CERTIFICATION OF APPROVAL

Stress Intensity Factor for a Crack Emanating From a Shaft

by

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Approved by,

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UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

KHOO SZE WEI

Abstract

Shaft is a rotational body used to transmit power or motion. Due to cyclic loading conditions, i.e. axial, bending and torsional load, surface cracks frequently grow in the shaft. Normally these cracks will propagate with a semi-circular shape and cause damage or premature failure to the whole system. These premature failures require expensive repair or replacement cost, and sometimes even worst the operators get severe injury when the shaft failed during it services. The objective of this project is to determine the stress intensity factor (SIF) for a crack emanating from a shaft by using finite element method and also to verify the finite element results with those obtained semi-analytically. The scope of this study is focused on the semi-circular crack on the shaft and the calculation of SIF for Mode I (Opening) and Mode III (Tearing/Torsion) crack loading. The study is divided into two phases. For the first phase, modelling of the cracked shaft is carried out in the ANSYS software, while for the second phase; verification is carried out between the finite element results and those obtained semi-analytically. In the results and discussion section, the relationship between the dimensionless stress intensity factor and the normalized relative crack depth is presented. The results obtained semi-analytically and numerically had been verified and the deviation in term of percentage is relatively small. In conclusion, the stress intensity factor of a shaft determined by the numerical method was verified to be accurate.

ACKNOWLEDGEMENTS

There are many parties that I would like to dedicate my deepest thankfulness as for the past two semesters of my final year project. I have received tremendous help from them in completing my project. I am deeply grateful for their remarkable assistance offered unconditionally to me. Hence, I would like to take this golden opportunity to thank each and every one of them.

The utmost gratitude is especially dedicate to my supervisor, Dr. Saravanan Karuppanan for his excellent guidance and support. I would like to thank him personally as his supervision and caring whenever I faced problem have more or less given me encouragement to finish the final year project. Besides, his confidences over me by giving me numerous of challenging tasks, have given me a lot of learning opportunities.

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Last but no means least, I would like to thank my family, my friends and fellow coursemates who have contributed in a way or another to the completion of my final year project.

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ABBREVIATIONS AND NOMENCLATURES

a	radius of uncracked region
a/c	crack aspect ratio
a/D	crack depth ratio
b	shaft's diameter
c	crack width
F_I	boundary correction factor
FEM	finite element method
FEA	finite element analysis
G	geometry factor
K	stress intensity factor
K_I	stress intensity factor for Mode I loading
K_{III}	stress intensity factor for Mode III loading
M	applied moment
P	applied axial load
r	radius of the cracked shaft
Rx	rotation in x-axis
Ry	rotation in y-axis
Rz	rotation in z-axis
SIF	stress intensity factor
T	applied torque
Ux	displacement in x-direction
Uy	displacement in y-direction
Uz	displacement in z-direction
σ	stress
σ_N	normal stress
τ	shear stress
ζ/h	normalized coordinate

CHAPTER 1
INTRODUCTION

1.1 BACKGROUND OF STUDY

Shaft is a rotational body used to transmit power or motion. It provides the axis of rotation for gears, pulleys, flywheels and etc. Due to cyclic loading conditions, such as axial, bending and torsional load, surface cracks or flaws frequently grow in the shaft. If these surface cracks or flaws reached their critical stage, the cracks will expand at the speed of sound and cause undesirable catastrophic failures.

Normally these cracks will propagate with a semi-circular or semi-elliptical shape and cause damage or premature failure to the whole system. In order to ensure the safety of the shafts, engineers or designers are always required to perform an assessment on the cracked shafts. By using the linear elastic theory, engineers are able to predict the cracks growth behavior with the introduction of stress intensity factor (SIF). The stress intensity study of cracks is a relatively new field in mechanical engineering known as fracture mechanic.

The fundamental principle of fracture mechanics [1] is that the stress field ahead of a crack in a structural member can be characterized as a single parameter, K, which is the stress intensity factor. For a cracked body, it is clear that at the crack tip (r = 0), the stress is singular since $\sigma \rightarrow \infty$ as $r \rightarrow 0$. So, the stress concentration approach is inappropriate for this problem due to this singularity. By using the stress intensity factor approach, the quantity $\sigma\sqrt{2r}$ is introduced, since this factor remains finite as $r \rightarrow 0$. A factor π is introduced to this quantity and the new factor is defined as:

$$K = \sigma\sqrt{2\pi r} = (\sigma\sqrt{a}/\sqrt{2r})\sqrt{2\pi r} = \sigma\sqrt{\pi a}$$

where K is the stress intensity factor with unit of MPa$\sqrt{\text{m}}$, $\boldsymbol{\sigma}$ is the stress, r is the crack tip radius and a is the crack length.

Stress intensity factor, K, is used to more accurately describe the stress state near the tip of a crack caused by a remote load or residual stresses. The magnitude of K depends on the size and position of the crack, the geometry of the sample, distribution of loads, and the temperature on the shaft. By performing experiments on a shaft with a known flaw size, engineers can determine the value of K that will cause the flaw to propagate and cause failure.

The crack propagation in a body can be subjected to three different types of loading as shown in Figure 1.1. These load types can be categorized as Mode I, Mode II and Mode III. For mode I (Opening), the load is applied normal to the crack plane and tends to open the crack. This opening mode of deformation is the most important mechanism which controls failure of homogeneous, isotropic materials. Mode II refers to in-plane shear loading or sliding. Mode III corresponds to out-of-plane loading or tearing [2]. According to the research on shafts, many failures that happened in rotor shafts are due to mixed-mode loading when the rotor shafts are subjected to cyclic Mode I loading combined with steady Mode III loading.

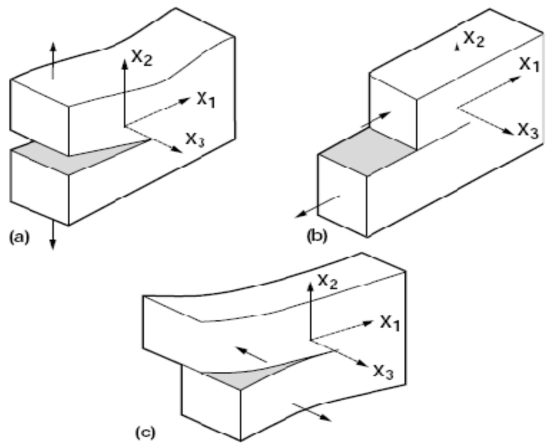


Figure 1.1: Three types of loading on a cracked body. (a) Mode I. (b) Mode II. (c) Mode III.

1.2 PROBLEM STATEMENT

For the power generation industries, the growth of fatigue cracks in the rotating components especially the pump shafts, drive shafts and etc is a significant economic and safety concern. These fatigue cracks in the shaft will always require expensive repair or replacement cost. Sometimes even worst when the shaft failed during it services and caused severe injury to the operators. Since the cracks normally propagate at the speed of sound when they reached critical stage, there must be a monitoring system that provides an early warning to the plant operator before catastrophic incident happen. In order to ensure the safety of a shaft, engineers or designers are always required to perform an assessment on the cracked shaft. This assessment was to calculate the stress intensity factor and to check the safety level of the cracked shaft.

Due to economic purposes, the engineers are always searching for the fastest way to calculate the stress intensity factor. The semi-analytical method is only applicable to simple geometry because there is no formula for complex geometry. While for the experimental method, the cost to determine the stress intensity factor is considered far too expensive. At the same time, it is impossible to test all the cracked models experimentally since they require expensive set-up cost.

In order to solve both problems above, stress intensity factor determination by using numerical method is preferred in the 21st century. This is because the analyzing time reduces dramatically without sacrificing the accuracy of the results. By using numerical method, engineers or designers can perform any kind of simulations on the cracked shaft with different crack parameters easily. Meanwhile, a faster decision can be made when solving the cracked shaft's problem.

1.3 OBJECTIVES

The objectives of this study are:

- To model and to determine the stress intensity factor for a crack emanating from a shaft by using finite element method (FEM).
- To compare the finite element method (FEM) results with those obtained semi-analytically.

1.4 SCOPE OF STUDY

In this project, the scope of study is focused on the semi-circular crack on the shaft subjected to axial, bending and torsional load. The stress intensity factor of a crack on a shaft is mainly influenced by a few factors which include the geometry factor, applied stress and the crack dimension. At the same time, the geometrical parameters or load type will also affect the value of the stress intensity factor. Since the shaft can be subjected to axial, bending and torsional load, the scope of study of this project is mainly focused on the Mode I (Opening) and Mode III (Tearing/Torsional) crack loading.

1.5 SIGNIFICANCE OF THE WORK

High accuracy and efficient way to determine the stress intensity factor are the most important criteria in fracture mechanics. By using the conventional method such as the analytical and experimental method, high accuracy of results can be obtained but the engineers might consume a lot of time to perform all these calculations. With the introduction of finite element method in determining the stress intensity factor, all the problems above can be solved in a faster way without scarifying the accuracy of the result. At the same time, all the calculations of stress intensity factor in complicated geometry is relatively easy to perform. Drastically, the cost of getting the stress intensity factor is lower and brings more profit to the industries.

The present study is to model and determine the stress intensity factor of a cracked shaft by using finite element analysis (FEA) software package. Different crack width will be modelled to show the relationship of the crack width with the stress intensity factors' value. Besides, validation of the results with the semi-analytical and the numerical results is carried out in order to prove the accuracy of the finite element method.

CHAPTER 2
LITERATURE REVIEW

In 1999, Manuel da Fonte and Manuel de Freitas had presented a paper [3] which is related to the stress intensity factors for semi-elliptical surface cracks in round bars under bending and torsional loading. The stress intensity factor for semi-elliptical surface cracks subjected to Mode I and Mode III loading was found by using a three dimensional finite element method. For the Mode I (Bending) loading, the result was compared with the available literature results in order to validate the proposed model. While for the Mode III (Torsional) loading, no validation was made since no solutions for the stress intensity factors were available during that moment.

A total of eight semi-elliptical surface cracks were considered with a constant *b/s* ratio, whereby the angle being tested is from the range of 10º to 80º. Figure 2.1 shows the geometry of the semi-elliptical surface crack which was used in the presented paper. The relative difference of shorter cracks depths was between 2% to 5% if compared to the literature value. While for the larger crack depths, the difference was between 12% and 15% and this value was still considered as a reasonable result. Besides, the results showed that at maximum crack depth, the pure Mode III exists and had the highest value. This result has impact for the crack growth rate predictions of the semi-elliptical surface cracks in round bars under bending and torsional loading.

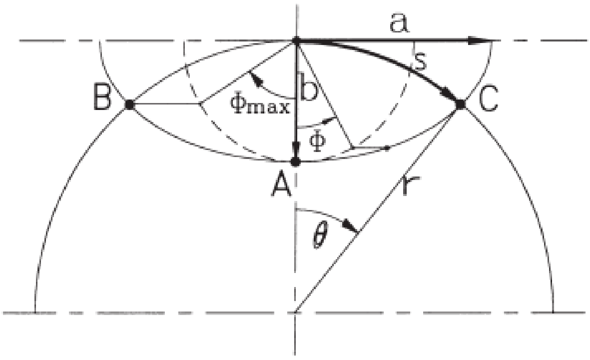


Figure 2.1: Geometry of the semi-elliptical surface crack.

Yan-Shin Shih and Jien-Jong Chen (2002) carried out a study which was related to stress intensity factor of an elliptical cracked shaft [4]. The numerical model of a round bar was evaluated by collapsed singular element with detailed mesh on crack front. The mesh of the three-dimensional finite element model of cracked bar is constructed by employing 20-node regular and collapsed singular element as illustrated in Figure 2.2. For this study, the ratio of crack depth to shaft diameter was considered in the range between 0.1 to 0.6 while the elliptical ratio of crack area was in the range between 0.0 to 1.0.

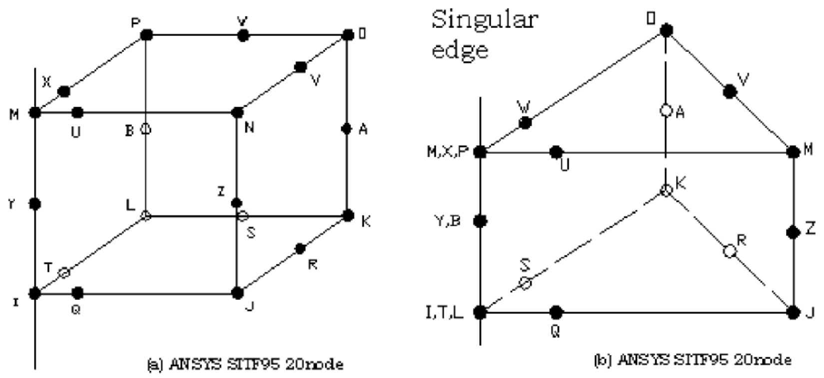


Figure 2.2: Three-dimensional crack front element.

There were some assumptions being used in conducting this study:

i. The round bar is made of homogenous, isotropic and linear elastic material.
ii. The square-root stress singularity is filled within the vicinity of the crack front.
iii. An elliptical-arc surface crack is located at the half-length of the round bar.
iv. Only the Mode I (Opening) fracture is considered.

For the dimensionless SIF relationship, the formula below was derived for various elliptical crack profiles,

$$\tilde{K}_{I,j} = \tilde{K}_{I,j}\left(\frac{a}{D}, \frac{a}{c}, \frac{\zeta}{h}\right)$$

where *a/D* is the crack depth ratio, *a/c* is the crack aspect ratio, and *ζ/h* is the normalized coordinate. This SIF results are divided into two regions, the surface area when $\zeta/h = 0$ and the interior area when $1 \geq \zeta/h > 0$. For the tensile load, the dimensionless SIF of an elliptical crack increases as the crack depth ratio increases, while decreases as the crack aspect ratio increases. For the bending load, the dimensionless SIF of an elliptical crack decreases as the crack depth ratio increases, while increases for the case of small crack depth ratio.

In 2005, A. Vaziri and H. Nayeb-Hashemi had presented a paper [5] entitled "The effect of crack surface interaction on the stress intensity factor in Mode III crack growth in round shafts". In this paper, the effective stress intensity factor in circumferentially cracked round shafts has been evaluated for a wide range of applied torsional load. This evaluation was done by considering a pressure distribution between mating fracture surfaces. The results showed that the pressure profile not only depends on the fracture surface roughness, but also depends on the magnitude of the applied Mode III stress intensity factor.

A schematic diagram of a cylindrical bar with a circumferential crack subjected to a torque, *T* is shown in Figure 2.3. The shaft material is assumed to be linear elastic perfectly plastic. Besides, the height and wavelength of these asperities were affected by the applied stress intensity factor, specimen geometry and the material properties. This crack pattern results in interaction between crack surfaces which decreases the effective stress intensity factor when the shaft is subjected to a torsional load.

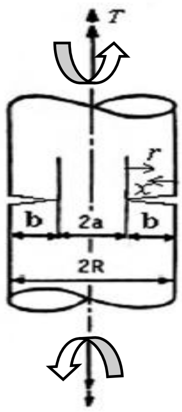


Figure 2.3: Cylinder bar with a circumferential crack under torsional load.

From this analysis, it had been concluded that the fracture surface interaction in circumferential cracked shafts could result in a significant reduction in the effective Mode III stress intensity factor. While for the frictional stress intensity factor, its value depends on the shaft radius, crack length, asperities height and wavelength and the shaft material properties. Besides, the effective stress intensity is considered as the crack driving force if the Mode III crack growth followed the Paris Law.

M. da Fonte, L. Reis, F. Romeiro, B. Li, M. de Freitas (2006) had carried out research [6] on “The effect of steady torsional on fatigue crack growth in shafts”. In this paper, long cracks growth tests have been carried out on cylindrical specimens in DIN Ck45k steel and two types of testing was accomplished. The testing was rotary or alternating bending combined with steady torsional in order to simulate the real conditions on power rotor shafts. The cylindrical specimen surface was measured for several loading conditions to understand the growth and the shape evolution of semi-elliptical surface cracks.

A three dimensional finite element analysis was used to obtain the Mode I and the Mode III stress intensity factors along the front of semi-elliptical surface cracks in shafts. The shaft sizes were 80mm diameter and 120mm in length. The surface crack in this study was on a normal plane to the axis of the shaft and the mesh is shown in

Figure 2.4. The symmetry conditions are not valid since the presence of torsional loading. The SIFs results of pure bending were compared with the available results. While for the surface cracks in round bars subjected to torsional loading, no comparison was made since there were no available results.

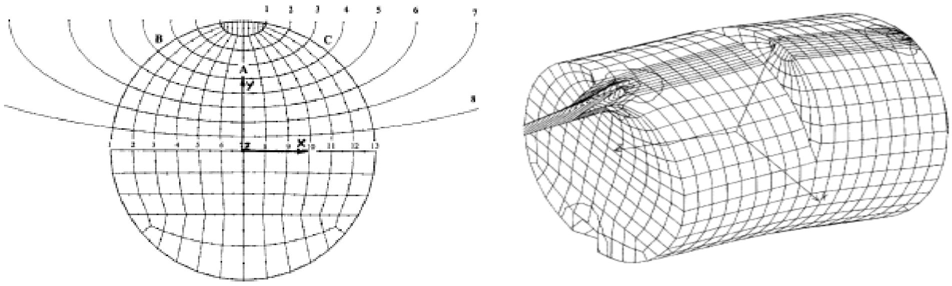


Figure 2.4: Finite element mesh.

From the results of this study, a superimposed steady Mode III loading to a crack growing in cyclic Mode I had led to significant fatigue crack growth retardation. The crack growth rate was decreased as the Mode I (ΔK_I) and Static Mode III (K_{III}) loading was increased. The values of stress intensity factor in semi-elliptical cracks depend on the direction of the steady torque applied. This explains why the fatigue crack front profile rotates during the fatigue crack propagation when a steady torsional loading is applied.

In 2006, C. J. Lissenden, S. P. Tissot, M. W. Trethewey, and K. P Maynard had presented a paper [7] entitled “Torsion response of a cracked stainless steel shaft”. In this paper, the author had focused on the relationship between cracks which propagated due to bending loads, and the torsional stiffness of the shaft. Besides, the author assumed that these cracked stainless steel shafts are susceptible to fatigue cracking when run under near-continuous operation. An analytical method to determine the compliance associated with a crack has been implemented. Figure 2.5 shows the fatigue crack grown in a stainless steel shaft using three-point bend apparatus.

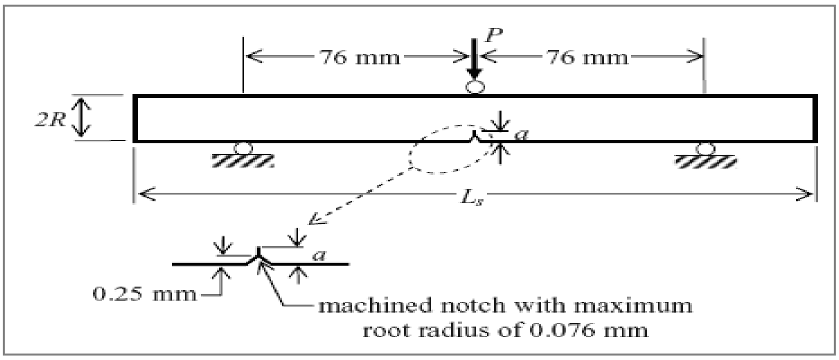


Figure 2.5: Three-point bend apparatus.

A 3-D finite element model of a shaft section with a crack as shown in Figure 2.6 has also been used to predict the effect of a crack on the shaft's stiffness. For the finite element analysis, thirteen different cracks depths ranging from 0 to 1.3R were analyzed. One end of the shaft is restrained in the z-axis while the other end is constrained except for rotation about z-axis. This model's crack size was varied with crack depth and had approximately 4220 elements and 16850 degrees of freedom. A mesh convergence study on the model having the deepest crack indicated that this model is sufficiently refined.

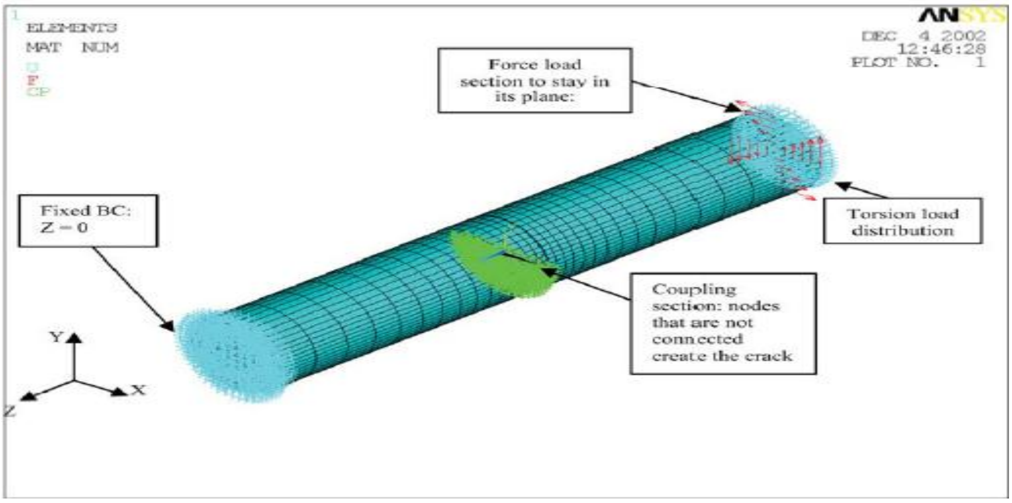


Figure 2.6: Finite element model of a cracked shaft.

In 2008, J. Toribio, J. C. Matos, B. Gonzalez, J. Escuadra had presented a paper [8] entitled "Numerical modelling of crack shape evolution for surface flaws in round bars under tensile loading". In this paper, the authors had studied how the aspect ratio (relation between the semi-axes of the ellipse) changes with the relative crack depth and the model used in the study is shown in Figure 2.7. According to the Paris-Erdogan Law, each point at the crack front advances in the direction perpendicular to such a front and this was represented in a numerical modelling.

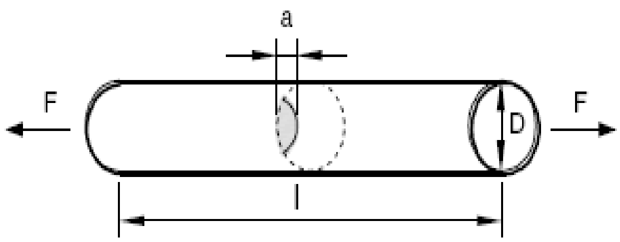


Figure 2.7: Semi-elliptical crack in round bars under tensile loading.

A computer program in Java Programming language was developed to determine iteratively the geometric evolution of the crack front when the round bars were subjected to tensile loading. Few assumptions were made for this study, amongst others were the basic hypothesis of the modelling that consisted a crack shape of an ellipse which centre is located at the bar surface. Figure 2.8 shows the fatigue crack growth direction which is perpendicular to the crack front and this follows the Paris-Erdogan Law. The results of this study shows that for crack depths between 0.7*D* and 0.8*D*, the crack shape evolution is slightly increased for free sample ends and decreasing for constrained sample ends.

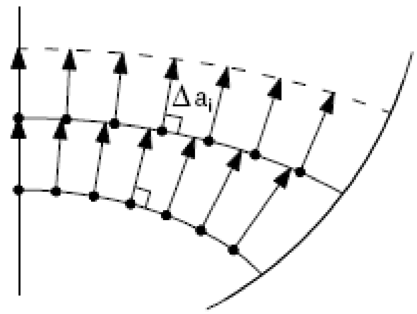


Figure 2.8: Scheme of fatigue crack growth at each point of the crack front.

CHAPTER 3
THEORY

3.1 GENERAL EQUATIONS

The determination of stress intensity factor for a cracked shaft by using semi-analytical method is very crucial in this project. It is necessary to obtain an equation or formula either from the stress handbook or literature reference. Since the results obtained from the semi-analytical method are set to be the reference value, it must provide high degree of accuracy. In this study, a total of three general equations were obtained from the stress and strain handbook [1]. These general equations were used to calculate the stress intensity factor of a cracked shaft under axial, bending and torsional load. Since the modelling of a cracked shaft in numerical method is under the same conditions as shown in the handbook, it is reasonable to assume that the results obtained from both methods should be the same.

3.1.1 General Equation for a Cracked Shaft Subject to Axial Load [1]

Figure 3.1 shows a cracked shaft under axial load. The axial load, P is applied along its axis of rotation and tends to "open" the crack. This is the reason why a cracked shaft under axial load is been categorized as Mode I crack loading.

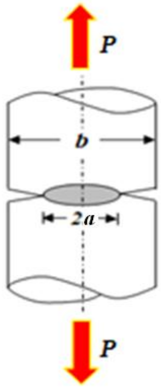


Figure 3.1: Cracked shaft under axial load.

The stress intensity factor, K_I, of a cracked shaft under axial load can be calculated by

$$K_I = \sigma_N \sqrt{\pi c}\, F_I\,(a/b)$$

where σ_N is the normal stress, c is the crack width ($c = b - 2a$) and $F_I\,(a/b)$ is the boundary correction factor.

The normal stress, σ_N, experienced by the cracked shaft is given by the equation below

$$\sigma_N = \frac{P}{\pi r^2}$$

where P is the axial load and r is the radius of the cracked shaft.

The boundary correction factor, $F_I\,(a/b)$, is given by

$$F_I\,(a/b) = \sqrt{1 - \frac{2a}{b}}\, G(a/b)$$

where $G(a/b)$ is the geometry factor, a is the radius of the uncracked region and b is the shaft's diameter.

The geometry factor, $G(a/b)$, of the cracked shaft is given by

$$G(a/b) = \frac{1}{2}\left[1 + \frac{1}{2}\,\frac{2a}{b} + \frac{3}{8}\left(\frac{2a}{b}\right)^2 - 0.363\left(\frac{2a}{b}\right)^3 + 0.731\left(\frac{2a}{b}\right)^4\right]$$

3.1.2 General Equation for a Cracked Shaft Subject to Bending Load [1]

Figure 3.2 shows a cracked shaft under bending load. The moment, M is applied at the end of the cracked shaft and tends to “open” the crack. This is the reason why a cracked shaft under bending load is been categorized as Mode I crack loading.

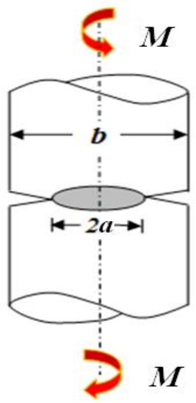


Figure 3.2: Cracked shaft under bending load.

The stress intensity factor, K_I, of a cracked shaft under bending load can be calculated by

$$K_I = \sigma\sqrt{\pi c}\, F_I(a/b)$$

where σ is the bending stress, c is the crack width ($c = b - 2a$) and $F_I(a/b)$ is the boundary correction factor.

The bending stress, σ, experienced by the cracked shaft is given by

$$\sigma = \frac{4M}{\pi r^3}$$

where M is the moment applied on the cracked shaft and r is the radius of the cracked shaft.

The boundary correction factor, $F_I\,(a/b)$, is given by

$$F_I\,(a/b) = \sqrt{1 - \frac{2a}{b}}\,G(a/b)$$

where $G(a/b)$ is the geometry factor, a is the radius of the uncracked region and b is the shaft's diameter.

The geometry factor, $G(a/b)$, of the cracked shaft is given by

$$G(a/b) = \frac{3}{8}\left[1 + \frac{1}{2}\,\frac{2a}{b} + \frac{3}{8}\left(\frac{2a}{b}\right)^2 + \frac{5}{16}\left(\frac{2a}{b}\right)^3 + \frac{35}{128}\left(\frac{2a}{b}\right)^4 + 0.537\left(\frac{2a}{b}\right)^5\right]$$

3.1.3 General Equation for a Cracked Shaft Subject to Torsional Load [1]

Figure 3.3 shows a cracked shaft under torsional load. The torque, T is applied at the end of the cracked shaft and tends to "tear" the crack. This is the reason why a cracked shaft under torsional load is been categorized as Mode III crack loading.

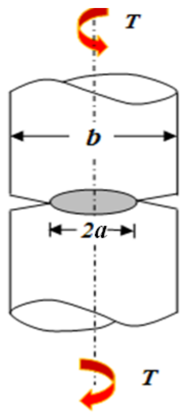


Figure 3.3: Cracked shaft under torsional load.

The stress intensity factor, K_{III}, of a cracked shaft under torsional load can be calculated by

$$K_{III} = \tau\,\sqrt{\pi c}\,F_I\,(a/b)$$

where τ is the shear stress, c is the crack width ($c = b - 2a$) and $F_I\,(a/b)$ is the boundary correction factor.

The shear stress, τ, experienced by the cracked shaft is given by

$$\tau = \frac{2T}{\pi r^3}$$

where τ is the torque applied on the cracked shaft and *r* is the radius of the cracked shaft.

The boundary correction factor, $F_I\,(a/b)$, is given by

$$F_I\,(a/b) = \sqrt{1 - \frac{2a}{b}}\, G(a/b)$$

where *G(a/b)* is the geometry factor, *a* is the radius of the uncracked region and *b* is the shaft's diameter.

The geometry factor, *G(a/b)*, of the cracked shaft is given by

$$G(a/b) = \frac{3}{8}\left[1 + \frac{1}{2}\frac{2a}{b} + \frac{3}{8}\left(\frac{2a}{b}\right)^2 + \frac{5}{16}\left(\frac{2a}{b}\right)^3 + \frac{35}{128}\left(\frac{2a}{b}\right)^4 + 0.208\left(\frac{2a}{b}\right)^5\right]$$

CHAPTER 4
METHODOLOGY

During the past few years, the preferred choice to determine the SIF is either by using analytical or experimental method. There were only a small group of engineers who opted for the numerical method to determine the stress intensity factors. The reason was because of lack of high performance workstation during that moment. In the recent years, the performances of the workstation and the finite element analysis software have been upgraded. This significant improvement will encourage engineers or designers to use the finite element analysis software to conduct the analysis. For this project, the two methods used to determine the stress intensity factor of a cracked shaft are the semi-analytical method and the numerical method. Lastly, verification of the results was carried out between the semi-analytical and the numerical method.

4.1 ANALYSIS TECHNIQUE

For this project, the analysis was conducted by a semi-analytical method and a numerical method:

a) Semi-Analytical Method

   Stress intensity factors of a cracked shaft were calculated by using the general equations obtained from the stress handbook.

b) Numerical Method

   Stress intensity factors were determined by simulating the finite element model in FEA software ANSYS.

4.2 RESEARCH METHODOLOGY

The work flow followed for the Final Year Project is as shown in Figure 4.1. Besides, two Gantt Charts shown in Figure 4.2 and Figure 4.3 were the detail activities in the final year project I and final year project II respectively.

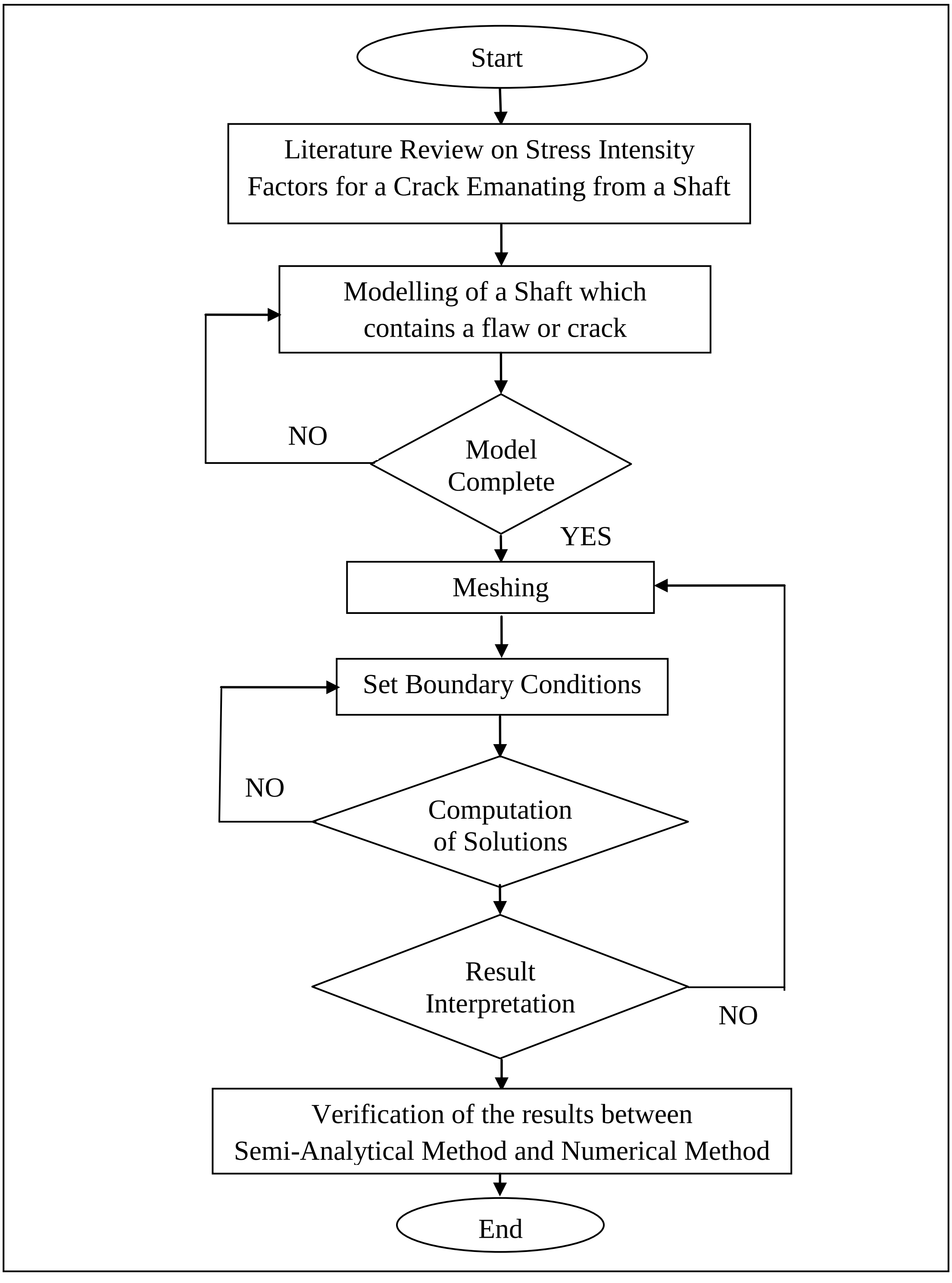


Figure 4.1: Project work flow for the final year project.

4.3 GANTT CHART (FINAL YEAR PROJECT I)

No	Detail/ Week	1	2	3	4	5	6	7	8	9	10		11	12	13	14
1	**Selection of Project Topic**											**MID SEMESTER BREAK**				
2	**Preliminary Research Work** • Research on stress intensity factors of shaft • Learning of FEA software, ANSYS															
3	**Submission of Preliminary Report**															
4	**Modelling of the Cracked Shaft** • Set the dimension and crack location, load case															
5	**Submission of Progress Report**															
6	**Computation for Solutions** • Run the load case in Solver Analysis															
7	**Submission of Interim Report**															
8	**Oral Presentation**															

Suggested milestone

Process

Figure 4.2: Suggested milestone for the first half of final year project.

GANTT CHART (FINAL YEAR PROJECT II)

No	Detail/ Week	1	2	3	4	5	6	7	8	9		10	11	12	13	14
1	**Computation for Solutions** • Run the load case in Solver Analysis										MID SEMESTER BREAK					
2	**Submission of Progress Report 1**				●											
3	**Result Interpretation** • Study the results from ANSYS															
4	**Submission of Progress Report 2 and Seminar**								●							
5	**Verification of Results between Semi-Analytical and Numerical Method**															
6	**Poster Exhibition**											●				
7	**Submission of Dissertation (Soft Bound)**													●		
8	**Oral Presentation**														●	
9	**Submission of Project Dissertation (Hard Bound)**															●

● Suggested milestone

Process

Figure 4.3: Suggested milestone for the second half of final year project.

4.4 MODELLING PROCESSES

For this project, the analysis of a cracked shaft was conducted by using numerical method. Three sets of modelling were created to simulate the cracked shaft under axial, bending and torsional load. These three sets of modelling have the same material properties, i.e. the material used was steel which have the modulus of elasticity of 206 GPa or 30×10^6 psi and poisson ratio of 0.3. Besides, *Solid 95* was assigned for the elements within the three sets of modelling. For each set of the modelling, a total of 9 models had been created, starting from 1 mm crack width to 9 mm crack width as shown in Table 4.1 below.

Table 4.1: Parameters for a cracked shaft under axial, bending and torsional load

	Cracked Shaft Under Axial Load	Cracked Shaft Under Bending Load	Cracked Shaft Under Torsional Load
Crack Width	1 mm – 9 mm	1 mm – 9 mm	1 mm – 9 mm
Shaft's Length	100 mm	100 mm	100 mm
Shaft's Diameter	10 mm	10 mm	10 mm
Type of Loading	Axial Load, P = 1000 N	Moment, M = 5 Nm	Torque, T = 10 Nm
Symmetrical Condition	Yes	Yes	No

The modelling processes of a cracked shaft under axial, bending and torsional load were almost the same. A total of three phases were involved during the modelling of a cracked shaft, i.e. Pre-Processing Phase, Solver or Processing Phase and lastly Post-Processing Phase. All these phases, especially the Pre-Processing Phase, must be carried out carefully in order to get accurate results.

4.4.1 Pre-Processing Phase

A solid shaft was created and followed by a crack which was located on the surface of the shaft. For the modelling of a cracked shaft under axial and bending load, only half of the solid shaft was modelled due to symmetrical shape and loading in the *z*-direction. While for the cracked shaft under torsional load, modelling of the whole shaft was necessary since the problem is not in symmetrical condition. In addition, a rigid body was created at the end of the cracked shaft under axial and torsional load; it was used to distribute the axial and torsional load evenly on the cracked shaft.

Solid 95 was assigned for the elements and the material properties of steel were used in the modelling of cracked shaft under axial, bending and torsional load. Meshing was carried out for the cracked shaft and a few criterions had been set in the ANSYS software. The mesh used in the cracked shaft under axial load as shown in Figure 4.4 was extremely fine near the crack tip since the study of stress intensity factors was focused on it, while the mesh is coarse for the part far away from the crack tip. These meshing criterions were also applied for a cracked shaft under bending and torsional load as shown in Figure 4.5 and Figure 4.6 respectively. Although fine mesh will increase the solver time, but it provides high accuracy of the results if compared with coarse mesh. Normally coarse mesh is appropriate to be used only on the area where no study is been carried out.

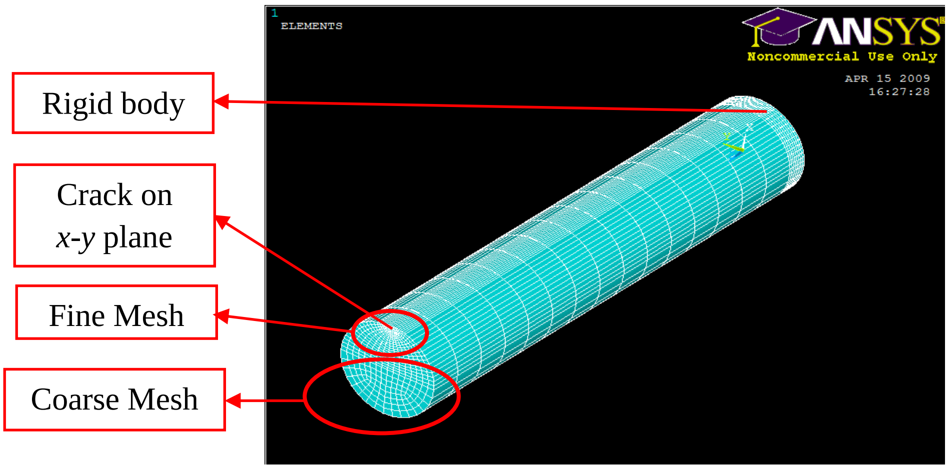


Figure 4.4: Meshing of 3 mm cracked shaft under axial load.

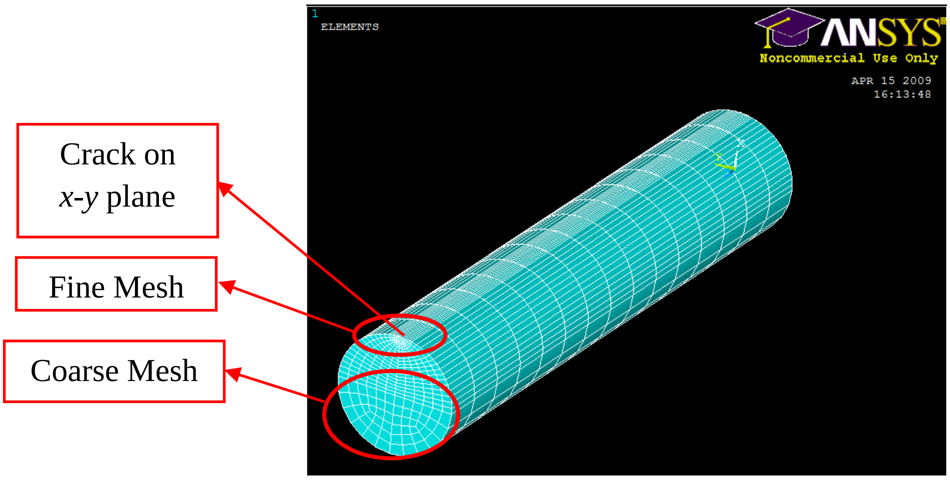


Figure 4.5: Meshing of 3 mm cracked shaft under bending load.

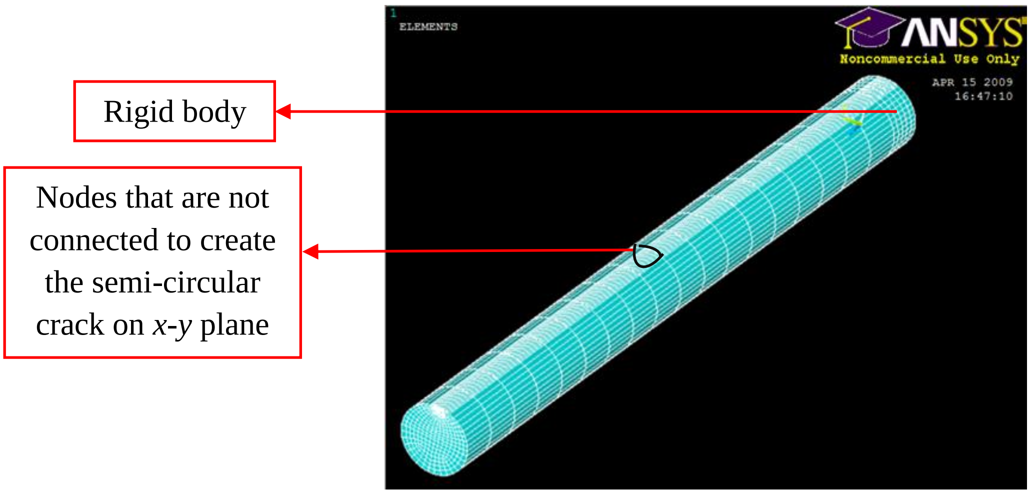


Figure 4.6: Meshing of 3 mm cracked shaft under torsional load.

When all the tasks above were accomplished, boundary conditions were set for the modelling above. For the cracked shaft under axial load, the displacement in the *z*-direction must be set to zero since the shaft is symmetrical in *z*-direction and the force is applied in *z*-direction. Besides, the rotation in *x* and *y*-axis must be set to zero to prevent any rotation in *x* and *y*-axis. Figure 4.7 shows the force of 1000 N being applied in negative *z*-direction at the end of the shaft in order to create an axial load.

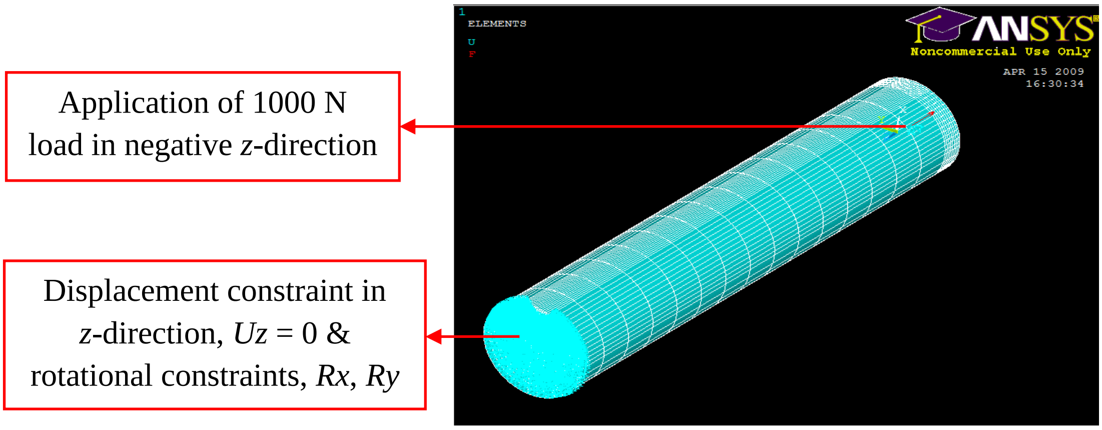


Figure 4.7: Boundary condition for 3 mm cracked shaft under axial load.

For the cracked shaft under bending load, the displacement in *x* and *z*-direction must be set to zero since the shaft is symmetrical in *z*-direction and the force is applied in *x*-direction. At the same time, the rotation in *x* and *y*-axis must be set to zero to prevent any rotation in *x* and *y*-axis. Figure 4.8 shows the force of 100 N being applied in negative *x*-direction at the end of the shaft in order to create a bending moment.

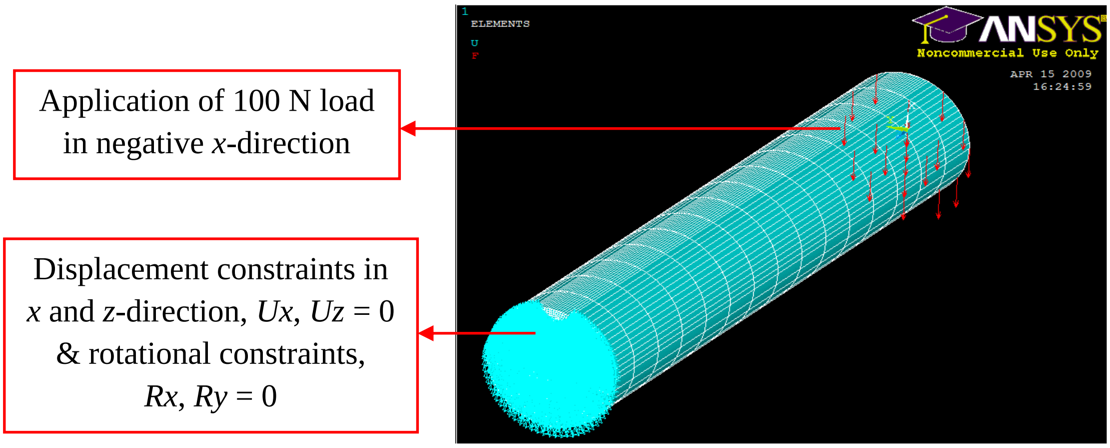


Figure 4.8: Boundary condition for 3 mm cracked shaft under bending load.

For the cracked shaft under torsional load, the displacement in *x* and *y*-direction at one end were set to zero and together with the rotational constraint in *z*-axis. In order to create 10 Nm of torque, forces at four different locations at the other end of the cracked shaft were applied as shown in Figure 4.9 below.

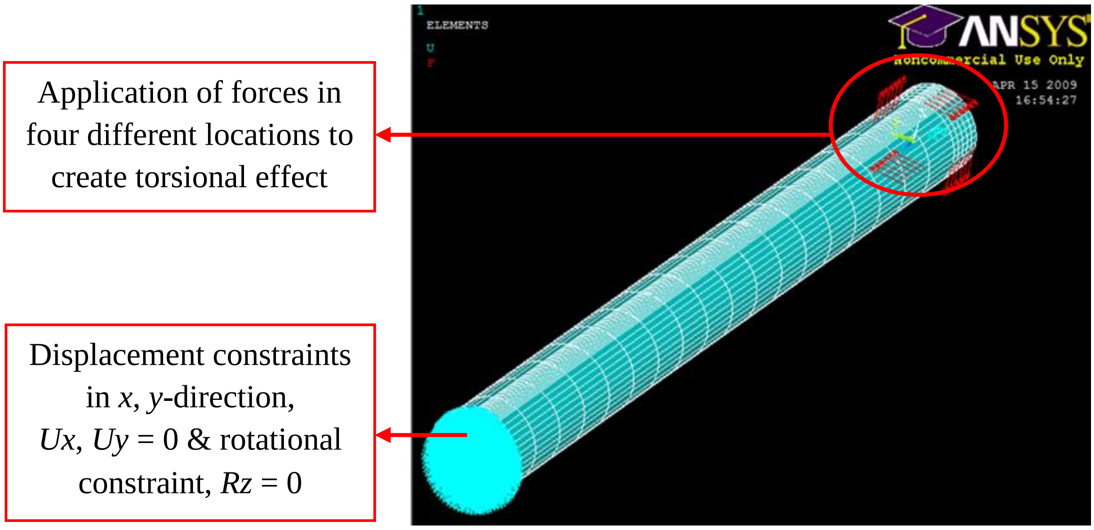


Figure 4.9: Boundary conditions for 3 mm cracked shaft under torsional load.

4.4.2 Processing Phase

During the Processing Phase, a series of calculations were performed by the ANSYS software. It was used to calculate the results of the modelling. In fact, the number of nodes and elements are directly proportional to the solver time, i.e. fine mesh which provides a large number of nodes and elements increases the solver time while coarse mesh which provides a small number of nodes and elements decreases the solver time.

4.4.3 Post-Processing Phase

During the Post-Processing Phase, result interpretation was carried out to determine the stress experienced by the cracked shaft. Besides, the stress intensity factor for a cracked shaft under axial, bending and torsional load is obtained by using the *KCAL* command in the ANSYS software. For the 3 mm cracked shaft under axial load, the normal stress can be obtained from the diagram plotted by ANSYS software. Figure 4.10 shows the maximum normal stress of 17.703 MPa located at the crack tip.

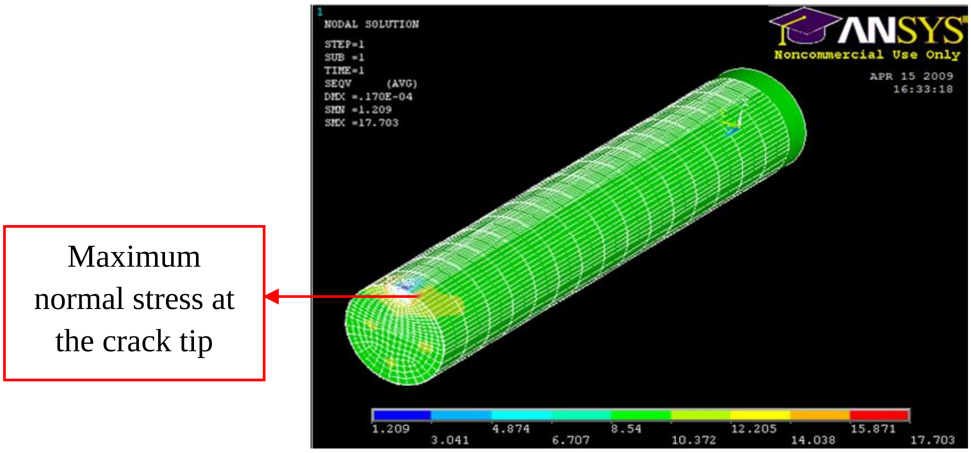


Figure 4.10: Normal stress for 3 mm cracked shaft under axial load.

For a cracked shaft under bending and torsional load, the steps to determine the bending stress and the shear stress are the same as the steps to determine the normal stress for a cracked shaft under axial load. Figure 4.11 and Figure 4.12 show the maximum bending stress and the shear stress for a 3 mm cracked shaft under bending and torsional load respectively.

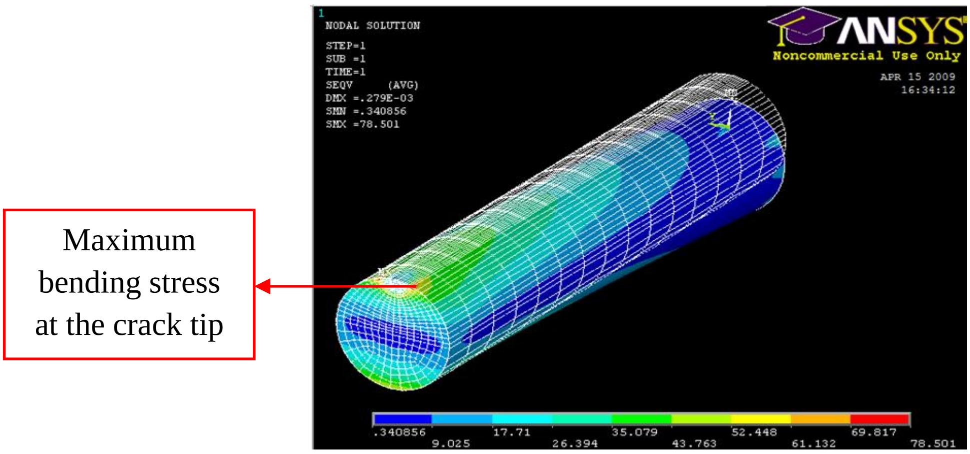


Figure 4.11: Bending stress for 3 mm cracked shaft under bending load.

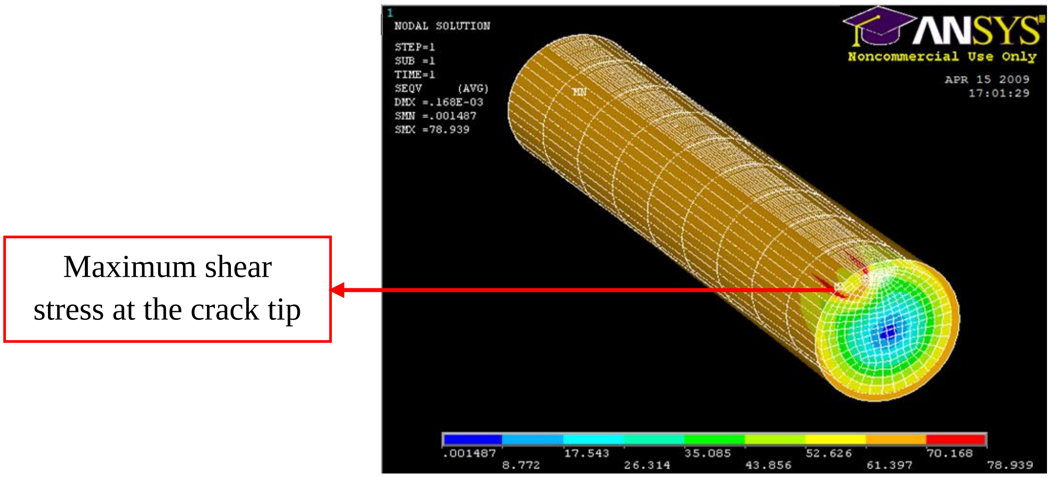


Figure 4.12: Shear stress for 3 mm cracked shaft under torsional load.

After obtaining the maximum normal stress at the crack tip, path operations had to be defined by selecting 3 nodes at the crack tip. These 3 nodes were then extrapolated by ANSYS Software and the stress intensity factor value was calculated. By referring to Figure 4.13, the stress intensity factor for 3 mm cracked shaft under specified axial load (1000N) is 0.6975 MPa$\sqrt{\text{m}}$ (refer Appendix *A* for details) and this value will be then compared with the result from the semi-analytical method.

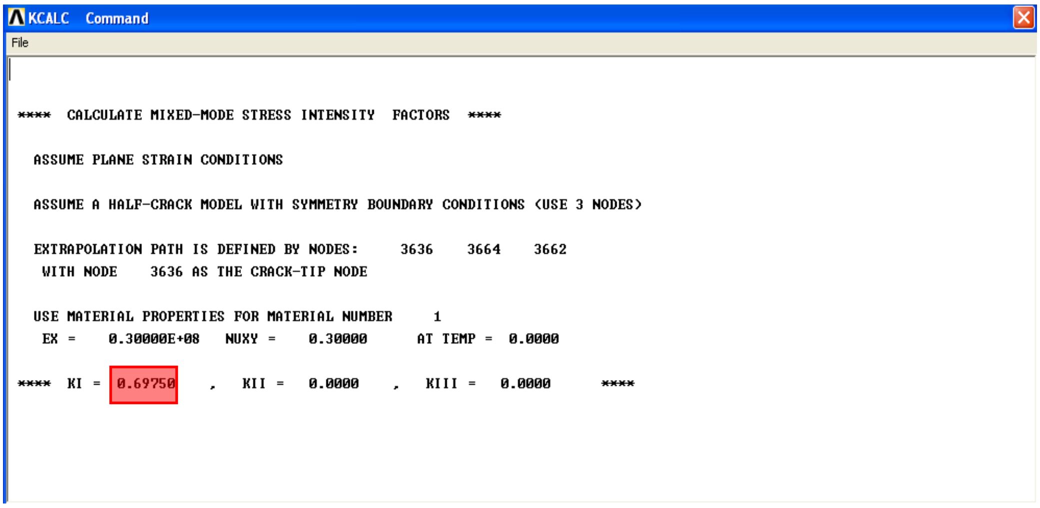


Figure 4.13: Stress intensity factor for 3 mm cracked shaft under axial load.

For a cracked shaft under bending and torsional load, the steps to determine the stress intensity factor are the same as the steps to determine the stress intensity factor for a cracked shaft under axial load. Figure 4.14 and Figure 4.15 show the stress intensity factor for a 3 mm cracked shaft under bending and torsional load respectively (refer Appendix *B* and Appendix *C* for details).

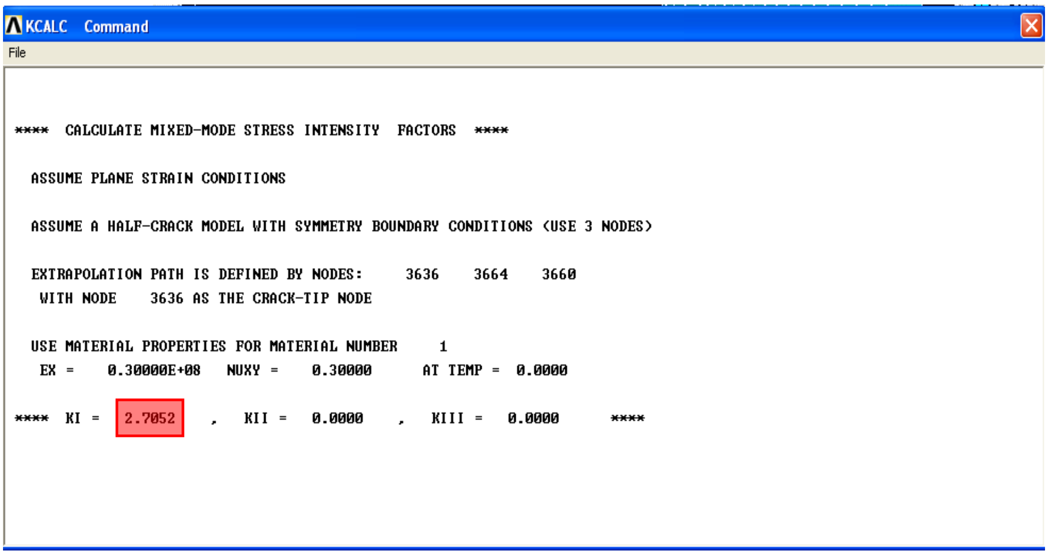


Figure 4.14: Stress intensity factor for 3 mm cracked shaft under bending load.

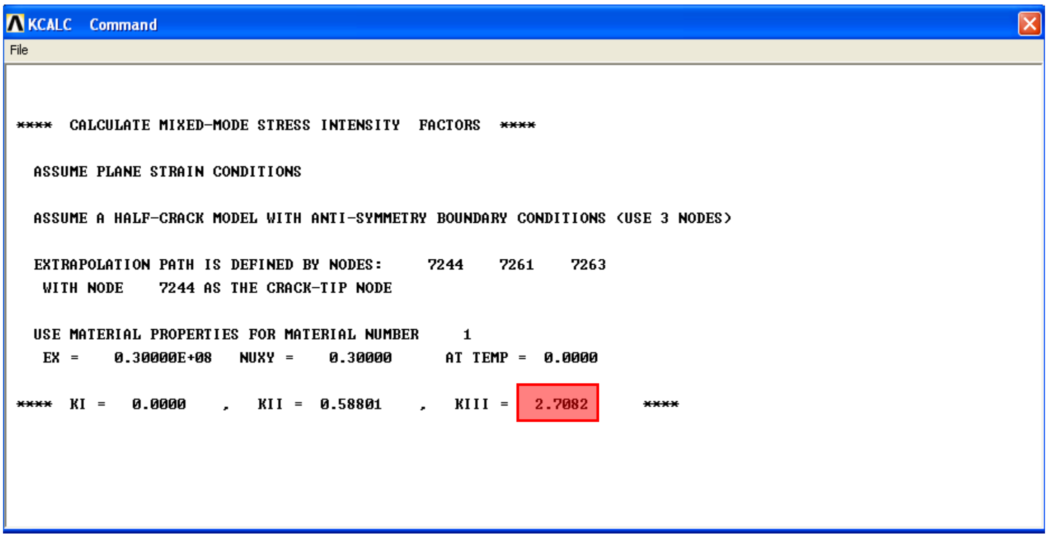


Figure 4.15: Stress intensity factor for 3 mm cracked shaft under torsional load.

CHAPTER 5
RESULTS AND DISCUSSION

5.1 SIF DETERMINED BY USING THE SEMI-ANALYTICAL METHOD

By referring to the equations obtained from the stress handbook [1], the stress intensity factors of a cracked shaft under axial, bending and torsional load were calculated. Since the results calculated from the semi-analytical method were considered the most accurate, they directly served as the reference results for the numerical method. All the semi-analytical method equations were solved by using *Microsoft Excel*. A total of 9 models from each crack loading had been calculated, starting from 1 mm crack width to 9 mm crack width. The stress intensity factors obtained by using the semi-analytical method were then grouped in dimensionless form as shown in Table 5.1.

Table 5.1: Dimensionless stress intensity factors for a cracked shaft determined by using the semi-analytical method

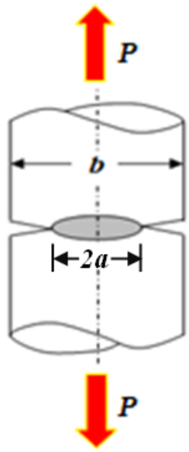
Normalised Relative Crack Depth (b/r)	**Dimensionless SIF (K_I/K_0) (Axial Load)**	**Dimensionless SIF (K_I/K_0) (Bending Load)**	**Dimensionless SIF (K_{III}/K_0) (Torsional Load)**
0.1	0.3113	0.2939	0.2708
0.2	0.3921	0.3502	0.3321
0.3	0.4340	0.3691	0.3577
0.4	0.4590	0.3747	0.3686
0.5	0.4752	0.3757	0.3729
0.6	0.4862	0.3754	0.3745
0.7	0.4936	0.3752	0.3749
0.8	0.4979	0.3750	0.3750
0.9	0.4997	0.3750	0.3750

According to the results above, the dimensionless stress intensity factors for a cracked shaft under axial, bending and torsional load increases as the normalised relative crack depth increases. This is because when the crack width is increasing, the normal stress, bending stress and the shear stress experienced by the cracked shaft is higher at the crack tip.

Example calculation for 3 mm cracked shaft under axial load is as shown below:

Given condition:

Shaft's diameter, b = 10 mm
Diameter of uncracked region, $2a$ = 7 mm
Crack width, c = 3 mm
Axial Load, P = 1000 N
Ratio of $\frac{2a}{b} = \frac{7\text{ mm}}{10\text{ mm}} = 0.7$
Average radius of the cracked shaft, r = 4.371 mm (Refer Appendix D for details)



Normal Stress, $\sigma_N = \frac{P}{\pi r^2}$

$$= \frac{1000}{\pi(0.004371)^2}$$

$$= 16.661 \text{ MPa}$$

Geometry factor, $G(a/b) = \frac{1}{2}\left[1 + \frac{1}{2}\frac{2a}{b} + \frac{3}{8}\left(\frac{2a}{b}\right)^2 - 0.363\left(\frac{2a}{b}\right)^3 + 0.731\left(\frac{2a}{b}\right)^4\right]$

$$= \frac{1}{2}\left[1 + \frac{1}{2}(0.7) + \frac{3}{8}(0.7)^2 - 0.363(0.7)^3 + 0.731(0.7)^4\right]$$

$$= 0.792$$

Boundary Correction Factor, $F_I(a/b) = \sqrt{1 - \frac{2a}{b}}\, G(a/b)$

$$= \sqrt{1 - 0.7} \times 0.792$$

$$= 0.434$$

Stress intensity factor for Mode I loading,

$$K_I = \sigma_N \sqrt{\pi c}\, F_I(a/b)$$

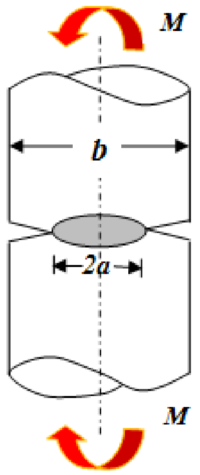
$$= 16.661 \times 10^6 \times \sqrt{\pi(0.003)} \times 0.434$$

$$= 0.702 \text{ MPa}\sqrt{\text{m}}$$

Example calculation for 3 mm cracked shaft under bending load is as shown below:

Given condition:

Shaft's diameter, b = 10 mm
Diameter of uncracked region, $2a$ = 7 mm
Crack width, c = 3 mm
Moment, M = 5 Nm
Ratio of $\frac{2a}{b} = \frac{7\ mm}{10\ mm} = 0.7$
Average radius of the cracked shaft, r = 4.371 mm (Refer Appendix D for details)



Bending Stress, $\sigma = \frac{4M}{\pi r^3}$,

$$= \frac{4\ (5)}{\pi (0.004371)^3}$$

$$= 76.232 \text{ MPa}$$

Geometry Factor, $G(a/b) = \frac{3}{8}\left[1 + \frac{1}{2}\frac{2a}{b} + \frac{3}{8}\left(\frac{2a}{b}\right)^2 + \frac{5}{16}\left(\frac{2a}{b}\right)^3 + \frac{35}{128}\left(\frac{2a}{b}\right)^4 + 0.537\left(\frac{2a}{b}\right)^5\right]$

$$= \frac{3}{8}\left[1 + \frac{1}{2}(0.7) + \frac{3}{8}(0.7)^2 + \frac{5}{16}(0.7)^3 + \frac{35}{128}(0.7)^4 + 0.537(0.7)^5\right]$$

$$= 0.674$$

Boundary Correction Factor, $F_I(a/b) = \sqrt{1 - \frac{2a}{b}}\, G(a/b)$

$$= \sqrt{1 - 0.7} \times 0.674$$

$$= 0.369$$

Stress Intensity Factor for Mode I loading,

$$K_I = \sigma \sqrt{\pi c}\, F_I(a/b)$$

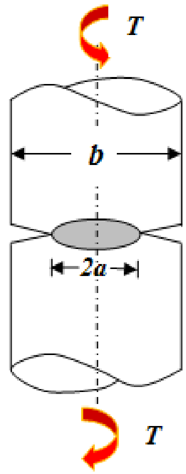
$$= 76.232 \times 10^6 \times \sqrt{\pi\ (0.003)} \times 0.369$$

$$= 2.731 \text{ MPa}\sqrt{\text{m}}$$

Example calculation for 3 mm cracked shaft under torsional load is as shown below:

Given Condition,

Shaft's diameter, b = 10 mm
Diameter of uncracked region, $2a$ = 7 mm
Crack width, c = 3 mm
Torque, T = 10 Nm
Ratio of $\frac{2a}{b} = \frac{7\ mm}{10\ mm} = 0.7$
Average radius of the cracked shaft, r = 4.371 mm (Refer Appendix D for details)



Shear Stress, $\tau = \frac{2T}{\pi r^3}$

$$= \frac{2(10)}{\pi(0.004371)^3}$$

$$= 76.232 \text{ MPa}$$

Geometry Factor, G (a/b) $= \frac{3}{8}\left[1 + \frac{1}{2}\frac{2a}{b} + \frac{3}{8}\left(\frac{2a}{b}\right)^2 + \frac{5}{16}\left(\frac{2a}{b}\right)^3 + \frac{35}{128}\left(\frac{2a}{b}\right)^4 + 0.208\left(\frac{2a}{b}\right)^5\right]$

$$= \frac{3}{8}\left[1 + \frac{1}{2}(0.7) + \frac{3}{8}(0.7)^2 + \frac{5}{16}(0.7)^3 + \frac{35}{128}(0.7)^4 + 0.208(0.7)^5\right]$$

$$= 0.653$$

Boundary Correction Factor, $F_I(a/b) = \sqrt{1 - \frac{2a}{b}}\, G(a/b)$

$$= \sqrt{1 - 0.7} \times 0.653$$

$$= 0.358$$

Stress Intensity Factor for Mode III loading,

$$K_{III} = \tau\sqrt{\pi c}\ \ F_I(a/b)$$

$$= 76.232 \times 10^6 \times \sqrt{\pi(0.003)} \times 0.358$$

$$= 2.647 \text{ MPa}\sqrt{\text{m}}$$

5.2 SIF DETERMINED BY USING THE NUMERICAL METHOD

During the finite element analysis on a cracked shaft under axial and bending load, only half of the shaft was modelled due to the symmetrical condition. While for the finite element analysis on a cracked shaft under torsional load, a whole shaft was modelled due to the non-symmetrical condition. A total of 9 models had been created, starting from 1 mm crack width to 9 mm crack width. The stress intensity factors' values calculated from ANSYS software's Post-Processing phase are summarized in dimensionless form as shown in Table 5.2.

Table 5.2: Dimensionless stress intensity factors for a cracked shaft determined by using the numerical method

Normalised Relative Crack Depth (b/r)	**Dimensionless SIF (K_I/K_0) (Axial Load)**	**Dimensionless SIF (K_I/K_0) (Bending Load)**	**Dimensionless SIF (K_{III}/K_0) (Torsional Load)**
0.1	0.2899	0.3076	0.2717
0.2	0.4042	0.3456	0.3232
0.3	0.4312	0.3655	0.3659
0.4	0.4611	0.3667	0.3561
0.5	0.4727	0.3689	0.3640
0.6	0.4910	0.3752	0.3841
0.7	0.5039	0.3670	0.3782
0.8	0.5074	0.3698	0.3834
0.9	0.5019	0.3648	0.3842

Referring to the results above, the dimensionless stress intensity factors determined by using the numerical method is increasing when the normalised relative crack depth increases. Based on the results, the increasing trend is the same if compared with the dimensionless stress intensity factors determined semi-analytically. Detail comparisons of the results obtained semi-analytically and numerically will be discussed in the next section.

5.3 RESULTS COMPARISON

5.3.1 Results Comparison for a Cracked Shaft under Axial Load

Mode I stress intensity factors obtained by using semi-analytical and numerical methods were then compared with each other in dimensionless form. For the cracked shaft under axial load, the deviation of results in term of percentage is in the range of minimum 0.4% to the maximum of 6.9% as shown in Table 5.3. These deviations are considered acceptable since the maximum difference is less than 10%.

Table 5.3: Dimensionless SIF comparison for a cracked shaft under axial load

Normalised Relative Crack Depth (*b/r*)	Dimensionless SIF by Semi-Analytical Method (K_I/K_0)	Dimensionless SIF by Numerical Method (K_I/K_0)	Deviation (%)
0.1	0.3113	0.2899	6.9
0.2	0.3921	0.4042	3.1
0.3	0.4340	0.4312	0.6
0.4	0.4590	0.4611	0.5
0.5	0.4752	0.4727	0.5
0.6	0.4862	0.4910	1.0
0.7	0.4936	0.5039	2.1
0.8	0.4979	0.5074	1.9
0.9	0.4997	0.5019	0.4

In order to present the results in a better way, a graph of dimensionless stress intensity factor versus normalised relative crack depth, *b/r* had been plotted as shown in Figure 5.1. The graph shows that the results obtained from the semi-analytical and the numerical methods are close to each other. The results are almost identical for the relative crack depth of 0.3, 0.4, 0.5 and 0.9. In conclusion, the results obtained from the numerical method were verified to be correct.

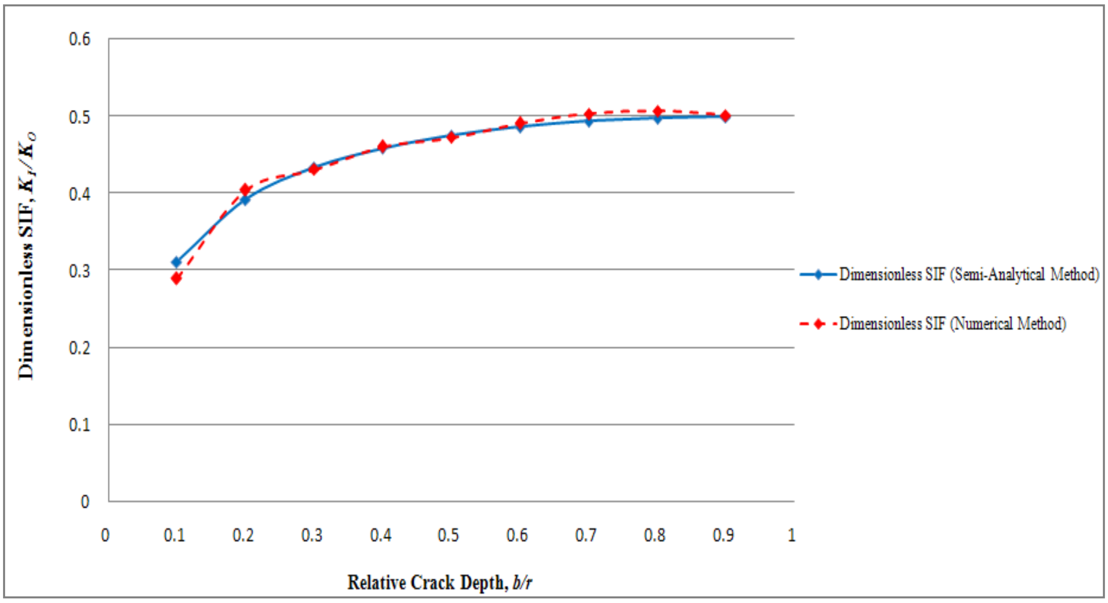


Figure 5.1: Dimensionless stress intensity factor versus normalised relative crack depth under axial load.

5.3.2 Results Comparison for a Cracked Shaft under Bending Load

Mode I stress intensity factors obtained by using semi-analytical and numerical methods were then compared with each other in dimensionless form. For the cracked shaft under bending load, the deviation of the results in term of percentage is in the range of minimum 0.1% to the maximum of 4.7% as shown in Table 5.4. These deviations are considered acceptable since the maximum difference is less than 5%.

Table 5.4: Dimensionless SIF comparison for a cracked shaft under bending load

Normalised Relative Crack Depth (*b/r*)	**Dimensionless SIF by Semi-Analytical Method (K_I/K_0)**	**Dimensionless SIF by Numerical Method (K_I/K_0)**	**Deviation (%)**
0.1	0.2939	0.3076	4.7
0.2	0.3502	0.3456	1.3
0.3	0.3691	0.3655	1.0
0.4	0.3747	0.3667	2.1
0.5	0.3757	0.3689	1.8
0.6	0.3754	0.3752	0.1
0.7	0.3752	0.3670	2.2
0.8	0.3750	0.3698	1.4
0.9	0.3750	0.3648	2.7

In order to present the results in a better way, a graph of dimensionless stress intensity factor versus normalized relative crack depth, *b/r* had been plotted as shown in Figure 5.2. The graph shows that the results obtained from the semi-analytical and the numerical methods are close to each other. The results are almost identical for the relative crack depth of 0.2, 0.3, 0.5, 0.6 and 0.8. In conclusion, the results obtained from the numerical method were verified to be correct.

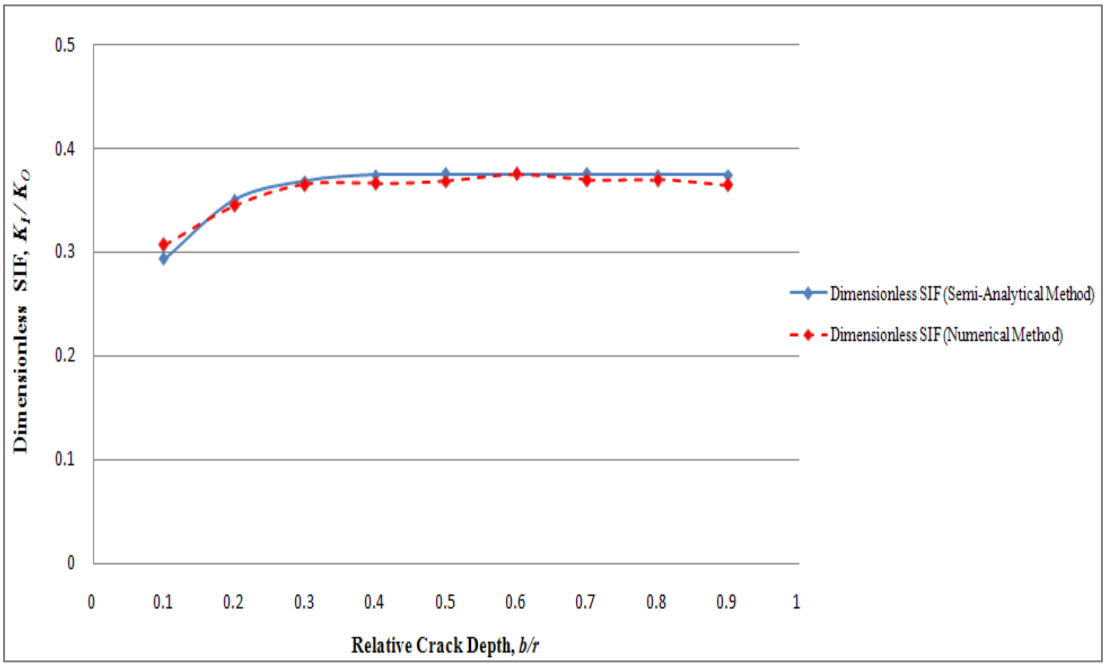


Figure 5.2: Dimensionless stress intensity factor versus normalised relative crack depth under bending load.

5.3.3 Results Comparison for a Cracked Shaft under Torsional Load

Mode III stress intensity factors obtained by using semi-analytical and numerical methods were then compared with each other in dimensionless form. For the cracked shaft under torsional load, the deviation of the results in term of percentage is in the range of minimum 0.3% to the maximum of 3.4% as shown in Table 5.5. These deviations are considered acceptable since the maximum difference is less than 5%.

Table 5.5: Dimensionless SIF comparison for a cracked shaft under torsional load

Normalised Relative Crack Depth (*b/r*)	**Dimensionless SIF by Semi-Analytical Method (K_{III}/K_0)**	**Dimensionless SIF by Numerical Method (K_{III}/K_0)**	**Deviation (%)**
0.1	0.2708	0.2717	0.3
0.2	0.3321	0.3232	2.7
0.3	0.3577	0.3659	2.3
0.4	0.3686	0.3561	3.4
0.5	0.3729	0.3640	2.4
0.6	0.3745	0.3841	2.6
0.7	0.3749	0.3782	0.9
0.8	0.3750	0.3834	2.2
0.9	0.3750	0.3842	2.5

In order to present the results in a better way, a graph of dimensionless stress intensity factor versus normalised relative crack depth, *b/r* had been plotted as shown in Figure 5.3. The graph shows that the results obtained from the semi-analytical and the numerical methods are close to each other. The results are almost identical for the relative crack depth of 0.1, 0.3, 0.5, 0.6 and 0.8. In conclusion, the results obtained from the numerical method were verified to be correct.

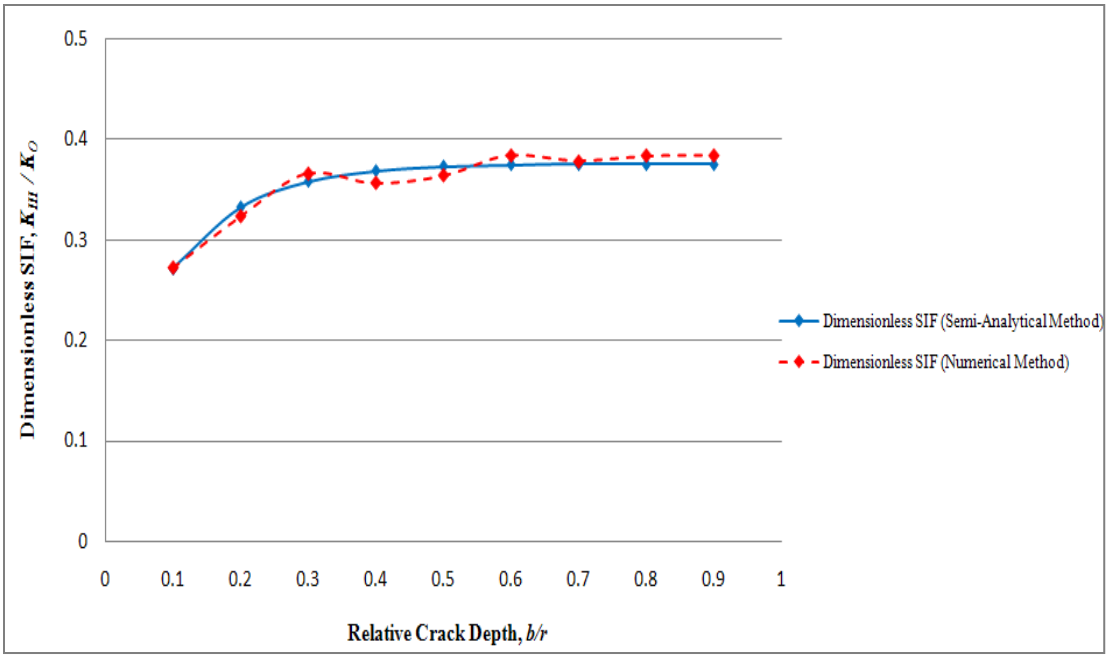


Figure 5.3: Dimensionless stress intensity factor versus normalised relative crack depth under torsional load.

5.4 IMPROVING ACCURACY BY CONVERGENCE ANALYSIS

The number of elements in a model plays an important role when accuracy is considered. When the numbers of elements are increased, the deviation of the results is decreased. In order to prove that this concept is valid, four same models of 8 mm cracked shaft had been modelled and all the parameters were set to be the same except the number of elements. By using fine mesh, the results are closer to the results obtained semi-analytically. By referring to Figure 5.4, the convergence study on the cracked models indicated that the model in this project is sufficiently refined.

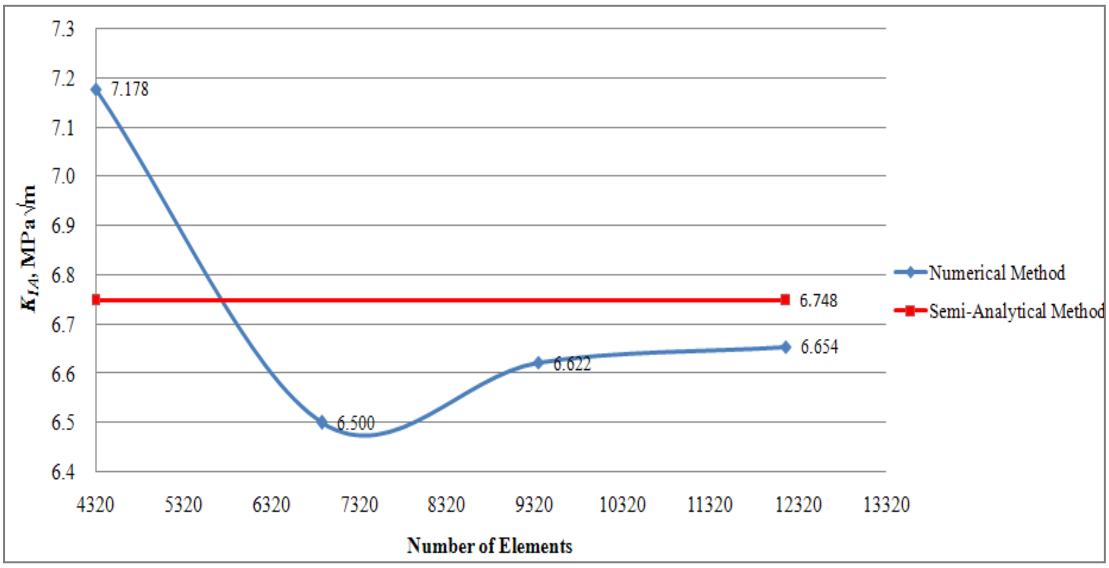


Figure 5.4: Stress intensity factor for 8 mm cracked shaft under bending load versus number of elements.

Based on Figure 5.4, the model which had 4320 elements experienced the highest deviation, which was 6.4% if compared with the result obtained semi-analytically. When the number of elements was increased to 6900, followed with another model which had 9360 elements, the results started to converge to the results obtained semi-analytically. Lastly, the finest model which had 12180 elements experienced the least deviation which was 1.4% if compared to the results obtained semi-analytically. The convergence analysis proved that the number of elements play an important role in obtaining an accurate results.

CHAPTER 6
CONCLUSION AND RECOMMENDATIONS

6.1 CONCLUSION

Fatigue cracks in the rotating components especially the pump shafts, drive shafts and etc are a significant economic and safety concern. These fatigue cracks in the shafts require expensive repair or replacement cost and sometimes even worst it will caused severe injury to the operators when the shafts failed during their services. Hence, an assessment of the cracked shafts was carried out to calculate the stress intensity factor and to check the safety level of the cracked shafts.

For this project, the cracked shafts were subjected to axial, bending and torsional load respectively. These three sets of calculations were performed by using semi-analytical and numerical method. For the semi-analytical method, the stress intensity factors of a cracked shaft were calculated by using the general equations obtained from the stress handbook. While for the numerical method, three sets of modelling under axial, bending and torsional load were modelled successfully.

By comparing the results obtained semi-analytically and numerically, the deviation in term of percentage had been found to be relatively small. In conclusion, the stress intensity factors of a cracked shaft determined by using the numerical method were verified to be accurate. By using the numerical method, it significantly shortens the stress intensity factors computing time and save money.

6.2 RECOMMENDATIONS

After completing this project, there are several pieces of work which merit further study in order to get a better understanding about a cracked shaft condition. The recommendations prompted by this project which include the following:

i. Analyze the cracked shaft under dynamic load

   In a real-world condition, shafts are normally operated under dynamic load instead of static load which was applied in this project. Hence, it is necessary to analyze a cracked shaft under dynamic load in order to get a better understanding or perspective of it.

ii. Optimize the meshing of a cracked shaft

   Based on section 5.4, mesh optimization of a cracked shaft during modelling is essential since it gives accurate results. For example, it is necessary to mesh the point of interest by using the fine mesh, while coarse mesh is applied for the part far away from it. Although the aim of using finite element method in determining stress intensity factor is to reduce the computing time, it is also necessary to get a balance point in between accuracy and computing time.

REFERENCES

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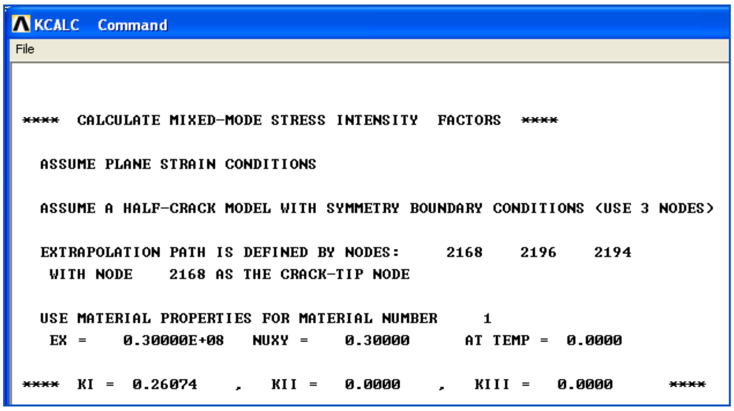
[11] R. J. Sanford, “Principle of Fracture Mechanics”, 2003, Prentice Hall

APPENDICES

Appendix A: Stress intensity factor for a cracked shaft under axial load

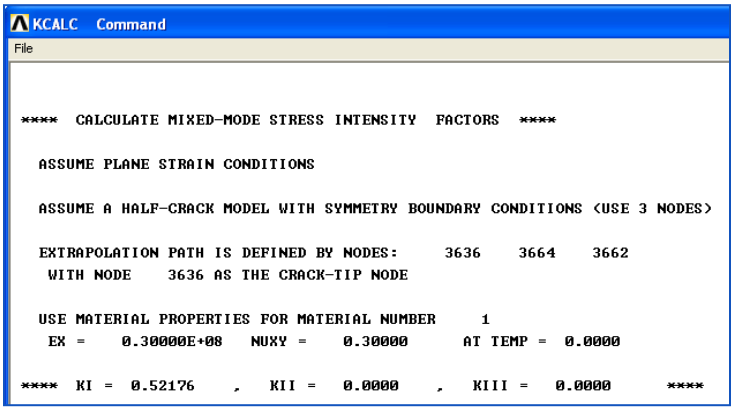
SIF for 1 mm cracked shaft



KCALC Command
File

```
****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS  ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:     2168     2196     2194
   WITH NODE     2168 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER      1
   EX =     0.30000E+08    NUXY =     0.30000      AT TEMP =  0.0000

****  KI =  0.26074     ,   KII =    0.0000    ,   KIII =    0.0000       ****
```

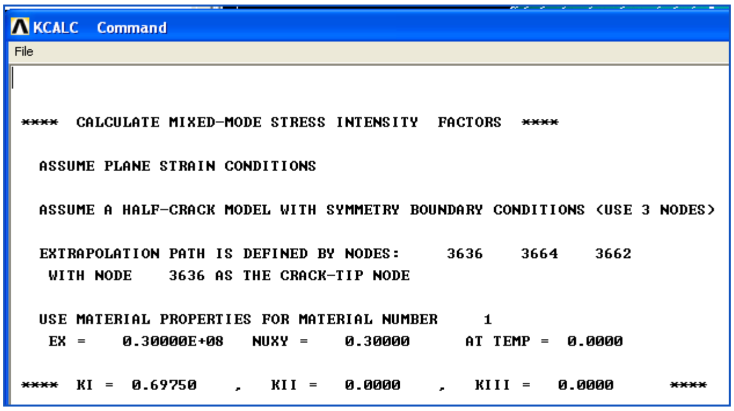
SIF for 2 mm cracked shaft



KCALC Command
File

```
****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS  ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:     3636     3664     3662
   WITH NODE     3636 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER      1
   EX =     0.30000E+08    NUXY =     0.30000      AT TEMP =  0.0000

****  KI =  0.52176     ,   KII =    0.0000    ,   KIII =    0.0000       ****
```

SIF for 3 mm cracked shaft



KCALC Command
File

```
****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS  ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:     3636     3664     3662
   WITH NODE     3636 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER      1
   EX =     0.30000E+08    NUXY =     0.30000      AT TEMP =  0.0000

****  KI =  0.69750     ,   KII =    0.0000    ,   KIII =    0.0000       ****
```

SIF for 4 mm cracked shaft

```
KCALC Command
File

**** CALCULATE MIXED-MODE STRESS INTENSITY FACTORS ****

 ASSUME PLANE STRAIN CONDITIONS

 ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

 EXTRAPOLATION PATH IS DEFINED BY NODES:     3636    3664    3660
  WITH NODE    3636 AS THE CRACK-TIP NODE

 USE MATERIAL PROPERTIES FOR MATERIAL NUMBER     1
  EX =    0.30000E+08    NUXY =    0.30000      AT TEMP =  0.0000

**** KI =  0.89072     ,  KII =   0.0000     ,  KIII =   0.0000       ****
```

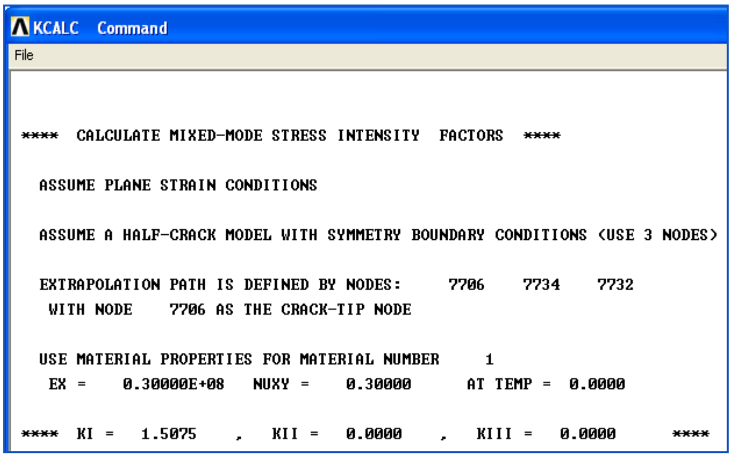
SIF for 5 mm cracked shaft

```
KCALC Command
File

**** CALCULATE MIXED-MODE STRESS INTENSITY FACTORS ****

 ASSUME PLANE STRAIN CONDITIONS

 ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

 EXTRAPOLATION PATH IS DEFINED BY NODES:     5482    5510    5502
  WITH NODE    5482 AS THE CRACK-TIP NODE

 USE MATERIAL PROPERTIES FOR MATERIAL NUMBER     1
  EX =    0.30000E+08    NUXY =    0.30000      AT TEMP =  0.0000

**** KI =   1.0650     ,  KII =   0.0000     ,  KIII =   0.0000       ****
```

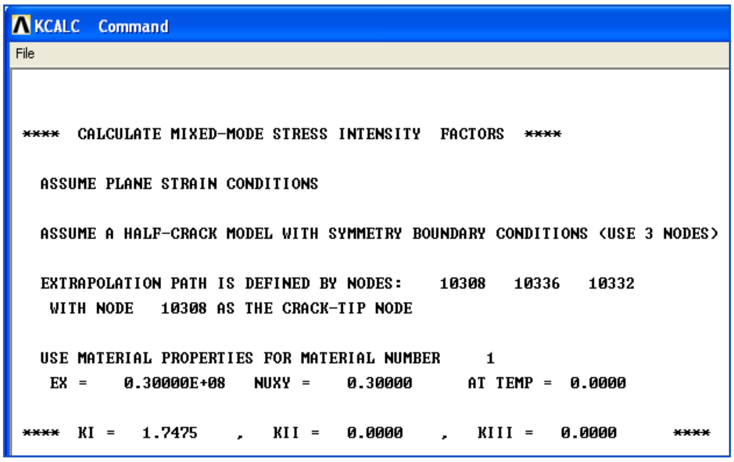
SIF for 6 mm cracked shaft

```
KCALC Command
File

**** CALCULATE MIXED-MODE STRESS INTENSITY FACTORS ****

 ASSUME PLANE STRAIN CONDITIONS

 ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

 EXTRAPOLATION PATH IS DEFINED BY NODES:     5482    5510    5508
  WITH NODE    5482 AS THE CRACK-TIP NODE

 USE MATERIAL PROPERTIES FOR MATERIAL NUMBER     1
  EX =    0.30000E+08    NUXY =    0.30000      AT TEMP =  0.0000

**** KI =   1.2770     ,  KII =   0.0000     ,  KIII =   0.0000       ****
```

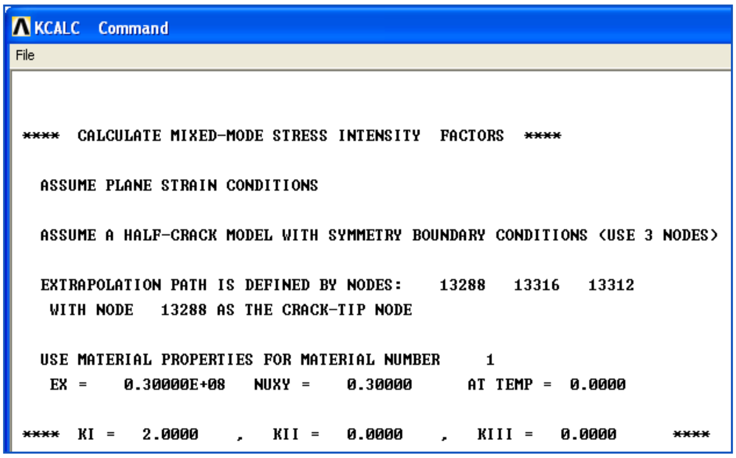
SIF for 7 mm cracked shaft



SIF for 8 mm cracked shaft

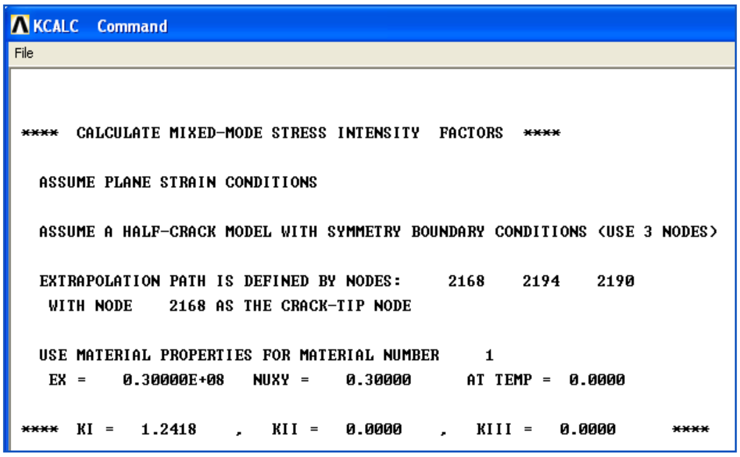


SIF for 9 mm cracked shaft

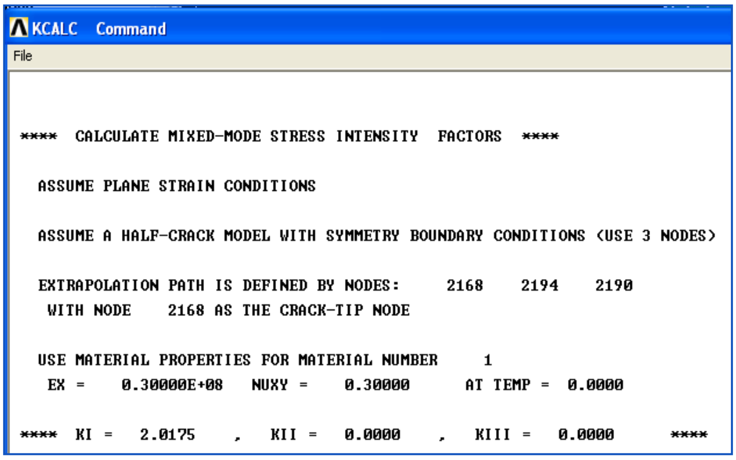


Appendix B: Stress intensity factor for a cracked shaft under bending load

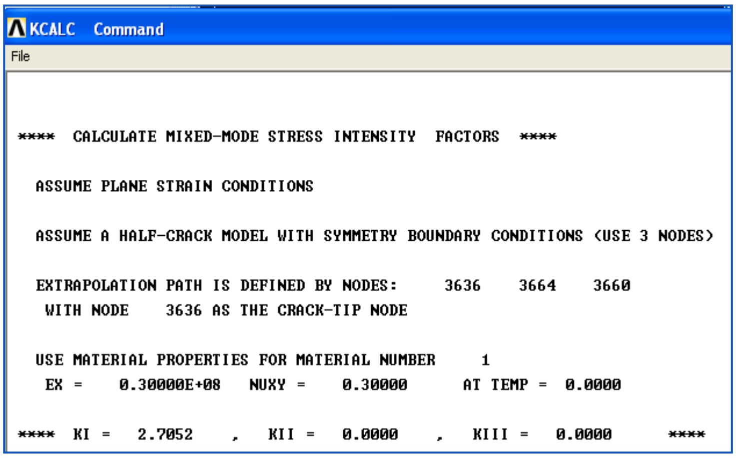
SIF for 1 mm cracked shaft



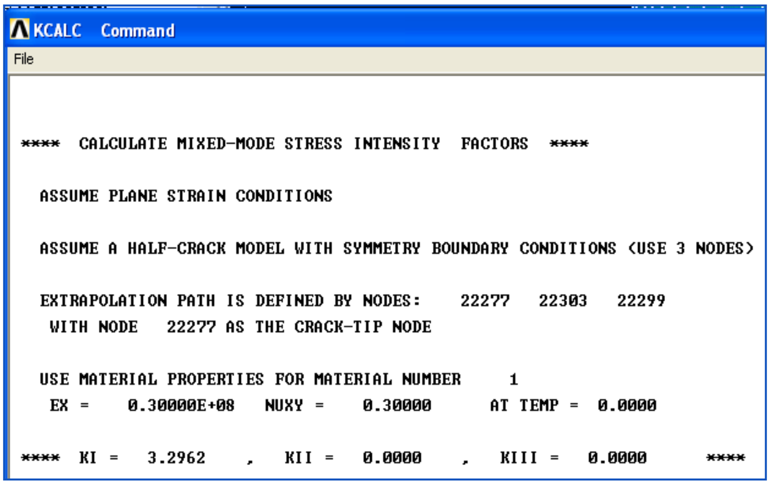
SIF for 2 mm cracked shaft



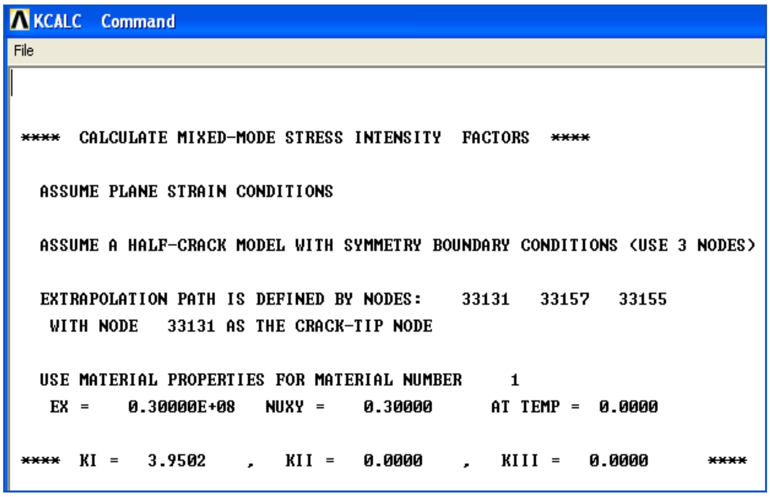
SIF for 3 mm cracked shaft



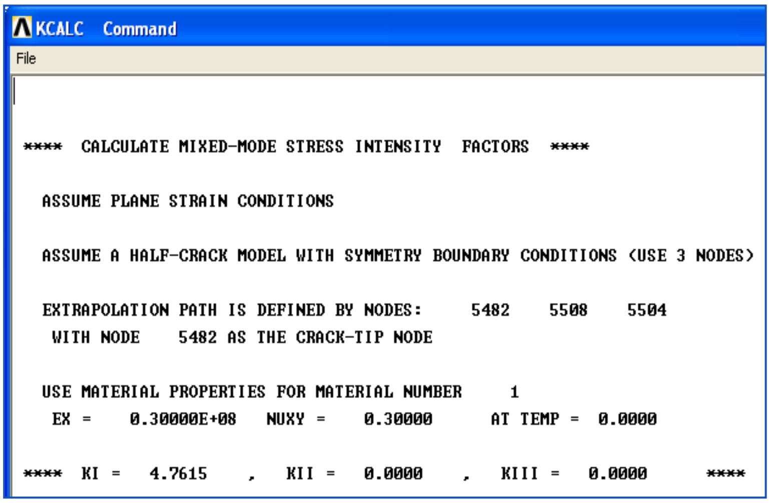
SIF for 4 mm cracked shaft



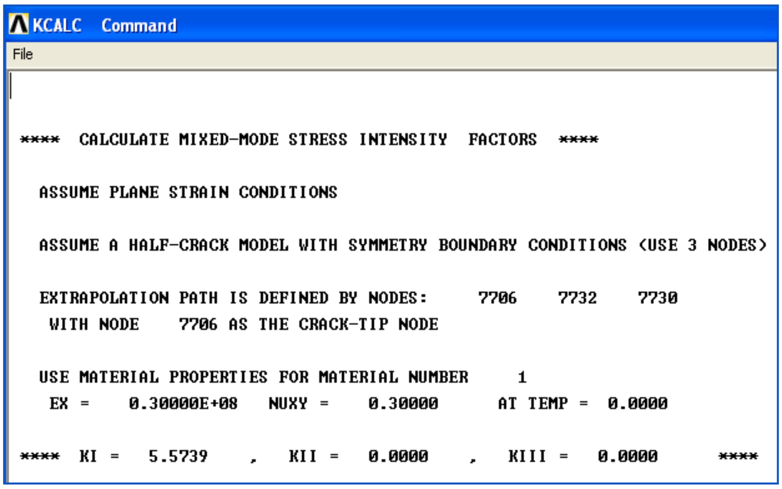
SIF for 5 mm cracked shaft



SIF for 6 mm cracked shaft

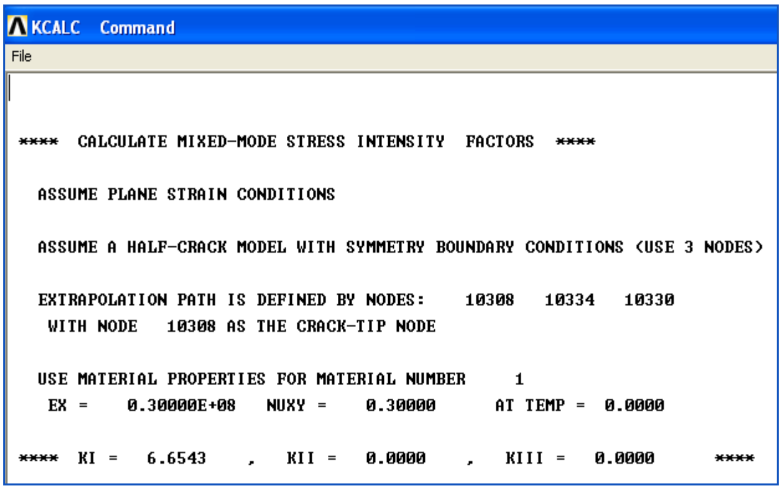


SIF for 7 mm cracked shaft



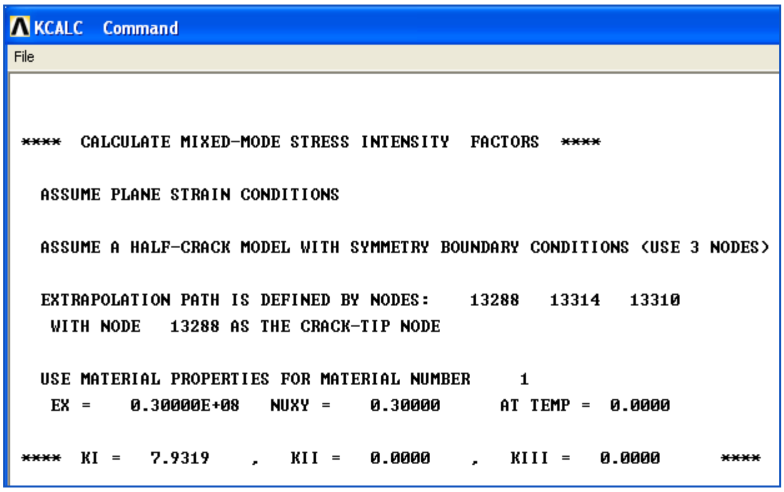
```
KCALC   Command
File

****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS  ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:     7706    7732    7730
   WITH NODE     7706 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER      1
   EX =    0.30000E+08    NUXY =    0.30000      AT TEMP =  0.0000

****  KI =    5.5739     ,  KII =   0.0000     ,  KIII =   0.0000       ****
```

SIF for 8 mm cracked shaft



```
KCALC   Command
File

****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS  ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:    10308    10334    10330
   WITH NODE    10308 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER      1
   EX =    0.30000E+08    NUXY =    0.30000      AT TEMP =  0.0000

****  KI =    6.6543     ,  KII =   0.0000     ,  KIII =   0.0000       ****
```

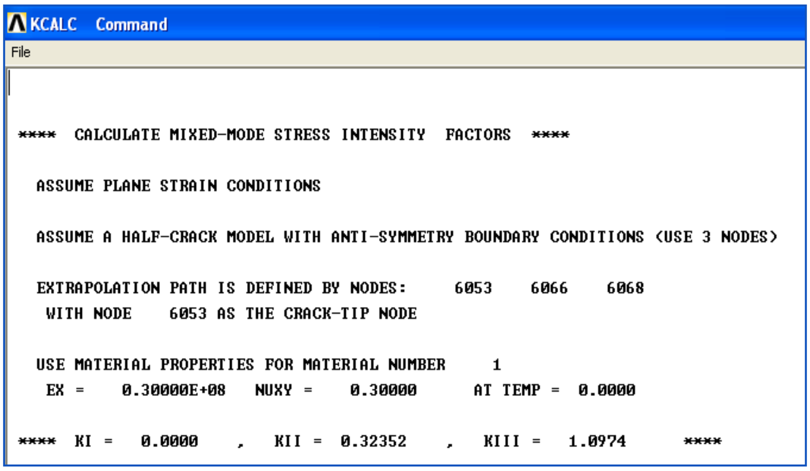
SIF for 9 mm cracked shaft



```
KCALC   Command
File

****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS  ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:    13288    13314    13310
   WITH NODE    13288 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER      1
   EX =    0.30000E+08    NUXY =    0.30000      AT TEMP =  0.0000

****  KI =    7.9319     ,  KII =   0.0000     ,  KIII =   0.0000       ****
```

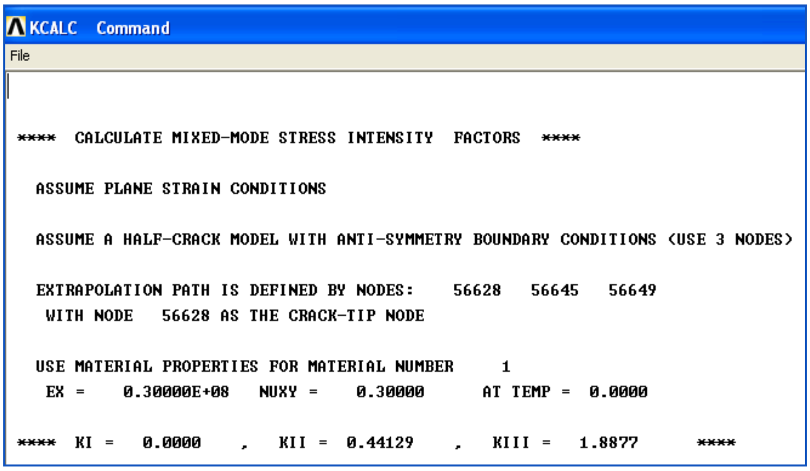
Appendix C: Stress intensity factor for a cracked shaft under torsional load

SIF for 1 mm cracked shaft



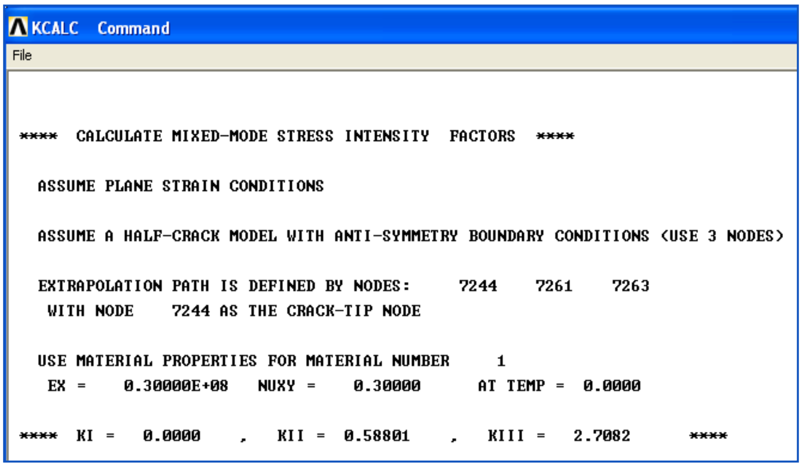
```
KCALC Command
File

**** CALCULATE MIXED-MODE STRESS INTENSITY FACTORS ****

ASSUME PLANE STRAIN CONDITIONS

ASSUME A HALF-CRACK MODEL WITH ANTI-SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

EXTRAPOLATION PATH IS DEFINED BY NODES:     6053     6066     6068
 WITH NODE     6053 AS THE CRACK-TIP NODE

USE MATERIAL PROPERTIES FOR MATERIAL NUMBER     1
 EX =     0.30000E+08    NUXY =     0.30000     AT TEMP =  0.0000

**** KI =    0.0000    ,  KII =  0.32352    ,  KIII =   1.0974     ****
```

SIF for 2 mm cracked shaft



```
KCALC Command
File

**** CALCULATE MIXED-MODE STRESS INTENSITY FACTORS ****

ASSUME PLANE STRAIN CONDITIONS

ASSUME A HALF-CRACK MODEL WITH ANTI-SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

EXTRAPOLATION PATH IS DEFINED BY NODES:     56628    56645    56649
 WITH NODE    56628 AS THE CRACK-TIP NODE

USE MATERIAL PROPERTIES FOR MATERIAL NUMBER     1
 EX =     0.30000E+08    NUXY =     0.30000     AT TEMP =  0.0000

**** KI =    0.0000    ,  KII =  0.44129    ,  KIII =   1.8877     ****
```

SIF for 3 mm cracked shaft



```
KCALC Command
File

**** CALCULATE MIXED-MODE STRESS INTENSITY FACTORS ****

ASSUME PLANE STRAIN CONDITIONS

ASSUME A HALF-CRACK MODEL WITH ANTI-SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

EXTRAPOLATION PATH IS DEFINED BY NODES:     7244     7261     7263
 WITH NODE     7244 AS THE CRACK-TIP NODE

USE MATERIAL PROPERTIES FOR MATERIAL NUMBER     1
 EX =     0.30000E+08    NUXY =     0.30000     AT TEMP =  0.0000

**** KI =    0.0000    ,  KII =  0.58801    ,  KIII =   2.7082     ****
```

SIF for 4 mm cracked shaft

```
KCALC  Command
File

****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS  ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH ANTI-SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:     7244     7261     7263
   WITH NODE     7244 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER      1
   EX =    0.30000E+08    NUXY =    0.30000      AT TEMP =  0.0000

****  KI =    0.0000    ,   KII =    3.3158    ,   KIII =    3.2012      ****
```

SIF for 5 mm cracked shaft

```
KCALC  Command
File

****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS  ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH ANTI-SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:    12317    12338    12340
   WITH NODE    12317 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER      1
   EX =    0.30000E+08    NUXY =    0.30000      AT TEMP =  0.0000

****  KI =    0.0000    ,   KII =    7.2363    ,   KIII =    3.8982      ****
```

SIF for 6 mm cracked shaft

```
KCALC  Command
File

****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS  ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH ANTI-SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:    12317    12338    12346
   WITH NODE    12317 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER      1
   EX =    0.30000E+08    NUXY =    0.30000      AT TEMP =  0.0000

****  KI =    0.0000    ,   KII =    8.7332    ,   KIII =    4.8750      ****
```

SIF for 7 mm cracked shaft

KCALC Command

File

```
****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS   ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH ANTI-SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:    12317   12338   12342
   WITH NODE   12317 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER     1
   EX =    0.30000E+08    NUXY =     0.30000      AT TEMP =  0.0000

****  KI  =    0.0000     ,   KII  =   15.163     ,   KIII  =   6.0074      ****
```

SIF for 8 mm cracked shaft

KCALC Command

File

```
****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS   ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH ANTI-SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:    20212   20237   20239
   WITH NODE   20212 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER     1
   EX =    0.30000E+08    NUXY =     0.30000      AT TEMP =  0.0000

****  KI  =    0.0000     ,   KII  =   23.273     ,   KIII  =   6.8983      ****
```

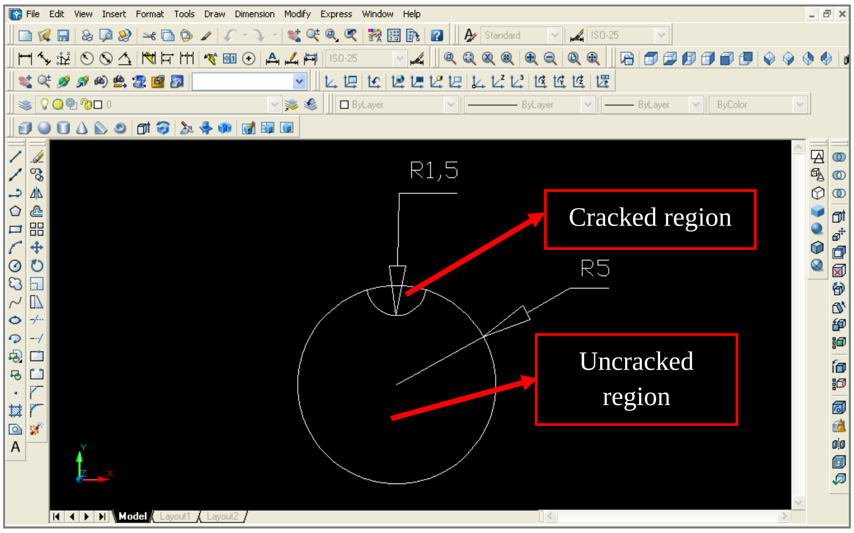
SIF for 9 mm cracked shaft

KCALC Command

File

```
****  CALCULATE MIXED-MODE STRESS INTENSITY  FACTORS   ****

  ASSUME PLANE STRAIN CONDITIONS

  ASSUME A HALF-CRACK MODEL WITH ANTI-SYMMETRY BOUNDARY CONDITIONS (USE 3 NODES)

  EXTRAPOLATION PATH IS DEFINED BY NODES:    22088   22113   22119
   WITH NODE   22088 AS THE CRACK-TIP NODE

  USE MATERIAL PROPERTIES FOR MATERIAL NUMBER     1
   EX =    0.30000E+08    NUXY =     0.30000      AT TEMP =  0.0000

****  KI  =    0.0000     ,   KII  =   23.386     ,   KIII  =   8.3540      ****
```

Appendix D: Average Radius of the 3mm Cracked Shaft Determined by using the AutoCAD software



Area for the uncracked region = Area for the shaft – Area for the cracked region

$$\pi r^2 = \pi R^2 - \pi \check{r}^2$$

where r is the average radius of the 3 mm cracked shaft, R is the radius of the shaft and $\check{r}$ is the radius of the cracked region. Average radius, r, of the 3 mm cracked shaft determined by using AutoCAD software was 4.371 mm.