Petroleum Refinery Planning Under Uncertainty: A Multiobjective Optimization Approach with Economic and Operational Risk Management

by

Van Fu Shen

Dissertation submitted in partial fulfillment of
the requirements for the
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(Chemical Engineering)

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CERTIFICATION OF APPROVAL

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Approved by,

(MR. KHOR CHENG SEONG)

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

(VAN FU SHEN)

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Throughout the whole period conducting my Final Year Research Project, many had provided ample amount of guidance, assistances, advice and support. Therefore, I would like to take this opportunity to thank everyone whom had given their support and help throughout the whole period of completing this project.

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ABSTRACT

In the current modernized globalization era, crude oil prices have reached a record high of USD 147 per barrel according to the NYMEX exchange on June 2008. It is forecast to spiral upwards (with the current graph trend) to a much higher price level. The current situation of fluctuating high petroleum crude oil prices is affecting the markets and industries worldwide by the uncertainty and volatility of the petroleum industry. As oil refining is the downstream of the petroleum industry, it is increasingly important for refineries to operate at an optimal level in the presence of volatility of crude oil prices. Downstream refineries must assess the potential impact that may affect its optimal profit margin by considering the costs of purchasing the raw material of crude oils and prices of saleable intermediates and products as well as production yields. With optimization, refinery will be able to operate at optimal condition.

In this work, we have attempted to solve model formulation concerning the petroleum refinery planning under uncertainty. We use stochastic programming optimization incorporating the weighted sum method as well as the epsilon constraint method to solve the model formulation of the petroleum refinery planning under uncertainty.

The objective of this research project is to formulate a deterministic model followed by a two stage stochastic programming model with recourse problem for a petroleum refinery planning. The two stage stochastic risk model is then reformulated using Mean Absolute Deviation as the risk measure. After formulating the stochastic model using Mean Absolute Deviation, the problem is then investigated using the Pareto front solution of efficient frontier of the resulting multiobjective optimization problem by using the Weighted Sum Method as well as the ε-constraint method in order to obtain the Pareto Optimal Curve which generates a wide selection of optimization solutions for our problem. The implementation of the multiobjective optimization problem is then automated to report the model solution by capturing the solution values using the GAMS looping system. Note that some of the major parameters used throughout the formulated stochastic

programming model include prices of the raw material crude oil and saleable products, market demands for products, and production yields.

The main contribution on this work in the first part is to conduct a further study/research on the implementation of the model formulation in Khor et al. (2008) where the model formulated by Khor et al. (2008) uses variance as the risk measure. The results obtain in the previous paper will be compared with the method in this paper that incorporates Mean Absolute Deviation as the risk measure. To further study the model formulated, the solution obtain is further enhanced using the Weighted Sum Method as well as the Epsilon constraint method to obtain the Pareto Optimal Curve generation. Hence, most of the exposition on the model formulation and solution algorithms are taken directly from the original paper so as to provide the readers with the most accurate information possible.

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ABBREVIATIONS AND NOMENCLATURE

Indices

i	for the set of materials or products
j	for the set of processes
t	for the set of time periods

Sets

I	set of materials or products
J	set of processes
T	set of time periods

Parameters

$d_{i,t}$	demand for product i in time period t
$d_{i,t}^{\mathrm{L}}, d_{i,t}^{\mathrm{U}}$	lower and upper bounds on the demand of product i during period t, respectively
$p_t^{\mathrm{L}}, p_t^{\mathrm{U}}$	lower and upper bounds on the availability of crude oil during period t, respectively
$I_{i,t}^{\mathrm{fmin}}, I_{i,t}^{\mathrm{fmax}}$	minimum and maximum required amount of inventory for material i at the end of each time period
$b_{i,j}$	stoichiometric coefficient for material i in process j
$\gamma_{i,t}$	unit sales price of product type i in time period t
λ_t	unit purchase price of crude oil in time period t
$\tilde{\gamma}_{i,t}$	value of the final inventory of material i in time period t

$\tilde{\lambda}_{i,t}$	value of the starting inventory of material *i* in time period *t* (may be taken as the material purchase price for a two-period model)
$\alpha_{j,t}$	variable-size cost coefficient for the investment cost of capacity expansion of process *j* in time period *t*
$\beta_{j,t}$	fixed-cost charge for the investment cost of capacity expansion of process *j* in time period *t*
r_t, o_t	cost per man-hour of regular and overtime labour in time period *t*

Variables

$x_{j,t}$	production capacity of process *j* (j = 1, 2, …, *M*) during time period *t*
$x_{j,t-1}$	production capacity of process *j* (j = 1, 2, …, *M*) during time period $t-1$
$y_{j,t}$	vector of binary variables denoting capacity expansion alternatives of process *j* in period *t* (1 if there is an expansion, 0 if otherwise)
$CE_{j,t}$	vector of capacity expansion of process *j* in time period *t*
$S_{i,t}$	amount of (commercial) product *i* (i = 1, 2, …, *N*) sold in time period *t*
$L_{i,t}$	amount of lost demand for product *i* in time period *t*
P_t	amount of crude oil purchased in time period *t*
$I_{i,t}^{s}, I_{i,t}^{f}$	initial and final amount of inventory of material *i* in time period *t*
$H_{i,t}$	amount of product type *i* to be subcontracted or outsourced in time period *t*
R_t, O_t	regular and overtime working or production hours in time period *t*

Superscripts

$(\ )^{L}$	lower bound
$(\ )^{U}$	upper bound

Nomenclature and Notations for the Numerical Example (as depicted in Figure 5.1)

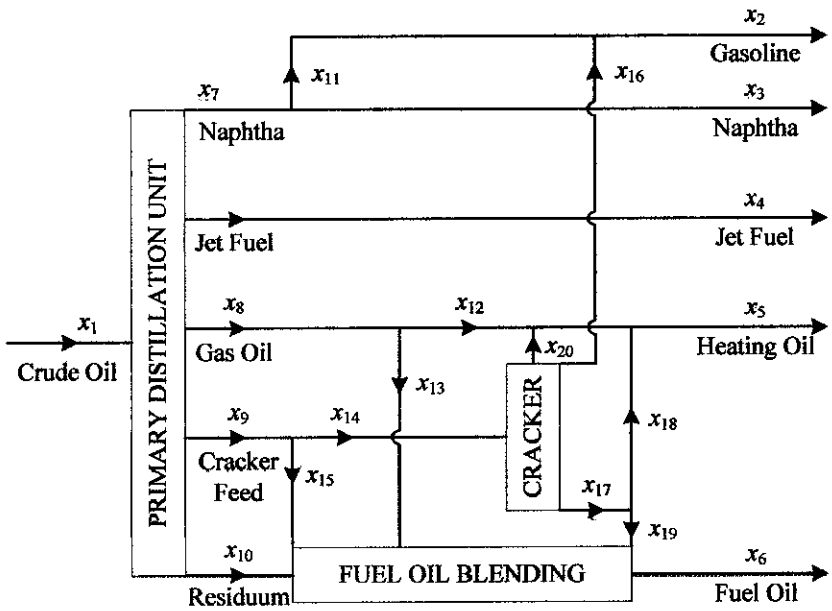

Figure 5.1: Simplified representation of a petroleum refinery production from crude oil (Khor et al. 2008)

x_1	mass flow rate (in ton/day) of crude oil stream
x_2	mass flow rate (in ton/day) of gasoline in combined streams of x_{11} and x_{16}
x_3	mass flow rate (in ton/day) of naphtha stream after a splitter
x_4	mass flow rate (in ton/day) of jet fuel stream
x_5	mass flow rate (in ton/day) of heating oil stream
x_6	mass flow rate (in ton/day) of fuel oil stream
x_7	mass flow rate (in ton/day) of naphtha stream exiting the primary distillation unit (PDU)
x_8	mass flow rate (in ton/day) of gas oil stream
x_9	mass flow rate (in ton/day) of cracker feed stream
x_{10}	mass flow rate (in ton/day) of residuum stream
x_{11}	mass flow rate (in ton/day) of gasoline stream after splitting of naphtha stream exiting the PDU
x_{12}	mass flow rate (in ton/day) of gas oil stream after a splitter

x_{13} mass flow rate (in ton/day) of gas oil stream entering the fuel oil blending facility

x_{14} mass flow rate (in ton/day) of cracker feed stream after a splitter

x_{15} mass flow rate (in ton/day) of cracker feed stream entering the fuel oil blending facility

x_{16} mass flow rate (in ton/day) of gasoline stream exiting the cracker unit

x_{17} mass flow rate (in ton/day) of stream exiting the cracker unit into a splitter

x_{18} mass flow rate (in ton/day) of heating oil stream after splitting of cracker output

x_{19} mass flow rate (in ton/day) of cracker output stream

x_{19} mass flow rate (in ton/day) of heating oil stream exiting the cracker unit

CHAPTER 1
INTRODUCTION

1.1 BACKGROUND STUDY

Petroleum or crude oil is a naturally occurring, flammable liquid found in rock formations in the Earth consisting of a complex mixture of hydrocarbons of various molecular weights plus other organic compounds. The composition hydrocarbon in crude oil mixture is highly variable and ranges from as much as 97% by weight in the lighter oils to as little as 50% in the heavier oils and bitumen. The hydrocarbons in crude oil are mostly alkanes, cycloalkanes and various aromatic hydrocarbons. The composition of weights is shown below:-

Table 1.1: Table of Composition of Crude Oil by Weight Percentage

Element	Percent Range
Carbon	83 to 87%
Hydrogen	10 to 14%
Nitrogen	0.1 to 2%
Oxygen	0.1 to 1.5%
Sulfur	0.5 to 6%
Metals	Less than 1000 ppm

Petroleum is the raw material for many chemical products, including pharmaceuticals, solvents, fertilizers, pesticides and plastics. The industry is divided into the major components: upstream and downstream. Petroleum is vital to many industries thus is critical concern to many nations. The world currently consumes energy at a rate of 200 million barrels of oil per day, with 87 percent supplied by oil, gas and coal. Topping the oil consumers largely consists of developed nations; in fact 24% of the oil consumed in 2004 went to the United States alone. The graph below shows World Energy consumption (in Quadrillion Btu):-

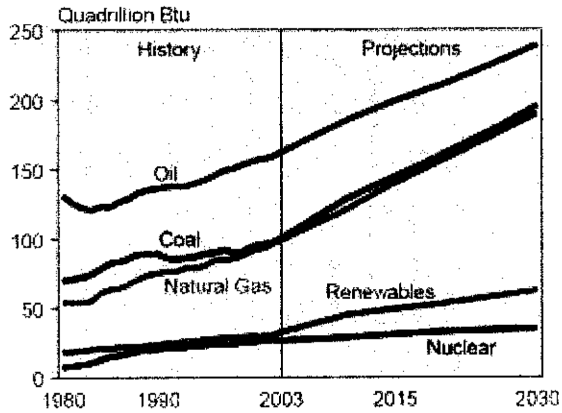

Figure 1.1: World Marketed Energy Use by Energy Type, 1980 – 2030

**Source: History: Energy Information Administration (EIA), International Energy Annual 2003 (May-July 2005), website www.eia.doe.gov/iea/. Projections: EIA, System for the Analysis of Global Energy Markets (2006)*

The price of crude oil has reached a record high of USD147.27 according to the NYMEX Exchange which occurred on 11th July 2008. At high fluctuation rate of crude oil price, it is essential to have refinery optimization to maximize profit from oil sales. The price comparison between years is tabulated into a graph as below:-

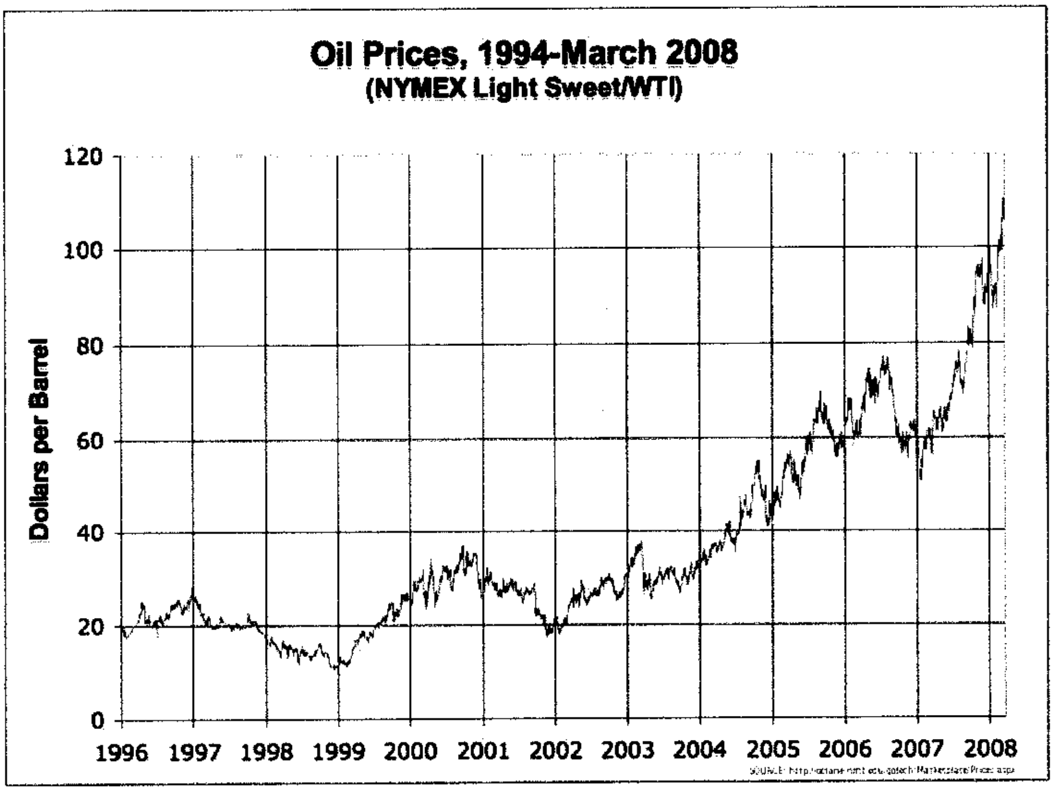

Figure 1.2: Oil Prices from 1994 to March 2008 (NYMEX Light Sweet/WTI)

*Source: http://octane.nmt.edu/gotech/Marketplace/Prices.aspx

1.2 OPTIMIZATION

Optimization is part of life. In our day to day lives we make decisions that we believe can maximize or minimize our set of objectives. This is known as optimization. However, as the system becomes more complicated involving more and more decisions and becoming constrained by various factors, it is difficult to take optimal decisions. Further, many times the stakes are high and there are multiple stake holders to be satisfied (Urmila Diwekar, 2003).

Optimization is the use of specific methods to determine the most cost effective and efficient solution to a problem or design for a process. This technique is one of the major quantitative tools in industrial decision making. A wide variety of problems in the design, construction, operation and analysis of chemical plants can be resolved by optimization (Edgar et. al., 2001). A typical engineering design problem is always involved with the objective function of maximizing profit and/or minimizing cost. Therefore, mathematical optimization theory provides a better alternative for decision making in these situations provided one can represent the decisions and the system mathematically (Urmila Diwekar, 2003).

For optimization of the crude oil refinery, we are using the Stochastic Programming which focuses on the Weighted Sum Method as well as the Epsilon Constraint method. Both methods will be explained in the Literature Review of the introduction section.

1.3 PROBLEM STATEMENT

In view of the current situation, crude oil prices have fluctuated to a record high of USD 147 per barrel according to the NYMEX Exchange. The midterm production planning problem for petroleum refineries would be on how to determine maximum-profit optimal midterm refinery planning. For our problem statement, we are given the available process units and their capacities as well as the crude oil and refinery products. What is the amount of materials processed at each time, in each unit, in each stream under uncertainties in:

- Prices of crude oil + saleable products

  ⇒ **(objective coefficients)**
- Market demand for products

  ⇒ **RHS coefficients of constraints**
- Product/Production **yields** of crude oil in Crude Distillation Unit (CDU)

  ⇒ **LHS coefficients of constraints**

In determining the problem statements, our objective is to determine the amount of materials that are processed in each process units by considering the following uncertain parameters:-

a) Market demands for products. Examples are the productions amounts of the desired products.
b) Prices of crude oil and the saleable products.
c) Product (or production) yields of crude oil from chemical reactions in the primary crude distillation unit.

It is now more important than ever for petroleum refineries to operate at an optimal level in the present dynamic global economy. This situation calls for a more robust planning of the refinery operations to be undertaken by considering possible uncertainties in the major parameters that primarily include prices of the raw material crude oil and saleable products, market demands for products, and production yields.

1.4 RESEARCH OBJECTIVES

The main objectives of research are as below:-

1. To formulate a deterministic optimization model for petroleum refinery planning;
2. To transform the deterministic model into a two-stage stochastic programming with fixed recourse formulation that accounts for uncertainty in the objective function coefficients of prices, the right-hand side constraint coefficients of product demands, and the left-hand side constraint coefficients of yields by implementing a suitable scenario generation approach.
3. To formulate two stage stochastic programming model with recourse using Mean-Absolute Deviation as risk measure.
4. To solve the stochastic programming model using the modeling language GAMS;
5. To automate the procedure for reporting the model solution by capturing the solution values using the GAMS looping system;
6. To investigate the Pareto front solution of efficient frontier (consisting of efficient or non-dominated points) of the resulting multiobjective optimization problem by using an automated recursive statement (such as loop) in GAMS.
7. To investigate further the multiobjective optimization problem by incorporating the Weighted Sum Method (WS method) as well as the ε-constraint method. Both methods are reformulated into the optimization model. Both optimization models will be compared with the original model as formulated by Khor et al. (2008) to know whether the new method produces a more evenly distributed Pareto Optimal Curve.

CHAPTER 2
LITERATURE REVIEW

2.1 PREVIOUS WORK

Table 2.1: Table of Previous Work from Previous Researchers

No	Author (Year)	Optimization Model Type	Solution Strategy	Application	Limitations
1	Guillén-Gosálbez and Grossmann (in press)	Stochastic MINLP	ε-constraint	Chemical supply chain design under uncertainty with environmental considerations	Does not consider risk management
2	You and Grossman (2008)	Stochastic MINLP	ε-constraint	Responsive supply chain design under demand uncertainty	Does not consider multiple objectives & risk management
3	Hugo et al. (2005)	Deterministic MILP	parametric programming	Strategic planning of hydrogen infrastructure with environmental considerations	Does not consider important uncertainty factors
4	Hugo and Pistikopoulos (2005)	Deterministic MILP	ε-constraint & parametric programming	Long-range design and planning of supply chain network	Does not consider important multiple objectives & uncertainty

2.2 INTRODUCTION TO STOCHASTIC PROGRAMMING

Process optimization is a manufacturing process to optimize some specified set of parameters without violating some constraint. The most common goals of process optimization are minimizing cost, maximizing profit and/or maximizing efficiency. Therefore, the main goal of optimizing a process is to maximize one or more of the process specifications, while keeping all others within their constraints. The main components of optimization under uncertainty (**Figure 2.1**) are as below:-

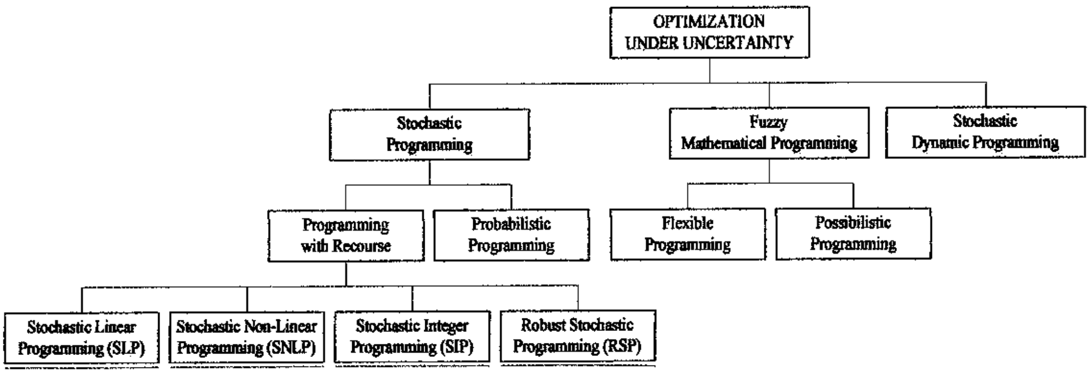

Figure 2.1: Established optimization techniques under uncertainty

Stochastic programming is an optimization method based on the probability theory. Stochastic programming is a framework for modeling optimization problems that involve uncertainty whereas deterministic optimization problems are formulated with known parameters). Uncertainty is usually characterized by a probability distribution on the parameters. Stochastic programming takes advantage of the fact that probability distributions governing the data are known or can be estimated. The goal of stochastic programming is to find the most feasible possible data that maximizes the expectation of function of the decisions and the random variables.

In constructing a mathematical model of a decision making situation, we should use approaches to reflect the randomness or the ambiguity involving parameters in a situation (Sakawa et al. 2001). Stochastic programming is a typical approach for such decision making problems involving uncertainty. What makes stochastic programming good is because it allows the decision maker to analyze multiple scenarios of an uncertain future, each with an associated probability of occurrence. Optimization maximizes net profit while minimizing various expected costs. What makes stochastic programming good is because it allows the decision maker to analyze multiple scenarios of an uncertain future, each with an associated probability of occurrence (Khor et al. 2008). Optimization maximizes net profit while minimizing various expected costs.

2.3 TWO-STAGE STOCHASTIC PROGRAMMING WITH RECOURSE SUBPROBLEM

The most widely applied and studied stochastic programming models are two-stage linear programs. In this section, the decision maker takes some action in the first stage, after which a random event occurs affecting the outcome of the first stage decision. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experience as a result of the first-stage decision. The optimal policy from such a model is a single first-stage policy and a collection of recourse decision defining which second-stage action should be taken in response to each random outcome.

Recourse models result when some of the decisions must be fixed before information relevant to the uncertainties is available, while some of them can be delayed until afterward. Stochastic programming with recourse is often used to model uncertainty, giving rise to large-scale mathematical programs that require the use of decomposition methods and approximation schemes for their solution. The term 'recourse' refers to the opportunity to adapt a solution to the specific outcome observed (Higle, 2005). Recourse problems are always presented as problems in which there are two or more decision stages.

It is highly evident that in production system, demand forecasts are often critical to the planning process. When demand is assumed to be known with certainty, an optimal deterministic production plan can be obtained easily. However, in reality demand is rarely known with absolute certainty. Thus, the two-stage production planning process is used to model problem that arises with uncertainty.

A Two-Stage Stochastic Programming with recourse subproblem can be expressed as below:-

$$\begin{aligned} \min\ & c^T x + E_{\xi}\left[Q\left(x,\xi(\omega)\right)\right] \\ & \text{s.t. to } Ax = b \\ & x \in X \geq 0 \end{aligned} \qquad (1)$$

$$\begin{aligned} Q\left(x,\xi(\omega)\right) = \text{minimize } & q^T(\omega)y(\omega) \\ \text{subject to } & Wy(\omega) = h(\omega) - T(\omega)x \\ & y(\omega) \geq 0 \end{aligned} \qquad (2)$$

With the notation:

$x \in \mathbb{R}^n$	: Vector of first-stage decision variables, size $(n \times 1)$
C	: First-stage column vector of cost coefficient, sizes $(n \times 1)$
A	: First-stage coefficient matrix, size $(m \times n)$
b	: Corresponding right-hand side vectors, size $(m \times 1)$
$\omega \in \Omega$	: Random events or scenario
$\xi(\omega)$	: Random vector
$q(\omega)$	: Second stage vector of recourse cost coefficient vectors size $(k \times 1)$
$h(\omega)$	: Second stage right-hand side vectors, size $(l \times 1)$
$T(\omega)$	: Matrix that ties the two stages together, size $(l \times k)$
$W(\omega)$	: Random recourse coefficient matrix, size $(l \times k)$
$y(\omega)$	: Vector of second-stage decision variables, size $(k \times 1)$

From the Two-Stage Stochastic Programming above, Equation **(1)** is known as the first stage, where x is referred to as the "here-and-now" decision. Note that x does not response to ω. Meanwhile, y represents the second stage variable with a "wait and see" approach. y is determined only after observations regarding ω have been obtained.

2.3.1 Two-Stage Stochastic Programming with Simple Recourse Subproblem

Simple recourse problems feature a very special form of the recourse matrix when the constraint coefficients in the second stage model, W, form an identity matrix. Deviations from a target value are penalized by a linear penalty. A simple recourse problem arises in many situations. For example, when 'target values' can be identified, and a primary concern involves minimizing deviations from these target values (although these might be weighted deviations), a simple recourse problems result.

2.3.2 Two-Stage Stochastic Programming with Fixed Recourse Subproblem

A fixed recourse problem is one in which the constraint matrix in the recourse subproblem is not subject to uncertainty (i.e., it is fixed). Meaning to say, fixed recourse model arises when the constraint coefficients matrix W (ω) in the second-stage problem is not subject to uncertainty, that is, it is fixed and hence is denoted simply as the matrix W. For a Fixed Recourse Subproblem, Equation **(2)** coefficient $W(\omega)$ is fixed, which means the value of W is determined and not subject to uncertainty.

2.3.3 Two-Stage Stochastic Programming with Complete Recourse Subproblem

A two-stage stochastic programming with complete recourse subproblem is said to have ***complete recourse*** if the recourse cost for every possible uncertainty remains finite (has a value), independent of the nature of the first-stage decisions (Khor et al. 2008). If a problem has complete recourse, the recourse function is necessarily finite. To ensure complete recourse in any problem, penalty functions (of costs) for deviations from constraint satisfaction of prescribed limits are used.

CHAPTER 3
METHODOLOGY

3.1 METHODOLOGY (GANTT CHART)

3.1.1 Semester 1 (July 2008)

No.	Task	Week													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Topic Selection	■	■												
2	Submission of Proposal		■												
3	Literature Review		■	■	■	■	■	■	■						
4	Preliminary Report Submission					■									
5	Stochastic Model Formulation with MAD					■	■	■	■	■					
6	Submission of Progress Report									■					
7	Computational Studies with GAMS											■	■	■	
8	Seminar I											■			
9	Submission of Interim Report and Final Oral Presentation													■	■

Next Semester: model reformulation & computational studies using:

- **Weighted Sum Method**
- **ε-constraint Method**

3.1.2 Semester 2 (January 2009)

No.	Task	Week													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Discussion with lecturer	■													
2	Project work commence		■	■	■	■	■	■	■						
3	Progress Report Submission				■										
4	Weighted Sum Method Model Formulation & GAMS					■	■	■	■						
5	Epsilon Constraint Method Model Formulation & GAMS						■	■	■	■					
6	Progress Report II Submission									■					
7	Pre-EDX										■				
8	EDX												■		
9	Submission of Final Report														■
10	Final Oral Presentation (Week 18 & 19)														
11	Submission of Hardbound (Week 20)														

3.2 METHODOLOGY (FLOW CHART)

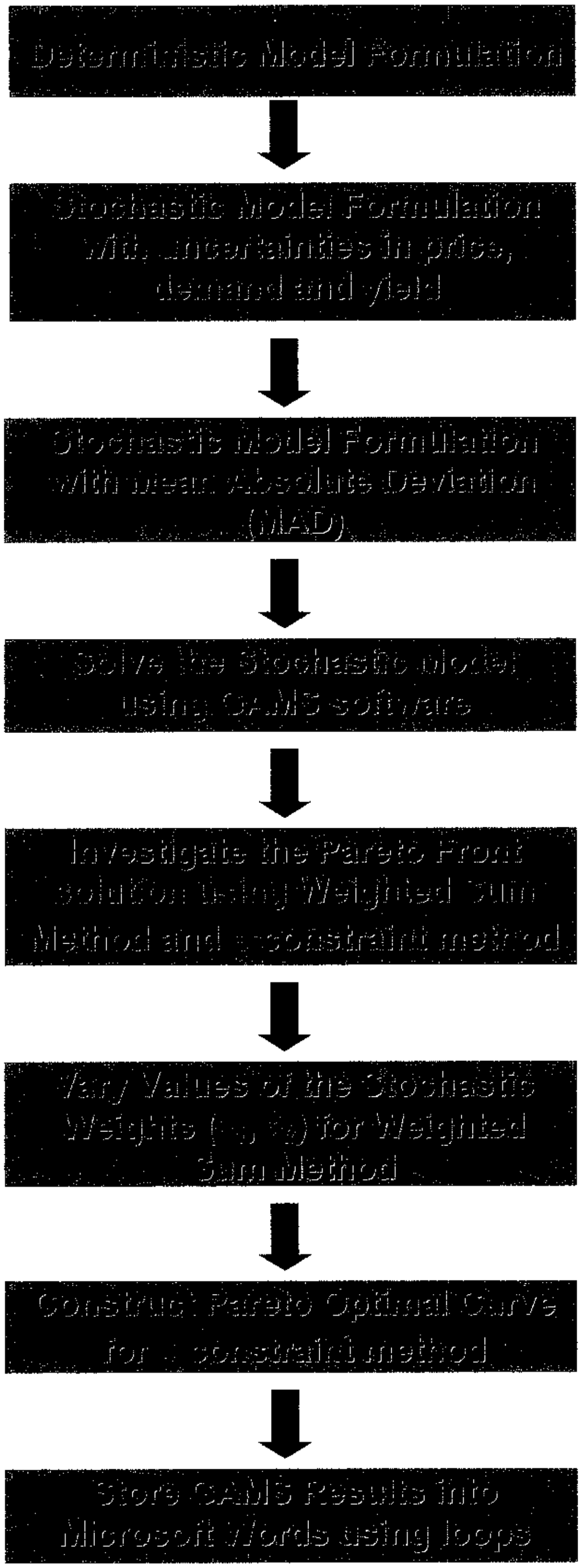

Figure 3.1: Methodology flow chart for the model formulation problem

CHAPTER 4

MODEL FORMULATION

Stochastic Programming Optimization; one of its heaviest users has been the petroleum refining industry. Refining operations are routinely planned by formal optimization, often on a daily or even hourly basis. Our goal in optimization model is to identify an optimal solution which is the most feasible choice satisfying all constrains (Rardin, 1998).

4.1 STOCHASTIC MODEL FORMULATION FOR DEVIATION OF RECOURSE PENALTY USING MEAN ABSOLUTE DEVIATION (MAD)

The mean-absolute deviation (MAD) is the average absolute deviation from the mean. The mean-absolute deviation (MAD) is defined as:

$$\text{MAD} = \frac{1}{n}\sum_{i=1}^{n} f_i \left| x_i - \bar{x} \right|$$

where n is the sample size, x_i are the values of the samples, $\bar{x}$ is the mean, and f_i is the absolute frequency. The use of Mean Absolute Deviation serves to overcome the computational difficulties and therefore enables large scale problems to be solved faster and more efficiently. Below shows the penalty functions for Mean Absolute Deviation when we maximize and minimize the objective function to obtain the penalty and return values:-

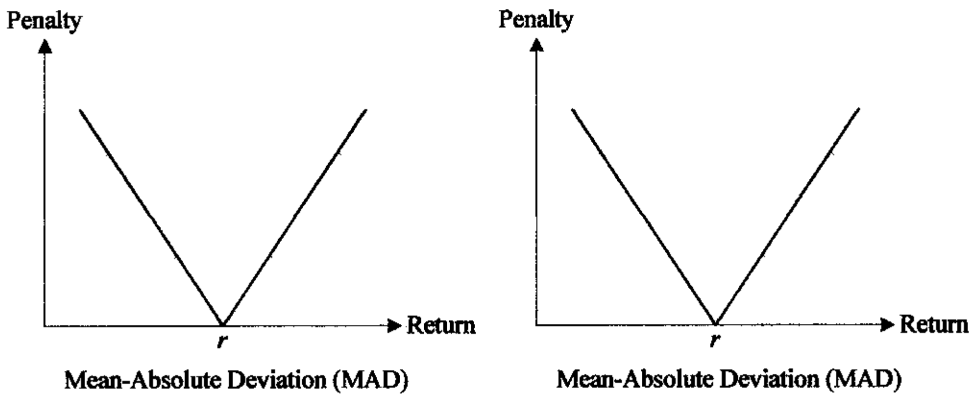

Figure 4.1: Penalty functions for Mean Absolute Deviation

As presented by Khor et al. (2008), Risk Model III model formulation using Mean Absolute Deviation for deviation of recourse penalty is given by

$$\max z \ = E(z_0) - \theta_1 V(z_0) - E_s - \theta_3 W_s \qquad \textbf{(3)}$$

Where,

$E(z_0)$ = Expected Profit

$\theta_1 V(z_0)$ = Deviation of Profit

E_s = Expected Recourse Penalty

$\theta_3 W_s$ = Deviation of Recourse Penalty

θ_1, θ_2 = Component weights of the objective function or risk

Based on Equation (3), the term W_s corresponds to the Mean Absolute Deviation (MAD) of the expected penalty costs due to violations of constraints for maximum demands and yield. The MAD of the expected penalty costs is formulated as below:

$$W_s = \sum_{s\in S} p_s \left| \xi_s - E_{s'} \right| = \sum_{s\in S} p_s \left| \xi_s - \sum_{s'\in S} p_{s'} \xi_{s'} \right|$$

$$\Rightarrow W_s = \sum_{s\in S} p_s \left| \sum_{i\in I} \begin{bmatrix} \left(c_i^+ z_{i,s}^+ + c_i^- z_{i,s}^-\right) \\ +\left(q_i^+ y_{i,s}^+ + q_i^- y_{i,s}^-\right) \end{bmatrix} - \sum_{i\in I} \sum_{s'\in S'} p_{s'} \begin{bmatrix} \left(c_i^+ z_{i,s'}^+ + c_i^- z_{i,s'}^-\right) \\ +\left(q_i^+ y_{i,k,s'}^+ + q_i^- y_{i,k,s'}^-\right) \end{bmatrix} \right| \qquad \textbf{(4)}$$

defined and W_s must then satisfy the following conditions:

$$W_s \geq -\sum_{s\in S} p_s \left\{ \sum_{i\in I} \begin{bmatrix} \left(c_i^+ z_{i,s}^+ + c_i^- z_{i,s}^-\right) \\ +\left(q_i^+ y_{i,s}^+ + q_i^- y_{i,s}^-\right) \end{bmatrix} - \sum_{i\in I} \sum_{s'\in S'} p_{s'} \begin{bmatrix} \left(c_i^+ z_{i,s'}^+ + c_i^- z_{i,s'}^-\right) \\ +\sum_{k\in K}\left(q_i^+ y_{i,k,s'}^+ + q_i^- y_{i,k,s'}^-\right) \end{bmatrix} \right\} \qquad \textbf{(5)}$$

$$W_s \geq \sum_{s\in S} p_s \left\{ \sum_{i\in I} \begin{bmatrix} \left(c_i^+ z_{i,s}^+ + c_i^- z_{i,s}^-\right) \\ +\left(q_i^+ y_{i,s}^+ + q_i^- y_{i,s}^-\right) \end{bmatrix} - \sum_{i\in I} \sum_{s'\in S'} p_{s'} \begin{bmatrix} \left(c_i^+ z_{i,s'}^+ + c_i^- z_{i,s'}^-\right) \\ +\sum_{k\in K}\left(q_i^+ y_{i,k,s'}^+ + q_i^- y_{i,k,s'}^-\right) \end{bmatrix} \right\} \qquad \textbf{(6)}$$

and the non-negativity constraints for W_s:

$$W \geq 0$$

Meanwhile, based on Equation (3) the term E_s corresponds to the expected recourse penalty for the second-stage costs due to yield uncertainty. The expected recourse penalty, E_s for the second-stage costs is given by:

$$E_{s,demand} = \sum_{i \in I} \sum_{s \in S} p_s (c_i^+ z_{i,s}^+ + c_i^- z_{i,s}^-)$$

$$E_{s,yield} = \sum_{i \in I} \sum_{s \in S} \sum_{k \in K} p_s (q_{i,k}^+ y_{i,k,s}^+ + q_{i,k}^- y_{i,k,s}^-)$$

Therefore;

$$E_s = E_{s,demand} + E_{s,yield}$$

$$E_s = \sum_{i \in I} \sum_{s \in S} p_s \left[(c_i^+ z_{i,s}^+ + c_i^- z_{i,s}^-) + \sum_{k \in K} (q_{i,k}^+ y_{i,k,s}^+ + q_{i,k}^- y_{i,k,s}^-) \right] = \sum_{i \in I} \sum_{s \in S} p_s \xi_s \quad (7)$$

Where; $\xi_s = (c_i^+ z_{i,s}^+ + c_i^- z_{i,s}^-) + \sum_{k \in K} (q_{i,k}^+ y_{i,k,s}^+ + q_{i,k}^- y_{i,k,s}^-)$

Thus, the Stochastic Model formulation using Mean-Absolute Deviation (MAD) for deviation of recourse penalty is formulated as Equation (3) by substituting E_s and W_s with Equation (4) and (7) respectively.

4.2 INTRODUCTION TO WEIGHTED SUM METHOD FORMULATION

The weighted sum method is used to approximate the non-dominated set through the identification of extreme points along the non-dominated surface. The idea of the weighting methods (Gass and Saaty, 1955; and Zadeh, 1963) is to associate each objective function with a weighting coefficient and minimize the weighted sum of the objectives. In this way, the multi-objective optimization problem is transformed into a series of single objective optimization problems.

The weights of each constraint should be a value greater than zero to satisfy the optimal solution of the weighted problem is a non-dominated solution. As long as the values of the weights are greater than zero, the multiobjective optimization will produce solutions between these two points. For our model formulation we incorporate the risk model as presented by Khor et al. (2008). The risk model is reformulated using Mean Absolute Deviation incorporating θ_1 and θ_2 values which represent the weights of the components of the objective function or risk factor.

As represented in equation (6), the MAD(z_0) is weighted by the operational risk factor , which is varied over the entire range of (0, ∞) to generate a set of feasible decisions that have maximum return for a given level of risk. This feasible decisions set is equivalent to the "efficient frontier" portfolios introduced by Markowitz (1952; 1959) for financial investment applications. The parameter θ_2 can be seen as reflecting the decision maker's attitude towards variability; in other words, it signifies the risk attitude of the decision maker.

4.3 STOCHASTIC PROGRAMMING MODEL FORMULATION OF REFINERY PLANNING PROBLEM USING MEAN-ABSOLUTE DEVIATION AS THE RISK MEASURE

We propose to extend the model formulation of Risk Model III as presented in Khor et al. (2008) to incorporate the L1 risk of mean-absolute deviation as a measure of deviation from the expected profit. Thus, the objective function of the model is reformulated replacing $V(z_0)$ with MAD(z_0) in equation **(3)** which is represented as below:

$$\mathrm{MAD}(x) = E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right]$$

$$\max\ z = E\left[z_0\right] - \theta_1 \mathrm{MAD}(z_0) - E_s - \theta_2 \mathrm{MAD}_s \qquad \textbf{(8)}$$

Where:

θ_1, $\theta_2 \in (0, 1]$ are weights of the components of the objective function or risk factor

4.3.1 Model reformulation using MAD(z_0) as risk measure for deviation from deterministic profit

$$\mathrm{MAD}(z_0) = \sum_{s \in S} p_s \left| z_0 - E\left[z_0\right] \right| \qquad \textbf{(9)}$$

where:

$$\text{Profit } z_0 = \sum_{t \in T} \left[\begin{array}{l} \sum_{i \in I} \gamma_{i,t} S_{i,t} + \sum_{i \in I} \tilde{\gamma}_{i,t} I^{f}_{i,t} - \sum_{i \in I} \lambda_{i,t} P_{i,t} - \sum_{i \in I} \tilde{\lambda}_{i,t} I^{s}_{i,t} - \sum_{j \in J} C_{j,t} x_{j,t} - \sum_{i \in I} h_{i,t} H_{i,t} \\ -\sum_{j \in J} \left(\alpha_{j,t} CE_{j,t} + \beta_{j,t} y_{j,t} \right) - \left(r_t R_t + o_t O_t \right) \end{array} \right] \qquad \textbf{(10)}$$

$$\mathrm{E}(z_0) = \sum_{t \in T} \left[\begin{array}{l} \sum_{i \in I} \sum_{s \in S} p_s c_{i,s,t} S_{i,t} + \sum_{i \in I} \tilde{\gamma}_{i,t} I^{f}_{i,t} - \sum_{i \in I} \sum_{s \in S} p_s c_{ir,s,t} P_{i,t} - \sum_{i \in I} \tilde{\lambda}_{i,t} I^{s}_{i,t} - \sum_{j \in J} C_{j,t} x_{j,t} - \sum_{i \in I} h_{i,t} H_{i,t} \\ -\sum_{j \in J} \left(\alpha_{j,t} CE_{j,t} + \beta_{j,t} y_{j,t} \right) - \left(r_t R_t + o_t O_t \right) \end{array} \right] \qquad \textbf{(11)}$$

Substituting **(10)** and **(11)** into **(9)**, and **(9)** into **(8)**, we have the complete Stochastic model which the deviation for profit term is expressed in Mean Absolute Deviation. Refer to Chapter 4: Numerical Example part for further Mean Absolute Deviation MAD(z_0) formulation discussion.

4.4 STOCHASTIC PROGRAMMING MODEL FORMULATION OF PETROLEUM REFINERY PLANNING PROBLEM

For simplicity of Stochastic Programming model formulation, we assume that no alternative source of production hence if there is a shortfall in production, the demand is actually lost. Therefore, we need to anticipate the production of the refinery at the beginning of planning that is production variable x is fixed (meaning all unmet demand is considered lost).

In second-stage stochastic programming, we take into account the recourse problem which takes into account penalty of surplus or shortfall. The representation of stochastic programming surplus/shortfall is as follow:-

$$\max\left(E\left[\text{Profit}\right]-\text{Deviation}\left[\text{Profit}\right]-E\begin{bmatrix}\text{Recourse}\\\text{Penalty}\end{bmatrix}-\text{Deviation}\begin{bmatrix}\text{Recourse}\\\text{Penalty}\end{bmatrix}\right)$$

$$\text{max profit} = E\left[z_0\right]-\theta_1\text{MAD}(z_0)-E_s-\theta_2\text{MAD}_s \qquad \textbf{(12)}$$

$$= \sum_{i\in I}\sum_{s\in S} p_s C_i x_i - \theta_1 \sum_{i\in I} x_i^2 MAD(z_0) - \sum_{i\in I}\sum_{s\in S} p_s\left[\left(c_i^+ z_{i,s}^+ + c_i^- z_{i,s}^-\right)+\left(q_i^+ y_{i,s}^+ + q_i^- y_{i,s}^-\right)\right]-\theta_2 MAD_s$$

where $z_{i,s}^+, z_{i,s}^-, y_{i,s}^+, y_{i,s}^-$ = second stage recourse decision/variables (amount underproduced or overproduced)

$z_{i,s}^+, z_{i,s}^-, y_{i,s}^+, y_{i,s}^-$ 2nd stage recourse cost (penalties for producing surplus or shortfall

Therefore from the deterministic equation stated previously, we formulate the risk model for the petroleum refinery planning. The expectation operator or mean of a discrete random variable for a discrete non-uniform distribution is given by:

$$E\left[z_0\right]=\sum_x xf(x) \qquad \textbf{(13)}$$

where in our problem formulation, x refers to the objective function of scenario s and $f(x)$ represents the probability of scenario s.

The L_1 risk of the absolute deviation function is given as follows (Konno and Koshizuka, 2005; Konno and Yamazaki, 1991):

$$\text{MAD}(x) = E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right] \quad \textbf{(14)}$$

With the notation;

R : Unit price or unit cost of material (either raw material of crude oil or the refinery products)

x_j : Amount of money invested in an asset j refers to the production flowrate of materials in refinery

Therefore, the mean-absolute deviation (MAD) function of equation **(14)** can be formulated as below:-

$$\text{MAD}(x) = E\left[\left|\sum_{j=1}^{n} R_j x_j - E\left[\sum_{j=1}^{n} R_j x_j\right]\right|\right]$$

$$\text{MAD}(z_0) = E\left[\left|z_{0,s} - E[z_0]\right|\right]$$

$$= E\left[\left|z_{0,s} - \sum_{s\in S} p_s z_{0,s}\right|\right]$$

$$= \sum_{s\in S} p_s \left[\left|z_{0,s} - \sum_{s\in S} p_s z_{0,s}\right|\right]$$

$$\text{MAD}(z_0) = \sum_{s\in S} p_s \left[\left|\sum_{i\in I} c_{i,s} x_{i,s} - \sum_{i\in I}\sum_{s\in S} p_s c_{i,s} x_{i,s}\right|\right] \quad \textbf{(15)}$$

4.5 FORMULATION OF THE PARETO FRONT SOLUTION OF EFFICIENT FRONTIER FOR THE EXPECTED RECOURSE TERM

4.5.1 Definition of Pareto Front Solution

Many optimization models are formulated with single multiobjective function, a criterion to be maximized or minimized. When such multiobjective is required, we emphasize on efficient solutions known as the Pareto Front solution formulation.

In this section, we develop the concept of efficient point and the efficient frontier also known as Pareto Optima which help to characterize the "best" feasible solutions in multiobjective models.

a) Efficient Point

   A feasible solution to a multiobjective optimization models is an efficient point if no other feasible solution scores at least as well in all objective functions and strictly better in one. (Rardin 1998)

b) Efficient Frontier

   The efficient frontier of a multiobjective optimization model is the collection of efficient points for the model. (Rardin 1998)

4.5.2 Adaptive weighted sum method for bi-objective optimization

In this section, we are to develop the bi-objective adaptive weighted sum method, which determines uniformly-spaced Pareto optimal solutions. However the method could solve only problems with two objective functions. In the first stage, a weighted sum method is performed on the formulated model solution. Subsequently, the adaptive weighted sum method is applied where each Pareto solution is then refined by imposing additional constraints that will produce a well-distributed Pareto front for effective visualization and find solutions in non-convex regions (Kim and Weck 2005).

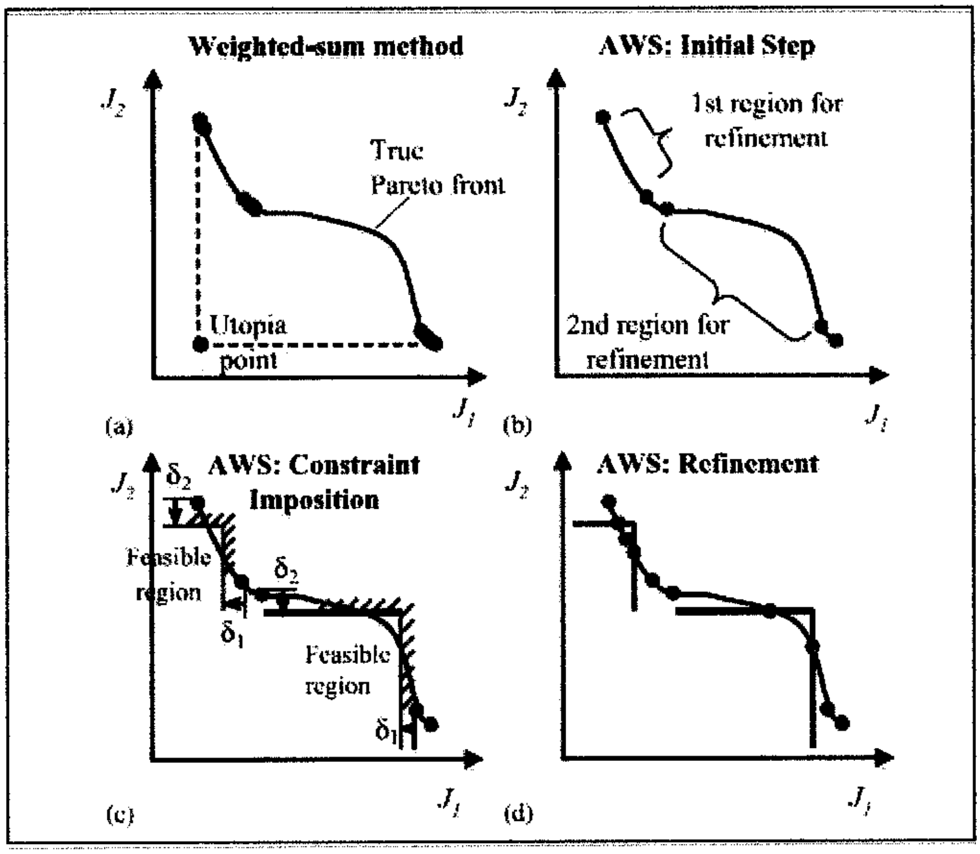

Figure 4.2: (a) Weighted sum method, (b) Initial step of adaptive weighted sum, (c) Adaptive weighted sum constraint imposition, (d) Pareto front refinement (Kim and Weck, 2005)

To compare both weighted sum method for convex Pareto front are as below:-

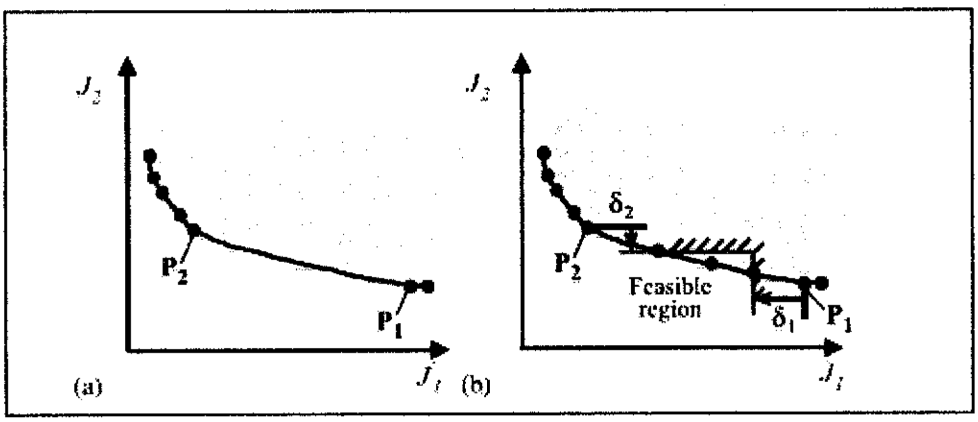

Figure 4.3: Adaptive weighted sum method for convex Pareto front:

(a) Solutions with weighted sum method only,

(b) Additional refinement with adaptive weighted sum method

(Kim and Weck, 2005)

The adaptive weighted sum method can effectively solve multiobjective optimization problems whose Pareto front has:

i) convex regions with non-uniform curvature

ii) non-convex regions of non-dominated solutions

iii) non-convex regions of dominated solutions

In summary, the adaptive weighted sum method produces evenly distributed solutions, finds Pareto optimal solutions in non-convex regions, and neglects non-Pareto optimal solutions in non-convex regions.

4.5.3 Literature Review on Adaptive weighted sum method (Pareto Front generation): Procedures

To formulate the adaptive weighted sum method to produce graphs of Figure 2 and Figure 3, we need to perform certain procedures to formulate the adaptive weighted sum method. The procedures follow step by step which are as below:-

Step 1

- Determine the objective functions which are J_1 (expected profit) and J_2 (MAD)

$$J_1 = -8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}$$

$$J_2 = \mathrm{MAD}(z_0) = \sum_{s\in S} p_s \left| \sum_{i\in I} c_{i,s}x_i - \sum_{s\in S} p_s \sum_{i\in I} c_{i,s}x_i \right|$$

$$= \underbrace{(0.35)\left| \begin{matrix} (-8.8x_1 + 20.35x_2 + 8.8x_3 + 13.75x_4 + 15.95x_5 + 6.6x_6 - 1.65x_{14}) \\ - \begin{bmatrix} (0.35)(-8.8x_1 + 20.35x_2 + 8.8x_3 + 13.75x_4 + 15.95x_5 + 6.6x_6 - 1.65x_{14}) \\ +(0.45)(-8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}) \\ +(0.20)(-7.2x_1 + 16.65x_2 + 7.2x_3 + 11.25x_4 + 13.05x_5 + 5.4x_6 - 1.35x_{14}) \end{bmatrix} \end{matrix} \right|}_{\text{Scenario 1}}$$

$$+ \underbrace{(0.45)\left| \begin{matrix} (-8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}) \\ - \begin{bmatrix} (0.35)(-8.8x_1 + 20.35x_2 + 8.8x_3 + 13.75x_4 + 15.95x_5 + 6.6x_6 - 1.65x_{14}) \\ +(0.45)(-8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}) \\ +(0.20)(-7.2x_1 + 16.65x_2 + 7.2x_3 + 11.25x_4 + 13.05x_5 + 5.4x_6 - 1.35x_{14}) \end{bmatrix} \end{matrix} \right|}_{\text{Scenario 2}}$$

$$+(0.20)\left| \begin{matrix} (-7.2x_1 + 16.65x_2 + 7.2x_3 + 11.25x_4 + 13.05x_5 + 5.4x_6 - 1.35x_{14}) \\ - \begin{bmatrix} (0.35)(-8.8x_1 + 20.35x_2 + 8.8x_3 + 13.75x_4 + 15.95x_5 + 6.6x_6 - 1.65x_{14}) \\ +(0.45)(-8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}) \\ +(0.20)(-7.2x_1 + 16.65x_2 + 7.2x_3 + 11.25x_4 + 13.05x_5 + 5.4x_6 - 1.35x_{14}) \end{bmatrix} \end{matrix} \right|$$

Step 2

- Number of divisions $\eta_{\text{initial}} = 10$
- Uniform step size of the weighting factor λ is determined by the number of divisions:

$$\Delta\lambda = \frac{1}{\eta_{\text{initial}}} = \frac{1}{10}$$
$$= 0.1$$

(the greater the number of divisions, the smaller the step size, hence, more solutions on the Pareto front are obtained)

Step 3

- to compute lengths of the segments between all neighboring solutions
- Fix prescribed distance $\varepsilon = 0.01$. If the distance among solutions is less than a prescribed distance (ε), then all solutions except one are deleted.

Step 4

- To determine number of further refinements in each of the regions
- $n_i = \text{round}\left(C \dfrac{l_i}{l_{avg}} \right)$

  C = constant of the algorithm

Step 5

If $n_i \le 1$, no further refinement is required.

If $n_i > 1$, go to Step 6.

Step 6

- To determine the offset distances from the two end points of each segment
- A piecewise linearized secant line is made by connecting the end points P1 and P2 similar as diagram on Figure 4
- The user selects the offset distance along the piecewise linearize Pareto front, δ_j

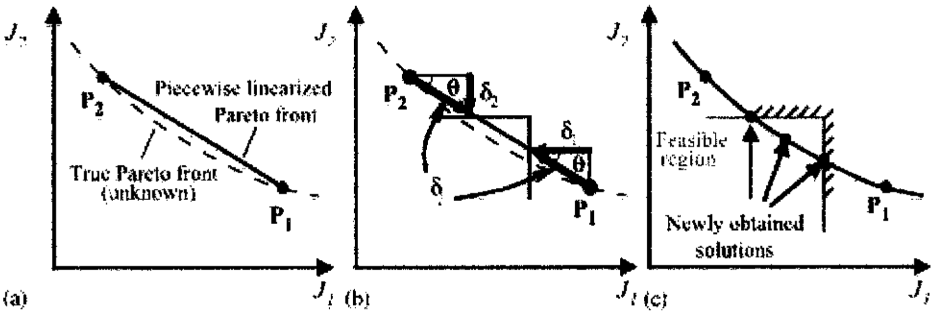

Figure 4.4: Determining the offset distances, δ_1 and δ_2 based on δ_J

(Kim and Weck, 2005)

$$\tan\theta = -\left(\frac{P_1^y - P_2^y}{P_1^x - P_2^x}\right) \text{ where } \begin{array}{l} \mathbf{P}_1 = \left(P_1^x, P_1^y\right) \\ \mathbf{P}_2 = \left(P_2^x, P_2^y\right) \end{array}$$

P_i^x and P_i^y are the x(J1) and y(J2) positions of the end points P1 and P2 respectively

Thus,

$\delta_1 = \delta_J \cos\theta$ and $\delta_2 = \delta_J \sin\theta$

Step 7

- Impose additional inequality constraints and conduct sub-optimization with the weighted sum method in each feasible region

$$\min\left[\lambda\frac{J_1(x)}{sf_{1,0}(x)} + (1-\lambda)\frac{J_2(x)}{sf_{2,0}(x)}\right]$$

Subject to; $\quad J_1(x) \le P_1^x - \delta_1$

$$J_2(x) \le P_2^y - \delta_2$$

- δ_1 and δ_2 are offset distance obtained in Step 6
- $sf_{1,0}(x)$ and $sf_{2,0}(x)$ are scaling factors
- λ is the uniform step size determines is obtained from Step 4

Step 8

- Compute the length of the segments between all the neighboring solutions
- Delete overlapping solutions
- If all segments length are less than δ_J terminate the optimization procedure
- If segment length greater than δ_J, go back to Step 4 and iterate

4.5.4 ε-constraint method for bi-objective optimization

In this section, we are to develop the ε-constraint method, which extends and fills in the gaps between adjacent points along the Pareto surface using a gradient-based local optimizer (such as GAMS/CONOPT3). ε-constraint method converts all but one of the objectives into inequality constraints and solving for all possible values of the inequality constraints. Each set of values represent a subproblem that if solved to global optimality, yields a point in the Pareto optimal set. The number of subproblems that one must solve to identify the complete Pareto-optimal surface grows exponentially with the number of objective functions (Siirola et. al., 2004; Miettinen, 1999).

However, it is important to note that the ε-constraint method can neither guarantee feasibility nor efficiency (that is, it can be complex and time consuming) and both conditions need to be verified once the complete set of solutions has been obtained. The major advantage of ε-constraint method approach developed and employed does not require the a priori articulation of preferences by the decision maker. Instead, the aim is to generate the full set of trade-off solutions and not to present only one single alternative. From the set of alternatives, the decision maker can then further investigate interesting trade-offs and ultimately select a particular supply chain design and capacity planning strategy that best satisfies his or her willingness to compromise (Hugo and Pistikopoulos, 2005).

As mentioned by Rangavajhala Et. Al, 2008, an approach called Generate First and Choose Later (GFCL) can be used to generate the Pareto curve. This approach GFCL generally involves generating a large number of Pareto solutions first, followed by choosing the most attractive of them. By generating a large pool of solutions, the researcher can decide on well-informed decision which proves a better optimization solution. However, generating a large number of potential solutions can be computationally expensive.

CHAPTER 5
COMPUTATIONAL EXPERIMENTS AND NUMERICAL RESULTS

5.1 NUMERICAL EXAMPLE

For numerical example, the implementation of the proposed stochastic model formulations on the petroleum refinery planning linear programming model will be demonstrated. The original single-objective linear programming model is first solved deterministically and is then reformulated with addition of the stochastic dimension.

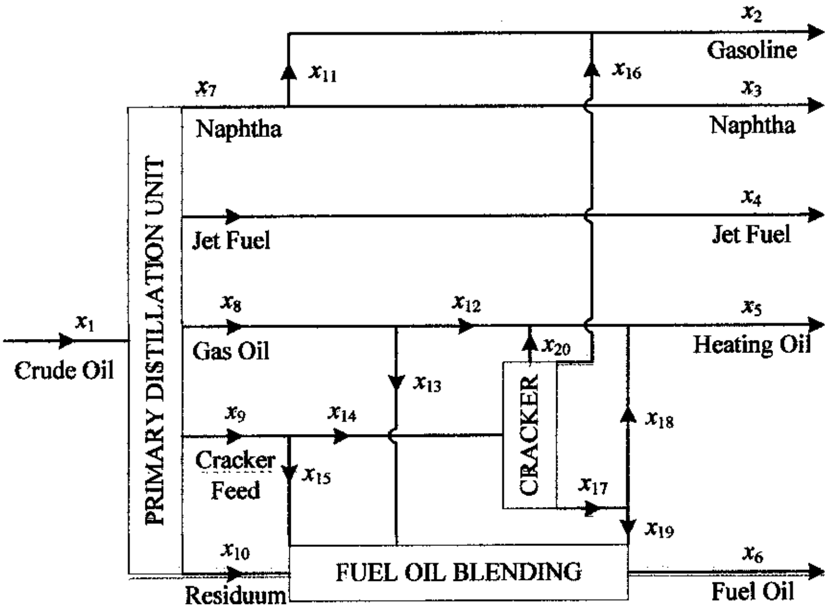

Figure 5.1: Simplified representation of a petroleum refinery production from crude oil (Khor et al. 2008)

Figure 5.1 is a simplified representation of a petroleum refinery that consist mainly the primary distillation unit which processes crude oil (x_1) and cracker feed (x_{14}) to produce gasoline (x_2), naphtha (x_3), jet fuel (x_4), heating oil (x_5) and fuel oil (x_6). The complete scenario representation of the Price Uncertainty, Demand uncertainty and Yield Uncertainty are provided in Table 5.1, Table 5.2 and Table 5.3 which are shown below:-

Table 5.1: Complete scenario condition for refinery production (**Price Uncertainty**)

Scenario (s) / Product Type (i)	Scenario 1 ($/tan)	Scenario 2 ($/tan)	Scenario 3 ($/tan)
Crude Oil (1)	-8.8	-8.0	-7.2
Gasoline (2)	20.35	18.5	16.65
Naphtha (3)	8.8	8.0	7.2
Jet Fuel (4)	13.75	12.5	11.25
Heating Oil (5)	15.95	14.5	13.05
Fuel Oil (6)	6.6	6.0	5.4
Cracker Feed (14)	-1.65	-1.5	-1.35

Table 5.2: Complete scenario condition for refinery production (**Demand Uncertainty**)

Scenario (s) / Product Type (i)	Scenario 1 ($/tan)	Scenario 2 ($/tan)	Scenario 3 ($/tan)
Gasoline (2)	2835	2700	2565
Naphtha (3)	1155	1100	1045
Jet Fuel (4)	2415	2300	2185
Heating Oil (5)	1785	1700	1615
Fuel Oil (6)	9975	9500	9025

Table 5.3: Complete scenario condition for refinery production (**Yield Uncertainty**)

Scenario (s) / Product Type (i)	Scenario 1 ($/tan)	Scenario 2 ($/tan)	Scenario 3 ($/tan)
Naphtha (3)	−0.1365	−0.13	−0.1235
Jet Fuel (4)	−0.1575	−0.15	−0.1425
Gas Oil (8)	−0.231	−0.22	−0.209
Cracker Feed (9)	−0.21	−0.20	−0.19
Residuum (10)	−0.265	−0.30	−0.335
Probability (P_s)	0.35	0.45	0.20

5.2 DETERMINISTIC MODEL FORMULATION OF PETROLEUM REFINERY PLANNING PROBLEM

Deterministic model is a model where it is reasonable to assume all problem data to be known with certainty. We employ deterministic models because they often produce valid enough results to be useful and because deterministic models are almost always easier to analyze than are their stochastic counterparts. The deterministic objective function of the Linear Programming model is given by (based on **Table 5.1** figures of price uncertainty):

$$\underset{x_i}{\text{maximize}}\ z = -8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14} \quad \textbf{(16)}$$

With the notation,

z	: Profit	x_4	: Jet Fuel
x_1	: Crude Oil	x_5	: Heating Oil
x_2	: Gasoline	x_6	: Fuel Oil
x_3	; Naphtha	x_{14}	: Cracker Feed

The equation z left-hand-side coefficients represent the cost or price of the associated materials. In which the negative coefficient denote the purchasing of feed and operating costs while the positive coefficient are the sales prices of products. Therefore, we can write the objective function (z) corresponding with price (c) and production flowrate (x) as:

$$z = c^T x = \sum_{s\in S}(\sum_{i\in I} c_{i,s}x_i), i = \{1,2,3,4,5,6,14\} \in I_{price}^{random} \subseteq I, s = \{1,2,3\} \in S \quad \textbf{(17)}$$

Where;

s = Scenario

i = Product Type

Hence, for the numerical example:

Objective function:

$$\text{maximize}\ z = -8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}$$

Based on equation (8) Chapter 4: Model Formulation, we try to formulate the risk measure of the Mean Absolute Deviation constraint. The expectation of the objective function value is given by the original objective function itself:

$$"E(aX \pm bY) = aE[X] \pm bE[Y]"$$

$$\Rightarrow "E(aX) = aE[X]"$$

$$E[z_0] = E(-8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14})$$

$$= E(-8.0x_1) + E(18.5x_2) + E(8.0x_3) + E(12.5x_4) + E(14.5x_5) + E(6.0x_6) + E(-1.5x_{14})$$

$$= -8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}$$

$$E[z_0] = -8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}$$

$$E[z_0] = -8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}$$

$$\text{MAD}(z_0) = \underbrace{(0.35)\left| \begin{array}{l} (-8.8x_1 + 20.35x_2 + 8.8x_3 + 13.75x_4 + 15.95x_5 + 6.6x_6 - 1.65x_{14}) \\ - \begin{bmatrix} (0.35)(-8.8x_1 + 20.35x_2 + 8.8x_3 + 13.75x_4 + 15.95x_5 + 6.6x_6 - 1.65x_{14}) \\ + (0.45)(-8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}) \\ + (0.20)(-7.2x_1 + 16.65x_2 + 7.2x_3 + 11.25x_4 + 13.05x_5 + 5.4x_6 - 1.35x_{14}) \end{bmatrix} \end{array} \right|}_{\text{Scenario 1}}$$

$$+ \underbrace{(0.45)\left| \begin{array}{l} (-8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}) \\ - \begin{bmatrix} (0.35)(-8.8x_1 + 20.35x_2 + 8.8x_3 + 13.75x_4 + 15.95x_5 + 6.6x_6 - 1.65x_{14}) \\ + (0.45)(-8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}) \\ + (0.20)(-7.2x_1 + 16.65x_2 + 7.2x_3 + 11.25x_4 + 13.05x_5 + 5.4x_6 - 1.35x_{14}) \end{bmatrix} \end{array} \right|}_{\text{Scenario 2}}$$

$$+ \underbrace{(0.20)\left| \begin{array}{l} (-7.2x_1 + 16.65x_2 + 7.2x_3 + 11.25x_4 + 13.05x_5 + 5.4x_6 - 1.35x_{14}) \\ - \begin{bmatrix} (0.35)(-8.8x_1 + 20.35x_2 + 8.8x_3 + 13.75x_4 + 15.95x_5 + 6.6x_6 - 1.65x_{14}) \\ + (0.45)(-8.0x_1 + 18.5x_2 + 8.0x_3 + 12.5x_4 + 14.5x_5 + 6.0x_6 - 1.5x_{14}) \\ + (0.20)(-7.2x_1 + 16.65x_2 + 7.2x_3 + 11.25x_4 + 13.05x_5 + 5.4x_6 - 1.35x_{14}) \end{bmatrix} \end{array} \right|}_{\text{Scenario 3}}$$

From the numerical example stated above, the model formulation for mean absolute deviation can be simplified to the equation as below:

$$\text{MAD}(z_0) = \sum_{s \in S} p_s \left| \sum_{i \in I} c_{i,s} x_i - \sum_{s \in S} p_s \sum_{i \in I} c_{i,s} x_i \right|$$

5.3 STOCHASTIC PROGRAMMING MODEL FORMULATION OF REFINERY PROBLEM (WEIGHTED SUM METHOD)

5.3.1 Solution Strategy 1: Result and Discussion for Weighted Sum Method Formulation

Consider inserting the graph of the Pareto optimal curve for objective function Z2 for different sets of values of θ_1 and θ_2. θ_1 and θ_2 are increasingly varied accordingly to range values from 0 to 1000. The table of the model formulated is tabulated as the table below (given the equation of calculating the objective function):-

$$\text{Objective Function } Z_2 = E[z_0] - \theta_1 \text{MAD}(z_0) - E_s - \theta_3 \text{MAD}_s$$

$$\text{Range: } 0 < \theta_1 \text{ and } \theta_2 < 1000$$

Table 5.4: Computational results for the Stochastic Model using Weighted Sum method

θ_1	θ_2	*E*[Profit] E(z_0)	Deviation[Profit] MAD(z_0)	E[Recourse Penalty] E_s	Deviation[Recourse Penalty] MAD_s	Objective Function Z2	$\sigma = \sqrt{MAD(z_0) + MAD_s}$
0	0	94669.050	5549.565	121910	78337.380	-27250	289.632
0.1	0.1	94669.050	5549.565	121920	78337.380	-35640	289.632
0.2	0.2	94669.050	5549.565	121920	78337.380	-44030	289.632
0.3	0.3	94669.050	5549.565	121920	78337.381	-52410	289.632
0.4	0.4	94669.050	5549.565	121920	78337.380	-60800	289.632
0.5	0.5	94669.050	5549.565	121920	78337.380	-69190	289.632
0.6	0.6	94669.050	5549.565	121920	78337.380	-77580	289.632

θ_1	θ_2	E[Profit] E(z_0)	Deviation[Profit] MAD(z_0)	E[Recourse Penalty] E_s	Deviation[Recourse Penalty] MAD_s	Objective Function Z2	$\sigma = \sqrt{MAD(z_0) + MAD_s}$
0.7	0.7	94669.050	5549.565	121920	78337.380	-85970	289.632
0.8	0.8	29314.353	1718.428	96717	27937.399	-91130	172.209
0.9	0.9	29314.353	1718.428	96717	27937.399	-94090	172.209
1.0	1.0	26042.558	1526.633	95629	25760.769	-96870	165.189
2.0	2.0	18900.020	1107.932	95524	21295.224	-121400	149.677
3.0	3.0	–	–	111040	5880.955	-128700	76.687
4.0	4.0	–	–	111420	5767.028	-134500	75.941
5.0	5.0	–	–	115050	4915.477	-139600	70.110
7.0	7.0	–	–	115050	4915.477	-149500	70.110
10.0	10.0	–	–	115050	4915.477	-164200	70.110
30.0	30.0	–	–	204310	5.670	-204500	2.381
50.0	50.0	–	–	204310	5.670	-204600	2.381
100.0	100.0	–	–	204310	5.670	-204900	2.381
1000.0	1000.0	–	–	204310	5.670	-210000	2.381

5.3.2 Formulation of Weighted Sum Graph; Expected Profit versus Profit and Recourse Penalty Costs Risk

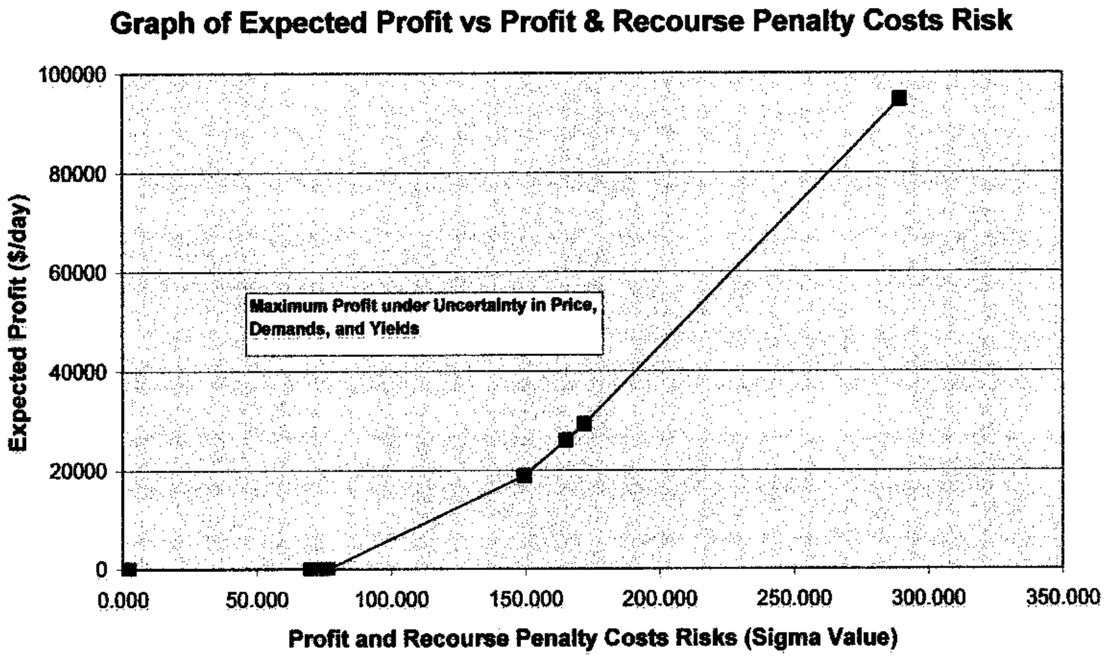

Figure 5.2: Graph of Expected Profit versus Profit and Recourse Penalty Costs Risk measured by Deviation of Profit and Deviation of Recourse Penalty

5.3.3 Analysis of Results for Weighted Sum Method

As for the numerical result of Weighted Sum Method, the value of θ_1 and θ_2 denotes the weights of the components of the objective function or risk factor. θ_1 and θ_2 represents the importance of risk in the model as contributed by variation in deviation profit and variation in deviation recourse penalty costs, respectively, in comparison with the corresponding expected values of the model's objective. From the results observed, reducing values of θ_1 implicates higher profit deviation. The graph plotted shows a typical Pareto Optimal Curve where the profit decreases periodically with increasing risk measure which is represented by deviation of profit.

One of the reasons the reducing values of θ_1 and θ_2 leads to increasing expected profit is that both θ_1 and θ_2 values corresponds to a decrement in variation σ of the recourse penalty. With small values of σ, it will further strengthened the model; which increases the value of our objective function Z_2. This again demonstrates that a proper selection of the operating range of θ_1 and θ_2 is crucial in varying the tradeoffs between the desired degree of model robustness and solution robustness, to ultimately obtain optimality between expected profit and expected production feasibility. (Khor et al., 2008)

The values of θ_1 and θ_2 denotes the importance of risk in the model as contributed by variation in profit and variation in recourse penalty costs, respectively, in comparison with the corresponding expected values of the model's objective. From graph of **Figure 5.2**, we can see that the objective function increases as the sigma value of profit and recourse penalty cost risk increases. Increasingly smaller θ_1 and θ_2 corresponds to higher expected profit which implies less uncertainty and risk to the model. A proper selection of θ_1 and θ_2 operating range will translate the model formulation to a more robust model.

5.4 STOCHASTIC PROGRAMMING MODEL FORMULATION OF PETROLEUM REFINERY PROBLEM (EPSILON CONSTRAINT METHOD)

5.4.1 Solution Strategy 2: Epsilon Constraint Method

We employ the procedure suggested by You and Grossmann (2008) for applying the ε-constraint method for multiobjective optimization problems. In this model formulation section, it is shown from the equation that we have four objective functions to obtain the objective function. The mean absolute deviation model formulation is as below (as formulated previously):-

$$\max\ z = E[z_0] - \theta_1 \mathrm{MAD}(z_0) - E_s - \theta_2 \mathrm{MAD}_s$$

In order to obtain the Pareto curve using epsilon constraint method, we can manually prescribe the constraints (Rangavajhala Et. Al, 2008). In other words, to solve the model we reduce the formulation from four objective functions to a bi-objective function. This will lead to a reduced problem dimensionality (from four objectives to two objectives) and facilitates visualization. To generate the Pareto curve using epsilon constraint method, we follow the steps as below:-

$$\begin{aligned} \max\quad & E(z_0) \\ \text{s.t.}\quad & \mathrm{MAD}(z_0) \le \varepsilon_1 \\ & E_s \le \varepsilon_2 \\ & \mathrm{MAD}_s \le \varepsilon_3 \\ & \text{other constraints} \end{aligned}$$

To enforce acceptable tolerance or limits in this profit maximization program, upper bound values of ε_1, ε_2, ε_3 are specified for each of the parameters MAD(z_0), E_s, and MAD$_s$, respectively within the range of the minimum value and the maximum value for the respective parameter. The next steps will be to determine the range for each of the parameters:-

Step 1

$$\begin{array}{ll} \max & E(z_0) \\ \text{s.t.} & \text{constraints} \end{array}$$

Consider objective function of maximizing E(z_0) which is the expected profit, that in turn yields the largest Pareto-optimal deviation. So, in this step we obtain the largest value for MAD(z_0) and the largest value for E(z_0). So, in this step we obtain the maximum value of MAD(z_0), which we indicate as MAD$(z_0)_{max}$, and the maximum value of the expected profit $E(z_0)$, which we indicate as $E(z_0)_{max}$ to represent the maximum expected profit. Preliminary computational results on GAMS maximizing E(z_0) using epsilon-constraint method:

$$\text{MAD}(z_0)_{max} = 7140.000$$

$$E(z_0)_{max} = 94\,669.050$$

Step 2:

We consider the objective function of minimizing MAD(z_0), in order to obtain the lowest deviation from the expected profit, which in turn yields the lowest Pareto-optimal profit (since the metric of MAD only penalizes downside deviation, therefore, minimum upside deviation corresponds to minimum profit). This lowest Pareto-optimal profit corresponds to the minimum value of the expected profit. So, in this step we obtain the minimum value of MAD(z_0), which we indicate as MAD$(z_0)_{min}$, and the minimum value of the expected profit E(z_0), which we indicate as E$(z_0)_{min}$ to represent the lowest expected profit. Preliminary computational results on GAMS/CONOPT3 for minimizing MAD(z_0)

$$\text{MAD}(z_0)_{min} = 5549.565$$

$$\text{E}(z_0)_{min} = -121800$$

$$\begin{array}{ll} \max & E(z_0) \\ \text{s.t.} & \text{MAD}(z_0) \le \varepsilon_1 \\ & E_s \le \varepsilon_2 \\ & \text{MAD}_s \le \varepsilon_3 \\ & \text{other constraints} \end{array}$$

To enforce acceptable tolerance or limits in this profit maximization program, upper bound values of ε_1, ε_2, ε_3 are specified for each of the parameters MAD(z_0), E_s, and MAD$_s$, respectively within the range of the minimum value and the maximum value for the respective parameter. The next steps will be to determine the range for each of the parameter.

Step 3

Finally, repeat Step 1 to Step 2 by changing the objective function from MAD(z_0) to E_s and then MAD$_s$. By repeating step 1 to step 2, we will obtain the lower bound and upper bound of each objective functions of MAD(z_0), E_s, MAD$_s$. Note: For epsilon constraint method, we reduce the objective function in GAMS from four objective functions to two objective functions. One of the objective function should be the Expected Profit meanwhile the other objective function will be the constraint, either MAD(z_0), E_s or MAD$_s$.

5.4.2 Epsilon Constraint Method Summary

The model formulation of Model III as presented in Khor et al. (2008) is reformulated to introduce Mean Absolute Deviation MAD(z_0) as the measure for deviation of profit. The method proposed in this work is to further study Model III proposed using the epsilon-constraint method which fully utilize the Mean Absolute Deviation MAD(z_0) as the Deviation of Profit. This epsilon constraint method is to eliminate the use of the weighting factors θ_1 and θ_2 from the model formulation presented in **equation (11)**, in which θ_1 and θ_2 are weights of the components of the multiple objective functions that acts alternatively as the risk factors of the problem under investigation.

Based on the recent work by Guillen-Gosalbez and Grossmann (2008), consider the solution of a set of single-objective-function problems for different values of the parameter ε:

$$\begin{aligned} \max \quad & \text{profit} = V(z_0) \\ \text{s.t.} \quad & E(z_0) \le \varepsilon \\ & \text{other model constraints} \end{aligned}$$

In this formulation, the lower and upper limits (or bounds) that define the interval within which the epsilon parameter must fall, i.e., $\varepsilon \in [\varepsilon^L, \varepsilon^U]$ can be obtained by solving each objective separately:

The ε-constraint formulation proposed by Guillen-Gosalbez and Grossmann (2008) is similar to the formulation by You and Grossman (2008). Both formulation practices the method to maximize profit $E(z_0)$ and minimizing $MAD(z_0)$ in order to obtain the Pareto-optimal curve in which each of the Pareto efficient frontiers points is determined by the values of $E(z_0)$ and $MAD(z_0)$.

Using the epsilon-constraint method as proposed earlier in section 4.5.4, in order to obtain the Pareto optimal curve, each component of the objective function is correspondingly/appropriately minimized and maximized using the GAMS modeling software. We minimize and maximize each objective function individually to obtain the lower and upper bound of each objective function. The objective functions that are minimized and maximized are listed as below:

a) Deviation of Profit, $MAD(z_0)$
b) Expected Recourse Penalty, E_s
c) Deviation of Recourse Penalty, MAD_s

The minimum and maximum values of each parameter are as listed below:-

a) Deviation of Profit, MAD(z_0) *(NOTE: Objective Function = OF)*

	Expected Profit, Ep	**Expected Recourse Penalty, E_s**
Maximize OF Upper Bound on MAD(z_0)	94669.050	7140.000
Minimize OF Lower Bound on MAD(z_0)	−121800.000	5549.565

1. Maximize MAD(z_0) to obtain $\text{MAD}^{\text{U}}(z_0)$ = 7140.000, which corresponds to $E_p^{\text{L}} = 94669.050$
2. Minimize MAD(z_0) to obtain $MAD^{\text{L}}(z_0)$ = 5549.565, which corresponds to $Ep^{\text{L}}[z_0] = -121800$

b) Expected Recourse Penalty, E_s

	Expected Profit, Ep	**Expected Recourse Penalty, E_s**
Maximize OF Upper Bound on E_s	94669.050	279420.000
Minimize OF Lower Bound on E_s	−121800.000	121920.000

1. Maximize E_s to obtain $E_s^{\text{U}} =$ = 279420, which corresponds to $Ep^{\text{U}}(z_0)$ = 94669.050
2. Minimize E_s to obtain $E_s^{\text{L}} =$ = 121920, which corresponds to $Ep^{\text{L}}(z_0)$ = −121800

c) Deviation of Recourse Penalty, MAD_s

	Expected Profit, Ep	Deviation of Recourse Penalty, MAD_s
Maximize OF Upper Bound on MAD_s	94669.050	150000.000
Minimize OF Lower Bound on MAD_s	−121800.000	78337.380

1. Maximize MAD_s to obtain MAD_s^U = 150000, which corresponds to $Ep^U[z_0]$ = 94669.050
2. Minimize MAD_s to obtain MAD_s^L = 78337.380, which corresponds to $Ep^L[z_0]$ = −121800

Obtaining all this four objective function lower and upper bound, we then combine all the lower and upper bound values of each objective function to construct the Pareto optimal curve as drawn on **Figure 5.3**, **Figure 5.4** and **Figure 5.5**. These three graphs represent the expected profit of the model versus the constraints which are the Deviation of Profit, Expected Recourse Penalty and finally Deviation of Recourse Penalty.

5.4.3 Results and Discussion of Epsilon-Constraint Method Formulation

a) **Part 1** – Varying MAD(z_0) value while maintaining E_s and MAD_s values

Table 5.5: Values of Ep, E_s and MAD_s after varying the MAD(z_0) value

Deviation of Profit, MAD(z_0)	Expected Recourse Penalty, E_s	Deviation of Recourse Penalty, MAD_s	Expected Profit, Ep	Objective Function, Z2
5549.565	121917.44	78337.38	94669.05	94669.05
5000	131703.225	69100.994	85294.118	85294.118
4000	151311.068	52294.271	68235.294	68235.294
3000	151652.011	39507.86	51176.471	51176.471
2000	140057.491	26323.42	34117.647	34117.647
1000	122126.37	24162.932	17058.824	17058.824
4.64E-10	102884.535	11000.692	7.93E-09	7.93E-09

Note: When MAD(z_0) value is less than 0, the formulation is infeasible.

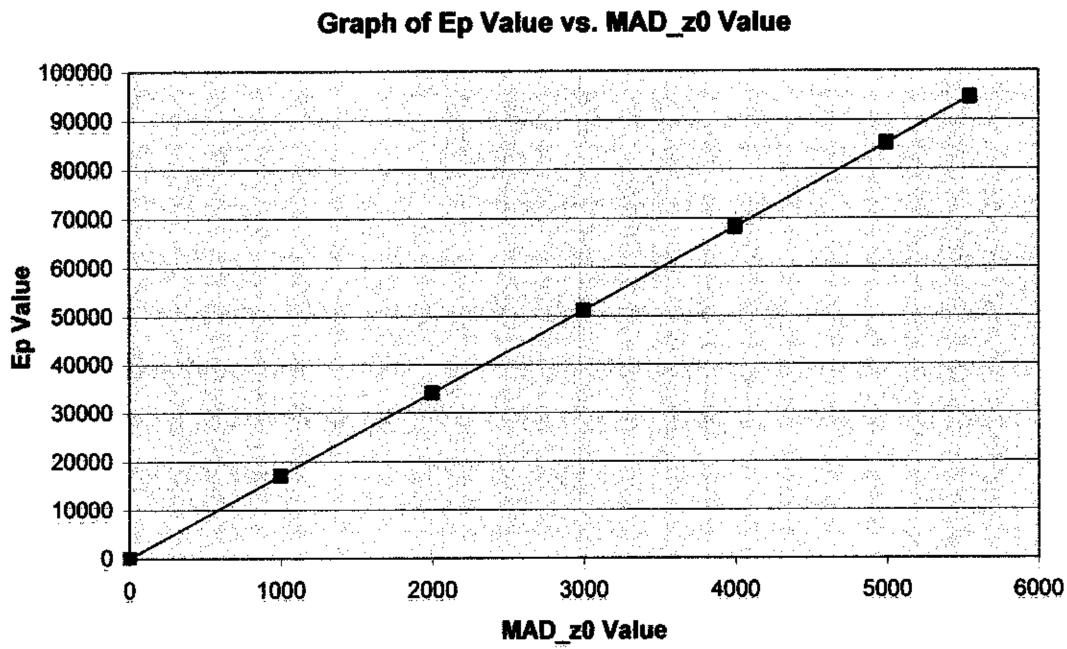

Figure 5.3: Graph of Pareto Curve Optimal Solution for Ep versus MAD(z_0)

Part 1 Graph Interpretation

- The graph trend shows a linear relationship between the Expected Profit, Ep and Deviation of Expected Profit, MAD(z_0)
- Expected Profit, Ep increases as Deviation of Expected Profit, MAD(z_0) increases
- This means that the Expected Profit, Ep increases when the Deviation of Expected Profit, MAD(z_0) increases

b) **Part 2** – Varying E_s value while maintaining MAD(z_0) and MAD_s values

Table 5.6: Values of Ep, MAD(z_0) and MAD_s after varying the E_s value

Deviation of Profit, MAD(z_0)	**Expected Recourse Penalty, E_s**	**Deviation of Recourse Penalty, MAD_s**	**Expected Profit, Ep**	**Objective Function, Z2**
5549.56	121917.44	78337.38	94669.05	94669.05
5258.058	120000	74502.5	89696.281	89696.281
4497.911	115000	64502.5	76729.074	76729.074
3737.765	110000	54502.5	63761.868	63761.868
2977.618	105000	44502.5	50794.661	50794.66
2217.471	100000	34502.5	37827.454	37827.45
1388.915	95000	24502.5	23693.248	23693.248
166.053	92643	19788.5	2832.671	2832.671

Note: When E_s *value is less than 92643, the formulation is infeasible.*

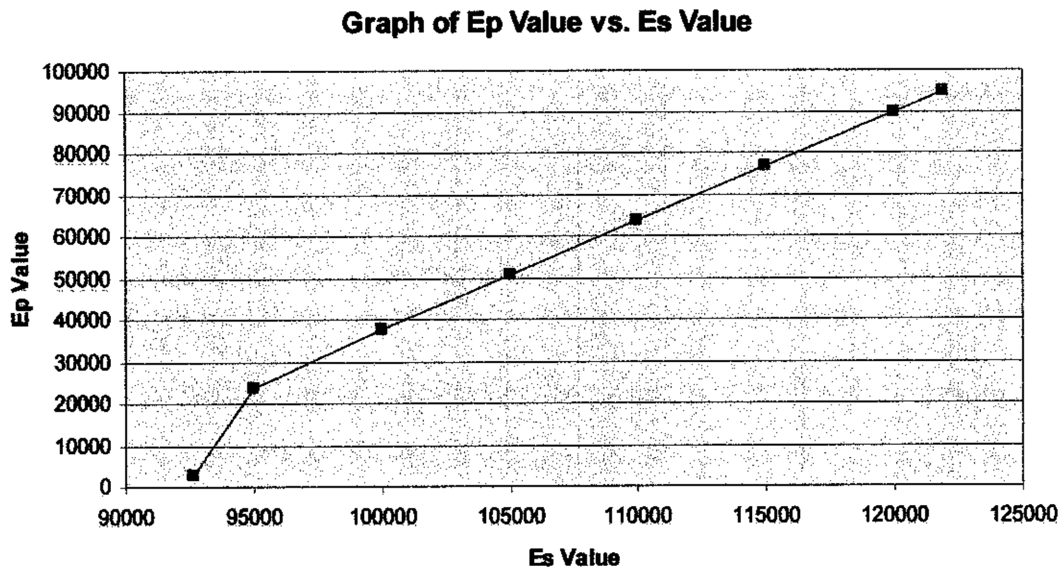

Figure 5.4: Graph of Pareto Curve Optimal Solution for Ep versus E_s

Part 2 Graph Interpretation

- The graph trend shows an increase of Expected Profit, Ep when we increase our risk which is the Expected Recourse Penalty, E_s
- This means that the Expected Profit, Ep increases when the Expected Recourse Penalty, E_s increases
- The rate of increase of Expected Profit, Ep reduces for Expected Recourse Penalty, E_s greater than 95,000
- The larger the Expected Recourse Penalty, E_s the rate of increase of Expected Profit, Ep reduces

c) **Part 3** – Varying MAD_s value while maintaining $MAD(z_0)$ and E_s values

Table 5.7: Values of Ep, $MAD(z_0)$ and E_s after varying the MAD_s value

Deviation of Profit, $MAD(z_0)$	**Expected Recourse Penalty, E_s**	**Deviation of Recourse Penalty, MAD_s**	**Expected Profit, Ep**	**Objective Function, Z2**
5549.565	121917.44	78337.38	94669.05	94669.05
5053.491	130654.384	70000	86206.609	86206.609
4458.491	142321.051	60000	76056.609	76056.609
3863.491	153987.718	50000	65906.609	65906.609
3268.491	167469.806	40000	55756.609	55756.609
2673.491	181019.379	30000	45606.609	45606.609
2078.491	192686.046	20000	35456.609	35456.609
1238.88	200983.329	10000	21133.843	21133.843
619.089	202649.995	5000	10560.927	10560.927
0.208	204316.179	4	-3.55	-3.55

Note: When MAD_s value is less than 4, the formulation is infeasible

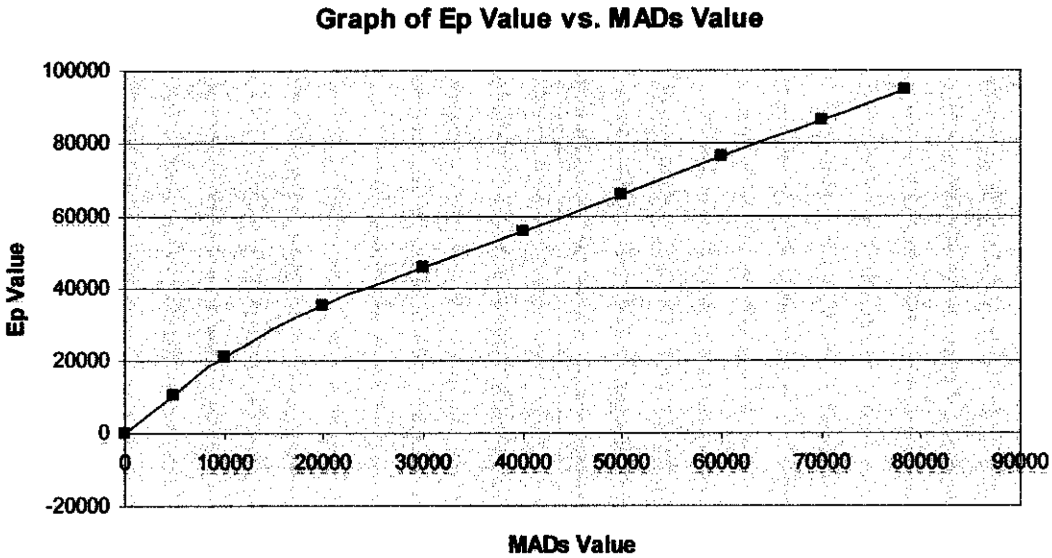

Figure 5.5: Graph of Pareto Curve Optimal Solution for Ep versus MAD_s

Part 3 Graph Interpretation

- The graph trend shows an increase of Expected Profit, Ep when we increase our risk which is the Deviation of Recourse Penalty, MAD_s
- This means that the Expected Profit, Ep increases when the Deviation of Recourse Penalty, MAD_s increases
- The rate of increase of Expected Profit, Ep reduces for Deviation of Recourse Penalty, MAD_s greater than 20,000
- The larger the Deviation of Recourse Penalty, MAD_s the rate of increase of Expected Profit, Ep reduces
- Graph 2 and Graph 3 show similar graph trend relationship

5.4.4 Analysis of Results for Epsilon-Constraint Method

From the graph trends of all three graphs, we can see that all three graphs objective function (Expected Profit Ep) increases with respect to the increasing values of $MAD(z_0)$, E_s and MAD_s respectively. The higher the risk of the model as reflected by higher values of $MAD(z_0)$, E_s and MAD_s value, the lower the expected profit E_p. From the graphs, all three graphs utilize the epsilon constraint method approach for its multiobjective optimization problem. In this epsilon constraint method, it extends the solution range of its optimization model as well as fills in the gaps between the adjacent points along the Pareto optimal curve. The advantage of this epsilon constraint method is that it is able to generate a full set of solutions and not to the present one single alternative solution only.

5.5 SUMMARY OF NUMERICAL RESULTS

5.5.1 GAMS Numerical Results

Objective function: For max $z = E(z_0) - \theta_1 \text{MAD}(z_0) - E_s - \theta_2 \text{MAD}_s$

Table 5.8: Summary of Numerical Results

Weight for risk measure, θ_1	0.100
Weight for risk measure, θ_2	0.100
Expected Profit, E_p for model using MAD	$94,669.050 /day
Deviation of Profit, $\text{MAD}(z_0)$	$5,549.565
Expected Recourse Penalty, E_s	$121,920.000
Deviation of Recourse Penalty, MAD_s	$78,337.380

5.5.2 Computational Statistics

Table 5.9: Summary of Computational Statistics

Solver	GAMS/CONOPT3
Number of single variables	81
Number of nonlinear variables	37
Number of constraints	45
Number of iterations	10
CPU time/Resource usage	0.094s

CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

Stochastic programming is an optimization method used in manufacturing process to optimized specified set of parameters without violating some constrain. Stochastic programming is good because it allows the decision maker to analyze multiple scenarios of an uncertain future, maximizing net profit while minimizing various expected costs.

The risk model is reformulated in the form of mean-absolute deviation (MAD) where MAD is the average absolute deviation from the mean. A Risk Model is a measure of operational risk provides the computational linear property. Therefore, the problem for petroleum refinery planning under uncertainty with multiobjective optimization approach and financial risk management is reformulated as the equation below [refer to Equation (8)]:-

$$\max\ z = E(z_0) - \theta_1 \mathrm{MAD}(z_0) - E_s - \theta_2 \mathrm{MAD}_s$$

Our objective of this study is to reformulate the equation above using different methods to obtain the Pareto Optimal Curve. From the equation above, we apply the two methods which are the weighted sum method and the ε-constraint method in order to obtain the Pareto front generation. The first method studied is know as the weighted sum method, emphasizes on θ_1 and θ_2 values which represents the importance of risk in the model. From the results observed, reducing values of θ_1 and θ_2 implicates higher profit deviation and reduces uncertainty as well as risk to the model. A proper selection of θ_1 and θ_2 operating range will translate the model formulation to a more robust model.

The second method studied is the ε-constraint method which generally extends and fills in the gaps between adjacent points along the Pareto front. The epsilon-constraint method maximize profit $E(z_0)$ and minimizing MAD(z_0), E_s and MAD$_s$ in order to obtain the Pareto-optimal curve in which each of the Pareto efficient frontiers points is determine by the values of $E(z_0)$ & MAD(z_0), $E(z_0)$ & E_s and $E(z_0)$ & MAD$_s$. The higher the risk as reflected by higher MAD(z_0), E_s and MAD$_s$ values, the lower the expected profit of $E(z_0)$. From the results obtained, the major advantage of epsilon constraint method it is able to generate a wide range of solutions from the MAD(z_0), E_s and MAD$_s$ constraints. From the range of solutions available, the researcher will select a planning strategy to choose the most attractive solution range on well-informed decision which proves a better optimization solution.

In conclusion, both weighted sum method and ε-constraint method produces a more evenly distributed Pareto Optimal Curve (model solutions), giving more accuracy and precision to the solution produced. Stochastic programming is proven to be very suitable for optimization models that involve uncertainties and risk.

6.2 RECOMMENDATIONS FOR FUTURE WORK

Some recommendations for future work can be conducted to further improve the model formulated by this study. The recommendations are as following:-

- To develop a more systematic approach in determining the values of θ_1 and θ_2 which are the weights for the objective function or risk measures of MAD(z_0), E_s and MAD$_s$.
- To develop a better approach; to implement "spider diagram" or "radar charts" approach to display all four objectives graphically as compared to the epsilon constraint method model formulation where we can only display two objectives graphically. The idea here is to optimize each objective and display in a cross the maximum (or minimum) value for each objective. From this, we can see how far we can stretch or contract each objective.
- To analyze and interpret the Pareto Optimal Curve graph in order to obtain accurate and precise solutions that is able to satisfy the model formulated.
- Formulate a proper loop system for the weighted sum method and epsilon constraint method to store the formulated solutions into Microsoft Excel environment.

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APPENDIX A

Weighted Sum Method GAMS Input File

```
$TITLE Approach 4: Risk Model III of Two-Stage Stochastic Programming with Fixed
Recourse for Minimization of the Expected Value and the Mean-Absolute Deviation (MAD)
of the Variation in Recourse Penalty Costs

SETS
I        types of materials  / 1*20 /

S        Scenarios  / S1*S3 /

ID(I)    types of materials subject to demand uncertainty / 2*6 /

IY(I)    types of materials subject to demand uncertainty  / 4,7,8,9,10 /

K        production shortfall and surplus or yield decrement or increment  / K1, K2 /
;

ALIAS(S,SC)
;

PARAMETER

P(S)  Probability of the realization of scenario
/
S1  0.35,
S2  0.45,
S3  0.20
/

V(I)  Variance of Price
/
1     0.352,
2     1.882375,
3     0.352,
4     0.859375,
5     1.156375,
6     0.198,
14    0.012375
/

Table PRICE(I,S)  Table of Price Uncertainty
        S1         S2        S3
1       -8.8       -8.0      -7.2
2       20.35      18.5      16.65
3       8.8        8.0       7.2
4       13.75      12.5      11.25
5       15.95      14.5      13.05
6       6.6        6.0       5.4
14      -1.65      -1.5      -1.35;

Table DEMAND(ID,S)  Table of Demand Uncertainty
        S1         S2        S3
2       2835       2700      2565
3       1155       1100      1045
4       2415       2300      2185
5       1785       1700      1615
6       9975       9500      9025;

Table YIELD(IY,S)  Table of Yield Uncertainty
        S1         S2        S3
4       -0.1575    -0.15     -0.1425
7       -0.1365    -0.13     -0.1235
8       -0.231     -0.22     -0.209
9       -0.21      -0.20     -0.19
10      -0.265     -0.30     -0.335;

Table PENALTY_DEMAND(ID,K)  Table of Penalty Demand
        K1         K2
2       25         20
3       17         13
4       5          4
```

```
5       6          5
6       10         8;

Table PENALTY_YIELD(IY,K)  Table of Penalty Yield
        K1         K2
4       5          3
7       5          4
8       5          3
9       5          3
10      5          3;

VARIABLES
Z2   Maximize Profit for Z
Ecv
;

POSITIVE VARIABLES
Z(ID,S,K)          stochastic variables on production shortfall and surplus (amount of
unsatisfied demand for product i due to underproduction or overproduction per
realization of scenario s)
Y(IY,S,K)          stochastic variables on production shortfall and surplus (amount of
unsatisfied yield for product i due to underproduction or overproduction per
realization of scenario s)
X                production flowrates of materials
MAD_z0, MADs, Es, Ep, DEVIATIONprofit, Tshortfall, Tsurplus
;

EQUATIONS
OBJ          Objective function to maximiaze profit
Feed1        Feed equation limitation for Crude Oil
Feed14       Feed equation limitation for Cracker Feed
FY14_16      Fixed Yield of Cracker for X(14) and X(16)
FY14_17      Fixed Yield of Cracker for X(17) and X(17)
FY14_20      Fixed Yield of Cracker for X(20) and X(20)
FB2_11       Fixed Blend of Gasoline Blending for X(2) and X(11)
FB2_16       Fixed Blend of Gasoline Blending for X(2) and X(16)
FB5_12       Fixed Blend of Heating Oil Blending for X(5) and X(12)
FB5_18       Fixed Blend of Heating Oil Blending for X(5) and X(18)
UB3          Unrestricted Balance for Naphtha
UB8          Unrestricted Balance for Gas Oil
UB14         Unrestricted Balance for Cracker Feed
UB17         Unrestricted Balance for Cracked Oil
UB6          Unrestricted Balance for Fuel Oil
CONS1
CONS2
CONS3
CONS4

YIELDstoc(IY,S)      uncertain or stochastic fixed yield of primary distillation unit
DEMANDstoc(ID,S)     uncertain or stochastic fixed demand of primary distillation unit
;

OBJ..   Z2 =E= Ep - 0.1*MAD_z0 - Es - 0.1*MADs;

CONS1.. Ep =E= SUM((I,S), P(S)*Price(I,S)*X(I));

CONS2..  MAD_z0  =E=  SUM(SC,  P(SC)*ABS(SUM(I,  PRICE(I,SC)*X(I))  -  SUM((I,S),
P(S)*PRICE(I,S)*X(I))));

CONS3.. Es =E= SUM(S, P(S)*(SUM((ID,K), PENALTY_DEMAND(ID,K)*Z(ID,S,K)) + SUM((IY,K),
PENALTY_YIELD(IY,K)*Y(IY,S,K))));

CONS4..  MADs  =E=  SUM(S,  P(S)*ABS(SUM((ID,K),  PENALTY_DEMAND(ID,K)*Z(ID,S,K))  +
SUM((IY,K), PENALTY_YIELD(IY,K)*Y(IY,S,K))
                 -  SUM((ID,K),   PENALTY_DEMAND(ID,K)*Z(ID,S,K))   +   SUM((IY,K),
PENALTY_YIELD(IY,K)*Y(IY,S,K))));

**LIMITATIONS OF PLANT CAPACITY
Feed1..       X('1') =L= 15000;
Feed14..      X('14') =L= 2500;

*************************************************************************************
*FIXED YIELDS FOR CRACKER (deterministic constraints)
*************************************************************************************
FY14_16..    -0.40*X('14') + X('16') =E= 0;
FY14_17..    -0.55*X('14') + X('17') =E= 0;
FY14_20..    -0.05*X('14') + X('20') =E= 0;
```

```
FB2_11..      0.5*X('2') + X('11') =E= 0;
FB2_16..      0.5*X('2') + X('16') =E= 0;
FB5_12..      0.75*X('5') + X('12') =E= 0;
FB5_18..      0.25*X('5') + X('18') =E= 0;
UB3..         -X('7') + X('3') + X('11') =E= 0;
UB8..         -X('8') + X('12') + X('13') =E= 0;
UB14..        -X('9') + X('14') + X('15') =E= 0;
UB17..        -X('17') + X('18') + X('19') =E= 0;
UB6..         -X('10') - X('13') - X('15') - X('19') + X('6') =E= 0;

***********************************************************************************
**CONSTRAINTS ON PRODUCTION DEMANDS
***********************************************************************************
DEMANDstoc(ID,S).. X(ID) + Z(ID,S,'K1') - Z(ID,S,'K2') =E= DEMAND(ID,S);

***********************************************************************************
**CONSTRAINTS ON PRODUCTION YIELD
***********************************************************************************
YIELDstoc(IY,S).. YIELD(IY,S)*X('1') + X(IY) + Y(IY,S,'K1') - Y(IY,S,'K2') =E= 0;

*Initial values
X.L('1') = 12500;
X.L('2') = 2000;
X.L('3') = 625;
X.L('4') = 1875;
X.L('5') = 1700;
X.L('6') = 6175;
X.L('7') = 1625;
X.L('8') = 2750;
X.L('9') = 2500;
X.L('10') = 3750;
X.L('11') = 1000;
X.L('12') = 1275;
X.L('13') = 1475;
X.L('14') = 2500;
X.L('15') = 0;
X.L('16') = 1000;
X.L('17') = 1375;
X.L('18') = 425;
X.L('19') = 950;
X.L('20') = 125;

Z.L(ID,S,K) = 0;
Y.L(IY,S,K) = 0;

* Upper bounds of variables
X.UP('1') = 15000;
X.UP('2') = 2700;
X.UP('3') = 1100;
X.UP('4') = 2300;
X.UP('5') = 1700;
X.UP('6') = 9500;
X.UP('7') = 1950;
X.UP('8') = 3300;
X.UP('9') = 3000;
X.UP('10') = 3000;
X.UP('11') = 1350;
X.UP('12') = 1275;
X.UP('13') = 3300;
X.UP('14') = 3000;
X.UP('15') = 3000;
X.UP('16') = 1200;
X.UP('17') = 1650;
X.UP('18') = 425;
X.UP('19') = 1650;
X.UP('20') = 150;

* Lower bounds of variables
X.LO('1') = 10;

MODEL REFINERY / all /;
SOLVE REFINERY USING DNLP MAXIMIZING Z2;
```

APPENDIX B

Weighted Sum Method GAMS Output File

```
               S O L V E      S U M M A R Y

     MODEL   REFINERY              OBJECTIVE  Z2
     TYPE    DNLP                  DIRECTION  MAXIMIZE
     SOLVER  CONOPT                FROM LINE  209

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      2 LOCALLY OPTIMAL
**** OBJECTIVE VALUE           -35637.0848

 RESOURCE USAGE, LIMIT          0.109      1000.000
 ITERATION COUNT, LIMIT        10         10000
 EVALUATION ERRORS              0             0

    C O N O P T 3   x86/MS Windows version 3.14S-017-061
    Copyright (C)   ARKI Consulting and Development A/S
                    Bagsvaerdvej 246 A
                    DK-2880 Bagsvaerd, Denmark

 Using default options.


    The model has 85 variables and 49 constraints
    with 251 Jacobian elements, 37 of which are nonlinear.
    The Hessian of the Lagrangian has 37 elements on the diagonal,
    156 elements below the diagonal, and 37 nonlinear variables.

 ** Optimal solution. There are no superbasic variables.


 CONOPT time Total                              0.109 seconds
   of which: Function evaluations               0.000 =  0.0%
             1st Derivative evaluations         0.000 =  0.0%

 Workspace          =     0.38 Mbytes
    Estimate        =     0.38 Mbytes
    Max used        =     0.10 Mbytes

                       LOWER     LEVEL     UPPER    MARGINAL

---- EQU OBJ             .         .         .        1.000
---- EQU Feed1         -INF     10.000 15000.000       .
---- EQU Feed14        -INF        .     2500.000      .
---- EQU FY14_16         .         .         .       17.795
---- EQU FY14_17         .         .         .       12.454
---- EQU FY14_20         .         .         .         EPS
---- EQU FB2_11          .         .         .      105.130
---- EQU FB2_16          .         .         .      -17.795
---- EQU FB5_12          .         .         .       31.660
---- EQU FB5_18          .         .         .      -12.454
---- EQU UB3             .         .         .        4.800
---- EQU UB8             .         .         .       12.454
---- EQU UB14            .         .         .       12.454
---- EQU UB17            .         .         .       12.454
---- EQU UB6             .         .         .       12.454
---- EQU CONS1           .         .         .        1.000
---- EQU CONS2           .         .         .       -0.100
---- EQU CONS3           .         .         .       -1.000
---- EQU CONS4           .         .         .       -0.100

  OBJ  Objective function to maximiaze profit
  Feed1  Feed equation limitation for Crude Oil
  Feed14  Feed equation limitation for Cracker Feed
  FY14_16  Fixed Yield of Cracker for X(14) and X(16)
  FY14_17  Fixed Yield of Cracker for X(17) and X(17)
  FY14_20  Fixed Yield of Cracker for X(20) and X(20)
  FB2_11  Fixed Blend of Gasoline Blending for X(2) and X(11)
  FB2_16  Fixed Blend of Gasoline Blending for X(2) and X(16)
  FB5_12  Fixed Blend of Heating Oil Blending for X(5) and X(12)
```

```
  FB5_18  Fixed Blend of Heating Oil Blending for X(5) and X(18)
  UB3  Unrestricted Balance for Naphtha
  UB8  Unrestricted Balance for Gas Oil
  UB14  Unrestricted Balance for Cracker Feed
  UB17  Unrestricted Balance for Cracked Oil
  UB6  Unrestricted Balance for Fuel Oil

---- EQU YIELDstoc  uncertain or stochastic fixed yield of primary distillation
                    unit

         LOWER     LEVEL     UPPER    MARGINAL

4 .S1      .         .         .        1.260
4 .S2      .         .         .        1.620
4 .S3      .         .         .        0.720
7 .S1      .         .         .        1.680
7 .S2      .         .         .        2.160
7 .S3      .         .         .        0.960
8 .S1      .         .         .        1.260
8 .S2      .         .         .        1.620
8 .S3      .         .         .        0.720
9 .S1      .         .         .        1.260
9 .S2      .         .         .        1.620
9 .S3      .         .         .        0.720
10.S1      .         .         .        1.260
10.S2      .         .         .        1.620
10.S3      .         .         .        0.720

---- EQU DEMANDstoc  uncertain or stochastic fixed demand of primary distillatio
                     n unit

        LOWER     LEVEL     UPPER    MARGINAL

2.S1  2835.000  2835.000  2835.000    -8.750
2.S2  2700.000  2700.000  2700.000   -11.250
2.S3  2565.000  2565.000  2565.000    -5.000
3.S1  1155.000  1155.000  1155.000    -5.950
3.S2  1100.000  1100.000  1100.000    -7.650
3.S3  1045.000  1045.000  1045.000     2.600
4.S1  2415.000  2415.000  2415.000    -1.750
4.S2  2300.000  2300.000  2300.000    -2.250
4.S3  2185.000  2185.000  2185.000     0.800
5.S1  1785.000  1785.000  1785.000    -2.100
5.S2  1700.000  1700.000  1700.000    -2.700
5.S3  1615.000  1615.000  1615.000    -1.200
6.S1  9975.000  9975.000  9975.000    -3.500
6.S2  9500.000  9500.000  9500.000    -4.500
6.S3  9025.000  9025.000  9025.000     1.600

                      LOWER     LEVEL     UPPER    MARGINAL

---- VAR Z2            -INF  -3.564E+4     +INF        .

  Z2  Maximize Profit for Z

---- VAR Z  stochastic variables on production shortfall and surplus (amount of
            unsatisfied demand for product i due to underproduction or overprodu
            ction per realization of scenario s)

           LOWER     LEVEL     UPPER    MARGINAL

2.S1.K1      .      2835.000    +INF        .
2.S1.K2      .          .       +INF     -15.750
2.S2.K1      .      2700.000    +INF        .
2.S2.K2      .          .       +INF     -20.250
2.S3.K1      .      2565.000    +INF        .
2.S3.K2      .          .       +INF      -9.000
3.S1.K1      .        55.000    +INF        .
3.S1.K2      .          .       +INF     -10.500
3.S2.K1      .          .       +INF        .
3.S2.K2      .          .       +INF     -13.500
3.S3.K1      .          .       +INF      -6.000
3.S3.K2      .        55.000    +INF        .
4.S1.K1      .       115.000    +INF        .
4.S1.K2      .          .       +INF      -3.150
4.S2.K1      .          .       +INF        .
4.S2.K2      .          .       +INF      -4.050
```

4.S3.K1	.	.	+INF	-1.800
4.S3.K2	.	115.000	+INF	.
5.S1.K1	.	1785.000	+INF	.
5.S1.K2	.	.	+INF	-3.850
5.S2.K1	.	1700.000	+INF	.
5.S2.K2	.	.	+INF	-4.950
5.S3.K1	.	1615.000	+INF	.
5.S3.K2	.	.	+INF	-2.200
6.S1.K1	.	675.000	+INF	.
6.S1.K2	.	.	+INF	-6.300
6.S2.K1	.	200.000	+INF	.
6.S2.K2	.	.	+INF	-8.100
6.S3.K1	.	.	+INF	-3.600
6.S3.K2	.	275.000	+INF	.

---- VAR Y stochastic variables on production shortfall and surplus (amount of unsatisfied yield for product i due to underproduction or overproduc tion per realization of scenario s)

	LOWER	LEVEL	UPPER	MARGINAL
4 .S1.K1	.	.	+INF	-3.360
4 .S1.K2	.	2298.425	+INF	.
4 .S2.K1	.	.	+INF	-4.320
4 .S2.K2	.	2298.500	+INF	.
4 .S3.K1	.	.	+INF	-1.920
4 .S3.K2	.	2298.575	+INF	.
7 .S1.K1	.	.	+INF	-3.780
7 .S1.K2	.	1098.635	+INF	.
7 .S2.K1	.	.	+INF	-4.860
7 .S2.K2	.	1098.700	+INF	.
7 .S3.K1	.	.	+INF	-2.160
7 .S3.K2	.	1098.765	+INF	.
8 .S1.K1	.	.	+INF	-3.360
8 .S1.K2	.	3297.690	+INF	.
8 .S2.K1	.	.	+INF	-4.320
8 .S2.K2	.	3297.800	+INF	.
8 .S3.K1	.	.	+INF	-1.920
8 .S3.K2	.	3297.910	+INF	.
9 .S1.K1	.	.	+INF	-3.360
9 .S1.K2	.	2997.900	+INF	.
9 .S2.K1	.	.	+INF	-4.320
9 .S2.K2	.	2998.000	+INF	.
9 .S3.K1	.	.	+INF	-1.920
9 .S3.K2	.	2998.100	+INF	.
10.S1.K1	.	.	+INF	-3.360
10.S1.K2	.	2997.350	+INF	.
10.S2.K1	.	.	+INF	-4.320
10.S2.K2	.	2997.000	+INF	.
10.S3.K1	.	.	+INF	-1.920
10.S3.K2	.	2996.650	+INF	.

---- VAR X production flowrates of materials

	LOWER	LEVEL	UPPER	MARGINAL
1	10.000	10.000	15000.000	-4.315
2	.	.	2700.000	.
3	.	1100.000	1100.000	14.272
4	.	2300.000	2300.000	12.213
5	.	.	1700.000	.
6	.	9300.000	9500.000	.
7	.	1100.000	1950.000	.
8	.	3300.000	3300.000	8.854
9	.	3000.000	3000.000	8.854
10	.	3000.000	3000.000	8.854
11	.	.	1350.000	-109.930
12	.	.	1275.000	-44.114
13	.	3300.000	3300.000	.
14	.	.	3000.000	.
15	.	3000.000	3000.000	.
16	.	.	1200.000	.
17	.	.	1650.000	.
18	.	.	425.000	.
19	.	.	1650.000	.
20	.	.	150.000	.

```
                                LOWER      LEVEL      UPPER     MARGINAL

---- VAR MAD_z0                   .        5549.565     +INF        .
---- VAR MADs                     .       78337.380     +INF        .
---- VAR Es                       .       1.2192E+5     +INF        .
---- VAR Ep                       .       94669.050     +INF        .


**** REPORT SUMMARY :               0     NONOPT
                                    0 INFEASIBLE
                                    0  UNBOUNDED
                                    0     ERRORS


EXECUTION TIME       =       0.000 SECONDS      2 Mb  WIN226-149 Dec 19, 2007


USER: course license                                  S060628:0842AL-WIN
      Phd course about mathematical programming                   DC5953
      License for teaching and research at degree granting institutions
```

APPENDIX C

Epsilon Constraint Method GAMS Input File

```
$TITLE Approach 4: Risk Model III of Two-Stage Stochastic Programming with Fixed
Recourse for Minimization of the Expected Value and the Mean-Absolute Deviation (MAD)
of the Variation in Recourse Penalty Costs

SETS

I        types of materials  / 1*20 /

S        Scenarios  / S1*S3 /

ID(I)    types of materials subject to demand uncertainty / 2*6 /

IY(I)    types of materials subject to demand uncertainty  / 4,7,8,9,10 /

K        production shortfall and surplus or yield decrement or increment  / K1, K2 /
;

ALIAS(S,SC)
;

PARAMETER

P(S)  Probability of the realization of scenario
/
S1  0.35,
S2  0.45,
S3  0.20
/

V(I)  Variance of Price
/
1      0.352,
2      1.882375,
3      0.352,
4      0.859375,
5      1.156375,
6      0.198,
14     0.012375
/

Table PRICE(I,S)  Table of Price Uncertainty
       S1          S2          S3
1      -8.8        -8.0        -7.2
2      20.35       18.5        16.65
3      8.8         8.0         7.2
4      13.75       12.5        11.25
5      15.95       14.5        13.05
6      6.6         6.0         5.4
14     -1.65       -1.5        -1.35;

Table DEMAND(ID,S)  Table of Demand Uncertainty
       S1          S2          S3
2      2835        2700        2565
3      1155        1100        1045
4      2415        2300        2185
5      1785        1700        1615
6      9975        9500        9025;

Table YIELD(IY,S)  Table of Yield Uncertainty
       S1          S2          S3
4      -0.1575     -0.15       -0.1425
7      -0.1365     -0.13       -0.1235
8      -0.231      -0.22       -0.209
9      -0.21       -0.20       -0.19
10     -0.265      -0.30       -0.335;

Table PENALTY_DEMAND(ID,K)  Table of Penalty Demand
       K1          K2
2      25          20
3      17          13
4      5           4
```

```
5       6          5
6       10         8;

Table PENALTY_YIELD(IY,K)  Table of Penalty Yield
        K1         K2
4       5          3
7       5          4
8       5          3
9       5          3
10      5          3
;

VARIABLES

Z2    Maximize Profit for Z
Z3
Z4
Ecv
;

POSITIVE VARIABLES

Z(ID,S,K)       stochastic variables on production shortfall and surplus (amount of
unsatisfied demand for product i due to underproduction or overproduction per
realization of scenario s)
Y(IY,S,K)       stochastic variables on production shortfall and surplus (amount of
unsatisfied yield for product i due to underproduction or overproduction per
realization of scenario s)
X               production flowrates of materials
Ep, DEVIATIONprofit, Tshortfall, Tsurplus
;

PARAMETER MAD_z0_value, Es_value, MADs_value, Ep_value;

EQUATIONS
OBJ         Objective function to maximiaze profit
Feed1       Feed equation limitation for Crude Oil
Feed14      Feed equation limitation for Cracker Feed
FY14_16     Fixed Yield of Cracker for X(14) and X(16)
FY14_17     Fixed Yield of Cracker for X(17) and X(17)
FY14_20     Fixed Yield of Cracker for X(20) and X(20)
FB2_11      Fixed Blend of Gasoline Blending for X(2) and X(11)
FB2_16      Fixed Blend of Gasoline Blending for X(2) and X(16)
FB5_12      Fixed Blend of Heating Oil Blending for X(5) and X(12)
FB5_18      Fixed Blend of Heating Oil Blending for X(5) and X(18)
UB3         Unrestricted Balance for Naphtha
UB8         Unrestricted Balance for Gas Oil
UB14        Unrestricted Balance for Cracker Feed
UB17        Unrestricted Balance for Cracked Oil
UB6         Unrestricted Balance for Fuel Oil

MAD_z0
Es
MADs

YIELDstoc(IY,S)     uncertain or stochastic fixed yield of primary distillation unit
DEMANDstoc(ID,S)    uncertain or stochastic fixed demand of primary distillation unit
;

**LIMITATIONS OF PLANT CAPACITY
Feed1..      X('1') =L= 15000;
Feed14..     X('14') =L= 2500;

*********************************************************************************
*FIXED YIELDS FOR CRACKER (deterministic constraints)
*********************************************************************************
FY14_16..    -0.40*X('14') + X('16') =E= 0;
FY14_17..    -0.55*X('14') + X('17') =E= 0;
FY14_20..    -0.05*X('14') + X('20') =E= 0;
FB2_11..     0.5*X('2') + X('11') =E= 0;
FB2_16..     0.5*X('2') + X('16') =E= 0;
FB5_12..     0.75*X('5') + X('12') =E= 0;
FB5_18..     0.25*X('5') + X('18') =E= 0;
UB3..        -X('7') + X('3') + X('11') =E= 0;
UB8..        -X('8') + X('12') + X('13') =E= 0;
UB14..       -X('9') + X('14') + X('15') =E= 0;
UB17..       -X('17') + X('18') + X('19') =E= 0;
```

```
UB6..       -X('10') - X('13') - X('15') - X('19') + X('6') =E= 0;

**************************************************************************************
**CONSTRAINTS ON PRODUCTION DEMANDS
**************************************************************************************
DEMANDstoc(ID,S).. X(ID) + Z(ID,S,'K1') - Z(ID,S,'K2') =E= DEMAND(ID,S);

**************************************************************************************
**CONSTRAINTS ON PRODUCTION YIELD
**************************************************************************************
YIELDstoc(IY,S).. YIELD(IY,S)*X('1') + X(IY) + Y(IY,S,'K1') - Y(IY,S,'K2') =E= 0;

*Initial values
X.L('1') = 12500;
X.L('2') = 2000;
X.L('3') = 625;
X.L('4') = 1875;
X.L('5') = 1700;
X.L('6') = 6175;
X.L('7') = 1625;
X.L('8') = 2750;
X.L('9') = 2500;
X.L('10') = 3750;
X.L('11') = 1000;
X.L('12') = 1275;
X.L('13') = 1475;
X.L('14') = 2500;
X.L('15') = 0;
X.L('16') = 1000;
X.L('17') = 1375;
X.L('18') = 425;
X.L('19') = 950;
X.L('20') = 125;
Z.L(ID,S,K) = 0;
Y.L(IY,S,K) = 0;

* Upper bounds of variables
X.UP('1') = 15000;
X.UP('2') = 2700;
X.UP('3') = 1100;
X.UP('4') = 2300;
X.UP('5') = 1700;
X.UP('6') = 9500;
X.UP('7') = 1950;
X.UP('8') = 3300;
X.UP('9') = 3000;
X.UP('10') = 3000;
X.UP('11') = 1350;
X.UP('12') = 1275;
X.UP('13') = 3300;
X.UP('14') = 3000;
X.UP('15') = 3000;
X.UP('16') = 1200;
X.UP('17') = 1650;
X.UP('18') = 425;
X.UP('19') = 1650;
X.UP('20') = 150;

Ep.L = 0;

* Lower bounds of variables
X.LO('1') = 10;

OBJ..      Z2 =E= SUM((I,S), P(S)*Price(I,S)*X(I));

MAD_z0.. SUM(SC, P(SC)*ABS(SUM(I, PRICE(I,SC)*X(I)) - SUM((I,S),
P(S)*PRICE(I,S)*X(I)))) =L= 7140;

Es..      SUM(S, P(S)*(SUM((ID,K), PENALTY_DEMAND(ID,K)*Z(ID,S,K)) + SUM((IY,K),
PENALTY_YIELD(IY,K)*Y(IY,S,K)))) =L= 279420;

MADs..    SUM(S, P(S)*ABS(SUM((ID,K), PENALTY_DEMAND(ID,K)*Z(ID,S,K)) + SUM((IY,K),
PENALTY_YIELD(IY,K)*Y(IY,S,K))
          - SUM((ID,K), PENALTY_DEMAND(ID,K)*Z(ID,S,K)) + SUM((IY,K),
PENALTY_YIELD(IY,K)*Y(IY,S,K)))) =L= 150000;

MODEL REFINERY / all /;
```

```
SOLVE REFINERY USING DNLP MAXIMIZING Z2;

Ep_value = SUM((I,S), P(S)*Price(I,S)*X.L(I));

MAD_z0_value = SUM(SC, P(SC)*ABS(SUM(I, PRICE(I,SC)*X.L(I)) - SUM((I,S),
P(S)*PRICE(I,S)*X.L(I))));

Es_value = SUM(S, P(S)*(SUM((ID,K), PENALTY_DEMAND(ID,K)*Z.L(ID,S,K)) + SUM((IY,K),
PENALTY_YIELD(IY,K)*Y.L(IY,S,K))));

MADs_value =  SUM(S, P(S)*ABS(SUM((ID,K), PENALTY_DEMAND(ID,K)*Z.L(ID,S,K)) +
SUM((IY,K), PENALTY_YIELD(IY,K)*Y.L(IY,S,K))
        - SUM((ID,K), PENALTY_DEMAND(ID,K)*Z.L(ID,S,K)) + SUM((IY,K),
PENALTY_YIELD(IY,K)*Y.L(IY,S,K))));

DISPLAY Ep_value, MAD_z0_value, Es_value, MADs_value;
```

APPENDIX D

Epsilon Constraint Method GAMS Output File

```
               S O L V E      S U M M A R Y

     MODEL   REFINERY              OBJECTIVE  Z2
     TYPE    DNLP                  DIRECTION  MAXIMIZE
     SOLVER  CONOPT                FROM LINE  216

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      2 LOCALLY OPTIMAL
**** OBJECTIVE VALUE            94669.0500

 RESOURCE USAGE, LIMIT          0.048      1000.000
 ITERATION COUNT, LIMIT        11         10000
 EVALUATION ERRORS              0             0

    C O N O P T 3   x86/MS Windows version 3.14S-017-061
    Copyright (C)   ARKI Consulting and Development A/S
                    Bagsvaerdvej 246 A
                    DK-2880 Bagsvaerd, Denmark

 Using default options.


    The model has 81 variables and 48 constraints
    with 243 Jacobian elements, 37 of which are nonlinear.
    The Hessian of the Lagrangian has 37 elements on the diagonal,
    156 elements below the diagonal, and 37 nonlinear variables.

 ** Optimal solution. There are no superbasic variables.


 CONOPT time Total                                  0.032 seconds
   of which: Function evaluations                   0.016 = 50.0%
             1st Derivative evaluations             0.000 =  0.0%

 Workspace            =     0.36 Mbytes
    Estimate          =     0.36 Mbytes
    Max used          =     0.10 Mbytes

                      LOWER      LEVEL      UPPER     MARGINAL

---- EQU OBJ            .          .          .         1.000
---- EQU Feed1        -INF      10.000 15000.000         .
---- EQU Feed14       -INF        .      2500.000         .
---- EQU FY14_16        .          .          .       -37.555
---- EQU FY14_17        .          .          .         6.090
---- EQU FY14_20        .          .          .          EPS
---- EQU FB2_11         .          .          .          EPS
---- EQU FB2_16         .          .          .        37.555
---- EQU FB5_12         .          .          .        21.653
---- EQU FB5_18         .          .          .        -6.090
---- EQU UB3            .          .          .          EPS
---- EQU UB8            .          .          .          EPS
---- EQU UB14           .          .          .         6.090
---- EQU UB17           .          .          .         6.090
---- EQU UB6            .          .          .         6.090
---- EQU MAD_z0       -INF    5549.565  7140.000         .
---- EQU Es           -INF  1.2192E+5 2.7942E+5          .
---- EQU MADs         -INF  78337.380 1.5000E+5          .

  OBJ  Objective function to maximiaze profit
  Feed1  Feed equation limitation for Crude Oil
  Feed14  Feed equation limitation for Cracker Feed
  FY14_16  Fixed Yield of Cracker for X(14) and X(16)
  FY14_17  Fixed Yield of Cracker for X(17) and X(17)
  FY14_20  Fixed Yield of Cracker for X(20) and X(20)
  FB2_11  Fixed Blend of Gasoline Blending for X(2) and X(11)
  FB2_16  Fixed Blend of Gasoline Blending for X(2) and X(16)
  FB5_12  Fixed Blend of Heating Oil Blending for X(5) and X(12)
  FB5_18  Fixed Blend of Heating Oil Blending for X(5) and X(18)
  UB3  Unrestricted Balance for Naphtha
```

UB8 Unrestricted Balance for Gas Oil
UB14 Unrestricted Balance for Cracker Feed
UB17 Unrestricted Balance for Cracked Oil
UB6 Unrestricted Balance for Fuel Oil

---- EQU YIELDstoc uncertain or stochastic fixed yield of primary distillation unit

	LOWER	LEVEL	UPPER	MARGINAL
4 .S1	.	.	.	EPS
4 .S2	.	.	.	EPS
4 .S3	.	.	.	EPS
7 .S1	.	.	.	EPS
7 .S2	.	.	.	EPS
7 .S3	.	.	.	EPS
8 .S1	.	.	.	EPS
8 .S2	.	.	.	EPS
8 .S3	.	.	.	EPS
9 .S1	.	.	.	EPS
9 .S2	.	.	.	EPS
9 .S3	.	.	.	EPS
10.S1	.	.	.	EPS
10.S2	.	.	.	EPS
10.S3	.	.	.	EPS

---- EQU DEMANDstoc uncertain or stochastic fixed demand of primary distillation unit

	LOWER	LEVEL	UPPER	MARGINAL
2.S1	2835.000	2835.000	2835.000	EPS
2.S2	2700.000	2700.000	2700.000	EPS
2.S3	2565.000	2565.000	2565.000	EPS
3.S1	1155.000	1155.000	1155.000	EPS
3.S2	1100.000	1100.000	1100.000	EPS
3.S3	1045.000	1045.000	1045.000	EPS
4.S1	2415.000	2415.000	2415.000	EPS
4.S2	2300.000	2300.000	2300.000	EPS
4.S3	2185.000	2185.000	2185.000	EPS
5.S1	1785.000	1785.000	1785.000	EPS
5.S2	1700.000	1700.000	1700.000	EPS
5.S3	1615.000	1615.000	1615.000	EPS
6.S1	9975.000	9975.000	9975.000	EPS
6.S2	9500.000	9500.000	9500.000	EPS
6.S3	9025.000	9025.000	9025.000	EPS

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR Z2	-INF	94669.050	+INF	.

Z2 Maximize Profit for Z

---- VAR Z stochastic variables on production shortfall and surplus (amount of unsatisfied demand for product i due to underproduction or overproduction per realization of scenario s)

	LOWER	LEVEL	UPPER	MARGINAL
2.S1.K1	.	2835.000	+INF	.
2.S1.K2	.	.	+INF	EPS
2.S2.K1	.	2700.000	+INF	.
2.S2.K2	.	.	+INF	EPS
2.S3.K1	.	2565.000	+INF	.
2.S3.K2	.	.	+INF	EPS
3.S1.K1	.	55.000	+INF	.
3.S1.K2	.	.	+INF	EPS
3.S2.K1	.	.	+INF	.
3.S2.K2	.	.	+INF	EPS
3.S3.K1	.	.	+INF	EPS
3.S3.K2	.	55.000	+INF	.
4.S1.K1	.	115.000	+INF	.
4.S1.K2	.	.	+INF	EPS
4.S2.K1	.	.	+INF	.
4.S2.K2	.	.	+INF	EPS
4.S3.K1	.	.	+INF	EPS
4.S3.K2	.	115.000	+INF	.

5.S1.K1	.	1785.000	+INF	.
5.S1.K2	.	.	+INF	EPS
5.S2.K1	.	1700.000	+INF	.
5.S2.K2	.	.	+INF	EPS
5.S3.K1	.	1615.000	+INF	.
5.S3.K2	.	.	+INF	EPS
6.S1.K1	.	675.000	+INF	.
6.S1.K2	.	.	+INF	EPS
6.S2.K1	.	200.000	+INF	.
6.S2.K2	.	.	+INF	EPS
6.S3.K1	.	.	+INF	EPS
6.S3.K2	.	275.000	+INF	.

---- VAR Y stochastic variables on production shortfall and surplus (amount of unsatisfied yield for product i due to underproduction or overproduc tion per realization of scenario s)

	LOWER	LEVEL	UPPER	MARGINAL
4 .S1.K1	.	.	+INF	EPS
4 .S1.K2	.	2298.425	+INF	.
4 .S2.K1	.	.	+INF	EPS
4 .S2.K2	.	2298.500	+INF	.
4 .S3.K1	.	.	+INF	EPS
4 .S3.K2	.	2298.575	+INF	.
7 .S1.K1	.	.	+INF	EPS
7 .S1.K2	.	1098.635	+INF	.
7 .S2.K1	.	.	+INF	EPS
7 .S2.K2	.	1098.700	+INF	.
7 .S3.K1	.	.	+INF	EPS
7 .S3.K2	.	1098.765	+INF	.
8 .S1.K1	.	.	+INF	EPS
8 .S1.K2	.	3297.690	+INF	.
8 .S2.K1	.	.	+INF	EPS
8 .S2.K2	.	3297.800	+INF	.
8 .S3.K1	.	.	+INF	EPS
8 .S3.K2	.	3297.910	+INF	.
9 .S1.K1	.	.	+INF	EPS
9 .S1.K2	.	2997.900	+INF	.
9 .S2.K1	.	.	+INF	EPS
9 .S2.K2	.	2998.000	+INF	.
9 .S3.K1	.	.	+INF	EPS
9 .S3.K2	.	2998.100	+INF	.
10.S1.K1	.	.	+INF	EPS
10.S1.K2	.	2997.350	+INF	.
10.S2.K1	.	.	+INF	EPS
10.S2.K2	.	2997.000	+INF	.
10.S3.K1	.	.	+INF	EPS
10.S3.K2	.	2996.650	+INF	.

---- VAR X production flowrates of materials

	LOWER	LEVEL	UPPER	MARGINAL
1	10.000	10.000	15000.000	-8.120
2	.	.	2700.000	.
3	.	1100.000	1100.000	8.120
4	.	2300.000	2300.000	12.688
5	.	.	1700.000	.
6	.	9300.000	9500.000	.
7	.	1100.000	1950.000	.
8	.	3300.000	3300.000	.
9	.	3000.000	3000.000	6.090
10	.	3000.000	3000.000	6.090
11	.	.	1350.000	.
12	.	.	1275.000	-21.653
13	.	3300.000	3300.000	6.090
14	.	.	3000.000	-19.285
15	.	3000.000	3000.000	.
16	.	.	1200.000	.
17	.	.	1650.000	.
18	.	.	425.000	.
19	.	.	1650.000	.
20	.	.	150.000	.

**** REPORT SUMMARY : 0 NONOPT

```
                         0 INFEASIBLE
                         0  UNBOUNDED
                         0     ERRORS

GAMS Rev 149  x86/MS Windows                        06/05/09 03:06:40 Page 6
Approach 4: Risk Model III of Two-Stage Stochastic Programming with Fixed Recourse
for Minimization of the Expected Value and the Mean-Absolute Deviation (MAD) of the
Variation in Recourse Penalty Costs
E x e c u t i o n


----    228 PARAMETER Ep_value              =    94669.050
            PARAMETER MAD_z0_value          =     5549.565
            PARAMETER Es_value              =   121917.440
            PARAMETER MADs_value            =    78337.380


EXECUTION TIME       =        0.032 SECONDS      3 Mb  WIN226-149 Dec 19, 2007


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