DEVELOPMENT OF DESIGN WORK SHEETS FOR ANALYSIS AND DESIGN OF FLEXIBLE RAFT FOUNDATIONS BASED ON BEAMS ON ELASTIC FOUNDATION

By

MOHD NOR AZZUAN BIN ZAWAWI

FINAL PROJECT REPORT

Submitted to the Civil Engineering Programme
in partial fulfillment of the requirements
for the Degree
Bachelor of Engineering (Hons)
(Civil Engineering)

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
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Approved:

Associate Professor Dr. Nasir Shafiq
Project Supervisor

UNIVERSITI TEKNOLOGI PETRONAS
TRONOH, PERAK

June 2007

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

MOHD NOR AZZUAN BIN ZAWAWI

ABSTRACT

Raft foundations are one of the most appropriate foundation systems in order to prevent the differential settlement particularly when column loads are large and variable. The behavior of raft foundation is either rigid and/or flexible depending on the characteristics of the soil at which the foundation rests. For analysis and design of rigid raft, traditional methods of reinforced concrete design are available in a number of literatures; however, the analysis of flexible raft is too tedious and quite cumbersome, which involve very long formula and complex mathematical functions. This study is mainly focused on the development of excel work sheets for analysis of flexible rafts based on beams on elastic foundations that may ease out the tedious calculations and solutions of complex mathematical functions.

ACKNOWLEDGEMENT

I hereby would like to grab this opportunity to express my deepest gratitude and gratefulness firstly to God, The Almighty for giving me a chance to acquire new knowledge and with His will, this project was successfully completed. My special thanks and appreciation to my project supervisor, Associate Professor Dr. Nasir Shafiq for all the guidance, advice and commitment in assisting me through out this project. A warmness thanks to all panels that involved in my project's evaluation and improvement. Same goes to my fellow friends for all the help, support and opinion given in completing this project. Last but not least I would like to thank my beloved parents, brothers and sister for giving me moral supports and all parties that involved in completing this project.

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CHAPTER 1

INTRODUCTION

1.1. Background of study

Every single one of engineered construction resting on the earth must be carried by some kind of interfacing element called a *foundation*. The foundation (also called as substructure) is the part of an engineered system that transmits to, and into, the underlying soil or rock the loads supported by the foundation and it's self-weight. The term superstructure is commonly used to describe the engineered part of the system bringing load to the foundation. This term has particular significance for building and bridge; however, foundations also may carry only machinery, support industrial equipment (pipes, towers, and tanks), act as sign bases, and the like. For these reasons it is better to describe a foundation as the part of the engineered system that interfaces the load-carrying components to the ground.

Even the ancient builders knew that the most carefully designed structures can fail if they are not supported by suitable foundations. The Tower of Pisa in Italy (perhaps the world's most successful foundation "failure") reminds people of this truth. Although builders have acknowledged the importance of firm foundations for countless generations, and the history of foundation engineering as people know today did not begin to develop until the late nineteenth century. Early foundation designs were based solely on precedent, intuition, and common sense. In the course of trial-and-error, builders developed rules for sizing and constructing foundations. For example, load-bearing masonry walls built on compact gravel in New York City during the nineteenth century were supported on spread footings that had a width 1.5 times that of the wall. Those built on sand or stiff clay were three times the width of the wall (Powell, 1884).

As structures prolonged to become larger and heavier, engineers continued to gain knowledge more about foundation design and construction. Instead of simply developing new empirical rules, they began to investigate the behavior of foundations and develop more rational methods of design, hence establishing the discipline of foundation engineering. The development of geotechnical engineering, which began in earnest during the 1920s, offered a better theoretical base for foundation engineering. It also provided advanced methods of exploring and testing soil and rock. These developments carried on throughout the twentieth century. Many new methods of foundation construction also have been developed, making it possible to build foundations at sites where construction had previously been impossible or impractical.

Today, people's knowledge of foundation design and construction is much better than it was one hundred years ago. It is now possible to build reliable, cost-effective, high capacity foundations for all types of modern structures. Foundation can be divided into two broad categories; *shallow foundations* and *deep foundations.* Shallow foundations transmit the structural loads to the near-surface soils, consists of spread footings (square, circular, rectangular), combined footings, continuous footings or mats/raft foundation. Deep foundations transmit some or all the loads to deeper soils, e.g piles, anchors, auger-cast piles, drilled shafts, drilled caissons, pressure-injected footing or manderel-driven thin shells filled with concrete.

In this project, the research focused on the *raft or mat foundation* which is in the shallow foundations classification. Shallow foundation usually is the more economical option. As a general rule, consider deep foundations only when shallow foundations do not give satisfactory design. A raft foundation is a large concrete slab used to interface one column, or more than one column in several lines, with the base soil. It may en compass the entire foundation area or only a portion. The foundation's behavior is governed by two systems based on the soil properties which are *rigid* or *flexible*. Both behaviors were investigated, but this project mainly emphasized on the flexible raft foundation.

1.2 Problem Statement

1.2.1 Problem Identification

The soil properties will influence the type of foundation to be used. In most difficult soil, construction practitioners usually adopt pile foundation for safety and comfort of design even though it is reliable to used shallow foundation such raft foundation. They always refer to piling system since lack of knowledge of raft foundation. Piles are very expensive compare to shallow foundation like raft. Therefore the studies and research of this project focusing on the raft foundation behavior to introduce its usage and applications, enhance and publicize the knowledge for the student himself and also other people. By having knowledge on this matter, the foundation can be used and hence, the cost will be reduced.

Advantages of using shallow foundation (Mat foundation);

1- Cost (affordable compared to deep foundation)
2- Construction Procedure (simple and efficient time management)
3- Materials (mostly concrete)
4- Labor (does not need expertise)

Focusing on raft foundation, there are two behaviors regarding this type of foundation which are rigid and flexible. Both flexible and rigid is the stress distribution system beneath the foundation that needs to be determined before designing any foundation. The analysis and design of rigid raft are available in a number of literatures compare to the flexible raft which is less in the market and involve tedious calculations and solutions of complex mathematical functions.

In raft foundation design, the rigid approach is simpler than flexible. It assumes the mat is much more rigid than the underlying soils, which means any distortions in the mat are too small to significantly impact the distribution of bearing pressure. Unfortunately, flexible approach is more difficult to implement because they require consideration of *soil-structure interaction* and because the bearing pressure is not as simple. For that reason, this project mainly focused on the **flexible raft foundation.**

1.2.2 Significant of the Project

One of approach in designing the raft foundation which is being focused in this project is a "flexible system". This approach will be much influenced by the soil properties and governed by the soil parameters which are the modulus of subgrade reaction, k_s. k_s is the conceptual relationship between soil pressure and deflection of foundation members. This k_s is very important in the determination of moment, shear and deflection of the raft foundation which later will be used reinforced concrete design. These three elements are the most significant matters to take care of before proceed for any reinforced concrete design. In order to obtain the k_s, moment, shear and deflection, there will be lots of other parameters to be verified first. All of these involved tedious calculations and solutions of complex and long mathematical functions.

This project focused on the development of design work sheets to ease the determination of all parameters discussed above in flexible raft foundation analysis and design. The flexible raft foundation analysis and design are based on the beams on elastic foundation. When flexural rigidity of the footing is taken into account, a solution can be used that is based on some form of a beam on an elastic foundation. With aids of these design work sheets, they will ease out the tedious calculations and solutions of complex mathematical functions and compute the result of calculations with only few input data instead of long manual calculations, thus saving time. They also will produce the precise value of the analysis and avoid mistakes in calculations. The reinforced concrete design can be proceed faster whenever the moment, shear and deflection are obtained. This is the main target for this project.

Each of the tedious and long formulae involved in the analysis were entered into the excel spreadsheet to create a program or software. When foundation design is made, the user can enter the related design and soil properties data into these work sheets. With just a few input data, the user can get lots of important parameters needed for the RC design like which have been discussed before. For the analysis matter, many of the data and parameters already prepared and calculated in the work sheets. Only few input data needed from the designer. There are also several conditions need to be fulfilled in order to proceed with the calculations, but all of these already encountered by the design work sheets with the practiced of if-else function. Without thinking too much, the user can enter the data according to the simple instructions given.

1.3 Objective and Scope of Study

1.3.1 The Relevancy of the Project

The objectives of this project are:-

1) To investigate behavior (rigid and flexible) of raft foundation based on soil properties
   * Both behaviors were investigated to know each characteristic, but detail investigations more focusing on flexible behavior and analysis
2) To develop **Excel Work Sheets** for analysis of flexible rafts based on beams on elastic foundations to:-
   - Ease out tedious calculations and solutions of complex mathematical functions
   - Compute the result of calculations with only few input data instead of long manual calculations, thus saving time
   - Produce the precise value of the analysis and avoid mistakes in calculations

1.3.2 Feasibility of the Project within the Scope and Time Frame

The feasibility study of the project within the scope is to get the best way how to manage the task given and gather the required information about the raft foundation (rigid/flexible) for the behavior investigation. This is the challenges in order to attain all the needed information from any source or references as well as the expertise in the tools or programs involved.

The project scopes mainly focused on the development of excel work sheets for analysis of flexible rafts based on beams on elastic foundations. Along with this, the investigation of both behaviors of raft foundation (rigid and flexible) based on soil properties was carried out but precisely focused more on the flexible raft foundation behavior. This is because of the research focusing on the development of flexible raft foundation design worksheets for analysis. The research also emphasized on the concept in determining the modulus of subgrade reaction and moment, shear and deflection of the foundation. These three elements are the important matters for the reinforced concrete design. In order to obtain the modulus of subgrade reaction and moment, shear and deflection, there are several other parameters need to be determined first. Therefore this matter will be included in the project scope. The soil properties/data and design parameters were assumed in the analysis. The analysis of these design work sheets is based on a single concentrated load.

Last semester, for the Final Year Project I, the scope focused more on the research for the project. The research and studies were conducted on basic of foundation, shallow foundation (raft foundation), rigid and flexible behavior, modulus of subgrade reaction, moment, shear and deflection of foundation and other related parameters that involved in the analysis. In addition to that, excel program usage enhancement and practices have been done through out the semester. These are to develop skills in using the program and also adept to the application of if-else and other complicated functions. The relevant researches were included in the literature review of what theory that have been used and also the discussion on the findings chapter. The researches from last semester still continue until this semester because of the author still upgrading his studies on this project time after time. Additional findings are always adapted. There were few drafts on the development of the design work sheets done during last semester. The first design worksheets developed were only consider one type of soil data at any particular area.

This semester for the Final Year Project II, the scope focused on the development of the design work sheets. The first drafts of the design worksheets from the last semester were improved throughout this semester. The target are to consider more than one type of soil to be analyzed because in a logical aspects, there usually exists more than one type of soil in any particular area being considered for the design. These already been implemented into the design worksheets. The input data slots were re-arranged to ease the user in entering the input organizely. Throughout this semester, the improvement also focusing more on to reduce as many as possible the input data so that more time saving program can be obtained.

The references use for this project includes foundation analysis & design handbooks, journals of previous research that have been done, websites and etc. The skills enhancement in the excel program usage was achieved through self-learning and also with reference to the expertise in this field especially the author's supervisor, A.P. Dr. Nasir Shafiq. Discussion from time to time with the supervisor detected any weaknesses and lead to the improvement of the project. All these contributed to the project completion with success achievement. The project started within the time frame using all the knowledge and information that have been acquired. The design work sheets were completed at the end of three quarter period of this semester.

CHAPTER 2

LITERATURE REVIEW

2.1. Mat foundation (Shallow foundation)

A mat foundation is one of the shallow foundations. It is a large concrete slab used to interface one column, or more than one column in several lines, with the base soil. It may encompass the entire foundation area or only a portion. A mat maybe used to support on-grade storage tanks or several pieces of industrial equipments. Mats are commonly used beneath silo clusters, chimneys and various tower structures. It becomes a matter definition as to when the dimensions of a spread footing make the transition into being called a mat. Figure 2.11.1a illustrates several mat configurations as might be used for buildings; (Those shown encompass the entire building plan, but this is not a requirement)

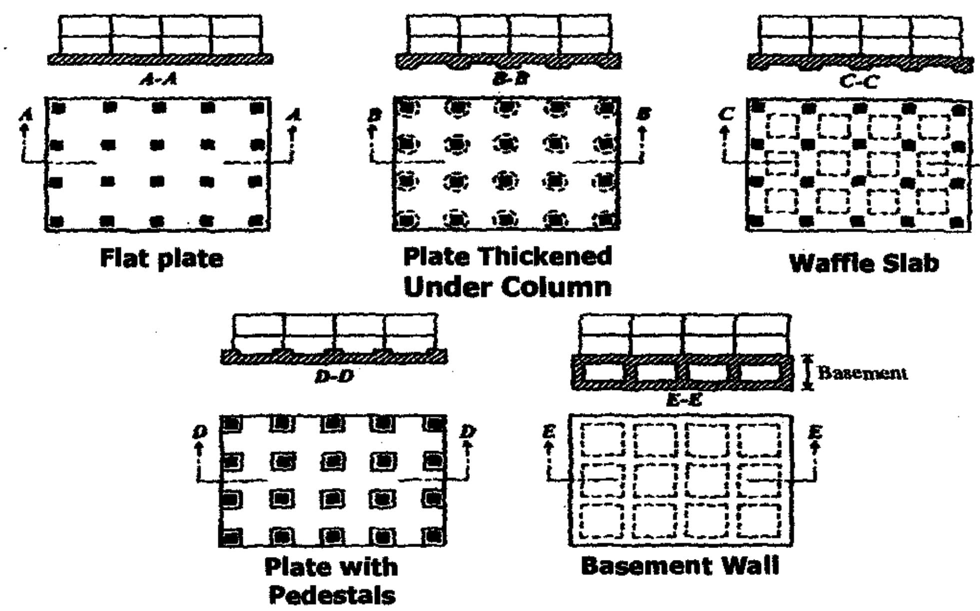

Figure 2.1.1a - Common types of mat foundations

A mat foundation may be used where the base soil has a low bearing capacity and/or the column loads are so large that more than 50 percent of the area is covered by conventional spread footings. It is common to use mat foundations for deep basements both to spread the column loads to a more uniform pressure distribution and to provide the floor slab for the basement. A particular advantage for basement at or below the GWT is to provide a water barrier. Depending on local costs, and nothing that a mat foundation requires both positive and negative reinforcing steel, one may find it more economical to use spread footings. The mat contact stresses will penetrate the ground to a greater depth or have greater relative intensity at a shallower depth. Both factors tend to increase settlements unless there is stress compensation from excavated soil so that the net increase in pressure is controlled.

2.2. Designing Mat foundation

1- Determine the bearing capacity of the foundation
2- Determine the settlement of the foundation
3- Determine the differential settlement
4- Determine the stress distribution beneath the foundation

From Step (4)

a- The mat foundation is assumed to be **a Rigid foundation**

b- The mat foundation is assumed to be **a Flexible Foundation**; here use Beam on Elastic

5- Design the structural components of the mat foundation using the stress distribution obtained from (4).

For step 4, here the rigid and flexible approaches become the matter of concern when determining the stress distribution or stress system beneath the mat foundation to be designed. The rigid and flexible behavior of the mats will be governed by the modulus of subgrade reaction, K_s. The higher value of modulus of subgrade reaction, the more possibility towards flexible system approach. These parameters will be discussed later in the discussion on findings chapter.

2.3. Rigid and flexible Raft Foundation

Before going further, it's better to know about the fundamental concepts and differences of raft foundation behavior. In can be divided into two categories: Rigid and flexible.

2.3.1 Rigid Raft Foundation

The simplest approach to structural design of mats is the rigid method. This method assumes the mat is more rigid than the underlying soils, which means any distortions in the mats are too small to significantly impact the distribution of bearing pressure. Therefore, the magnitude and distribution of bearing pressure depends only on the applied loads and the weight of the mat, and is either uniform across the bottom of the mat (if the normal load acts through the centroid and no moment load is present) or varies linearly across the mat (if eccentric or moment loads are present).

This simple distribution makes it easy to compute the flexural stresses and deflections (deferential settlements) in the mat. For analysis purposes, the mat becomes an inverted and simply loaded two-way slab, which means the shears, moments and deflections may be easily computed using the principles of structural mechanics. The engineer can then select the appropriate mat thickness and reinforcement. Refer to figure 2.3.1a for typical rigid behavior of raft foundation.

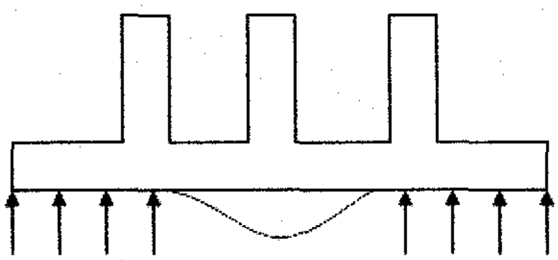

Figure 2.3.1a – Rigid Raft Foundation

2.3.2 Flexible Raft Foundation

The inaccuracies of the rigid method by using analyses that considers deformations in the mat and their influence on the bearing pressure distribution. These are called the flexible method and produce more accurate values of mat deformations and stresses. Unfortunately, flexible analyses also are more difficult to implement because they require consideration of soil-structure interaction and because the bearing pressure distribution is not as simple as the rigid raft foundation analyses. Refer to figure 2.3.2a for the typical flexible raft foundation.

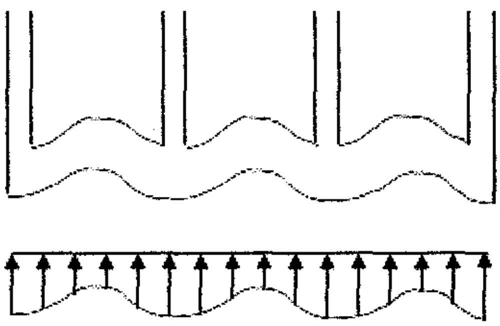

Figure 2.3.2a – Flexible Raft Foundation

2.4. Supporting Information (Relevant previous researches journal)

There're also few journal of previous researches that have been carried out for analysis. Among them are *Rational Analysis of Raft Foundation* by Hain, S.J.; Lee, I.K., Journal of the Soil Mechanics and Foundations Division. Vol. 100, no. 7, pp. 843-860. July 1974. An analysis of a raft foundation is developed which takes into account the interaction of the three elements of the system, i.e., the structure, the raft, and the supporting soil. The stiffness of the structure is incorporated into the stiffness matrix of the raft which is treated as a thin elastic plate supported on a soil mass modeled either by the Wrinkler (spring) or by the linear elastic model. A three-bay by three-bay multistory space frame supported on a square raft is analyzed and the effects of structural stiffness, relative raft-soil stiffness, and soil stiffness are considered in detail for both the Winkler and linear elastic soil models. It is shown that use of the linear elastic model leads to the conclusion that the settlement profile of a flexible raft is concave, thus there is a transfer of load to the outer columns compared with a rigid raft. In contrast, the Winkler analysis predicts a convex settlement profile. Special methods of analysis are developed for rigid structures and for highly flexible rafts.

The second one, *The behavior of an impermeable flexible raft on a deep layer of consolidating soil* by Booker, J R; Small, J C International Journal for Numerical and Analytical Methods in Geomechanics. Vol. 10, no. 3, pp. 311-328. 1986. In their attempt, analytic solutions to the problem of the time-settlement behavior of raft foundations have been limited in the past to flexible or rigid loadings, and have treated the foundation as being completely permeable. In this paper, solutions are presented for smooth circular rafts of any flexibility causing consolidation of a deep homogeneous clay layer, where the raft may be considered permeable or impermeable. Results for the time-dependent behavior of contact stresses, pore pressures, raft displacements and moment in the raft are presented. But in this project, the student is focusing on the rectangular raft foundation.

The third one, *The Effect of Spread Footing Flexibility on Structural Response* by Sami W. Tabsh, Associate Professor, Civil Engineering Dept., American Univ. of Sharjah, and 2) Raouf Al Shawa, Project Engineer, ABB Transmission and Distribution, Al Ghaith Tower, Abu Dhabi, United Arab Emirates; Pract. Periodical on Struct. Des. and Constr., Volume 10, Issue 2, pp. 109-114 (May 2005). In their attempt, Spread footings are normally used under individual columns of buildings and bridge piers. They are economical to use and are applicable for any soil conditions where the bearing capacity for the applied loads is adequate. Structural design codes and specifications allow a linear soil pressure distribution to be assumed for the design of spread footings. This approach is valid for infinitely rigid footings. Past experience has shown that the assumption of a linear pressure distribution is satisfactory for most footings; however, there are some cases in which a shallow foundation must be analyzed as a flexible structure, particularly if the footing is excessively long/wide and thin. In this study, a relative stiffness factor, $K^{r^{f}}$, is developed that can determine whether a footing can be considered rigid for the purposes of structural analysis and design. This factor is a modified version of an expression first proposed by Meyerhof in 1953, but takes into account the size of the column supported on the footing. The study is based on modeling square and rectangular spread footings subjected to concentric and eccentric loadings by finite elements. The footings are modeled using thick rectangular plate elements and the soil with elastic springs. The results of the study showed that a footing with K^{r} factor greater than 1.0 indicates that it can be analyzed as a rigid footing with reasonable accuracy. This includes determination of soil pressures, vertical footing displacements, shear forces, and bending moments. The study also showed that maximum shear forces within a spread footing are less sensitive to changes in the stiffness of a footing than bending moments.

These journals were found through the journal discovery search engine in the webpage. From the webpage, the instruction urge the users to seek for the journal in their respective library in university or others for the full contents of the particular journals. The full research contents were not in the webpage. However, the journals could not be found in Universiti Teknologi Petronas Library during the findings. The full texts of the journals were not available in UTP library including the OPAC. Only a brief description available for these journals except for the third one in the journal discovery search engine. These journals help a little bit on this project since several facts were taken and compared with the current research.

CHAPTER 3

METHODOLOGY AND PROJECT WORK

3.1. Project identification

Basically, the flow of the project is to study and investigate the behavior of mat/raft foundation with both approaches rigid and flexible. From an engineering point of view, the key of this foundation behavior investigation is to know which design approach is suitable for particular soil properties and any design parameters. The project continued with the analysis of flexible raft foundation which is being focused in this project. The analysis of the flexible raft foundation's behavior helps in developing the design work sheets which act as a program that compute several parameters that important in the reinforced concrete design afterwards. Next, by using assumption's soil parameters and design data, one set of result obtained. Manual calculations also were done to compare with the one computed by the program for the comparison in terms of preciseness. Time after time, the enhancement of the design worksheets developed were carried out to improve any weaknesses detected and acquire the most practical program.

The methodology entailed in this research can be summarized into several steps; project planning, literature studies, computer program's development, manual calculations and finally came out with the report and presentation.

3.1.1 Project Planning

- All defined scope and work is outlined along specific time frame to keep track with on-going and planned activities. It is vital to keep updates with the outlined activities and its time frame to minimize behind schedule activities.
- Project planning is very important because sometime whatever that already planned, not executed as planned because of certain

uncertainties. But it doesn't mean that the actual tasks need to be lagged behind the schedule because of this. There's lot of ways to overcome this. Probably, the tasks were conducted according to the current condition, availability of material, limitation and others. Therefore, prevention it is always better than cure. The project needs to be planned properly.

- For this Final Year Project, the planning part was done accurately to make sure the progress of the project run smoothly. Although the author took quite plenty time for the planning part initially, the result yield a positive progress of the project. The project run smoothly according to the schedule and completed succesfully.

3.1.2 Literature Study

- Most of the information was gathered from technical handbooks, journal, internet/websites and library and through discussion with supervisor and colleagues. This includes the basics of foundation and any other relevant and related theories regarding the flexible and rigid approaches of raft foundation. The studies were done precisely towards flexible raft foundation analysis in order to develop the it's design work sheets.
- Available research (journals as supporting information) by individual on related topics serves as a useful reference for basic of understanding.
- In addition to that, practices and tutorials of the related program applications and skills enhancement were carried out with reference to the proper channels and expertise.
- The information which have been gathered and studied includes all the related theories and concepts were applied to develop the design work sheets.

3.1.3 Computer Program Development and Software Usage

- The design work sheets of flexible raft foundation based on beams on elastic foundation were developed using the excel spreadsheet program for the project.
- The purpose is to determine all related parameters for the foundation analysis in the fastest way, time saving and precise answers. There are not much software available for the flexible foundation analysis compared to the rigid foundation analysis. Therefore creating own spreadsheet with reference to the establish theory, the concept is as same as the software and the analysis could be carried out. Hence, by inserting any particular soil parameters and raft foundation design data, the result could be determined easily and fast.
- For the Final Year Project I, tutorial on basis of spreadsheet program development was learnt through the handbooks and excel website. They include all the application of inserting formulae, functions, numbers, and many other useful features. Many spreadsheets were developed during the basis learning as the practices approach. The student also being asked by the supervisor to master the excel program especially in the function usage and inserting formulae. These are very important since sometimes, there is a case with several assumptions and different condition to be met. Here, the IF-ELSE function is very important since there are many conditions of mathematical functions. This also ensures the ease of the user in terms of inserting the inputs data without confuse and no need to think much. Means that, everything already prepared and the user only inserting data according to the stated instruction.
- For last semester (FYP I) also, few spreadsheets already developed for flexible analysis. However, the work sheets still need an improvement, expansion and continuation time after time.

- Therefore this semester for Final Year Project II (FYP II), the design work sheets were improved to obtain the more practical program.
- Improvements were made according to the latest fingdings, wekneasses detected and based on the matters which have been raised up by the panels in the FYP I presentation last semester.

3.1.4 Manual Calculations

- The analysis of this project also being carried out in term of manual calculations.
- The purpose is to compare with the result computed by the computer program in term of result's precision, checking mistakes and ensured that the formulae applied were correct. Perhaps to prove that the manual calculations were more complicated, long and consume more times.

3.1.5 Report

- Reports were produced as a requirement of the study and as a platform for discussions, findings and future references.
- For the FYP I in last semester, the author submitted weekly reports every week to the supervisor for progress supervision. With that also, the preliminary, progress I and interim report were partially submitted throughout the last semester as a requirement for final year project.
- While for FYP II, same like the previous semester, weekly reports were submitted every week for progress supervision.. Progress report I and progress report II were partially submitted to report the latest findings and discussions. Full dissertation report are completed and submitted at the end of this semester for evaluation purpose and to conclude the project.

3.1.6 Presentation

- Presentation also made to the supervisor for each research and findings during the meeting to ensure the rightness and smooth progress flow
- For the FYP I in last semester, the author attended a presentation regarding the research that have been done throughout the semester. The presentation was judge by the author's supervisor and UTP internal panels.
- This semester in FYP II, the author will attend the final presentation to present his completed project as well as the product. Evaluation will be done during the presentation as the final year project's requirement.

3.2. Tools / Equipment Used

3.2.1 Tools

1) *PC Pentium 4* (as a medium for computer program and software usage)

3.2.2 Software

1) *Excel Spreadsheet Program* [as the interface program (spreadsheet) for the analysis especially flexible foundation analysis].

CHAPTER 4

RESULT AND DISCUSSION

This chapter discussed on the findings during the research from last semester until up to this date. The discussion includes the analysis of flexible raft foundation based on beams on elastic foundation, theories involve in determining the modulus of subgrade reaction, k_s, moment, shear and deflection of the foundation and any other theories related in flexible analysis. The flow of the design work sheets development and samples also discussed in this chapter.

4.1. Findings and Discussion

4.1.1 Modulus of Subgrade Reaction, k_s

The modulus of subgrade reaction is a conceptual relationship between soil pressure and deflection that is widely used in the structural analysis of foundation members. It is used for continuous footings, mats and various types of pilings. It also governs the behavior of the foundation which is flexible or rigid. Higher value of k_s means more possibility towards flexible system. It is very important and also used in determining the moment, shear and deflection of the foundation. After this three elements obtained, the reinforced concrete design can be carried out. The ratio was defined on figure 4.1.1a and the basic equation when using the plate-load test data is;

$$k_s = \frac{q}{\delta}$$

with terms identified on figure 4.1.1a, 4.1.1b and 4.1.1c.

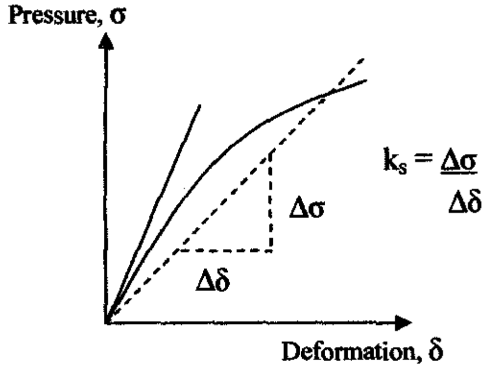

Figure 4.1.1a – Modulus of Subgrade Reaction

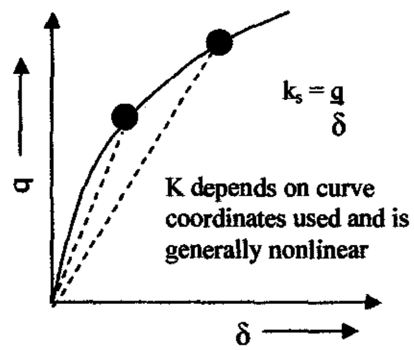

Figure 4.1.1b

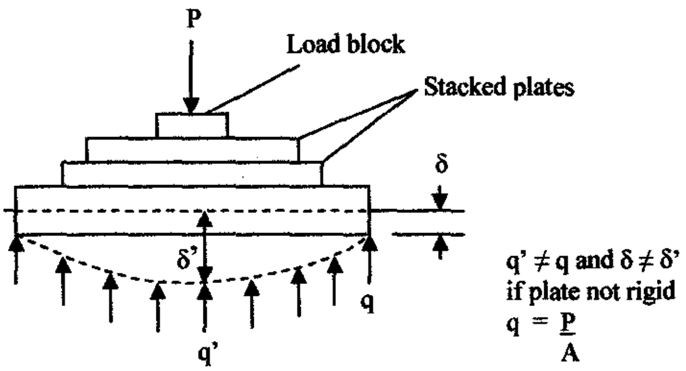

Figure 4.1.1c

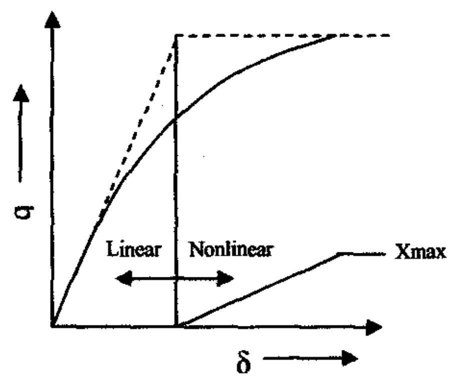

Figure 4.1.1d

Plots of q versus δ from load tests give curves of the type qualitatively shown in figure 4.1.1b. If this type of curve is used to obtain k_s in the preceding equation, it is evident that the value depends on whether it is tangent or secant modulus and on the location of coordinates of q and δ.

It is difficult to make a plate load test except for very small plates because of reaction load required. Even with small plates of, say, 450-, 600-, and 700-mm diameter it is difficult to obtain d since the plate tends to be less than rigid so that a constant deflection across the larger ones tends to increase the rigidity, but in any case the plot is of load divided by plate contact area (nominal P/A) and the average measured deflection. Figure 4.1.1d is a representation of k_s where k_s taken as a constant up to a deflection X_{max}. Beyond X_{max} the soil pressure is a constant value defined by

$$q_{con} = k_s(X_{max})$$

One of the early contributions was that of Terzaghi (1955), who proposed that k_s for full-sized footings could be obtained from plate-load tests using the following equations;

$$k_s = k_1 \frac{B_1}{B}$$, for footings on clay

$$k_s = k_1 \left(\frac{B + B_1}{2B}\right)^2$$, for footings on sand

$$k_s = k_1 \frac{m + 0.5}{1.5m}$$, for rectangular footing on stiff clay or medium dense sand (dimensions, B x L with m = L/B)

where; k_s = desired value of modulus of subgrade reaction for the full-size fdn.
k_1 = value obtained from a plate-load test using a 0.3 x 0.3 or other size

However, these three equations are presented primarily for historical purposes and not recommended for general use.

In 1961, Vesic proposed that the modulus of subgrade reaction could be computed using the stress-strain modulus Es as;

$$k'_s = 0.65 \sqrt[12]{\frac{E_s B^4}{E_f I_f}} \frac{E_s}{1-\mu^2}$$

where; E_s, E_f = modulus of soil and footing
B, I_f = footing width and its moment of inertia based on cross section (not plan)

k_s can be obtained from k'_s as;

$$k_s = \frac{k'_s}{B}$$

Since the twelfth root of any value x 0.65 will be close to 1, for all practical purposes the Vesic equation reduces to;

$$k_s = \frac{E_s}{B(1-\mu^2)}$$

μ = poisson ratio of soil

From part settlement ΔH, allowable pressure q_a and stress-strain modulus E_s, few equation which are;

$$\Delta H = q_0 B' \frac{1-\mu^2}{E_s} m I_S I_F$$

$$E's = \frac{1-\mu^2}{E_s}$$

$$\frac{\Delta H_2}{\Delta H_1} = \frac{q_{02}\, B'_2\, mI_{S2}\, I_{F2}\, E'_{S2}}{q_{01}\, B'_1\, mI_{S1}\, I_{F1}\, E'_{S1}} \qquad q_{02} = q_{01} \frac{B'_1\, mI_{S1}\, I_{F1}\, E'_{S1}}{B'_2\, mI_{S2}\, I_{F2}\, E'_{S2}}$$

Can be rearranged and become;

$$\Delta H = \Delta q\, B\, E'_S\, mI_S\, I_F$$

Since k_s is defined as **Δq / ΔH,** obtain;

$$k_s = \frac{1}{B\, E'_S\, mI_S\, I_F} \qquad \text{units kN/m}^3\ \text{(SI)} \qquad \text{eqn. 4.1.1a}$$

where,

k_s = modulus of subgrade reaction
B = Base dimension of foundation
E'_s = Modulus elasticity of soil with effect of poisson's ratio
m = Numbers of corners contributing to settlement ΔH
I_S, I_F = Influence factors which depend on L'/B', thickness of stratum H, Poisson's ratio μ, and base embedment depth D

This is the most general and recommended equation to be used to determine the modulus of subgrade reaction and also be compared by $k_{s,SF}$ based on safety factor (developed after recognizing that bending moments and the computed soil pressure are not very sensitive to what is used for k_s because the structural member stiffness is usually 10 or more times as great as the soil stiffness);

$$k_s = 40(SF)q_a \qquad \text{units kN/m}^3\ \text{(SI)} \qquad \text{eqn. 4.1.1b}$$

where,

q_a = allowable bearing pressure
SF = safety factor

4.1.2 Other related parameters

Actually, there are still many other parameters that need to be determined first to obtain the modulus of subgrade reaction and the one to be used in the calculations.

4.1.2.1 k_s for centre and corner

From the eqn. 4.1.2a, there were few complicated parameters to be determined first before the modulus of subgrade reaction can be obtained. There are the E'_s, m, I_S and I_F. This k_s need to be determine for the centre and corner. The description and formulae are such below:-

from eqn. 4.1.1a, $k_S = \dfrac{1}{B E'_S m I_S I_F}$

a) Steinbrenner influence factor, $I_S = I_1 + \dfrac{[1-2\mu]}{1-\mu} I_2$

$$I_1 = \frac{1}{\pi}\left[M \ln \frac{(1+\sqrt{M^2+1})(\sqrt{M^2+N^2})}{M(1+\sqrt{M^2+N^2+1})} + \ln \frac{(M+\sqrt{M^2+1})(\sqrt{1+N^2})}{M+(\sqrt{M^2+N^2+1})} \right]$$

$$I_2 = \frac{N}{2\pi}\left(\frac{M}{N(\sqrt{M^2+N^2+1})}\right)$$

where, $M = \dfrac{L'}{B'}$ $N = \dfrac{H}{B'}$

For center;	For corner;
$B' = B/2$	$B' = B$
$L' = L/2$	$L' = L$

This Steinbrenner influence factor, I_S also can be obtain through the table 5.2 (Joseph E. Bowles), but the process is even more complicated than using the formulae above.

b) Fox influence factor, I_F = no formulae available, need to use the figure 4.1.2.1a

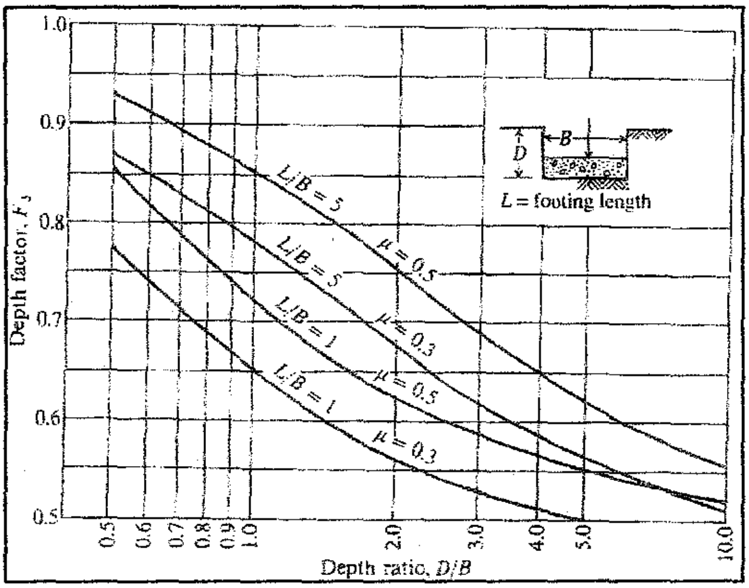

Figure 4.1.2.1a – Fox Chart to determine I_F
(figure 5-7 in Joseph E. Bowles)

c) Numbers of corners contributing to settlement ΔH, m; Centre, **m = 4**
Side, **m = 2**
Corner, **m = 1**

d) Modulus Elasticity of soil with effect of poisson's ratio, $\mathbf{E'_S = \frac{1-\mu^2}{E_s}}$

* After all parameters above obtained, and then the k_S for corner and centre can be determined.

4.1.2.2 k_s weighting

This k_S can be find after the k_S for corner and centre already obtained;

$$k_{s\ weighting} = \frac{[4\ k_S(\text{centre}) + k_S(\text{corner})]}{5}$$

4.1.2.3 $k_{s\,SF}$ based on safety factor

from eqn. 4.1.1b, $k_s = 40(SF)q_a$

Allowable bearing pressure of soil, q_a is one of the inputs data that need to be inserted in order to obtain the modulus of subgrade reaction, k_s. It is use to find the ks based on safety factor which is used to compare with the k_s weighting. The comparison is to find the average ks. Based on Terzhagi;

$$q_{ult} = c\, N_c\, s_c + q_{bar}\, N_q + s_\gamma\, \gamma\, B\, N_\gamma$$
$$q_a = q_{ult}/S.F$$

Based on Vessic;

$$q_{ult} = c\, N_c\, s_c\, \underline{dc} + q_{bar}\, N_q\, s_q\, d_q + 0.5\, \gamma'\, B\, N_\gamma\, s_\gamma\, d_\gamma$$
$$q_a = q_{ult}/S.F$$

Both are the established equation but the Vessic equation is more updated and latest compared to Terzhagi. Several coefficient and parameters need to be computed to obtain the allowable bearing pressure, q_a. It is not a simple parameter that people can assume or something. It needs to be determined following the true condition of the particular soil to be chosen. Therefore it helps much in saving time and avoid calculation mistake to develop a design worksheet for q_a determination. The Vessic method were used in the calculation of bearing capacity to determine the modulus of subgrade reaction in the design worksheets.

4.1.2.4 k_s Average

This is the final modulus of subgrade reaction which is the average of k_S weighting and k_S based on safety factor. k_s average is the value that will later used to determine the moment, shear and deflection of the foundation. It can be concluded such below:-

$$k_{s\ average} = \frac{k_{s\ weighting} + k_{s\ SF}}{2}$$

Table below are taken from the reference book and consist of established value of k_S according to soil types. It can be used for guidance and comparison when using approximate equations

Table 4.1.2.4a : Range of modulus of subgrade reaction, k_S

Soil	k_s, kN/m^3
loose sand	4800-16000
Medium dense sand	9600-80000
Dense sand	64000-128000
Clayey medium dense sand	32000-80000
Silty medium dense sand	24000-48000
Clay soil:	
$q_a \leq 200kPa$	12000-24000
$200 < q_a \leq 800$ kPa	24000-48000
$q_a > 800$ kPa	>48000

4.1.2.5 k_s Critical or Maximum

k_S critical is determined from the maximum value obtained among the k_s average of few soils sample which are considered in the analysis. The maximum value will be considered as the critical value of the modulus of subgrade reaction and will be used in the next calculations to determine the moment, shear and deflection. This feature of selecting the $k_{s\ Maximum}$ already included in the design worksheets as part of improvement.

4.1.3 Moment, Shear and Deflection

These three parameters, moment, M; shear, Q; & deflection, y are the most important parameters that need to be determined as the foundation design using flexible approach. When flexural rigidity of the footing is taken into account, a solution can be used that is based on some form of a beam on an elastic foundation. This may be the classical Winkler solution of about 1867, in which the foundation is considered as a bed springs ("Winkler foundation").

The classical solutions, being of closed form are not so general in application as the finite element method. The basic equation is

$$EI\frac{d^4y}{dx^4} = q = -k'_s y$$

where $k'_s = k_s B$ (This showed the modulus of subgrade reaction is very important to be determined), figure 4.1.3a. In solving the equations a variable is introduced:

$$\lambda = \sqrt[4]{\frac{k'_s}{4EI}} \quad \text{or} \quad \lambda L = \sqrt[4]{\frac{k'_s L^4}{4EI}}$$

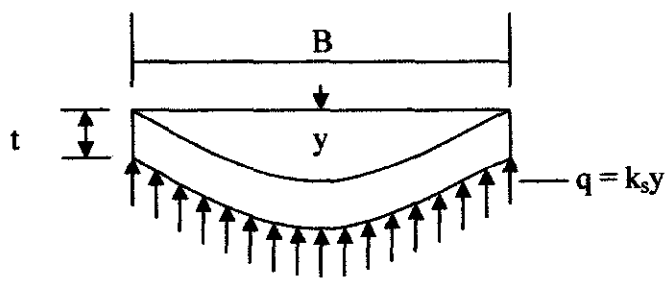

Figure 4.1.3a - $k'_S = k_S B$ (includes effect of B)

There are 2 condition in determining the moment, shear and deflection; a) Finite length beam on elastic foundation (Hetenyi 1946) and b) Infinite length beam on an elastic foundation with mid or center loading (Winkler 1867)

a) Finite length beam on elastic foundation

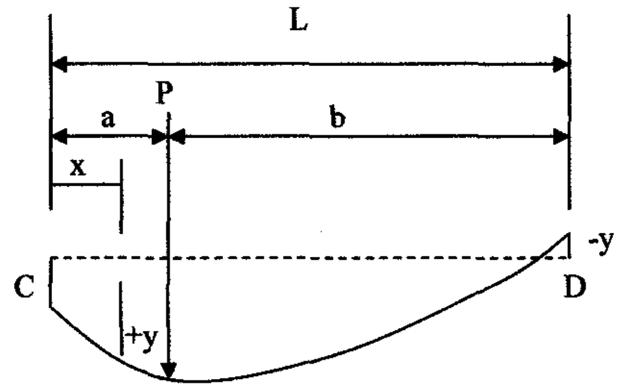

Figure 4.1.3b – Finite length beam on elastic foundation

The equations are;

Deflection, y =

$$\frac{P\lambda}{k'_s(\sinh^2\lambda L - \sin^2\lambda L)}\{2\cosh\lambda x\cos\lambda x(\sinh\lambda L\cos\lambda a\cosh\lambda b - \sin\lambda L\cosh\lambda a\cos\lambda b) + (\cosh\lambda x\sin\lambda x + \sinh\lambda x\cos\lambda x)[\sinh\lambda L(\sin\lambda a\cosh\lambda b - \cos\lambda a\sinh\lambda b) + \sin\lambda L(\sinh\lambda a\cos\lambda b - \cosh\lambda a\sin\lambda b)]\}$$

Moment, M =

$$\frac{P}{2\lambda(\sinh^2\lambda L - \sin^2\lambda L)}\{2\sinh\lambda x\sin\lambda x(\sinh\lambda L\cos\lambda L\cosh\lambda b - \sin\lambda L\cosh\lambda a\cos\lambda b) + (\cosh\lambda x\sin\lambda x - \sinh\lambda x\cos\lambda x)[\sinh\lambda L(\sin\lambda a\cosh\lambda b - \cos\lambda a\sinh\lambda b) + \sin\lambda L(\sinh\lambda a\cos\lambda b - \cosh\lambda a\sin\lambda b)]\}$$

Shear, Q =

$$\frac{P}{\sinh^2\lambda L - \sin^2\lambda L}\{(\cosh\lambda x\sin\lambda x + \sinh\lambda x\cos\lambda x) + (\sinh\lambda L\cos\lambda a\cosh\lambda b - \sin\lambda L\cosh\lambda a\cos\lambda b) + \sinh\lambda x\sin\lambda x[\sinh\lambda L(\sin\lambda a\cosh\lambda b - \cos\lambda a\sinh\lambda b) + \sin\lambda L(\sinh\lambda a\cos\lambda b - \cosh\lambda a\sin\lambda b)]\}$$

The equation for the slope, θ of the beam at any point is not presented since it is of little value in the design of a footing. The value x to use in the equations is from the end of the beam to the point for which the deflection, moment or shear is desired. If x less than distance, a of figure 4.1.3b, use the equation given and measure x from C. If x is larger than a, replace a with b in the equations and measure x from D.

b) Infinite length beam on an elastic foundation with mid or centre loading.

The equations involved are;

Concentrated Load at end;

a) deflection, $y = \frac{2V_1\lambda}{k's} D_{\lambda x}$

b) Slope, $\theta = \frac{-2V_1\lambda^2}{k's} A_{\lambda x}$

c) Moment, $M = \frac{-V_1}{\lambda} B_{\lambda x}$

d) Shear, $Q = -V_1 C_{\lambda x}$

Moment at end;

a) deflection, $y = \frac{-2M_1\lambda}{k's} C_{\lambda x}$

b) Slope, $\theta = \frac{4M_1\lambda^3}{k's} D_{\lambda x}$

c) Moment, $M = M_1 A_{\lambda x}$

d) Shear, $Q = -2M_1\lambda B_{\lambda x}$

Concentrated Load at Centre;

a) deflection, $y = \frac{P\lambda}{2k's} A_{\lambda x}$

b) Slope, $\theta = \frac{-P\lambda^2}{k's} B_{\lambda x}$

c) Moment, $M = \frac{P}{4} C_{\lambda x}$

d) Shear, $Q = \frac{-P}{2} D_{\lambda x}$

Moment at Centre;

a) deflection, $y = \frac{M_0\lambda}{k's} B_{\lambda x}$

b) Slope, $\theta = \frac{M_0\lambda^3}{k's} C_{\lambda x}$

c) Moment, $M = \frac{M_0}{2} D_{\lambda x}$

d) Shear, $Q = -\frac{M_0\lambda}{2} A_{\lambda x}$

Where the coefficients;

$A_{\lambda x} = e^{-\lambda x}(\cos \lambda x + \sin \lambda x)$
$B_{\lambda x} = e^{-\lambda x}\sin \lambda x$
$C_{\lambda x} = e^{-\lambda x}(\cos \lambda x - \sin \lambda x)$
$D_{\lambda x} = e^{-\lambda x}\cos \lambda x$

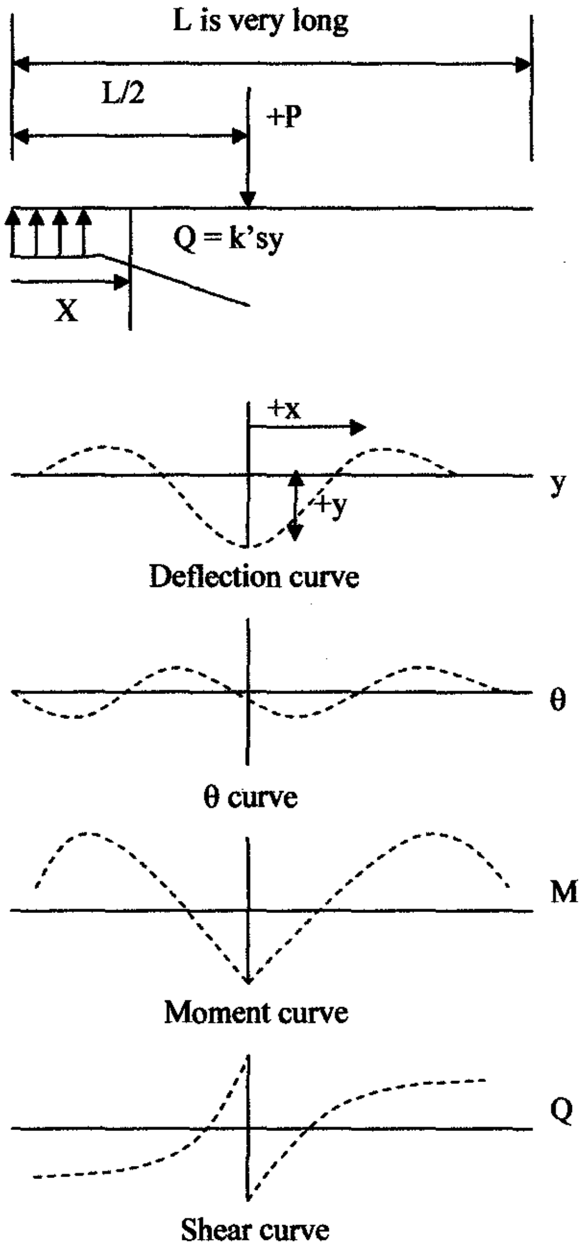

Figure 4.1.3c – Infinite length beam on an elastic foundation with mid or center loading

4.1.4 Excel Spreadsheet Program Practices and Development

Throughout the last semester, the author was asked by the project's supervisor to learn the basis of the Excel program, practice on how to insert variety of complicated formulae into the program to develop the solution in the easiest way, developing a simple spreadsheet program and others.

The practices were carried out with reference to excel books, tutorial in Excel websites and also from colleagues. Basic of Excel include; function; [Sum, Average, Count, If, Max, Hyperlink, Sin and many others], Manage excel into appropriate and readable database, inserting formulae and etc.

All the related formulae were extracted and transferred to the Excel program to develop the design work sheets. The main aims are to obtained the deflection, moment and shear for the flexible foundation design of finite and infinite beam. These three parameters are the main important thing in designing a flexible foundation. In obtaining these three parameters, there are several other parameters that need to be obtained first which have been discussed before. The sequence in obtaining the parameters until finally getting the moment, shear and deflection of the design flexible foundation is related to each other. The author's aim is to prepare a spreadsheet that can generate result of the analysis with only a few inputs of data. Therefore, a combination of few sets of spreadsheets into a single spreadsheet could decrease the inputs of data and hence, save much more time of the users.

With all the researches that have been done during last semester in FYP I, the author had developed few spreadsheets for the first draft on how to obtain the k_s, moment, shear and the deflection and all parameters involved in the analysis by just inserting few parameters of the foundation and soil properties. The design worksheets only considered one type of soil data to be analyzed. Any references that needed in the calculations and could not be calculated by the program were prepared in the design worksheets. Enclosed below are the first draft of the design worksheets development:-

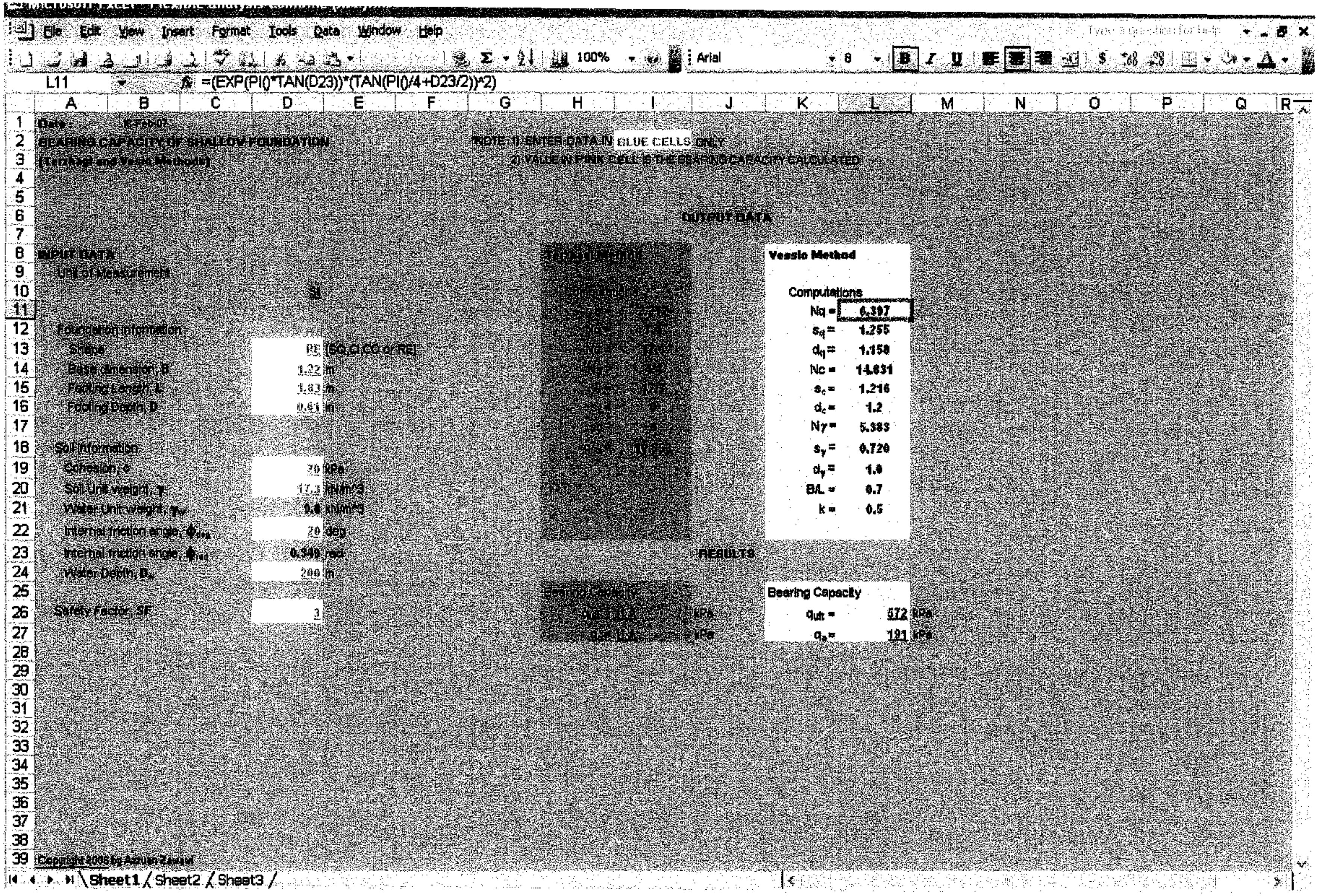

Figure 4.1.4a – Design Work Sheet of allowable bearing pressure, q_a (1st Draft)

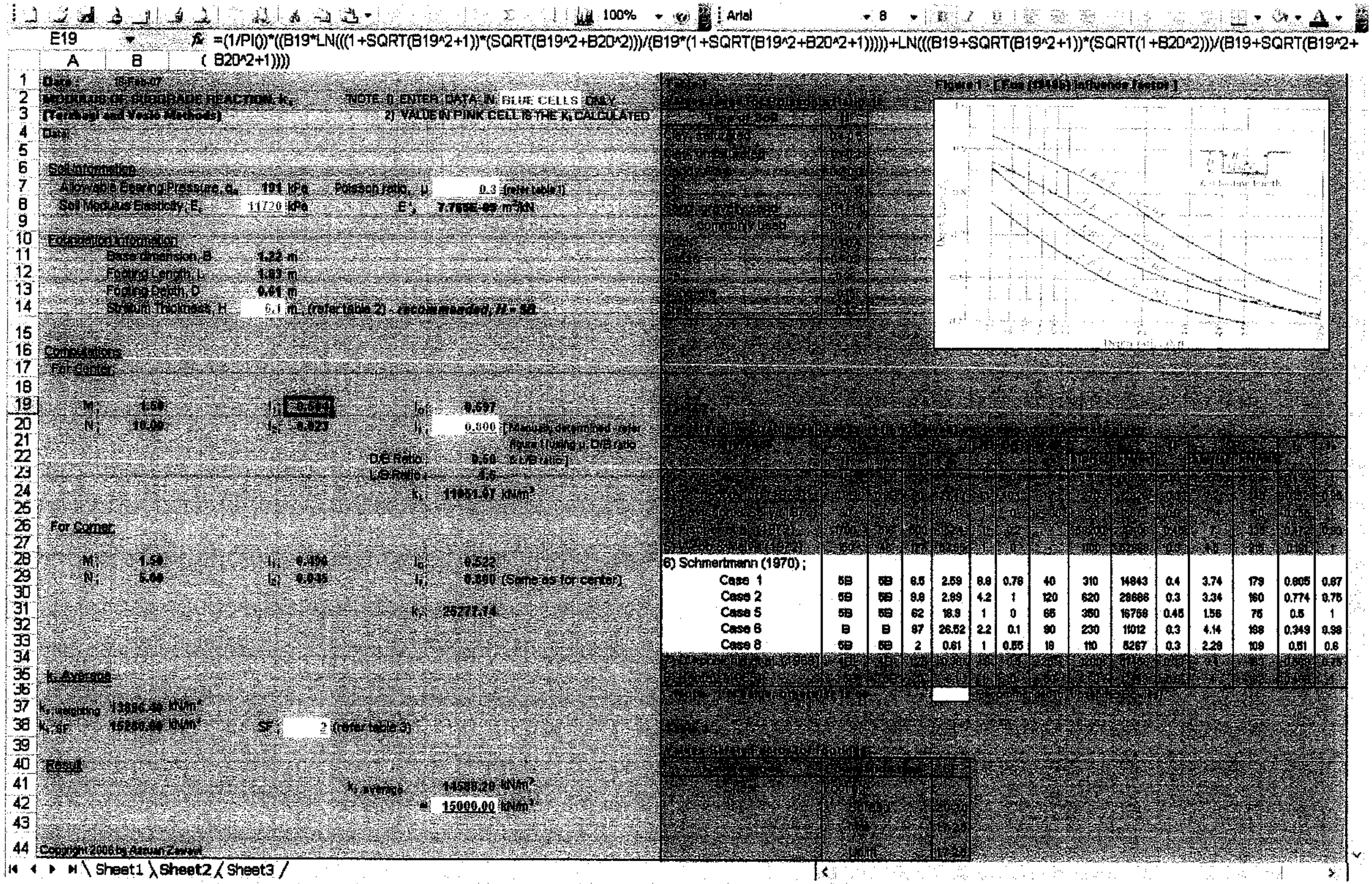

Figure 4.1.4b – Design Work Sheet of Modulus of Subgrade Reaction, k_s (1st Draft)

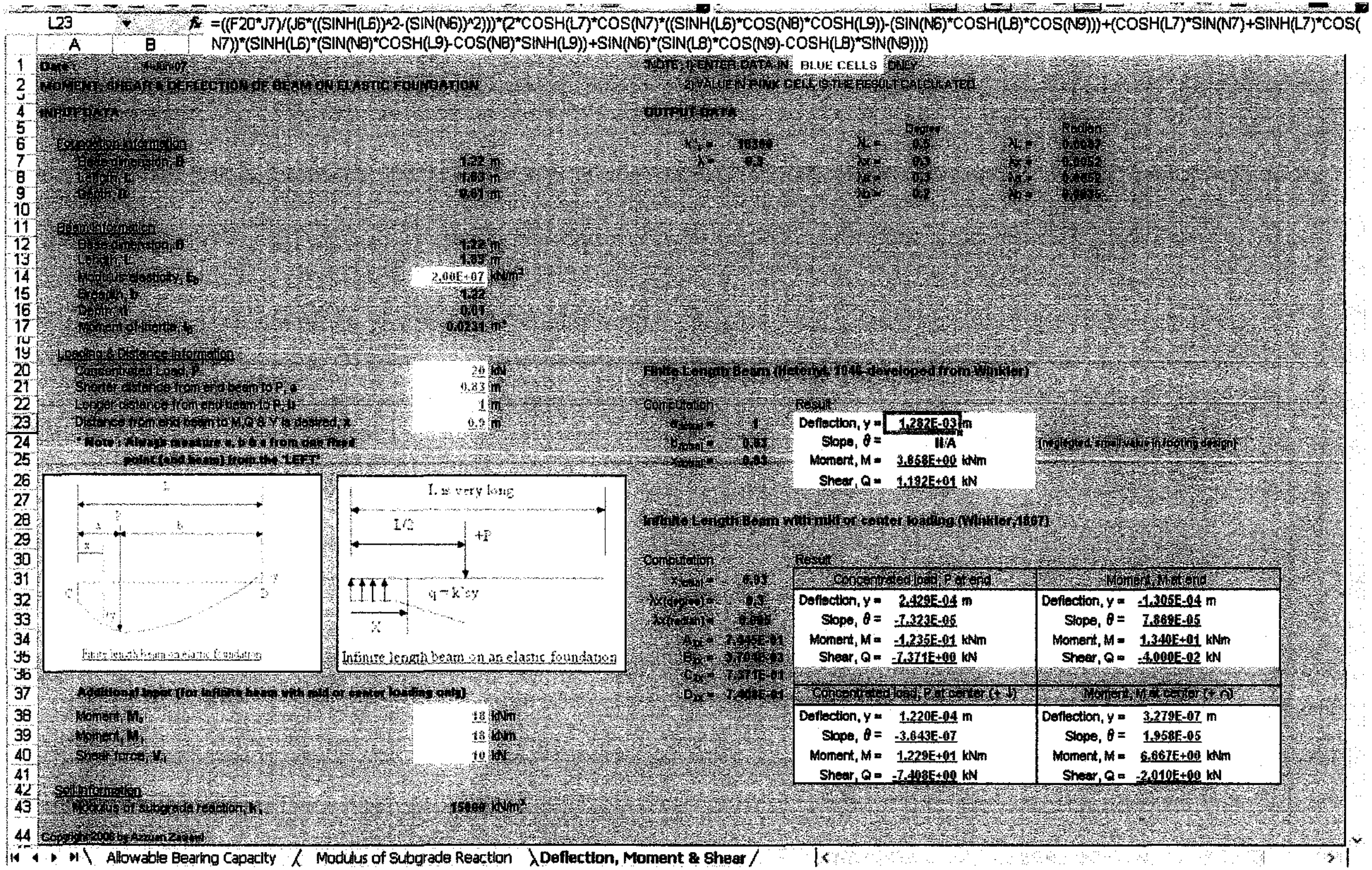

Figure 4.1.4c – Design Work Sheet of Moment, Shear and Deflection (1st Draft)

The spreadsheets contain several steps and formulae in determining the important parameters for flexible foundation analysis with some reference table and figure for the input value in blue. The cell in pink showed the calculated value for each worksheet. Each of worksheet was in separate for the initial development. Throughout the enhancement of these spreadsheets, they have been combined together so that the input value could be decreased and hence, result with time saving and more practical. Furthermore, there will be no repetition for the input value. As described before, the development of the first draft design worksheets only considered one type of soil data to be analyzed. This will limit the usage of this design worksheets since in terms of logical aspect, there are always more than one type of soils exist in any particular area.

Therefore, in FYP II for this semester, the design worksheets have been enhanced. Consideration made for few types of soil. The design worksheets allowed maximum of six types of soils at any particular area to be considered in the analysis. The determination of moment, shear and deflection would take the maximum value of modulus of subgrade reaction obtained among the soils. The maximum value will be considered as the critical one. This means that the RC design later will be safe since the critical value of the present soils was taken into consideration.

Same like the previous one, the separate design workheets of bearing capacity, modulus of subgrade reaction and moment, shear and deflection determination were combined into one. The combination made to reduce the user's data input by avoiding repetition of the input value. In addition, the input slots have been rearranged to ease the user in inserting the input data organizely. The input slots were rearranged to the top side of the design worksheets. All the computations done by the design worksheets also were rearranged in organize way to make sure the result data that computed easy to be read and checked. The final design work sheets attached as below:-

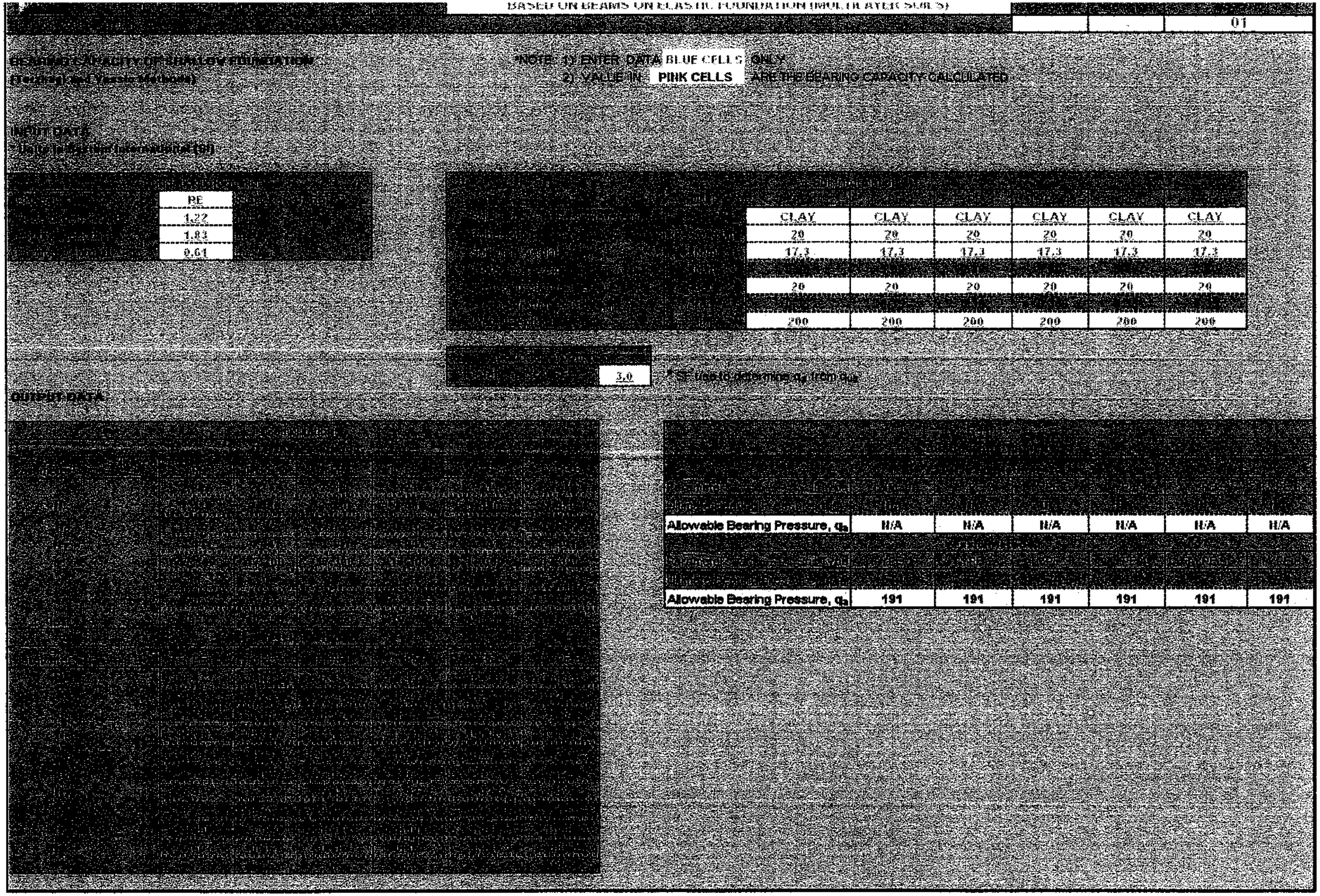

Figure 4.1.4d – Design Work Sheet of allowable bearing pressure, q_a (Final Draft)

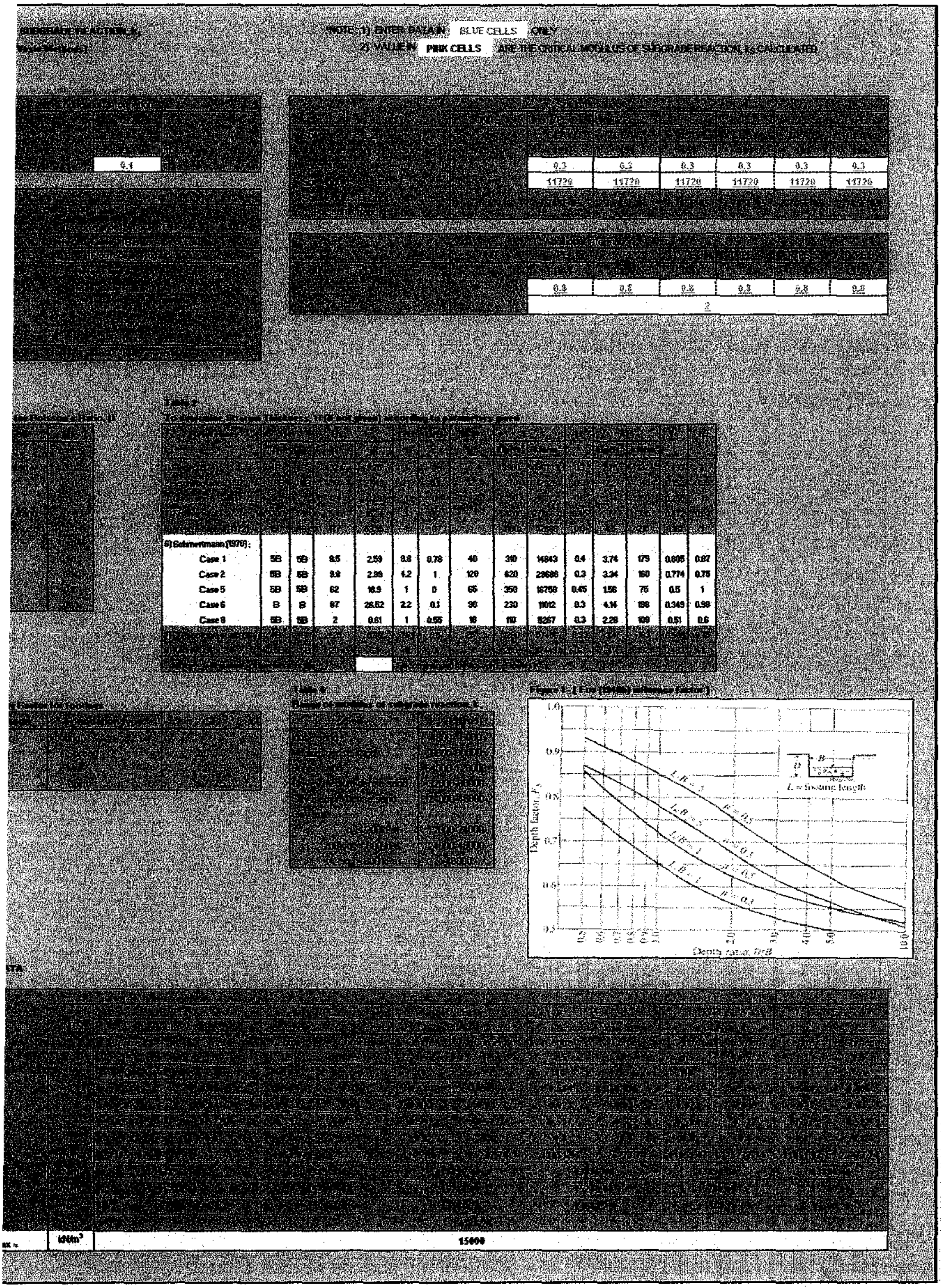

Figure 4.1.4e – Design Work Sheet of Modulus of Subgrade Reaction, k_s (Final Draft)

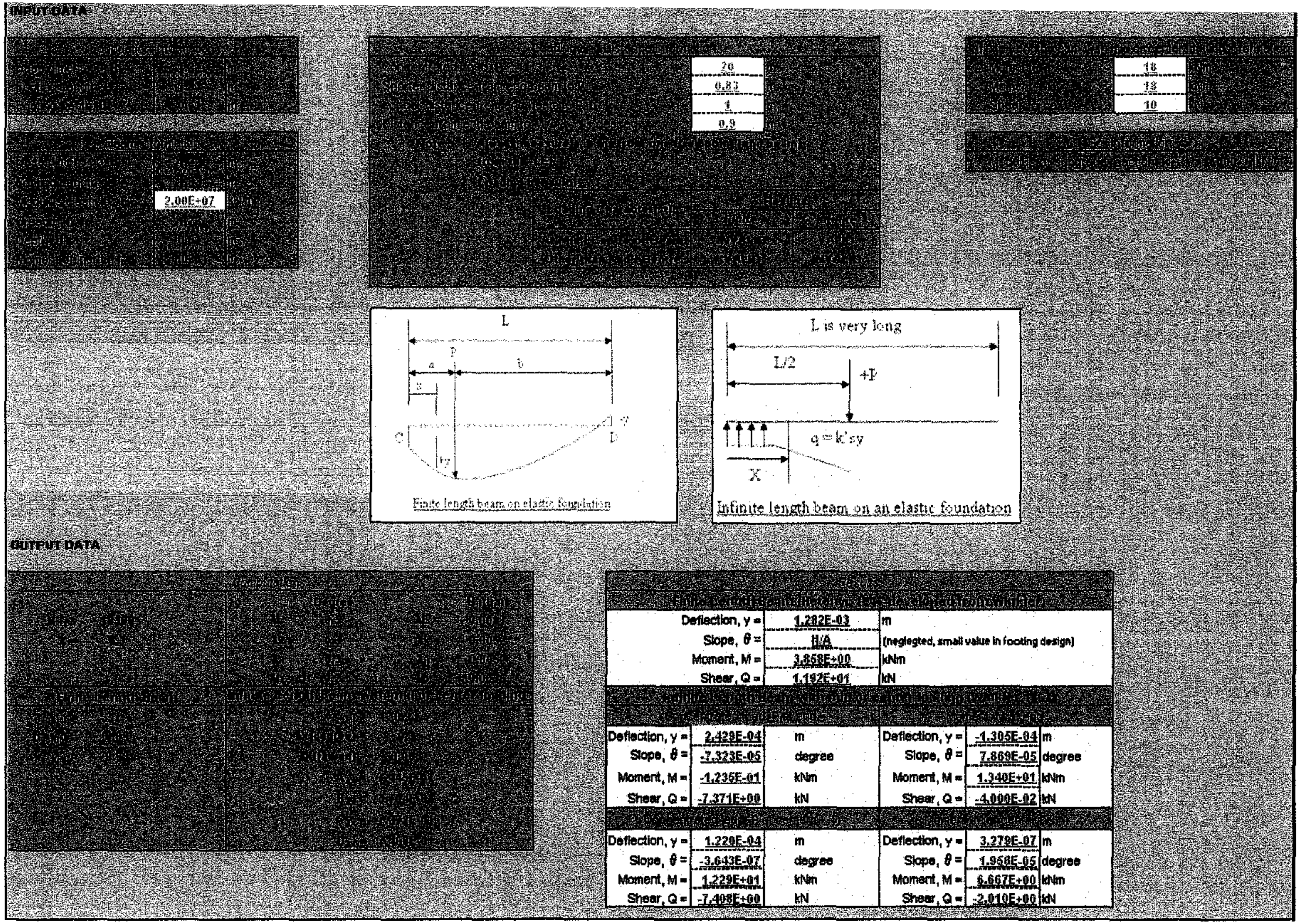

Figure 4.1.4f – Design Work Sheet of Moment, Shear and Deflection (Final Draft)

4.1.5. Manual Calculation

Manual calculations of the analysis have been done to compare with the value obtain by the design work sheets. The calculations were done currently based on one type of soil sample. The results obtained were approximately the same. The difference occurred cause by the human error when conducting manual calculation. These also because of when manual calculations done, the value of the result were rounded to few decimal point. Therefore the precision not as accurate as the one calculated by the program. The precisions were consistent and accurate. Table below showed the design worksheet versus manual calculation:-

Table 4.1.5a – Comparison of Design Worksheet and Manual Calculation Results

PARAMETER	SYM	UNIT	SPREADSHEET	MANUAL
Bearing Capacity	q_{ult}	kN/m^2	**574**	573
	q_a	kN/m^2	191	191
Modulus Subgrade Reaction	k_s	kN/m^3	15000	15000
Finite Length Beam	y	m	1.28E-03	1.736E-03
	θ	-	N/A	N/A
	M	kNm	3.86	4.08
	Q	kN	11.92	10.22
Infinite Length Beam				
Concentrated Load at End	y	m	2.429E-04	2.823E-04
	θ	-	-7.323E-05	-8.436E-05
	M	kNm	-1.235E-01	-1.140E-01
	Q	kN	-7.371E+00	-7.460E+00
Concentrated Load at Centre	y	m	1.220E-04	1.318E-04
	θ	-	-3.643E-07	-4.219E-07
	M	kNm	1.229E+01	1.166E+01
	Q	kN	-7.408E+00	-7.500E+00
Moment at End	y	m	-1.305E-04	-1.503E-04
	θ	-	7.869E-05	8.869E-05
	M	kNm	1.340E+01	1.352E+01
	Q	kN	-4.000E-02	-4.200E-02
Moment at Centre	y	m	3.279E-07	3.797E-07
	θ	-	1.958E-05	2.404E-05
	M	kNm	6.667E+00	6.753E+00
	Q	kN	-2.010E+00	-2.172E+00

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1. Conclusion

As a conclusion, the selection of raft foundation, one type of shallow foundation in design should be publicized to reduce cost in construction instead of using piling system that is far more expensive. Therefore the studies and research of this project focusing on the raft foundation behavior to introduce its usage and applications, enhance and publicize the knowledge of it. As long as the design is acceptable for a particular soil condition, there's no doubt of using this approach. The objectives of the project were successfully achieved. The design work sheets for analysis of flexible rafts based on beams on elastic foundations were fully developed. The weaknesses which detected in the first design worksheets were encountered and improved in the final design worksheets. The author feels that the design worksheets that already developed are practical and can be used to ease out the user's or designer's works in designing a shallow foundation that is the flexible raft foundation.

5.2. Recommendation

The design worksheets according to the author's Final Year Project scopes were completed accordingly. However, the input data can be reduced more in the future to enhance the efficiency of this design worksheets. For example, the modulus elasticity of soil, E_s. Soil data from field test from geotechnical investigation needed in the determination of E_s such as standard penetration test. From these data, there is still several steps to determine the E_s and involving lots of formula and complex mathematical function. If the worksheets can include the determination of this E_s or make it simpler to be determined, it is better and can save more times for the users. Same goes to other input data, if possible to be reduced, the design worksheets will be more efficient and better.

Afterall, part of the project's objectives was to reduce the input data as much as possible and hence, saving time.

This design worksheets were developed based on the theory and concept. However, it still need to be applied to the real design of foundation to test on it's effectiveness. With the actual soil parameters at the considered design site, the result of the design worksheets computations will be used for the RC design of the foundation. At the same time, the common way to determine the same parameters are carried out to compare with the design worksheets. From this mehod, the efficiency of the design worksheets can be determined.

This design worksheets were developed a little bit with reference to the reinforced concrete council design work sheets. Based on the author's research, the the design worksheets for flexible raft foundation still not exist. Perhaps, through all recommendations which are discussed before will bring the author's design worksheets to complete the lacking.

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> http://spreadsheets.about.com/

APPENDIX

MANUAL CALCULATION

Assumption; 1) Use one sample of data for manual calculation
2) Unit of measurement = S.I
3) Reference for tables and figures; "Foundation Analysis & Design", by Joseph E. Bowles, P.E, S.E, 5th Edition, McGraw-Hill International Edition

1. Bearing Capacity of Shallow Foundation (Terzaghi and Vessic Method)

Input Data;

Foundation Information;

Shape	= Rectangular (RE)
Base, B	= 1.22 m
Footing Length, L	= 1.83 m
Footing Depth, D	= 0.61m

Additional Information;

Safety Factor, SF = 3

Soil Information;

Cohesion, c	= 20 kN/m^2
Soil Unit Weight, γ	= 17.3 kN/m^3
Water Unit Weight, γ_w	= 9.8 kN/m^3
Internal Friction Angle, ϕ	= 20 ^{0}
Water Depth, D_w	= 200 m

Terzhaghi Method (Reference, table 4-1, 4-2)

Equation;

$$q_{ult} = c\,N_c\,s_c + q_{bar}\,N_q + s_\gamma\,\gamma\,B\,N_\gamma$$
$$q_a = q_{ult}/S.F$$

Nq;

$$Nq = \frac{a^2}{A\cos^2(45+\phi/2)} = \frac{2.34^2}{2.34\cos^2(45+20/2)} = \underline{7.11}$$

$$a = e^{\pi(0.75-\phi/360)(\tan\phi)} = e^{\pi(0.75-20/360)(\tan 20)} = 2.34$$

Nc;

$$Nc = (Nq-1)\cot\phi = (7.11-1)\frac{1}{\tan 20} = \underline{16.8}$$

* $\phi = 0$, Nc = 5.7
ϕ = Other than 0, Nc = (Nq-1)cot ϕ

N_γ;

$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$
$$= \frac{\tan 20}{2}\left(\frac{25}{\cos^2 20} - 1\right)$$
$$= \underline{4.97}$$

@

$$N_\gamma = \frac{2(Nq + 1)\tan\phi}{1 + 0.4\sin 4\phi}$$
$$= \frac{2(7.11 + 1)\tan 20}{1 + 0.4\sin 4(20)}$$
$$= \underline{4.24}$$

* $\phi = 20^0$, $K_{p\gamma} = 25$

γ_{actual};

$$\gamma_{actual} = \gamma$$
$$= \underline{17.3}\ kN/m^3$$

* $Dw <= D$ then, $\gamma_{actual} = \gamma - \gamma_w$
$D + B <= Dw$ then, $\gamma_{actual} = \gamma$
Other than above; then,
$$\gamma_{actual} = (\gamma - \gamma_w)\left[1 - \frac{(Dw - D)}{B}\right]$$

s_c;

$$s_c = \underline{0}$$

* Shape; SQ → $s_c = 1.3$
CO → $s_c = 1$
CI → $s_c = 1.3$
RE → $s_c = 0$

s_γ;

$$s_\gamma = \underline{0}$$

* Shape; SQ → $s\gamma = 0.4$
CO → $s\gamma = 0.5$
CI → $s\gamma = 0.3$
RE → $s\gamma = 0$

q_{bar};

$$q_{bar} = D \cdot \gamma_{actual}$$
$$= 0.61(17.3)$$
$$= \underline{10.55}$$

* $Dw > D$ then, $q_{bar} = D \cdot \gamma_{actual}$
Other than above; then,
$$q_{bar} = D \cdot \gamma_{actual} - \gamma_w(D - Dw)$$

q_{ult};

$$q_{ult} = \cancel{cNcs_c}^{\,0} + q_{bar}Nq + \cancel{s_\gamma \gamma BNq}^{\,0}$$
$$= q_{bar}Nq$$
$$= 10.55(7.11)$$
$$= \underline{75.01\ kN/m^2}$$

q_a;

q_a = q_{ult} / SF
= 75.01/3
= **25 kN/m^2**

Note : Since according to the theory, if the shape is rectangular, bearing capacity for Terzhaghi Method will not be available,

q_{ult} = N/A
q_a = N/A

Vessic Method

$$qult = cNcscdc + \bar{q}NqSqdq + 0.5\gamma act BN_\gamma S_\gamma d_\gamma$$

$$qa = \frac{qult}{S.F}$$

Nq, (same as Mayerhof(1963))

$$Nq = e^{\pi \tan\phi} \tan^2\left(45 + \frac{\phi}{2}\right)$$
$$= e^{\pi \tan 20} \tan^2\left(45 + \frac{20}{2}\right)$$
$$= 6.4$$

$$Sq = 1 + \frac{B}{L}(\tan\phi)$$
$$= 1 + \frac{1.22}{1.83}(\tan 20)$$
$$= 1.24$$

$$dq = 1 + 2k\tan\phi(1 - \sin\phi)^2$$
$$= 1 + 2(0.5)\tan 20(1 - \sin 20)^2$$
$$= 1.16$$

$* k, \frac{D}{B} \le 1$ then $k = \frac{D}{B}$

$\frac{D}{B} > 1$ then $k = \tan^{-1}\left(\frac{D}{B}\right)$

$$\frac{D}{B} = \frac{0.61}{1.22} = 0.5 \le 1$$
$$k = 0.5$$

$$Nc = (Nq - 1)\cot\phi$$
$$= (6.4 - 1)\frac{1}{\tan 20}$$
$$= 14.84$$

$* \phi = 0°$ then $Nc = 5.14$

$\phi > 0°$ then $Nc = (Nq - 1)\cot\phi$

$$Sc = 1 + k\frac{Nq}{Nc}$$
$$= 1 + 0.5\frac{6.5}{14.84}$$
$$= 1.22$$

$$dc = 1 + 0.4k$$
$$= 1 + 0.4(0.5)$$
$$= 1.2$$

$$N\gamma = 2(Nq + 1)\tan\phi$$
$$= 2(6.4 + 1)\tan 20$$
$$= 5.39$$

$$S\gamma = 1 - 0.4\left(\frac{B}{L}\right)$$
$$= 1 - 0.4\left(\frac{1.22}{1.83}\right)$$
$$= 0.73$$

$\frac{B}{L}$, Shape Sq, $\frac{\mathrm{B}}{\mathrm{L}} = 1$

Cl, $\frac{\mathrm{B}}{\mathrm{L}} = 1$

C0, $\frac{\mathrm{B}}{\mathrm{L}} = 1$

RE, $\frac{\mathrm{B}}{\mathrm{L}} = 1$

$$d\gamma = 1.0$$

$$qult = \mathrm{cNcscdc} + \bar{\mathrm{q}}\mathrm{NqSqdq} + 0.5\gamma act\mathrm{BN}rS\gamma d\gamma$$
$$= 20(14.84)(1.22)(1.2) + 10.55(6.4)(1.24)(1.16) + 0.5(17.3)(1.22)(5.39)(0.73)(1.0)$$
$$= 573\frac{kN}{m^2}$$

$$qa = \frac{qult}{S.F}$$
$$= \frac{573}{3}$$
$$= 191\frac{kN}{m^2}$$

2. Modulus of Subgrade Reaction, Ks

Input data,

Soil information;

Allowable bearing pressure, qa = 191 kN/m2 (from previous calculations)
Soil modulus elasticity, Es = 11720 kN/m2
Poisson ratio, μ = 0.3 * table 2-7,pg123 (clay)

Foundation Information;

Base,B = 1.22m
Length, L = 1.83m
Depth, D = 0.61m
Stratum thickness,

H = 5B
= 5(1.22)
= 6.1m

* recommended by Joseph e.Bowles H=5B (table 5-3, pg307)

$$Es' = \frac{1-\mu^2}{Es}$$

$$= \frac{1-0.3^2}{11720}$$

$$= 7.76\times10^{-5}\frac{m^2}{kN}$$

$$ks = \frac{1}{B'E'smIsIf}$$

$$Is = I1 + \frac{1-2\mu}{1-\mu}I2$$

$$I1 = \frac{1}{\pi}\left[M\frac{\ln\left(1+\sqrt{M^2+1}\right)\left(\sqrt{M^2+N^2}\right)}{M\left(1+\sqrt{M^2+N^2+1}\right)} + \frac{\ln\left(M+\sqrt{M^2+1}\right)\left(\sqrt{1+N^2}\right)}{M+\sqrt{M^2+N^2+1}}\right]$$

$$* \quad M = \frac{L'}{B'} \qquad B' = \frac{B}{2}(centre), B' = B(corner)$$

$$N = \frac{H}{B'} \qquad L' = \frac{L}{2}(centre), L' = L(corner)$$

$$I2 = \frac{N}{2\pi}\left(\frac{M}{N\sqrt{M^2 + N^2 + 1}}\right)$$

ks (centre);

$$M = \frac{L'}{B'} = \frac{L/2}{B/2} = \frac{L}{B} = \frac{1.83}{1.22} = 1.5$$
$$N = \frac{H}{B'} = \frac{H}{B/2} = \frac{2H}{B} = \frac{2(6.1)}{1.22} = 10$$

$$I1 = \frac{1}{\pi}\left[1.5\frac{\ln\left(1+\sqrt{1.5^2+1}\right)\left(\sqrt{1.5^2+10^2}\right)}{1.5\left(1+\sqrt{1.5^2+10^2+1}\right)} + \frac{\ln\left(1.5+\sqrt{1.5^2+1}\right)\left(\sqrt{1+10^2}\right)}{1.5+\sqrt{1.5^2+10^2+1}}\right]$$
$$= 0.586$$

$$I2 = \frac{10}{2\pi}\left(\frac{1.5}{10\sqrt{1.5^2+10^2+1}}\right)$$
$$= 0.023$$

$$Is = I1 + \frac{1-2\mu}{1-\mu}I2$$
$$= 0.586 + \frac{1-2(0.3)}{1-0.3}(0.023)$$
$$= 0.599$$

$If = 0.8$

*Figure 5-7 , pg, 303

$$\frac{D}{B}ratio = \frac{0.61}{1.22} = 0.5$$

1.

$$\frac{L}{B}ratio = \frac{1.83}{1.22} = 1.5$$
$$\mu = 0.3$$

m; Number of corners contributing to settlement ΔH

m = 4

$centre, m = 4$

$side, m = 2$ pg 306

$corner, m = 1$

$$ks = \frac{1}{B'E'smIsIf}$$
$$= \frac{1}{1.22/2(7.76\times10^{-5})(4)(0.599)(0.8)}$$
$$= 11021\left(\frac{kN}{m^3}\right)$$

Ks (corner)

$$M = \frac{L'}{B'} = \frac{L/2}{B/2} = \frac{L}{B} = \frac{1.83}{1.22} = 1.5$$
$$N = \frac{H}{B'} = \frac{H}{B/2} = \frac{2H}{B} = \frac{6.1}{1.22} = 5$$

$$I1 = \frac{1}{\pi}\left[1.5\frac{\ln\left(1+\sqrt{1.5^2+1}\right)\left(\sqrt{1.5^2+5^2}\right)}{1.5\left(1+\sqrt{1.5^2+5^2+1}\right)} + \frac{\ln\left(1.5+\sqrt{1.5^2+1}\right)\left(\sqrt{1+5^2}\right)}{1.5+\sqrt{1.5^2+5^2+1}}\right]$$
$$= 0.496$$

$$I2 = \frac{5}{2\pi}\left(\frac{1.5}{5\sqrt{1.5^2+5^2+1}}\right)$$
$$= 0.045$$

$$Is = I1 + \frac{1-2\mu}{1-\mu}I2$$
$$= 0.496 + \frac{1-2(0.3)}{1-0.3}(0.045)$$
$$= 0.522$$

$If = 0.8$ (same with centre)

m = 1 (corner)

$$ks = \frac{1}{B'E'smIsIf}$$
$$= \frac{1}{1.22(7.76\times10^{-5})(1)(0.522)(0.8)}$$
$$= 25294\frac{kN}{m^3}$$

Ks (average)

$$Ksweighting = \frac{[4Ks(centre) + 1Ks(corner)]}{5}$$

$$= \frac{4(11021) + 25294}{5}$$

$$= 13876 \frac{kN}{m^3}$$

$$Ks(s.F) = 40(S.F)(qa)$$

$$= 40(2)(191)$$

$$= 15280$$

* safety factor = 2

$$\text{Ks(average)} = \frac{\text{Ksweighting} + \text{ksS.F}}{2}$$

$$= \frac{13876 + 15280}{2}$$

$$= 14578 \frac{kN}{m^3}$$

$$\cong 15000 \frac{kN}{m^3}$$

3. Moment, Shear & Deflection of Beam on Elastic foundation

Input data;

Foundation information

Base, B = 1.22m
Length, L = 1.83m
Depth, D = 0.61m

Soil Information

Ks = 15000 kN/m2 (from previous calculations)

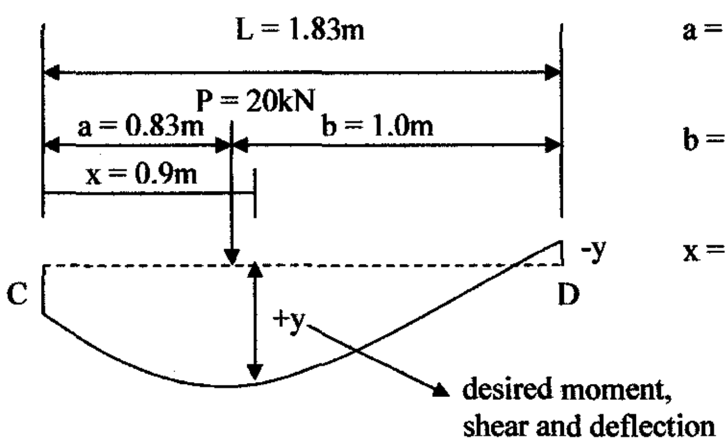
Beam information (foundation design as beam)

Length, L = 1.83m
Breadth, b = 1.22m
Depth, d = 0.61m

Moment Inertia, $I = \frac{bd^3}{12} = \frac{1.22(0.61)^3}{12} = 0.0231m^4$

Modulus elasticity, $E = 2x10^7$ kN/m^2

Finite Length Beam



a = shorter distance from end beam, A to P

b = longer distance from end beam, B to P

x = distance from end beam A to moment, shear and deflection desired

* a, b and x measured from fixed point (end beam) A

Note: 1. if $x \leq a$, then a = a, b = b, x measured from A
Note: 2 if $x > a$, then a = b, b = a, x measured from B

Here ;

$x > a$, $a_{act} = 1.0$
$b_{act} = 0.83$
$x_{act} = 1.83-0.9$
$= 0.93m$

Estimate k's, λ and λL.

$$k's = ksB$$
$$= 15000(1.22)$$
$$= 18300\frac{kN}{m^3}$$

$$\lambda = 4\sqrt{\frac{k's}{4EI}}$$
$$= 4\sqrt{\frac{18300}{4(2\times10^7)(0.0231)}}$$
$$= 0.32$$

$$\lambda L = 4\sqrt{\frac{k'sL^4}{4EI}}$$
$$= 4\sqrt{\frac{18300(1.83)^4}{4(2\times10^7)(0.0231)}}$$
$$= 0.58$$

$$\lambda x = 0.32(0.93)$$
$$= 0.3$$
$$\lambda a = 0.32(1.0)$$
$$= 0.32$$
$$\lambda b = 0.32(0.83)$$
$$= 0.27$$

Deflection, y;

$$y = \frac{p\lambda}{k's(\sinh^2\lambda L - \sin^2\lambda L)}$$
$$\left\{\begin{matrix}(2\cosh\lambda x\cos\lambda x)(\sinh\lambda L\cos\lambda a\cosh\lambda b - \sin\lambda L\cosh\lambda a\cos\lambda b)+ \\ (\cosh\lambda x\sin\lambda x + \sinh\lambda x\cos\lambda x)\begin{bmatrix}\sinh\lambda L(\sin\lambda a\cosh\lambda b - \cos\lambda a\sinh\lambda b)+ \\ \sin\lambda L(\sinh\lambda a\cos\lambda b - \cosh\lambda a\sin\lambda b)\end{bmatrix}\end{matrix}\right\}$$

For simplification.

$$a = \frac{p\lambda}{k's(\sinh^2\lambda L - \sin^2\lambda L)}$$
$$b = (2\cosh\lambda x\cos\lambda x)(\sinh\lambda L\cos\lambda a\cosh\lambda b - \sin\lambda L\cosh\lambda a\cos\lambda b)$$
$$c = (\cosh\lambda x\sin\lambda x + \sinh\lambda x\cos\lambda x)$$
$$d = \sinh\lambda L(\sin\lambda a\cosh\lambda b - \cos\lambda a\sinh\lambda b)$$
$$e = \sin\lambda L(\sinh\lambda a\cos\lambda b - \cosh\lambda a\sin\lambda b)$$

thus, $y = a[b + c(d + e)]$

$$a = \frac{(20)(0.32)}{18300(\sinh^2(0.58) - \sin^2(0.58))}$$
$$= 9.31 \times 10^{-4}$$
$$b = (2\cosh 0.3 \cos 0.3)(\sinh 0.58 \cos 0.32 \cosh 0.27 - \sin 0.58 \cosh 0.32 \cos 0.27)$$
$$= 1.31$$
$$c = (\cosh 0.3 \sin 0.3 + \sinh 0.3 \cos 0.3)$$
$$= 0.31$$
$$d = \sinh 0.58(\sin 0.32 \cosh 0.27 - \cos 0.32 \sinh 0.27)$$
$$= -0.16$$
$$e = \sin 0.58(\sinh 0.32 \cos 0.27 - \cosh 0.32 \sin 0.27)$$
$$= 5.02 \times 10^{-5}$$

$$y = a[b + c(d + e)]$$
$$= 9.31 \times 10^{-4}[1.31 + 0.31(-0.16 + 5.02 \times 10^{-5})]$$
$$= 1.173 \times 10^{-3} m$$

Moment, M

$$M = \frac{P}{2\lambda(\sinh^2 \lambda L - \sin^2 \lambda L)} \left\{ \begin{array}{l} 2\sinh \lambda x \sin \lambda x(\sinh \lambda L \cos \lambda a \cosh \lambda b - \sin \lambda L \cosh \lambda a \cos \lambda b) + \\ (\cosh \lambda x \sin \lambda x - \sinh \lambda x \cos \lambda x)\left[\begin{array}{l} \sinh \lambda L(\sin \lambda a \cosh \lambda b - \cos \lambda a \sinh \lambda b) \\ + \sin \lambda L(\sinh \lambda a \cos \lambda b - \cosh \lambda a \sin \lambda b) \end{array} \right] \end{array} \right\}$$

For simplification

$$a = \frac{P}{2\lambda(\sinh^2 \lambda L - \sin^2 \lambda L)}$$
$$b = 2\sinh \lambda x \sin \lambda x(\sinh \lambda L \cos \lambda a \cosh \lambda b - \sin \lambda L \cosh \lambda a \cos \lambda b)$$
$$c = (\cosh \lambda x \sin \lambda x - \sinh \lambda x \cos \lambda x)$$
$$d = \sinh \lambda L(\sin \lambda a \cosh \lambda b - \cos \lambda a \sinh \lambda b)$$
$$e = \sin \lambda L(\sinh \lambda a \cos \lambda b - \cosh \lambda a \sin \lambda b)$$

$$M = a[b + c(d + e)]$$

$$a = \frac{20}{2(0.32)(\sinh^2 0.58 - \sin^2 0.58)}$$
$$= 83.17$$
$$b = 2\sinh 0.32 \sin 0.3(\sinh 0.58 \cos 0.32 \cosh 0.27 - \sin 0.58 \cosh 0.32 \cos 0.27)$$
$$= 2.13 \times 10^{-3}$$
$$c = (\cosh 0.3 \sin 0.3 - \sinh 0.3 \cos 0.3)$$
$$= -0.299$$
$$d = \sinh 0.58(\sin 0.32 \cosh 0.27 - \cos 0.32 \sinh 0.27)$$
$$= -0.16$$
$$e = \sin 0.58(\sinh 0.32 \cos 0.27 - \cosh 0.32 \sin 0.27)$$
$$= 3.24 \times 10^{-3}$$

$$M = a[b + c(d + e)]$$
$$= 83.17[2.13 \times 10^{-3} - 0.299(-0.16 + 3.24 \times 10^{-3})]$$
$$= 4.08kNm$$

Shear, Q

$$Q = \frac{P}{\left(\sinh^2 \lambda L - \sin^2 \lambda L\right)} \left\{ \begin{aligned} &(\cosh \lambda x \sin \lambda x + \sinh \lambda x \cos vx) \\ &(\sinh \lambda L \cos \lambda a \cosh \lambda b - \sin \lambda L \cosh \lambda a \cos \lambda b) \\ &+ \sinh \lambda x \sin \lambda x \begin{bmatrix} \sinh \lambda L(\sin \lambda a \cosh \lambda b - \cos \lambda a \sinh \lambda b) \\ + \sin \lambda L(\sinh \lambda a \cos \lambda b - \cosh \lambda a \sin \lambda b) \end{bmatrix} \end{aligned} \right\}$$

for simplification.

$$a = \frac{P}{\left(\sinh^2 \lambda L - \sin^2 \lambda L\right)}$$
$$b = (\cosh \lambda x \sin \lambda x + \sinh \lambda x \cos \lambda x)$$
$$c = (\sinh \lambda L \cos \lambda a \cosh \lambda b - \sin \lambda L \cosh \lambda a \cos \lambda b)$$
$$d = \sinh \lambda x \sin \lambda x$$
$$e = \sinh \lambda L(\sin \lambda a \cosh \lambda b - \cos \lambda a \sinh \lambda b)$$
$$f = \sin \lambda L(\sinh \lambda a \cos \lambda b - \cosh \lambda a \sin \lambda b)$$

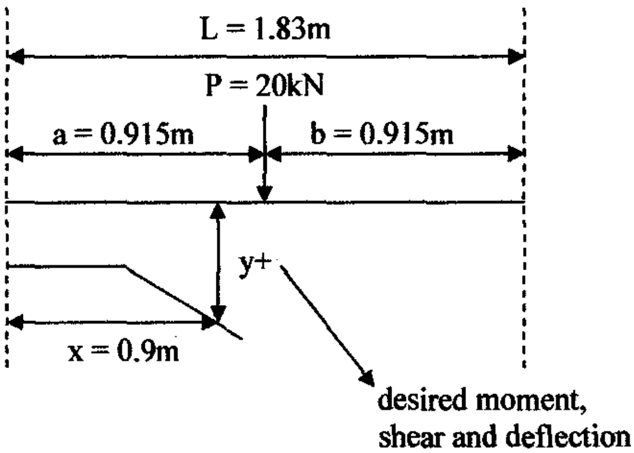
$$a = \frac{20}{\left(\sinh^2 0.58 - \sin^2 0.58\right)}$$
$$= 53.23$$
$$b = (\cosh 0.3 \sin 0.3 + \sinh 0.3 \cos 0.3)$$
$$= 0.31$$
$$c = (\sinh 0.58 \cos 0.32 \cosh 0.27 - \sin 0.58 \cosh 0.32 \cos 0.27)$$
$$= 0.62$$
$$d = \sinh 0.3 \sin 0.3$$
$$= 1.59 \times 10^{-3}$$
$$e = \sinh 0.58(\sin 0.32 \cosh 0.27 - \cos 0.32 \sinh 0.27)$$
$$= -0.16$$
$$f = \sin 0.58(\sinh 0.32 \cos 0.27 - \cosh 0.32 \sin 0.27)$$
$$= 5.02 \times 10^{-5}$$

.

$$Q = a[bc + d(e + f)]$$
$$= 53.23[0.31(0.62) + 1.59 \times 10^{-3}(-0.16 + 5.02 \times 10^{-5})]$$
$$= 10.22 kN$$

Slope, Θ = Not available, small values in footing design.

Infinite length Beam with mid/center loading



Estimate; Aλx, Bλx Cλx and Dλx (constant)

$$
\begin{aligned}
A\lambda x &= e^{-\lambda x}(\cos\lambda x + \sin\lambda x)\\
&= e^{-0.288}(\cos 0.288 + \sin 0.288)\\
&= 0.754
\end{aligned}
$$

$$
\begin{aligned}
B\lambda x &= e^{-\lambda x}\sin\lambda x\\
&= e^{-0.288}\sin 0.288\\
&= 3.77\times 10^{-3}
\end{aligned}
$$

$$
\begin{aligned}
C\lambda x &= e^{-\lambda x}(\cos\lambda x - \sin\lambda x)\\
&= e^{-0.288}(\cos 0.288 - \sin 0.288)\\
&= 0.746
\end{aligned}
$$

$$
\begin{aligned}
D\lambda x &= e^{-\lambda x}\cos\lambda x\\
&= e^{-0.288}\cos 0.288\\
&= 0.75
\end{aligned}
$$

*Assume Mo = 18, M1=18, V1=10kN

1) Concentrated load at end

a)

$$y = \frac{2V1\lambda}{k's} D\lambda x$$
$$= \frac{2(10)(0.32)(0.75)}{18300}$$
$$= 2.623 \times 10^{-4}\, kNm$$

b)

$$\theta = -\frac{2V1\lambda^2 A\lambda x}{k's}$$
$$= \frac{-2(10)(0.32)^2(0.754)}{18300}$$
$$= -8.438 \times 10^{-5}$$

c)

$$M = \frac{-V1B\lambda x}{\lambda}$$
$$= \frac{-10(3.77 \times 10^{-3})}{0.32}$$
$$= -0.118 kNm$$

d)

$$Q = -V1C\lambda x$$
$$= -10(0.746)$$
$$= -7.46 kN$$

2) Concentrated land at center

a)

$$y = \frac{P\lambda A\lambda x}{2k's}$$
$$= \frac{20(0.32)(0.754)}{2(18300)}$$
$$= 1.318 \times 10^{-4}$$

b)

$$\theta = -\frac{P\lambda^2 B\lambda x}{k's}$$
$$= \frac{-20(0.32)^2(0.754)}{18300}$$
$$= -4.219\times10^{-7}$$

c)

$$M = \frac{PC\lambda x}{4\lambda}$$
$$= \frac{20(0.746)}{4(0.32)}$$
$$= 11.66kNm$$

d)

$$Q = \frac{-PD\lambda x}{2}$$
$$= \frac{-20(0.75)}{2}$$
$$= -7.5kN$$

3) Moment at end

a)

$$y = \frac{-2M1\lambda^2 C\lambda x}{k's}$$
$$= \frac{-2(18)(0.32)^2(0.746)}{18300}$$
$$= 1.503\times10^{-4}m$$

b)

$$\theta = \frac{4M1\lambda^3 D\lambda x}{k's}$$
$$= \frac{4(18)(0.32)^3(0.75)}{18300}$$
$$= 9.669\times10^{-5}$$

c)

$$M = M1A\lambda x$$
$$= 18(0.754)$$
$$= 13.572 kNm$$

d)

$$Q = -2M1\lambda B\lambda x$$
$$= -2(18)(0.32)(3.77 \times 10^{-3})$$
$$= -0.043 kN$$

4) Moment at center

a)

$$y = \frac{M0\lambda^2 B\lambda x}{k's}$$
$$= \frac{18(0.32)^2(3.77 \times 10^{-3})}{18300}$$
$$= -3.797 \times 10^{-7} m$$

b)

$$\theta = \frac{M0\lambda^3 C\lambda x}{k's}$$
$$= \frac{18(0.32)^3(0.746)}{18300}$$
$$= 2.404 \times 10^{-5}$$

c)

$$M = \frac{M0D\lambda x}{2}$$
$$= \frac{18(0.75)}{2}$$
$$= 6.75 kNm$$

d)

$$Q = \frac{-M0\lambda A\lambda x}{2}$$
$$= \frac{-18(0.32)(0.754)}{2}$$
$$= -2.172 kN$$