density and viscosity, the salinity by itself is of a secondary importance in the offshore structure design as the mass density of water is a function of salinity. The values of these quantities change with the change of the temperature. Determining the forces on a structure can be done by knowing the density values and the difference in the density in different layers of water with depth may contribute to internal waves in the deeper region, which has a very important effect on a submerged structures.

## 2.1 Wave theories

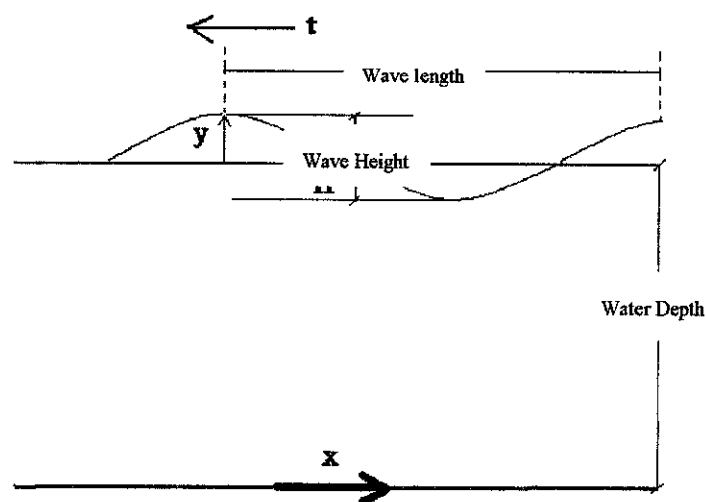
Wave theories describe the kinematics of waves on the basis of potential theory. In particular, they serve to calculate water particles properties such as particle velocities, accelerations, and the dynamic pressure as functions of the surface elevation of the wave. Three parameters are needed in the describing any wave theory. They are:

1. Period ( $T$ ), which is the time taken for two successive crests to pass a stationary point.
2. Height ( $H$ ), which the vertical distance between the crest and the following trough. For a linear wave, the crest amplitude is equal to the trough amplitude, while they are unequal for a non-linear wave.
3. Water depth ( $d$ ), which represents the vertical distance from the mean water level to the mean ocean floor. For wave theories the floor is assumed to be horizontal and flat. (See **figure 2.1**).

Several other quantities that are important in water wave theory may be computed from these parameters. They are:

1. Wave Length ( $L$ ), which is the horizontal distance between successive crests.
2. Wave celerity or phase speed ( $c$ ), which represents the propagation speed of the wave crest.

3. Frequency ( $f$ ), which is the reciprocal of the period.
4. Wave elevation ( $\eta$ ), which represents the instantaneous elevation of the wave from the still water level (SWL) or the mean water level (MWL).
5. Horizontal water particle velocity ( $u$ ), which is the instantaneous velocity along x axis of a water particle.
6. Vertical water particle velocity ( $v$ ), which is the instantaneous velocity along y axis of a water particle.
7. Horizontal water particle acceleration ( $\dot{u}$ ), which is the instantaneous acceleration along x of a water particle, and
8. Vertical water particle acceleration ( $\dot{v}$ ), which is the instantaneous acceleration along y of water particle.



**Figure 2.1** two-dimensional (x,y) wave motion over flat bottom

Different wave theories of varying complexity were developed on the basis of simplifying assumptions, are appropriate for different ranges of the wave parameters.

Among the most common theories are:

- 1- The linear Airy theory.
- 2- The Stokes fifth-order theory.
- 3- The solitary wave theory.
- 4- The cnoidal theory.
- 5- Dean's stream function theory.
- 6- The numerical theory by Chappellear.

Since there are many theories API (2000) developed a method for the selection of the most appropriate theory as shown in Figure 2.2,

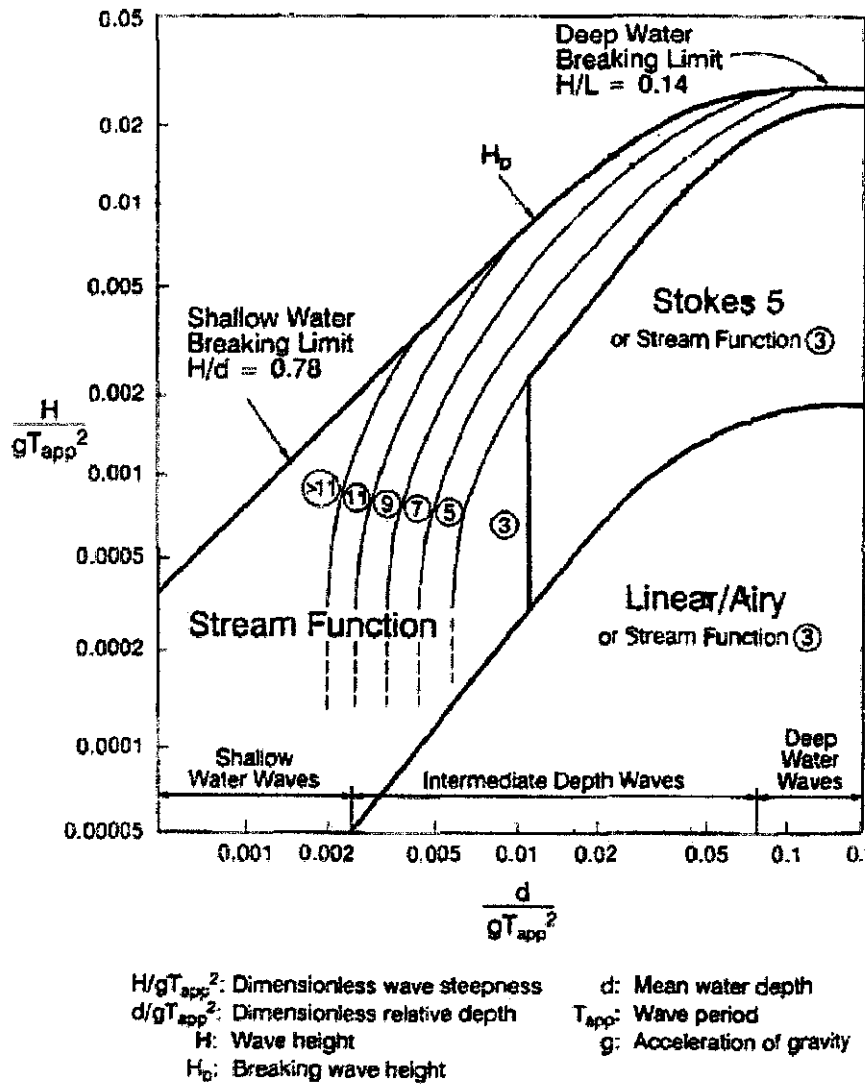


Figure 2.2 Region of application of wave theories (API, 2000)

Since the ratio  $H/d$  is very small, Linear Airy wave theory is appropriate for water particles properties calculation for this project, Appendix C shows linear wave theory formulas.

## 2.2 Wave Load Calculation

In order to predict hydrodynamic forces acting on a cylinder in an ocean environment more realistically, the random nature of the waves should be considered, but it can be simplified if we assume that these random nature consists of a number of regular sea waves. Researchers consider the Morison equation as the suitable choice for calculating sea waves' load in surge direction; this equation can be expressed as:

$$F_x = \frac{1}{2} \rho D C_D U |U| + \frac{1}{4} \rho \pi D^2 C_M \frac{dU}{dt} \quad (1)$$

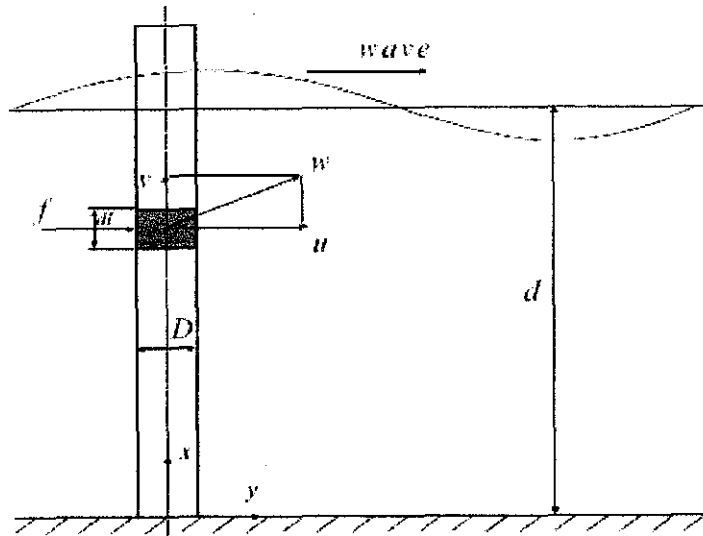
Where  $F_x$  is the force per unit length;  $U$  is the instantaneous flow velocity taken at the section centre;  $D$  the diameter of the cylinder;  $\rho$  the density of the fluid;  $C_D$  the drag coefficient; and  $C_M$  the inertia coefficient.

To calculate the force in the y direction  $F_y$ , the following equation is used:

$$F_y = \rho g \frac{\cosh kl}{\cosh kd} \times \frac{\pi D^2}{4} \times \eta \quad (2)$$

Where  $\eta$  is the wave profile and can be calculated by using the linear airy theory with the following equation:

$$\eta = \frac{H}{2} \cos[k(x - ct)] \quad (3)$$



**Figure 2.3** Morison Force on a vertical pile (Hydrodynamics of offshore structure 1994)

It is well-known that for regular waves or sinusoidal flows  $C_D$  and  $C_M$  generally depend on the characteristics of the flow such as  $Kc$ , the Keulegan-Carpenter number, and  $Re$ , the Reynolds number.

**Table 2.1** Important non-dimensional quantities

Parameters	Formula
Reynolds number	$Re = u_o D / \nu$
Keulegan-Carpenter number	$Kc = u_o T / D$
Relative Surface Roughness	$e = K / D$
Diffraction Parameter	$\pi D / L$
Reduced velocity	$V_R = U / f_s D$

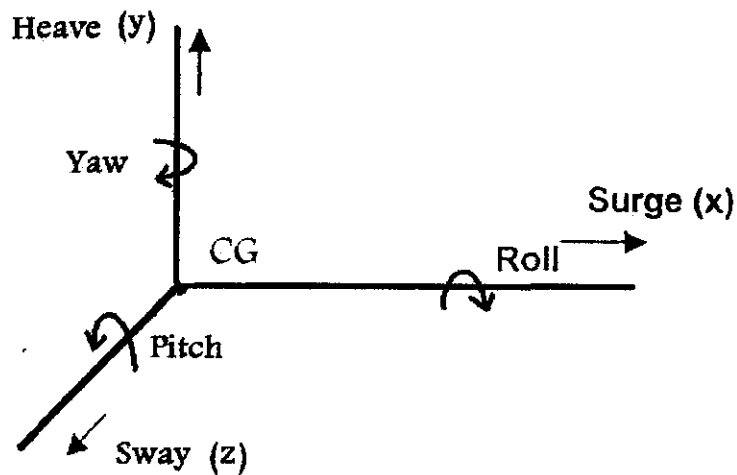
Where  $u_o$  = water particle velocity amplitude,  $D$  = member diameter,  $\nu$  = kinematic viscosity of water,  $e$  = relative roughness parameter,  $K$  = surface roughness

parameter,  $V_R$  = reduce velocity,  $U$  = current velocity,  $f_s$  = structure vibration frequency.

### 2.3 Frequency Domain Analysis

RAO (Response Amplitude Operator) is an expression to statically estimate the movements of the Spar as a function of the six degrees of freedom (Surge, Sway, Heave, Roll, Pitch, and Yaw) as shown in **Figure 2.4**, since only regular wave is considered for this project it is expected that RAO will be consistent at all times

The equations used to estimate these profiles are shown **Appendix C**.



**Figure 2.4** The six degrees of freedom

## **CHAPTER 3**

### **METHODOLOGY**

#### **3.1 General**

Although the spar platform is exposed to wave, wind, current and tides which contribute to the structure's response, wave load is considered to have the most predominant effect. The focus of this project is determining the spar responses due to regular waves. This chapter describes the process of determining the spar motion in Surge, Heave, and Pitch.

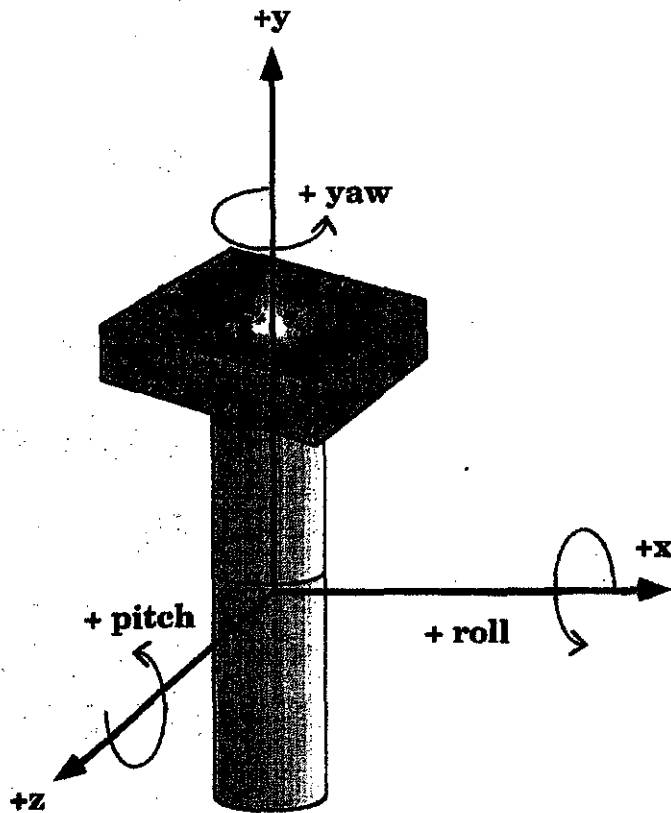
FYP Gantt chart as attached in **Appendix A** shows the key milestones and project activities which can be summarized into 5 main activities:

1. Literature review and Data gathering
2. Wave Properties Calculation
3. Wave Load calculation.
4. Frequency Domain Analysis.
5. Parametric studies

For achieving these activities the following procedures were followed and implemented.

#### **3.2 Spar platform structural model**

The Spar platform is modelled as a rigid cylinder with six degrees-of-freedom (i.e. three displacement degrees-of-freedom i.e. Surge, Sway and Heave along X, Y and Z axis and three rotational degree-of-freedom i.e. Roll, Pitch and Yaw about X, Y and Z axis) at its centre of gravity, CG as shown in **Figure 3.1**



**Figure 3.1** Spar six degrees of freedom around centre of gravity CG

An existing Spar platform Neptune was chosen in order to perform a simple dynamic analysis in time domain. Neptune Spar Platform (See **Figure 1.1**) is located 217.26 km (135 miles) south-east of New Orleans, in the Gulf of Mexico. The water depth ranges from 358.14 m (1175ft) to 982.98 m (3,225ft). To develop the field, Oryx used the world's first production spar as a base for well-production operations.

The spar supports a three-level integrated deck above the top of the hull, consisting of a clear work over deck, a mezzanine production deck and a main production deck, Deck elevations are 35.36 m (116ft), 28.35 m (93ft) and 21.6 m (71ft), respectively. A two-stage, three-phase separation system is used to produce the well stream.

The spar hull of the Neptune was fabricated in 17 sections; it is essentially a 22m (72ft)-diameter cylinder, which is 214.88m (705ft) long with a draft of 198.12m (650ft). The centre well inside the hull accommodates 16 buoyancy-supported risers, grouped in four rows. Buoyancy tanks extend from the cellar deck to the bottom of



the variable ballast tanks, 67m (220ft) below the waterline. It is subdivided by four vertical radial bulkheads and horizontal decks into smaller watertight compartments.

The spar is held in position by a six-line mooring. Securing each mooring leg to the seafloor is a 213.36 cm (84in)-diameter by 54.86 m (180ft)-long pile. This is connected to 220ft of 4,3/4in chain, 2,400ft of 4,3/4in jacketed spiral-strand steel rope and 1,050ft of 4,3/4in chain, leading through a fairlead, to a chain jack at the spar's deck. Breaking strengths for the 4,3/4in wire and 4,3/4in chain are 2,750kips and 2,846kips, respectively.

**Table 3.1** Data for multi component catenary mooring line

Diameter of mooring line and anchor line	0.089 <i>m</i> *
Diameter of clump weight	1.055 <i>m</i>
Effective area of mooring line and anchor line	0.0032 <i>m</i> <sup>2</sup>
Effective area of clump weight	0.874 <i>m</i> <sup>2</sup>
Weight of mooring line and anchor line	293.2 <i>N/m</i>
Weight of clump weight	25000 <i>N/m</i>
Length of clump weight	40 <i>m</i>
Length of anchor line	800 <i>m</i>
Mean sea level	914.4 <i>m</i>
Height of fairlead point	808.8 <i>m</i>
Angle of inclination at the fairlead point	30 <i>degree</i>

\*Data obtained from Offshore Technology Research Centre (OTRC)

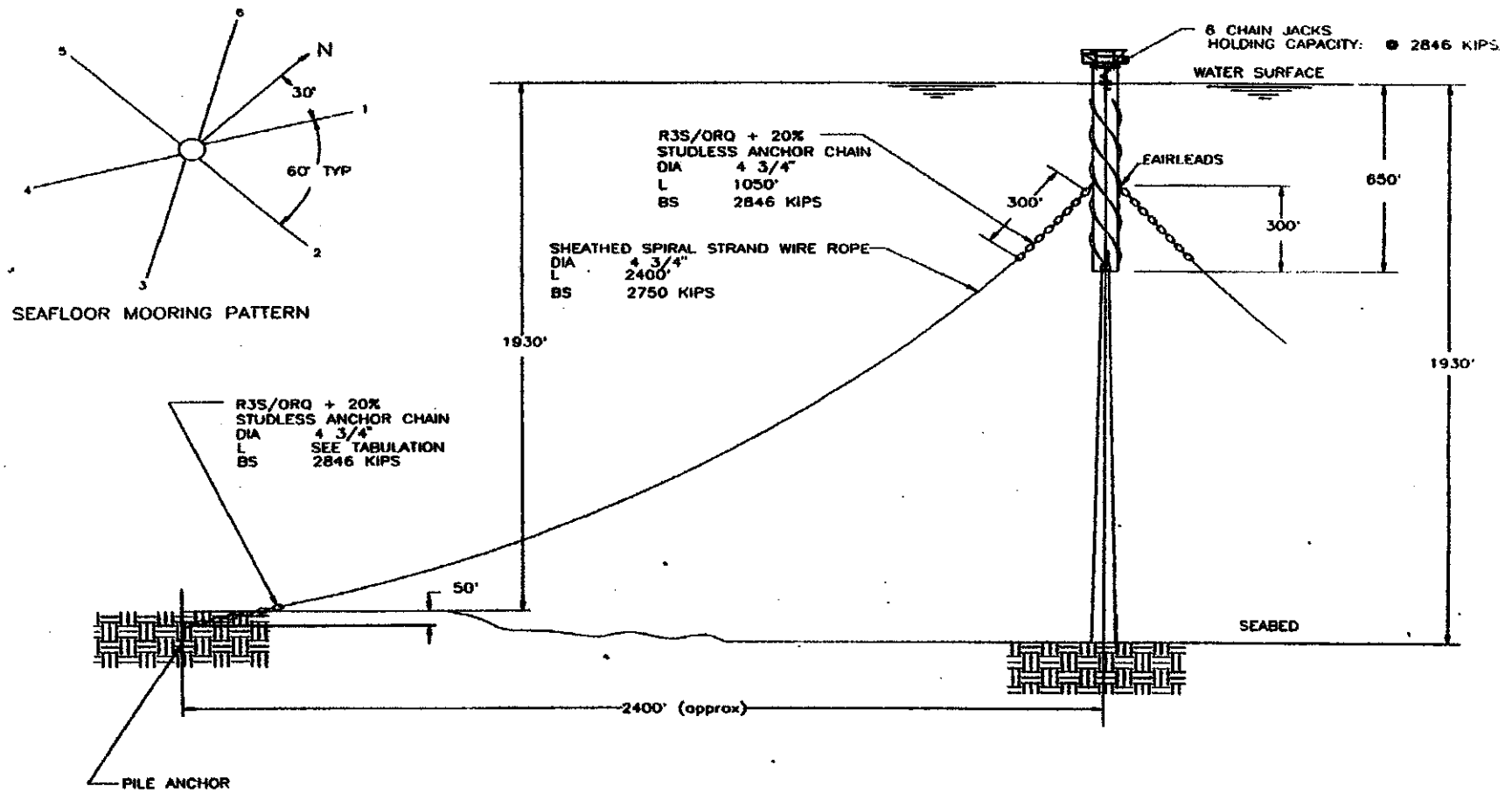


Figure 3.2 Neptune Spar Mooring

**Table 3.2** Dimensions of Spar platform and wave data

Weight of the structure	$2.6 \times 10^6 \text{ kN}^*$
Height of the Spar platform	214.88 m
Radius of the Spar platform	22 m
Distance of centre of gravity to buoyancy	6.67 m
Distance of centre of gravity from keel	92.4 m
Distance of centre of gravity to fairleads	0.17 m
Structural damping ratio	0.05 & 0.03
Wave period	14 sec
Wave height	3.6 m
Drag coefficient ( $C_D$ )	1.0
Inertia coefficient ( $C_M$ )	2.0

\*Data obtained from Offshore Technology Research Centre (OTRC)

**Table 3.1** Typical gravity loads on the deck

Item	% Weight
Deck Structure	5.0
Helideck	1.5
Living quarter	3.5
Topside equipment and facilities	60.0
Drilling rig	12.0
Hook load	6.0
Jacket support structure	12.0
<b>Total Payload</b>	<b>100.0</b>

### 3.3 Water particle properties calculation

For Horizontal/vertical velocity and acceleration calculation Linear Airy wave theory was suitable for this case, the following data are considered in order to calculate these parameters:

$$H = 7 \text{ m}$$

$$T = 12.5 \text{ sec}$$

$y$  = varies from 0 to -199.6  $m$  (Neptune Platform length beneath water surface)

Given the water depth for Neptune as 588.26  $m$

The equations used to determine  $(u)$ ,  $(v)$ ,  $(u)$  and  $(v)$  are given in **Appendix C** for Deep water linear airy theory equations.

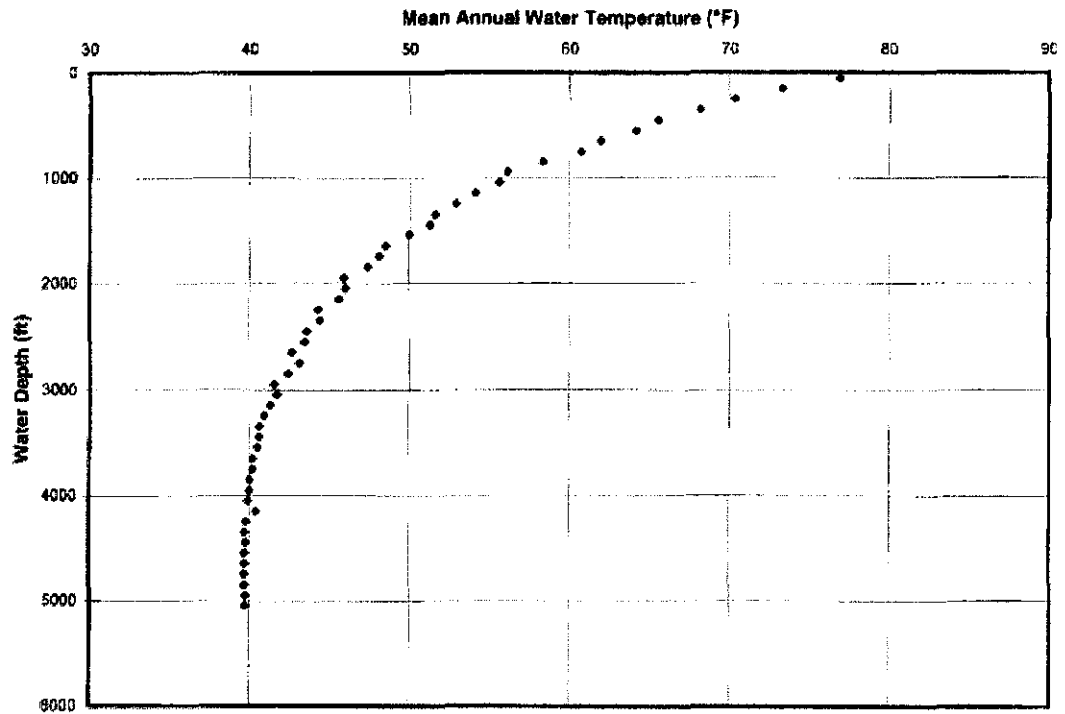
### 3.4 Wave loads on the structure

Wave load is calculated by using Morison Equation

$$F = \frac{1}{2} \rho D C_D U |U| + \frac{1}{4} \rho \pi D^2 C_M \frac{dU}{dt} \quad (1)$$

Knowing that  $(\rho)$  Density and  $(\nu)$  kinematic viscosity can be obtained using **Appendix B**

The Annual Temperature in Gulf of Mexico region is obtained from **Figure 3.2** Mean annual water temperature vs. Depth, From the this graph Temperature can be assumed as 64° F for water depth of nearly 600  $m$ .



**Figure 3.3** Mean annual water temperature vs. depth averaged for 100-foot intervals of depth in the northern Gulf of Mexico. Data from the NOAA World Ocean Database (Forrest, Marcucci and Scott 2007)

The values of  $C_D$ ,  $C_M$  can be assumed as 1.0, 2.0 relatively, using the above parameters Morison equation is used to calculate wave force at a certain time (t), depth (y), and displacement (x)

### 3.5 Moments about the centre of gravity

Moment calculation about (z) axis can be carried out knowing the magnitude of force with reference to the location of the centre of gravity which is located at 92.4 m from the bottom end of the platform structure, and with assuming a free board (portion above water) of 15.24 m

### 3.6 Frequency Domain analysis

RAO can be calculated for Surge, Heave, and Pitch as a function of time. The following steps are used to calculate RAO:

**Step 1** Results from **Appendix F** indicate that  $T_n$  for Surge, Heave, and Pitch are 392.23 and 39.79 sec, and 50.84 sec relatively, from this  $\omega_n$  can be calculated using:

$$\omega_n = \frac{2\pi}{T_n} \quad (6)$$

**Step 2** By having  $\omega_n$  the stiffness K can be calculated by using Equations.

$$K_{11} = \omega_n^2 m_{11} \quad (13)$$

$$K_{22} = \omega_n^2 m_{22} \quad (20)$$

$$K_{33} = \omega_n^2 m_{33} \quad (27)$$

**Step 3** RAO is calculated using the following equations.

$$RAO_{surge} = \frac{F_x / (H / 2)}{[(K_{11} - m_{11}\omega^2)^2 + (C_{11}\omega^2)^2]^{1/2}} \quad (15)$$

$$RAO_{heave} = \frac{F_y / (H / 2)}{[(K_{22} - m_{22}\omega^2)^2 + (C_{22}\omega^2)^2]^{1/2}} \quad (22)$$

$$RAO_{pitch} = \frac{M_z / (H / 2)}{[(K_{33} - m_{33}\omega^2)^2 + (C_{33}\omega^2)^2]^{1/2}} \quad (29)$$

**Step 4** The Responses  $\eta_x$ ,  $\eta_y$  and  $\eta_z$  with respect to time ( $t$ ) is calculated as follows

$$\eta_{surge}(t) = X \cos(\omega t + \beta_{11}) \quad (17)$$

$$\eta_{heave}(t) = Y \cos(\omega t + \beta_{22}) \quad (24)$$

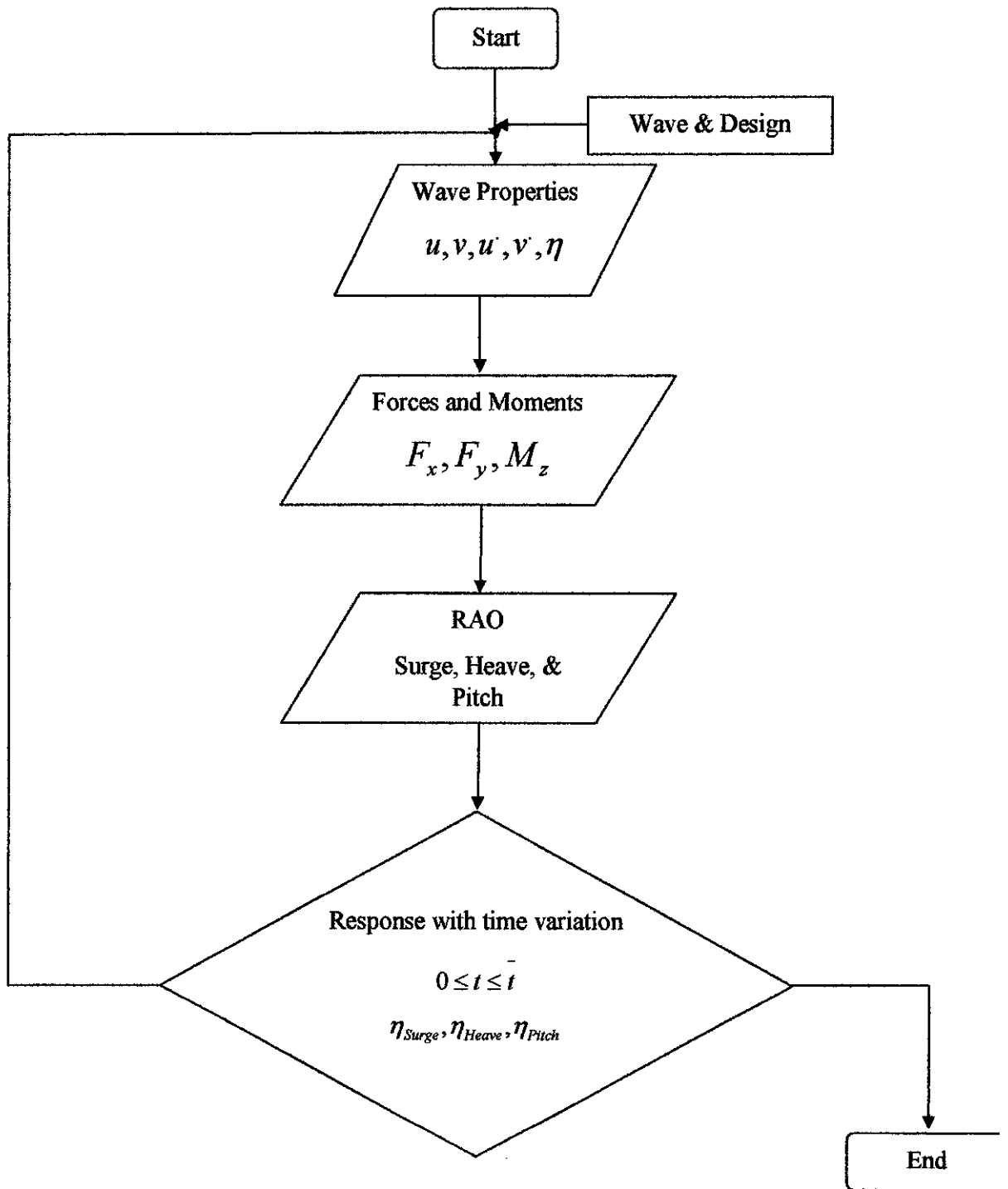
$$\eta_{pitch}(t) = Z \cos(\omega t + \beta_{33}) \quad (31)$$

### **3.7 Parametric Studies**

Parametric Studies are important for establishing the relationship between spar responses and different parameters such as wave height, wave period, and water depth. Water depth effect can be studied by calculating the responses with the same wave parameters in different water depths

Wave parameters effect such as wave height and wave period is established by comparing the responses of the same platform in different sea conditions.

**Figure 3.3** shows the process flow diagram for the project.



**Figure 3.4** Process Flow Diagram



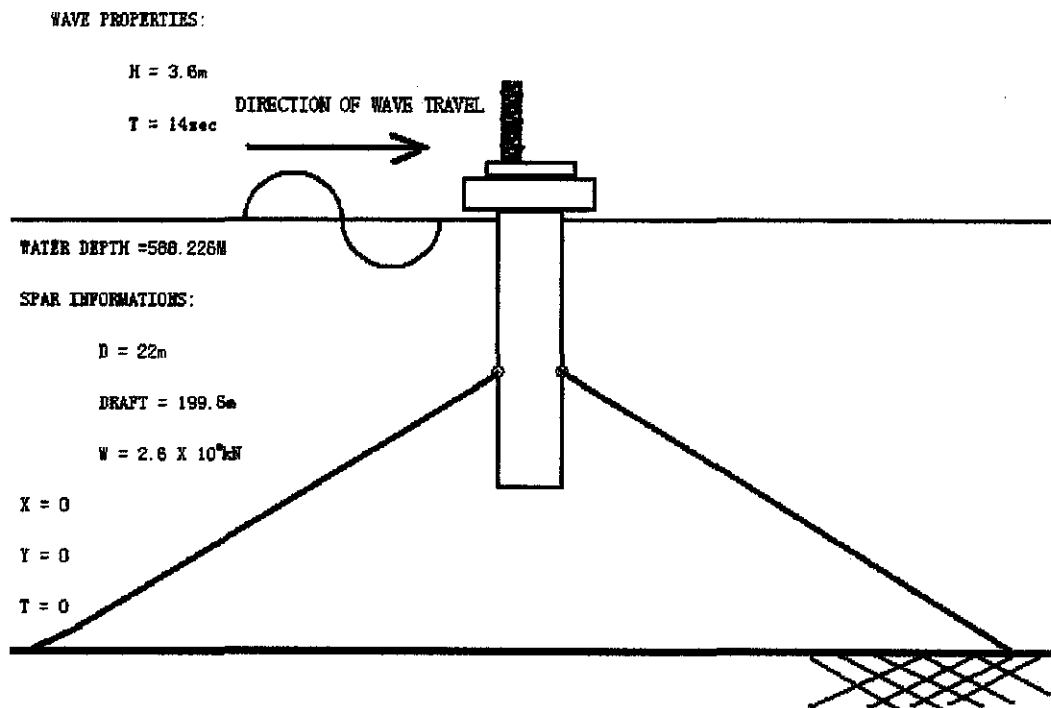
## CHAPTER 4

### RESULTS AND DISCUSSION

#### 4.1 General

The spar experiences periodic displacement when it is subjected to periodic wave loads. A simplified analysis including the determination of these loads and associated particle velocity and acceleration are discussed in this chapter. It also includes parametric study to observe the effect of different parameters such as wave characteristics and water depth on responses.

#### 4.2 Calculations



**Figure 4.1** Initial Conditions to calculate water particles properties

The dimensions and data for the Neptune Spar Platform used for analysis are as follows.

Weight	$2.6 \times 10^6$ kN
Platform dimension	
- Diameter	22m
- Draft	199.6m
- Distance from keel to CG	92.4 m
- Radius of gyration $r_x$	66.2m
- Natural period in surge	392.23 sec
- Natural period in heave	39.79 sec
- Natural period in pitch	50.84 sec
Water depth	588.26 m
Wave height	3.6 m
Wave period	7 sec
Water Density	$1000 \text{ kg/m}^3$
Drag coefficient ( $C_D$ )	1
Inertia coefficient ( $C_M$ )	2
Structural damping ratio	0.04

#### 4.2.1 Water particles Properties Calculations

$$L = \frac{gT^2}{2\pi} \quad (4)$$

$$L = \frac{9.81T^2}{2\pi} = 76.54 \text{ m}$$

$$k = \frac{2\pi}{L_o} = 0.082 \text{ m}^{-1} \quad (5)$$

$$\omega = \frac{2\pi}{T} = 0.897 \text{ rad/sec} \quad (6)$$

Horizontal velocity as described by Linear Airy theory for deep water:

$$u = \frac{gkH}{2\omega} \exp(ky) \cos[k(x - ct)] \quad (7)$$

But for initial conditions  $(x, y, t) = (0, 0, 0)$

$$u = \frac{gkH}{2\omega} = 1.637 \text{ m/sec}$$

Horizontal acceleration as described by Linear Airy theory for deep water:

$$u' = \frac{gkH}{2} \exp(ky) \cos[k(x - ct)] \quad (8)$$

But for initial conditions  $(x, y, t) = (0, 0, 0)$

$$u' = \frac{gkH}{2} = 1.469 \text{ m/sec}^2$$

Vertical velocity as described by Linear Airy theory for deep water:

$$v = \frac{gkH}{2\omega} \exp(ky) \cos[k(x - ct)]$$

But for initial conditions  $(x, y, t) = (0, 0, 0)$

$$v = \frac{gkH}{2\omega} = 1.637 \text{ m/sec}$$

Vertical acceleration as described by Linear Airy theory for deep water:

$$v' = \frac{gkH}{2} \exp(ky) \cos[k(x - ct)]$$

But for initial conditions  $(x, y, t) = (0, 0, 0)$

$$v' = \frac{gkH}{2} = 1.469 \text{ m/sec}^2$$

#### 4.2.2 Force calculation

##### a- surge

Force at the surface ( $y = 0$ )

Force acting on x axis is calculated by using Morison Equation

$$F = \frac{1}{2} \rho D C_D U |U| + \frac{1}{4} \rho \pi D^2 C_M \frac{dU}{dt} \quad (1)$$

$$u = 1.637 \text{ m/s}, u = 1.469 \text{ m/s}^2, \rho = 1000 \text{ kg/m}^3, D = 22 \text{ m}, C_D, C_M(\text{max}) = 1.0, 2.0$$

$$\therefore F_x = 11238.921 \text{ kN at } y = 0$$

With repeating above calculation steps with  $x_{\text{initial}} = 0$ ,  $t_{\text{initial}} = 0$  and  $y$  varies from 0 to  $-199.64 \text{ m}$  (all calculation is shown in Appendix D) the following graph shows the variation of force with depth and from this graph it is found the total force would be:

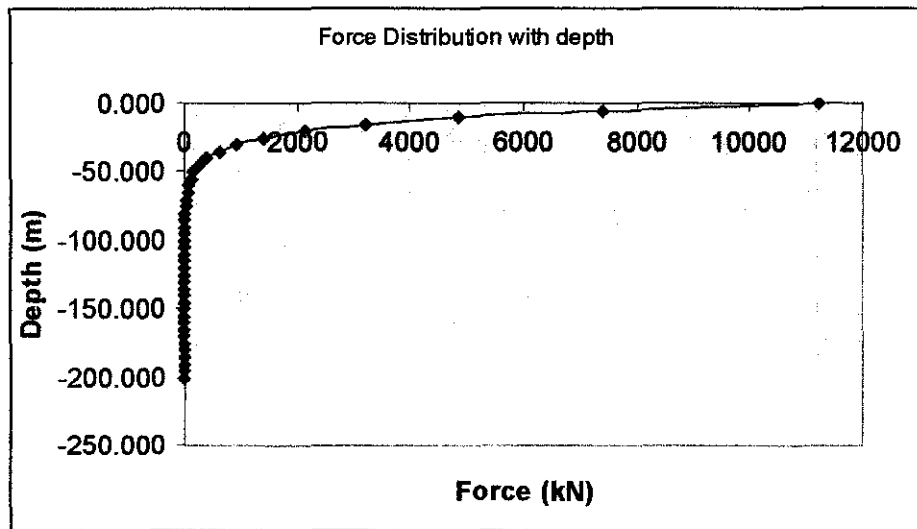
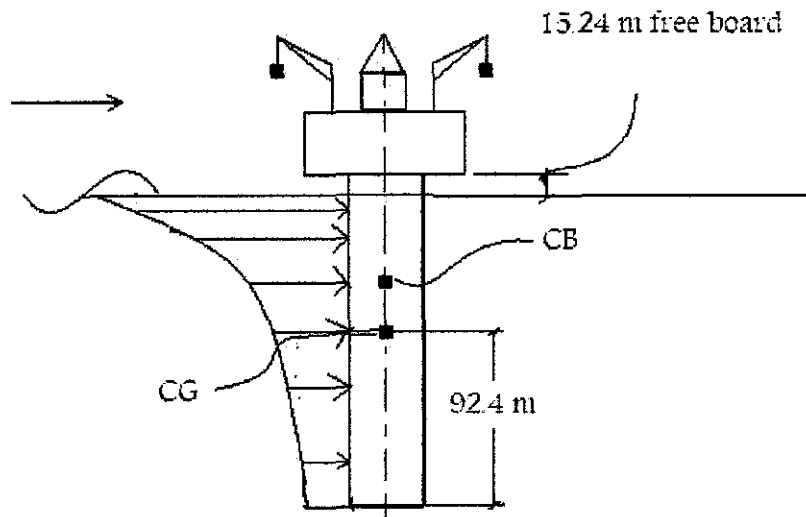


Figure 4.2 Force Distribution with depth



**Figure 4.3** Force distribution on spar Hull

For initial conditions  $(x, t) = (0, 0)$

$$\therefore \Sigma F_x = 33056.551 \text{ kN}$$

**b- Heave**

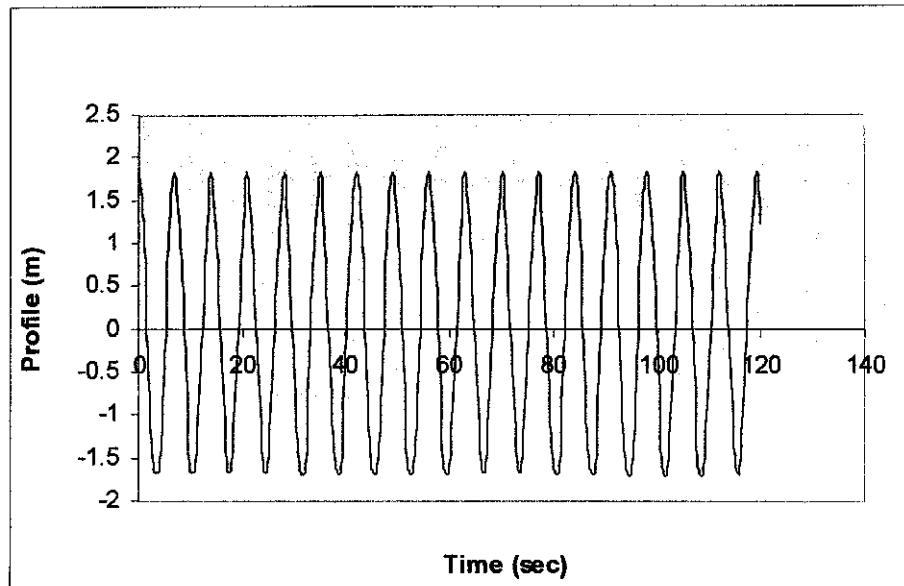
Linear Airy theory defines wave profile for deep water by the following equation:

$$\eta = \frac{H}{2} \cos[k(x - ct)]$$

But for initial conditions  $(x, y, t) = (0, 0, 0)$

$$\eta = \frac{H}{2} = 1.8\text{m}$$

But as time varies the general graph of the wave profile can be shown as a relation of vertical displacement vs. time as shown in **Figure 4.4**



**Figure 4.4** Wave Profile for Regular wave ( $H = 3.6m$ ,  $T = 7s$ )

Morison equation defines the vertical force acting on a cylinder by

$$F_y = \rho g \frac{\cosh kl}{\cosh kd} \times \frac{\pi D^2}{4} \times \eta \quad (2)$$

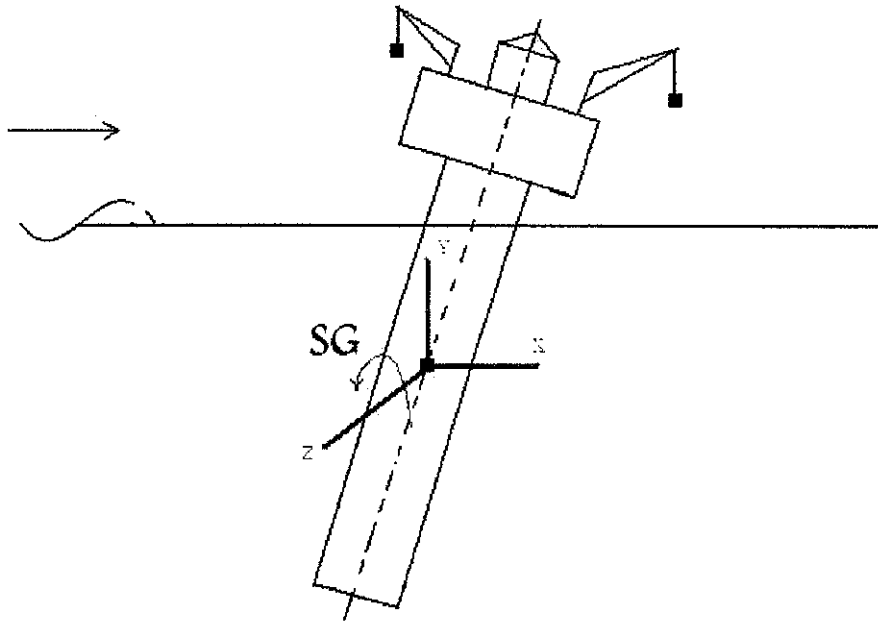
With initial conditions  $(x, y, t) = (0, 0, 0)$ ,  $\eta = 1.8 m$ ,  $\rho = 1000 \text{ kg/m}^3$ ,  $k = 0.082 \text{ m}^{-1}$ ,  $l = 199.64m$ ,  $d = 588.26 m$ ,  $D = 22m$   
 $\therefore \sum F_y = 96.32 \text{ kN}$

#### 4.2.3 Moment Calculation

Calculating moments around centre of gravity (CG) due to surge forces is important to know Pitch responses, and these moments can be calculated by using the following equation:

$$m = \pm Fy' \quad (9)$$

$$\therefore \sum m = \sum Fy' = -3234049.166 \text{ kN.m}$$



**Figure 4.5** Rotation due to surge around SG

#### 4.2.4 Stiffness

Stiffness  $K_{11}, K_{22}, K_{33}$  are derived from natural time period  $T_{n11}, T_{n22}, T_{n33}$  for spar platform from Appendix F as follows

$$\omega_{n11} = \frac{2\pi}{T_{n11}} \quad (6)$$

$$\omega_{n22} = \frac{2\pi}{T_{n22}} \quad (6)$$

$$\omega_{n33} = \frac{2\pi}{T_{n33}} \quad (6)$$

$$K_{11} = \omega_{n11}^2 m_{11} \quad (13)$$

$$K_{22} = \omega_{n22}^2 m_{22} \quad (20)$$

$$K_{33} = \omega_{n33}^2 I_{33} \quad (27)$$

#### 4.2.5 Surge Response Calculation

Surge Added mass = Volume of Platform hull x density of water

$$\text{Surge Added mass} = \frac{\pi}{4} l D^2 \rho = 75.89 \times 10^6 \text{ kg} \quad (11)$$

Total Surge mass  $m_{11}$  = mass of platform + added mass

$$\therefore m_{11} = 3.34 \times 10^6 \text{ kg}$$

$$C_{11} = 2\zeta\omega_{n11}m_{11} \quad (12)$$

$$\tan \beta_{22} = \frac{C_{11}\omega}{K_{11} - m_{11}\omega^2} \quad (14)$$

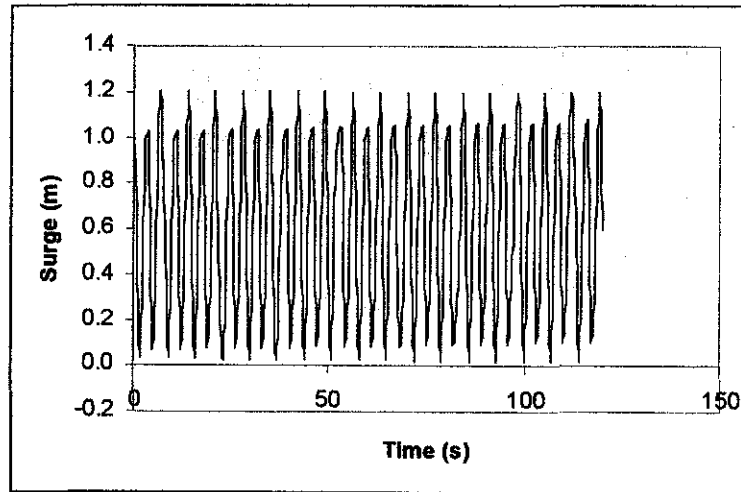
$$RAO_{surge} = \frac{F_x / (H/2)}{[(K_{11} - m_{11}\omega^2)^2 + (C_{11}\omega^2)^2]^{1/2}} \quad (15)$$

$$X = RAO_{surge} \times H/2 \quad (16)$$

$$\eta_x(t) = X \cos(\omega t + \beta_{11}) \quad (17)$$

**Appendix E-2** Shows the results for surge motion within  $0 \leq t \leq 120$  **Figure 4.6**

Summarize the results obtained for Surge responses



**Figure 4.6** Spar Surge Responses in Regular waves (H = 3.6, T = 7s)



#### 4.2.6 Heave Response Calculation

$$\text{Heave Added mass} = \frac{\pi D^3}{12} \rho = 2787639.881 \text{ kg} \quad (18)$$

Total Surge mass  $m_{22}$  = mass of platform + added mass

$$\therefore m_{22} = 2.63 \times 10^6 \text{ kg}$$

$$C_{22} = 2\zeta\omega_{n22}m_{22} \quad (19)$$

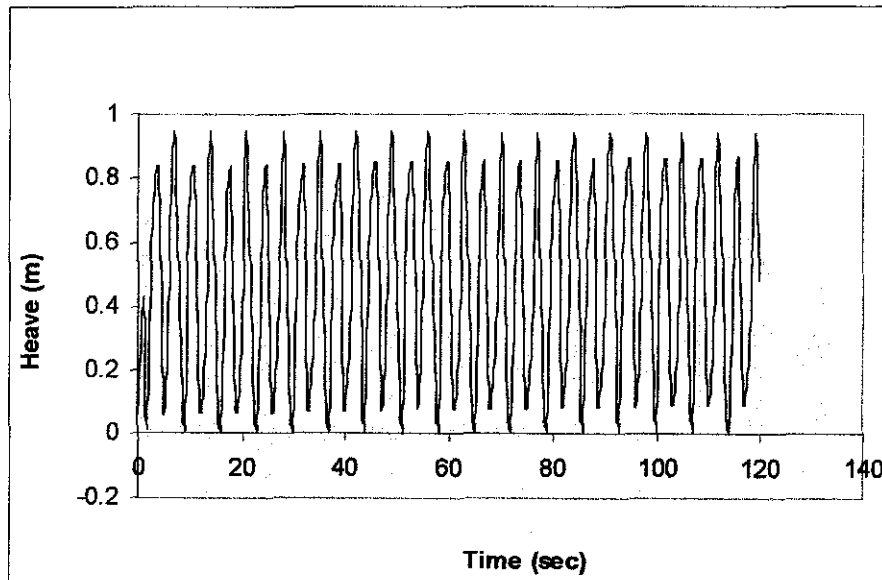
$$\tan \beta_{22} = \frac{C_{22}\omega}{K_{22} - m_{22}\omega^2} \quad (21)$$

$$RAO_{\text{Heave}} = \frac{F_y / (H/2)}{[(K_{22} - m_{22}\omega^2)^2 + (C_{22}\omega^2)^2]^{1/2}} \quad (22)$$

$$Y = RAO_{\text{Heave}} \times H/2 \quad (23)$$

$$\eta_y(t) = Y \cos(\omega t + \beta_{22}) \quad (24)$$

**Appendix E-2** Shows the results for surge motion within  $0 \leq t \leq 120$  **Figure 4.7**  
Summarize the results obtained for Heave responses



**Figure 4.7** Spar Heave Responses in Regular waves ( $H = 3.6$ ,  $T = 14s$ )

#### 4.2.7 Pitch Response Calculations

$$I_z = m_{22} \times r^2 \quad (25)$$

$$m_{22} = 2.63 \times 10^6 \text{ kg}$$

$$\therefore I = 174 \times 10^6 \text{ kg.m}$$

$$C_{33} = 2\zeta\omega_{n33}I_{33} \quad (26)$$

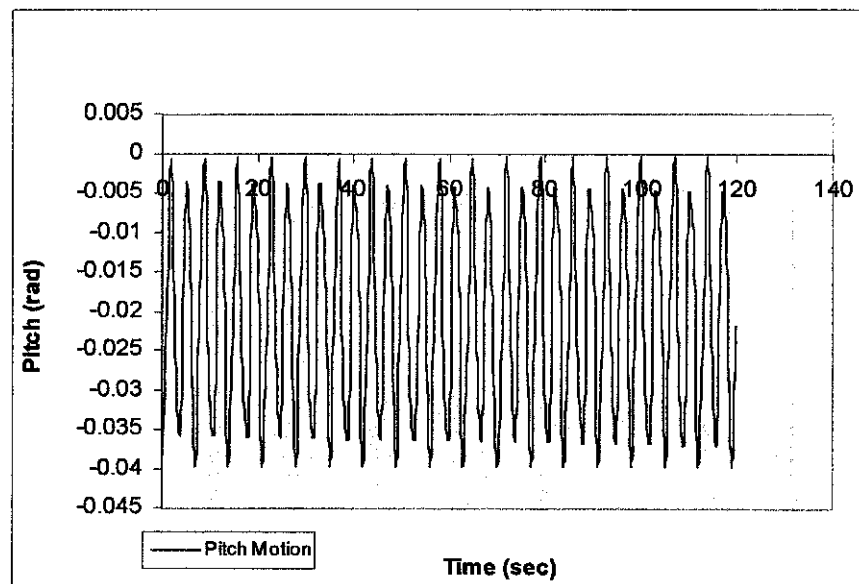
$$\tan \beta_{33} = \frac{C_{33}\omega}{K_{33} - I_{33}\omega^2} \quad (28)$$

$$RAO_{Pitch} = \frac{M_z / (H/2)}{[(K_{33} - I_{33}\omega^2)^2 + (C_{33}\omega^2)^2]^{1/2}} \quad (29)$$

$$Z = RAO_{Pitch} \times H/2 \quad (30)$$

$$\eta_z(t) = Z \cos(\omega t + \beta_{33}) \quad (31)$$

**Appendix E-2** Shows the results for Pitch Response within  $0 \leq t \leq 200$  **Figure 4.8**  
Summarize the results obtained for Pitch responses



**Figure 4.8** Spar Pitch Responses in Regular waves ( $H = 3.6$ ,  $T = 14s$ )

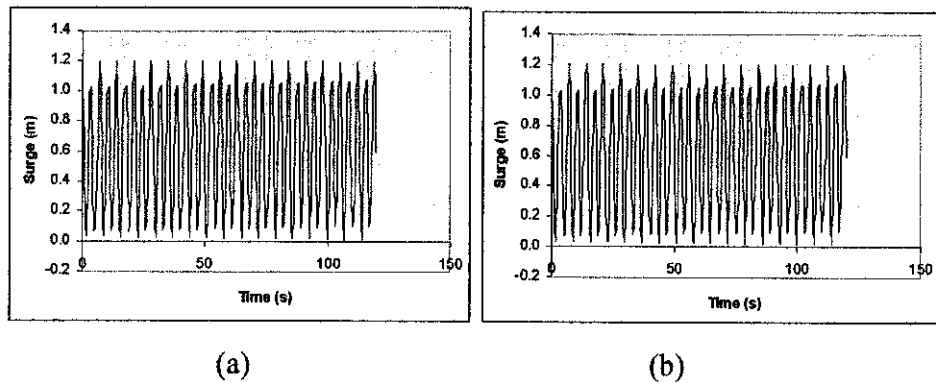
### 4.3 Parametric Studies

To understand the effect of different parameters in Spar responses, parametric Studies were performed by using different parameters such as wave heights, periods and water depths.

#### 4.3.1 Depth effect in Spar responses

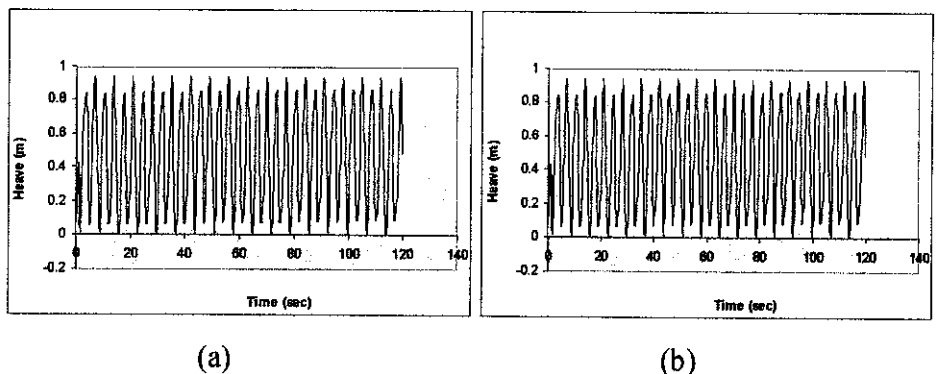
Water depth effect was established by comparing spar response under the same sea condition when varying water depth, as shown in **Figures 4.9, 4.10, 4.11** For wave height of 3.6m, and wave period of 7s with different water depths (588.226m, 789.443m and 238.62m) it was clearly observed that water depth has very small or no effect on Spar responses due to regular waves.

##### a- Surge



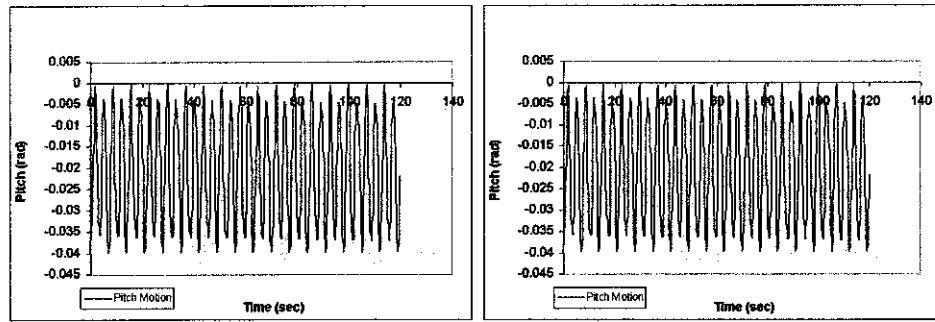
**Figure 4.9** Surge Response ( $H = 3.6\text{m}$ ,  $T = 7\text{s}$ ); (a) water depth of 789.443m, (b) water depth of 2382.62m

##### b- Heave



**Figure 4.10** Heave Responses ( $H = 3.6\text{m}$ ,  $T = 7\text{s}$ ); (a) water depth of 789.443m, (b) water depth of 2382.62m

**c- Pitch**



**Figure 4.11** Pitch Responses ( $H = 3.6\text{m}$ ,  $T = 7\text{s}$ ); (a) water depth of 789.443m, (b) water depth of 2382.62m

**4.3.2 Wave characteristic effects in Spar responses**

To establish wave characteristics effects in Spar motions different possible cases of wave heights and wave periods are used as shown in **Table 4.1**

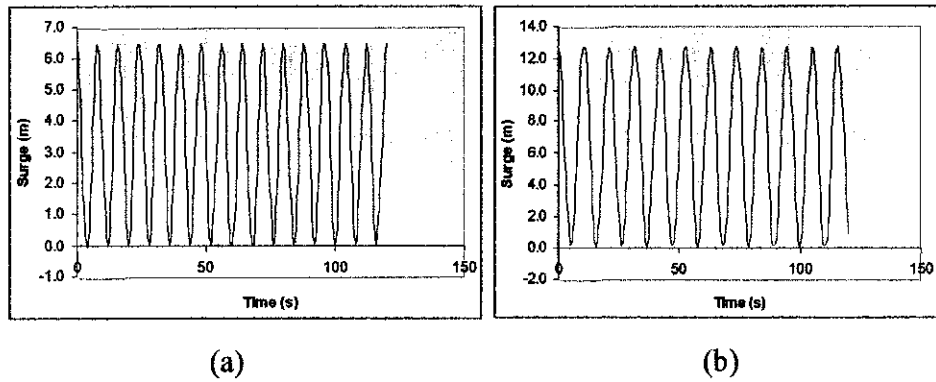
**Table 4.1** Regular waves

Cases	H (m)	T (sec)
Case 1 (original)	3.6	7*
Case 2	4.6	16
Case 3	6	21

\*Data obtained from Offshore Technology Research Centre (OTRC)

From the results obtained in **Figures 4.12, 4.13, 4.14** it is observed that with an increase in wave characteristics such as wave Height and Period, Spar responses increase

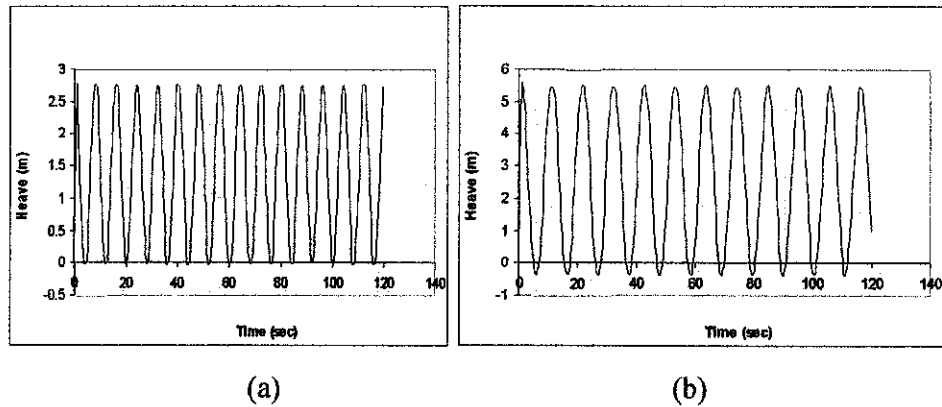
**a- Surge**



**Figure 4.12** Spar Surge in regular waves; (a) Case 2 ( $H = 4.6\text{m}$ ,  $T = 16\text{s}$ ); (b) Case 3 ( $H = 6\text{m}$ ,  $T = 21\text{s}$ )

As observed from **Figures 4.6, 4.12(a), 4.12(b)** The maximum amount of Surge response observed in Case 1 (where  $H = 3.6\text{m}$ ,  $T = 7\text{m}$ ) is (1.2m), increases up to (6m) in Case 2 (where  $H = 4.6\text{m}$ ,  $T = 16\text{s}$ ), and up to (12.4m) in Case 3 (where  $H = 6\text{m}$ ,  $T = 21\text{s}$ ).

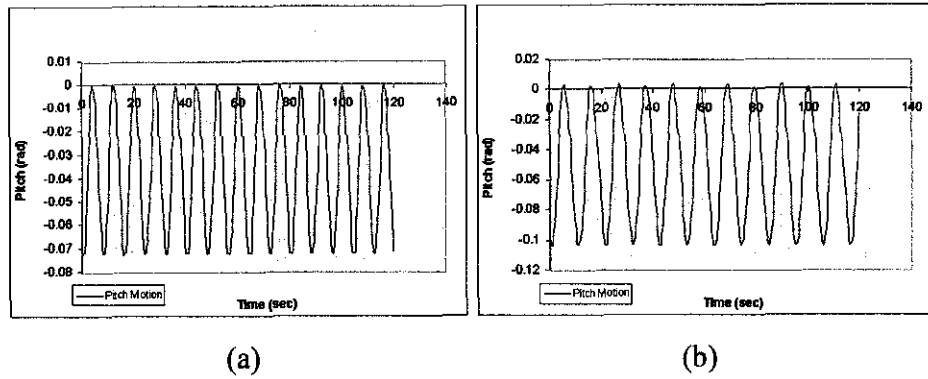
**b- Heave**



**Figure 4.13** Spar Heave in regular waves; (a) Case 2 ( $H = 4.6\text{m}$ ,  $T = 16\text{s}$ ); (b) Case 3 ( $H = 6\text{m}$ ,  $T = 21\text{s}$ )

As observed from **Figures 4.7, 4.13(a), 4.13(b)** The maximum amount of Heave response observed in Case 1 (where  $H = 3.6\text{m}$ ,  $T = 7\text{m}$ ) is (0.95m), increases up to (2.8m) in Case 2 (where  $H = 4.6\text{m}$ ,  $T = 16\text{s}$ ), and up to (5.56m) in Case 3 (where  $H = 6\text{m}$ ,  $T = 21\text{s}$ ).

c- **Pitch**



**Figure 4.14** Spar Heave in regular waves; (a) Case 2 ( $H = 4.6\text{m}$ ,  $T = 16\text{s}$ ); (b) Case 3 ( $H = 6\text{m}$ ,  $T = 21\text{s}$ )

As observed from **Figures 4.8, 4.14(a), 4.14(b)** The maximum amount of Pitch response observed in Case 1 (where  $H = 3.6\text{m}$ ,  $T = 7\text{m}$ ) is  $(-0.04\text{rad})$ , increases up to  $(-0.07\text{rad})$  in Case2 (where  $H = 4.6\text{m}$ ,  $T = 16\text{s}$ ), and up to  $(-0.1\text{rad})$  in Case 3 (where  $H = 6\text{m}$ ,  $T = 21\text{s}$ ).

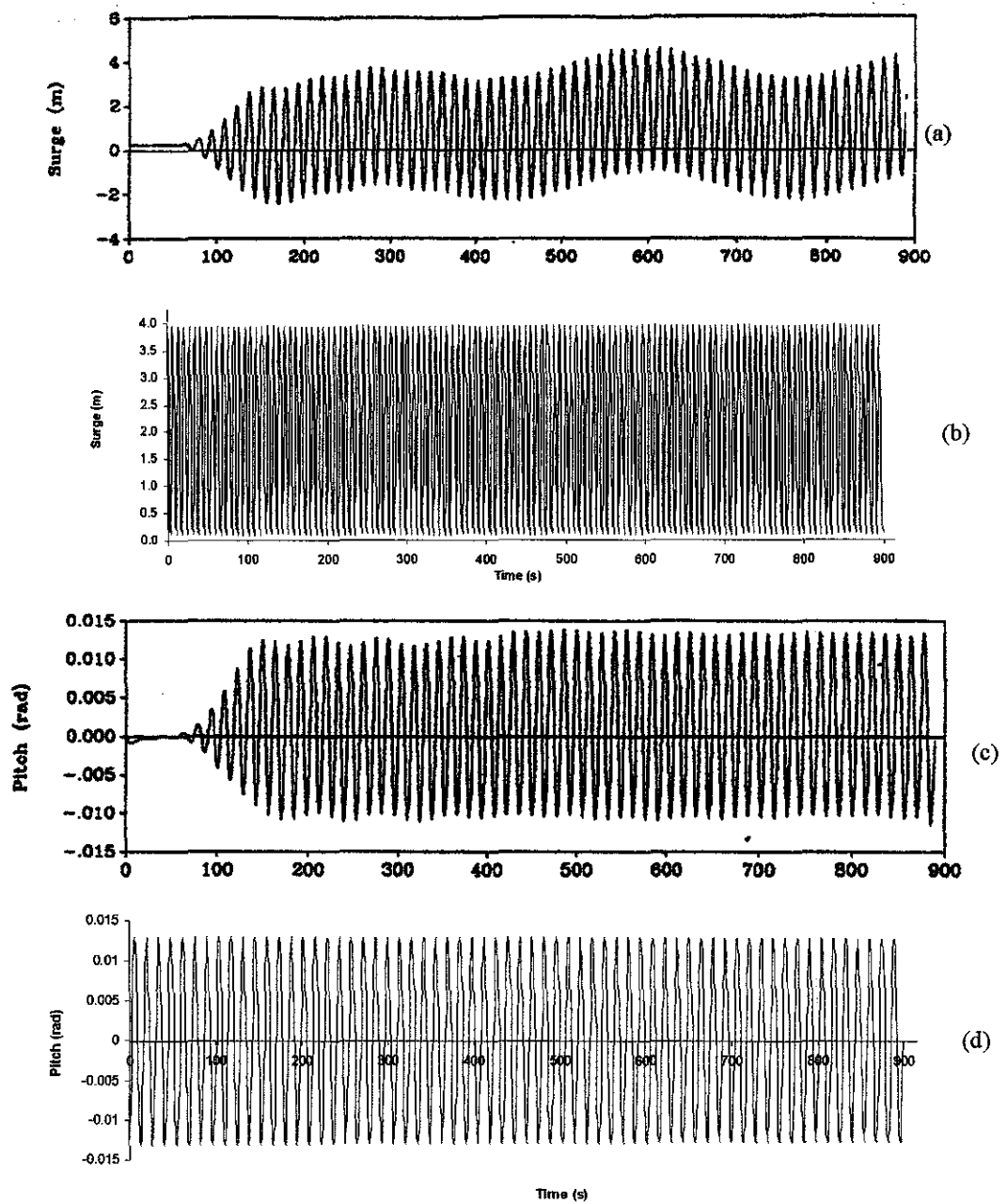
**4.4 Comparison with experiment**

According to *Z. Ran et al, (1996)*, experiment values as shown in **Figure 4.15** Indicates that the maximum response of Surge due to regular wave for GENESIS Spar platform with diameter of  $40\text{m}$  and Hull length of  $199.4\text{m}$  ( $H = 3\text{m}$  and  $T = 14\text{s}$ ) for a study period of  $900\text{ sec}$  was  $4\text{m}$ , and a maximum pitch response of  $0.014\text{rad}$ . Whereas the maximum computed response for the surge for a study period of  $900\text{ sec}$  was  $3.9\text{m}$ , and the maximum computed pitch response for a period of  $900\text{ sec}$  was  $0.013\text{rad}$ . **Table 4.2** shows the summarized comparison between theory and experiment

**Table 4.2** Comparison between experiment and theory

	experiment	theory	Error (%)
Maximum Surge (m)	4	3.9	2.5
Maximum Pitch (rad)	0.014	0.013	7.14

Figure 4.15 shows the comparison between Surge and pitch computed values vs. experimental values.



**Figure 4.15** Spar responses in regular waves ( $H = 3\text{m}$ ,  $T = 14\text{s}$ ); (a) experimental surge (Z. Ran, *et al* 1996), (b) calculated surge, (c) experimental pitch(Z. Ran, *et al* 1996), and (d) calculated pitch

## **CHAPTER 5**

### **CONCLUSION AND RECOMMENDATIONS**

#### **5.1 Conclusions**

- A frequency-domain dynamic analysis computer program was developed to numerically simulate the responses of a Spar platform subjected to regular sea waves, and the results obtained agreed with similar studies done earlier.
- The maximum surge, Heave & pitch responses for Neptune Spar platform were 1.2m, 0.9m & 0.039rad respectively and these responses are considered to be within the permissible limits
- Parametric study was performed to establish the effect of different parameters such as water depth and wave characteristics, and it is found that:
  - Water depth has no effect on spar responses due to regular waves
  - Increasing sea state values such as wave height and wave period will result in an increment on spar responses due to regular waves.

#### **5.2 Recommendations**

- Further research is required for more accurate results in the general behaviour of the Spar, and that might be due to:
  - Different theories might have different results.
  - In the real life situation sea waves are considered to be random at all times and will have different properties than obtained earlier therefore more variables need to be considered.



- Model testing is very important in order to assure the theoretical results on the dynamic analysis as well as understanding the behaviour of a classic Spar Platform subjected to regular waves.

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## **APPENDICES**

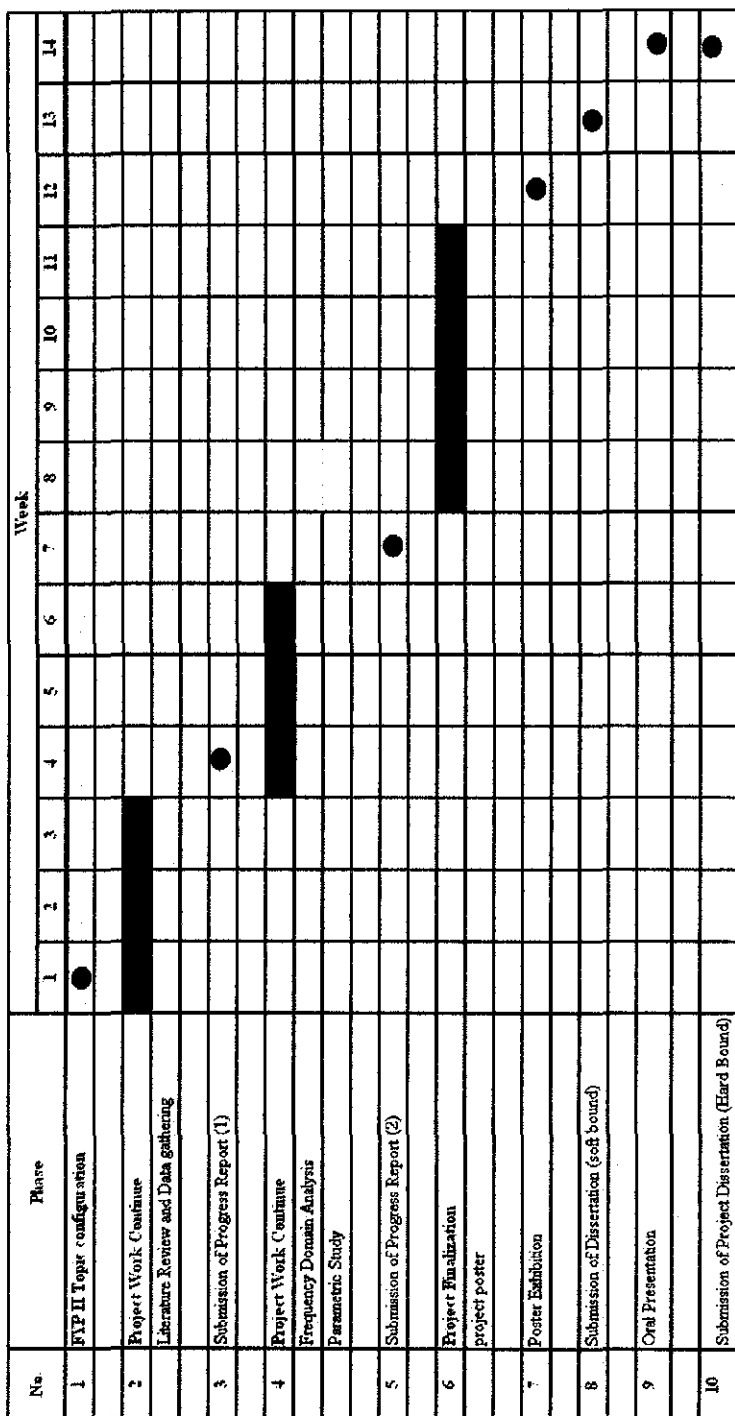
<b>APPENDIX A</b>	FYP Gantt Chart
	A-1    FYP I Gantt Chart
	A-2    FYP II Gantt Chart
<b>APPENDIX B</b>	density and viscosity vs. temperature of fresh and sea-water
<b>APPENDIX C</b>	Formulas
<b>APPENDIX D</b>	Surge Force Results
<b>APPENDIX E</b>	Profiles ( $\eta$ )
	E-1    Wave profile at the surface
	E-2    Surge and Heave Profiles
<b>APPENDIX F</b>	Natural time period for spar platform with different initial horizontal force (SEC)
<b>APPENDIX G</b>	MATLAB PROGRAM

**APPENDIX A**  
**A-1 FYP I Gantt Chart**

No.	Phase	Week No.														Stud. Week	Ex W 1	Ex W 2	Ex W 3	Ex W 4
		1	2	3	4	5	6	7	8	9	10	11	12	13	14					
1	SELECTION OF PROJECT TOPIC	■	■																	
2	RESEARCH WORK			■	■	■	■													
	Project Objectives																			
3	Submission of Preliminary Report					●														
4	Project work continue																			
	Literature Review and Data gathering						■	■	■	■	■	■	■	■						
	Wave Properties Calculation										■	■	■	■	■					
5	Submission of Progress Report										●									
6	Project work continue																			
	Wave Load Calculation														■	■	■	■		
8	Submission of Interim Report Final Draft																		●	
9	Submission of Interim Report																			●
	Oral Presentation																			●

● Reports Submissions  
■ Progress

## A-2 FYP II Gantt chart



● Reports Submissions  
■ Progress

**APPENDIX B**  
**DENSITY AND VISCOSITY VS. TEMPERATURE OF FRESH**  
**AND SEA-WATER**

Water		Mass Density		Mass Density		Kinematic Viscosity		Kinematic Viscosity	
Temp.		Fresh water		Salt Water @ 35%		Fresh water, x 10 <sup>-5</sup>		salt water @ 35%, x 10 <sup>-5</sup>	
°F	°C	lbm/ft <sup>3</sup>	Kg/m <sup>3</sup>	lbm/ft <sup>3</sup>	Kg/m <sup>3</sup>	ft <sup>2</sup> /s	m <sup>2</sup> /s	ft <sup>2</sup> /s	m <sup>2</sup> /s
32	0	1.9399	31.075	1.9947	31.953	1.9291	0.1792	1.9681	0.1828
34	1.11	1.94	31.076	1.9946	31.951	1.8565	0.1725	1.8974	0.1763
36	2.22	1.9401	31.078	1.9944	31.948	1.7883	0.1661	1.8309	0.1701
38	3.33	1.9401	31.078	1.9942	31.945	1.7242	0.1602	1.7683	0.1643
40	4.44	1.9401	31.078	1.994	31.941	1.6638	0.1546	1.7091	0.1588
42	5.56	1.9401	31.078	1.9937	31.937	1.6068	0.1493	1.6568	0.1539
44	6.67	1.94	31.076	1.9934	31.932	1.553	0.1443	1.6035	0.1490
46	7.78	1.9399	31.075	1.9931	31.927	1.5021	0.1395	1.5531	0.1443
48	8.89	1.9398	31.073	1.9928	31.922	1.4538	0.1351	1.5053	0.1398
50	10	1.9396	31.070	1.9924	31.916	1.408	0.1308	1.4599	0.1356
52	11.11	1.9394	31.067	1.9921	31.911	1.3646	0.1268	1.4168	0.1316
54	12.22	1.9392	31.064	1.9917	31.905	1.3233	0.1229	1.3758	0.1278
56	13.33	1.9389	31.059	1.9912	31.897	1.284	0.1193	1.3368	0.1242
58	14.44	1.9386	31.054	1.9908	31.890	1.2466	0.1158	1.2996	0.1207
60	15.56	1.9383	31.049	1.9903	31.882	1.2109	0.1125	1.2641	0.1174
62	16.67	1.9379	31.043	1.9898	31.874	1.1769	0.1093	1.2303	0.1143
64	17.78	1.9375	31.036	1.9893	31.866	1.1444	0.1063	1.1979	0.1113
66	18.89	1.9371	31.030	1.9888	31.858	1.1133	0.1034	1.1669	0.1084
68	20	1.9367	31.023	1.9882	31.848	1.0836	0.1007	1.1372	0.1056
70	21.11	1.9362	31.015	1.9876	31.839	1.0552	0.0980	1.1088	0.1030
72	22.22	1.9358	31.009	1.987	31.829	1.0279	0.0955	1.0816	0.1005

## APPENDIX C

### FORMULAS

Linear wave theory for deep water

Quantity	Deep water formula
Dispersion Relationship	$\omega^2 = gk$
Wave profile	$\eta = \frac{H}{2} \cos[k(x - ct)]$
Horizontal velocity	$u = \frac{gkH}{2\omega} \exp(ky) \cos[k(x - ct)]$
Vertical velocity	$v = \frac{gkH}{2\omega} \exp(ky) \cos[k(x - ct)]$
Horizontal acceleration	$u' = \frac{gkH}{2} \exp(ky) \cos[k(x - ct)]$
Vertical acceleration	$v' = \frac{gkH}{2} \exp(ky) \cos[k(x - ct)]$
Dynamic pressure	$p = \rho g \frac{H}{2} \exp(ky) \cos[k(x - ct)]$

$$F_x = \frac{1}{2} \rho D C_D U |U| + \frac{1}{4} \rho \pi D^2 C_M \frac{dU}{dt} \quad (1)$$

$$F_Y = \rho g \frac{\cosh kl}{\cosh kd} \times \frac{\pi D^2}{4} \times \eta \quad (2)$$

$$\eta = \frac{H}{2} \cos[k(x - ct)] \quad (3)$$

$$L = \frac{gT^2}{2\pi} \quad (4)$$

$$k = \frac{2\pi}{L_o} \quad (5)$$

$$\omega = \frac{2\pi}{T} \quad (6)$$

$$u = \frac{gkH}{2\omega} \exp(ky) \cos[k(x - ct)] \quad (7)$$

$$u' = \frac{gkH}{2} \exp(ky) \cos[k(x - ct)] \quad (8)$$

$$m = \pm Fy' \quad (9)$$

$$\sum m = \sum Fy' \quad (10)$$

$$\text{Added mass for } m_{11} = \frac{\pi}{4} l D^2 \rho \quad (11)$$

$$C_{11} = 2\zeta\omega_{n11}m_{11} \quad (12)$$

$$K_{11} = \omega_{n11}^2 m_{11} \quad (13)$$

$$\tan \beta_{11} = \frac{C_{11}\omega}{K_{11} - m_{11}\omega^2} \quad (14)$$

$$RAO_{surge} = \frac{F_x l (H/2)}{[(K_{11} - m_{11}\omega^2)^2 + (C_{11}\omega^2)^2]^{1/2}} \quad (15)$$

$$X = RAO_{surge} \times H/2 \quad (16)$$

$$\eta_x(t) = X \cos(\omega t + \beta_{11}) \quad (17)$$

$$\text{Added mass for } m_{22} = \frac{\pi D^3}{12} \rho \quad (18)$$

$$C_{22} = 2\zeta\omega_{n22}m_{22}$$

(19)

$$K_{22} = \omega_{n22}^2 m_{22} \quad (20)$$

$$\tan \beta_{22} = \frac{C_{22}\omega}{K_{22} - m_{22}\omega^2} \quad (21)$$

$$RAO_{heave} = \frac{F_y l (H/2)}{[(K_{22} - m_{22}\omega^2)^2 + (C_{22}\omega^2)^2]^{1/2}} \quad (22)$$

$$Y = RAO_{heave} \times H/2 \quad (23)$$

$$\eta_y(t) = Y \cos(\omega t + \beta_{22}) \quad (24)$$

$$\text{Mass + added mass for Pitch } I = m_{22} \times r^2 \quad (25)$$

$$C_{33} = 2\zeta\omega_{n33}I_{33}$$

(26)

$$K_{33} = \omega_{n33}^2 I_{33} \quad (27)$$



$$\tan \beta_{33} = \frac{C_{33}\omega}{K_{33} - m_{33}\omega^2} \quad (28)$$

$$RAO_{pitch} = \frac{M_z / (H/2)}{[(K_{33} - I_{33}\omega^2)^2 + (C_{33}\omega^2)^2]^{1/2}} \quad (29)$$

$$Z = RAO_{pitch} \times H/2 \quad (30)$$

$$\eta_z(t) = Z \cos(\omega t + \beta_{33}) \quad (31)$$

**APPENDIX D**  
**FORCE AND MOMENT CALCULATIONS**

sec	x (m)	y (m)	u (m/sec)	v (m/sec)	$\dot{u}$ (m/sec <sup>2</sup> )	$\dot{v}$ (m/sec <sup>2</sup> )	Fx (kN)	M (kN.m)
000	0.0000	0.0000	1.6373	1.6373	1.4689	1.4689	11238.9213	-1209307.9320
000	0.0000	-5.0000	1.0863	1.0863	0.9746	0.9746	7392.4345	-758463.7779
000	0.0000	-10.0000	0.7208	0.7208	0.6466	0.6466	4876.4449	-475941.0242
000	0.0000	-15.0000	0.4782	0.4782	0.4291	0.4291	3223.0025	-298450.0291
000	0.0000	-20.0000	0.3173	0.3173	0.2847	0.2847	2132.9506	-186846.4737
000	0.0000	-25.0000	0.2105	0.2105	0.1889	0.1889	1412.7861	-116696.1340
000	0.0000	-30.0000	0.1397	0.1397	0.1253	0.1253	936.3152	-72658.0561
000	0.0000	-35.0000	0.0927	0.0927	0.0832	0.0832	620.7746	-45068.2383
000	0.0000	-40.0000	0.0615	0.0615	0.0552	0.0552	411.6768	-27829.3501
000	0.0000	-45.0000	0.0408	0.0408	0.0366	0.0366	273.0563	-17093.3231
000	0.0000	-50.0000	0.0271	0.0271	0.0243	0.0243	181.1326	-10433.2396
000	0.0000	-55.0000	0.0180	0.0180	0.0161	0.0161	120.1638	-6320.6147
000	0.0000	-60.0000	0.0119	0.0119	0.0107	0.0107	79.7208	-3794.7124
000	0.0000	-65.0000	0.0079	0.0079	0.0071	0.0071	52.8913	-2253.1708
000	0.0000	-70.0000	0.0052	0.0052	0.0047	0.0047	35.0919	-1319.4545
000	0.0000	-75.0000	0.0035	0.0035	0.0031	0.0031	23.2828	-759.0187
000	0.0000	-80.0000	0.0023	0.0023	0.0021	0.0021	15.4478	-426.3600
000	0.0000	-85.0000	0.0015	0.0015	0.0014	0.0014	10.2495	-231.6387
000	0.0000	-90.0000	0.0010	0.0010	0.0009	0.0009	6.8005	-119.6885
000	0.0000	-95.0000	0.0007	0.0007	0.0006	0.0006	4.5121	-56.8524
000	0.0000	-100.0000	0.0004	0.0004	0.0004	0.0004	2.9938	-22.7526
000	0.0000	-105.0000	0.0003	0.0003	0.0003	0.0003	1.9864	-5.1645
000	0.0000	-110.0000	0.0002	0.0002	0.0002	0.0002	1.3179	3.1631
000	0.0000	-115.0000	0.0001	0.0001	0.0001	0.0001	0.8745	6.4710
000	0.0000	-120.0000	0.0001	0.0001	0.0001	0.0001	0.5802	7.1945
000	0.0000	-125.0000	0.0001	0.0001	0.0001	0.0001	0.3850	6.6983
000	0.0000	-130.0000	0.0000	0.0000	0.0000	0.0000	0.2554	5.7215
000	0.0000	-135.0000	0.0000	0.0000	0.0000	0.0000	0.1695	4.6435
000	0.0000	-140.0000	0.0000	0.0000	0.0000	0.0000	0.1124	3.6432
000	0.0000	-145.0000	0.0000	0.0000	0.0000	0.0000	0.0746	2.7903
000	0.0000	-150.0000	0.0000	0.0000	0.0000	0.0000	0.0495	2.0989
000	0.0000	-155.0000	0.0000	0.0000	0.0000	0.0000	0.0328	1.5568
000	0.0000	-160.0000	0.0000	0.0000	0.0000	0.0000	0.0218	1.1419
000	0.0000	-165.0000	0.0000	0.0000	0.0000	0.0000	0.0145	0.8300
000	0.0000	-170.0000	0.0000	0.0000	0.0000	0.0000	0.0096	0.5986
000	0.0000	-175.0000	0.0000	0.0000	0.0000	0.0000	0.0064	0.4290
000	0.0000	-180.0000	0.0000	0.0000	0.0000	0.0000	0.0042	0.3058
000	0.0000	-185.0000	0.0000	0.0000	0.0000	0.0000	0.0028	0.2169
000	0.0000	-190.0000	0.0000	0.0000	0.0000	0.0000	0.0019	0.1532
000	0.0000	-195.0000	0.0000	0.0000	0.0000	0.0000	0.0012	0.1078
000	0.0000	-200.0000	0.0000	0.0000	0.0000	0.0000	0.0008	0.0756
000	1.2050	0.0000	1.1427	1.1427	1.0252	1.0252	7783.0990	-837461.4556
000	1.2050	-5.0000	0.7582	0.7582	0.6802	0.6802	5132.6224	-526607.0633
000	1.2050	-10.0000	0.5031	0.5031	0.4513	0.4513	3391.6430	-331024.3606
000	1.2050	-15.0000	0.3338	0.3338	0.2995	0.2995	2244.2567	-207818.1684
000	1.2050	-20.0000	0.2215	0.2215	0.1987	0.1987	1486.3793	-130206.8261
000	1.2050	-25.0000	0.1469	0.1469	0.1318	0.1318	985.0305	-81363.5156
000	1.2050	-30.0000	0.0975	0.0975	0.0875	0.0875	653.0471	-50676.4583
000	1.2050	-35.0000	0.0647	0.0647	0.0580	0.0580	433.0675	-31440.7029
000	1.2050	-40.0000	0.0429	0.0429	0.0385	0.0385	287.2393	-19417.3790
000	1.2050	-45.0000	0.0285	0.0285	0.0255	0.0255	190.5388	-11927.7296
000	1.2050	-50.0000	0.0189	0.0189	0.0170	0.0170	126.4029	-7280.8067

sec	x (m)	y (m)	u (m/sec)	v (m/sec)	$\dot{u}$ (m/sec <sup>2</sup> )	$\dot{v}$ (m/sec <sup>2</sup> )	Fx (kN)	M (kN.m)
0000	1.2050	-55.0000	0.0125	0.0125	0.0112	0.0112	83.8597	-4411.0188
0000	1.2050	-60.0000	0.0083	0.0083	0.0075	0.0075	55.6371	-2648.3248
0000	1.2050	-65.0000	0.0055	0.0055	0.0050	0.0050	36.9135	-1572.5157
0000	1.2050	-70.0000	0.0037	0.0037	0.0033	0.0033	24.4914	-920.8755
0000	1.2050	-75.0000	0.0024	0.0024	0.0022	0.0022	16.2497	-529.7399
0000	1.2050	-80.0000	0.0016	0.0016	0.0014	0.0014	10.7815	-297.5700
0000	1.2050	-85.0000	0.0011	0.0011	0.0010	0.0010	7.1535	-161.6685
0000	1.2050	-90.0000	0.0007	0.0007	0.0006	0.0006	4.7463	-83.5349
0000	1.2050	-95.0000	0.0005	0.0005	0.0004	0.0004	3.1492	-39.6794
0000	1.2050	-100.0000	0.0003	0.0003	0.0003	0.0003	2.0895	-15.8799
0000	1.2050	-105.0000	0.0002	0.0002	0.0002	0.0002	1.3864	-3.6045
0000	1.2050	-110.0000	0.0001	0.0001	0.0001	0.0001	0.9198	2.2076
0000	1.2050	-115.0000	0.0001	0.0001	0.0001	0.0001	0.6103	4.5163
0000	1.2050	-120.0000	0.0001	0.0001	0.0001	0.0001	0.4049	5.0213
0000	1.2050	-125.0000	0.0000	0.0000	0.0000	0.0000	0.2687	4.6750
0000	1.2050	-130.0000	0.0000	0.0000	0.0000	0.0000	0.1783	3.9932
0000	1.2050	-135.0000	0.0000	0.0000	0.0000	0.0000	0.1183	3.2409
0000	1.2050	-140.0000	0.0000	0.0000	0.0000	0.0000	0.0785	2.5427
0000	1.2050	-145.0000	0.0000	0.0000	0.0000	0.0000	0.0521	1.9475
0000	1.2050	-150.0000	0.0000	0.0000	0.0000	0.0000	0.0345	1.4649
0000	1.2050	-155.0000	0.0000	0.0000	0.0000	0.0000	0.0229	1.0866
0000	1.2050	-160.0000	0.0000	0.0000	0.0000	0.0000	0.0152	0.7970
0000	1.2050	-165.0000	0.0000	0.0000	0.0000	0.0000	0.0101	0.5793
0000	1.2050	-170.0000	0.0000	0.0000	0.0000	0.0000	0.0067	0.4178
0000	1.2050	-175.0000	0.0000	0.0000	0.0000	0.0000	0.0044	0.2994
0000	1.2050	-180.0000	0.0000	0.0000	0.0000	0.0000	0.0029	0.2134
0000	1.2050	-185.0000	0.0000	0.0000	0.0000	0.0000	0.0020	0.1514
0000	1.2050	-190.0000	0.0000	0.0000	0.0000	0.0000	0.0013	0.1069
0000	1.2050	-195.0000	0.0000	0.0000	0.0000	0.0000	0.0009	0.0753
0000	1.2050	-200.0000	0.0000	0.0000	0.0000	0.0000	0.0006	0.0528
0000	0.5313	0.0000	-0.2930	-0.2930	-0.2628	-0.2628	-1968.4500	211805.2177
0000	0.5313	-5.0000	-0.1944	-0.1944	-0.1744	-0.1744	-1303.9981	133790.2099
0000	0.5313	-10.0000	-0.1290	-0.1290	-0.1157	-0.1157	-864.2920	84354.9024
0000	0.5313	-15.0000	-0.0856	-0.0856	-0.0768	-0.0768	-573.0567	53065.0526
0000	0.5313	-20.0000	-0.0568	-0.0568	-0.0509	-0.0509	-380.0465	33292.0748
0000	0.5313	-25.0000	-0.0377	-0.0377	-0.0338	-0.0338	-252.0831	20822.0611
0000	0.5313	-30.0000	-0.0250	-0.0250	-0.0224	-0.0224	-167.2228	12976.4918
0000	0.5313	-35.0000	-0.0166	-0.0166	-0.0149	-0.0149	-110.9372	8054.0435
0000	0.5313	-40.0000	-0.0110	-0.0110	-0.0099	-0.0099	-73.6002	4975.3730
0000	0.5313	-45.0000	-0.0073	-0.0073	-0.0065	-0.0065	-48.8308	3056.8071
0000	0.5313	-50.0000	-0.0048	-0.0048	-0.0043	-0.0043	-32.3979	1866.1204
0000	0.5313	-55.0000	-0.0032	-0.0032	-0.0029	-0.0029	-21.4954	1130.6604
0000	0.5313	-60.0000	-0.0021	-0.0021	-0.0019	-0.0019	-14.2620	678.8698
0000	0.5313	-65.0000	-0.0014	-0.0014	-0.0013	-0.0013	-9.4627	403.1112
0000	0.5313	-70.0000	-0.0009	-0.0009	-0.0008	-0.0008	-6.2785	236.0698
0000	0.5313	-75.0000	-0.0006	-0.0006	-0.0006	-0.0006	-4.1657	135.8028
0000	0.5313	-80.0000	-0.0004	-0.0004	-0.0004	-0.0004	-2.7640	76.2850
0000	0.5313	-85.0000	-0.0003	-0.0003	-0.0002	-0.0002	-1.8339	41.4456
0000	0.5313	-90.0000	-0.0002	-0.0002	-0.0002	-0.0002	-1.2168	21.4152
0000	0.5313	-95.0000	-0.0001	-0.0001	-0.0001	-0.0001	-0.8073	10.1723
0000	0.5313	-100.0000	-0.0001	-0.0001	-0.0001	-0.0001	-0.5357	4.0710
0000	0.5313	-105.0000	-0.0001	-0.0001	0.0000	0.0000	-0.3554	0.9241

sec	x (m)	y (m)	u (m/sec)	v (m/sec)	$\dot{u}$ (m/sec <sup>2</sup> )	$\dot{v}$ (m/sec <sup>2</sup> )	Fx (kN)	M (kN.m)
0000	0.5313	-110.0000	0.0000	0.0000	0.0000	0.0000	-0.2358	-0.5660
0000	0.5313	-115.0000	0.0000	0.0000	0.0000	0.0000	-0.1565	-1.1578
0000	0.5313	-120.0000	0.0000	0.0000	0.0000	0.0000	-0.1038	-1.2873
0000	0.5313	-125.0000	0.0000	0.0000	0.0000	0.0000	-0.0689	-1.1985
0000	0.5313	-130.0000	0.0000	0.0000	0.0000	0.0000	-0.0457	-1.0237
0000	0.5313	-135.0000	0.0000	0.0000	0.0000	0.0000	-0.0303	-0.8309
0000	0.5313	-140.0000	0.0000	0.0000	0.0000	0.0000	-0.0201	-0.6519
0000	0.5313	-145.0000	0.0000	0.0000	0.0000	0.0000	-0.0133	-0.4993
0000	0.5313	-150.0000	0.0000	0.0000	0.0000	0.0000	-0.0089	-0.3755
0000	0.5313	-155.0000	0.0000	0.0000	0.0000	0.0000	-0.0059	-0.2786
0000	0.5313	-160.0000	0.0000	0.0000	0.0000	0.0000	-0.0039	-0.2043
0000	0.5313	-165.0000	0.0000	0.0000	0.0000	0.0000	-0.0026	-0.1485
0000	0.5313	-170.0000	0.0000	0.0000	0.0000	0.0000	-0.0017	-0.1071
0000	0.5313	-175.0000	0.0000	0.0000	0.0000	0.0000	-0.0011	-0.0768
0000	0.5313	-180.0000	0.0000	0.0000	0.0000	0.0000	-0.0008	-0.0547
0000	0.5313	-185.0000	0.0000	0.0000	0.0000	0.0000	-0.0005	-0.0388
0000	0.5313	-190.0000	0.0000	0.0000	0.0000	0.0000	-0.0003	-0.0274
0000	0.5313	-195.0000	0.0000	0.0000	0.0000	0.0000	-0.0002	-0.0193
0000	0.5313	-200.0000	0.0000	0.0000	0.0000	0.0000	-0.0001	-0.0135
0000	0.0443	0.0000	-1.4716	-1.4716	-1.3202	-1.3202	-10075.1106	1084081.8959
0000	0.0443	-5.0000	-0.9764	-0.9764	-0.8760	-0.8760	-6632.6584	680510.7511
0000	0.0443	-10.0000	-0.6478	-0.6478	-0.5812	-0.5812	-4377.7979	427273.0731
0000	0.0443	-15.0000	-0.4298	-0.4298	-0.3856	-0.3856	-2894.5558	268035.8692
0000	0.0443	-20.0000	-0.2852	-0.2852	-0.2559	-0.2559	-1916.0853	167849.0764
0000	0.0443	-25.0000	-0.1892	-0.1892	-0.1698	-0.1698	-1269.3623	104849.3252
0000	0.0443	-30.0000	-0.1256	-0.1256	-0.1126	-0.1126	-841.3587	65289.4371
0000	0.0443	-35.0000	-0.0833	-0.0833	-0.0747	-0.0747	-557.8614	40500.7394
0000	0.0443	-40.0000	-0.0553	-0.0553	-0.0496	-0.0496	-369.9737	25010.2195
0000	0.0443	-45.0000	-0.0367	-0.0367	-0.0329	-0.0329	-245.4038	15362.2770
0000	0.0443	-50.0000	-0.0243	-0.0243	-0.0218	-0.0218	-162.7929	9376.8723
0000	0.0443	-55.0000	-0.0161	-0.0161	-0.0145	-0.0145	-107.9988	5680.7358
0000	0.0443	-60.0000	-0.0107	-0.0107	-0.0096	-0.0096	-71.6509	3410.5815
0000	0.0443	-65.0000	-0.0071	-0.0071	-0.0064	-0.0064	-47.5376	2025.1002
0000	0.0443	-70.0000	-0.0047	-0.0047	-0.0042	-0.0042	-31.5399	1185.9017
0000	0.0443	-75.0000	-0.0031	-0.0031	-0.0028	-0.0028	-20.9262	682.1942
0000	0.0443	-80.0000	-0.0021	-0.0021	-0.0019	-0.0019	-13.8843	383.2065
0000	0.0443	-85.0000	-0.0014	-0.0014	-0.0012	-0.0012	-9.2121	208.1939
0000	0.0443	-90.0000	-0.0009	-0.0009	-0.0008	-0.0008	-6.1122	107.5746
0000	0.0443	-95.0000	-0.0006	-0.0006	-0.0005	-0.0005	-4.0554	51.0983
0000	0.0443	-100.0000	-0.0004	-0.0004	-0.0004	-0.0004	-2.6908	20.4498
0000	0.0443	-105.0000	-0.0003	-0.0003	-0.0002	-0.0002	-1.7853	4.6418
0000	0.0443	-110.0000	-0.0002	-0.0002	-0.0002	-0.0002	-1.1846	-2.8429
0000	0.0443	-115.0000	-0.0001	-0.0001	-0.0001	-0.0001	-0.7859	-5.8160
0000	0.0443	-120.0000	-0.0001	-0.0001	-0.0001	-0.0001	-0.5215	-6.4663
0000	0.0443	-125.0000	-0.0001	-0.0001	0.0000	0.0000	-0.3460	-6.0204
0000	0.0443	-130.0000	0.0000	0.0000	0.0000	0.0000	-0.2296	-5.1424
0000	0.0443	-135.0000	0.0000	0.0000	0.0000	0.0000	-0.1523	-4.1736
0000	0.0443	-140.0000	0.0000	0.0000	0.0000	0.0000	-0.1011	-3.2745
0000	0.0443	-145.0000	0.0000	0.0000	0.0000	0.0000	-0.0671	-2.5079
0000	0.0443	-150.0000	0.0000	0.0000	0.0000	0.0000	-0.0445	-1.8864
0000	0.0443	-155.0000	0.0000	0.0000	0.0000	0.0000	-0.0295	-1.3993
0000	0.0443	-160.0000	0.0000	0.0000	0.0000	0.0000	-0.0196	-1.0263

sec)	x (m)	y (m)	u (m/sec)	v (m/sec)	ü (m/sec^2)	v̇ (m/sec^2)	Fx (kN)	M (kN.m)
0000	0.0443	-165.0000	0.0000	0.0000	0.0000	0.0000	-0.0130	-0.7460
0000	0.0443	-170.0000	0.0000	0.0000	0.0000	0.0000	-0.0086	-0.5381
0000	0.0443	-175.0000	0.0000	0.0000	0.0000	0.0000	-0.0057	-0.3856
0000	0.0443	-180.0000	0.0000	0.0000	0.0000	0.0000	-0.0038	-0.2748
0000	0.0443	-185.0000	0.0000	0.0000	0.0000	0.0000	-0.0025	-0.1949
0000	0.0443	-190.0000	0.0000	0.0000	0.0000	0.0000	-0.0017	-0.1377
0000	0.0443	-195.0000	0.0000	0.0000	0.0000	0.0000	-0.0011	-0.0969
0000	0.0443	-200.0000	0.0000	0.0000	0.0000	0.0000	-0.0007	-0.0680
0000	0.9670	0.0000	-1.5279	-1.5279	-1.3707	-1.3707	-10469.8712	1126558.1428
0000	0.9670	-5.0000	-1.0137	-1.0137	-0.9095	-0.9095	-6890.5132	706966.6531
0000	0.9670	-10.0000	-0.6726	-0.6726	-0.6034	-0.6034	-4547.0932	443796.3009
0000	0.9670	-15.0000	-0.4463	-0.4463	-0.4004	-0.4004	-3006.0947	278364.3688
0000	0.9670	-20.0000	-0.2961	-0.2961	-0.2657	-0.2657	-1989.7442	174301.5926
0000	0.9670	-25.0000	-0.1965	-0.1965	-0.1763	-0.1763	-1318.0821	108873.5812
0000	0.9670	-30.0000	-0.1304	-0.1304	-0.1169	-0.1169	-873.6170	67792.6808
0000	0.9670	-35.0000	-0.0865	-0.0865	-0.0776	-0.0776	-579.2352	42052.4737
0000	0.9670	-40.0000	-0.0574	-0.0574	-0.0515	-0.0515	-384.1421	25968.0058
0000	0.9670	-45.0000	-0.0381	-0.0381	-0.0342	-0.0342	-254.7988	15950.4049
0000	0.9670	-50.0000	-0.0253	-0.0253	-0.0227	-0.0227	-169.0240	9735.7814
0000	0.9670	-55.0000	-0.0168	-0.0168	-0.0150	-0.0150	-112.1320	5898.1419
0000	0.9670	-60.0000	-0.0111	-0.0111	-0.0100	-0.0100	-74.3928	3541.0951
0000	0.9670	-65.0000	-0.0074	-0.0074	-0.0066	-0.0066	-49.3566	2102.5906
0000	0.9670	-70.0000	-0.0049	-0.0049	-0.0044	-0.0044	-32.7468	1231.2784
0000	0.9670	-75.0000	-0.0032	-0.0032	-0.0029	-0.0029	-21.7269	708.2966
0000	0.9670	-80.0000	-0.0022	-0.0022	-0.0019	-0.0019	-14.4155	397.8686
0000	0.9670	-85.0000	-0.0014	-0.0014	-0.0013	-0.0013	-9.5646	216.1597
0000	0.9670	-90.0000	-0.0009	-0.0009	-0.0009	-0.0009	-6.3461	111.6905
0000	0.9670	-95.0000	-0.0006	-0.0006	-0.0006	-0.0006	-4.2106	53.0533
0000	0.9670	-100.0000	-0.0004	-0.0004	-0.0004	-0.0004	-2.7937	21.2322
0000	0.9670	-105.0000	-0.0003	-0.0003	-0.0002	-0.0002	-1.8536	4.8194
0000	0.9670	-110.0000	-0.0002	-0.0002	-0.0002	-0.0002	-1.2299	-2.9517
0000	0.9670	-115.0000	-0.0001	-0.0001	-0.0001	-0.0001	-0.8160	-6.0386
0000	0.9670	-120.0000	-0.0001	-0.0001	-0.0001	-0.0001	-0.5414	-6.7137
0000	0.9670	-125.0000	-0.0001	-0.0001	0.0000	0.0000	-0.3592	-6.2507
0000	0.9670	-130.0000	0.0000	0.0000	0.0000	0.0000	-0.2384	-5.3391
0000	0.9670	-135.0000	0.0000	0.0000	0.0000	0.0000	-0.1581	-4.3333
0000	0.9670	-140.0000	0.0000	0.0000	0.0000	0.0000	-0.1049	-3.3998
0000	0.9670	-145.0000	0.0000	0.0000	0.0000	0.0000	-0.0696	-2.6039
0000	0.9670	-150.0000	0.0000	0.0000	0.0000	0.0000	-0.0462	-1.9586
0000	0.9670	-155.0000	0.0000	0.0000	0.0000	0.0000	-0.0306	-1.4528
0000	0.9670	-160.0000	0.0000	0.0000	0.0000	0.0000	-0.0203	-1.0656
0000	0.9670	-165.0000	0.0000	0.0000	0.0000	0.0000	-0.0135	-0.7745
0000	0.9670	-170.0000	0.0000	0.0000	0.0000	0.0000	-0.0090	-0.5586
0000	0.9670	-175.0000	0.0000	0.0000	0.0000	0.0000	-0.0059	-0.4004
0000	0.9670	-180.0000	0.0000	0.0000	0.0000	0.0000	-0.0039	-0.2853
0000	0.9670	-185.0000	0.0000	0.0000	0.0000	0.0000	-0.0026	-0.2024
0000	0.9670	-190.0000	0.0000	0.0000	0.0000	0.0000	-0.0017	-0.1430
0000	0.9670	-195.0000	0.0000	0.0000	0.0000	0.0000	-0.0012	-0.1006
0000	0.9670	-200.0000	0.0000	0.0000	0.0000	0.0000	-0.0008	-0.0706
0000	1.0197	0.0000	-0.5000	-0.5000	-0.4486	-0.4486	-3370.7877	362696.7605
0000	1.0197	-5.0000	-0.3317	-0.3317	-0.2976	-0.2976	-2230.4934	228848.6183
0000	1.0197	-10.0000	-0.2201	-0.2201	-0.1975	-0.1975	-1477.2800	144182.5263

sec	x (m)	y (m)	u (m/sec)	v (m/sec)	ü (m/sec <sup>2</sup> )	v̇ (m/sec <sup>2</sup> )	Fx (kN)	M (kN.m)
0000	1.0197	-15.0000	-0.1460	-0.1460	-0.1310	-0.1310	-979.0074	90656.0888
0000	1.0197	-20.0000	-0.0969	-0.0969	-0.0869	-0.0869	-649.0572	56857.4108
0000	1.0197	-25.0000	-0.0643	-0.0643	-0.0577	-0.0577	-430.4230	35552.9392
0000	1.0197	-30.0000	-0.0427	-0.0427	-0.0383	-0.0383	-285.4859	22153.7065
0000	1.0197	-35.0000	-0.0283	-0.0283	-0.0254	-0.0254	-189.3760	13748.6943
0000	1.0197	-40.0000	-0.0188	-0.0188	-0.0168	-0.0168	-125.6316	8492.6946
0000	1.0197	-45.0000	-0.0125	-0.0125	-0.0112	-0.0112	-83.3480	5217.5854
0000	1.0197	-50.0000	-0.0083	-0.0083	-0.0074	-0.0074	-55.2976	3185.1437
0000	1.0197	-55.0000	-0.0055	-0.0055	-0.0049	-0.0049	-36.6883	1929.8053
0000	1.0197	-60.0000	-0.0036	-0.0036	-0.0033	-0.0033	-24.3420	1158.6772
0000	1.0197	-65.0000	-0.0024	-0.0024	-0.0022	-0.0022	-16.1506	688.0139
0000	1.0197	-70.0000	-0.0016	-0.0016	-0.0014	-0.0014	-10.7158	402.9122
0000	1.0197	-75.0000	-0.0011	-0.0011	-0.0010	-0.0010	-7.1098	231.7806
0000	1.0197	-80.0000	-0.0007	-0.0007	-0.0006	-0.0006	-4.7173	130.1987
0000	1.0197	-85.0000	-0.0005	-0.0005	-0.0004	-0.0004	-3.1299	70.7368
0000	1.0197	-90.0000	-0.0003	-0.0003	-0.0003	-0.0003	-2.0767	36.5501
0000	1.0197	-95.0000	-0.0002	-0.0002	-0.0002	-0.0002	-1.3779	17.3615
0000	1.0197	-100.0000	-0.0001	-0.0001	-0.0001	-0.0001	-0.9142	6.9482
0000	1.0197	-105.0000	-0.0001	-0.0001	-0.0001	-0.0001	-0.6066	1.5771
0000	1.0197	-110.0000	-0.0001	-0.0001	-0.0001	-0.0001	-0.4025	-0.9659
0000	1.0197	-115.0000	0.0000	0.0000	0.0000	0.0000	-0.2670	-1.9761
0000	1.0197	-120.0000	0.0000	0.0000	0.0000	0.0000	-0.1772	-2.1971
0000	1.0197	-125.0000	0.0000	0.0000	0.0000	0.0000	-0.1176	-2.0455
0000	1.0197	-130.0000	0.0000	0.0000	0.0000	0.0000	-0.0780	-1.7472
0000	1.0197	-135.0000	0.0000	0.0000	0.0000	0.0000	-0.0518	-1.4180
0000	1.0197	-140.0000	0.0000	0.0000	0.0000	0.0000	-0.0343	-1.1126
0000	1.0197	-145.0000	0.0000	0.0000	0.0000	0.0000	-0.0228	-0.8521
0000	1.0197	-150.0000	0.0000	0.0000	0.0000	0.0000	-0.0151	-0.6410
0000	1.0197	-155.0000	0.0000	0.0000	0.0000	0.0000	-0.0100	-0.4754
0000	1.0197	-160.0000	0.0000	0.0000	0.0000	0.0000	-0.0067	-0.3487
0000	1.0197	-165.0000	0.0000	0.0000	0.0000	0.0000	-0.0044	-0.2535
0000	1.0197	-170.0000	0.0000	0.0000	0.0000	0.0000	-0.0029	-0.1828
0000	1.0197	-175.0000	0.0000	0.0000	0.0000	0.0000	-0.0019	-0.1310
0000	1.0197	-180.0000	0.0000	0.0000	0.0000	0.0000	-0.0013	-0.0934
0000	1.0197	-185.0000	0.0000	0.0000	0.0000	0.0000	-0.0009	-0.0662
0000	1.0197	-190.0000	0.0000	0.0000	0.0000	0.0000	-0.0006	-0.0468
0000	1.0197	-195.0000	0.0000	0.0000	0.0000	0.0000	-0.0004	-0.0329
0000	1.0197	-200.0000	0.0000	0.0000	0.0000	0.0000	-0.0002	-0.0231
0000	0.0868	0.0000	1.0082	1.0082	0.9045	0.9045	6852.0294	-737278.3603
0000	0.0868	-5.0000	0.6689	0.6689	0.6001	0.6001	4521.8307	-463939.8310
0000	0.0868	-10.0000	0.4438	0.4438	0.3982	0.3982	2989.4525	-291770.5659
0000	0.0868	-15.0000	0.2945	0.2945	0.2642	0.2642	1978.7547	-183232.6890
0000	0.0868	-20.0000	0.1954	0.1954	0.1753	0.1753	1310.8137	-114827.2845
0000	0.0868	-25.0000	0.1296	0.1296	0.1163	0.1163	868.8047	-71763.2659
0000	0.0868	-30.0000	0.0860	0.0860	0.0772	0.0772	576.0467	-44701.2217
0000	0.0868	-35.0000	0.0571	0.0571	0.0512	0.0512	382.0285	-27735.2693
0000	0.0868	-40.0000	0.0379	0.0379	0.0340	0.0340	253.3973	-17129.6575
0000	0.0868	-45.0000	0.0251	0.0251	0.0225	0.0225	168.0945	-10522.7139
0000	0.0868	-50.0000	0.0167	0.0167	0.0150	0.0150	111.5154	-6423.2878
0000	0.0868	-55.0000	0.0111	0.0111	0.0099	0.0099	73.9837	-3891.5448
0000	0.0868	-60.0000	0.0073	0.0073	0.0066	0.0066	49.0852	-2336.4573
0000	0.0868	-65.0000	0.0049	0.0049	0.0044	0.0044	32.5667	-1387.3431

sec	x (m)	y (m)	u (m/sec)	v (m/sec)	$\dot{u}$ (m/sec <sup>2</sup> )	$\dot{v}$ (m/sec <sup>2</sup> )	Fx (kN)	M (kN.m)
0000	0.0868	-70.0000	0.0032	0.0032	0.0029	0.0029	21.6074	-812.4401
0000	0.0868	-75.0000	0.0021	0.0021	0.0019	0.0019	14.3363	-467.3628
0000	0.0868	-80.0000	0.0014	0.0014	0.0013	0.0013	9.5120	-262.5314
0000	0.0868	-85.0000	0.0009	0.0009	0.0008	0.0008	6.3112	-142.6323
0000	0.0868	-90.0000	0.0006	0.0006	0.0006	0.0006	4.1874	-73.6988
0000	0.0868	-95.0000	0.0004	0.0004	0.0004	0.0004	2.7784	-35.0072
0000	0.0868	-100.0000	0.0003	0.0003	0.0002	0.0002	1.8434	-14.0101
0000	0.0868	-105.0000	0.0002	0.0002	0.0002	0.0002	1.2231	-3.1801
0000	0.0868	-110.0000	0.0001	0.0001	0.0001	0.0001	0.8115	1.9477
0000	0.0868	-115.0000	0.0001	0.0001	0.0001	0.0001	0.5385	3.9846
0000	0.0868	-120.0000	0.0001	0.0001	0.0000	0.0000	0.3573	4.4301
0000	0.0868	-125.0000	0.0000	0.0000	0.0000	0.0000	0.2370	4.1246
0000	0.0868	-130.0000	0.0000	0.0000	0.0000	0.0000	0.1573	3.5230
0000	0.0868	-135.0000	0.0000	0.0000	0.0000	0.0000	0.1044	2.8593
0000	0.0868	-140.0000	0.0000	0.0000	0.0000	0.0000	0.0692	2.2433
0000	0.0868	-145.0000	0.0000	0.0000	0.0000	0.0000	0.0459	1.7182
0000	0.0868	-150.0000	0.0000	0.0000	0.0000	0.0000	0.0305	1.2924
0000	0.0868	-155.0000	0.0000	0.0000	0.0000	0.0000	0.0202	0.9586
0000	0.0868	-160.0000	0.0000	0.0000	0.0000	0.0000	0.0134	0.7031
0000	0.0868	-165.0000	0.0000	0.0000	0.0000	0.0000	0.0089	0.5111
0000	0.0868	-170.0000	0.0000	0.0000	0.0000	0.0000	0.0059	0.3686
0000	0.0868	-175.0000	0.0000	0.0000	0.0000	0.0000	0.0039	0.2642
0000	0.0868	-180.0000	0.0000	0.0000	0.0000	0.0000	0.0026	0.1883
0000	0.0868	-185.0000	0.0000	0.0000	0.0000	0.0000	0.0017	0.1336
0000	0.0868	-190.0000	0.0000	0.0000	0.0000	0.0000	0.0011	0.0943
0000	0.0868	-195.0000	0.0000	0.0000	0.0000	0.0000	0.0008	0.0664
0000	0.0868	-200.0000	0.0000	0.0000	0.0000	0.0000	0.0005	0.0466
0000	0.4502	0.0000	1.6360	1.6360	1.4677	1.4677	11229.6450	-1208309.8036
0000	0.4502	-5.0000	1.0855	1.0855	0.9738	0.9738	7386.3836	-757842.9542
0000	0.4502	-10.0000	0.7202	0.7202	0.6461	0.6461	4872.4759	-475553.6460
0000	0.4502	-15.0000	0.4779	0.4779	0.4287	0.4287	3220.3892	-298208.0357
0000	0.4502	-20.0000	0.3171	0.3171	0.2844	0.2844	2131.2255	-186695.3574
0000	0.4502	-25.0000	0.2104	0.2104	0.1887	0.1887	1411.6454	-118601.9137
0000	0.4502	-30.0000	0.1396	0.1396	0.1252	0.1252	935.5600	-72599.4584
0000	0.4502	-35.0000	0.0926	0.0926	0.0831	0.0831	620.2744	-45031.9188
0000	0.4502	-40.0000	0.0614	0.0614	0.0551	0.0551	411.3452	-27806.9343
0000	0.4502	-45.0000	0.0408	0.0408	0.0366	0.0366	272.8364	-17079.5594
0000	0.4502	-50.0000	0.0271	0.0271	0.0243	0.0243	180.9868	-10424.8405
0000	0.4502	-55.0000	0.0179	0.0179	0.0161	0.0161	120.0671	-6315.5272
0000	0.4502	-60.0000	0.0119	0.0119	0.0107	0.0107	79.6567	-3791.6583
0000	0.4502	-65.0000	0.0079	0.0079	0.0071	0.0071	52.8488	-2251.3575
0000	0.4502	-70.0000	0.0052	0.0052	0.0047	0.0047	35.0636	-1318.3926
0000	0.4502	-75.0000	0.0035	0.0035	0.0031	0.0031	23.2640	-758.4079
0000	0.4502	-80.0000	0.0023	0.0023	0.0021	0.0021	15.4354	-426.0169
0000	0.4502	-85.0000	0.0015	0.0015	0.0014	0.0014	10.2413	-231.4523
0000	0.4502	-90.0000	0.0010	0.0010	0.0009	0.0009	6.7950	-119.5922
0000	0.4502	-95.0000	0.0007	0.0007	0.0006	0.0006	4.5085	-56.8066
0000	0.4502	-100.0000	0.0004	0.0004	0.0004	0.0004	2.9914	-22.7343
0000	0.4502	-105.0000	0.0003	0.0003	0.0003	0.0003	1.9848	-5.1604
0000	0.4502	-110.0000	0.0002	0.0002	0.0002	0.0002	1.3169	3.1605
0000	0.4502	-115.0000	0.0001	0.0001	0.0001	0.0001	0.8738	6.4658
0000	0.4502	-120.0000	0.0001	0.0001	0.0001	0.0001	0.5797	7.1887



sec)	x (m)	y (m)	u (m/sec)	v (m/sec)	$\dot{u}$ (m/sec <sup>2</sup> )	$\dot{v}$ (m/sec <sup>2</sup> )	Fx (kN)	M (kN.m)
0000	0.4502	-125.0000	0.0001	0.0001	0.0001	0.0001	0.3847	6.6929
0000	0.4502	-130.0000	0.0000	0.0000	0.0000	0.0000	0.2552	5.7168
0000	0.4502	-135.0000	0.0000	0.0000	0.0000	0.0000	0.1693	4.6398
0000	0.4502	-140.0000	0.0000	0.0000	0.0000	0.0000	0.1124	3.6403
0000	0.4502	-145.0000	0.0000	0.0000	0.0000	0.0000	0.0745	2.7881
0000	0.4502	-150.0000	0.0000	0.0000	0.0000	0.0000	0.0495	2.0972
0000	0.4502	-155.0000	0.0000	0.0000	0.0000	0.0000	0.0328	1.5556
0000	0.4502	-160.0000	0.0000	0.0000	0.0000	0.0000	0.0218	1.1410
0000	0.4502	-165.0000	0.0000	0.0000	0.0000	0.0000	0.0144	0.8293
0000	0.4502	-170.0000	0.0000	0.0000	0.0000	0.0000	0.0096	0.5982
0000	0.4502	-175.0000	0.0000	0.0000	0.0000	0.0000	0.0064	0.4287
0000	0.4502	-180.0000	0.0000	0.0000	0.0000	0.0000	0.0042	0.3055
0000	0.4502	-185.0000	0.0000	0.0000	0.0000	0.0000	0.0028	0.2167
0000	0.4502	-190.0000	0.0000	0.0000	0.0000	0.0000	0.0019	0.1531
0000	0.4502	-195.0000	0.0000	0.0000	0.0000	0.0000	0.0012	0.1077
0000	0.4502	-200.0000	0.0000	0.0000	0.0000	0.0000	0.0008	0.0756

**APPENDIX E**  
**PROFILES ( $\eta$ )**

**E-1**

**WAVE PROFILE**

H (m)	T(sec)	t (sec)	c (m/sec)	k	$\eta$ (m)
3.65	7	0	10.93471	0.082045	1.825
3.65	7	1	10.93471	0.082045	1.138518
3.65	7	2	10.93471	0.082045	-0.40448
3.65	7	3	10.93471	0.082045	-1.64319
3.65	7	4	10.93471	0.082045	-1.64571
3.65	7	5	10.93471	0.082045	-0.41015
3.65	7	6	10.93471	0.082045	1.133969
3.65	7	7	10.93471	0.082045	1.824991
3.65	7	8	10.93471	0.082045	1.143056
3.65	7	9	10.93471	0.082045	-0.39881
3.65	7	10	10.93471	0.082045	-1.64065
3.65	7	11	10.93471	0.082045	-1.64821
3.65	7	12	10.93471	0.082045	-0.41581
3.65	7	13	10.93471	0.082045	1.129408
3.65	7	14	10.93471	0.082045	1.824963
3.65	7	15	10.93471	0.082045	1.147581
3.65	7	16	10.93471	0.082045	-0.39314
3.65	7	17	10.93471	0.082045	-1.63809
3.65	7	18	10.93471	0.082045	-1.6507
3.65	7	19	10.93471	0.082045	-0.42147
3.65	7	20	10.93471	0.082045	1.124836
3.65	7	21	10.93471	0.082045	1.824917
3.65	7	22	10.93471	0.082045	1.152096
3.65	7	23	10.93471	0.082045	-0.38746
3.65	7	24	10.93471	0.082045	-1.63552
3.65	7	25	10.93471	0.082045	-1.65317
3.65	7	26	10.93471	0.082045	-0.42712
3.65	7	27	10.93471	0.082045	1.120253
3.65	7	28	10.93471	0.082045	1.824852
3.65	7	29	10.93471	0.082045	1.156598
3.65	7	30	10.93471	0.082045	-0.38177
3.65	7	31	10.93471	0.082045	-1.63294
3.65	7	32	10.93471	0.082045	-1.65562
3.65	7	33	10.93471	0.082045	-0.43277
3.65	7	34	10.93471	0.082045	1.115658
3.65	7	35	10.93471	0.082045	1.824769
3.65	7	36	10.93471	0.082045	1.161089
3.65	7	37	10.93471	0.082045	-0.37609
3.65	7	38	10.93471	0.082045	-1.63033
3.65	7	39	10.93471	0.082045	-1.65806
3.65	7	40	10.93471	0.082045	-0.43842
3.65	7	41	10.93471	0.082045	1.111052
3.65	7	42	10.93471	0.082045	1.824667
3.65	7	43	10.93471	0.082045	1.165568
3.65	7	44	10.93471	0.082045	-0.3704
3.65	7	45	10.93471	0.082045	-1.62771
3.65	7	46	10.93471	0.082045	-1.66048
3.65	7	47	10.93471	0.082045	-0.44406
3.65	7	48	10.93471	0.082045	1.106435
3.65	7	49	10.93471	0.082045	1.824546

H (m)	T(sec)	t (sec)	c (m/sec)	k	η (m)
3.65	7	50	10.93471	0.082045	1.170035
3.65	7	51	10.93471	0.082045	-0.3647
3.65	7	52	10.93471	0.082045	-1.62507
3.65	7	53	10.93471	0.082045	-1.66289
3.65	7	54	10.93471	0.082045	-0.44969
3.65	7	55	10.93471	0.082045	1.101806
3.65	7	56	10.93471	0.082045	1.824407
3.65	7	57	10.93471	0.082045	1.174491
3.65	7	58	10.93471	0.082045	-0.35901
3.65	7	59	10.93471	0.082045	-1.62242
3.65	7	60	10.93471	0.082045	-1.66527
3.65	7	61	10.93471	0.082045	-0.45533
3.65	7	62	10.93471	0.082045	1.097166
3.65	7	63	10.93471	0.082045	1.82425
3.65	7	64	10.93471	0.082045	1.178934
3.65	7	65	10.93471	0.082045	-0.3533
3.65	7	66	10.93471	0.082045	-1.61975
3.65	7	67	10.93471	0.082045	-1.66764
3.65	7	68	10.93471	0.082045	-0.46095
3.65	7	69	10.93471	0.082045	1.092515
3.65	7	70	10.93471	0.082045	1.824074
3.65	7	71	10.93471	0.082045	1.183366
3.65	7	72	10.93471	0.082045	-0.3476
3.65	7	73	10.93471	0.082045	-1.61706
3.65	7	74	10.93471	0.082045	-1.66999
3.65	7	75	10.93471	0.082045	-0.46657
3.65	7	76	10.93471	0.082045	1.087853
3.65	7	77	10.93471	0.082045	1.82388
3.65	7	78	10.93471	0.082045	1.187785
3.65	7	79	10.93471	0.082045	-0.34189
3.65	7	80	10.93471	0.082045	-1.61436
3.65	7	81	10.93471	0.082045	-1.67233
3.65	7	82	10.93471	0.082045	-0.47219
3.65	7	83	10.93471	0.082045	1.08318
3.65	7	84	10.93471	0.082045	1.823667
3.65	7	85	10.93471	0.082045	1.192193
3.65	7	86	10.93471	0.082045	-0.33618
3.65	7	87	10.93471	0.082045	-1.61164
3.65	7	88	10.93471	0.082045	-1.67465
3.65	7	89	10.93471	0.082045	-0.47781
3.65	7	90	10.93471	0.082045	1.078496
3.65	7	91	10.93471	0.082045	1.823436
3.65	7	92	10.93471	0.082045	1.196588
3.65	7	93	10.93471	0.082045	-0.33046
3.65	7	94	10.93471	0.082045	-1.6089
3.65	7	95	10.93471	0.082045	-1.67695
3.65	7	96	10.93471	0.082045	-0.48341
3.65	7	97	10.93471	0.082045	1.073801
3.65	7	98	10.93471	0.082045	1.823186
3.65	7	99	10.93471	0.082045	1.200971

H (m)	T(sec)	t (sec)	c (m/sec)	k	$\eta$ (m)
3.65	7	100	10.93471	0.082045	-0.32474
3.65	7	101	10.93471	0.082045	-1.60615
3.65	7	102	10.93471	0.082045	-1.67924
3.65	7	103	10.93471	0.082045	-0.48902
3.65	7	104	10.93471	0.082045	1.069095
3.65	7	105	10.93471	0.082045	1.822917
3.65	7	106	10.93471	0.082045	1.205342
3.65	7	107	10.93471	0.082045	-0.31902
3.65	7	108	10.93471	0.082045	-1.60338
3.65	7	109	10.93471	0.082045	-1.6815
3.65	7	110	10.93471	0.082045	-0.49461
3.65	7	111	10.93471	0.082045	1.064379
3.65	7	112	10.93471	0.082045	1.82263
3.65	7	113	10.93471	0.082045	1.209701
3.65	7	114	10.93471	0.082045	-0.3133
3.65	7	115	10.93471	0.082045	-1.6006
3.65	7	116	10.93471	0.082045	-1.68376
3.65	7	117	10.93471	0.082045	-0.50021
3.65	7	118	10.93471	0.082045	1.059651
3.65	7	119	10.93471	0.082045	1.822325
3.65	7	120	10.93471	0.082045	1.214047

**E-2**

**SURGE, HEAVE, & PITCH RESPONSES**

t (sec)	RAO11	X (m)	$\eta_x$ (m)	RAO22	Y (m)	$\eta_y$ (m)	RAO33	Z(rad)	$\eta_z$ (rad)
0	0.0660	0.1205	1.2050	0.0000	0.0000	0.0000	-0.1258	-0.2295	-0.0398
1	0.0459	0.0837	0.5313	0.0328	0.0598	0.0435	-0.0873	-0.1594	-0.0196
2	-0.0117	-0.0213	0.0443	-0.0116	-0.0212	0.0017	0.0222	0.0405	-0.0008
3	-0.0593	-0.1081	0.9670	-0.0473	-0.0863	0.0716	0.1129	0.2060	-0.0304
4	-0.0616	-0.1123	1.0197	-0.0473	-0.0864	0.0824	0.1172	0.2140	-0.0352
5	-0.0199	-0.0364	0.0868	-0.0118	-0.0215	0.0078	0.0380	0.0693	-0.0040
6	0.0404	0.0738	0.4502	0.0326	0.0595	0.0300	-0.0770	-0.1404	-0.0130
7	0.0660	0.1204	1.2039	0.0525	0.0958	0.0948	-0.1257	-0.2294	-0.0398
8	0.0460	0.0840	0.5351	0.0329	0.0600	0.0438	-0.0876	-0.1599	-0.0198
9	-0.0114	-0.0209	0.0428	-0.0115	-0.0209	0.0016	0.0218	0.0397	-0.0008
10	-0.0592	-0.1080	0.9640	-0.0472	-0.0861	0.0713	0.1127	0.2057	-0.0303
11	-0.0616	-0.1125	1.0224	-0.0474	-0.0865	0.0826	0.1174	0.2142	-0.0353
12	-0.0202	-0.0368	0.0888	-0.0120	-0.0218	0.0079	0.0384	0.0700	-0.0041
13	0.0402	0.0734	0.4464	0.0325	0.0593	0.0297	-0.0766	-0.1398	-0.0129
14	0.0660	0.1204	1.2037	0.0525	0.0958	0.0948	-0.1257	-0.2293	-0.0397
15	0.0462	0.0843	0.5389	0.0330	0.0602	0.0441	-0.0879	-0.1604	-0.0199
16	-0.0112	-0.0205	0.0413	-0.0113	-0.0206	0.0016	0.0213	0.0390	-0.0007
17	-0.0591	-0.1078	0.9609	-0.0471	-0.0860	0.0711	0.1125	0.2053	-0.0302
18	-0.0617	-0.1126	1.0250	-0.0475	-0.0867	0.0828	0.1175	0.2145	-0.0354
19	-0.0204	-0.0372	0.0909	-0.0121	-0.0221	0.0081	0.0388	0.0708	-0.0042
20	0.0401	0.0731	0.4426	0.0324	0.0591	0.0295	-0.0763	-0.1392	-0.0128
21	0.0660	0.1204	1.2035	0.0525	0.0958	0.0947	-0.1256	-0.2293	-0.0397
22	0.0463	0.0845	0.5427	0.0331	0.0605	0.0444	-0.0882	-0.1610	-0.0200
23	-0.0110	-0.0201	0.0398	-0.0111	-0.0203	0.0015	0.0209	0.0382	-0.0007
24	-0.0590	-0.1076	0.9579	-0.0470	-0.0859	0.0708	0.1123	0.2050	-0.0301
25	-0.0618	-0.1127	1.0277	-0.0476	-0.0868	0.0830	0.1176	0.2147	-0.0355
26	-0.0206	-0.0376	0.0930	-0.0123	-0.0224	0.0083	0.0392	0.0715	-0.0042
27	0.0399	0.0728	0.4388	0.0322	0.0588	0.0292	-0.0760	-0.1386	-0.0127
28	0.0660	0.1204	1.2032	0.0525	0.0958	0.0947	-0.1256	-0.2293	-0.0397
29	0.0465	0.0848	0.5465	0.0333	0.0607	0.0447	-0.0885	-0.1615	-0.0201
30	-0.0108	-0.0196	0.0384	-0.0110	-0.0200	0.0014	0.0205	0.0374	-0.0007
31	-0.0589	-0.1075	0.9548	-0.0470	-0.0857	0.0705	0.1122	0.2047	-0.0300
32	-0.0618	-0.1128	1.0303	-0.0476	-0.0869	0.0832	0.1178	0.2149	-0.0355
33	-0.0208	-0.0380	0.0951	-0.0124	-0.0227	0.0085	0.0396	0.0722	-0.0043
34	0.0397	0.0725	0.4350	0.0321	0.0586	0.0289	-0.0756	-0.1380	-0.0125
35	0.0659	0.1203	1.2029	0.0525	0.0958	0.0946	-0.1256	-0.2292	-0.0397
36	0.0466	0.0851	0.5504	0.0334	0.0610	0.0450	-0.0888	-0.1620	-0.0203
37	-0.0105	-0.0192	0.0370	-0.0108	-0.0197	0.0013	0.0201	0.0366	-0.0006
38	-0.0588	-0.1073	0.9518	-0.0469	-0.0856	0.0703	0.1120	0.2044	-0.0299
39	-0.0619	-0.1130	1.0329	-0.0477	-0.0870	0.0834	0.1179	0.2152	-0.0356
40	-0.0210	-0.0383	0.0973	-0.0126	-0.0230	0.0086	0.0400	0.0730	-0.0044
41	0.0395	0.0721	0.4312	0.0320	0.0583	0.0286	-0.0753	-0.1374	-0.0124
42	0.0659	0.1203	1.2026	0.0525	0.0958	0.0946	-0.1256	-0.2292	-0.0396
43	0.0468	0.0853	0.5542	0.0335	0.0612	0.0453	-0.0890	-0.1625	-0.0204
44	-0.0103	-0.0188	0.0356	-0.0107	-0.0194	0.0012	0.0196	0.0358	-0.0006
45	-0.0587	-0.1071	0.9487	-0.0468	-0.0855	0.0700	0.1118	0.2041	-0.0298
46	-0.0620	-0.1131	1.0355	-0.0478	-0.0872	0.0836	0.1180	0.2154	-0.0357

t (sec)	RAO11	X (m)	$\eta_x$ (m)	RAO22	Y (m)	$\eta_y$ (m)	RAO33	Z(rad)	$\eta_z$ (rad)
47	-0.0212	-0.0387	0.0995	-0.0128	-0.0233	0.0088	0.0404	0.0737	-0.0045
48	0.0394	0.0718	0.4274	0.0318	0.0581	0.0283	-0.0749	-0.1368	-0.0123
49	0.0659	0.1203	1.2023	0.0525	0.0958	0.0945	-0.1256	-0.2292	-0.0396
50	0.0469	0.0856	0.5580	0.0337	0.0614	0.0456	-0.0893	-0.1630	-0.0205
51	-0.0101	-0.0184	0.0343	-0.0105	-0.0191	0.0011	0.0192	0.0350	-0.0006
52	-0.0586	-0.1070	0.9456	-0.0467	-0.0853	0.0697	0.1116	0.2037	-0.0297
53	-0.0620	-0.1132	1.0380	-0.0478	-0.0873	0.0838	0.1182	0.2156	-0.0358
54	-0.0214	-0.0391	0.1017	-0.0129	-0.0236	0.0090	0.0408	0.0744	-0.0046
55	0.0392	0.0715	0.4236	0.0317	0.0578	0.0281	-0.0746	-0.1361	-0.0122
56	0.0659	0.1203	1.2019	0.0525	0.0958	0.0944	-0.1255	-0.2291	-0.0396
57	0.0471	0.0859	0.5618	0.0338	0.0617	0.0459	-0.0896	-0.1635	-0.0206
58	-0.0099	-0.0180	0.0329	-0.0103	-0.0188	0.0011	0.0188	0.0342	-0.0005
59	-0.0585	-0.1068	0.9425	-0.0467	-0.0852	0.0695	0.1115	0.2034	-0.0296
60	-0.0621	-0.1133	1.0406	-0.0479	-0.0874	0.0840	0.1183	0.2159	-0.0358
61	-0.0216	-0.0395	0.1039	-0.0131	-0.0239	0.0092	0.0412	0.0752	-0.0046
62	0.0390	0.0712	0.4199	0.0316	0.0576	0.0278	-0.0743	-0.1355	-0.0120
63	0.0659	0.1203	1.2016	0.0525	0.0958	0.0944	-0.1255	-0.2291	-0.0396
64	0.0472	0.0861	0.5656	0.0339	0.0619	0.0463	-0.0899	-0.1640	-0.0208
65	-0.0096	-0.0176	0.0316	-0.0102	-0.0185	0.0010	0.0183	0.0335	-0.0005
66	-0.0584	-0.1066	0.9393	-0.0466	-0.0850	0.0692	0.1113	0.2031	-0.0295
67	-0.0622	-0.1135	1.0431	-0.0480	-0.0875	0.0842	0.1184	0.2161	-0.0359
68	-0.0218	-0.0399	0.1061	-0.0133	-0.0242	0.0094	0.0416	0.0759	-0.0047
69	0.0388	0.0708	0.4161	0.0314	0.0574	0.0275	-0.0739	-0.1349	-0.0119
70	0.0659	0.1202	1.2012	0.0525	0.0958	0.0943	-0.1255	-0.2290	-0.0396
71	0.0473	0.0864	0.5694	0.0340	0.0621	0.0466	-0.0902	-0.1645	-0.0209
72	-0.0094	-0.0172	0.0303	-0.0100	-0.0182	0.0009	0.0179	0.0327	-0.0005
73	-0.0583	-0.1064	0.9362	-0.0465	-0.0849	0.0689	0.1111	0.2027	-0.0293
74	-0.0622	-0.1136	1.0456	-0.0480	-0.0877	0.0844	0.1185	0.2163	-0.0360
75	-0.0221	-0.0403	0.1084	-0.0134	-0.0245	0.0095	0.0420	0.0766	-0.0048
76	0.0386	0.0705	0.4124	0.0313	0.0571	0.0272	-0.0736	-0.1343	-0.0118
77	0.0659	0.1202	1.2007	0.0525	0.0957	0.0943	-0.1255	-0.2290	-0.0395
78	0.0475	0.0867	0.5733	0.0342	0.0624	0.0469	-0.0904	-0.1651	-0.0210
79	-0.0092	-0.0168	0.0291	-0.0098	-0.0179	0.0008	0.0175	0.0319	-0.0004
80	-0.0582	-0.1063	0.9330	-0.0464	-0.0847	0.0686	0.1109	0.2024	-0.0292
81	-0.0623	-0.1137	1.0481	-0.0481	-0.0878	0.0846	0.1187	0.2166	-0.0361
82	-0.0220	-0.0402	0.1095	-0.0136	-0.0248	0.0097	0.0420	0.0766	-0.0049
83	0.0385	0.0702	0.4087	0.0312	0.0569	0.0269	-0.0732	-0.1337	-0.0117
84	0.0659	0.1202	1.2003	0.0525	0.0957	0.0942	-0.1255	-0.2289	-0.0395
85	0.0476	0.0869	0.5771	0.0343	0.0626	0.0472	-0.0907	-0.1656	-0.0212
86	-0.0090	-0.0163	0.0279	-0.0097	-0.0176	0.0008	0.0170	0.0311	-0.0004
87	-0.0581	-0.1061	0.9299	-0.0464	-0.0846	0.0684	0.1107	0.2021	-0.0291
88	-0.0624	-0.1138	1.0506	-0.0482	-0.0879	0.0848	0.1188	0.2168	-0.0361
89	-0.0225	-0.0410	0.1129	-0.0137	-0.0251	0.0099	0.0428	0.0781	-0.0050
90	0.0383	0.0699	0.4049	0.0310	0.0566	0.0267	-0.0729	-0.1330	-0.0116
91	0.0658	0.1202	1.1998	0.0525	0.0957	0.0941	-0.1254	-0.2289	-0.0395
92	0.0478	0.0872	0.5809	0.0344	0.0628	0.0475	-0.0910	-0.1661	-0.0213
93	-0.0087	-0.0159	0.0267	-0.0095	-0.0173	0.0007	0.0166	0.0303	-0.0004



(sec)	RAO11	X (m)	$\eta_x$ (m)	RAO22	Y (m)	$\eta_y$ (m)	RAO33	Z(rad)	$\eta_z$ (rad)
94	-0.0580	-0.1059	0.9267	-0.0463	-0.0845	0.0681	0.1105	0.2017	-0.0290
95	-0.0624	-0.1139	1.0531	-0.0482	-0.0880	0.0850	0.1189	0.2170	-0.0362
96	-0.0227	-0.0414	0.1152	-0.0139	-0.0254	0.0101	0.0432	0.0788	-0.0051
97	0.0381	0.0695	0.4011	0.0309	0.0564	0.0264	-0.0725	-0.1324	-0.0115
98	0.0658	0.1201	1.1994	0.0524	0.0957	0.0941	-0.1254	-0.2288	-0.0394
99	0.0479	0.0875	0.5847	0.0345	0.0630	0.0478	-0.0913	-0.1666	-0.0214
100	-0.0085	-0.0155	0.0255	-0.0093	-0.0170	0.0006	0.0162	0.0295	-0.0004
101	-0.0579	-0.1057	0.9235	-0.0462	-0.0843	0.0678	0.1104	0.2014	-0.0289
102	-0.0625	-0.1140	1.0555	-0.0483	-0.0882	0.0852	0.1190	0.2172	-0.0363
103	-0.0229	-0.0418	0.1176	-0.0141	-0.0257	0.0103	0.0436	0.0795	-0.0052
104	0.0379	0.0692	0.3974	0.0308	0.0561	0.0261	-0.0722	-0.1317	-0.0113
105	0.0658	0.1201	1.1988	0.0524	0.0957	0.0940	-0.1254	-0.2288	-0.0394
106	0.0481	0.0877	0.5885	0.0347	0.0633	0.0481	-0.0915	-0.1671	-0.0215
107	-0.0083	-0.0151	0.0243	-0.0092	-0.0167	0.0006	0.0158	0.0287	-0.0003
108	-0.0578	-0.1056	0.9202	-0.0461	-0.0842	0.0675	0.1102	0.2010	-0.0288
109	-0.0626	-0.1142	1.0579	-0.0484	-0.0883	0.0854	0.1191	0.2174	-0.0364
110	-0.0231	-0.0422	0.1199	-0.0142	-0.0260	0.0105	0.0440	0.0803	-0.0053
111	0.0377	0.0689	0.3937	0.0306	0.0559	0.0259	-0.0718	-0.1311	-0.0112
112	0.0658	0.1201	1.1983	0.0524	0.0957	0.0939	-0.1253	-0.2287	-0.0394
113	0.0482	0.0880	0.5924	0.0348	0.0635	0.0484	-0.0918	-0.1675	-0.0217
114	-0.0081	-0.0147	0.0232	-0.0090	-0.0164	0.0005	0.0153	0.0280	-0.0003
115	-0.0577	-0.1054	0.9170	-0.0460	-0.0840	0.0673	0.1100	0.2007	-0.0287
116	-0.0626	-0.1143	1.0603	-0.0484	-0.0884	0.0855	0.1193	0.2177	-0.0364
117	-0.0233	-0.0425	0.1223	-0.0144	-0.0263	0.0107	0.0444	0.0810	-0.0053
118	0.0375	0.0685	0.3900	0.0305	0.0556	0.0256	-0.0715	-0.1305	-0.0111
119	0.0658	0.1201	1.1978	0.0524	0.0957	0.0938	-0.1253	-0.2287	-0.0394
120	0.0484	0.0882	0.5962	0.0349	0.0637	0.0487	-0.0921	-0.1680	-0.0218

**APPENDIX F**  
**NATURAL TIME PERIOD FOR SPAR PLATFORM WITH**  
**DIFFERENT INITIAL HORIZONTAL FORCE (sec)**

Time Instant	Case	H <sub>0</sub> (kN)	Surge	Sway	Heave	Roll	Pitch	Yaw
Response at t = 0	A	2500	215.43	215.43	28.04	50.84	50.84	102.97
	B	2000	254.60	254.60	28.04	50.84	50.84	115.13
Steady state Response	A	2500	392.23	215.43	39.79	50.84	50.84	102.97
	B	2000	360.41	254.60	39.79	50.84	50.84	115.13

## APPENDIX G

### MATLAB PROGRAM

```

H = 3.6; T = 7; d = 588.26; D = 22; g = 9.81;
y = linspace(0, -199.64, 200); p = 1000; Cd = 0.6; Cm=2;
l = linspace(588.26, 388.62, 200); wn1 = 0.016; wn2 = 0.158; wn3 = 0.124;
r_x = 62.228
yM = linspace(-107.6, 92.4, 200); E = 0.05;
W = 2.6*(10^6); L_s = 199.64;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L = g*(T).^2/2*pi ;
k = 2*pi/L ;
w = 2*pi/T;
c = L/T ;
x = 0;
m1 = 0.25*pi*L_s *D*D*p*g/1000;
m11 = m1 + W;
C11 = 2*E*wn1*m11;
K11 = wn1*wn1*m11/g;
B11 = atan(C11*w/(K11-((m11*w*w)/9.81)));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
m2 = pi*D*D*D*p/12;
m22 = m2 + W;
C22 = 2*E*wn2*m22;
K22 = wn2*wn2*m22/g;
B22 = atan(C22*w/(K22-((m22*w*w)/9.81)));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
I33 = m22*r_x*r_x;
C33 = 2*E*wn3*I33;
K33 = wn3*wn3*I33/g;
B33 = atan(C33*w/(K33-((I33*w*w)/9.81)));

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for il = 0:200
    i = il+1;
    nw(i) = (H/2)*cos(k*(x - c*il));
    for j = 1:200

        u(i, j) = (g*k*H/2*w)*exp(k.*y(j))*cos(k*(x - c*il));
        a(i, j) = u(i, j).*w;
        Fx1(i, j) = 0.5*p*D*Cd.*u(i, j).*abs(u(i, j));
        Fx2(i, j) = 0.25*p*pi*D*D*Cm.*a(i, j);
        Fx(i, j) = g.*(Fx1(i, j) + Fx2(i, j));
        Fx_Sum(i) = sum(Fx(:, j));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        Fy(i, j) = p*g.*(cosh(k.*l(j))./cosh(k*d)).*(0.25*pi*D^2*nw(i));
        Fy_sum(i) = sum(Fy(:, j));
        M(i, j) = Fx(i, j).*yM(j);
        M_sum(i) = sum(M(:, j));
    end
    RA011(i) = (Fx_Sum(i)/(H/2))/sqrt((K11-((m11*w*w)/9.81))^2 +
(C11*w)^2);
    X(i) = RA011(i)*H*0.5;
    x = (X(i) .* cos(w.*il + B11));
    nx(i) = x ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    RA022(i) = (Fy_sum(i)/(H/2))/sqrt((K22-((m22*w*w)/9.81))^2 +
(C22*w)^2);
    Y(i) = RA022(i)*H*0.5;
    ny (i) = (Y(i) .* cos(w.*il + B22));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    RA033(i) = (M_sum(i)/(H/2))/sqrt((K33-((I33*w*w)/9.81))^2 + (C33*w)^2);

```

```

m(i) = RA033(i)*H*0.5;
nM (i) = (m(i) .* cos(w.*i1 + B33));
end
plot([0:200],nx)
title('Surge Responses for 200 sec');
xlabel('Time (sec)');
ylabel('Surge (m)');
figure,plot([0:200],nx)
title('Heave responses for 200 sec');
xlabel('Time (sec)');
ylabel('Heave(m)');
figure,plot([0:200],nx)
title('Pitch response for 200 sec');
xlabel('Time (sec)');
ylabel('Pitch(rad)');

```