DYNAMIC MODELING AND OPEN-LOOP CONTROL OF A TWIN ROTOR MIMO SYSTEM

By

NOR HAYATI YAACOB

FINAL PROJECT REPORT

Submitted to the Electrical & Electronics Engineering Programme
in Partial Fulfillment of the Requirements
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Universiti Teknologi Petronas
Bandar Seri Iskandar
31750 Tronoh
Perak Darul Ridzuan

CERTIFICATION OF APPROVAL

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Approved:

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Associate Professor Dr. Mohd Noh Karsiti
Project Supervisor

UNIVERSITI TEKNOLOGI PETRONAS
TRONOH, PERAK

December 2005

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

Nor Hayati Yaacob

ABSTRACT

The Twin Rotor MIMO System is an aero-dynamical model of a helicopter with significant cross-couplings between longitudinal and lateral directional motions. In this project there are two main critical parts, which are the development of dynamic models for the characterization of 1-DOF horizontal and vertical part and also the design stage of the state-feedback controller to control the main and tail rotor of the TRMS. The dynamic models of the main and tail rotor of the TRMS were obtained by applying a step input to the rotors independently, one at a time. The step responses were then evaluated to find the relevant information and quantities to develop second-order transfer function. From these linearized models, the state-feedback controllers were designed independently for the main and tail rotor by selecting desired pole locations and calculating the feedback gains. Real-time experiments were then performed using the feedback gains obtained to evaluate the performance of the state-feedback controllers designed. Strong interactions between the tail and main rotor also seen by performing 2-DOF real-time experiments.

ACKNOWLEDGEMENTS

Many people have been contributing ideas, guidance, supervision, support and technical assistance during the accomplishment of this project entitled 'Dynamic Modeling and Open-Loop Control of a Twin Rotor MIMO System'.

Here, I would like to express my utmost gratitude to my project supervisor Associate Professor Dr. Mohd Noh Karsiti for spending his time to guide me in the completion of this project.

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Last but not least, thank you to my beloved family and friends who have been very supportive from the beginning of the project and all other people who are not mentioned here.

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LIST OF ABBREVIATIONS

DC	-	Direct Current
DOF	-	Degree of Freedom
PD	-	Proportional & Derivative
PID	-	Proportional Integral & Derivative
RTWT	-	Real-Time Workshop Target
TRMS	-	Twin Rotor MIMO System

CHAPTER 1
INTRODUCTION

1.1 Background of Study

Dynamic Modeling and Open-Loop Control of a Twin Rotor MIMO System (TRMS) project, as the topic implies requires the development of a dynamic model of TRMS and design of a control strategy to control the behavior of the TRMS.

TRMS consists of main and tail rotor in which there are significant cross-couplings between the actions of the rotors, with each rotor influencing both positions angles. The main rotor is responsible for the motion in vertical plane while the tail rotor is responsible for the movement in the horizontal plane.

Creation of two 1-DOF separate models for horizontal and vertical plane is necessary since there is no natural way to split 2-DOF complex model into two independent parts.

The development of dynamic model for the main and tail rotor of the TRMS is very important since it gives the input and output relationship of the system. By having a good model for the tail and main rotor of the TRMS a better control strategy can be introduced to the system to achieve desired response.

The design of controllers for such a system is based on de-coupling. For a decoupled system an independent control input can be applied for each co-ordinate of the system.

1.2 Problem Statement

1.2.1 Problem Identification

The Twin Rotor MIMO System (TRMS) is a high-order nonlinear system with significant cross-coupling. In some aspects, its behavior resembles that of a helicopter, with significant cross-coupling between longitudinal and lateral directional motions. The approach to control problems connected with TRMS involves some theoretical knowledge of laws of physics which results in complicated mathematical modeling of the TRMS. Since this project requires the development of a control strategy to control the behavior of the TRMS, hence the TRMS need to be modeled first before the control strategy can be introduced to the system.

1.2.2 Significance of the Project

The significance of this project is to obtain linearized model for the main and tail rotor of the TRMS using the experimental approach and design a control strategy to control the behavior of the rotors.

1.3 Objectives and Scope of Study

1.3.1 Objectives of the Project

By the end of the semester, the project is expected to meet the following objectives:

- To obtain linearized models for 1-DOF main rotor and 1-DOF tail rotor of the TRMS
- To design a controller so that the state vector of the closed-loop system is stabilized around a desired point of the state-space and follows a given trajectory.

1.3.2 Scope of the Project

Scope of study for this project is narrowed down to the design of state-feedback controller using the pole placement method.

1.3.3 Relevancy of the Project

The design process of this project requires a strong basic knowledge in control system design and analysis. As the TRMS is a model of a helicopter, this project is very much relevant to the real control systems design for helicopter in which the TRMS can be used to model the helicopter. A controller can be designed based on this model and the performance of this controller can be analyzed by using Matlab before applied to the real system. Hence, by doing this project, student can have a better understanding in the process of designing a control system.

CHAPTER 2
LITERATURE REVIEW

2.1 Control Systems[1]

The portion of a system to be controlled is called the plant or process. It is affected by applied signals called inputs, and produces signals of particular interest called outputs, as indicated in Figure 1. The plant is fixed as far as the control-designer is concerned.

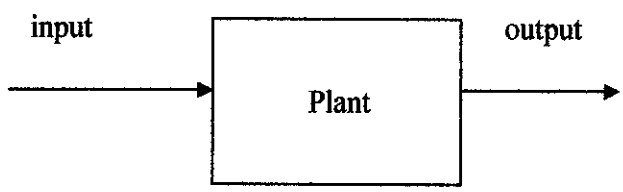


Figure 1 Plant to be controlled

A controller maybe used to produce desired behavior of the plant. The controller generates plant input signals designed to produce desired outputs.

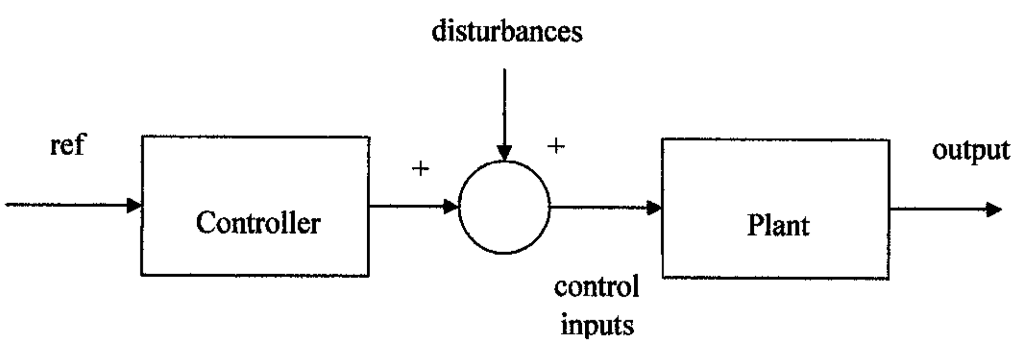


Figure 2 Open-loop system

Those systems in which the output has no effect on the control action are called open-loop systems as shown in Figure 2. In other words, in an open-loop system the output is neither measured nor fed back for comparison with the input.

In any open-loop control system the output is not compared with the reference input. Thus, to each reference input there correspond a fixed operating condition; as a result, the accuracy of the system depends on calibration. In the presence of disturbances, an open-loop control system will not perform the desired task. Open-loop control can be used in practice, only if the relationship between the input and output is known and there are neither internal nor external disturbances.

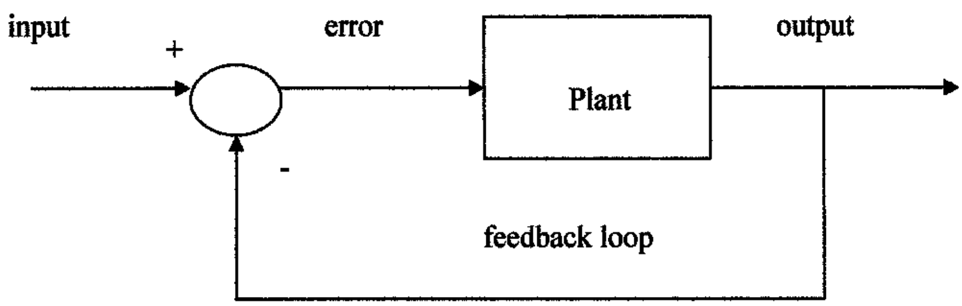


Figure 3 Closed- loop control system

Feedback control systems are often referred to as closed-loop control systems shown in Figure 3. In practice, the terms feedback control and closed-loop control are used interchangeably. In a closed-loop control system the actuating error signal, which is the difference between the feedback signal (which maybe the output signal itself or a function of the output signal and its derivatives and/or integrals), is fed to the controller as to reduce the error and bring the output of the system to a desired value. The term closed-loop control always implies the use of feedback control action in order to reduce the system error.

2.2 State-Feedback Controller Using Pole Placement Method[2]

2.2.1 Topology for Pole Placement

In the state space representation, a system or plant is represented by

$$\dot{x} = Ax + Bu \quad (1)$$

$$y = Cx \quad (2)$$

and shown pictorially in Figure 4, where light lines are scalars and the heavy lines are vectors. In a typical feedback control system the output, y is fed back to summing junction. It is that the topology of the design changes. Instead of feeding back y, all of the state variables are fed back to the summing junction.

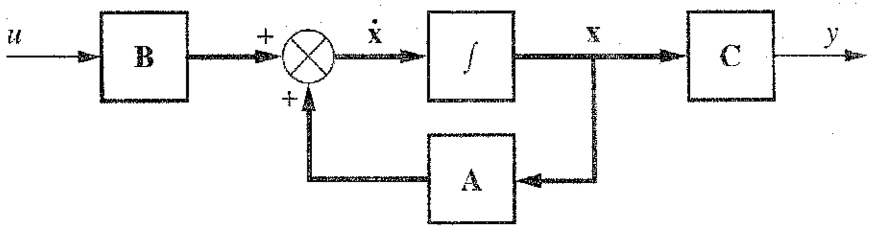


Figure 4 State space representation of a plant

If each state variable is fed back to the control, u through a gain k_i, there would be n gains, k_i that could be adjusted to yield the required closed-loop poles. The feedback through the gains k_i, is represented in Figure 5 by the feedback vector $-K$.

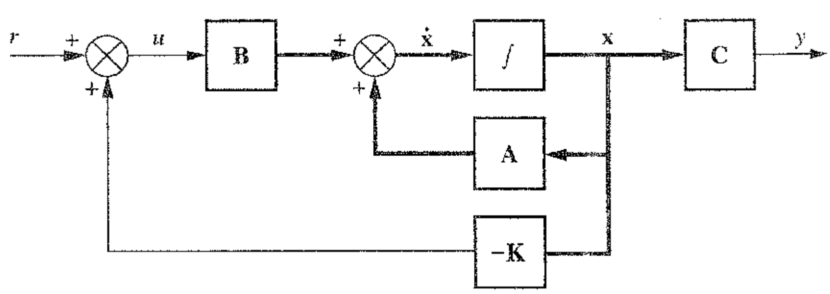


Figure 5 System with feedback

The state equations for the closed-loop system of Figure 5 can be written by inspection as

$$\dot{x} = Ax + B(-Kx + r) = (A - BK)x + Br \tag{3}$$

$$y = Cx \tag{4}$$

The design of state-feedback controller for closed-loop pole placement consists of equating the characteristic equation of a closed-loop system to a desired characteristic equation and then finding the values of the feedback gains, k_i.

2.2.2 *Pole Placement for System in Control Canonical Form*

To apply pole placement methodology to system represented in control canonical form, take the following steps:

1. Represent the system in control canonical form.
2. Feedback each phase variable to the input of the system through a gain, k_i.
3. Find the characteristic equation for the closed-loop system represented in step 2.
4. Decide upon all closed-loop pole locations and determine an equivalent characteristic equation.
5. Equate like coefficients of the characteristic equation s from step 3 and step 4 and solve for k_i.

Following these steps, the control canonical form representation of system is given by

$$A = \begin{bmatrix} 0 & 1 & 0 & .. & 0 \\ 0 & 0 & 1 & .. & 0 \\ : & : & : & .. & : \\ -a_0 & -a_1 & -a_2 & .. & -a_{n-1} \end{bmatrix};$$

$$B = \begin{bmatrix} 0 \\ 0 \\ : \\ 1 \end{bmatrix}; \quad C = \begin{bmatrix} c_1 & c_2 & .. & c_n \end{bmatrix} \tag{5}$$

The characteristic equation of the system is thus

$$s^n + a_{n-1}s^{n-1} + ... + a_1 s + a_0 = 0 \tag{6}$$

Now form the closed-loop system by feeding back each state variable to u, forming

$$u = -Kx \tag{7}$$

where

$$K = \begin{bmatrix} k_1 & k_2 & k_3 & .. & k_n \end{bmatrix} \tag{8}$$

The k_i 's are the phase variables' feedback gains. Using equation (3) with equations (5) and (8), the system matrix $A-BK$, for the closed loop system is

$$A-BK=\begin{bmatrix} 0 & 1 & 0 & .. & 0 \\ 0 & 0 & 1 & .. & 0 \\ \vdots & \vdots & \vdots & .. & \vdots \\ -(a_0+k_1) & -(a_1+k_2) & -(a_2+k_3) & .. & -(a_{n-1}+k_n) \end{bmatrix} \quad (9)$$

Since equation (9) is in phase variable form, the characteristic equation of the closed-loop system can be written by inspection as

$$\det(sI-(A-BK))=s^n+(a_{n-1}+k_n)s^{n-1}+(a_{n-2}+k_{n-1})s^{n-2}$$

$$..+...(a_1+k_2)s+(a_0+k_1)=0 \quad (10)$$

Notice the relationship between equations (6) and (10). For plants represented in phase variable form, the closed-loop characteristic equation can be written from the open loop characteristic equation by adding the appropriate k_i to each coefficient.

Now assume that the desired characteristic equation for proper pole placement is

$$s^n+d_{n-1}s^{n-1}+d_{n-2}s^{n-2}+......+d_2s^2+d_1s+d_0=0 \quad (11)$$

where the d_i 's are the desired coefficients. Equating equations (10) and (11), we obtain

$$d_i=a_i+k_{i+1} \qquad i=0,1,2,3,....,n-1 \quad (12)$$

from which

$$k_{i+1}=d_i-a_i \quad (13)$$

CHAPTER 3

THE TWIN ROTOR MIMO SYSTEM[3]

3.1 Introduction to Twin Rotor MIMO System

The Twin Rotor MIMO System is a laboratory set-up designed for control experiments as in Figure 6. In certain aspects its behavior resembles that of a helicopter. From the control point of view it exemplifies a high order non-linear system with significant cross couplings.

At both ends of the beam, pivoting on its base, there are two propellers driven by DC motors. The articulated joint allows the beam to rotate in such a way that it can rotate freely both in its horizontal and vertical planes. There is a counter-weight fixed to the beam and it determines a stable equilibrium position. The system is balanced in such a way, that when the motors are switched off, the main rotor end of beam is lowered.

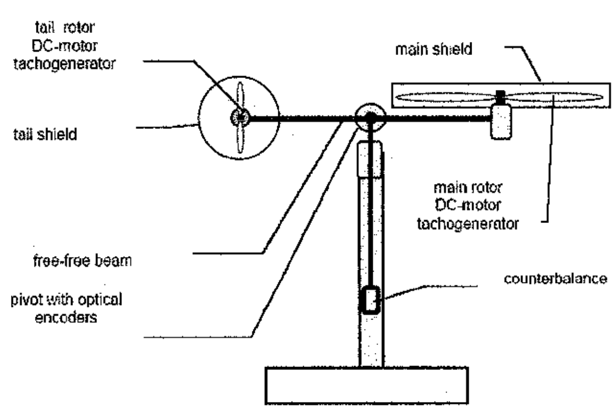


Figure 6 Aero-dynamical model of o Twin Rotor MIMO System

The TRMS helicopter system has main and tail rotor for generating vertical and horizontal propeller thrust, and requiring only two easily measured outputs, the horizontal and vertical angle (α_h and α_v) of the helicopter.

In a normal helicopter the aerodynamic force is controlled by changing the angle of attack. However in the TRMS model the angle of attack is fixed and the aerodynamic force are controlled by varying the speed of rotors.

The control outputs therefore are the voltages applied to the DC motors. A change in the voltage value will results in a change of the rotation speed of the propeller which results in a change of the corresponding position of the beam. There are significant cross-couplings between the actions of the rotors, with each influencing both the position angles.

3.2 Twin Rotor MIMO System Operating Modes

The TRMS can be set to operate in three modes

- A 1-DOF system using only the tail rotor – motion only in the horizontal plane
- A 1-DOF system using only the main rotor – motion only in the vertical plane
- A 2-DOF system using the tail and main rotor – motion in both horizontal and vertical planes

This is accomplished by setting the two nylon locking screws which clamp motion in either the horizontal or vertical plane.

The tail rotor horizontal motion can be controlled by mechanically blocking its freedom to move in the vertical plane (by tightening the horizontal axis blocking screw as shown in Figure 7). While the main rotor motion in the vertical plane can be set by mechanically blocking its freedom to move in the horizontal plane by tightening the vertical axis locking screw.

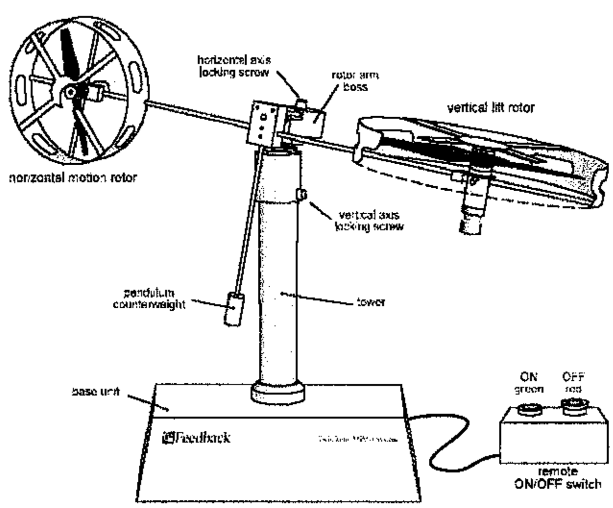


Figure 7 Vertical and horizontal locking screws of TRMS

The 2-DOF control motion can be carried out by releasing both of the vertical and horizontal set screws in which the main and tail rotor can be controlled simultaneously.

Motion of the main rotor upwards from the reference position is considered as motion in the positive direction and clockwise rotation of the tail (viewed from the top) is considered as positive.

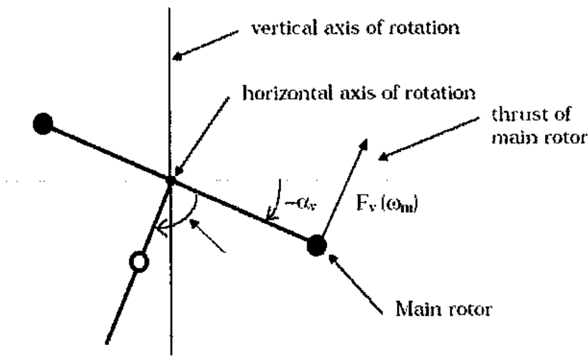


Figure 8 Main rotor positioning of TRMS

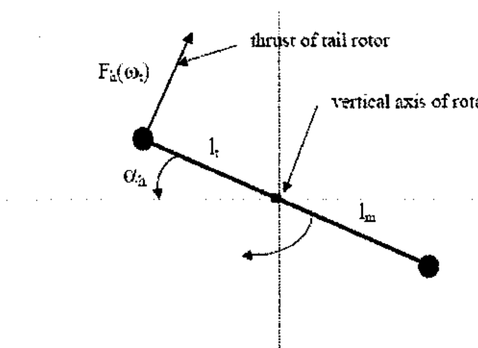


Figure 9 Tail rotor positioning of TRMS (viewed from above)

3.3 Description of the HelicopterPID RTWT Block Diagram

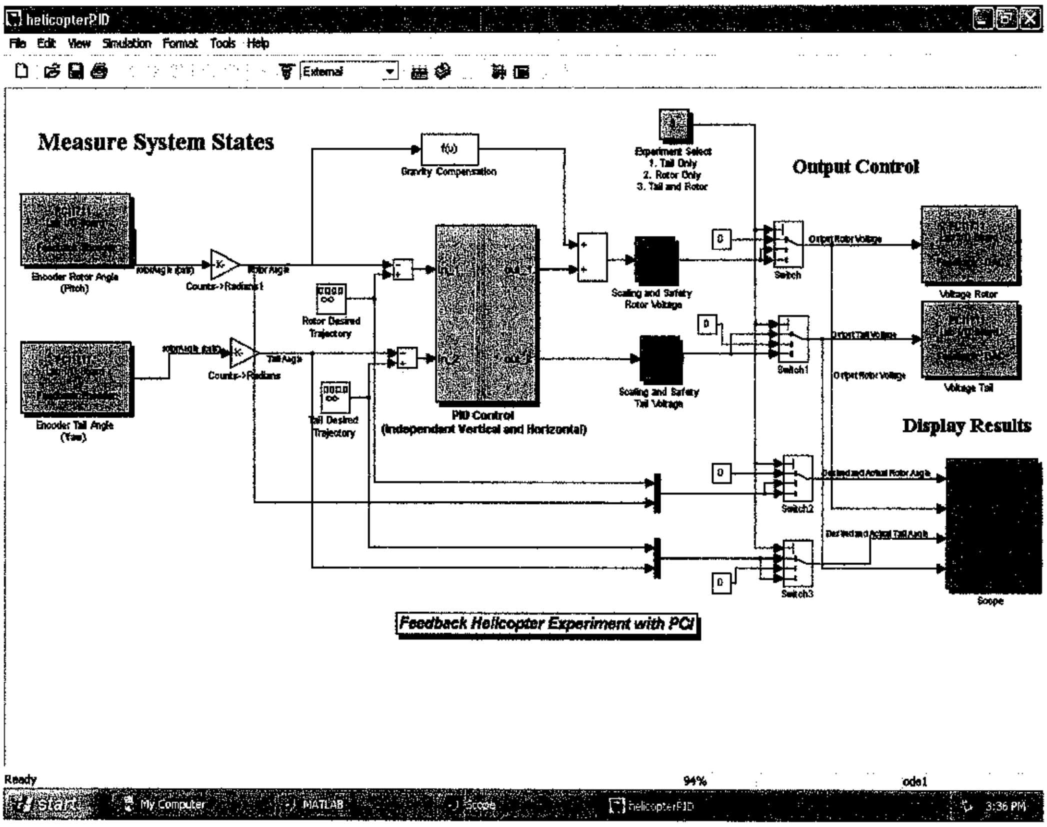


Figure 10 Block diagram of HelicopterPID controller

The HelicopterPID command is invoked in the MATLAB command window to load and execute the RTWT block diagram.

The RTWT block diagram for the HelicopterPID is shown in Figure 10 above. The model provides PID control for each of the 3 operating modes of the TRMS; 1-DOF tail rotor, 1-DOF main (vertical) rotor and 2-DOF (both main and tail rotor).

The selection is made by entering a constant value of 1, 2, or 3 in the "Experiment Select" box colored cyan in the top right corner of the block diagram window.

In Figure 10, the green blocks denote input/output operations of hardware and the red block denotes the safety subsystem. The green blocks labeled Encoder Rotor Angle and Encoder Tail Angle represent input of the vertical and horizontal positions as encoder counts from two incremental encoders.

Prior to the start of a control experiment the system must be at the reference position. To do this depress the STOP button and manually hold the counterweight arm vertical until the real-time target has been connected.

These reference vertical and horizontal positions will be referred to as 0 radians.

The scaling blocks convert from counts of the incremental encoders turning with the motor, to unit angle in radians.

The green blocks labeled Voltage Rotor and Voltage Tail represent output voltage to the main and tail rotor of the TRMS.

The Scaling block found in "Scaling and Safety" block is set to 1 so that a positive input creates a motor torque that acts to move the rotor in positive direction.

The Control Subsystem is shown as a blue block in the center of Figure 10. The two inputs represent the position error for the two rotors. Double click on this block to open it (Figure 11). The "PID Rotor" and "PID Tail" blocks are masked.

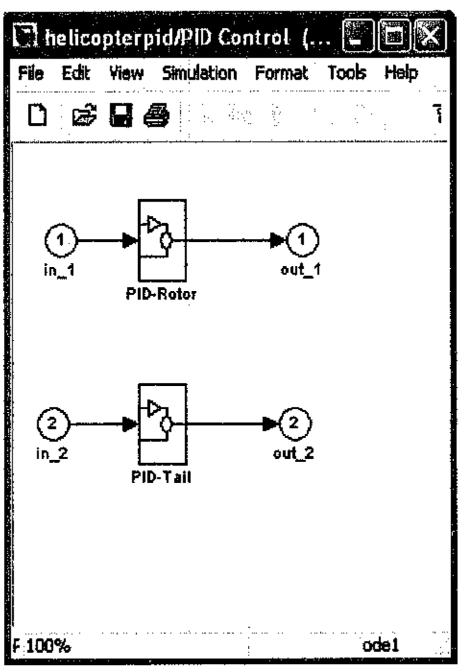


Figure 11 Control subsystem mask

To reveal the structure right click the mouse and select "Look under mask". The structures are shown in Figure 12 and Figure 13.

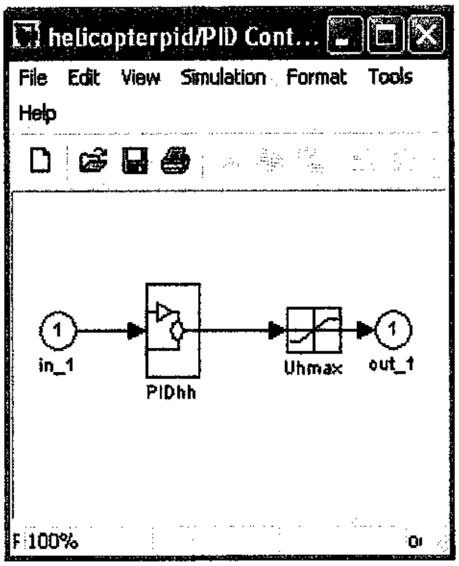


Figure 12 PID main rotor secondary block

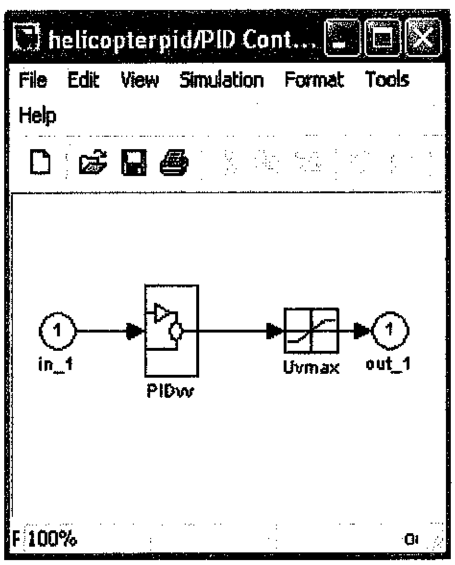


Figure 13 PID tail rotor secondary block

This operation is repeated on the PI blocks which appear to show the structure of the controllers shown in Figure 14 and Figure 15.

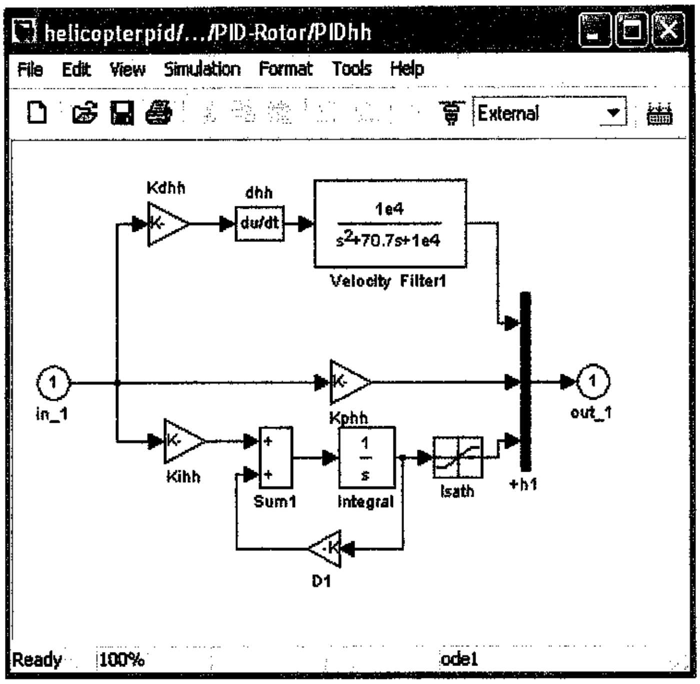


Figure 14 PID controller main rotor

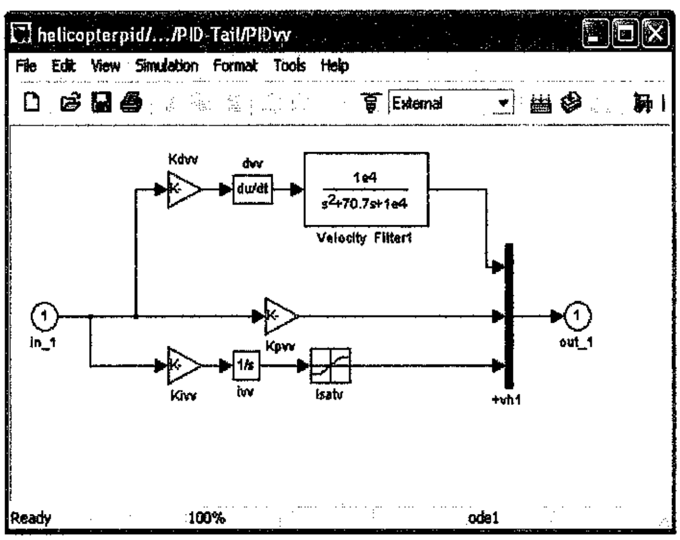


Figure 15 PID controller tail rotor

CHAPTER 4

METHODOLOGY AND PROJECT WORK

4.1 Twin Rotor MIMO System Modeling and Identification

i. System Modeling

The purpose of system modeling is to establish the relationships between parameters in physical systems and transient behavior of the systems. There are two ways in modeling a system, which are the mathematical approach or empirical approach.

The mathematical approach has limitations, which generally results from the complexity of mathematical models. Thus, modeling most realistic system requires a large engineering effort to formulate the equations, determine all parameter values and solve the equations, usually through numerical methods.

The empirical modeling is specifically designed for plant process control. But, this method can be applied also to any physical systems by applying a step input to the system. The resulting dynamic response is used to determine the model. A linear transfer function developed using this method is adequate for many system control designs and implementations.

Steps taken in identifying dynamic models of the TRMS are as follows:

1. A step change is introduced in the input variable.
2. Collect the output response until the output reaches steady-state.
3. Perform relevant calculations to determine the parameters for the second-order model.

There are two physically meaningful specifications for second-order systems. These quantities can be used to describe the characteristics of the second-order transient response just as time constants describe the first-order system response. The two quantities are called natural frequency, ω_n and damping ratio, ξ.

The natural frequency, ω_n of a second-order system is the frequency of oscillation of the system without damping. It relates to the speed of the response for a particular value of ξ.

- When $\xi < 1.0$, the system is said to be underdamped and will overshoot the final steady state value. If $\xi < 0.707$, the system will not only overshoot but will oscillate about the final steady-state value.
- When $\xi > 1.0$, the system is said to be overdamped and will not oscillate or overshoot the final steady-state value.
- When $\xi = 1.0$, the system is said to be critically damped and yields the fastest response without overshoot or oscillation.

The general second-order system transfer function looks like this:

$$T(s) = \frac{{\omega_n}^2}{s^2 + 2\omega_n \xi + {\omega_n}^2}$$

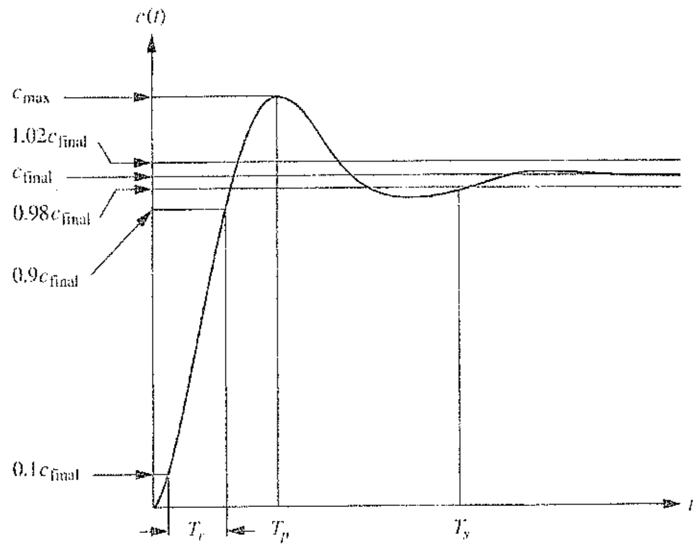


Figure 16 Second-order underdamped response specifications

For underdamped second-order system, the other parameters associated with it are percent overshoot, peak time, settling time, and rise time. These specifications are defined as follows (see Figure 16):

1. *Peak time,* T_p: The time required to reach the first, or maximum, peak.
2. *Percent Overshoot,* $\%OS$: The amount that the waveform overshoots the steady-state, or final, value at the peak time, expressed as a percentage of the steady-state value.
3. *Settling time,* T_s: The time required for the transient's damped oscillations to reach and stay within $\pm 2\%$ of the steady-state value.
4. *Rise time,* T_r: The time required for the waveform to go from 0.1 of the final value to 0.9 of the final value.

Refer to Appendix A for useful design formulas for underdamped second-order systems.

ii. Model Identification

During the identification process, the parameters of the model developed are tuned to obtain a satisfactory degree of conformity of the model with the actual system. The point is to tune the parameters of the model in such a way, that the outputs of the model fit the actual output of the real system. A good model is a model that can represent a small error when compared to the actual system. Identification procedures are very time consuming but it is necessary to carry them out precisely. Design of the controllers is more effective when a reliable identified model is used.

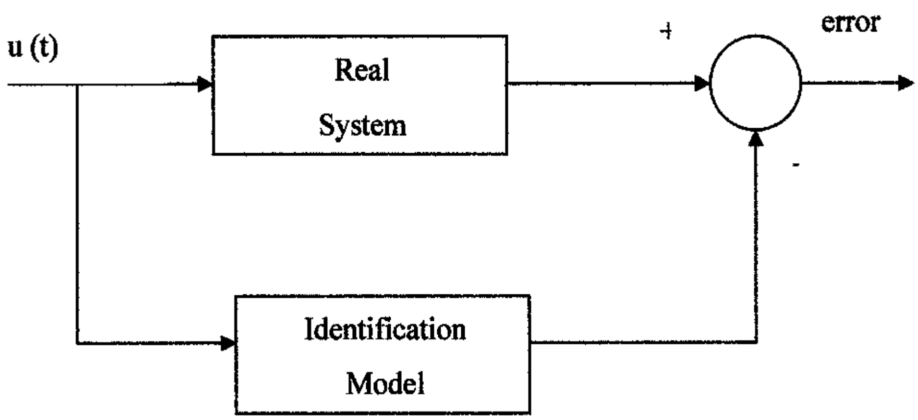


Figure 17 Block diagram of identification procedure

4.1.1 Tail Rotor Modeling and Identification

The RTWT Simulink block below is used for real-time experiments to determine the step response of the tail rotor. In this experiment, the beam is allowed to move in the horizontal plane only.

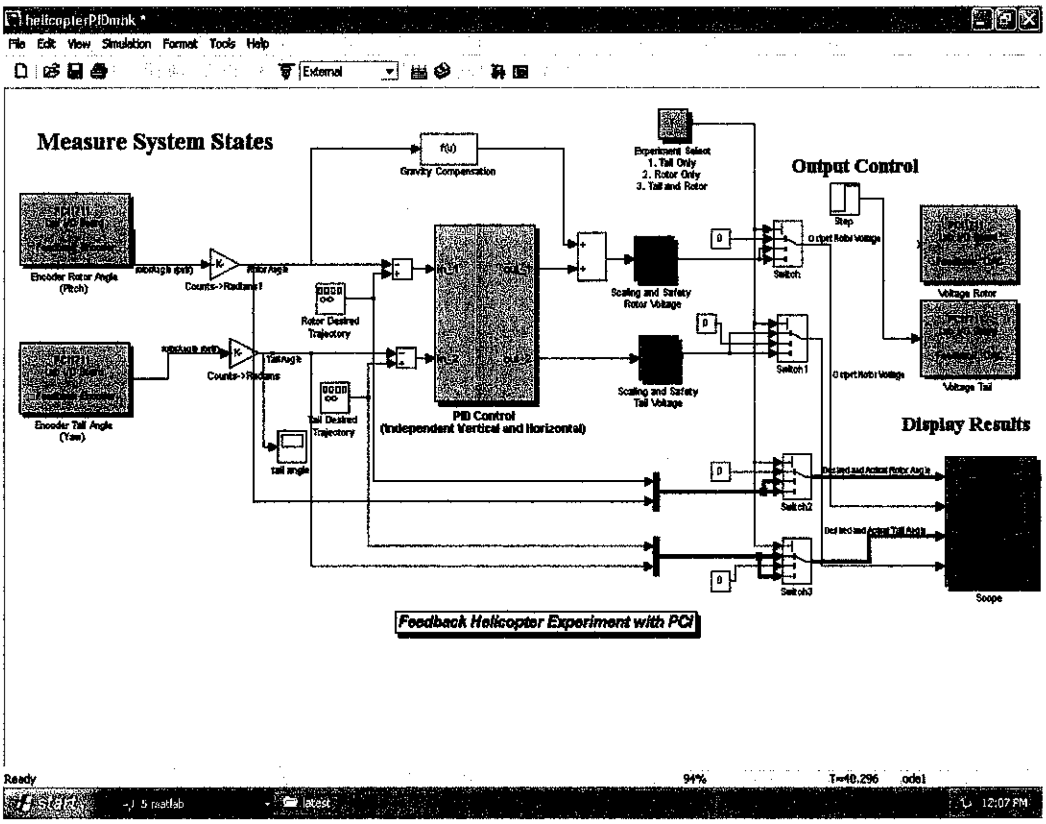


Figure 18 Simulink block diagram used to obtain step response of tail rotor

Before any other actions, the TRMS was set to 1-DOF horizontal plane by mechanically blocking its freedom to move in the vertical plane by tightening the horizontal axis locking screw. A constant value of 1 was entered in the "Experiment Select" box colored cyan in the top right corner of the block diagram window.

A step input with amplitude of 0.5 was applied to the DAC block of the tail rotor of the TRMS. The step input applied was reduced to 0.5 to avoid the tail rotor from reaching the limit which will give inaccurate response of the system.

Simulation was conducted until the step response reached steady-state value. The step response was saved to perform the calculation to determine the value of ξ and ω_n for second-order model system modeling.

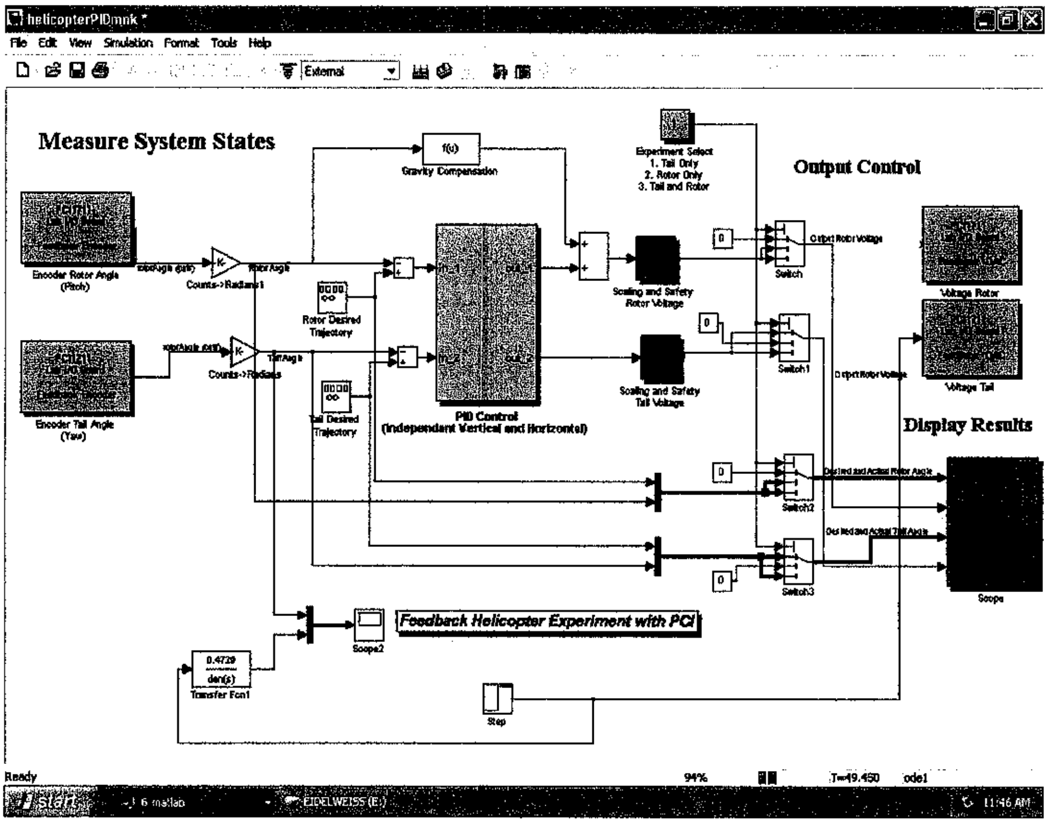


Figure 19 Tail rotor model identification

After the approximate second-order transfer function for the tail rotor has been obtained, model identification procedure was carried out as shown in Figure 19.

The identification process was carried out by tuning the second-order model parameters until the step response of the developed model match the step response of the real system.

4.1.2 Main Rotor Modeling and Identification

The Simulink block diagram shown in Figure 20 is used for real-time experiments to determine the step response for main rotor. In this experiment, the beam is only allowed to move in the vertical plane.

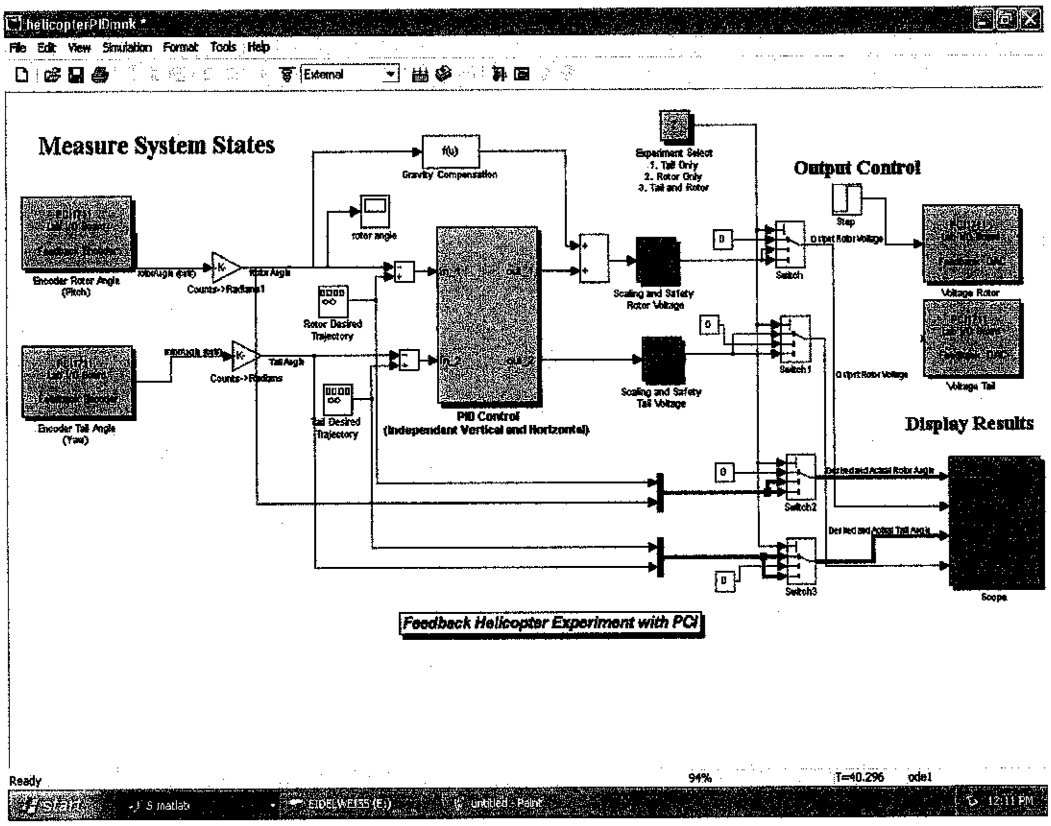


Figure 20 Simulink block diagram used to obtain step response of main rotor

Before any other actions, the TRMS was set to 1-DOF vertical plane by mechanically blocking its freedom to move in the horizontal plane. A constant value of 2 was entered in the "Experiment Select" box colored cyan in the top right corner of the block diagram window.

A step input with amplitude of 0.2 was introduced to the DAC block of the main rotor. The amplitude was reduced to 0.2 so that the step response obtained will capture the accurate behavior of the main rotor.

The real-time experiment was conducted as described previously for the tail rotor part. Calculation was then carried out based on the response obtained to determine the value of ξ and ω_n for second-order system modeling.

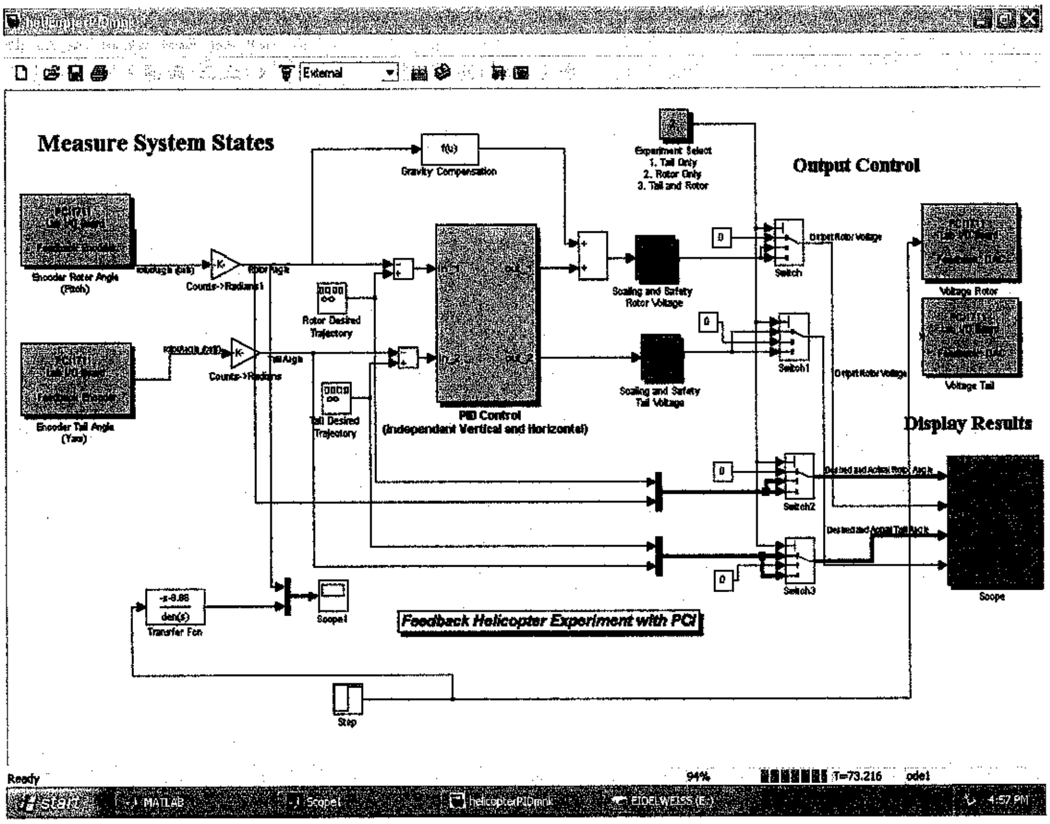


Figure 21   Main rotor model identification

From the second-order model developed, model identification was carried out using Simulink block connections as shown in Figure 21 to ensure that the model developed conformed to the actual system.

The identification process was done by tuning the parameters of the second-order model until the output of the main rotor model fit the actual output of the real system.

4.2 State Feedback Controller Design

The models developed earlier are used as the basis of the design of the state feedback controller for 1-DOF horizontal and vertical systems. The 1-DOF control problem can be formulated as follows. Design a controller that will stabilize the system, or make it follow a desired trajectory in one plane (one degree of freedom) while motion in the other plane is blocked mechanically or being controlled by another controller.

The design process comprises the followings:

- convert the transfer function into state-space representation, control canonical form ,
- calculate eigenvalues of the open linear system by typing in the MATLAB Command Window eig(A),
- select desired eigenvalues of the closed-loop system,
- calculate feedback gains using acker function,
- simulate the closed-loop state-feedback system with the linear model and check the behavior of the system,
- if it is necessary, change the desired pole locations and repeat the simulation,
- perform real-time experiment with the feedback gains obtained.

CHAPTER 5
RESULTS AND DISCUSSIONS

5.1 System Modeling and Identification

Open loop response test using a step input is carried out to define the system by determining the damping ratio, ξ and natural frequency, ω_n using the response obtained.

5.1.1 1-DOF Tail Rotor

The step response of the tail rotor for 1-DOF motion in horizontal plane using a step input of 0.5amplitude is shown below. The parameters for second-order underdamped system can be directly determined from the response obtained.

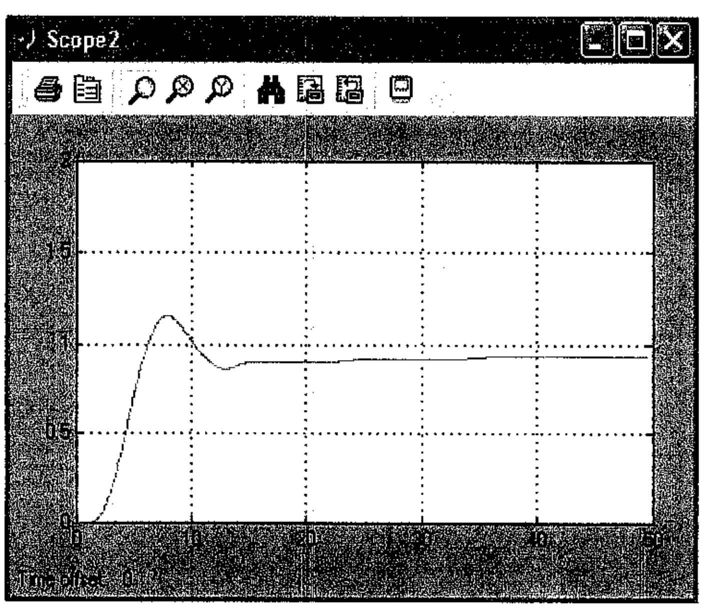


Figure 22 Step response of tail rotor

The value of damping ratio ξ and natural frequency ω_n cannot be directly found from the step response. Some simple calculations need to be carried out using appropriate formulas to determine these parameters. In order to minimize the calculations, the formulas for peak time and rise time are used for this purpose (refer to Appendix A).

The characteristics or specifications of the tail rotor step response are tabulated in Table 1 below. The rise time, settling time and peak time yield information about the speed of the transient response.

Table 1 The second-order specifications of tail rotor

	From step response	
1	Peak time, T_p	8 s
2	Percent Overshoot, $\%OS$	25 %
4	Settling time, T_s	15 s
	Calculated values	
5	Damping ratio, ξ	0.5618
6	Natural frequency, ω_n	0.4746

By using the general transfer function for second-order systems, the initial tail rotor transfer function obtained is as below:

$$T(s) = \frac{0.2252}{s^2 + 0.5333s + 0.2252}$$

In order to determine the accuracy of the transfer function developed earlier, model identification is carried out to obtain satisfactory degree of conformity of the model with the real system.

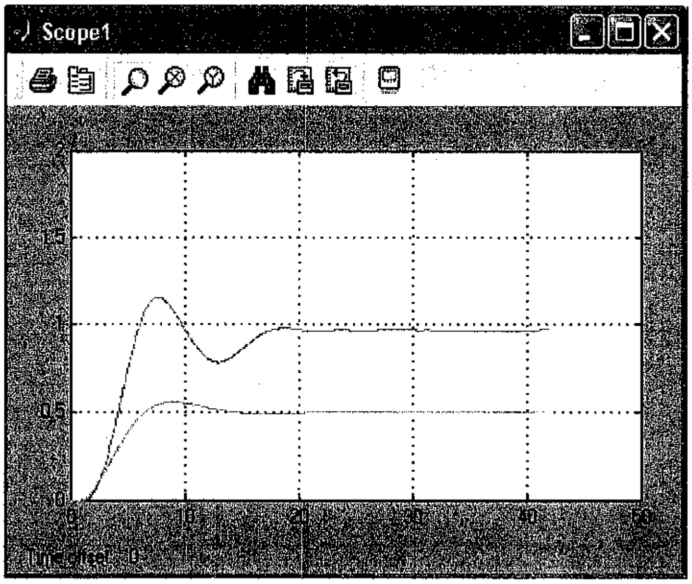


Figure 23 Model and system step responses of tail rotor before identification

The step responses of the tail rotor model and real system before identification are shown in Figure 23 above. Since the denominator of the transfer function will only affect the nature of response –exponential, damped sinusoid and so on, hence only the numerator of the transfer function need to be changed (since it affects the amplitude of a response component) so that the outputs of the model fit the real system outputs.

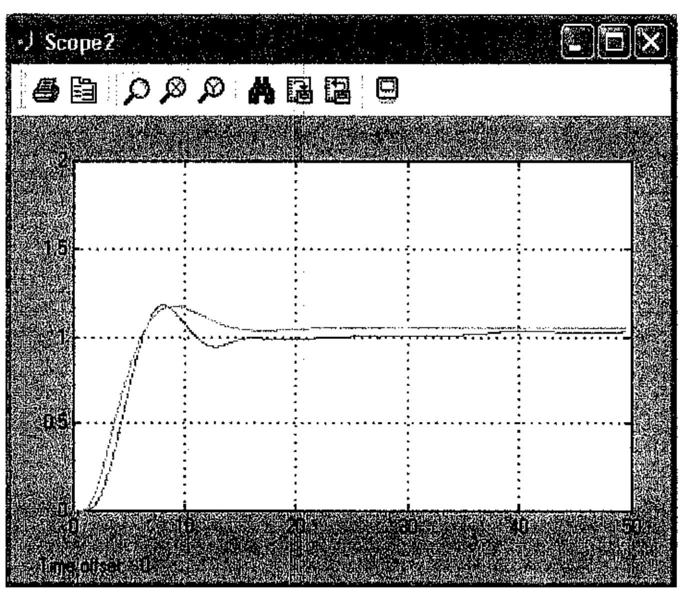


Figure 24 Model and system step responses of tail rotor after identification

The tail rotor model and system step responses after identification are shown in Figure 24. The best approximation of the tail rotor transfer function is given by:

$$T(s) = \frac{0.5180}{s^2 + 0.5333s + 0.2252}$$

5.1.2 1-DOF Main Rotor

The step response of the main rotor is shown in Figure 25 below for 1-DOF movement in vertical plane only. The step input with amplitude of 0.2 is applied to the main rotor to avoid getting inaccurate response of the main rotor.

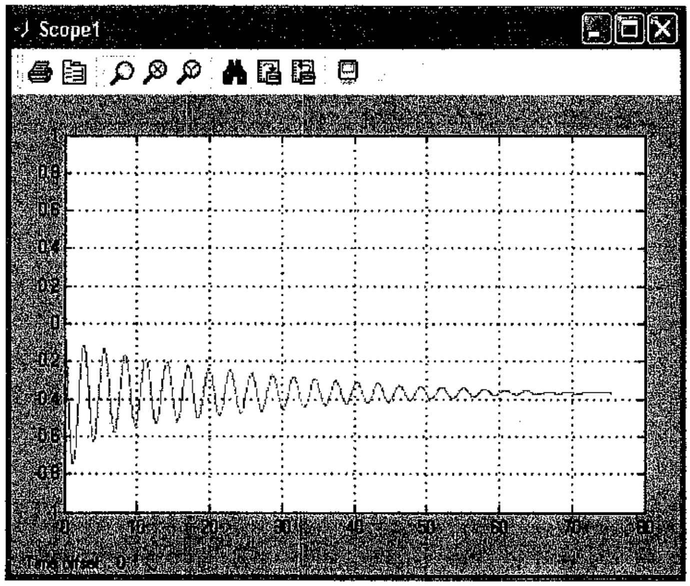


Figure 25 Step response of main rotor

The second-order parameters of the main rotor are tabulated in Table 2. Using the calculated damping ratio, ξ and natural frequency, ω_n obtained, the main rotor transfer function before identification is as below:

$$T(s) = \frac{-4.44}{s^2 + 0.1334s + 4.44}$$

Table 2 The second-order specifications of main rotor

From step response		
1	Peak time, T_p	1.5 s
2	Percent Overshoot, $\%OS$	53.3%
4	Settling time, T_s	60 s
Calculated values		
5	Damping ratio, ξ	0.0317
6	Natural frequency, ω_n	2.107

The model identification for main rotor is carried out using the linearized model obtained. Step responses of the main rotor model and system before identification are as shown in Figure 26 below using step input with amplitude of 0.2.

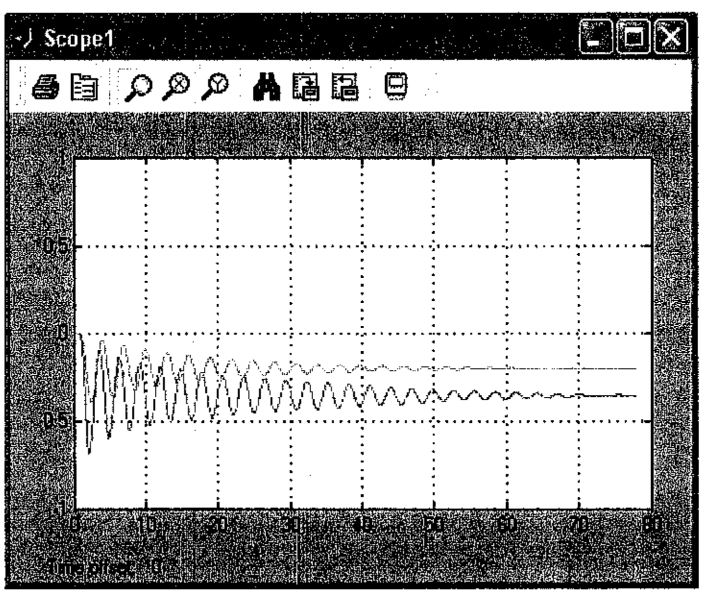


Figure 26 Model and system responses of main rotor before identification

Model identification is carried out to the model of the main rotor to make sure that the model develop conform to the real system. Since the amplitude of the model response differs from the actual system, the numerator of the transfer function model should be tuned until the outputs of the model fit the real system outputs. The main rotor model and system responses after identification are as shown in Figure 27 below.

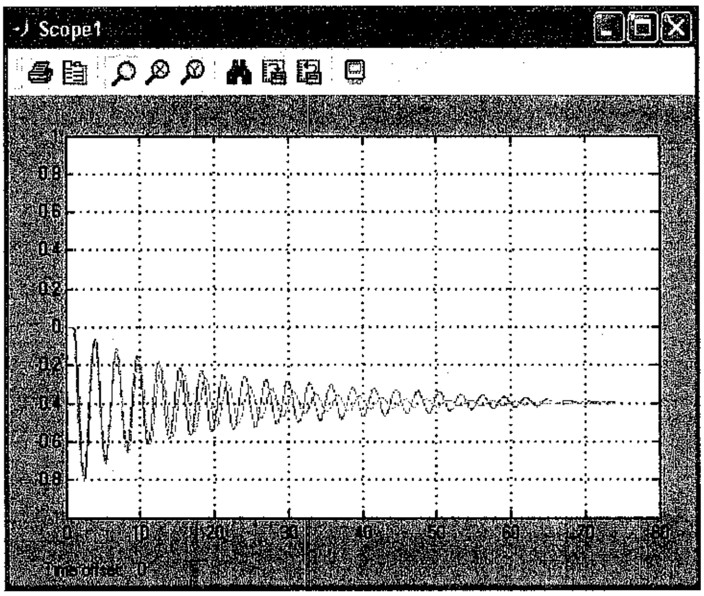


Figure 27 Model and system responses of main rotor after identification

Thus, the best approximation of the main rotor transfer function after identification is given by:

$$T(s) = \frac{-8.88}{s^2 + 0.1334s + 4.44}$$

5.2 State-Feedback Control of TRMS

5.2.1 1-DOF Tail Rotor

Pole placement is a viable design technique only for systems that are controllable. If any of the state variables cannot be controlled by the control u, then the poles cannot be placed at desired points.

The tail rotor transfer function is given by:

$$T(s) = \frac{0.5180}{s^2 + 0.5333s + 0.2252}$$

The state variables x_1 and x_2 are defined as follows:

$$x_1 = y = \alpha_t \quad \text{where } \alpha_t \text{ is the tail angle}$$

$$x_2 = \dot{x}_1$$

Then the state space representation of the tail rotor in control canonical form becomes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.2252 & -0.5333 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0.5180 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The controllability of the system can be determined by examining the controllability matrix of the system. The rank of

$$M = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -0.5333 \end{bmatrix}$$

is 2. Hence, the system is completely state controllable. The open-loop poles of the tail rotor are at

$-0.2665 + j0.3927$ and

$-0.2665 - j0.3927$

The desired locations to place the eigenvalues of the closed-loop tail rotor system are selected to be at

$$-2 + j0.5 \text{ and}$$

$$-2 - j0.5$$

to have a faster settling time to step input. The feedback gain matrix, K that achieves the desired closed-loop poles is

$$K = \begin{bmatrix} 4.028 & 3.467 \end{bmatrix}$$

The state-feedback system with linearized model is simulated to check the behavior of the closed-loop system. This simulation is done using MATLAB and the result of the simulation is shown below.

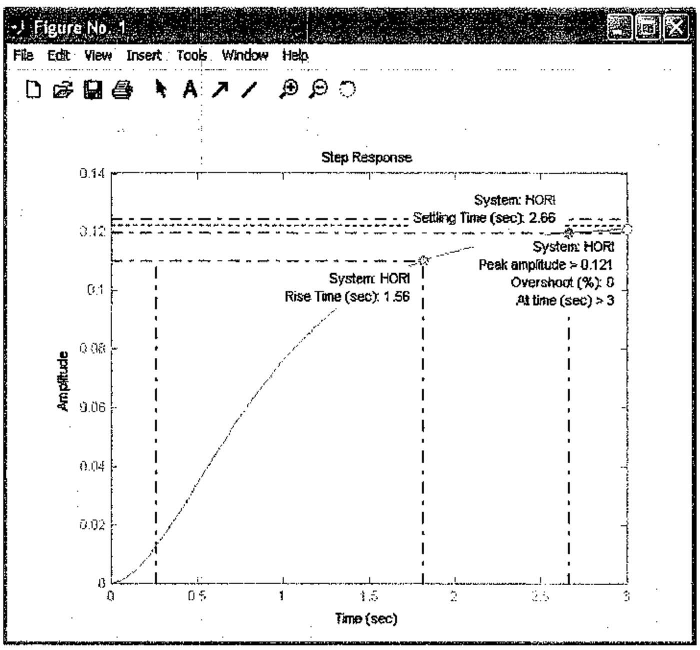


Figure 28 Simulation results of the closed-loop tail rotor model with state-feedback

After the simulation, a real-time experiment is performed by setting the controller parameters in the Control Subsystem block at the center of the HelicopterPID RTWT block diagram (refer to Appendix B for setting).

The real-time experiment conducted using 3 different types of reference signal which are sine wave, square wave and saw tooth. Results of the real-time experiment by using different types of desired trajectories are shown in Figure 29 to 31.

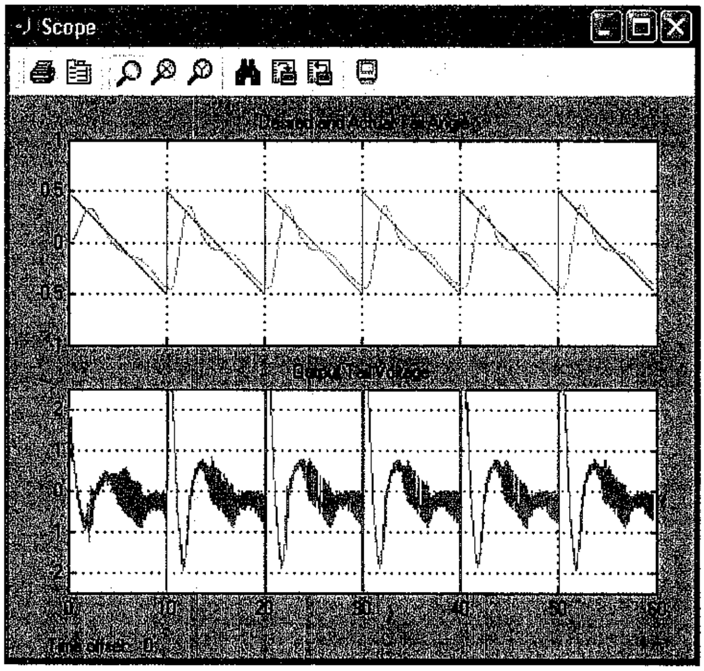


Figure 29 Tail rotor real-time experiment results using saw tooth as reference

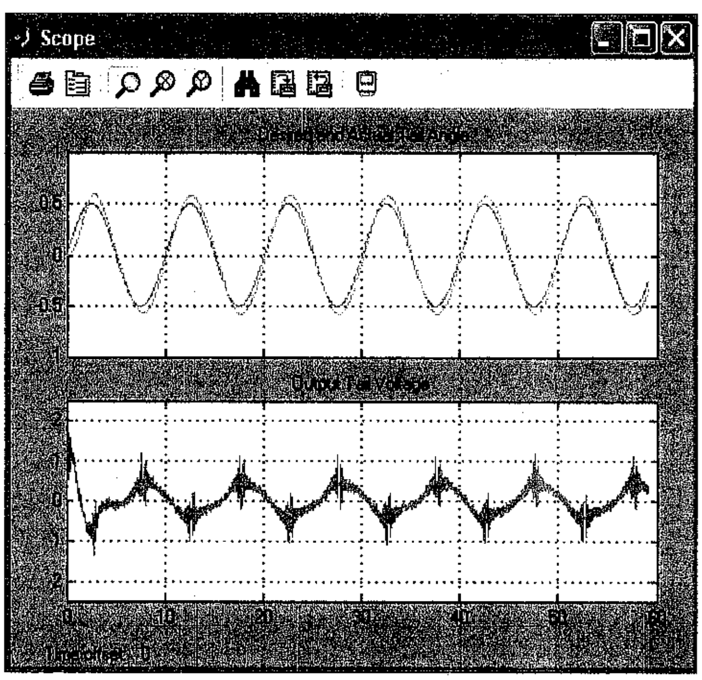


Figure 30 Tail rotor real-time experiment results using sine wave as reference

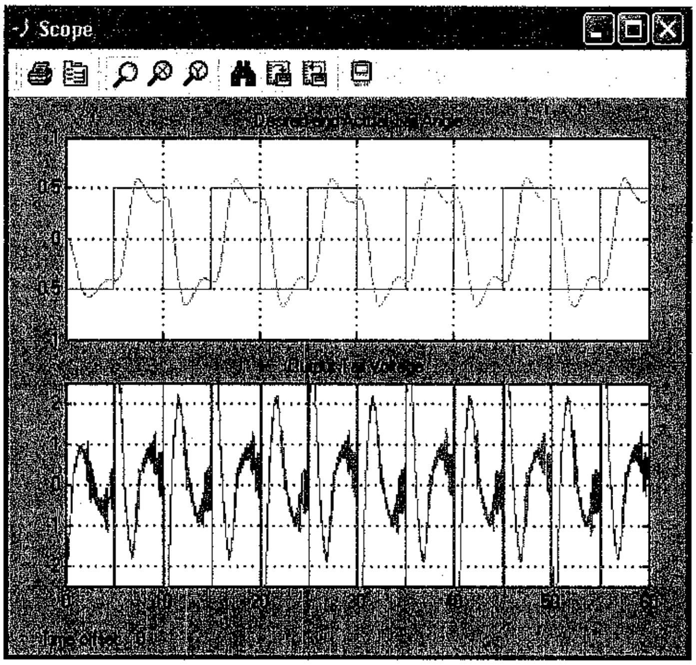


Figure 31 Tail rotor real-time experiment results using square wave as reference

The real-time experiment to study the system response to external disturbance is also performed by pushing the TRMS beam by hand in the horizontal plane when the system is stabilized. A result for this experiment is shown in Figure 32 below.

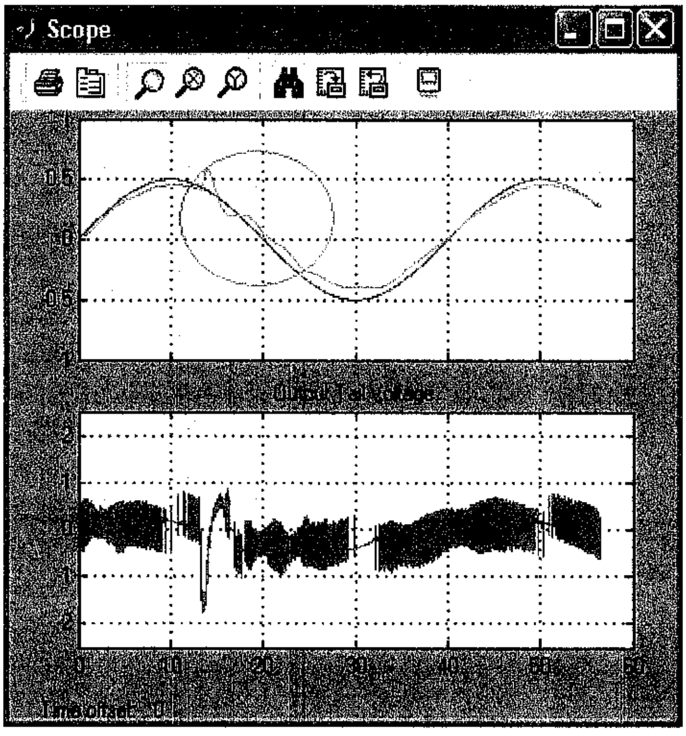


Figure 32 Tail rotor real-time experiment results with disturbance

5.2.2 1-DOF Main Rotor

The main rotor transfer function is given by:

$$T(s) = \frac{-8.88}{s^2 + 0.1334s + 4.44}$$

The state variables x_1 and x_2 for the main rotors are defined as follows:

$x_1 = y = \alpha_m$ where α_m is the main angle

$x_2 = \dot{x}_1$

Then the state space representation of the main rotor in control canonical form becomes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4.44 & -0.1334 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -8.88 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The controllability of the system can be determined by examining the controllability matrix of the system. The rank of

$$M = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -0.1344 \end{bmatrix}$$

is 2. Hence, the system is completely state controllable. The open linear system poles of the main rotor are at

$-0.0667 + j2.1061$ and

$-0.0667 - j2.1061$

The desired locations to place the eigenvalues of the closed-loop main rotor system are selected to be at -4 and -3 to have faster settling time to a unit step input. The feedback gain matrix, *K* that achieves the desired closed-loop poles is

$$K = \begin{bmatrix} 7.57 & 6.8666 \end{bmatrix}$$

The state-feedback system with linearized model is simulated to check the behavior of the closed-loop system. The result of the simulation is shown below.

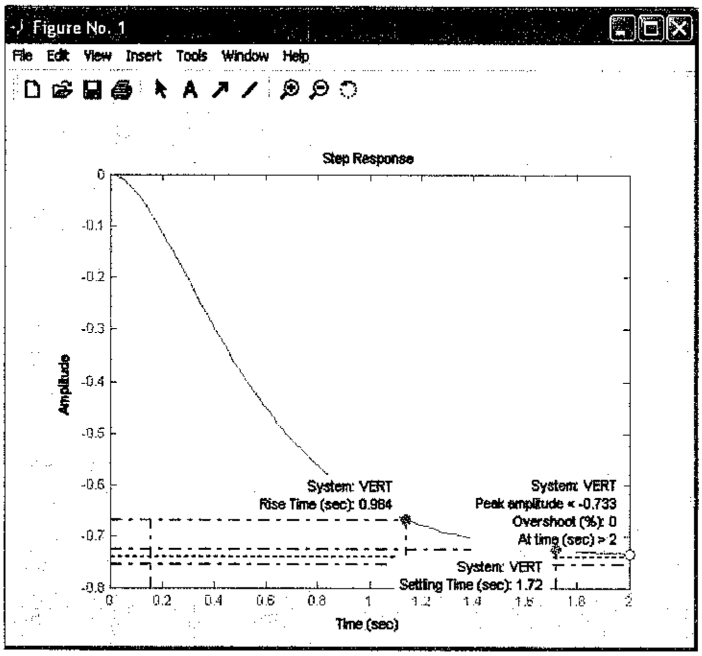


Figure 33 Simulation results for closed-loop main rotor model with state-feedback

Real-time experiment for main rotor is performed by setting the controller parameters in the Control Subsystem block in HelicopterPID RTWT. The results of real-time experiment using different types of trajectories are as shown in Figure 34 through 36.

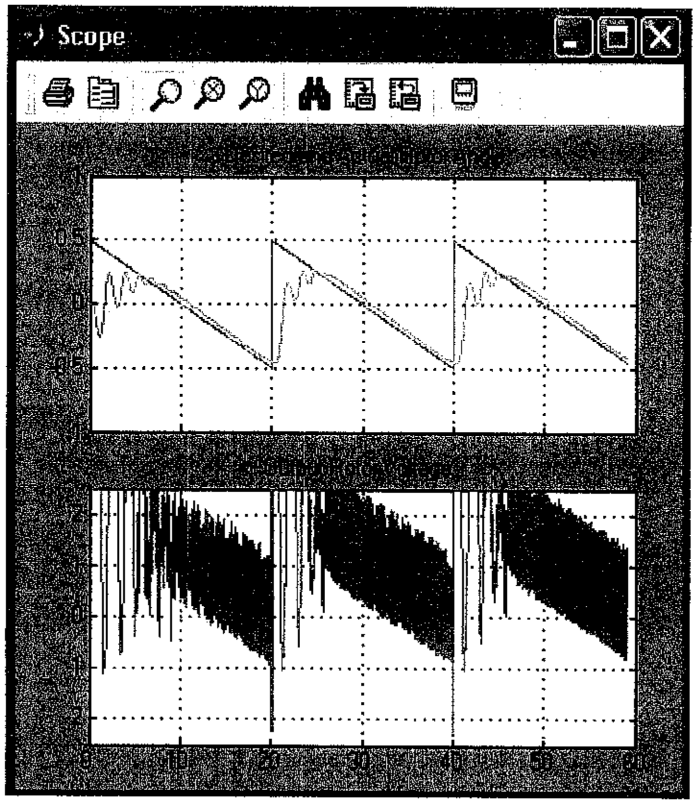


Figure 34 Main rotor real-time experiment results using saw tooth as reference

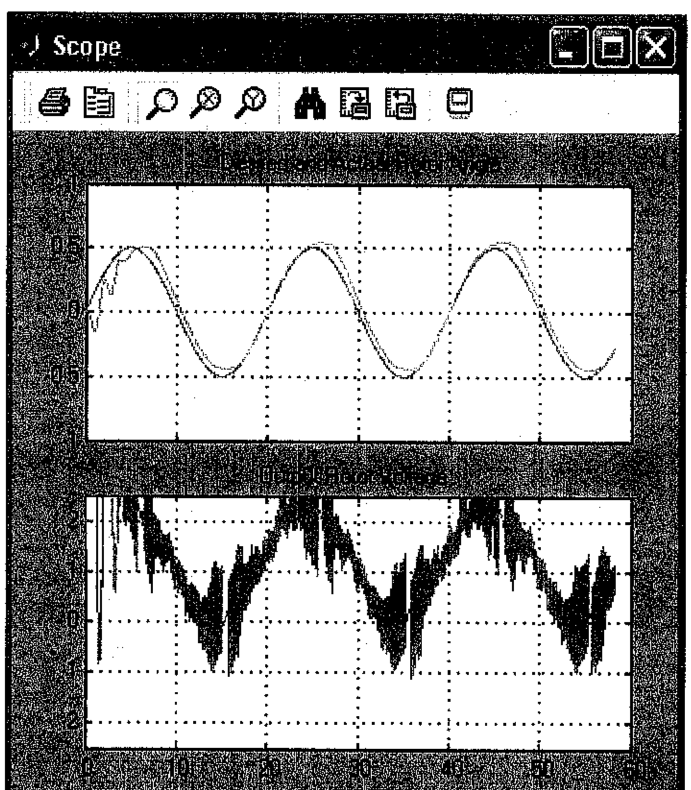


Figure 35 Main rotor real-time experiment results using sine wave as reference

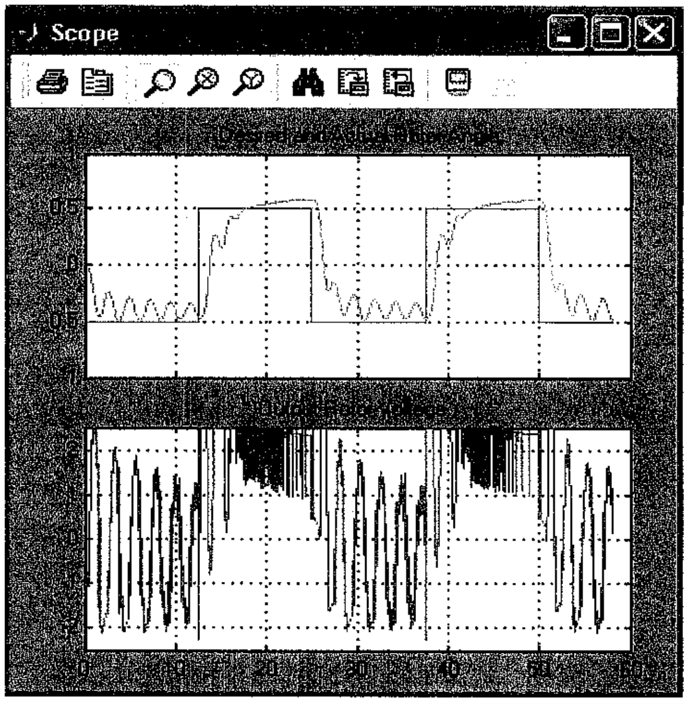


Figure 36 Main rotor real-time experiment results using square wave as reference

The real-time experiment to study the system response to external disturbance is also performed to main rotor by pushing the TRMS beam by hand in the vertical plane when the system is stabilized at 0 radian angle. A result for this experiment is shown in Figure 37 below.

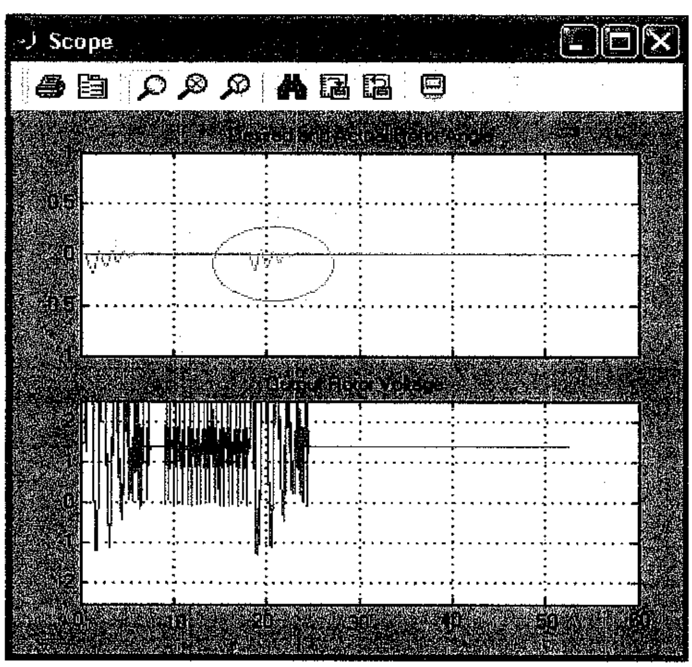


Figure 37 Main rotor real-time experiment results with disturbance

5.2.3 2-DOF Simultaneous Main and Tail Rotor

Real-time experiment to control both main and tail rotor simultaneously is also carried out. Figure 38 shows the responses of the simultaneous control in which the reference inputs are sinusoidal in the horizontal plane and a straight line at 0 radian angle in the vertical plane.

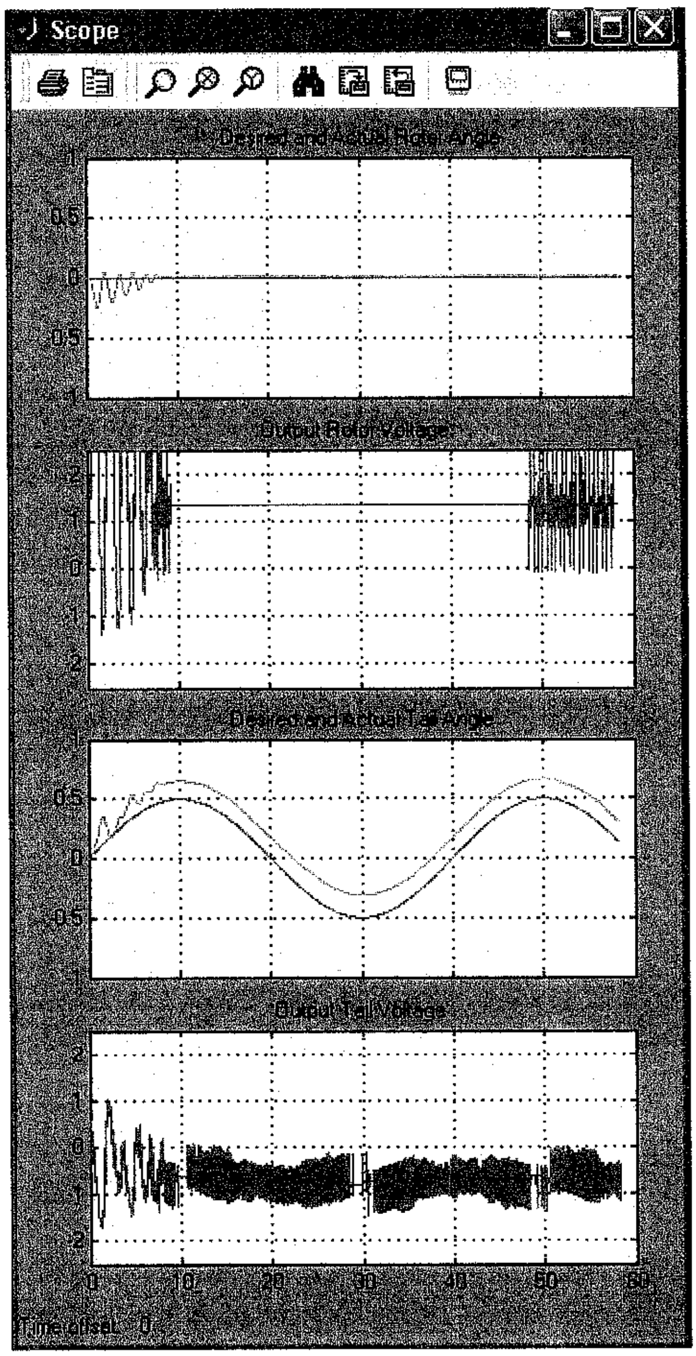


Figure 38 Simultaneous tail and main rotor control results

A real-time experiment to study the 2-DOF system responses to external disturbance is also performed by pushing the TRMS beam by hand in the vertical plane when the system is stabilized. A result for this experiment is shown in Figure 39 below. Notice that, there are strong interactions between the vertical and horizontal response in which when a disturbance applied to the main rotor, it will also influence the tail angle position.

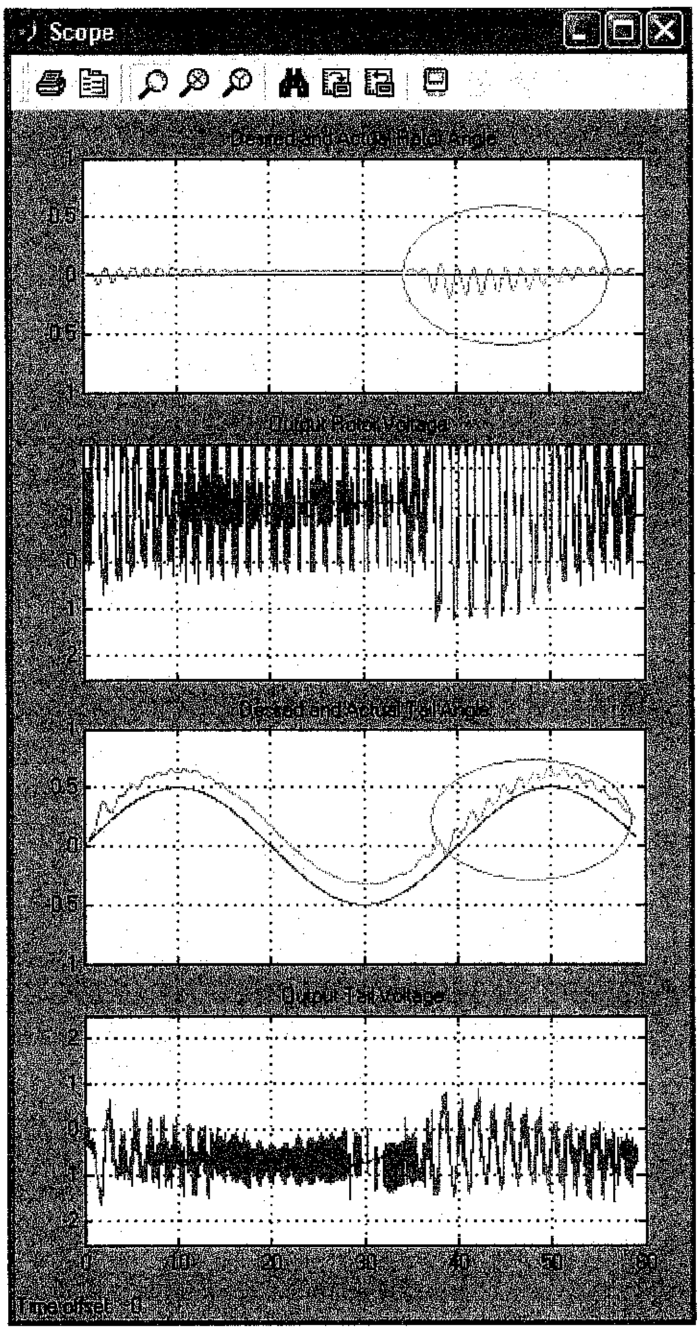


Figure 39 Simultaneous tail and main rotor control results with disturbance

5.3 Discussions and Findings

The model developed by using the step response gives a linear relationship between the input and output. Although it does not provide enough information to satisfy the analysis requirements, a linear transfer function model developed using this method are adequate for control design implementations.

In the state-feedback controller design using pole-placement method, the pole of the system can be placed at any arbitrary locations only if the control signal, u can control the behavior of each state variable. If any of the state variables cannot be controlled by the control u, it is not possible to place the poles at desired locations.

The reason for adding feedback is to improve the system characteristics or transient response such as rise time, overshoot, and settling time. The tail and main rotor systems transient responses after adding the state-feedback controller are much more improved compared than before.

In the state-feedback controller, the properties of the system are changed by the design of the controller gain matrix, K. The degree of freedom in choosing the pole locations is the main crux in the pole-placement method. The pole-placement strategy should be introduced to improve only the undesirable aspects of the open-loop response.

By selecting desired poles far into LHP of the s-plane, the system will have faster response and larger bandwidth. An increase in bandwidth will result in increase of system sensitivity to disturbances and measurement noise. Thus, the pole-placement strategy should avoid large increases in bandwidth.

The state-feedback controller leads to PD (Proportional & Derivative) type compensators. Since the state variables of the TRMS are the position angles and angular velocities of the tail and main rotors, these make the state-feedback controller to be the same as the PD compensators.

In this project, it is assumed that all states of the system are measurable. But, it is in fact the angular velocities of the rotors are reconstructed by differentiating and filtering the position angles of the rotors.

The reference input is introduced in such a way that the system output can track the external command with acceptable transient characteristics. The results of the real-time experiment show that both main and tail rotor able to follow the desired trajectory with significant steady-state errors.

From the individual real-time experiment of the tail and main rotor, it can be said that both systems are stable even when external disturbance is introduced by pushing the TRMS beam by hand in the horizontal and vertical plane the TRMS still capable to come back to track the external commands.

Strong interactions between the tail and main rotor are also seen by performing 2-DOF real-time experiment. When a disturbance introduced to the main rotor, it will also affect the motion of the tail rotor in the horizontal plane and thus the position angle of the tail rotor.

CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The state-feedback controllers designed in this project used the linearized models of the tail and main rotor due to the complexity of the mathematical modeling of the system. A linear transfer function model developed by evaluating the step responses of the tail and main rotors are proven to be adequate for control design implementations.

The state-feedback controllers for 1-DOF tail and main rotor designed in this project are able to follow the desired trajectory but with significant steady-state errors. The TRMS with state-feedback control is also stable even when a disturbance is applied to the system; it is still able to come back to follow the desired trajectory.

In conclusion, it can be said that the objectives of the project to obtain linearized models for 1-DOF main rotor and 1-DOF tail rotor of the TRMS and to design controller so that the state vector of the closed-loop system is stabilized around desired point of the state space and follows given trajectory have been achieved.

6.2 Recommendations

This project can be further improved by means of any other approach that will develop a model that can describe the real system much better. By having a more accurate model of the TRMS, a better control strategy can be introduced to the system.

The state-feedback controller designed for the TRMS can be further enhanced to eliminate the steady-state error by designing an integral controller for the TRMS. Integral control by the addition of an integration before the controlled plant can force the system so that the output follows the input command signals with zero steady-state error.

Since nowadays, artificial intelligence techniques such as neural network, fuzzy logic and genetic algorithm have become popular control system approach, these methods also can be applied to control the behavior of the TRMS.

REFERENCES

[1] Richard C. Dorf, Robert H. Bishop, "Modern Control Systems", Addison Wesley, 7^{tth} Edition, 1995

[2] Norman S. Nise, "Control Systems Engineering", John Wiley & Sons, 3rd Edition, 2000

[3] Twin Rotor MIMO System- Control Experiments 33-007-1C

APPENDICES

APPENDIX A

DESIGN FORMULAS FOR SECOND-ORDER SYSTEMS

The design formulas presented here are valid for second-order systems of the form

$$T(s) = \frac{\omega_n^{\ 2}}{s^2 + 2\omega_n \xi + \omega_n^{\ 2}}$$

- Settling time (to within 2% of the final value)

$$T_s = \frac{4}{\omega_n \xi}$$

- Percent Overshoot

$$\%OS = 100 e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

- Time to peak

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$

- Rise time (time to rise from 10% to 90% of final value)

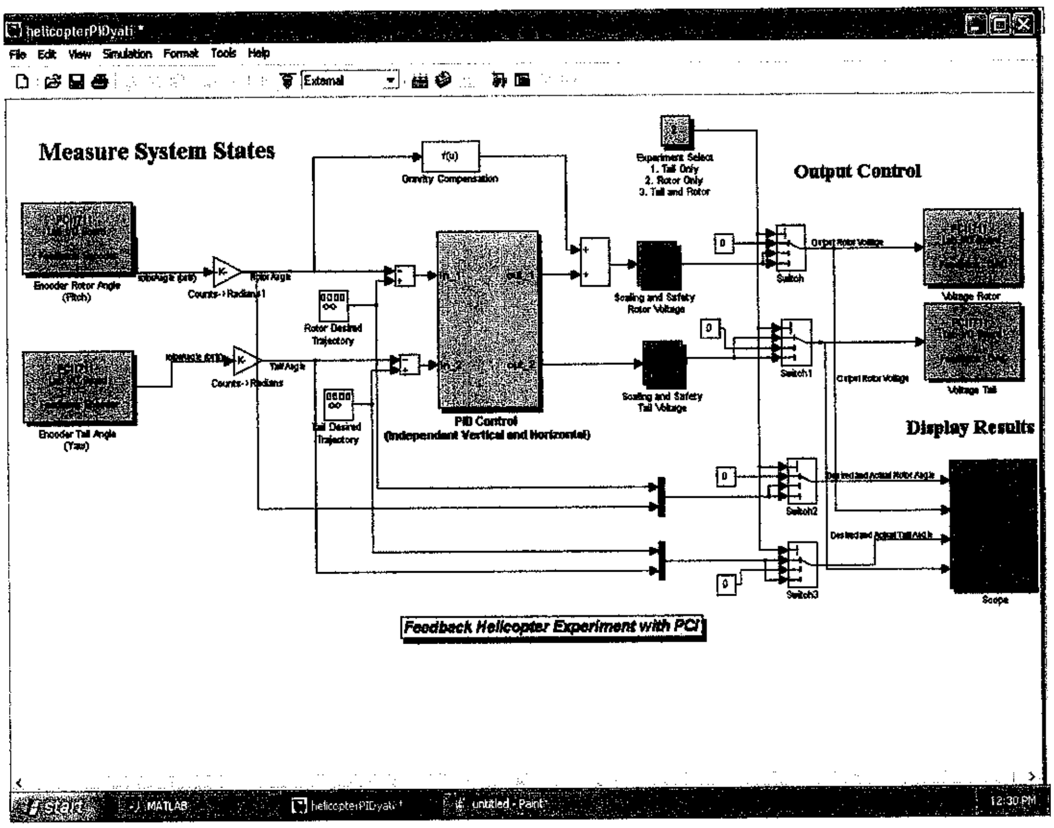
$$T_r = \frac{2.16\xi + 0.60}{\omega_n}$$

APPENDIX B

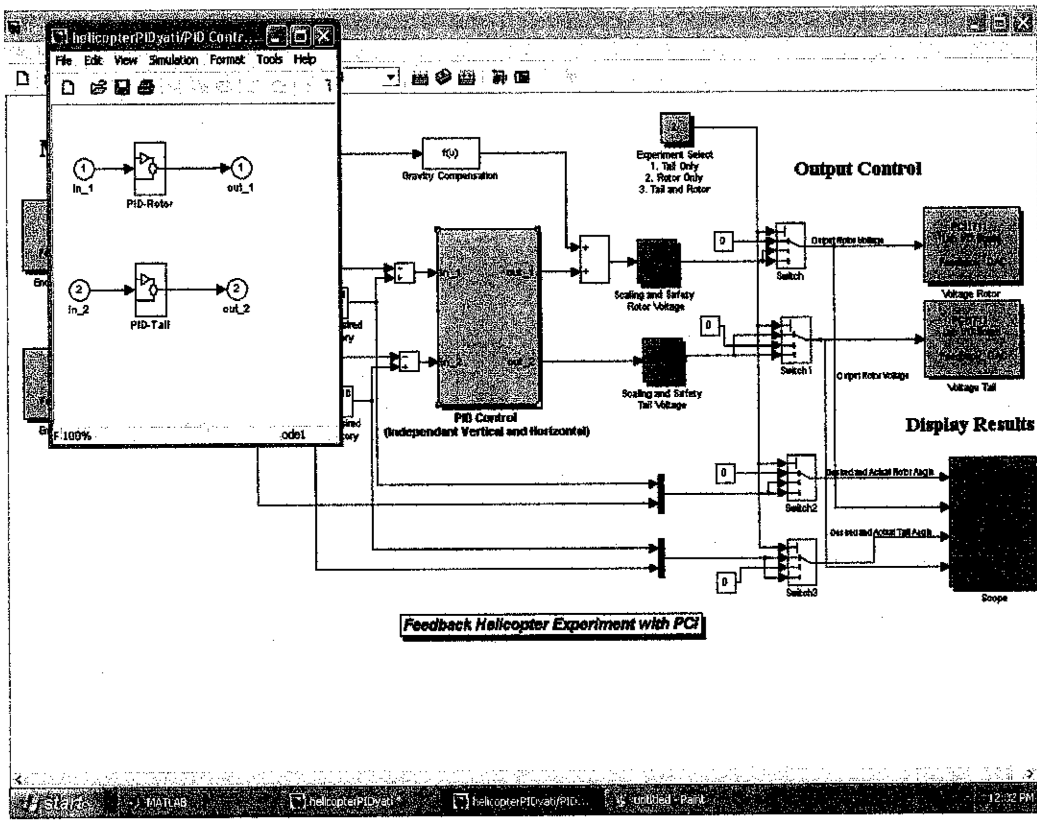
HELICOPTERPID CONTROLLER PARAMETERS SETTING

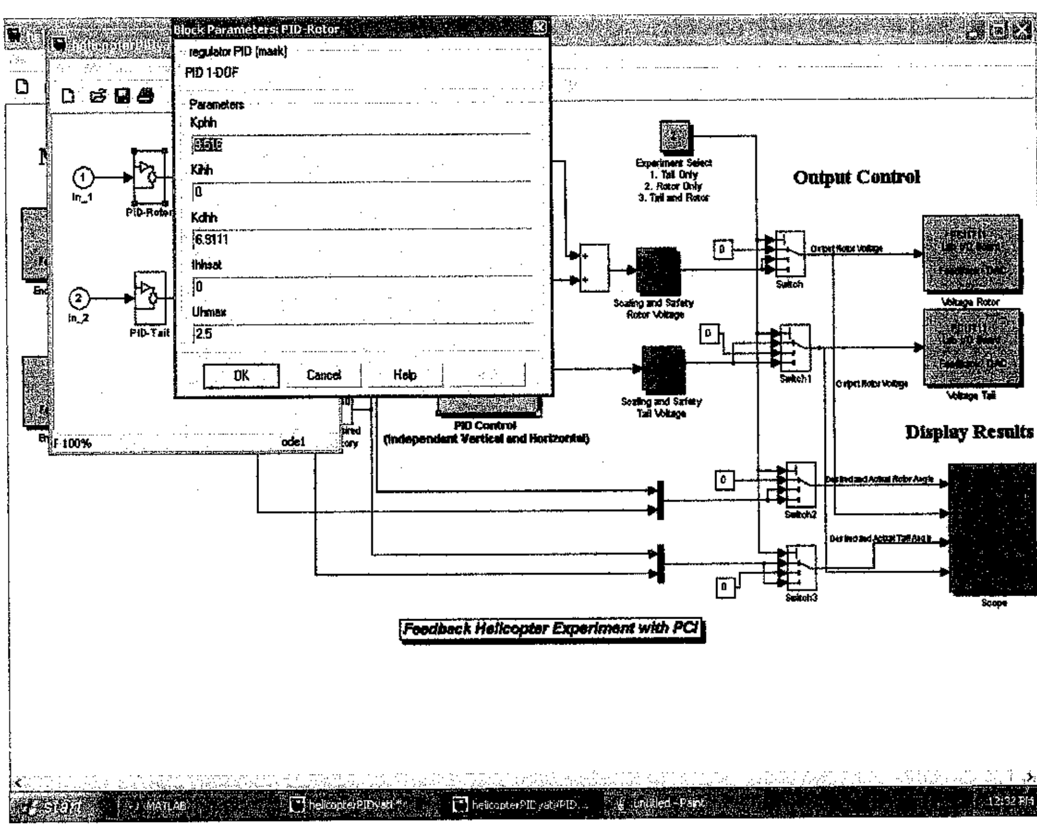
To set the controller parameters, follow the instructions below:

1. Double click the Control Subsystem mask colored blue at the middle of the HelicopterPID RTWT.



2. To set the gain K of the main rotor double click the PID-Rotor block and set the values of the feedback gains obtained.





3. To set the gain K of the tail rotor double click the PID-Tail block and set the values of the feedback gains obtained.