ANALOGY OF MECHANICAL AND TRANSPORT PROPERTIES IN DISPERSE COMPOSITES

by

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

(LOGESWARY A/P SEVALOGA NATHAN)

ABSTRACT

Analogy of the mechanical and transport properties on disperse composites is a literature research in identifying experimental data reported for mechanical and various transport properties in composite materials. The main objective is to explore the possibility to arrive at a common model for both effective thermal conductivity and shear modulus in terms of the properties of individual phases and the volume fraction. Poor utilization of one researcher's results in one field by other researchers is the problem that has been faced in developing approaches in prediction of the properties of disperse media. This study is concerned with particulate filled matrices constitute from a three and two dimensional composites. A large number of theoretical models and data gathered that had been proposed by earlier researcher are been studied and included in the literature review. Those models being identified in order to apply them to these experimental data and see how they compare. In the result part, the desired models, Eshelby and Halpin-Tsai which chosen to be predictive model will be discussed in further. The assumptions and explanation for other models also included in discussion part.

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CHAPTER 1

INTRODUCTION

1.0 INTRODUCTION

1.1 Background Studies

Imagine our life without composites. Every single day will be a total mess because all the staffs we are dealing with are composites. The world will turn upside down. Starting from our mobile phone which contains composite printed circuit boards to under the bonnet of the car, composites are everywhere. If they are not there yet, we can be pretty sure they are coming. Composites can be defined in a broad way. According to definition for Wikipedia website, **composite materials** (or **composites** for short) are engineered materials made from two or more constituent materials with significantly different physical or chemical properties and which remain separate and distinct on a macroscopic level within the finished structure [1]. Most composite materials are made from two (2) materials: a reinforcement material called fibre and a base material, called matrix material. At least one portion of each type is required.

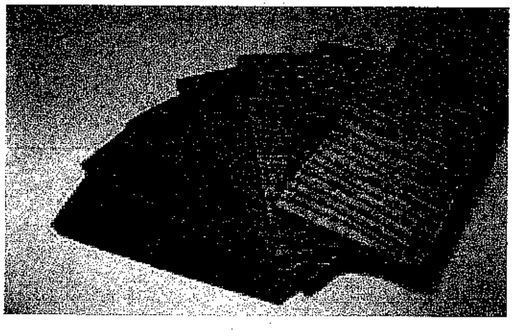


Figure 1.1: Engineering wood is a common composite material

Composite materials are usually formed in three different types: (1) fibrous composites, which consist of fibres of one material in a matrix material of another; (2) particulate composites, which are composed of macro size particles of one material on a matrix of another; and (3) laminated composites, which are made of layers of different materials, including composites of the first two types [2, 3].

This project is all about finding the relationship between the effective shear modulus (mechanical property) and thermal conductivity (transport property) in composite materials as both these phenomena are governed by the same set of equations. The main idea of this project is to be able to come up with a model which has a good agreement between the model and experimental values for elastic modulus and thermal conductivity. Elastic modulus and thermal conductivity have been chosen because available studies on the said fields. Studies show that Hashin – Shtrikman upper bound is the inverted – Maxwell limit and both these theories are related by the shear modulus and thermal conductivity. A much simplified version can be proposed by analyzing both elastic modulus and thermal conductivity. The finding will be studied closely to define the common elastic and thermal features that exist in composites.

After conducting a literature review on the findings, the next stage is to develop a better and improved model for both elastic and thermal properties for composite materials. This model will be developed based on the experimental data analysis and literature review data. Once the model is developed, the model will be applied to the experimental data and see how they compare. A better prediction to the shear modulus and thermal conductivity can be developed based on the degree of analogy exist between these two models. This prediction cannot exits the Hashin- Shtrikman and Maxwell bounds.

1.2 Problem Statement

1.2.1 Problem Identification

This project concerns with predicting the properties of composite materials, given the properties of the constituent materials and the volume fraction. The differential equations for several transport phenomena are alike with those governing mechanical load, stress and deformation. This has resulted in predictive equations being developed for each set of properties. However there are few problems which lead to poor prediction of the properties of disperses media.

- **Poor utilization of other researcher's result**

Poor utilization of one researcher's results in one field by other researchers has been issue in developing approaches in prediction of the properties of disperse media. It is either that the researcher is not aware about the existence of the other theories or the developer does not fully explore other researchers' work. Lots of effort and time has been expended in identifying the existence of theories and formula. For example Maxwell's 1873 result for magnetic permittivity has been rediscovered by Hashin and Shtrikman in 1962.

- **Insufficient theoretical models in predicting behaviour of all material and under all conditions.**

The problem of analytical prediction of effective properties of composites material is important from both practical and theoretical points of view. They can be determined by experimental data gathered on samples, or by prediction from the theoretical models. A large number of theoretical models proposed and presented by earlier researcher. However, no model is applicable to all materials and under all conditions, accounting for all effects that modify the effective moduli of composites.

1.2.2 Significance of the project

Regarding the above problem statements, we realize that developing the existing theories and formulas is a time consuming and at the same time, it shows that researchers are not fully explore this field before developing any models. Thus, this project is a literature research in identifying experimental data reported and theoretical models for thermal conductivity and shear modulus in composite materials. This project aims to come up with a better model to get a good agreement between the model and experimental values from literature research. Developing a project that meets the requirement of the analytical studies about the elastic moduli and thermal conductivity on disperse composites will lead to a better correlations and predictions.

1.3 Objective

- **Establish analogy between the mechanical and transport properties**

The main objective of this project is to study analytically and establish the analogy between the mechanical properties and their counterparts of the same tensorial order among the various transport properties. This is because both these properties are governed by the same set of equation which is Laplace equation.

- **Study applicability of Maxwell and Hashin Shtrikman limits for thermal conductivity and shear modulus, respectively in composites material**

In this study, the applicability of the limits for the direct and phase – inverted composite materials as predicted by Maxwell theory is studied. Comparisons between Hashin – Shtrikman bounds for mechanical properties and Maxwell's predictions for thermal conductivity are being analyzed for a better correlations and divination. The Maxwell and Hashin-Shtrikman bounds are the lines that cannot be exceeded by the proposed model.

- **Study and analyze theoretical models proposed by other researchers.**

Apart from Hashin-Shtrikman and Maxwell limits, other models such as those of 'self-consistent scheme', 'generalized self-consistent scheme' and 'method of cell' methods will also be taken into consideration in order to come out with a better prediction for shear modulus and thermal conductivity of composite materials.

- **Develop a model for both effective thermal conductivity and shear modulus**

The other objective of this project is to explore the possibility to arrive at a common model for both effective thermal conductivity and shear modulus in terms of the properties of individual phases and the volume fraction. The existing practical and theoretical models will be analyzed and those models which can superimpose with one another will be chosen as better model.

1.4 Scope of Studies

- Particle – filled matrices constitute form a three dimensional (3-D) composites.
- Fibre-reinforced matrices constitute form a two dimensional (2-D) composites
- Macroscopically homogeneous, microscopically heterogeneous, and continuous composites.
- The models taken into consideration for shear modulus are:
  - Isostress model
  - Isostrain model
  - Haskin – Shtrikman upper and lower bounds.
- The models taken into account for thermal conductivity are:
  - Isotherms model (parallel)
  - Constant heat flux model (series)
  - Maxwell and 'Maxwell Inverted' models.

1.5 Relevancy of Project

This project requires a through study of the literature on the relationship between mechanical properties and various transport properties in disperse composites. Available studies on mechanical and thermal behaviour of composite materials will be gathered and studies in through to obtain better information. These experimental reported data then will be compared to find the similarities and finally this will lead to development of predictive model. This model will be a better correlation for elastic modulus and thermal conductivity in composite materials which have been a long standing problem.

1.6 Feasibility of project

This project is a literature research in identifying experimental data reported for thermal conductivity and shear modulus. At the same time, models in literature also being identified in order to apply them to these experimental data and see how they compare. Thus, this project aims to come up with a better model to get a good agreement between the model and experimental values from literature research.

CHAPTER 2
LITERATURE REVIEW

2.0 LITERATURE REVIEW

2.1 Composite Material

Structural materials can be separated into four basic categories: metals, polymers, ceramics, and composites. Composites consist of two or more separate materials combined in a structural unit, are naturally made from various combinations of the other three materials. In the early days of modern man-made composite materials, the constituents were typically macroscopic [4, 5]. As macroscopic molecules are large, it is useful to consider the behaviour of molecule and to discuss the size of the molecule to qualify it as a macromolecule [6]. The advanced composites technology over the past few decades had steadily decreased the size of the constituent materials, particularly the reinforcement materials. Now the ongoing research more concerned with the microstructure of the composites.

Generally composite materials are microscopically heterogeneous and very anisotropic (properties in composite change as they move from matrix to fiber and as they change the direction along which they are measured). The physical properties of composite materials are generally not isotropic (independent on direction of applied force) in nature, but rather are typically orthotropic (different, depending on the direction of the applied force or load). For instance, the stiffness of a composite panel will often depend upon the direction of the applied forces and/or moments [7].

2.1.1 Fibres

The assembly of reinforcement material in forming a composite material take the following forms:

- Unidirectional: unidirectional tows, yarns, or tapes. Laminated composites are the one-dimensional system.
- Bidimensional: woven or nonwoven fabrics (Felts or mats). Fiber-reinforced composites form a two dimensional composites.

- Tridimensional: fabrics (sometimes called multidimensional fabrics) with fibers oriented along many directions (>2); Particle-filled matrices constitute the three-dimensional type of system [8].

The normalized specific stiffness and strength are reduced even further when the loading is in a direction other than along the fibres. Nevertheless, actual experience has shown that significant weight savings are possible in primary engineering structures through the use of advanced composites.

2.1.2 Matrix Materials

Polymers, metals, and ceramics are all used as matrix materials in continous fiber composites. Polymeric matrix materials can be further subdivided into thermoplastics and thermosets. The most common metals used as matrix materials are aluminium, titanium, and copper. Reasons for choosing a metal as the matrix material include higher use temperature range, higher transverse strength, toughness (as contrasted with the brittle behaviour of polymers and ceramics), and high thermal conductivity (cooper). The main reasons for choosing ceramics as the matrix include a very high use temperature range (>2000^{0}C, 3600^{0}F), high elastic modulus, and low density. The major disadvantage to ceramic matrix materials is their brittleness, which makes them susceptible to flaws. Carbon, silicon carbide, and silicon nitride are ceramics that have been used as matrix materials.

2.1.3 Composite Properties

Composites are used broadly because they have desirable properties that cannot be achieved by any of the constituent materials acting alone. The most common example is the fibrous composite consisting of reinforcing fibers embedded in a binder or matrix material. Composite materials may be selected to give remarkable combinations of stiffness, strength, weight, high–temperature performance, corrosion resistance, hardness, or conductivity. Fibers alone cannot support longitudinal compressive loads and their transverse mechanical properties are generally not as good as the equivalent longitudinal properties. Thus fibers need to be held together in a structural unit with a binder or matrix material and to provide a better stiffness. In composites, fibers are the load-carrying members, and the matrix material which keeps the fibers together, acts as a load-transfer medium between fibers. It also protects fibers from being exposed to the environment [7].

Most fiber – reinforced composites provide improved strength, fatigue resistance, Young's modulus, and strength-to-weight ratio by incorporating strong, stiff, but brittle fibers into a softer, more ductile matrix. The matrix material transmits the force to the fibers, which carry most the applied force, provides protection for the fiber surface and minimizes diffusion of species such as oxygen or moisture that can degrade the mechanical properties of fibers [7]. The reason why fiber – reinforced composites are much stronger and stiffer than the same material in bulk form is that the fine fibers contain fewer defects than does the bulk material.

This study concerned with three dimensional composites which is more complex than the one and two dimensional composites. In many instances, particulate reinforced composites can be thought as a feasible alternative. They are usually isotropic since the particles are added randomly. They can be used as either dual or multi-phase materials with the same advantage as monolithic materials in that they are easily processed to near net shape. At the same time, they have an improved stiffness, strength and fracture toughness that is characteristics of continuous fiber reinforced composites materials. Particles utilized for reinforcing have improved properties: they are capable of increasing the modulus and decreasing the permeability and the ductility. Particle used for reinforcing include ceramics and glasses such as fine mineral particles, metal particles such as aluminium, and amorphous materials, including polymers and carbon black [7].

Apart from particulate composites and fibre composites, flake composites are also widely used. Flake composites consist of flat reinforcements of matrices. Typical flake materials are glass, mica, aluminium, and silver. These types of composites provide advantages such as high out-of-plane flexural modulus, higher strength, and low cost. However, flakes cannot be oriented easily and only a limited number of materials are available for use. Figure 2.1 shows types of composites based on reinforcement shape while figure 2.2 shows the typical phases of 3-D composite material [8].

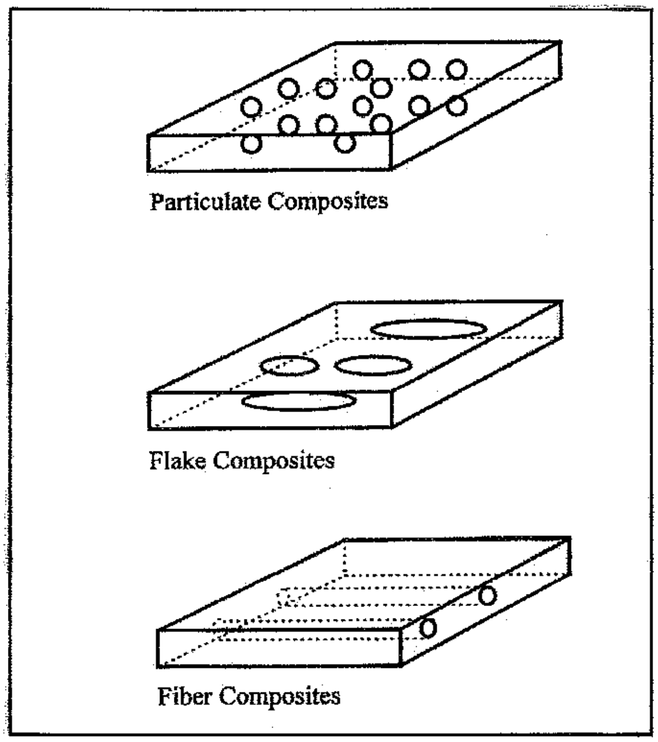


Figure 2.1: Types of composites based on reinforcement shape

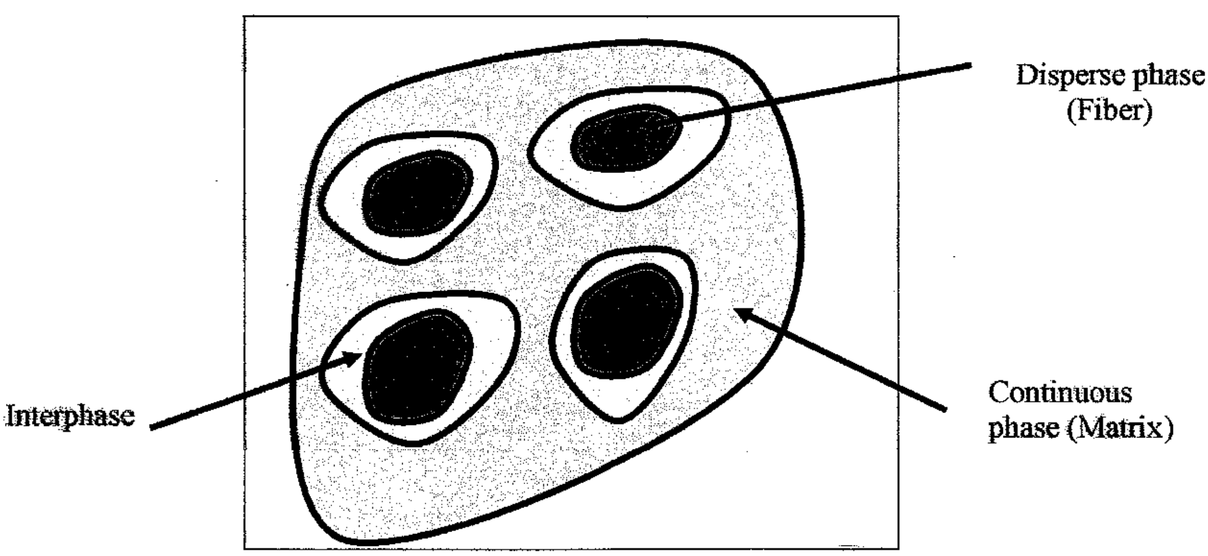


Figure 2.2: Phases of 3-D Composite Material

2.2 Effective Properties

Continuous fibre composite generally is orthotropic [27] with nine independent elastic constants. However, for a unidirectional composite which exhibits isotropic properties in a plane transverse to fibres (same properties in all direction in the x_2-x_3 plane), the effective response is transversely isotropic. In this case there are only five independent elastic constants. Layers of unidirectional composites with a large number of fibres through the layer thickness generally are considered to be transversely isotropic. When the full tensor notation is used for the stresses, σ_{ij}, and the strains, εij, the average or effective constitutive equations for a transversely isotropic material have the form

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} C_{11}^* & C_{12}^* & C_{12}^* & 0 & 0 & 0 \\ C_{12}^* & C_{22}^* & C_{23}^* & 0 & 0 & 0 \\ C_{12}^* & C_{23}^* & C_{22}^* & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C_{22}^* - C_{23}^*}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{66}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66}^* \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{Bmatrix} \tag{2.1}$$

where the five elastic coefficients C_{11}^*, C_{22}^*, C_{12}^*, C_{23}^*, and C_{66}^* are the effective stiffness coefficients of the equivalent homogenous material. They can be expressed in terms of the effective engineering properties. The goal of micromechanics, as far as elastic response is concerned, is to determine the effective (or average) stiffness, C_{ij}^*, in terms of the fibre and matrix properties, the fibre volume fraction, and the arrangement of fibres in the matrix.

2.3 Equivalent Homogeneity

All materials are heterogeneous [27] when evaluated on a sufficiently small scale. However, if the scale of interest is large enough, most materials exhibit statistical homogeneity. For a fibrous composite, the statistical homogeneity can be denied in terms of a characteristic dimension of the inhomogeneity. Let consider the fibre spacing, λ, as the characteristic dimension (Figure 2.3). Then there exists a length scale δ (sum of several λ) >> λ, over which the properties can be averaged in a meaningful way. If δ is small compared with the characteristic dimensions of the structure, the material can be idealized as being effectively homogeneous, and the problem can be analyzed using average or effective material properties.

We define a representative volume element (RVE) as a volume of material that exhibits statistically homogeneous material properties. A representative volume element and two nonrepresentative volume elements are shown in figure below. To be representative, the volume element must include a sufficient number of fibres and surrounding matrix to adequately represent the interaction between the phases. Obviously, a region of all fibre or all matrixes is not representative of the effective properties of the composite.

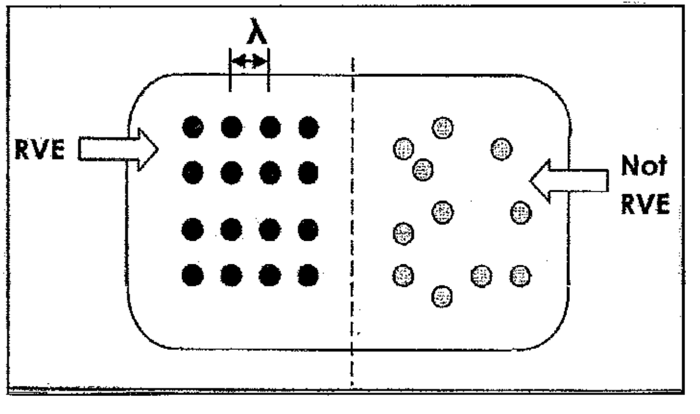


Figure 2.3: Representative volume element (RVE)

2.4 Analogy between Mechanical and Transport Properties

Analogies exist between mechanical and transport properties and this can be proven by Laplace equation. Laplace's equation is a partial differential equation named after Pierre-Simon Laplace who first studied its properties. The solutions of Laplace's equation are important in many fields of science, notably the fields of electromagnetism, astronomy, and fluid dynamics, because they describe the behavior of electric, gravitational, and fluid potentials. The general theory of solutions to Laplace's equation is known as potential theory. There are seven transport properties those analogies. Below is the table summarizes those properties:

Table 2.1: Analogy of transport properties

Properties	Analogous flow quantity
1.Electrical conductivity	Electric current
2. Thermal conductivity	Heat flux
3.Magnetic permittivity	Magnetic flux
4.Elastic moduli	Deflection
5. Dielectric constant	Microwaves
6. Refractive index	Light
7.Diffusion coefficient	Species (e.g., Absorption or drying)

All these phenomena governed by Laplace equation.Laplace equation are given below:

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+\frac{\partial^2 P}{\partial z^2}=0 \tag{2.2}$$

where P, the Potential, is the dependent variable; P = P (x,y,z)

The solution from Laplace equation can be presented in a simple form such as:

1. ***Ohm*'s Law** – for electricity
2. ***Hooke's* Law** – for strain
3. ***Fourier*'s Law** – for heat flow

Below are the governing equation for Ohm's, Hooke's and Fourier Law:

***Ohm's* Law**: Applies to electrical circuits; it states that the current through a conductor between two points is directly proportional to the potential difference or voltage across the two points, and inversely proportional to the resistance between them.

Ohm's Law (*electrical conductivity*)

$$\boxed{\frac{\text{voltage}}{\text{current}} = \text{electrical resistance}} \tag{2.3}$$

Taking its reciprocal,

$$\boxed{\frac{\text{current}}{\text{voltage}} = \text{electrical conductance}} \tag{2.4}$$

Hooke's **Law**: An approximation that states that the extension of a spring is in direct proportion with the load added to it as long as this load does not exceed the elastic limit. Materials for which Hooke's law is a useful approximation are known as linear-elastic or "Hookean" materials.

Hooke's Law (*mechanical properties*)

$$\boxed{\frac{\text{stress}}{\text{strain}} = \text{elastic modulus}} \tag{2.5}$$

Fourier's **Law**: The time rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area at right angles, to that gradient, through which the heat is flowing. We can state this law in two equivalent forms: the integral form, in which we look at the amount of energy flowing into or out of a body as a whole, and the differential form, in which we look at the flows or fluxes of energy locally.

Fourier's Law (*thermal conductivity*)

$$\boxed{\frac{\text{heat flux}}{\text{temp. gradient}} = \text{thermal conductivity}} \tag{2.6}$$

All the three laws above can be briefly described as:

Table 2.2: Property of Ohm's, Hooke's and Fourier's Law

Effect	Cause	Property = Effect / Cause
Current	Voltage	Electrical conductivity
Heat flux	Temperature gradient	Thermal conductivity
Strain	Stress	Compliance = Elastic modulus^{-1}

Since shear stress and thermal conductivity have the same tensorial order and governed by same set of equation, it is shown that analogies exist between these two properties.

2.5 Mechanical Properties

In this study, one of the mechanical properties, i.e., elastic modulus will be analyzed. There are three types of moduli categorized under elastic moduli which are tensile modulus, compressive modulus, and shear modulus.

2.5.1 Elements of mechanical behaviour of composites

This study is concerned with the analysis of both the micromechanical and the macromechanical behaviour of fiber-reinforced composite materials. As shown in figure 2.2, micromechanics are concerned with the mechanical behaviour of constituent materials (fiber and matrix materials), the interaction of these constituents, and the resulting behaviour of the basic composite (a single lamina in a laminate). Macromechanics is concerned with the gross mechanical behaviour of composite materials and structures (in this case, lamina, laminate, and structure), without regard for the constituent materials or their interactions.

As will seen later in this study, this macromechanical behaviour may be characterized by average stresses and strains and averaged, or "effective", mechanical properties in an equivalent homogeneous materials. As for micromechanical behaviour, it focuses on the relationships between the effective composite properties and the effective constituent properties.

The relationships between forces and deformations (or between stresses and strains) are complex in anisotropic composites than in isotropic materials, and this can lead to unexpected behaviour. For example, in an isotropic material, a normal stress (extensions and/or contractions), and a shear stress induces only shear strains (distortions). In an anisotropic composite, a normal stress may include both normal strains and shear strains, and a shear stress may induce both shear strains and normal strains. A temperature change in an anisotropic material may cause nonuniform expansion or contraction plus distortion. These so-called "coupling" effects have important implications not only for the analytical mechanics of composites, but for the experimental characterization of composites' behaviour as well [9].

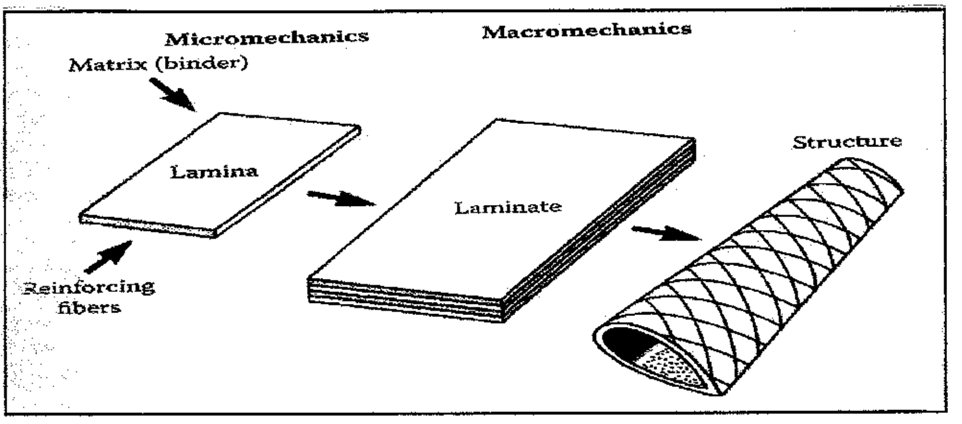


Figure 2.4: Micromechanics and macromechanics of composites.

2.5.2 Strength of Composite

The stiffness and strength of fibrous composites come from fibers which are stiffer and stronger. The basic mechanism of load transfer between the matrix and a fiber can be explained by considering a cylindrical bar of single fiber in a matrix material (Figure 2.2).When an applied load P on the matrix is tensile, shear stress develops on the outer surface of the fiber, and its magnitude decreases from a high value at the end of the fiber to zero at a distance from the end. The tensile stress in the fiber cross section has the opposite trend, starting from zero value at the end of the fiber to its maximum at a distance from the end. The two stresses together balance the applied load, P, on the matrix. The pure tensile state continues along the rest of the fiber.

When a compressive load is applied on the matrix, the stresses in the region of characteristic length are reversed in sign; in the compressive region, i.e., rest of the fiber length, the fiber tends to buckle, much like a wire subjected to compressive load. At this stage, the matrix provides a lateral support to reduce the tendency of the fiber to buckle. When a fiber is broken, the load carried by the fiber is transferred through shear stress to the neighbouring two fibers, elevating the fiber axial stress level [2, 3]

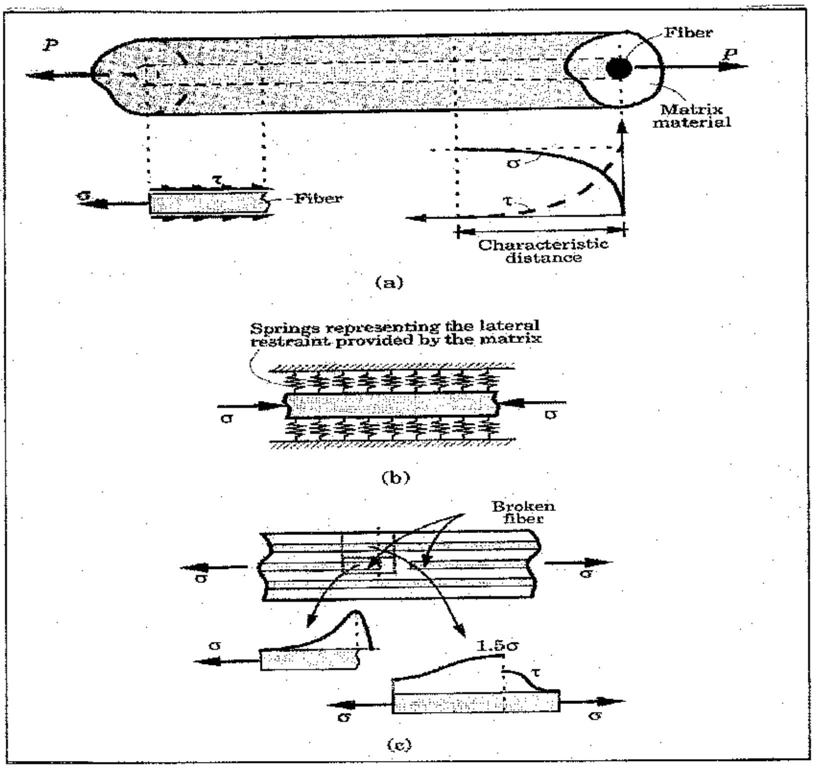


Figure 2.5: Load transfer and stress distribution in a single fiber embedded in a matrix material and subjected to an axial load.

2.5.3 Modulus of Elasticity

Micromechanical analyses are based on the mechanics of material approach or the elasticity theory. In mechanics of material approach, simplifying assumptions make it unnecessary to specify the details on the stress and strain distributions. The fiber packing geometry is normally subjective. Elasticity theory grips the solution of actual stresses and strains at the micromechanical level. Fiber packing geometry is also taken into consideration at this stage. It also involves numerical solutions because of the complex geometries and boundary conditions [10-12]. The figure below is considered for a detailed discussion on elastic moduli.

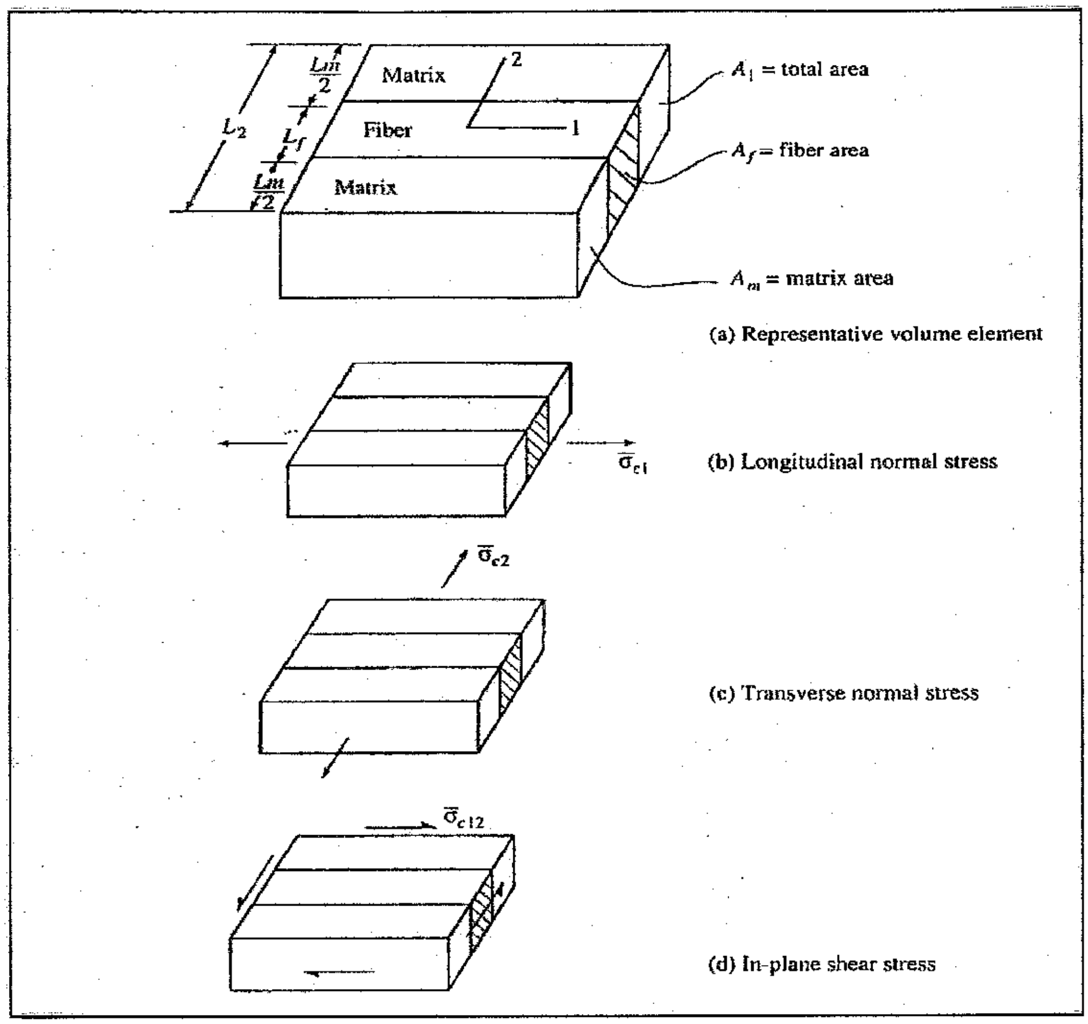


Figure 2.6: RVE and simple stress states used in elementary mechanics of materials models.

Shear Modulus (Fig.1(c))

The effective in – plane shear modulus is defined as

$$G_{12} = \frac{\sigma_{c12}}{\gamma_{c12}} \tag{2.7}$$

where σ_{c12} = average composite shear stress in the 12 plane

$\gamma_{c12} = 2\ \varepsilon_{c12}$, average engineering shear strain in the 12 plane

Geometric compatibility of the shear deformation, along with the assumption of equal shear stresses in fibers and matrix, leads to another inverse rule:

$$\frac{1}{G_{12}} = \frac{v_f}{G_{f12}} + \frac{v_m}{G_m} \tag{2.8}$$

where G_{f12} = shear modulus of fiber in the 12 plane

G_m = shear modulus of matrix

Practically, this equation is not very accurate because the shear stresses are not equal as assumed.

The equation of elasticity must be satisfied at every point in the model regardless of any simplifying assumptions about the stress and strain distribution. Fiber-packing geometry is generally specified in this approach. Numerical solutions of the governing elasticity equation are often necessary for complex structural geometries [12].

Adams and Doner [14, 15] state that the reinforcement effect for both effective in-plane shear modulus (G_{12}) and transverse modulus (E_2) only become significant for fiber volume fractions about 50% but the combinations of high fiber stiffness and high fiber volume fractions increase G_{12} and E_2. However, these combinations also generate very high stress concentration factors at the fiber/matrix interfaces.

2.6 Transport Properties

Heat energy can be transmitted through solids via electric carriers (electrons or holes), lattice waves (phonons), electromagnetic waves, spin waves, or other excitations. In metal, electrical carriers carry the majority of the heat, while in insulators lattice waves are the principal heat transporters. The thermal conductivities of solids vary dramatically both in magnitude and temperature dependence from one material to another [4].

Composites are usually subjected to changing environmental conditions during both initial fabrication and final use. For matrix – dominated properties, increased temperature causes a gradual softening of the polymer matrix material up to a point. However for fiber-reinforced composites, the fibers are not affected as much by temperature condition, the swelling or contraction of the matrix is resisted by the fibers and residual stresses develop in the composite [5].

2.6.1 Theory of Thermal Conductivity

Let consider a crystal with N_o unit cells, each of volume Ω. Let also identify a phonon with its wave vector q, polarization index s, frequency ω (qs), and group velocity $c_s(q)$. the heat current Q can be expressed by including contributions from phonons in all possible modes [28]

$$Q = \frac{1}{N_o\Omega} \Sigma\hbar\omega(qs)n_{qs}c_s q \qquad (2.9)$$

The quantity n_{qs}, assumes its equilibrium value n^*_{qs} characterized by the crystal temperature T. in the presence of a temperature gradient across the crystal it can be express

$$n_{qs} = n^*_{qs} + \delta n_{qs}, \qquad (2.10)$$

where δn_{qs} indicates deviation from the equilibrium value. Clearly, then, the heat current is governed by δn_{qs}, so that Eq. (2.9) can be expressed as

$$Q = \frac{1}{N_o\Omega} \sum_{qs}\hbar\omega(qs)\delta n_{qs}c_s(q) \qquad (2.11)$$

The deviation quantity δn_{qs}, which is significantly controlled by crystal anharmonicity, particularly at high temperatures, is in general unknown. Microstructure theories of lattice thermal conductivity attempt to address the quantity δn_{qs}.

2.6.2 Importance of Thermal Conductivity

A solid's thermal conductivity is one of its most fundamental and important physical parameters. Its manipulation and control have impacted an enormous variety of technical applications, including thermal management of mechanical, electrical, chemical, and nuclear system. Lattice thermal conductivity is the heat conduction via vibrations of the lattice ions in a solid. Lattice thermal conductivity of solids near ambient temperature can span an enormously wide range. "High" thermal conductivity of 0.03 $Wcm^{-1}K^{-1}$ would, for this class of solids, have a "high" thermal conductivity. On the other hand, such a value of thermal conductivity for an inorganic crystalline semiconductor (the thermoelectric material PbTe, for example) would be considered very "low". Frequently in the literature a value of thermal conductivity in excess of 1$Wcm^{-1}K^{-1}$ has been chosen, rather arbitrarily, as the lower limit for a high-thermal-conductivity solid. Because the main driver in the search for high-thermal-conductivity solids is for thermal management of electronics systems, a more suitable metric may be how the thermal conductivity compares to traditional materials used in these types of applications. By far most widely used material for thermal management in high-volume applications is crystalline alumina, with a thermal conductivity on the order of $0.5 Wcm^{-1}K^{-1}$. Thus the lower limit set to be "high" thermal conductivity at $0.5 Wcm^{-1}K^{-1}$. Even with this more relaxed criterion, the family of high-thermal-conductivity electrical insulators is still rather small.

2.6.3 Coefficient of Thermal Expansion (CTE)

Rosen [15] observed that for composites having high fiber volume fraction, the predicted longitudinal coefficient of thermal expansion (CTE) is almost zero. Measurement of such materials confirmed that longitudinal CTE is so small as to fluctuate between positive and negative values due to small changes in temperature or fiber volume fraction. Over the range of practical fiber volume fractions transverse CTE is much greater than longitudinal CTE. At the same time, at low fiber volume fractions transverse CTE can be greater than longitudinal CTE of matrix. The figure below shows the variations of longitudinal CTE (α_1) and transverse CTE (α_2) with fiber volume fraction for typical graphite/epoxy composite [1].

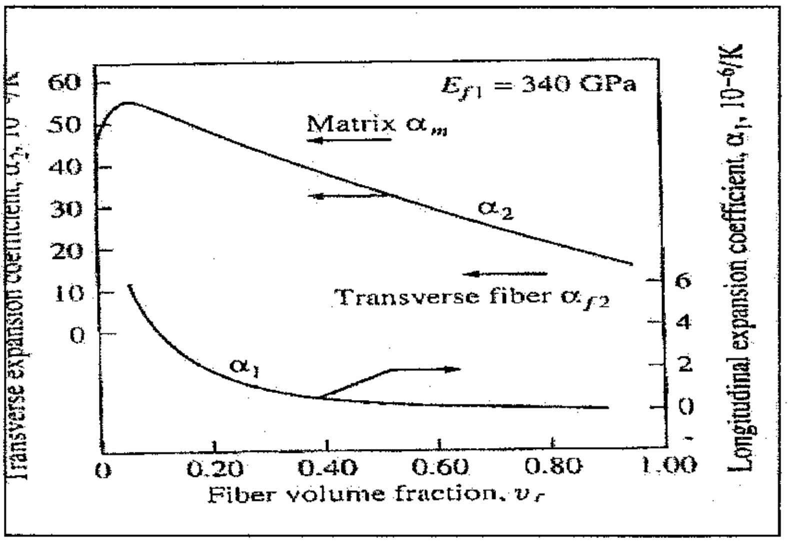


Figure 2.7: Variation of predicted longitudinal and transverse coefficients of thermal expansion with fiber volume fraction for typical unidirectional graphite/epoxy composite.

CHAPTER 3

METHODOLOGY

3.0 METHODOLOGY

3.1 Technique of Analysis

Figure below shows the summary of methodology that will be implemented in order to complete the project. It involves project activities which move from one phase to another.

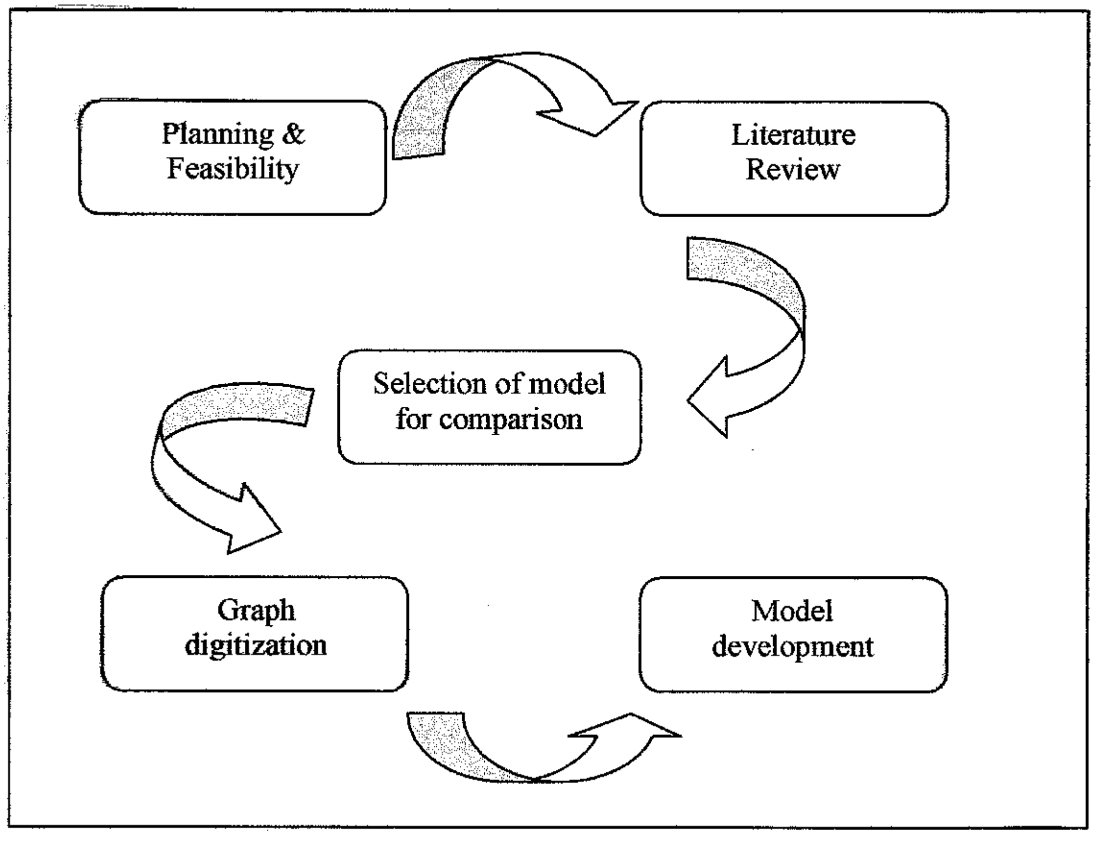


Figure 3.1: Summary of Methodology

3.1.1 Planning & Feasibility

This project concerned with the literature studies on analogy of mechanical and transport properties in composite materials. Information on the mechanical behaviour and thermal conductivity of composite will be studied thoroughly at this stage. The objectives and scope of study of the project were identified and the further activities were planned based on these information. Then the limitations to the proposed model were identified and this will guide to an enhanced union and divination.

3.1.2 Literature Review

Since this study is a literature research, the development of predictive model predominantly depends on the existing earlier theories. Abundant of information on the existing theories are needed in order to come out with a better prediction of proposed model. All the information mostly gathered form books written by early researchers and journals and articles found in Information Resource Centre (IRC) about the composite materials as well as discussion with lecturers. All the data from various sources will be gathered in order to be analyzed later.

3.1.3 Selection of model for comparison

The gathered data will be analyzed in through at this stage. The early existing theories such Rule of Mixtures, Maxwell prediction for thermal conductivity and Hashin-Shtrikman bounds for mechanical properties were analyzed at first place before moving on with other theories. Other theories such as Effective Moduli method and Raghavan-Martin model will be studied as project works continuous. The ideas and theories brought forward by these researchers will be studied in through and evaluate the similarities among those theories. This will lead to a much simpler prediction about the shear modulus and thermal conductivity can be developed by analyzing the analogy of mechanical and transport properties of disperse composite. This is the stage where the selected theories and practical data analyzed in thorough so that those models can be used later in graph digitization.

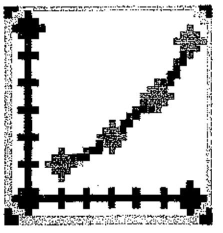
3.1.4 Graph digitization

In this part, both selected practical and theoretical data will be exported to "Engauge" software to be digitized. The points from the graph will be extracted to excel form and the exact points can be obtained. The same thing will be done for all selected models. Once all the points been extracted from the graph, effective properties for thermal conductivity and shear modulus will be evaluated. Those properties will be compared with other points extracted from other graphs. The analogy between those models will be figure out. Since both mechanical properties and thermal conductivity governs by the same set of equation, it is possible to develop a better model using any one of the property.

3.1.5 Model Development

At this stage, all the gathered data will be compared to each another in order to find the similarities among the existing model. Graphs of effective shear or thermal curves vs. volume fraction of disperse medium will be plotted. Then, the effective thermal models will be superimposed with effective shear curves and see the degree of analogy that exists between them. Those models which superimpose with one another will be identified at first place. The main aim is to identify whether the effective thermal models that predict effective shear models well and are also correct at the limit. The proposed model will be developed based on the model which gives a good prediction. The proposed model also cannot exceed the Maxwell and Hashin – Shtrikman bounds.

3.2 Tool required



Engauge Digitizer - Digitizing software

The software used in this project is "Engauge". This open source, digitizing software converts an image file showing a graph or map, into numbers. The image file can come from a scanner, digital camera or screenshot. The numbers can be read on the screen, and written or copied to a spreadsheet. The process starts with an image file containing a graph or map. The final result is digitized data that can be used by other tools such as Microsoft Excel and Gnumeric. Engauge (from en "make" and gauge "to measure") verb meaning to convert an image file containing a graph or map, into numbers. Below are some of the features of "Engauge" software:

- Automatic curve tracing of line plots
- Automatic point matching of point plots
- Automatic axes matching
- Support for drag-and-drop and copy-and-paste makes data transfer fast and easy
- Tutorials with pictures explain strategies for common operations
- Preview windows give immediate feedback while modifying settings
- Export support for common software packages such as Microsoft Excel, OpenOffice CALC, gnuplot, gnumeric, MATLAB and Mathematica
- Engauge is available for a wide variety of platforms (Linux, Mac OSX, Windows)
- Engauge Digitizer is completely open source and free courtesy of Sourceforge, Trolltech and FFTW

CHAPTER 4

RESULT AND DISCUSSION

4.0 RESULT AND DISCUSSION

4.1 Result

4.1.1 Two Dimensional Composite Materials

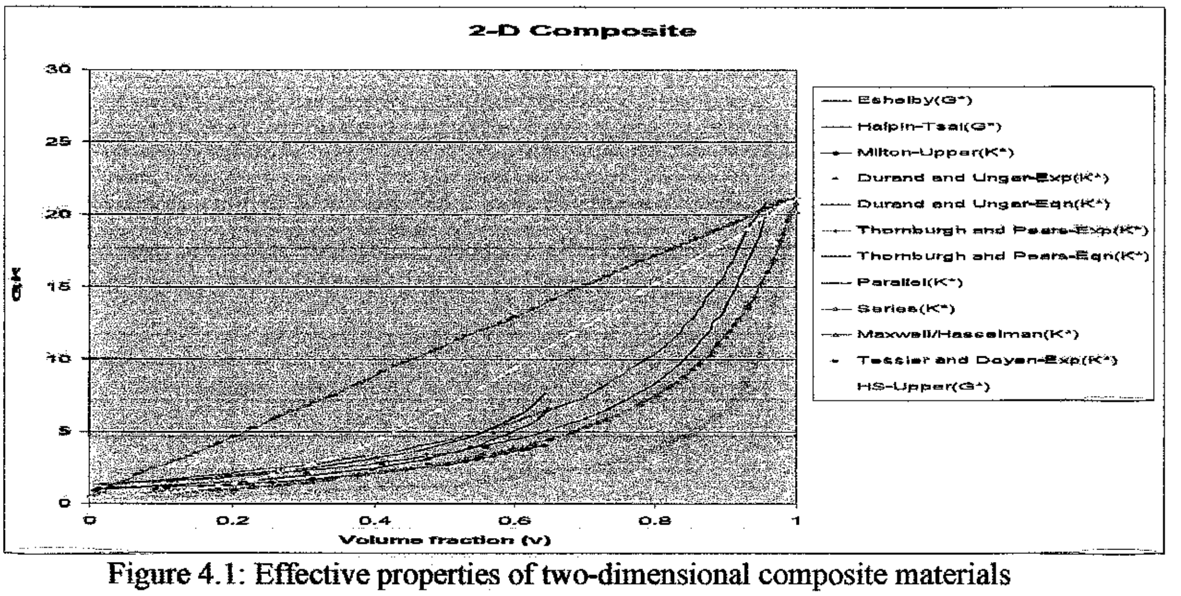


Figure 4.1: Effective properties of two-dimensional composite materials

4.1.2 Three Dimensional Composite Materials

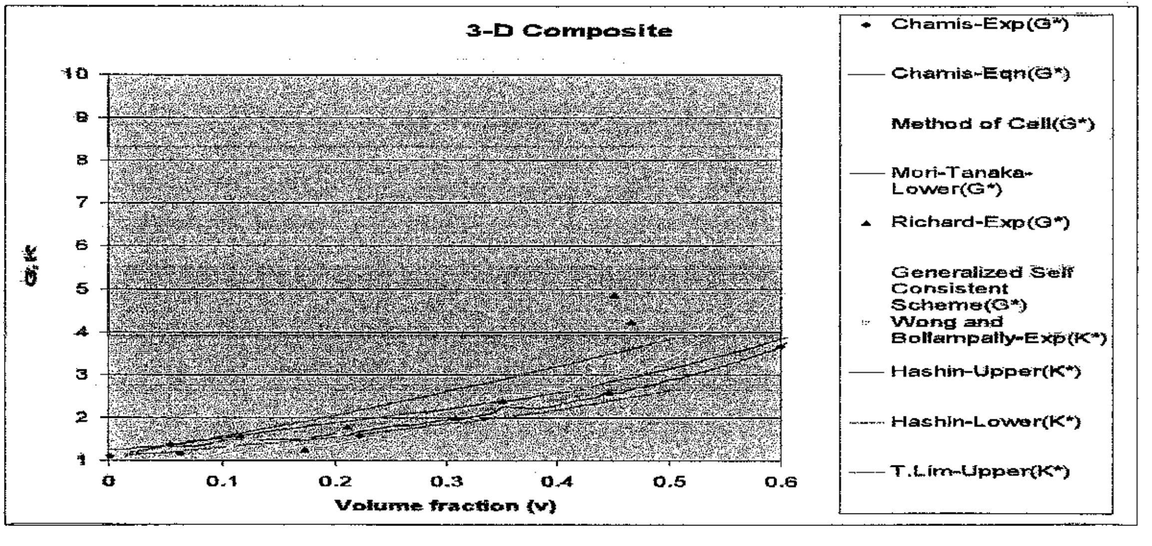


Figure 4.2: Effective properties of three-dimensional composite materials

4.2 Discussion

4.2.1 Rule of Mixtures (Strength of Materials Approach)

This approach was developed by Voigt and Reuss. The strain field approach introduced by Voigt states that strains is same over all composite and equal in each phase. The stress dual approach invented by Reuss states that constant stresses over all composites and equal in each phase. This constant stress and strain only valid when constituents are either in parallel or in series. In the theory of mechanical properties for composites, there are two "rules of mixing" that act on a composite [18]:

- Isostrain: Loading parallel to fibers: Isostrain is similar to springs in parallel as in Figure 4.3
- Isostress: Loading perpendicular to fibers: Isostress is similar to springs in series as shown in Figure 4.4

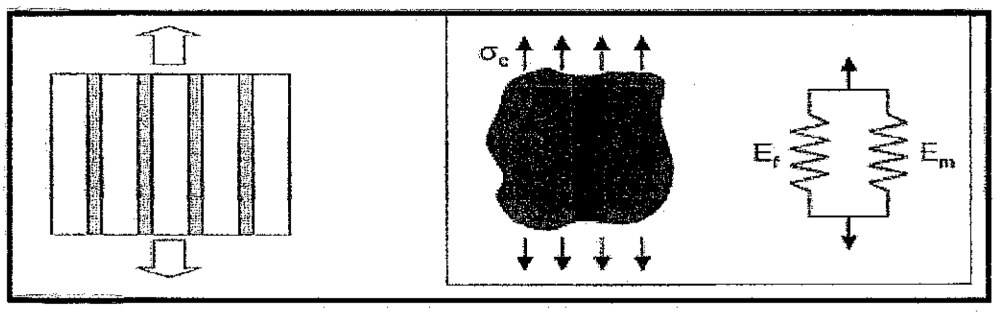


Figure 4.3: Isostrain condition – similar to springs in parallel.

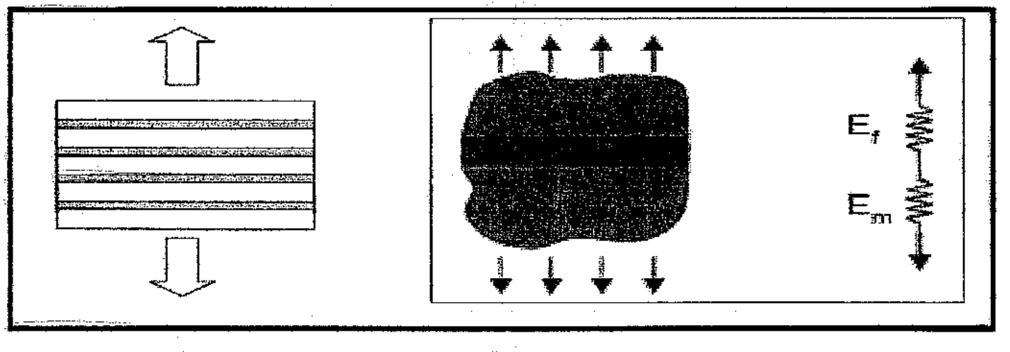


Figure 4.4: Isostress condition – similar to springs in series.

As for particulate composites, the rule of mixtures always predicts the density of fiber – reinforced composites [19]:

$$\rho_c = f_m \rho_m + f_f \rho_f \qquad (4.1)$$

where the subscripts c, m, and f refer to composite, matrix, and fiber. Note that

$$f_m = 1 - f_f \qquad (4.2)$$

This method does not consider fibre-packing geometry buy do consider representative volume element (RVE). The bonding at interface is perfect, so no slip occurs between fibre and matrix material. However the results obtained from this method are not accurate and agreement with experimental results are generally poor.

4.2.1.1 Modulus of Elasticity

The rule of mixtures is used to predict the modulus of elasticity when the fibers are continuous and unidirectional. Parallel to the fibers, the modulus of elasticity may be as high as:

$$E_{c,//} = f_m \cdot E_m + f_f \cdot E_f \quad (4.3)$$

However, when the applied stress is very large, the matrix begins to deform and the stress-strain curve is no longer linear (Figure 4.5). Since the matrix now contributes little to the stiffness of the composite, the modulus can be approximated by:

$$E_{c,//} = f_f \cdot E_f \quad (4.4)$$

When the load is applied perpendicular to the fibers, each component of the composite acts independently of the other. The modulus of the composite is now:

$$\frac{1}{E_c} = \frac{f_m}{E_m} + \frac{f_f}{E_f} \quad (4.5)$$

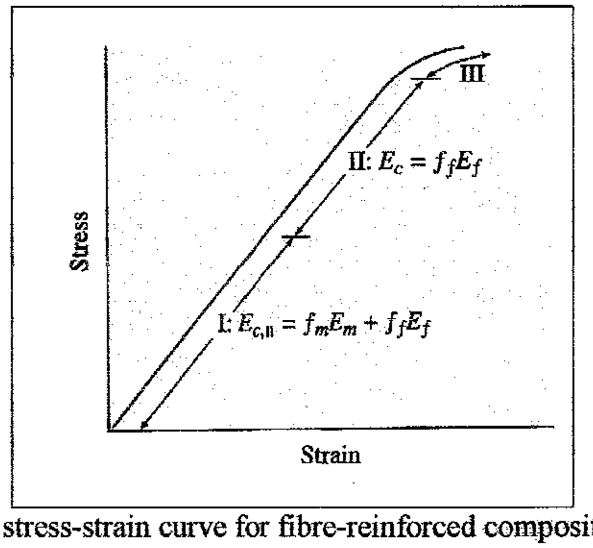


Figure 4.5: The stress-strain curve for fibre-reinforced composite

4.2.1.2 Thermal Conductivity

The rule of mixtures accurately predicts the thermal conductivity of fiber – reinforced composites along the fiber direction if the fibers are continuous and unidirectional:

$$K_c = f_m K_m + f_f K_f \tag{4.6}$$

where K is the thermal conductivity. Thermal energy can be transferred through the composite at a rate that is proportional to the volume fraction of the conductive material. In a matrix containing metallic fibers, energy would be transferred through the fibres.

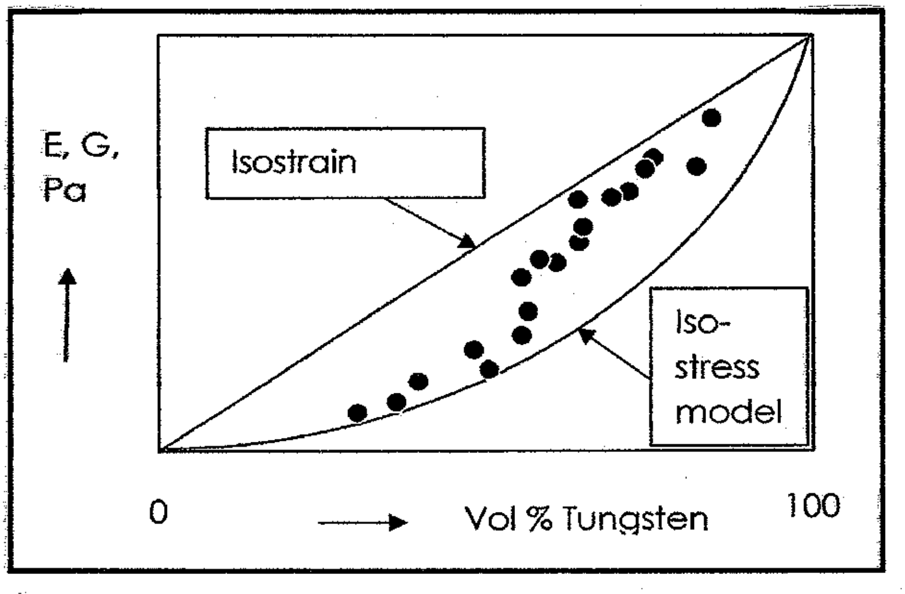


Figure 4.6: Sample data for Isostrain and Isostress for copper matrix tungsten particle.

4.2.2 Hashin – Shtrikman Model

The analytical expressions proposed by Hashin and Shrikman [20], provide bounds for the elastic constants of a heterogeneous material with a random isotropic distribution of phases from the properties and volume fraction of each phase. They are based on a variational principle which, combined with a hypothesis of isotropy, leads to a calculation of the average strain in one of the phases. The lower bound is built with the softer phase taken as the matrix and the upper bound corresponds to the harder phase taken as the matrix. The spherical shape of the inclusion reflects the isotropic phase distribution. The HS bounds are easy to compute when the constitutive phases are isotropic and have general validity. In the particular case of a composite made of spherical inclusions isotropically distributed in a matrix, the lower and upper bounds provide good estimation on effective properties of composites. Hashin-Shtrikman had narrowed the range of the earlier, wider bounds given by the Isostrain and Isostress models.

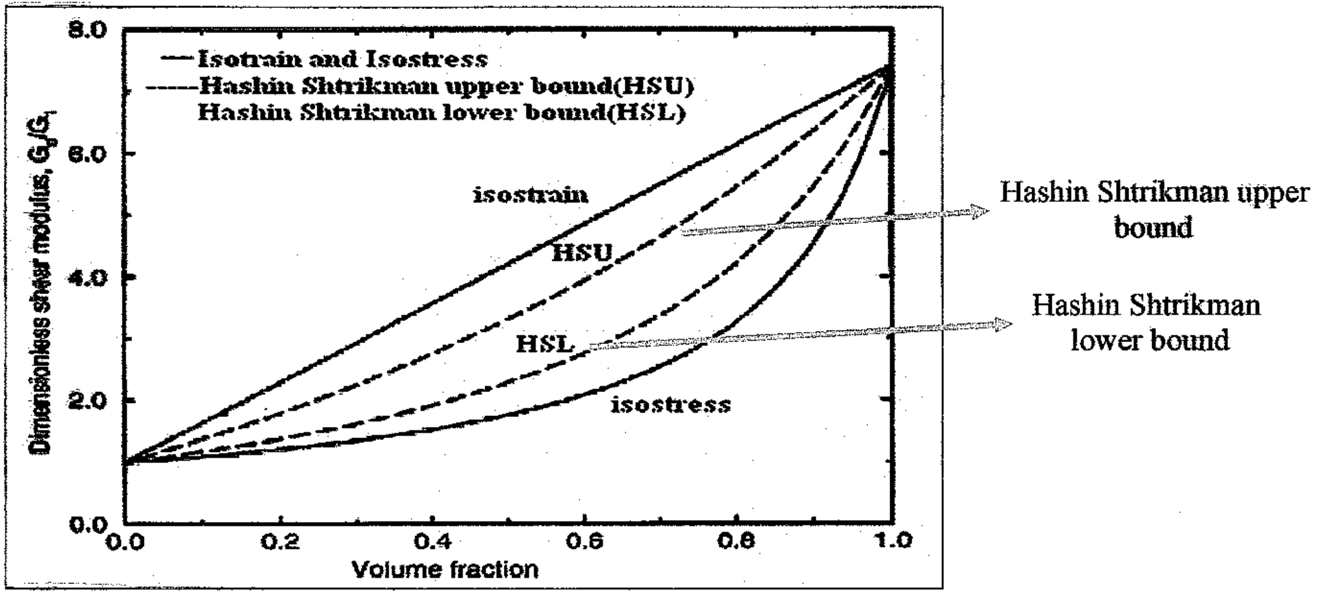


Figure 4.7: Hashin Shtrikman Upper and Lower bounds on the dimensionless effective shear modulus versus volume fraction for a two phase composite composed of particulate material.

The HS bounds can be used as a rapid check to find out whether the elastic properties of a particulate composite are reasonable or not, provided that its microstructure is in agreement with the HS hypothesis. The properties lay below lower bound-composite might contain fair amount of defects while properties which lay above upper bound-structure might be fibrous. The HS bounds are identical to the results of Maxwell which will be discussed next. The lower bound corresponding to the classical Maxwell result and the upper bound is equivalent again to the Maxwell result when the phases are inverted. This result is frequently referred as 'inverted Maxwell', though there is no difference between the two except for change of symbols.

4.2.2.1 Effective Axial Shear Modulus

The effective axial shear modulus, G^*_{12}, of a unidirectional fibrous composite is obtained from the boundary displacements [27]

$u_1 = \varepsilon^0_{12} x_2$

$u_2 = \varepsilon^0_{12} x_1$

$$u_3 = 0 \tag{4.7}$$

if this displacement field is applied to homogeneous, transversely isotropic circular cylinder with axial shear modulus G^*_{12}, the resulting strains are

$$[\varepsilon_{ij}] = \begin{bmatrix} 0 & \varepsilon^0_{12} & 0 \\ \varepsilon^0_{12} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.8}$$

And the stresses are

$$[\sigma_{ij}] = \begin{bmatrix} 0 & 2G^*_{12} & 0 \\ 2G^*_{12} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.9}$$

Thus the equivalent homogeneous cylinder is in a state of pure axial shear.

Solution of the concentric cylinder elasticity problem for the displacement boundary conditions gives the final result

$$\frac{G^*_{12}}{G_m} = \frac{G_f(1+V_f)+G_m(1-V_f)}{G_f(1-V_f)+G_m(1+V_f)} \tag{4.10}$$

From the above result, it is seen that the rule of mixtures is not a good approximation for the axial shear modulus. It is interesting to note that the effective shear modulus of the composite is a function of only the fiber and matrix shear moduli and not any other properties of the constituents.

4.2.3 Maxwell Prediction

Maxwell [21] solved for the effective conductivity, k_{eff}, of dilute suspension of conducting spheres in a conducting matrix. Maxwell used the idea of an effective medium to calculate the effective conductivity, of a two phase composite consisting of isotropic spheres embedded randomly in a homogeneous matrix. Maxwell considered a spherical particle in a large matrix and assumed that the concentration of the spherical inclusion, c, in the matrix is nearly zero, ($c \rightarrow 0$). Thus,

$k_{eff} = fn(\alpha, c)$, i.e., $k_{eff} = (k_d, k_c, c)$ (4.11)

where k_d = conductivity in dispersed area,

k_c = conductivity in continuous area

and $k_d/k_c = \alpha$

$k_{eff}/k_c = K = (1+2\beta)(1-\beta c)$ in which $\beta = (\alpha - 1)/(\alpha + 2)$ (4.12)

$c \rightarrow 0$ because of the vanishingly small concentration. The result may be approximated as $k \approx 1 + 3\beta$ for the effective property of the composite.

The true effective property lies between these bounds as seen in figure below. Its value depends on the individual property (elastic modulus/ thermal conductivity) of the constituent phases and on the volume concentration of the disperse phases.

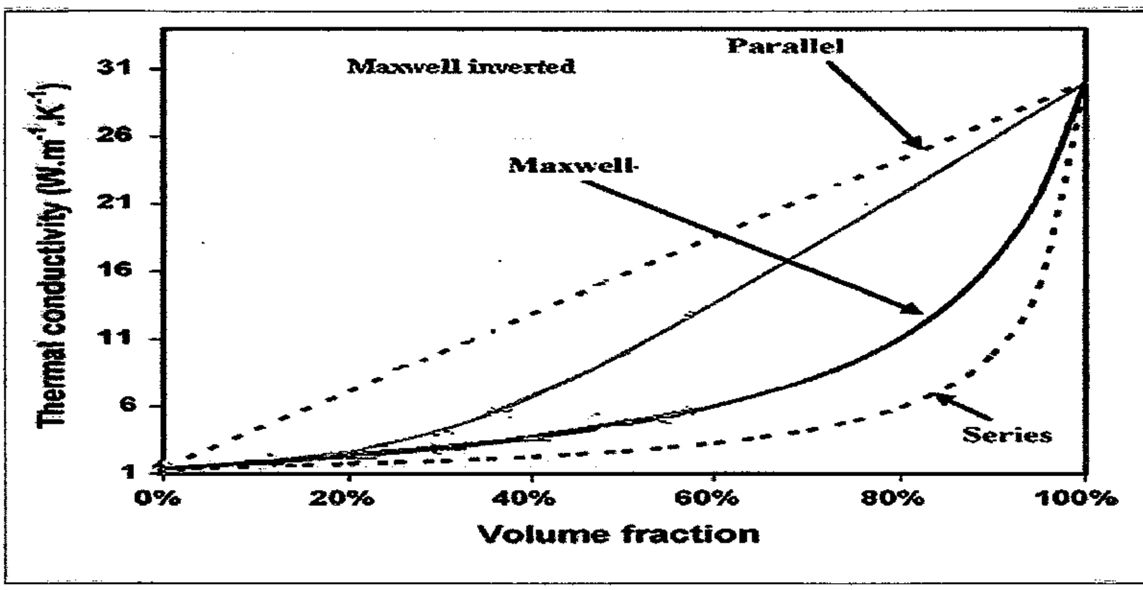


Figure 4.8: Maxwell and Maxwell-inverted limits, parallel and series lines in thermal conductivity.

4.2.4 The Eshelby inclusion method

Eshelby [1, 2, and 26] starts his research by assuming that the elastic stiffness matrices C_M and C_I of matrix and inclusions are both equal to C. As in fig 4.4, the inclusion is cut from the medium and taken "outside", where it can transform subject to no elastic constraint. Surface tractions are then applied to the inclusion, whereby it undergoes an elastic strain $-\varepsilon^T$ back to its original shape and size. Then it is returned to its hole, where it fits exactly. At this stage, the stress in the inclusion is equal to $-C\varepsilon^T$ and the matrix is unstressed. When the surface traction is finally removed by equal but opposite surface tractions, the inclusion reaches equilibrium with the elastic matrix after an additional elastic strain $\varepsilon^C - \varepsilon^T$ and displacement will be introduced in the matrix. This is to describe the analysis of stress and strain generated in infinite homogeneous linear elastic medium when inclusion undergoes transformation, which produces a homogeneous inelastic strain throughout the inclusion. The transformation strain results from differential thermoplastic deformation.

Eshelby method can provide a stress state solution in the case of heterogeneous materials and so composite materials. For the temperature gradient, since in most cases, thermal expansion coefficient of matrix is higher than fiber, a change in temperature produces a change in the shape of the inclusion. In sum, the analysis of the equivalent inclusion allows behaviour homogenization problem to be solved by substitution of a problem with heterogeneity into a problem of a homogeneous medium submitted to a localized free strain. Figure 4.9 shows the Eshelby's thought experiment.

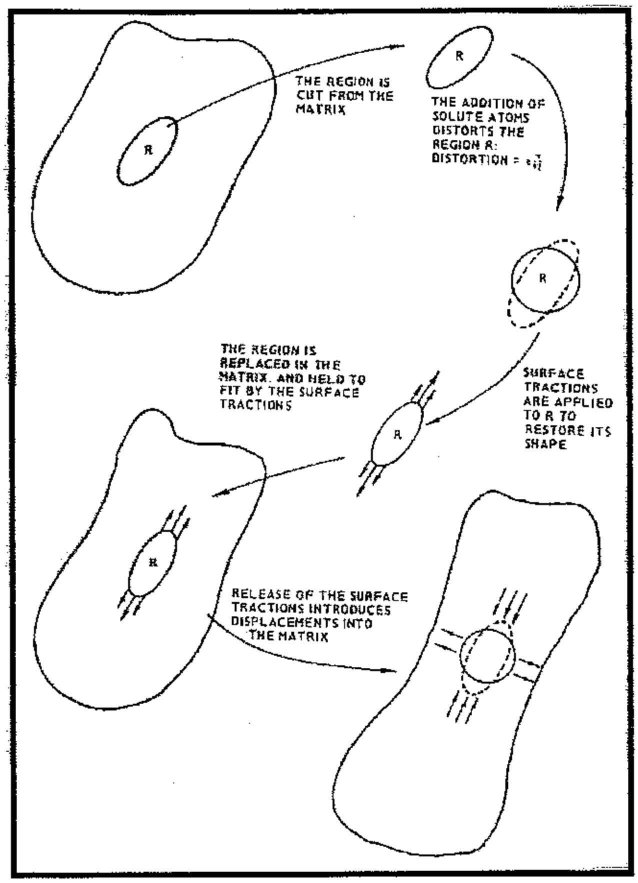


Figure 4.9: Eshelby's thought experiment, as illustrated by Brown and Ham

4.2.5 Semiempirical Model

This approach [1] involves the use of semiempirical equations which are adjusted to match experimental results or elasticity results by the use if curve fitting parameters. The equations are referred to as being "semiempirical" because, although they have terms containing curve-fitting perimeters, they also have some basis in mechanics.

4.2.5.1 Halpin – Tsai Equation

This model [1, 26] is a mathematical model for the prediction of elasticity of composite material based on the geometry and orientation of the filler and the elastic properties of the filler and matrix. The model is based on the self-consistent field method although often consider to be empirical. Homogenization method is used to obtain numerical results of the plane strain bulk modulus and the transverse shear modulus.

The appropriate equation for this method is,

$$G_{12} = \frac{G_m (1 + \xi\eta f)}{(1 - \eta f)} \tag{4.13}$$

In which, $\eta = \frac{\{G_f/G_m - 1\}}{\{G_f/G_m - \xi\}}$ (4.14)

and the parameter ξ is taken to have a value of around unity.

4.2.6 Elasticity Approach

This approach [1,26] considers equilibrium of forces, compatibility and Hooke's law relationship in three dimensions. This model also called as composite cylinder assemblage (CCA) models. This approach is about selecting a suitable RVE and subjecting the RVE to uniform stress of displacement at the boundary. The equation must be satisfied at each point in the model and no simplifying assumptions are made regarding the stress or strain distributions as in the mechanics of materials. Fibre packing geometry is specified. Complete stress and strain distributions in RVE are generated and calculation of stress concentration factors is possible. The governing equation for shear modulus is given below.

$$G_{12} = G_f \frac{G_f v_f + G_m (1 + v_m)}{G_f (1 + v_m) + G_m v_f} \tag{4.15}$$

where G_{12}, G_m, and G_f are representing shear modulus of composite, matrix and fibre respectively. v_m and v_f representing volume fraction of matrix and fibre respectively.

4.2.6.1 Method of Cell

This approach [26] depends on the assumption that the two phase composite has a periodic structure in which the reinforcing material (e.g. fibres) is arranged in a periodic manner thus forming a periodic array. This assumption allows the analysis of a single representative element rather than the whole composite with its many fibres. The equilibrium equations solved subjected to continuity of displacement and tractions at interfaces between subcells and between neighbouring cells in an average basis. This method produces lengthy equations but excellent agreement with experimental data on graphite/epoxy. This approach yields to in-plane lamina properties and through-the-thickness properties such as G_{23} and v_{23}. Figure 4.10 shows the periodic array of fibre in method of cell.

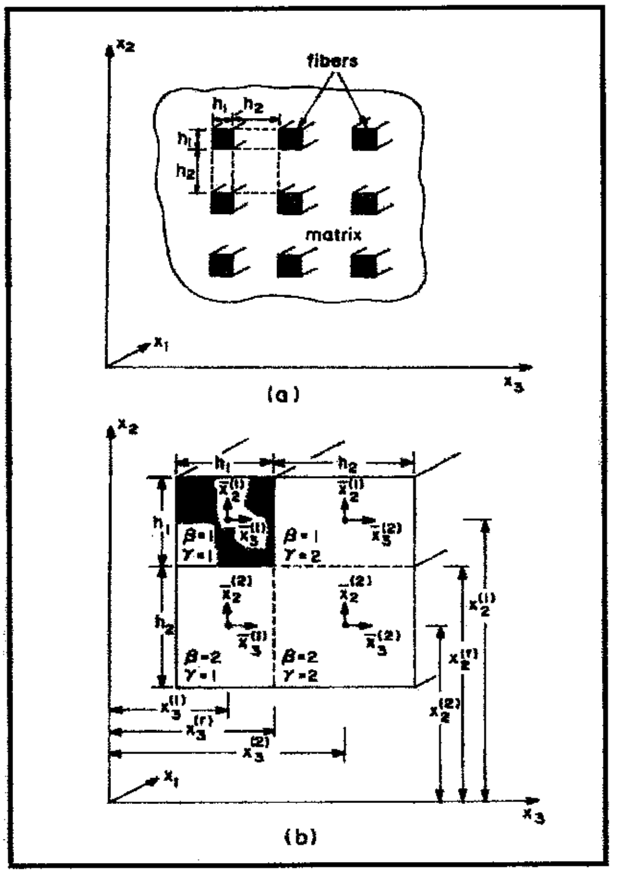


Figure 4.10 (a) Composite with doubly periodic array of fibres. (b) Representative cell with four subcells α, β= 1, 2

4.2.7 Self-Consistent Method

This method [26] does not consider an inclusion or the REV as isolated free-bodies. The purpose is to place these volumes in an infinite medium which is already homogenized and the properties of which have to be found. At the beginning, the properties of the composite are assumed so that the stress and strain fields in the REV can be computed. When the REV is homogenized to represent the composite, the resulting material properties in the REV must match those assumed previously for the composite. This approach is good for low volume fractions of heterogeneities because it considers an infinite medium with a single REV, cannot take into consideration interactions between the constituents [22]. Figure 4.11 shows the self-consistent scheme model.

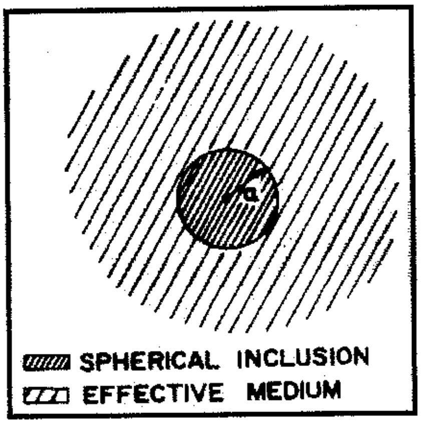


Figure 4.11: The self-consistent scheme model

4.2.7.1 The Mori – Tanaka Model

This method was introduced by Mori and Tanaka which lead to better description on real strain state in the matrix [25]. The main assumption of this model is that the strain in the inclusion is uniform. It is an accurate method to predict the effective moduli of the coated inclusion based composite materials. The average strain in a typical inclusion (fiber) is related to the average strain in the matrix by a fourth – order tensor T where T is defined to give the relation between the uniform strains in the inclusion embedded in an all-matrix material subjected to an imposed uniform strain at infinity. The fiber strain concentration factors are found to be

$$A_f = T[V_f T + (1-V_f)I]^{-1} \quad (4.16)$$

$$\text{where } T = [SC_m^{-1}(C_f - C_m) + I]^{-1} \quad (4.17)$$

The result obtained from this model is similar with lower bound Haskin-Shtrikman model for spheres, elongated inclusions with the same shape and anisotropic. However, the expansions of this model differ from Haskin-Strikman bounds for the case of elongated particles with different shapes and orientations because of the simplifications regarding inclusion interaction in this model.

4.2.8 The Generalized Self-Consistent Scheme

This procedure [26] also referred to as the three-phase model yields better results than the self-consistent scheme. For a particulate composite with randomly distributed spherical inclusion, this scheme consists in imbedding a composite sphere with an inclusion core and surrounded by a shell of the matrix material in an infinite medium of unknown effective properties. For the effective shear modulus, μ^* this model [26] considered uniform strain field c applied at infinity in the geometry of the generalized self-consistent scheme. By imposing the interfacial conditions of perfect bonding, these researchers obtained effective shear modulus is governed by a quadratic equation of the form

$$A\,(\mu^* / \mu_1)^2 + 2B\,(\mu^* / \mu_1) + C = 0 \tag{4.18}$$

where the coefficient A, B and C are complicated functions of the inclusion and matrix elastic properties. Figure 4.12 shows the generalized self-consistent scheme model.

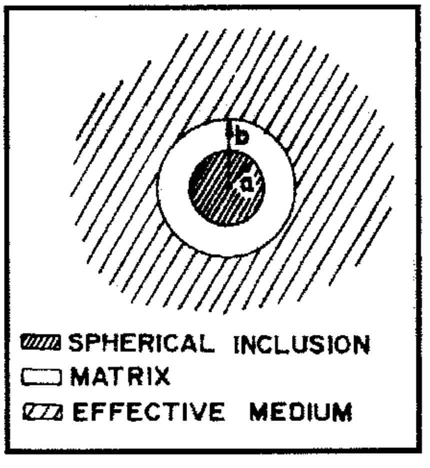


Figure 4.12: The generalized self-consistent scheme model

The weak point of the generalized self-consistent method (GSCM) is that its solution for the effective shear moduli involves determining the complicated displacement and strain fields in constituents. Furthermore, the effective moduli estimated by GSCM cannot be expressed in an explicit form.

4.2.9 Chamis Approach

This approach is based on "simplified micromechanics equations" (SME) which are based on subregions method whereby divide square array of fibers into subregions for more detailed analysis if convert to a square fiber having the same area as the round fiber. The longitudinal fiber shear modulus G_{12} are not actually measure but are inferred by substitution of measured composite properties and matrix properties in SME. Chamis's equation is

$$G_{12} = G_m \left[(1-v_f)^{1/2} + (v_f)^{1/2} / \{1-(v_f)^{1/2}(1-G_m/G_{f2})\}\right] \quad (4.19)$$

4.2.10 Milton Approach

Given only the phase volume fractions, conductivities, bulk moduli and shear moduli, denoted by $ø_1$ and $ø_2$, σ_1 and σ_2, k_1 and k_2, and G_1 and G_2, restrictive bounds on σ_e, k_e, and μ_e which include additional microstructural information on the transversely isotropic fiber-reinforced material obtained by Milton (1981, 1982). Milton's bounds on σ_e depend not only upon three-point probability function but upon S_4. this approach for disordered composites has been virtually nonexistent because of the difficulty involved in determining S_3 and S_4, either experimentally or theoretically. For transversely isotropic fiber-reinforced material, Milton demonstrated that both integrals may be expressed in terms of single intergrals ζ_2 which depends upon the three point probability function. The simplified form for transverse conductivity is expressed as below:

$$\sigma_L^{(3)} \leq \sigma_e \leq \sigma_U^{(3)} \quad (4.20)$$

where $$\sigma_U^{(3)} = \langle\sigma\rangle - \frac{ø_1 ø_2 (\sigma_2 - \sigma_1)^2}{\langle\sigma\rangle + \sigma\zeta} \quad (4.21)$$

and $$\sigma_L^{(3)} = \langle 1/\sigma\rangle - \frac{ø_1 ø_2 (1/\sigma_2 - 1/\sigma_1)^2}{\langle 1/\sigma\rangle + (1/\sigma)\zeta} \quad (4.22)$$

For the fourth order bounds which depend on σ_1, σ_2, $ø_2$ and ζ_2 and upon multidimensional intergral that involves fourth point probability function of S_4. Utilizing phase-interchange theorem for fiber-reinforced materials, Milton showed that the intergral involving S_4 can be expressed in terms of $ø_2$ and ζ_2 only.

$$\sigma_U^{(4)} = \frac{\sigma_2 (\sigma_1+\sigma_2)(\sigma_1 + \langle\sigma\rangle) - ø_2\zeta_1(\sigma_2-\sigma_1)^2}{(\sigma_1+\sigma_2)(\sigma_2 + \langle\sigma\rangle) - ø_2\zeta_1(\sigma_2-\sigma_1)^2} \quad (4.23)$$

and $\sigma_L^{(4)} = \dfrac{\sigma_1 (\sigma_1+\sigma_2) (\sigma_2+ <\sigma>) - \o_1\zeta_2(\sigma_2-\sigma_1)^2}{(\sigma_1+\sigma_2) (\sigma_1+ <\sigma>) - \o_1\zeta_2(\sigma_2-\sigma_1)^2}$ (4.24)

For the case of $\alpha = 0.1$, the Milton bounds provide similar improvement over Hashin bounds, except that most of the improvement is in the lower bound. For the range of $0.1\leq \alpha\leq 10$, the fourth order Milton bounds are sharp enough to give good estimate of σ_e/σ_1 for the entire range of volume fraction.

4.2.11 Predictive Model

The predictive model is a model which has less scatter between the practical and theoretical data gathered. The standard deviation between the points should be less and the experimental data should not deviate much from theoretical data. All the models discussed earlier are within the range and do not exceed the parallel and series limit, Hashin-Shtrikman and Maxwell lines. The theories lie behind all these models have been discussed earlier.

4.2.11.1 Bidimensional Composite Materials

Based on the Figure 4.1, the Milton upper limit identical to Maxwell lower limit. When volume fraction is smaller, Milton bounds provide similar improvement over Hashin-Shtrikman bounds, expect the most of the improvement is in the lower bound. It is good enough to give a good estimate of thermal conductivity for entire range of volume fractions. As noted above, bounds diverge as α is made large and Milton lower bound on conductivity yield a good estimate, with maximum error occurring at maximum volume fraction reported, i.e, at volume fraction of 0.65 or equivalently at 80% of closing-packing volume fraction.

All the models give good approximation when the volume fraction is lower. As for Thornburgh and Pears model, the theoretical data when compared with experimental data for two-dimensional composites gives excellent estimates of experimental data. This further supports claims about utility of bounds.

When the theoretical value of Durand and Ungar model to exact simulation results obtained using Boundary Element Method (BEM), it is seen to predict effective thermal conductivity extremely accurately. This supports the assertion that bounds, which incorporate nontrivial microstrucutral information on the medium, can be used to accurately estimate effective properties, even when the phase properties are widely different.

It can be seen that the Halpin-Tsai expression and Eshelby approach represents a fairly good approximation to the axial shear modulus even at higher volume fraction. Both the Halpin-Tsai and Eshelby theoretical value exhibits good agreement with experimental data proposed by other researchers. This supports the validity and reliability of micromechanical approach for the prediction of the effective moduli of bidimensional composites. This provides a critical check for the property of a micromechanical model. At the same time, Halpin-Tsai shear curve is not as low as the prediction of the equal stress model. Thus both this approach gives fairly good estimation for effective properties of composite materials.

4.2.11.2 Tridimensional Composite Material

The further analysis on Maxwell and Rule of Mixtures was unable due to insufficient model for three-dimensional model. All the models proposed by early researchers lie within Hashin-Shtrikman bound expect for experimental points measured by Richard (1975) (based on Figure 4.2). The points scatter outside Hashin-Shtrikman bounds and this indicates this model exceeds the limit of Maxwell and Hashin-Shtrikman limits.

It is seen that the method of cell is in a good agreement with the generalized self-consistent method. The agreement with the three phase model shown in the figure is significant since it supports the validity prediction of method of cell. At the same time, the effective moduli of three phase composite materials with randomly oriented disperse media predicted by method of cells, can be compared with the Mori-Tanaka method. It is noted that the Method of cell coincides with the corresponding lower bound of Mori-Tanaka method.

Figure 4.2 also shows that the lower bound of Mori-Tanaka model identical to Hashin-Shtrikman lower bound for spheres, elongated inclusions with the same shape and anisotropic. However, the expansions of this model differ from Haskin-Strikman bounds for the case of elongated particles with different shapes and orientations because of the simplifications regarding inclusion interaction in this model.

The higher scatter of experimental points is due to the particle size. The diameter of the fibers or particles used to develop composite materials is mostly not uniform. Inclusions do not interact enough to allow the establishment of thermally connected paths (percolation) for composite phase, even if the average distance separating two consecutive inclusions reduces as volume fraction increases. Examination of microstructure of a high composite content shows that each bead surrounded by matrix phase. in essence, a minimum thickness of this matrix phase limits significantly the possibility of physical contact. In fact, the process of fabrication ensures complete coating of beads during mixing because a good adherence of powder to the beads is obtained with organic additives.

Next is the shape of the particles or fibers impregnated in composite materials. Even though all the fibers and particles fabricated to be spherical or cylindrical in shape, defects in the manufacturing process make ruin the shape of the particles. The arrangement of the particles in the composite will not be in order when one or more of the fibers' shape has changed.

It is obvious that many other factors have an influence on the conductivity and modulus of these models: the location, orientation, size distribution, and whether the particles are in contact. There are other modifying factors for elasic moduli such as interfacial mismatch. The equivalent effect in the thermal problem would be the phonon mismatch between the phases. The formation of an oxide layer has also been found to influence the effective value of the property.

In short, fiber volume fractions are usually much lower due to processing limitations (e.g., the viscosity of the fiber/particles must be controlled for proper flow during molding) and the random orientation of the fibers and particles. Since fiber-packing geometry is never entirely repeatable from one piece to material to another, we should not expect our micromechanics predictions to be exact. Therefore, the experimental results always show a scatter and a deviation from identical models.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.0 CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The ground rules of the analogy between various mechanical and transport properties have been discussed in this report. It is shown that by virtue of the analogy, models developed for one set of properties can be used to obtain another set of properties. Properties of composites lie between the Maxwell and Hashin-Shtrikman's bounds. Both these lines are the limits that cannot be exceeded by the proposed model. Many models have been developed as linear combinations of these model pairs. It can be seen that there are several micromechanics models that provide identical or similar predictions for the effective elastic properties. In general, it may be stated that the Eshelby inclusion method and Halpin-Tsai equation give better approximation in wide range of volume fraction in most cases. The Reuss, Voigt, Mori-Tanaka bounds and generalized self-consistent models provide good results only in determining the rationale for limiting bounds.

Experimental results for the effective engineering properties of composites as a function of fibre volume fraction are quite scarce due to the difficulties in measuring these properties accurately. The experimental results are proposed by Durand and Ungar, Thornburgh and Pears and Tessier and Doyen can be seen in the result. When these experimental results compared with theoretical data presented by Eshelby and Halpin-Tsai the comparisons generally indicate that the experimental results are in line with stated the theoretical data. Thus it can be concluded that Eshelby and Halpin-Tsai models can give a better correlation between elastic moduli and thermal conductivity of composites.

However for three-dimensional composites, experimental data proposed by Richard shows a huge scatter to Method of cell, Mori-Tanaka bounds and generalized self-consistent scheme. Some of the points lie outside the limiting bounds. This is due to delamination of the composite materials. Chamis and Wong and Bollamplally's experimental data are in line with HS lower bound.

5.2 Recommendation

In most cases, the stiffness and conductivity of composite materials are mainly influenced by the shape, location, orientation, size and size distribution and whether the particles are in contact. In order to improve the properties of composite materials, it is useful to modify the fibre-packing geometries to triangular shape and shown in figure below.

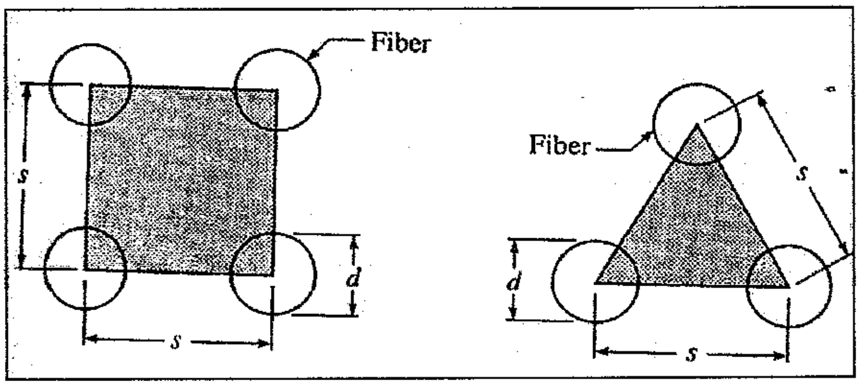


Figure 5.1: Representative area elements for idealized square and triangular fibre-packing geometries.

If the fibre spacing assumed to be s, and fibre diameter, d, do not change along the fibre length, and then the area fractions must be equal to the volume fractions. The fibre volume fraction for the square array is

$$V_f = \pi/4\,(d/s)^2$$

Thus the maximum theoretical fibre volume fraction occurs when s=d. In this case,

$$v_{fmax} = \pi/4 = 0.785$$

The triangular arrays shows that

$$V_f = \pi/2\sqrt{3}\ (d/s)^2$$

and when s=d, the maximum fibre volume fraction is

$$V_{fmax} = \pi/2\sqrt{3} = 0.907$$

The close packing of fibres required to produce these theoretical limits is generally not achievable in practice, however. In most continuous fibre composites the fibres are packed in a random fashion and the fibre volume fractions range from 0.5 to 0.8. Since fibre-packing geometry is never entirely repeatable from one piece of material to another, the micromechanical predictions are seen to give a window of uncertainty rather than exact values.

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APPENDIX

Gantt Chart

Table A.1 Milestone for the Final Year Project II

No.	Detail/ Week	1	2	3	4	5	6	7		8	9	10	11	12	13	14
1	Project Work Continue	■	■	■												
2	Submission of Progress Report 1				●											
3	Project Work Continue				■	■	■	■								
4	Submission of Progress Report 2									●						
5	Seminar (compulsory)									●						
5	Project work continue								■	■	■	■	■			
6	Poster Exhibition												●			
7	Submission of Dissertation (soft bound)													●		
8	Oral Presentation														●	
9	Submission of Project Dissertation (Hard															●

● Suggested milestone

■ Process

Table A.2: Milestone for the Final Year Project I

No.	Detail/ Week	1	2	3	4	5	6	7	8	9	10		11	12	13	14
1	Selection of Project Topic											Mid-semester break				
2	Preliminary Research Work															
3	Submission of Preliminary Report															
4	Project Work															
5	Submission of Progress Report															
6	Seminar (compulsory)															
7	Project work continues															
8	Submission of Interim Report Final Draft															
9	Oral Presentation															

Suggested milestone

Process

Figure A.1: Effective properties of two-dimensional composite materials

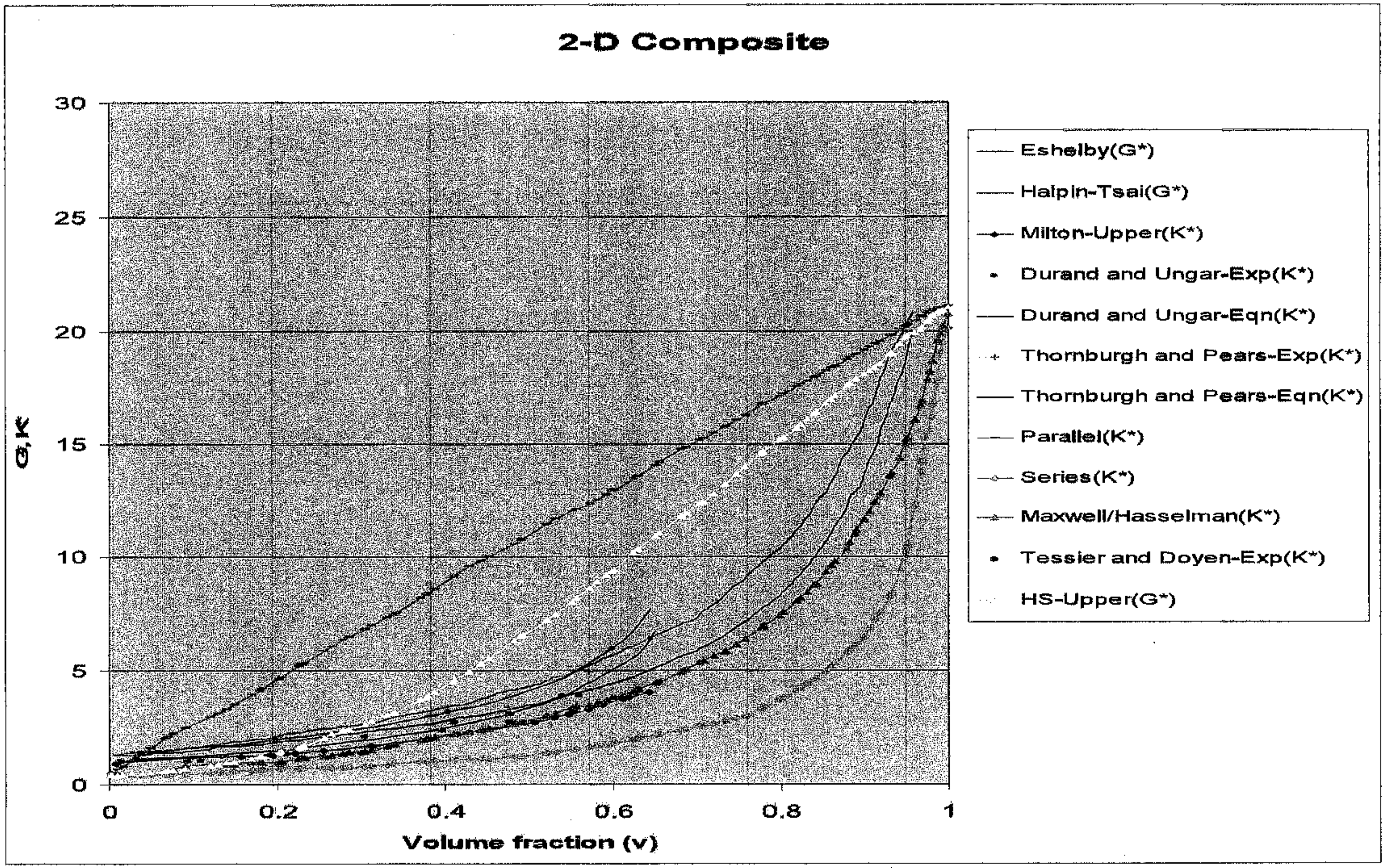

Figure A.2: Effective properties of three-dimensional composite materials